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Dig Limits Optimization Using Binary Integer Linear Programming Method in Open Pit Mines

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DIG LIMITS OPTIMIZATION USING BINARY INTEGER LINEAR PROGRAMMING METHOD IN OPEN PIT MINES

by

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ABSTRACT

Dig limits optimization is the process for classifying different materials (e.g., ore, stockpile material, and waste) into appropriately sized contiguous zones for open pit mining. The efficient determination of dig-limits is crucial for profitable and sustainable resource extraction in mining. Previous research has focused on defining dig-limits manually or using optimization approaches, but these methods are limited to only handling two material destinations (ore and waste). Thus, there is a need for operations research methods that consider the selectivity of mining equipment and can optimize dig-limits for metal mining operations with more than two material destinations. Consequently, the objective of this thesis was to find the optimal block boundaries that allow for multiple material categories and their designated destinations while maximizing the profit of a bench section. The problem is modeled using a binary-integer linear programming (BILP) formulation that accounts for the equipment size. The study evaluated the performance of the proposed BILP dig-limits optimization method and obtained an optimal solution for a 20×20 bench section with a 3×3 dig limit size, achieving an objective function value of $332,000 and a gap tolerance of 0.0% within approximately 64.02 seconds. Findings indicated that larger problem sizes led to longer solution times due to increased constraints and decision variables, while higher minimum mining dimensions decreased objective function values. Incorporating rectangular minimum mining width dimensions in the model provided flexibility and control. The analysis emphasized the substantial impact of mining width variations on profitability and efficiency.

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NOMENCLATURE

Symbol	Description
2D	Two-Dimensional Space
SA	Simulated Annealing
LP	Linear Programming
BILP	Binary Integer Linear Programming
MILP	Mixd Integer Linear Programming
GA	Genetic Algorithm
SMUs	Selective Mining Units
TSP	Traveling Salesman Problem
α_i	Minimum Minng Width Along i-Direction
β_j	Minimum Mining Width Along j-Direction

1. INTRODUCTION

1.1. BACKGROUND

Mine planning is the process that engineers use to specify how an ore deposit will be exploited during the course of a mining operation. It inherently depends on all information that planning engineers believe will eventually have an impact on the feasibility of the proposed mine and uses all geology-related data on the deposit as its starting point [1][2]. Making the most of a mineral resource requires an effective mining strategy. Mine planning engineers must use a combination of appropriate technical standards, rules, and procedures in the process of designing and scheduling a mine to ensure that all mining-related activities are accurately and effectively considered [1][3][4].

The mineral industry has long recognized mine planning as a value-creating activity. Mine planning is now a common practice, and mining firms have established planning divisions as a result of advancements in optimization [5], hardware, and software technologies [6]. Mine planning is often classified into long-term, medium-term, and short-term planning based on the time horizon of the planning activities [7]. Long-term mine planning covers anywhere from five years to the remaining life of the mine and depending on the circumstances, it might cover periods exceeding 30 years. A typical medium-term mine plan covers timeframes between one and five years. Medium-term planning provides more precise information that enables a more precise plan of the ore extraction from a specific part of the mine or information that enables the replacement of essential equipment or the acquisition of necessary machinery and equipment. Lastly, the short-term production planning time period might be anything between one month and one year. One day to one month is the length of the sub-periods that make up this time frame [8]. The objectives and planning environments differ for short-, medium-, and long-term mine planning [7]. Figure 1.1 summarizes the mine planning process.

Figure 1.1 Mine Planning Process

In short-term mine planning, the goal is to ensure the material is accurately defined (ore type), measured (grades), mined, fed to the processing plant, or moved to another destination [9]. Successful mining occurs when different types of material are utilized to maximize profit by optimizing the previous steps [10]. An important part of this process is defining the most feasible and economic dig-limits on each bench in an open pit mine. Grade control, ore control, dig-limits optimization, or quality control are all terms used to describe this process.

The first step in this process starts with building a block model that consists of multiple attributes stored in tiny building components known as selective mining units. The smallest material volumes on which choices about the categorization of ore-waste are based are known as SMUs. Figure 1.2 shows an example of a block model. These SMUs are too tiny to be mined on their own. As a result, we gather these SMUs inside a polygon (clusters) known as dig-limits allowing heavy equipment (loaders) to extract them. Figure 1.3 and Figure 1.4 illustrate the concept of dig-limits.

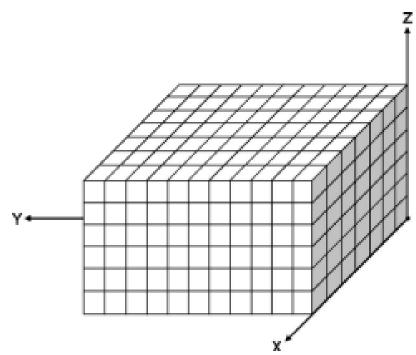

Figure 1.2 Block Model

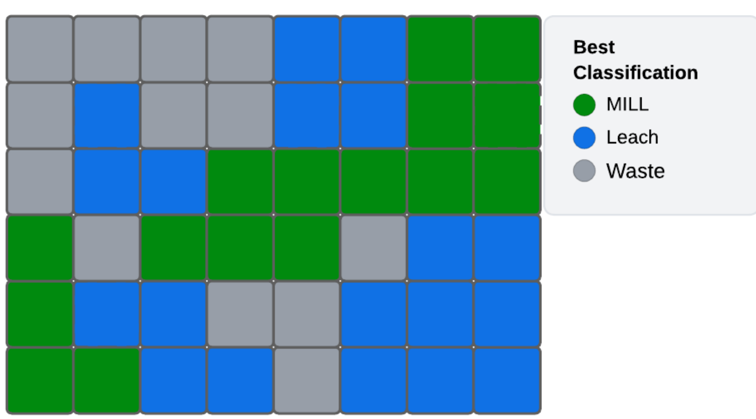

Figure 1.3 Ore-Waste Classification According to Ore-Type and Destination

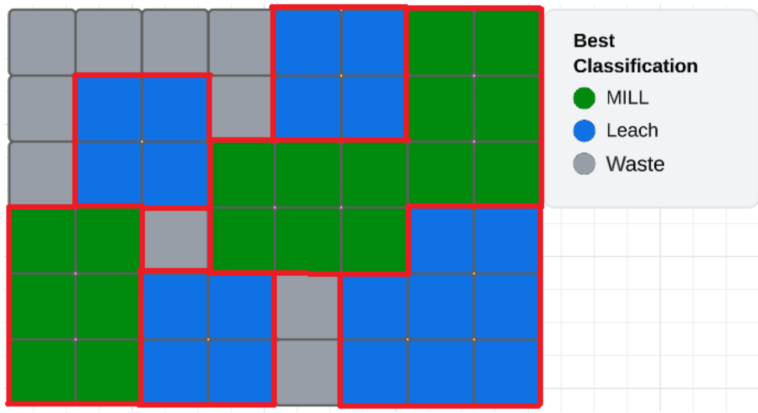

Figure 1.4 Ore-Waste Classification Based on Dig-Limits Optimization

1.2. PROBLEM STATEMENT

Dig-limits problem is one of the key elements to efficient resource extraction, and it is a crucial factor that might determine profitability. Therefore, mining companies must aim to define those limits properly to maximize profit and maintain sustainable development. The general problem is defining the most economic and feasible dig-limits on each bench in an open pit mine. This has been tackled by multiple authors in previous research [8]. However, the specific problem is that ore material can have multiple destinations (not just ore and waste). Typically, the ore is sent to the processing plant (or mill), but this is not always the case. In most open pit metal mines, valuable minerals in low-grade ores are recovered by heap leaching. Sometimes, sulfide and oxide ore are separated into different leach piles for heap leaching. At other times, some ore is crushed, and some is not crushed prior to heap leaching. All these can create multiple destinations for ore.

Traditionally, dig-limits are determined manually by geologists using blast holes' samples (in combination with the block model based on exploration drilling) to indicate the grades. The result of this traditional approach is a map of the grade information such as cut-off grade and the type of rock. However, this method of determining the boundaries between different material types (ore, stockpile material, waste etc.) using hand contouring has downsides [11]. Even then, in situations where the block model sizes are smaller than the selective mining unit of the loading equipment, engineers and geologists must manually assign small number of isolated blocks to other dig-limits to ensure operational feasibility. This is a decision-making problem that can be solved with operations research methods while accounting for the selectivity of the mining equipment. Previous research has attempted to solve this problem with several optimization approaches including heuristics, simulated annealing, and mixed-integer linear programming. Mathematical programming approaches that guarantee optimality have been limited to only classifying ore and waste [8]. These approaches are not useful for most metal mining operations that have more than two material destinations.

1.3. OBJECTIVES AND SCOPE

The overall objective of the research is to find the optimal block boundaries that allow for multiple material categories and their designated destinations while maximizing the profit of a bench section. The problem is modeled using binary-integer linear programming (BILP) formulation that accounts for the equipment size.

The overall objective is achieved by developing a mathematical (BILP) model, which can be used as a decision-making tool for selecting the optimal dig-limits in open pit mines.

While the entire dig-limits process includes several crucial steps, including sampling, and mining, the research focuses only on the classification and selectivity components of dig-limits.

1.4. STRUCTURE OF THESIS

This thesis has five chapters including this introduction. Chapter 2 covers the literature review, which is followed by the methodology & model formulation with a case study (Chapter 3), and then an evaluation of the proposed model with discussion (Chapter 4). Chapter 5 is the conclusion that summarizes the thesis's findings and recommendations for future work.

2. LITERATURE REVIEW

The objectives of this section are to (i) review the current knowledge on dig-limits optimization, in general, as well as the different models and algorithms used to solve dig-limits optimization problems; and (ii) identify the knowledge gaps related to dig-limits optimization that require further research. The relevant literature was found using keywords relating to the topic of this research, such as “dig-limits optimization,” “grade control,” and “selectivity of mining.” Google Scholar was the primary resource for finding review literature. Although most of the publications were journal articles, certain books were included to give background knowledge, particularly on the techniques and algorithms used to address such issues. When the desired content is background information, the author has no time constraint. For example, the nature of each algorithm is well known and has been studied in the past five or more decades. Therefore, this author did not place any time limit on how far back the literature review went. Regarding how to solve the dig-limit optimization problems, the author focused on best practices and models that result in optimal or near-optimal solutions. The methods described in this section are the ones that have been studied in the previous literature and show the most potential to solve the dig-limit problem optimally.

2.1. DIG LIMITS OPTIMIZATION

The “dig limits optimization” problem in open pit mines involves determining the optimal boundaries for excavating material from a given deposit, while achieving the desired objective(s), and respecting various constraints such as mining selectivity and ore types (Figure 1.4). The objective function typically includes maximizing the net present value (NPV) of the mine [8] or minimizing the dilution [12]. Most of the constraints are operational such as the capacity and availability of equipment, and the required grade and tonnage of the ore [13].

Past research has used several methods to formulate and solve the dig limit optimization problem, including linear [8], nonlinear programming [14][12], heuristic [15], and metaheuristic algorithms [11][16][17][18]. Table 2.1 summarizes some of the major attempts of previous researchers on solving this problem and shows the methods these researchers used.

Table 2.1 A Summary of Algorithms Used to Solve Dig Limits

Approach	Optimal
Simulated annealing [16][17][18]	Near-optimal
Genetic algorithm [11]	Near-optimal
Hierarchical clustering [14]	No
Heuristic approach [15]	No
Local search algorithm [12]	No
Mixed-integer linear programming [8]	Yes

One could broadly classify the approaches to formulate and solve the dig limits optimization problem into mathematical programming approaches (mainly, mixed integer linear programming) and heuristic/meta-heuristic methods. In the subsequent sub-sections of this section, the thesis presents discussions of the heuristic/meta-heuristic methods and mixed integer linear programming approaches. While the literature contains some simple heuristic algorithms such as floating circle and local search [12], these algorithms are very limited in the scale of problems they can solve and do not guarantee an optimal solution. Hence, the discussion of heuristic and meta-heuristic methods in this thesis does not include a discussion of these methods.

2.2. HEURISTIC AND METAHEURISTIC METHODS

Heuristics and metaheuristics are powerful techniques used for optimization problems. Heuristics are problem-solving strategies that provide a solution in a reasonable amount of time but without guaranteeing optimality [19]. Metaheuristics, on the other hand, are higher level problem-solving strategies that guide the search for optimal solutions by iteratively refining the candidate solutions. They are especially useful for problems where the search space is large, the evaluation of the solution is time-consuming, and the optimal solution is not known in advance [20]. These include methods such as simulated annealing, genetic algorithms, evolutionary algorithms, particle swarm optimization, and ant colony optimization [21]. Metaheuristic optimization methods have been used in many mining problems including production sequencing [22], exploration planning [23], and maintenance analysis of mining equipment [24]. Of the metaheuristic methods, simulated annealing and genetic algorithms are the ones that previous research has used to solve the dig-limits optimization problem.

2.2.1. Simulated Annealing. Kirkpatrick et al. and Cerny separately proposed the term annealing in combinatorial optimization in the early 1980s. Originally, a parallel between the physical annealing of solids and the issue of addressing huge combinatorial optimization problems prompted this notion [25]. A randomized search strategy is an example of simulated annealing. A randomized search technique, also known as a probabilistic search method, is an algorithm that considers randomized samples of candidate solutions in the set when searching the feasible set of an optimization problem [26]. While a problem solved with simulated annealing can be formulated with an objective function and contraints, in reality the constraints are incorporated into the objective function as penalties. The simulated annealing technique has proven its efficiency to solve a variety of optimization problems. The traveling salesman problem (TSP), which is known to be NP-hard, is one of the optimization problems that simulated annealing can efficiently solve [27]. Simulated annealing has been used in solving multiple engineering optimization problems [28]–[30]. It has also been applied in different mining applications, including image processing [31], production sequencing [32], mine phase design [33], and variography [34]. This shows its versatility and broad application.

Figure 2.1 shows the general simulated annealing algorithm. The algorithm starts with a candidate solution and performs a random walk in the solution space. At each step, the algorithm decides whether to accept or reject the new solution based on the energy of the solution and a temperature parameter. The temperature parameter decreases over time, allowing the algorithm to escape from local minima and converge to a near-optimal solution [25]. In the context of simulated annealing, the perturbation mechanism is commonly utilized to introduce random changes to the existing solution. The specific nature and magnitude of these perturbations may differ based on the particular problem being addresed [16].

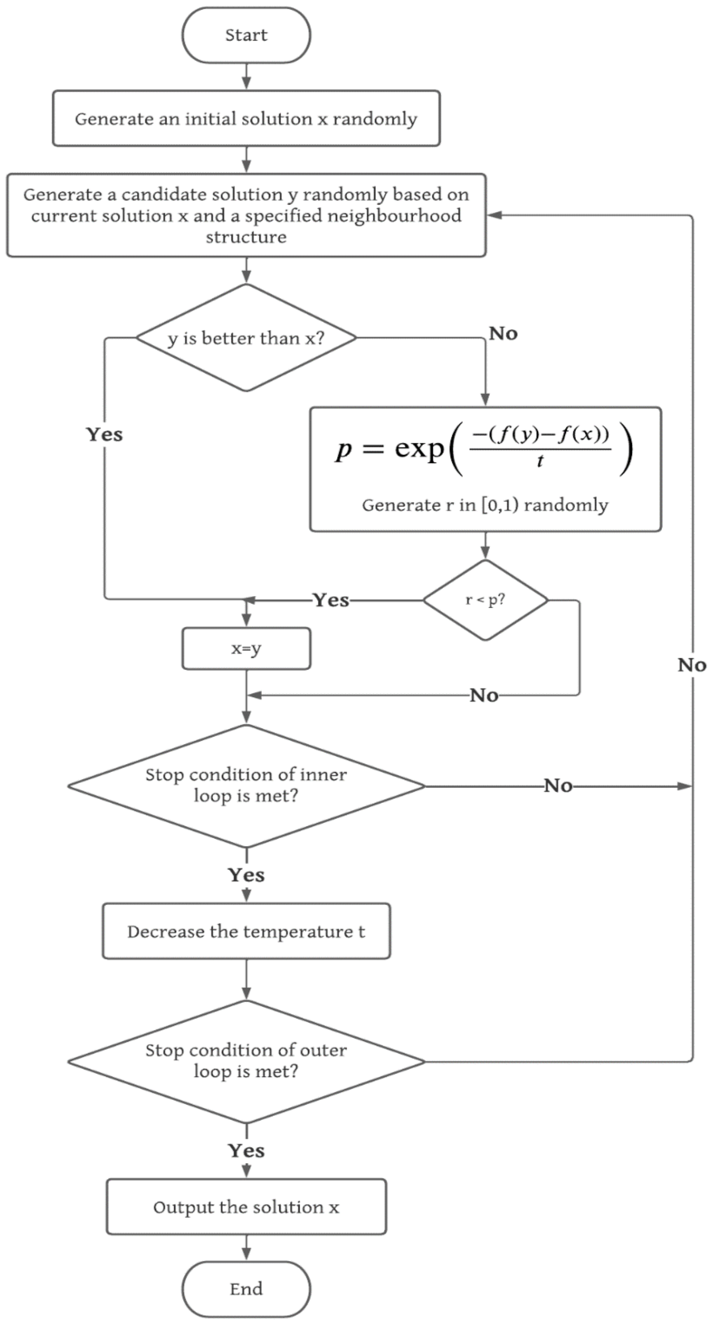

Figure 2.1 Simulated Annealing Algorithm (Adapted From [27])

Simulated annealing algorithms are beneficial when attempting to find best possible solution in optimization problems that have numerous local optima. These algorithms are particularly effective in situations where the objective function is noisy or where the data in the problem is uncertain. However, they may not be as successful in problems where the objective function is nonlinear or discontinuous [25][35]. Other drawbacks of simulated annealing, include the need for “adjustments” such managing temperature decreases; users must be knowledgeable about “good” adjustments. Additionally, the method can have high computational times. However, some researchers have implemented the method in parallel to overcome this limitation [36].

Because the objective function for dig limits is difficult to define and the solution space for determining optimal dig limits is large, some previous researchers have employed SA [17]. There are six basic components of SA: (1) the system, (2) the initial guess, (3) the objective function, (4) the perturbation mechanism, (5) the decision-making program, and (6) the annealing schedule [25]. In optimum dig limit selection, a map of expected profit, that is, the projected profit at each site if that area were marked as ore, is employed (note that the previous work only formulated a dig-limits problem with two material destinations – ore and waste). This calculation demands, among other things, an awareness of mining costs, treatment costs, pricing, and recoveries. The second component of SA is the initial assumption, which might come from any source: hand-drawn, computerized, or just by looking at the block borders. Next is a weighted sum of parameters that makes up the objective function (see Equation 2.1). In Equation 2.1, O_i represents a specific parameter or characteristic related to the dig limits problem. It is a variable that denotes a particular aspect or feature that contributes to the overall objective function. For example, these parameters could be block grade, block volume, mining cost, and treatment cost [17].

$$O_{profit} = \sum_{i=1}^{N} w_i \, . \, O_i \tag{2.1}$$

The dig limits are randomly changed using the perturbation mechanism. In order to account for the mining equipment's ability to dig the proposed limit (i.e., to impose the mining equipment constraint), a penalty function was implemented in Equation 2.2 [17]. The last component of the SA process is the decision making which represents the core of the SA.

$$O_{profit} = profit + penalty_{digability} \tag{2.2}$$

While simulated annealing, in general, can address multiple constraints, the algorithms implemented for the dig limits problem only addressed the equipment size constraint. The implemented model accounts for the equipment's ability to dig a specific polygon. Even with the implementation of this constraint, the model's penalty function (constraint) does appear to have the ability to enforce this constraint in real cases. The obtained results from the model are near-optimal for the specific case where it was tested [16] [17]. In summary, although, the use of simulated annealing algorithms to solve the dig-limits problem is perhaps computationally efficient when compared to say linear programming approaches (this has not been demonstrated yet with comparisons), the approach does not incorporate all constraints, and does not guarantee optimality.

2.2.2. Genetic Algorithms. Genetic algorithms, which are influenced by natural selection, are a form of metaheuristic optimization algorithm that is typically utilized to seek out optimal solutions to complicated problems, in cases where conventional optimization methods may be unsuccessful or impractical [37]. Genetic algorithms are used to find solutions to complex and multi-objective problems. They employ the concept of reproducing the fittest [38]. The general algorithm starts by creating an initial population of solutions, represented as chromosomes or sets of genes. These solutions are evaluated based on their fitness, which is determined by the objective function of the optimization problem. The fittest individuals are then selected to breed, and their genetic information is combined to produce new offspring. The offspring undergo genetic operations such as mutation and crossover, introducing new genetic information into the population of solutions. The new generation is then evaluated, and the process continues until a stopping criterion is met, typically a predefined number of generations or a satisfactory fitness level [39].

Genetic algorithms have been applied to various fields [36][40][41][42], and they are known for their ability to search large solution spaces, handle non-linear and non-convex problems, and deal with noisy or incomplete data [43]. It has been also utilized in various mining applications, including mine scheduling [44], production sequenceing [45], and geostatistical analysis [46]. However, genetic algorithms also have some limitations, such as the potential for premature convergence, sensitivity to parameter values, and the need for a well-defined objective function [43].

When using this algorithm to solve the dig-limit problem, fitness is referred to as the profit obtained from subtracting the mineable deviation from the possible dig-limit in a bench. It will “breed” solutions by merging points from two different solutions at random. Solutions with higher values as a consequence of the parents’ genes being mixed have a better chance of reproducing [47].

Genetic operators generate new viable dig-limits by perturbing existing ones. This “generation cycle” continues until the goal is met or a certain number of generations has passed [11]. The possibility that a heuristic search method might get stuck in a local maximum point applies to genetic algorithms [48], [49]. Adjusting the transformation ratio and generating a less prejudiced ‘predator’ are two ways to avoid local maxima. This method alleviates the stresses of population homogenization, providing accurate detailed searches of the solution domain [50], [39].

In the context of dig-limits optimization, every possible dig-limit, feasible or not, is considered as a solution. While every SMU is called a gene. Grades are assigned to each SMU as well as destinations. The first generation’s destinations are a combination of completely random solutions and ideal destinations based on “free choices” (i.e., those that are not constrained), allowing for flexibility and exploration of various possibilities. The algorithm's search radius is maintained big by creating a duplicate of primary-creation input dig-limits (the initial set of dig limits) that defines a distinct optimization direction (the specific goal that the algorithm aims to achieve). The destinations will be born and tweaked until they reach an optimal solution (i.e., the termination criteria are met) [11]. Figure 2.2 shows the steps of the model of genetic algorithms for solving the dig-limits problem.

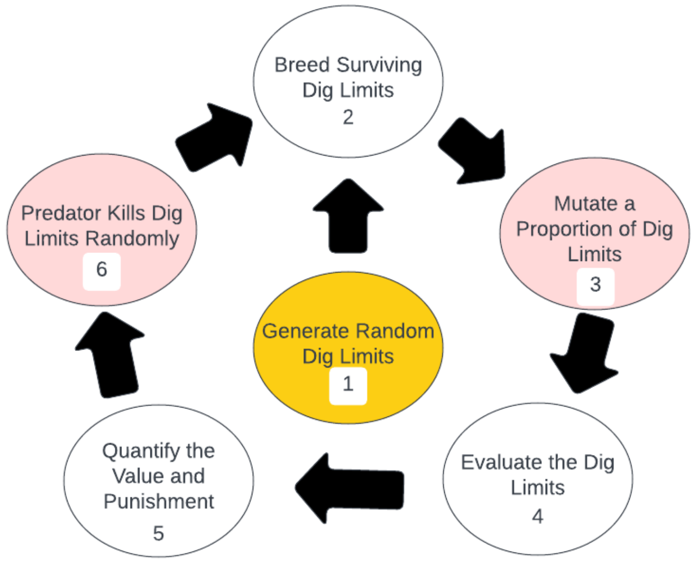

Figure 2.2 Genetic Algorithm Loop in Solving Dig Limits (Adapted From [11])

The steps are explained below:

1. A random set of feasible solutions (dig-limits) are generated to form the first population of solutions. Randomly chosen ore SMUs are expanded in this iteration to conform to the pre-generated cluster size.
2. Two different parents are combined together to find a new solution in a process called breeding. Each combined parent solution is used as a source of genes which has a 0.5 probability to be chosen.
3. The new dig-limit is made up of the SMU destinations of the two-parent solutions that have been chosen. The inserted transformation will abide to the clustering size to bypass impractical alterations, as determined by the mining equipment's clustering size.

4. The “preparer” function examines each SMU’s neighbors and applies a clustering deviation according to how much weight the clustering size is given.
5. The quantifier evaluates the SMU’s economic value and the clustering deviation. This evaluation is done when both solutions are reduced to a single integer.
6. A weighted roulette wheel selection mechanism is used by the predator. The collection of possible dig limits is arranged, with each minor timetable having a partially higher chance of being chosen than the one before it [11].

Genetic algorithms have proven to be an efficient tool for tackling complex optimization problems like the dig-limits problem, due to their ability to handle multi-objective problems, search large solution spaces, and solve non-linear and non-convex problems. Nonetheless, they have some limitations such as the potential for early convergence, sensitivity to parameter values, and the requirement for a well-defined objective function. Another concern is the possibility of getting stuck in a local maximum point, which can be mitigated by making adjustments to the algorithm. Genetic algorithms can be useful in finding near-optimal solutions, but further research is necessary to gain a deeper understanding of their strengths and weaknesses and to establish best practices for their use with regards to dig-limits optimization.

2.3. MIXED-INTEGER LINEAR PROGRAMMING

Linear programming (LP) and its extensions such as mixed integer, integer, and binary integer LP problems are well known optimization approaches that have been used to solve many diverse problems. Linear programming methods vary depending on the nature of the objective function, and the type of variables [51]. The general LP problem is stated as Equation 2.3 with decision variable **x**.

$$\begin{aligned}&\text{Minimize } \mathbf{c}^T\mathbf{x}\\&\text{Subject to:}\\&\mathbf{Ax} \le \mathbf{b}\\&\mathbf{x} \ge \mathbf{0}\end{aligned} \tag{2.3}$$

There are different versions of this problem that can be considered, such as maximizing instead of minimizing or equality rather than inequality constraints. However, all of these variations can be transformed into the general format of the problem [26].

Integer Linear Programming (ILP) is a variant of linear programming where the decision variables are required to take integer values. In ILP, the objective is to maximize or minimize a linear function subject to a set of linear constraints while ensuring that the decision variables take only integer values. ILP finds applications in various fields such as logistics [52], supply chain management [53], finance [54], telecommunications [55], and engineering, among others [56][57][58]–[60]. It is widely used to solve real-world problems that involve discrete decision-making such as selecting the best combination of products to manufacture, allocating resources to minimize costs, scheduling activities to minimize time, and optimizing transportation routes [61]–[64]. The effectiveness of ILP as an optimization tool is attributed to its ability to model and solve complex decision-making problems with discrete variables. The optimization problem can be formulated as a system of linear inequalities with an objective function (Equation 2.3), and the solutions to this system can be found by using specialized algorithms that take advantage of the integer constraints [65].

Mixed Integer Linear Programming (MILP) is a linear programming problem in which some (not all) of the decision variables must be integers [66]. MILP problems can be solved with a variety of solution methods including branch and cut and branch and bound [26][67]. MILP has been employed to address numerous engineering optimization problems [68]–[74]. It has been used in the mining industry as well. For instance, it has been successfully applied in mining for production scheduling of different mining methods [75]–[78], sequence optimization [79], tailings management [80][81], truck dispatching [82], and stockpiles blending [83]. This method has shown its ability among other methods to solve the dig limits problem optimally.

Sari and Kumral [8] were the first to use MILP to find the optimal dig-limits layout. The objective function of their model was to maximize the economic value of a bench. They assumed symmetrical block sizes; therefore, selective mining units (SMUs) were used in their approach.[1] Using the concept of SMUs in the problem gives an advantage when the equipment size is considered to be a constraint in the model. Therefore, the equipment size would be defined by the number of SMUs covered in both directions [8]. Figure 2.3 shows that every single SMU inside a frame must be from the same type of material ore or waste. The radius of an excavator's greatest reaching arm is

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[1] The term "selective mining unit" (SMU) which represents the minimum block model support for ore/waste allocation decisions, is typically much smaller than the sampling grid dimensions in the exploration stage [95].

shown in red circles in the middle of the frame. If the size of the equipment were not a restriction, in this case, it would be fair to directly mark SMUs as either ore or waste according to the cut-off grade. Because of the size of the mining equipment, many SMUs must be mined and sent to a destination at the same time. To put it another way, n × n nearby SMUs that match equipment size, must all be identified as ore or waste to fulfill this constraint [8]. One frame can share a selective mining unit with another frame, but in order for an SMU to be shared, it has to be placed in one acceptable frame. Although the search space is limited by equipment size, the goal is to increase bench sector revenue while staying within the equipment size restrictions [8].

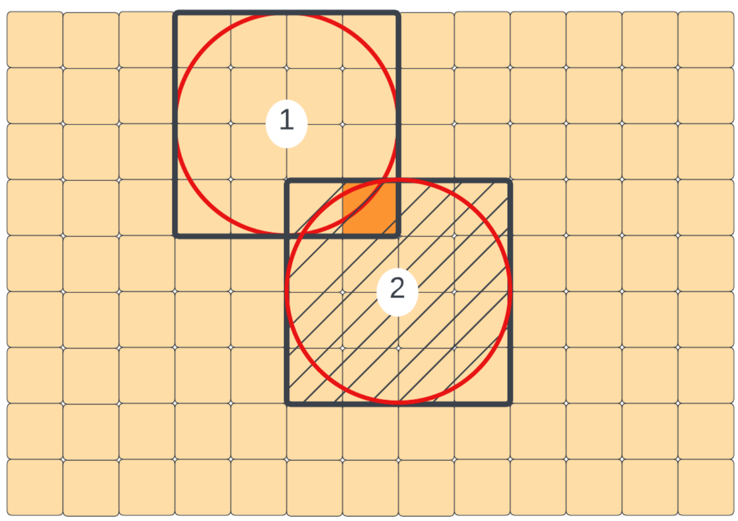

Figure 2.3 Frames with a Width of 4 × 4 SMUs (Adapted From [8])

The main decision variable in this model is (x_{ij}), and it is a binary variable that takes the value of 1 if SMU at that specific location (i, j) is ore, and a value of 0 if it is waste. The total number of SMU inside a frame is represented by the decision variable

($t_{ijf_xf_y}$) where the decision variable ($v_{ijf_xf_y}$) has the value of 1 if the frame is valid (a frame is considered valid when all of its SMUs consist of either ore or waste), 0 if not [8]. The objective function in this model seeks to maximize profit (revenue minus costs). Table 2.2 shows indices and sets of this model.

Table 2.2 Indices and Sets of Model by Sari and Kumral [8]

Indices	Sets
i	SMU index along X-direction
j	SMU index along Y-direction
f_x	Frame index along X-direction
f_y	Frame index along Y-direction
α	Offset index along X-direction in a frame
β	Offset index along Y-direction in a frame

The set of constraints that restricts the feasible solution space is divided into two major sets. Equations 2.4-2.8 show the set of constraints Sari and Kumral [8] used to ensure SMUs that are mined fit into a frame.

I. Frame constraints:

a. In each potential frame that the SMU located at position (ij) can belong to, the sum of (x_{ij}) values within that frame should be equal to the decision variable ($t_{ijf_xf_y}$).

$$t_{ijf_xf_y} = \sum_{\alpha}\sum_{\beta} x_{i-f_x+\alpha,j-f_y+\beta} \tag{2.4}$$

b. The decision variable ($t_{ijf_xf_y}$) is transformed into ($v_{ijf_xf_y}$) by assessing the validity of the frame.

$$v_{ijf_xf_y} = \begin{cases} 1, & t_{ijf_xf_y} = 0 \\ 0, & otherwise \end{cases} \tag{2.5}$$

c. It is necessary for every SMU to be assigned to at least one valid frame.

$$\sum_{f_x}\sum_{f_y} v_{ijf_xf_y} \geq 1 \tag{2.6}$$

II. Corner case handling: since the corner SMUs are part of incomplete frames, these frames should be ignored and not considered in the computation of valid frames.

$$t_{ijf_xf_y} = -1 \quad \forall\, ijf_y \tag{2.7}$$

$$t_{ijf_xf_y} = -1 \quad \forall\, ijf_x \tag{2.8}$$

However, the frame constraints or shape constraints are not efficient because it uses two decision variables per block, at least one of which ($t_{ijf_xf_y}$) is redundant because it is directly related to the main decision variable. The model limits the minimum mining dimensions to n × n frames and is unable to handle rectangular frames. This is indeed a drawback as real-world mining scenarios often involve irregularly shaped mining areas, and the model should be able to accommodate such cases. The decision variable, $t_{ijf_xf_y}$, has four indices, which lead to an exponential increase in the number of variables. This can result in scalability issues, especially for large scale mining problems, where the number of variables grows rapidly. It can significantly impact the computational time and memory requirements of the optimization process. The paper lacks sufficient explanation about the linearization of Equation (2.5) and its validity as a constraint. Linearization is an important step in formulating MILP models. It would be helpful to provide details on how this equation is linearized. Furthermore, the model does not incorporate different destinations of the ore such as oxide and heap leach ore, and using this approach is computationally expensive.

2.4. SHAPE CONSTRAINT

Shape constraints play a crucial role in mine planning because they ensure that physical shape of the mine or the bench meets operational requirements. Ignoring these constraints can lead to a loss of ore material, increase dilution, and suboptimal mining operations. For example, in an open pit mine, heavy equipment must be able to move around the bottom part of the mine, so ignoring this constraint could result in inadequate space for equipment operations, which can cause delays and safety hazards. Similarly, in determining dig limits for different ore types, the solution should include dig limits that provide enough space for the equipment to dig that specific shape. This is called the minimum mining width, which is often included in dig-limits determination manually by geologists and can result in significant financial loss.

It is challenging to formulate linear shape constraints as required for LP problems. Direct formulations using "natural" decision variables of such linear shape constraints require exponential order of constraints for the number of blocks [84]. Such constraints are inefficient resulting in long solution times. The shape constraints in dig limits optimization problems constitute the most constraints in the problem and have an outsized effect on solution times. Therefore, formulation of such constraints must be efficient to ensure reasonable solution times [85].

Queyranne and Wolsey's articles [84], [86] address the problem of scheduling tasks that have bounded up/down times (i.e., the tasks must start and end within a certain time frame). Additionally, the tasks have interval-dependent start-up times, which mean they can only begin at a certain time. They proposed two MIP models for solving this scheduling problem. Both models were tested on a set of instances with different parameters, and the results showed that the proposed models outperformed existing models in terms of solution quality and computation time. These models, which have been shown to valid and tighter formulations than the "natural formulation", can be applied to various problems and dig limits is one of them. They are superior to the formulation by Sari and Kumral [8].

We can divide their work into two major components that were utilized to formulate shape constraints in the proposed model for this thesis. Those two components are contiguity and mining width. To capture the contiguity and the mining width of a series of blocks on a bench scale:

- Assume a discrete (1D) series of blocks as in Figure 2.4.
- The width of the mined stope of that series of blocks must be at least α_t and at most β_t.
- Likewise, a pillar that begins with block t has a minimum length of γ_t and at most δ_t.

Define the binary decision variables:

- $y_t = 1$, if block *t* is mined (on); 0 otherwise (off).
- $z_t = 1$, if block *t* is the first (leftmost) of a stope.
- $z_t = 1$, if $y_{t-1} = 0$, and $y_t = 1$.
- $w_t = 1$, if block *t* is the first (leftmost) block of a pillar.
- $w_t = 1$, if $y_{t-1} = 1$, and $y_t = 0$.

Based on these decision variables, Queyranne and Wolsey proposed Equations (2.9 - 2.15) as tight MIP formulations of 1D constraints [84].

$$z_t \geq y_t - y_{t-1} \qquad t \in [1, n] \tag{2.9}$$

$$z_t \leq \sum_{u=t+1}^{t+\beta_t} 1 - y_u \qquad t: t \geq 0\ andt + \beta_t \tag{2.10}$$

$$w_t \leq \sum_{u=t+1}^{t+\delta_t} y_u \qquad t: t \geq 0\ and t + \delta_t \leq n \tag{2.11}$$

$$y_t - y_{t-1} = z_t - w_t \qquad t \in [1, n] \tag{2.12}$$

$$\sum_{\substack{u \in [0,t]: \\ u+\alpha_u > t}} z_u \leq y_t \qquad t \in [1, n] \tag{2.13}$$

$$\sum_{\substack{u \in [0,t]: \\ u+\gamma_u > t}} w_u \leq 1 - y_t \qquad t \in [1, n] \tag{2.14}$$

$$y, z, w \in \{0, 1\}^n \tag{2.15}$$

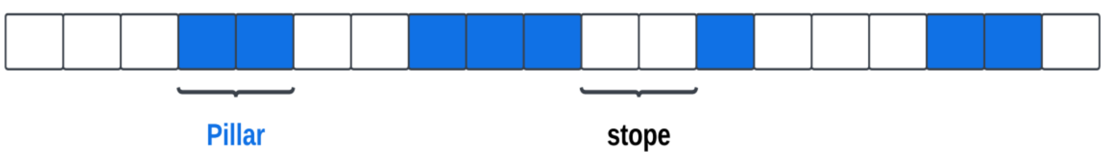

Figure 2.4 1D Series of Blocks

Equation (2.9) enforces the condition that if a block (t) is the first block after a leftmost block ($z_t = 1$), then the block (t) must be mined ($y_t = 1$) and the block immediately to the left of (t) must be not mined ($y_{t-1} = 0$). Equation (2.10) limits the width of the mined stope to at least β_t by ensuring that, if $z_t = 1$, then all blocks from (t + 1) to ($t + \beta_t$) must be mined. Equation (2.11) is similar to Equation (2.10) except that it ensures the width of a pillar is at least δ_t. Equation (2.12) relates the variables y_t, z_t, and w_t. This equation helps maintain consistency between the block variables and their corresponding leftmost block variables. Equation (2.13) works together with Equation (2.9) to ensure the proper relationship between z_t and y_t. Equation (2.14) ensures that each pillar is at least γ_u wide.

Overall, these equations form a mathematical representation of the 1D constraints related to contiguity and mining width, allowing for the formulation of a tight ILP model for solving the scheduling problem with bounded start and end times. Queyranne [84], [86] proposed that 2D rectangular constraints can be formulated by repeating these constraints in each dimension. Moving from 1D to 2D formulations requires additional decision variables to address constraints in each dimension. In a 2D grid of blocks, each block is represented by decision variables in each direction. For dig-limits optimization, because dig-limits are determined on each bench separately, this 2D extension is adequate to determine the optimal dig-limits. However, this introduces additional complexity and increases the number of decision variables and constraints. For example, when this approach is applied on 2D (grid of blocks), the backward constraints (Equations 2.13 and 2.14) cannot be applied for blocks at the boundary. To overcome this issue, we must add waste blocks at the boundary. For example, if we have a 20x20 grid size, we need to add waste blocks at the boundary in order to apply the backward constraints.

2.5. SUMMARY

The objectives of this section were to (i) review the current knowledge on dig-limits optimization, in general, as well as the different models and algorithms used to solve dig-limits optimization problems; and (ii) identify the knowledge gaps related to dig-limits optimization that require further research. The author discussed three major algorithms that other researchers have used to model the dig-limits optimization problem, which are simulated annealing, mixed-integer linear programming, and genetic algorithms. While all these approaches have been used to solve the dig-limits optimization problem, the review in this chapter shows that they all have certain limitations.

First, the simulated annealing algorithm in the literature addresses the dig-limits problem with respect to the equipment size alone. Ignoring other factors such as the material type and blasted material movement. However, the scale of the model was intended to account for the equipment's ability to dig a specific polygon as the only constraint. Another observation was the ability of the penalty function to be applied in a real case for proper testing. The obtained results from the model are near-optimal for the specific case where it was tested. The bottom line is that using the simulated annealing algorithm to solve the dig-limits problem is very complicated, and it does not account for all constraints.

Secondly, genetic algorithms have also been used in the literature to provide near-optimal solutions to the dig-limits problem. Solving the dig-limits using the genetic algorithm is relatively new and more research is needed to fully understand its usefulness in this area. This model is promising because it can account more than two destinations for the different ore types (beyond just ore and waste destinations).

Finally, this section reviewed the application of mixed-integer linear programming to solve the dig-limit optimization problem. The model seeks to maximize the profit while incorporating the equipment size constraint. The model reviewed assumes symmetrical block sizes (SMUs) allowing the model to include the equipment size as a function of block sizes, and it makes it easy to program. For example, the equipment size will be defined as 3 SMUs in a two-dimensional space. However, the model does not incorporate more than two destinations, which is common in metal mining, and is computationally expensive.

In the last part of this section, the work discussed the importance of shape constraints in mine planning to ensure operational requirements are met and to avoid financial losses. It highlighted the challenges of formulating linear shape constraints for LP problems and the need for efficient formulations to ensure reasonable solutions time. The work shows that Queyranne and Wolsey's articles [84], [86] proposed MIP models for scheduling tasks with bounded up/down times and interval-dependent start-up times can be applied to various problems, including dig-limits. Their formulation included two major components (contiguity and minimum width) that can be used to formulate shape constraints in a 1D series of blocks. However, expanding the problem to be in 2D adds more complexity to the problem. No work in the literature has used this approach to solve the dig-limits optimization problem.

3. BINARY INTEGER LINEAR PROGRAMMING FORMULATION OF DIG LIMITS OPTIMIZATION

The proposed binary integer linear programming formulation of dig-limit optimization in this thesis is going to be the main topic of this chapter. This chapter will introduce the model, as well as all important parameters, indices, sets, and decision variables. These will be utilized to define the objective function and constraints. At the end of this chapter, the work introduces a base case study to illustrate the proposed model.

3.1. BINARY INTEGER-LINEAR PROGRAMMING

BILP (Binary Integer Linear Programming) can be used to identify the optimal solution to an optimization problem [87]–[91]. The decision variables in a binary problem can only take a value of 0 or 1, which might be the selection or rejection of an option, the turning on or off switches, a yes/no response, or a variety of different circumstances [92]. Previous LP formulations of dig limits problem used a MILP formulation [8], which had its limitations as pointed out in the literature review. Therefore, the justification of this thesis project was to a BILP version of the dig-limits problem that guarantees optimality and incorporates efficient shape constraints to model the minimum mining width constraints.

3.2. DECISION VARIABLES, INDICES & SETS, AND PARAMETERS

Tables 3.1, 3.2, and 3.3 show the decision variables, indices and sets, and model-building parameters. This work uses the decision variables defined in this model, along with their related indices, to formulate the problem at hand. The indices provide a way to uniquely identify each decision variable, allowing for tracking throughout the optimization problem. Figure 3.1 shows a visual illustration of the decision variable *y* (*i*=2, *j*=4, and *k*=2) and its corresponding indices in the space of the problem.

Table 3.1 Model Parameters

Parameters	**Units**	**Meaning**
$\boldsymbol{M}$	t	tonnage of block
$\boldsymbol{G_{ij}}$	g/t	Grade of metal in the block (*ij*)
$\boldsymbol{P}$	\$/g	Price
$\boldsymbol{R^k}$	ratio	Processing recovery for material going to destination *k*
$\boldsymbol{C_k^p}$	\$/t	Unit processing cost for material going to destination *k*
$\boldsymbol{C_m}$	\$/t	Mining cost
$\boldsymbol{v_{ijk}}$	\$	The economic value of block (*ij*) mined to destination *k*
$\boldsymbol{\alpha_i}$	No. of blocks	Minimum mining width along *i*-direction
$\boldsymbol{\beta_j}$	No. of blocks	Minimum mining width along *j*-direction

Table 3.2 Model Indices

Indices	**Sets**
$\boldsymbol{i}$	Index for blocks along the Y-direction (1, 2, …., I).
$\boldsymbol{j}$	Index for blocks along the X-direction (1, 2, …., J).
$\boldsymbol{k}$	Index for destination (1, 2, …., K).

Table 3.3 Decision Variables

Decision variables	Meaning
y_{ijk}	[0,1] - 1 if block at (*ij*) is mined to destination k, 0 otherwise.
z^1_{ijk}	[0,1] - 1 if block at (*ij*) is the left-most block, 0 otherwise. (Along i-direction).
z^2_{ijk}	[0,1] - 1 if block at (*ij*) is the left-most block, 0 otherwise. (Along j-direction).

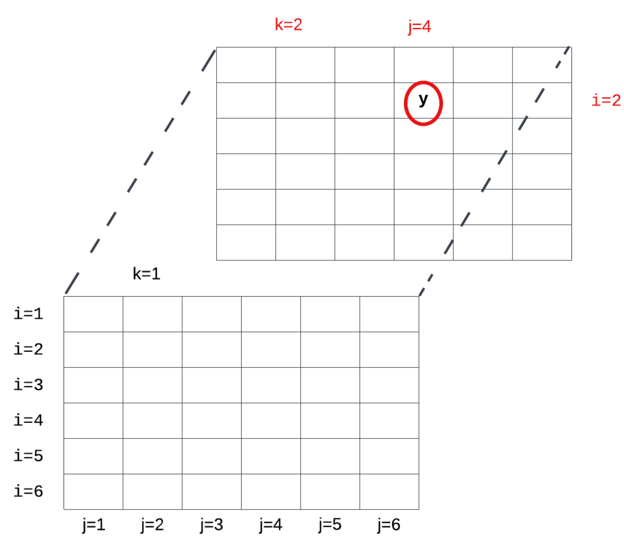

Figure 3.1 A Visual Illustration of the Problem Space

3.3. MODEL FORMULATION

The model's objective, as previously stated, is to determine the optimal dig limit that maximizes profits by delivering the material to the optimal destination. The profit here is the sum of the revenue from mining material and processing it based on the destination it is sent to minus the associated mining and processing costs. The economic block model and the minimum allowable mining widths are the two basic inputs to the dig-limits problem. The economic block model is a bench section (2D grid of regular blocks) of the 3D block model, where each block has variables (v_{ijk}) that represent the economic values for each of the potential targets. The values of each block are calculated by Equation (3.1) and then submitted to the model. Figure 3.2 summarizes the steps to get to the optimal solution.

$$v_{ijk} = \sum_{i=1}^{I} \sum_{j=1}^{J} (M \times G_{ij} \times P \times R - M \times (C_p + C_m)) \tag{3.1}$$

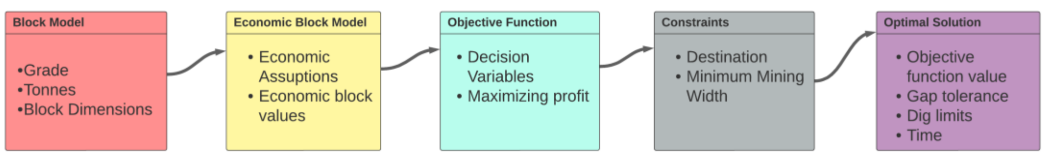

Figure 3.2 Framework of the Proposed Method

3.3.1. Objective Function. The objective function of this model is to maximize the profit obtained from a bench. Equation (3.2) shows the objective function.

$$Maximize \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} v_{ijk} * y_{ijk} \tag{3.2}$$

With this objective function, the optimization solution is likely to exclude the mining of waste blocks since their block values are negative unless those waste blocks (combined with ore blocks) meet the minimum mining width constraints and allow for an overall higher profit. Because of the ability to just leave out waste blocks, waste blocks do not have to meet the minimum mining width constraints. The approach in this thesis is to assume that waste blocks can always be mined after selectively mining the ore blocks.

3.3.2. Constraints. The constraints in this model are grouped into two main categories: destination and shape constraints.

3.3.2.1. Destination constraint. As stated before, the dig-limits optimization problem must consider multiple processing methods and therefore multiple material destinations (waste dump, stockpiles, and different processing destinations). Therefore, the first constraint, Equation (3.3), in the model will restrict each block to be sent at most to one destination.

$$\sum_{k=1}^{K} y_{ijk} \leq 1 \qquad \forall i, j \tag{3.3}$$

3.3.2.2. Shape constraints. As stated before, the goal of this thesis is to adapt the efficient shape constraints proposed by Queyranne & Wolsey [84], [86] to model the minimum mining dimensions constraint of the mining equipment. They demonstrated that direct formulations of such constraints generally require exponentially many constraints in the natural decision variables. With the help of auxiliary variables, we can overcome this and extended it to be solved in linear time. This extended formulation is ideal, and it controls the generated polygon “dig limits.”

To adapt Queyranne and Wolsey's work to the dig-limits optimization problem, two main adaptions are necessary. First, because dig-limits optimization does not require pillars, only Equations (2.9) and (2.13) of Equations (2.9)-(2.15) are necessary to model the minimum mining width in the 1D problem. Second, to adapt the 1D problem to a 2D problem (dig-limit optimization is a 2D problem as each bench is optimized separately), two variables are necessary to control the "leftmost" block (which now become the "leftmost" and "topmost" blocks) in a contiguous set of blocks.

The decision variable ($\boldsymbol{z^1_{ijk}}$) will be assigned to control the generated polygons "dig limits" along the *i*-direction and decision variable ($\boldsymbol{z^2_{ijk}}$) along the *j*-direction. Equations (3.4-3.7) represent two sets of constraints, one along each direction. These equations are used to initialize the values of ($\boldsymbol{z_{ijk}}$).

A. If block at (*ij*) is mined but the previous block is not, then block at (*ij*) is the leftmost or topmost block. This is analogous to Equation (2.9) in Queyranne and Wolsey's formulation.

$$z^1_{ijk} \geq y_{ijk} - y_{(i-1)jk} \quad \forall i, j, k \tag{3.4}$$

$$z^2_{ijk} \geq y_{ijk} - y_{i(j-1)k} \quad \forall i, j, k \tag{3.5}$$

One of the main inputs in the model is the minimum mining width. The minimum mining width of the mined blocks must be realistic for the equipment size. α_i and β_j control the width of the mined blocks. Equations (3.6-3.7) represent the contiguity of the mined blocks and are analogous to Equation (2.13). In this model, the constraint is repeated; one for each direction.

B. Contiguity constraint (minimum mining width)

$$\sum_{u=\max(1-\alpha+1,1)}^{i} z_{ujk}^{1} \leq y_{ijk} \quad \forall i, j, k \tag{3.6}$$

$$\sum_{u=\max(1-\beta+1,1)}^{j} z_{iuk}^{2} \leq y_{ijk} \quad \forall i, j, k \tag{3.7}$$

As a result of this ideal formulation, the extreme points of the corresponding polygons "dig limits" are 0-1 vectors representing all the contiguous solutions. Figure 3.3 shows the values of z_{ijk}^{1} and z_{ijk}^{2} at the boundaries of the generated dig limits. Blue arrows indicate blocks where variable z_{ijk}^{1}= 1. Red arrows indicate blocks where variable z_{ijk}^{2}= 1.

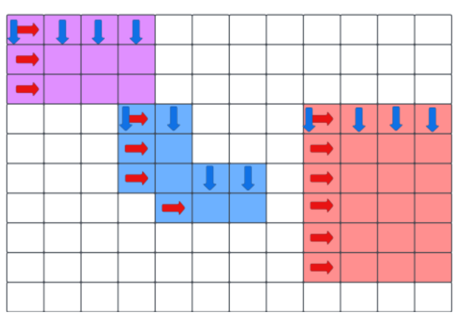

Figure 3.3 Values of z_{ijk}^{1} and z_{ijk}^{2} at the Boundaries of the Corresponding Dig Limits

In summary, Equation (3.8) presents the proposed optimization model.

$$
\begin{aligned}
&Maximize \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} v_{ijk} * y_{ijk} \\
&\sum_{k=1}^{K} y_{ijk} \leq 1 \qquad \forall i,j \\
&z_{ijk}^{1} \geq y_{ijk} - y_{(i-1)jk} \quad \forall i,j\,,k \\
&z_{ijk}^{2} \geq y_{ijk} - y_{i(j-1)k} \quad \forall i,j\,,k \\
&\sum_{u=\max\,(1-\alpha+1,1)}^{i} z_{ujk}^{1} \leq y_{ijk} \quad \forall i,j\,,k \\
&\sum_{u=\max\,(1-\beta+1,1)}^{j} z_{iuk}^{2} \leq y_{ijk} \quad \forall i,j\,,k
\end{aligned}
\tag{3.8}
$$

3.4. MODEL VERIFICATION

This work uses a base case study of benches extracted from a real geologic block model to verify the performance of BILP for dig-limits optimization. The structure of this section is derived from the framework shown in Figure 3.2. The model is implemented in MATLAB. This thesis uses Gurobi v9.5.2, which is one of the fastest solvers on the market, as the solver to solve the BILP model described in section 3.3 [93]. Gurobi v9.5.2 uses branch and bound to solve BILP problems [94]. To solve the model, this work uses the Gurobi MATLAB API. The MATLAB code (which can be found on Github at https://github.com/Somatalhi/Diglimits-Optimization.git) begins by setting up the data for the optimization problem, including the block model and input parameters. The data is provided by the user as an input file. The optimization problem is formulated using the MATLAB problem-based optimization workflow. The variables, objective function, and constraints are defined using the “optimvar” and “optimproblem” functions. The problem is then solved using Gurobi solver. The optimal solution, along with the objective value and exit flag, is returned to the MATLAB environment.

This thesis uses the geologic block model of a porphyry copper deposit to verify the model and MATLAB code. In this verification test, the work uses a 20×20 bench section to test the model. All the tests run in this thesis were run on the same computer. Table 3.4 shows the computer’s specifications.

Table 3.4 Computer Specifications

System type	64-bit operating system, x64-based processor
Processor	Intel(R) Xeon(R) Gold 5220R CPU @ 2.20GHz 2.19 GHz
Installed RAM	64.0 GB (63.6 GB usable)

3.4.1. Block Model. In the mining industry, block models are widely used to model ore deposits and guide the planning and operation of mines. The block model used to test the proposed BILP model is a copper-moly deposit with seven material destinations. Table 3.5 shows the multiple destinations of the ore deposit. All test cases in this thesis are extracts of bench sections (sections in the x-y plane) from this block model. Testing the mathematical model with a huge number of blocks in the block model will be computationally expensive. Therefore, the author decided to use a 20×20 extract from the bench at elevation 4025 feet above mean sea level for model verification. Figure 3.4

shows the copper grade for the blocks in this extract while Figure 3.5 shows the “best classification” of the blocks which shows the assigned destination based the geological and recovery parameters of each block.

Table 3.5 Target Classes

Target Class	Meaning
1	Mill Ore
2	Sulfide crushed leach
3	Oxide crushed leach
4	Suflide ROM leach
5	Oxide ROM leach
6	Low grade
7	Waste dump

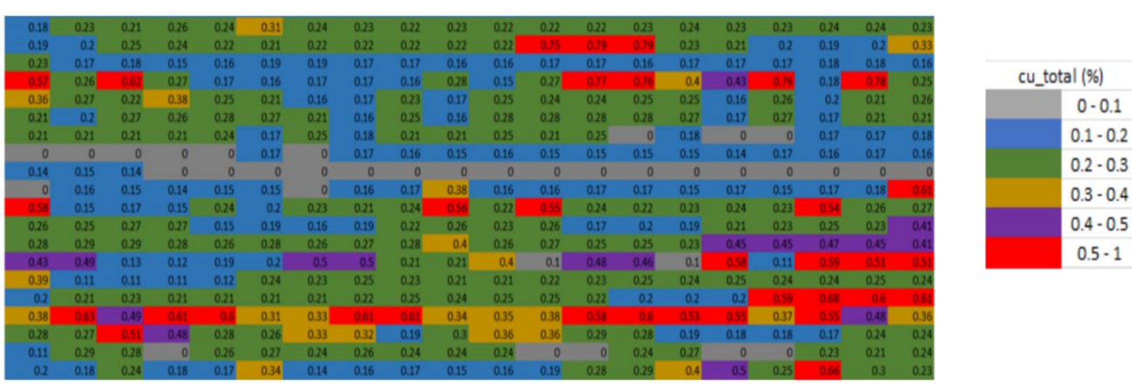

Figure 3.4 Copper Grade of Blocks of the 20×20 Bench Section Extract for Verification

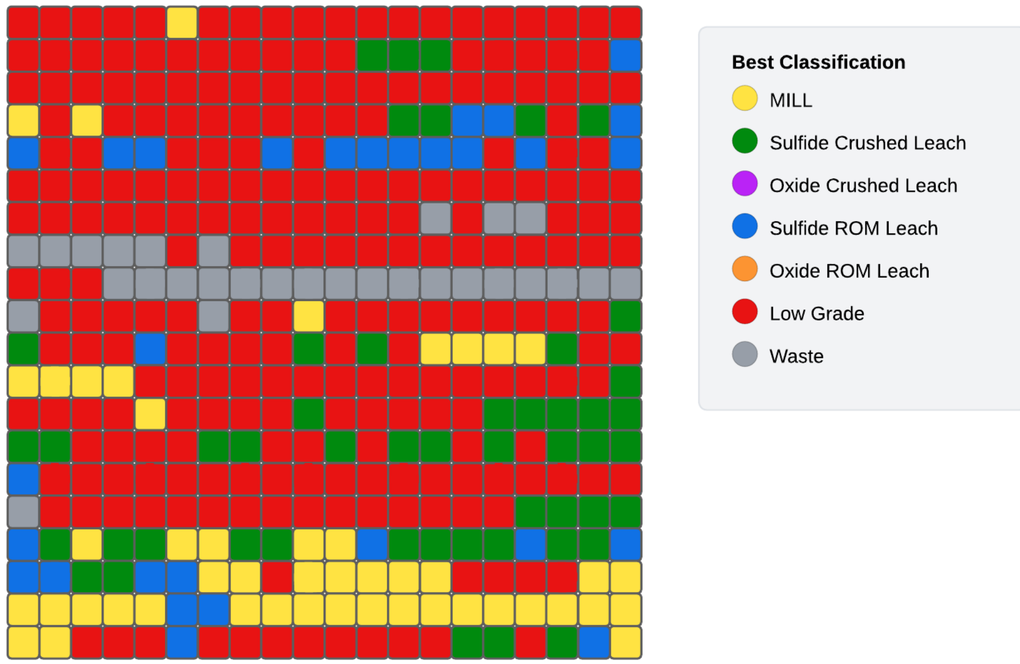

Figure 3.5 Best Classification of the 20×20 Bench Section Extract for Verification

3.4.2. Economic Block Model. The next step is to convert the geological block model into an economic block model using the economic parameters in Table 3.1 and the geological attributes within each block. This input is used in Equation 3.1 to find the economic block values (vijk) for each block in the geological block model. Although there are really seven destinations, the model input excludes the waste dump as a destination because the model then will be forced to create dig limits to waste blocks. In practice, we do not need to selectively mine waste blocks. Therefore, for this block model, each block will have six (6) economic values representing the six destinations (k) in the block model. All waste blocks get economic values less than 0 for all destinations to discourage their mining to other destinations. Because this author did not have access to the confidential recoveries for the mine that provided the block model (but had access to the mine's decisions on the best classification – Figure 3.5), the author assigned block values based on his understanding of the processes and materials that were consistent with the "best classifications" (i.e., the author assigned the highest economic value to destination "mill" if a block had "mill" as the best classification destination etc.). Table 3.6 shows the economic block values (vijk) for a small selection (seven out of 400) of blocks in the 20×20 block extract. The full set of economic block values used in the verification are available on the Github repository for this project (https://github.com/Somatalhi/Diglimits-Optimization). These economic values are the input to the Matlab code to find the optimal dig limits.

Table 3.6 Economic Block Values

Block Index-*i*	Block index- *j*	V1 ($)	V2 ($)	V3 ($)	V4 ($)	V5 ($)	V6 ($)
1	1	500	1,000	800	900	700	600
1	2	1,000	500	600	900	500	500
2	1	900	700	800	500	600	1,000
2	2	500	1,000	800	900	700	600
1	3	500	700	900	1,000	800	600
1	4	900	700	800	500	600	1,000
2	3	500	1,000	800	900	700	600

After assigning block values to the 400 blocks, and before sending the file to the Matlab code, the user must add waste blocks at the boundary (at i=1 & j=1) to ensure the backward-looking constraints (Equations 3.4 & 3.5) can be formulated for the problem. The waste blocks at the boundary will be deleted before displaying the results. The model has three inputs to be chosen by the user which are the number of targets, minimum mining width along i-direction (α), and minimum mining width along j-direction (ß). However, the user must make sure that the number of targets in his/her case is consistent with the economic values in the input file. Table 3.7 summarizes the verification problem.

Table 3.7 Summary of Verification Problem

Parameter	**Value**
Number of blocks	400
Number of Destinations	6
α (Number of blocks)	3
β (Number of blocks)	3
Number of Decision Variables	7,200
Number of Constraints	9,600

3.4.3. Results and Discussion. Figure 3.6 shows the optimal solution from the proposed BILP dig-limits model. Table 3.8 shows that the model achieved an optimal solution with a gap tolerance of 0.0% in about 64.02 seconds. As can be seen in Figure 3.6, most of the blocks in this bench extract are re-classified as low grade in the optimal solution, which results in an optimal objective function value of $ 332,000.

To illustrate that the results of this 20×20 bench section are optimal, we have divided bench section into six sectors as seen in Figure 3.7. We use each of these sectors to illustrate different decisions made by the algorithm.

Table 3.8 Optimization Results for the Base Case Scenario

Parameter	Value
Objective Function Value ($)	332,000
Solution Time (seconds)	64.02
Gap Tolerance	0.0%

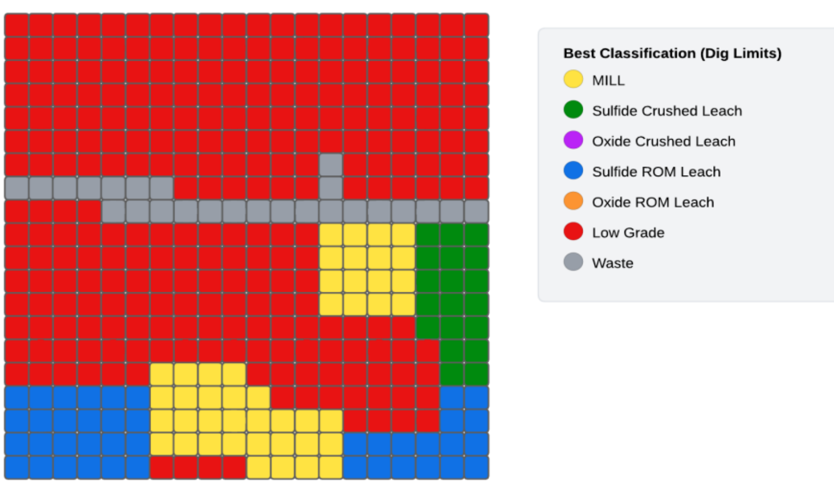

Figure 3.6 Optimal Dig Limits

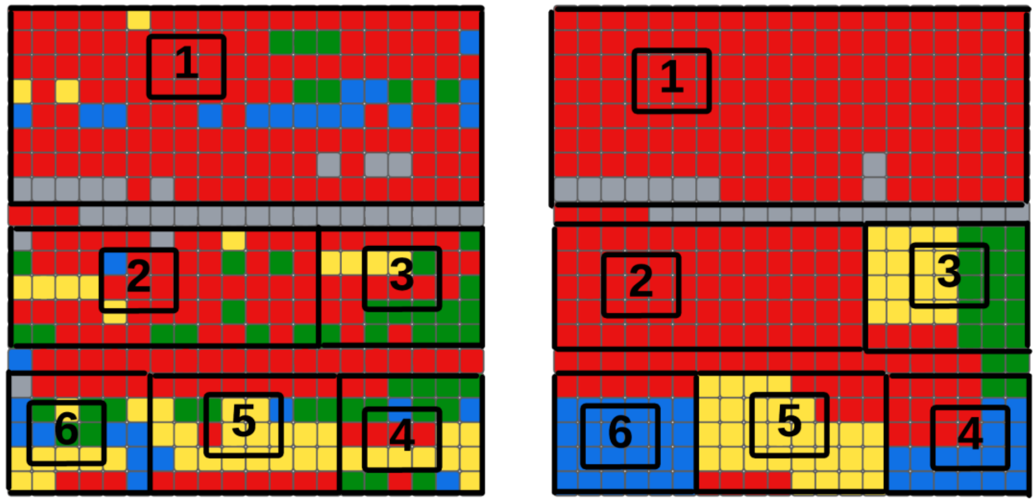

Figure 3.7 Before and After with Sectors Numbers

In sector 1, most of the blocks were initially classified as “low grade” and those blocks not classified as “low grade” would not meet the minimum mining width of 3×3 blocks to be mined selectively as any other type of material. Thus, the best classification was not a feasible solution. Given the block economic values, the optimal solution was to classify all these blocks as “low grade” ore. By doing so, the overall value of the bench section is maximized. For the “low grade” blocks to be classified as “sulfide ROM leach”, it will take the lowest economic values among the other targets. Therefore, the “sulfide ROM leach” blocks in the center of sector 1 were reclassified as “low grade” to ensure that the objective function is maximized. Sector 2 is similar to sector 1 were most the blocks are classified as “low grade”, and the rest will not satisfy the minimum mining width of 3×3 blocks.

In sector 3, the right-hand side has most the blocks classified as “sulfide crushed leach”, and it satisfies the minimum mining width at the same time. The second highest value for the “low grade” blocks is the “mill”, therefore, to maximize the value of the bench, the left side of this sector was reclassified as “mill”. For blocks classified as “mill” and “sulfide crushed leach”, the second highest value is to be mined to “sulfide ROM leach”. Therefore, in Sector 4, the reader can see that most blocks were reclassified as “sulfide ROM leach”. In sector 5, most blocks were classified as “mill”, and it satisfies the minimum mining width. However, at the bottom of this sector, you can see some blocks at the boundary were kept as “low grade” because they can be selectively mined due to their location at the boundary. The same logic in sector 4 goes for sector 6 that blocks classified as “mill” and “sulfide crushed leach” have the second highest value to be “sulfide ROM leach”.

Based on the above results, we can conclude that the reclassification was made to ensure the feasibility of the mining operations (minimum mining width) and optimize the overall value of the bench section. Thus, the proposed algorithm works as intended (i.e., is verified).

3.5. SUMMARY

The focus of this chapter was to propose a binary integer linear programming (BILP) formulation for dig-limits optimization. The model seeks to maximize the value of the blocks selected subject to destination (i.e., each block must be mined to only one destination) and minimum mining dimension constraints. The minimum mining dimensions (shape constraints) were inspired by previous work by Queyranne and Wolsey that modeled minimum mining width using auxiliary variables. The algorithm is implemented in MATLAB using the problem-based optimization workflow and the resulting problem is solved using Gurobi 9.5.2.

To verify the model, a 20×20 bench section was used with 3×3 dig limit size. The algorithm found an optimal solution with a gap tolerance of 0.0%, within approximately 64.02 seconds. A careful examination of the results shows that the model makes optimal decisions, and the optimal solution is feasible. The optimal objective function value amounted to $332,000.

4. EVALUATING THE MODEL'S SENSITIVITY TO PROBLEM SIZE AND MINING WIDTH DIMENSIONS

This section describes experiments carried out to assess the performance of the proposed BILP model for dig-limits optimization with changing size of the problem and minimum mining width dimensions. This work examined the size of the problem because it recognizes that computational time is an important aspect of dig-limits optimization from a practical standpoint and mine engineers are unlikely to use an algorithm that takes more than a few minutes to find a solution. The work also examined the dimensions of the minimum mining width because one of the strengths of this model is that it allows the user to generate any rectangular mining width. Therefore, the effect of this on solutions and solution times is of interest to any reader.

4.1. EFFECT OF SIZE OF THE PROBLEM

4.1.1. Problem Size Experiments. The experimental plan in this section adds two additional scenarios with different sizes to the base case scenario described in chapter 3 to test the effect of size of the problem. These scenarios involved grid sizes of 25×25 and 30×30 in addition to the 20×20 grid in the base case scenario. Similar to the base case, the author generated economic values for the block sections in these scenarios using the same methodology used in the base case and described in section 3.4. The complete input data for these scenarios can be accessed on GitHub. It is worth noting that these scenarios were solved using the same MATLAB code employed in the previous analysis (which can be found on Github at https://github.com/Somatalhi/Diglimits-Optimization.git), ensuring consistency and comparability in the results. Figure 4.1 and Figure 4.2 show the copper grade and the best classification of the 25×25 bench section, respectively. Figure 4.3 and Figure 4.4 show the copper grade and the best classification of the 30×30 bench section, respectively. Table 4.1 shows a summary of the two scenarios and the base case.

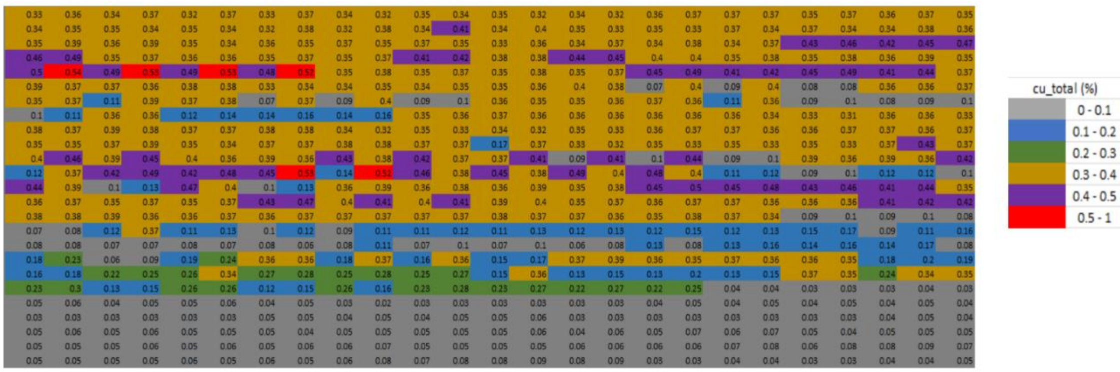

Figure 4.1 Copper Grade of Blocks of 25×25 Bench Section

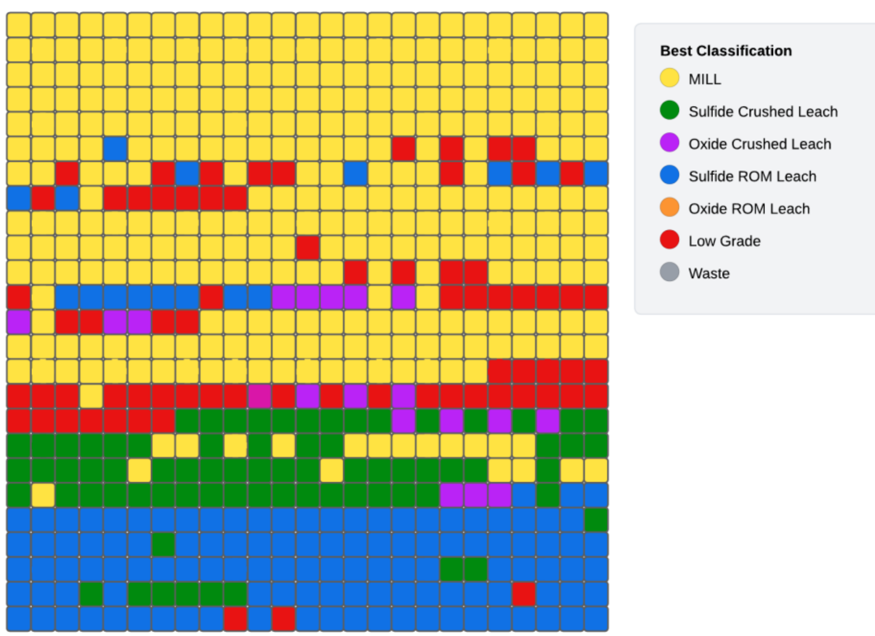

Figure 4.2 Best Classification of the 25×25 Bench Section

Figure 4.3 Copper Grade of Blocks of 30×30 Bench Section

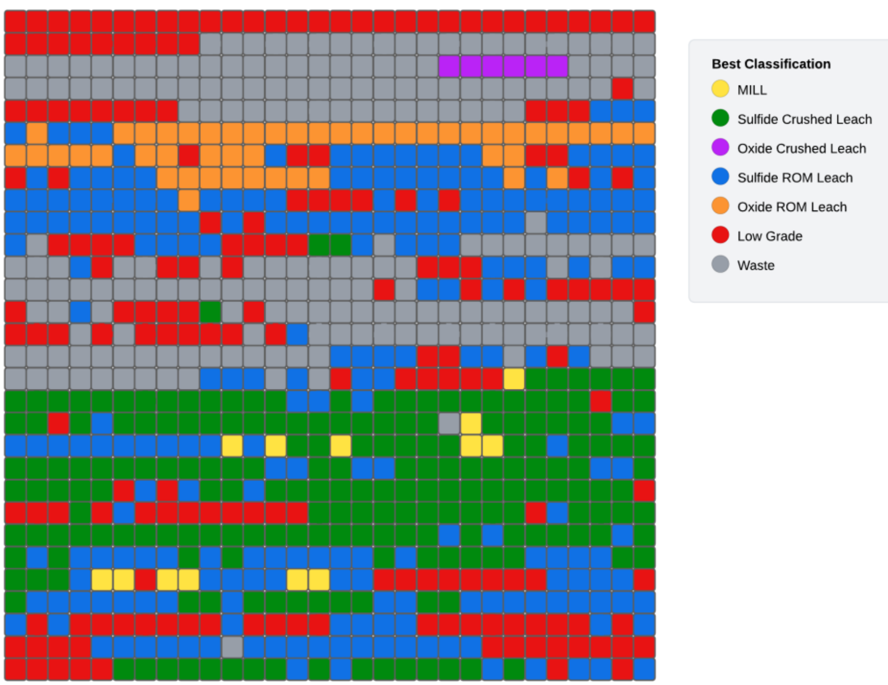

Figure 4.4 Best Classification of the 30×30 Bench Section

Table 4.1 Summary of the Three Scenarios

Parameter	20×20	25×25	30×30
Number of blocks	400	625	900
Number of Destinations	6	6	6
α (Number of blocks)	3	3	3
β (Number of blocks)	3	3	3
Number of Decision Variables	7,200	11,250	16,200
Number of Constraints	9,600	15,000	21,600

4.1.2. Results and Discussion. Figures 4.5 and 4.6 show the optimal solution for the 25×25 and 30×30 scenarios, respectively. Figure 3.6 shows the optimal solution for the base case problem. Table 4.2 shows the optimization results for the two scenarios and the base case.

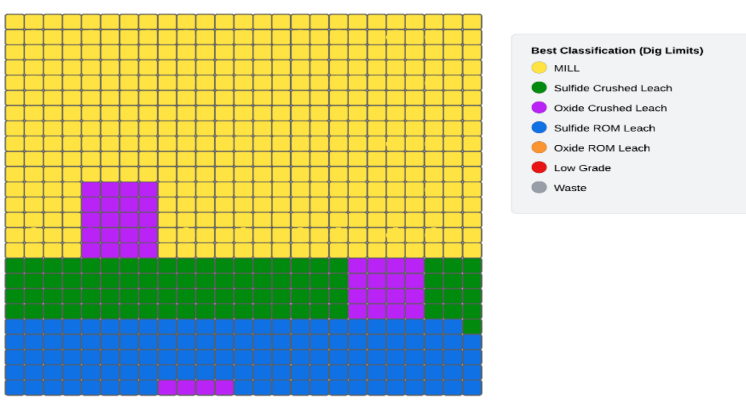

Figure 4.5 Optimal Dig Limits (25×25)

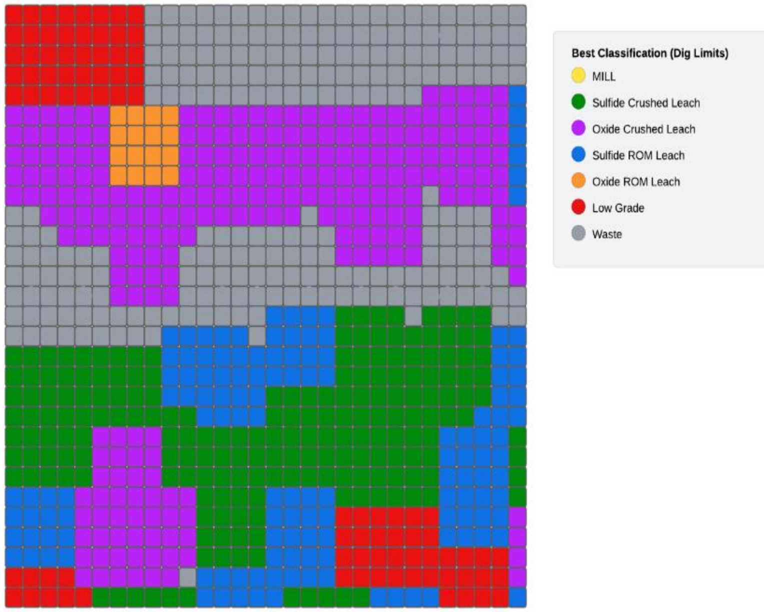

Figure 4.6 Optimal Dig Limits (30×30)

Table 4.2 Optimization Results for Effect of Problem Size Experiment

Parameter	20×20	25×25	30×30
Objective Function Value ($)	332,000	597,500	550,400
Solution Time (seconds)	64.02	125.73	233.39
Gap Tolerance	0.0%	0.0%	0.0%

All the generated dig limits respect the minimum mining width constraints and maximize the value (with gap tolerance of 0.0%). Thus, these results validate the model's performance for different sized problems.

The optimal solution obtained for the 25×25 grid size in Figure 4.5 closely resembles the best classification depicted in Figure 4.2. The model effectively directed the upper half of the section to be mined for the mill, ensuring compliance with the minimum mining width constraints while maximizing profit. However, the model reclassified certain blocks located near the midpoint between the left-hand edge and the center as oxide crushed leach, which corresponds to the third destination. To comprehend the reasoning behind this reclassification, it is necessary to examine the economic values for the third destination (V3) assigned to these blocks prior to the reclassification. Specifically, the economic values (V3) of the blocks originally designated for destinations one, four, and six were 800, 900, and 800, respectively. Consequently, reclassifying these blocks to be mined for destination three was the second and third best options, considering the economic values. This reclassification was contingent upon satisfying the minimum mining width constraints.

The optimal solution for the 30×30 section shows interesting results. Figure 4.7 presents a comparison of the best classification and the optimal dig limits for the 30×30 bench section. The most interesting thing for the reader to note is the fact that the optimal solution reclassifies the areas classified as sectors 1 and 2 to be mined to the third destination when there were no blocks classified to that destination prior to the reclassification. Again, the economic values for mining these blocks to destination three (V3), and the dimensions of the minimum mining width forced the model to pick the third destination. The rest of the generated dig limits by the model in the 30×30 bench section is very similar to the best classification.

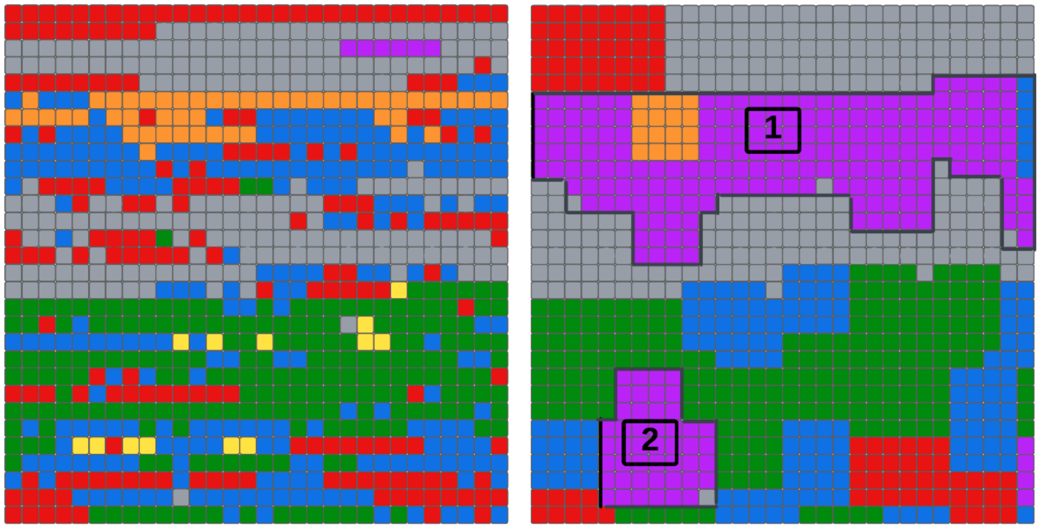

Figure 4.7 Before and After with Sector Numbers (30×30)

The results (Table 4.2) show that the solution time can increase significantly as the grid sizes increase. Large problems result in a greater number of blocks, constraints, and variables. Consequently, as the number of blocks, constraints, and variables increase, the time required to find the optimal solution also grows. For instance, you need around 4 minutes to get the optimal dig limits for a grid size of 30×30. The objective function, on the other hand, does not follow the same trend. It depends on the grade distribution of the bench. The 30×30 bench section has a lower objective function than the 25×25 bench section because of the number of waste blocks in it. While the 4 minutes solution time is acceptable, depending on the application, it is possible to have many more blocks on a bench than 900 (30×30). Such scenarios will require much more time than what is acceptable in short range mine planning tasks.

In order to assess the factors driving the increase in solution time, the author generated scatter plots for the number of blocks, number of variables, and number of constraints versus the solution time. All three scatter plots exhibited an identical pattern. It is challenging to determine which variable had the most impact on the solution time based on these results. Figure 4.8 – 4.10 show all three scatter plots.

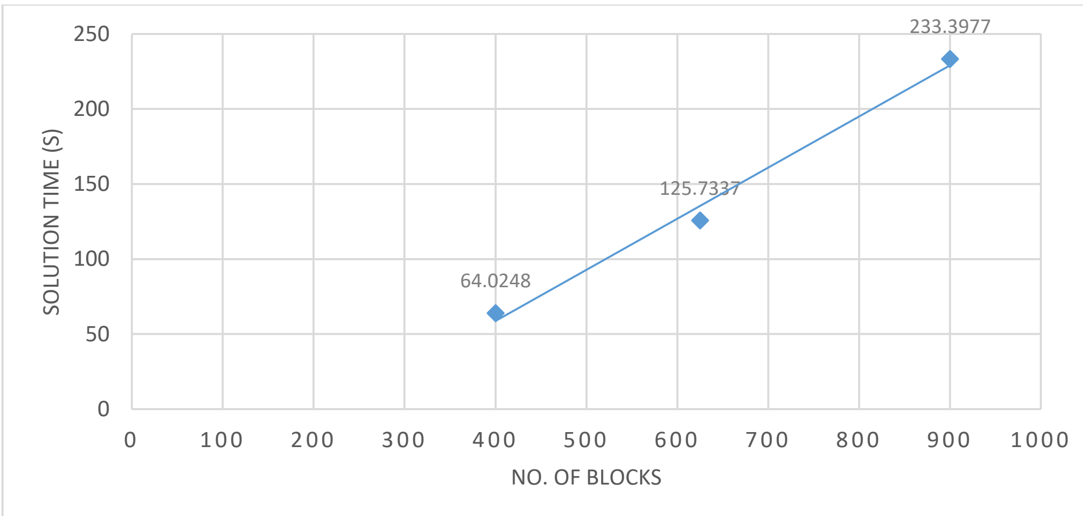

Figure 4.8 Relationship Between Solution Time and Number of Blocks

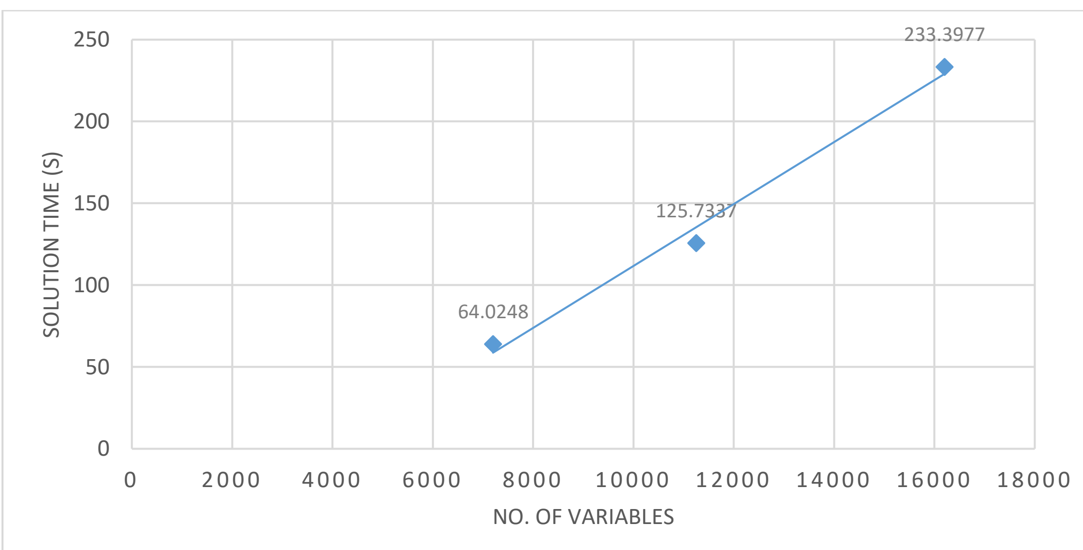

Figure 4.9 Relationship Between Solution Time and Number of Variables

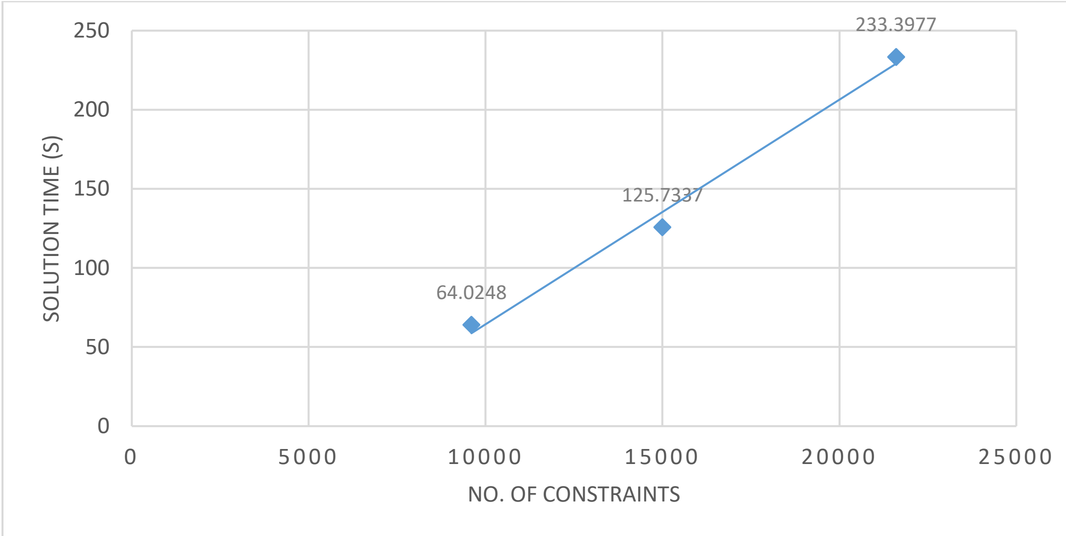

Figure 4.10 Relationship Between Solution Time and Number of Constraints

This finding indicates a strong dependence between the number of blocks (grid size), these three variables, and the complexity of the problem. One can infer that the problem size (grid size) affects equally the number of blocks, number of decision variables, and number of constraints, all of which play a crucial role in determining the solution time. As the size of the problem expands, demanding more blocks, variables, and constraints, the computational time required to discover the optimal solution also escalates. It is important to note that the conclusions drawn from the analysis should be interpreted with caution, as the results are based on a limited number of experiments. Conducting additional experiments with a wider range of problem sizes could provide a more comprehensive understanding of the relationship between these variables and solution time. Future work should explore more comprehensively the relationship between the size of the problem and the computational time.

4.2. EFFECT OF MINIMUM MINING WIDTH DIMENSIONS

4.2.1. Minimum Mining Width Dimensions Experiments. In the preceding sections, the minimum mining width dimensions for the "dig limits" were uniform square shapes and were of moderate size ($\boldsymbol{\alpha_i} = \boldsymbol{\beta_j}$). However, to further investigate the impact of varying the dimensions of the minimum mining width, this section extends the analysis by including additional dimensions. Using the same input data as in section 4.1, this experiment varies the values of $\boldsymbol{\alpha_i}$ and $\boldsymbol{\beta_j}$ to explore different mining width proportions.

In addition to the previous 3×3 minimum mining width, this section considers three additional minimum mining width dimensions: 2×5, 5×2, and 5×5. These variations allow for a more comprehensive evaluation of the model's performance with varying dimensions of the minimum mining width.

4.2.2. Results and Discussions. Table 4.3 shows the optimization results for these different minimum mining widths. In addition to exploring the effect of varying $\boldsymbol{\alpha_i}$ and $\boldsymbol{\beta_j}$ on the solution, objective function value, and computational time, it is crucial to visually examine the resulting solutions for different minimum mining width dimensions. Figure 4.11-Figure 4.13 showcase the solutions for the 30×30 bench section scenario (Figure 4.6 shows the optimal dig limits with minimum mining width 3×3). Appendix A shows similar figures for the other two bench sections. These figures demonstrate the effectiveness of the proposed approach in accommodating different minimum mining width dimensions.

Table 4.3 Optimization Results for Different Minimum Mining Width Dimensions

Bench Size	Minimum Mining Width ($\alpha_i x\ \beta_j$)	Objective Function ($)	Solution Time (s)
20×20	3×3	332,000	64.02
	2×5	334,100	64.29
	5×2	332,100	65.46
	5×5	328,400	67.16
25×25	3×3	597,500	125.73
	2×5	600,000	126.68
	5×2	592,500	128.17
	5×5	591,800	127.30
30×30	3×3	550,400	233.39
	2×5	564,900	237.14
	5×2	543,300	252.19
	5×5	534,800	257.22

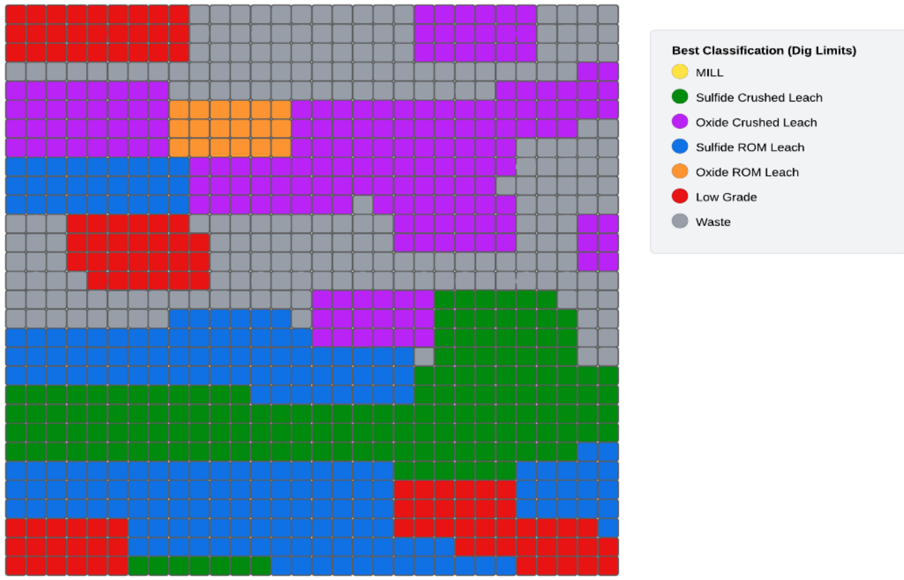

Figure 4.11 Optimal Dig Limits for Bench Section (30×30) with Minimum Mining Width (2×5)

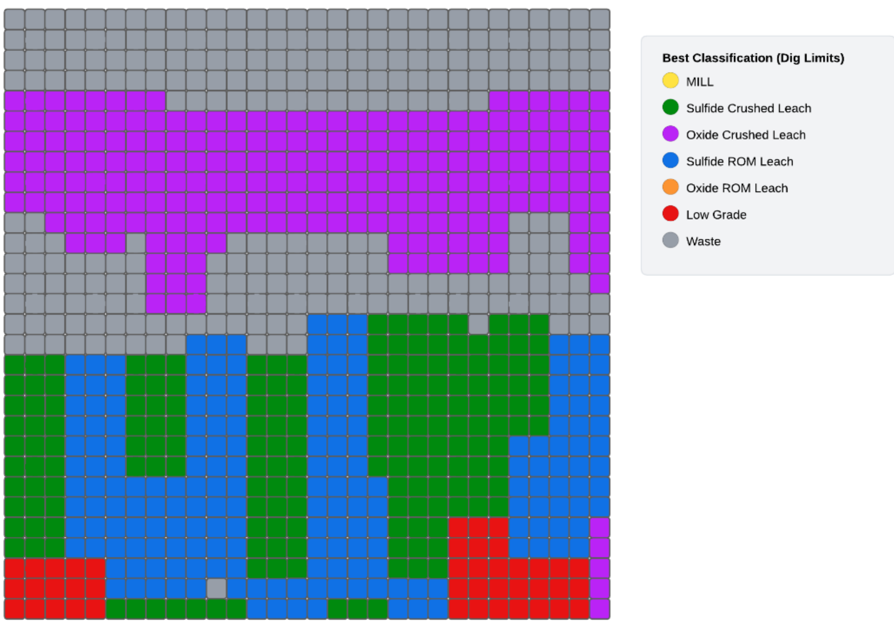

Figure 4.12 Optimal Dig Limits for Bench Section (30×30) with Minimum Mining Width (5×2)

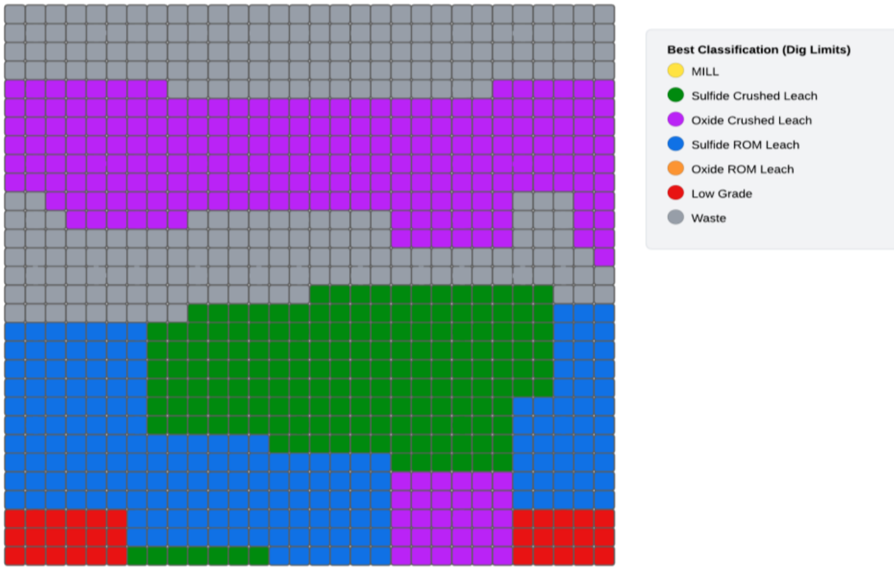

Figure 4.13 Optimal Dig Limits for Bench Section (30×30) with Minimum Mining Width (5×5)

First, Table 4.3 and Figure 4.11 – Figure 4.13 show that varying α_i and β_j has an effect on the solution, and objective function value. The figures show that the solution respected the different minimum mining width dimensions. However, the Table 4.3 shows that the objective function value decreases for all three bench section sizes (20×20, 25×25, and 30×30) when the minimum mining width changes from 3×3 to 5×5. Similarly, the objective function value increases in all situations when one dimension increases even if the other dimension stays constant. That is, the objective function decreases when the minimum mining width changes from 2×5 to 5×5 or from 5×2 to 5×5. This indicates that the size affects the objective function value. This is because, with higher minimum widths, the solution lacks selectivity and the solution differs even more from the "best" classification.

Additionally, these results (Table 4.3) suggest that when rectangular minimum mining width are employed, it is more advantageous to utilize the longer side of the minimum mining width in the β_j direction. For all bench sections, the problem 2×5 minimum mining dimensions has the highest objective function value. To understand this trend, the reader must recall the best classification as well as the copper grades distribution figures for bench section 30×30 (Figure 4.3 and Figure 4.4). The blocks of similar classification or grade tend to align along the j-direction. This is due to the geology of the bench sections of the block model used in this work. By setting the minimum mining width at 2×5 as in Figure 4.11, we are increasing the possibility "helping" the model to pick the same best destination and not looking at other options "destinations". This result shows that, in cases where the minimum mining width can be a rectangle based on equipment dimensions and operating specifications, such dimensions can be advantageous for dig limits optimization. When using rectangular minimum mining dimensions, the benefits depend on the orientation of the longer dimension relative to the geology.

Secondly, the relationship between solution times and the objective function value appears to be similar. That is, solutions with higher objective functions tend to also be the ones with lower computational time (Table 4.3). This indicates that the problems with larger minimum mining width dimensions are also more complicated problems.

Finally, this experiment highlights a significant contribution of the proposed BILP model, setting it apart from existing models in the literature. The unique feature of this model lies in its capability to incorporate rectangular minimum mining width dimensions, which provides mine engineers with a level of flexibility and control that was previously unavailable. This represents a notable advancement in dig-limits optimization models. The significance of this capability becomes particularly evident when analyzing the experimental results. The experiments with varying α_i and β_j dimensions clearly demonstrate the impact of different mining width proportions on the objective function value and solution time. The model's sensitivity to these variations indicates that the dimensions of the minimum mining width can significantly affect the profitability and efficiency of the mining operation. More importantly, the results show that when the geology of the model has clear trends and the equipment also allows for rectangular minimum mining width dimensions, aligning these two can increase the value of the objective function. Nonetheless, it is crucial to acknowledge that the model does not consider the potential rise in mining costs that may arise from utilizing rectangular dig limits, even if it enhances the value of the dig limits themselves. An elongated (narrow) minimum mining width might result in slower mining rate that increases the unit mining cost. This model does not account for that.

4.3. SUMMARY

This section conducted a series of experiments to evaluate the performance of the proposed BILP dig-limits optimization with varying problem size and minimum mining width dimensions. The objective was to evaluate the effect of problem size and minimum mining width dimensions on the solution (including objective function value) and computational time. Based on these experiments, the thesis makes the following conclusions:

- The results of the investigation on the problem size demonstrated the solution time increases when the size of the problem (grid sizes) increases. This can be attributed to the larger number of constraints and decision variables associated with larger problem. For example, the solution time for a 30×30 grid size was approximately 4 minutes compared to that of a 20×20 grid size which was approximately 1 minute.

- As the minimum mining width increases, the objective function of the optimal dig limits decreases because of the loss in selectivity. Consequently, mining engineers should ensure that the minimum mining widths they specify are the absolute minimum because these can lead to reduced value.
- In mining situations where the minimum mining width can be a rectangle based on equipment dimensions and operating specifications, such dimensions can be advantageous for dig limits optimization. When using rectangular minimum mining dimensions, the benefits depend on the orientation of the longer dimension relative to the geology.
- The experiment showcases a significant contribution of the proposed BILP model, distinguishing it from existing models. Its unique feature is the ability to integrate rectangular minimum mining width dimensions, granting engineers more flexibility and control. This advancement is particularly evident in the analysis of experimental results, which reveal the influence of varying mining width dimensions on objective function value and solution time. The model's sensitivity to these variations underscores the substantial impact of minimum mining width dimensions on mining operation profitability and efficiency.
- However, it is essential to note that the model does not account for potential increased mining costs associated with using rectangular dig limits, even if it improves the value of the dig limits themselves.

5. CONCLUSION, RECOMMENDATIONS & FUTURE WORK

5.1. OVERVIEW

The thesis focuses on solving the dig-limits optimization problem in open pit mining to efficiently extract resources and maximize profits. The specific problem is determining the most economic and feasible dig-limits for each bench in an open pit mine, taking into account the multiple destinations for ore material. Traditionally, dig-limits are determined manually by geologists using blast holes' samples and block models. Engineers and geologists often manually assign isolated blocks to different dig-limits to ensure operational feasibility. Previous research has attempted to solve this problem using various optimization approaches, but mathematical programming approaches have been limited to only classifying ore and waste. The objective of this research was to find the optimal block boundaries that allow for multiple material types and their designated destinations while maximizing the profit of a bench sector. This is achieved by developing a binary-integer linear programming (BILP) model that accounts for the size of the mining equipment. The model focuses on the classification and selectivity components of dig-limits and can be used as a decision-making tool for selecting the optimal dig-limits in open pit mines.

5.2. CONCLUSIONS

The author has successfully developed and demonstrated the application of a binary-integer linear programming (BILP) model for selecting optimal dig-limits that accounts for equipment size constraints. The proposed BILP model, implemented in

MATLAB and solved with Gurobi, demonstrated its effectiveness in finding optimal dig-limits within reasonable time frames. The work has illustrated the model's application by applying it to realistic mining benches of differing sizes and configurations. All solutions obtained were feasible solutions.

The work verified the model using a 20×20 bench section with 3×3 dig limit size. The algorithm found an optimal solution with optimal objective function value of $332,000 and a gap tolerance of 0.0%, within approximately 64.02 seconds. A careful examination of the results shows that the model makes optimal decisions, and the optimal solution is feasible.

Experiments to evaluate how problem size and minimum mining width dimensions affect the solution (including objective function value) and computational time yielded the following conclusions:

- Increasing the problem size (grid sizes) resulted in longer solution times due to the larger number of constraints and decision variables associated with larger problems. For instance, a 30×30 grid size took approximately 4 minutes to solve compared to approximately 1 minute for a 20×20 grid size.
- Higher minimum mining widths led to decreased objective function values of the optimal dig limits due to decreased selectivity. Therefore, mining engineers should ensure they specify the absolute minimum mining widths to avoid reducing value.
- In mining scenarios where the minimum mining width can be a rectangle based on equipment dimensions and operating specifications, such dimensions can offer advantages for dig limits optimization, depending on the orientation of the longer dimension relative to the geology.

Lastly, the experiments varying minimum mining width dimensions highlighted the significant contribution of the proposed BILP model, which sets it apart from existing models. Its unique feature is the ability to incorporate rectangular minimum mining width dimensions, granting engineers greater flexibility and control. The advantage of this feature is particularly evident in those experiments that used rectangular minimum mining dimensions. The experimental results show the objective function is higher for rectangular minimum mining widths in which the longer side of the rectangle is aligned with the geology (i.e., the best classifications are longer in that same direction). This shows that models that only allow for equal dimensions in the minimum mining width (even when the equipment specifications will support rectangular minimum mining widths) are likely to yield suboptimal results. This result also shows that mining engineers can generate more value by exploring rectangular minimum mining width dimensions when doing dig limit optimization.

5.3. CONTRIBUTIONS

The thesis is a significant contribution to the literature and mining industry because it addresses key challenges of the dig-limits optimization problem in open pit mines that have previously not been addressed.

- The proposed binary-integer linear programming (BILP) model efficiently determines the most economic and feasible dig-limits for each bench while allowing for multiple destinations for ore material. Unlike traditional manual methods, this model optimally classifies and select dig-limits, enabling the identification of optimal block boundaries for various material types and their designated destinations, thereby maximizing profits. Unlike, previous metaheuristic models, this model guarantees optimality and is superior to the previous mixed integer linear programming model [8] because it allows for multiple destinations for ore. This is the first model that guarantees optimality and allows for multiple ore destinations.

- This is the first model to model rectangular minimum mining width dimensions. The model's unique feature of allowing rectangular minimum mining width dimensions grants mining engineers greater flexibility and control, resulting in higher objective function values aligned with geology. All previous work used a square or circular minimum mining width dimension although some mining equipment are capable of mining in narrow, elongated narrow areas. This powerful decision-making tool not only enhances resource extraction efficiency but also offers valuable insights for mining engineers to make informed and strategic dig-limit optimization decisions, ultimately driving value and efficiency in open pit mining operations. Mining engineers can now explore the benefits of rectangular (elongated) minimum mining width dimensions on dig limits optimization now that this thesis has shown its advantages.
- This model is the first to demonstrate that dig limits optimization can lead to solutions with higher economic value by strategically aligning rectangular minimum mining widths to favor the geology. By accommodating elongated dig limits where mining equipment can operate efficiently, models such as the one proposed in this model (the first of its kind) will yield higher objective function values, if the elongated dig limits are aligned favorably with geological characteristics.

5.4. RECOMMENDATIONS FOR FUTURE WORK

To further improve and advance the proposed Binary-Integer Linear Programming model for optimizing the dig-limits, there are several avenues for future research.

- Future work should investigate approaches to reduce the computational time required to solve the BILP model, particularly for large and more complex block models.
- Future research should aim to incorporate the movements of blasted material into the BILP model, which could further enhance the accuracy and efficiency of mine planning and production.
- Additionally, future studies should explore the integration of uncertainty analysis and risk assessment into the dig-limits optimization process. By incorporating stochastic modeling and probabilistic techniques, the model can account for the inherent uncertainties in geological conditions, commodity prices, and operational constraints. This will enable mine planners to make more informed decisions and develop robust dig-limits strategies that are resilient to fluctuations and unforeseen circumstances.
- Furthermore, it is recommended to extend the applicability of the BILP model to consider other factors that impact mine planning and production, such as environmental constraints, and safety regulations. Incorporating these additional constraints and objectives into the optimization framework will provide a more comprehensive and sustainable approach to dig-limits optimization.

Overall, by addressing these research directions, the proposed BILP model can be further enhanced and tailored to meet the evolving needs and challenges of the mining industry, enabling more efficient and profitable open pit operations.

APPENDIX

Optimal Dig Limits (20×20)

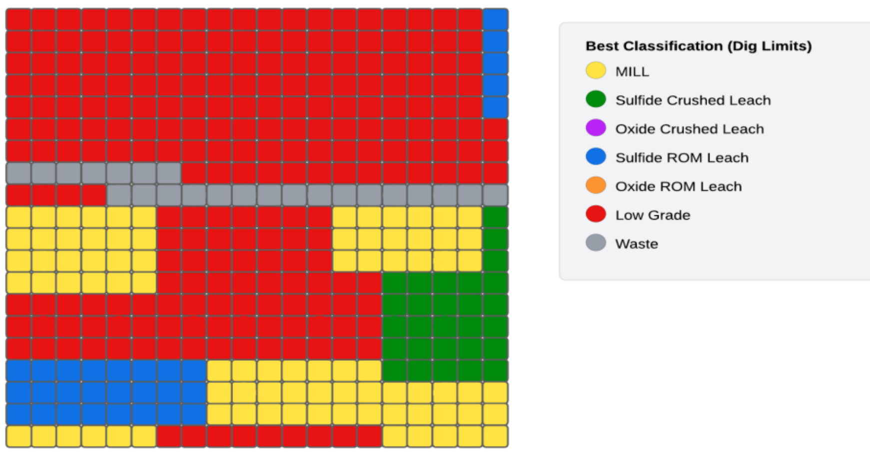

Figure A.1 Optimal Dig Limits (2×5)

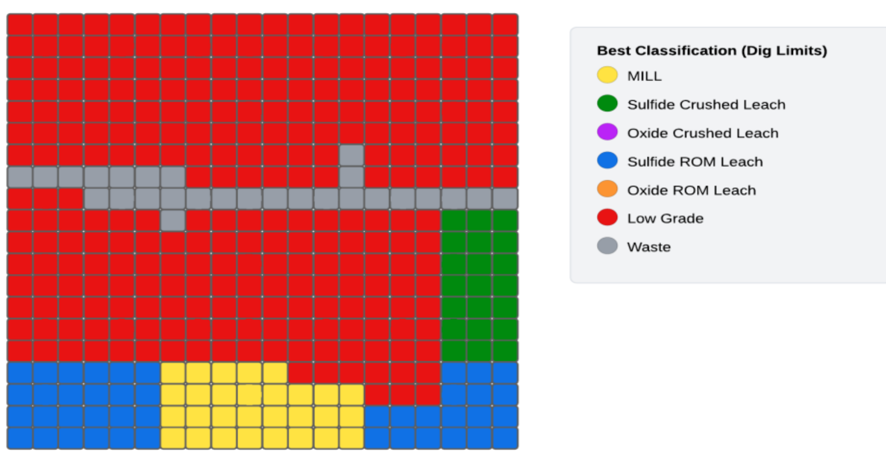

Figure A.2 Optimal Dig Limits (5×2)

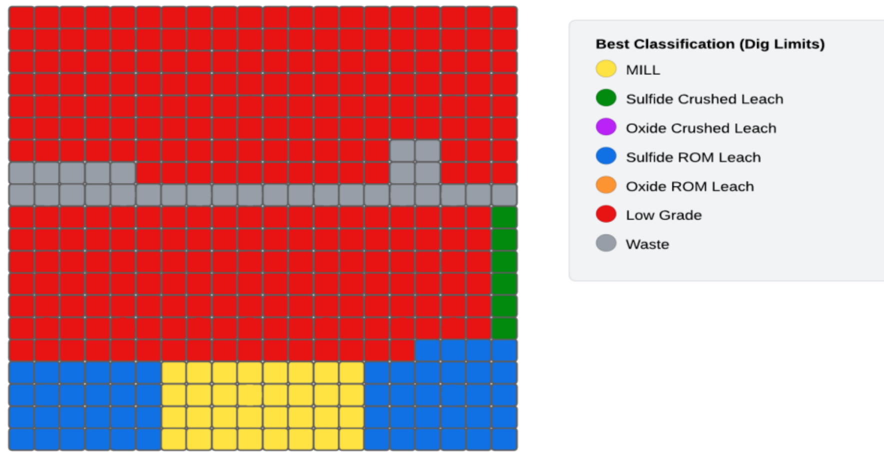

Figure A.3 Optimal Dig Limits (5×5)

Optimal Dig Limits (25×25)

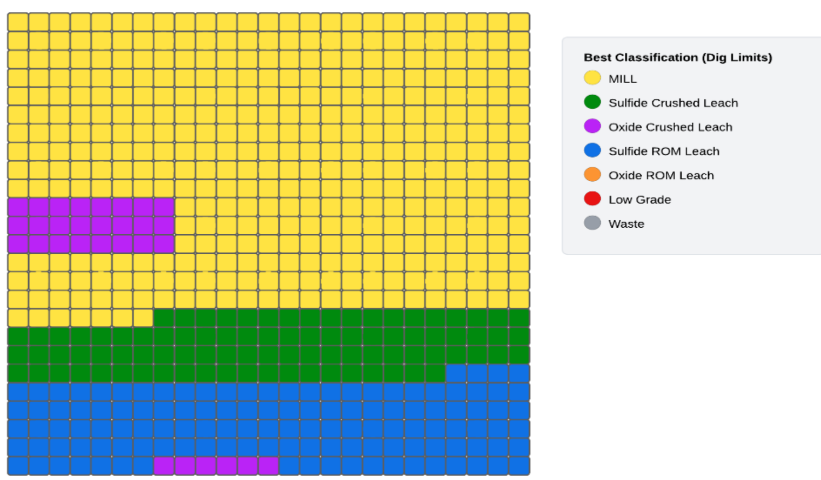

Figure A.4 Optimal Dig Limits (2×5)

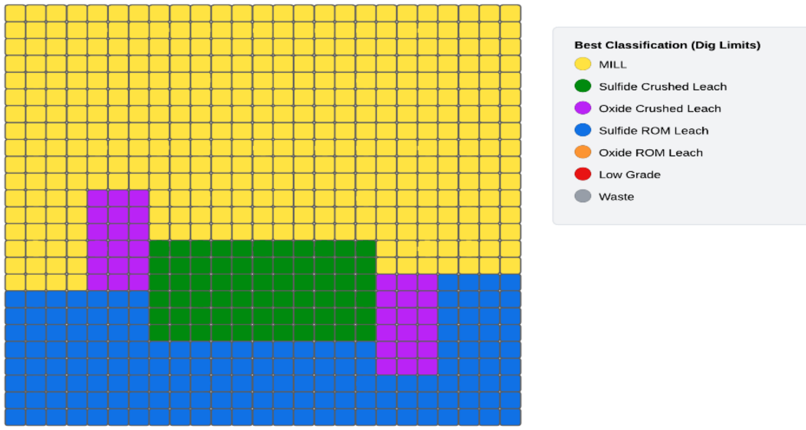

Figure A.5 Optimal Dig Limits (5×2)

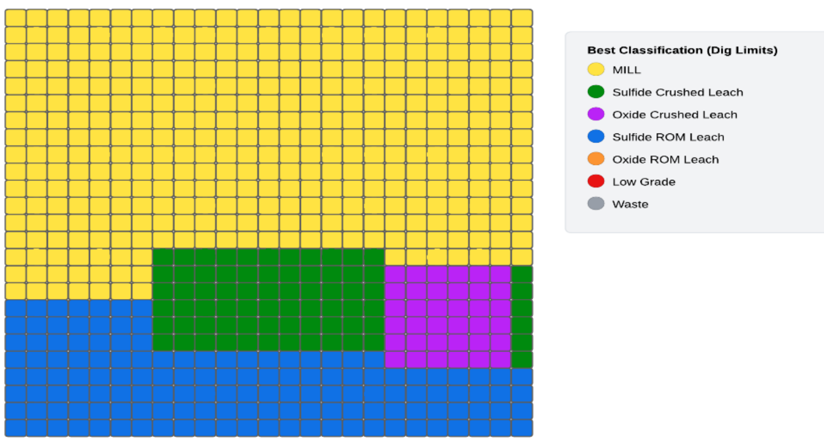

Figure A.6 Optimal Dig Limits (5×5)

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VITA

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After graduation, Hussam worked for a large international company called Llotor Arabia (EPSA) at Al-Sukhaybarat gold mine in Saudi Arabia. His job title was Production Engineer, and his main objective was to ensure a high degree of equipment availability, so he could guarantee clients that their production objectives would be met.

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