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AN INVESTIGATION OF THE ECONOMIC ORDER QUANTITY MODEL WITH QUANTITY DISCOUNTS UNDER AN ENVIRONMENTAL OBJECTIVE

by

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ABSTRACT

In a sustainable supply chain, retailers are the direct link between customers and products. Retailers play an important role by relaying feedback such as customer satisfaction, inventory improvement, or product improvement to the other key players in a supply chain. Their overall goal is to reduce supply chain costs, such as the cost of ordering product, transporting product, or holding product in inventory. Other costs associated in a supply chain can include environmental and operations costs. It is important to consider these costs due to the impact environmental operations play in the role of how sustainable a supply chain can be. By reducing supply chain costs, retailers can take advantage of maximizing their profit. This study investigates how a retailer may reduce costs while considering the impact of carbon emissions in a supply chain. From the inventory management perspective, retailers may order product in large quantities and take advantage of economies of scale. By using a bi-objective formulation of the economic order quantity model, the main goal is find order quantities that reduce costs and emissions. A two-part all-units discount approach offered from the supplier is applied to the model, yielding several cases in which the cost and/or emissions functions are minimized. A Pareto front numerical solution set explicitly characterizes that quantity discounts can either decrease costs and emissions of the retailer or decrease costs while increasing the emissions impact of the retailer, therefore, this study shows how quantity discounts do affect the environment.

ACKNOWLEDGMENTS

I would like to thank God first, for being the head of my life. A true spiritual connection with Him allowed me to unlock my wisdom and potential in great ways. He had a plan predestined for me, and with faith and perseverance, He made the path straight. I could not do it without Him at all.

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Finally, I would like to thank my mom, who has supported me all throughout college. She has always encouraged me to continue school, while lending a helping hand throughout my journey. She pushed me to the limits to help me see that life itself is not challenging; it is whether we are ready for the challenge. I appreciate her loving spirit and her God-fearing attitude, that helped, shaped me to be the woman I am today!

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1. INTRODUCTION

1.1. IMPORTANCE OF INVENTORY MANAGEMENT

Inventory management involves managing inventory to ensure a business's long-term survivability. Meeting demand, customer satisfaction, responsiveness, and efficiency are just a few goals most supply chains strive to succeed [21]. Supply chains manage their success through their ability to achieve profit. By planning and controlling inventory, supply chains can lower their cost of goods or increase their sales to contribute to their profitability [21]. Inventory management can help businesses become more profitable by effectively making decisions to meet their business goals.

Inventory management involves a wide range of decisions to overall satisfy customer needs. Decisions involving order quantities, times to restock and meeting demand relate managing inventory to business objectives. Supply chains are essentially a network of key players such as manufacturers, suppliers, distributors, retailers, and customers [21]. With the overall goal of a supply chain being profit, a supply chain benefits from working together rather than operating as separate entities. For example, if one key player, such as the manufacturer, had the opportunity to decrease the cost of materials to build a product but kept all other supply chain costs the same, the total supply chain costs would not decrease. Other players of the supply chain would need to decrease their costs as well in order for a supply chain to decrease costs across the entire network. How does inventory play a role in the cost relationship with a supply chain? Inventory goes through every key player in a supply chain [21]. Managing the product at each aspect of the supply chain allows the network an opportunity to decrease costs. Not every supply chain is the same; each one is different and may not involve all of the key players mentioned above. Regardless of the nature of the network, inventory levels should be managed to satisfy demand. Demand can be deterministic or random. In this case, forecasting techniques are typically used to estimate demand trends. Managing inventory plays an important role in reducing stockouts by achieving satisfactory levels of stored product [21]. It is not enough for supply chains to maintain a bulk of product because ordering and storing product is costly. Inventory management thus allows supply chains to achieve satisfactory levels of inventory within reasonable cost bounds.

1.1.1. Inventory Related Costs. Inventory decisions can affect how profitable businesses become. Efficient inventory decisions involve methods to decrease costs. There are three general costs associated with inventory management [21]. First, ordering costs, denoted by A^c, involve the costs associated with ordering and purchasing product [21]. Most businesses have transporting and receiving expenses associated as ordering costs due to product being transferred from one location to the receiving end. These costs can vary with the quantity of product ordered. In either fact, ordering costs can increase or decrease with the quantity of product ordered. In an insufficient supply chain, ordering large amounts of product not specific to the observed demand can cause an increase of costs due to the prolonged time of transferring product to the end user, the customer. Situations in which ordering costs decrease with an increased amount of product involve a supply chain taking advantage of economies of scale [21]. Distributors will often offer a discounted price for a large amount of product ordered, thus allowing the supply chain to reduce costs to increase profit. Holding costs either include the cost of storing product in warehouses or retail stores [21]. Businesses in the food industry, for example, may possess higher holding costs than businesses in other industries, such as fashion, due to the expenses associated with holding perishable foods. Holding costs increase with the quantity of product ordered and is normally expressed as a percentage or fraction of the product cost. Lastly, material costs, C, are the costs associated with purchasing a certain quantity of product (i.e., price per unit purchased) [21]. The quantity of product ordered, Q, has a direct relationship with the costs, as mentioned above. The sum of all three costs mentioned are known as the total cost function of the supply chain [21]. In a perfect supply chain, all demand is satisfied; therefore, this condition will be used throughout this study. Table 1.1 shows the parameters affecting the total cost, thus representing the amount the total cost of the supply chain. Equation 1 represents the total cost in relation to the parameters mentioned in Table 1.1 [21].

Table 1.1. Parameters for Total Cost Equation.

Parameter	*Description*
Q	Size of order
D	Demand per unit time
A^c	Cost of ordering product
h^c	Holding cost, (a fraction of the product cost)
p^c	Price per unit

$$\text{Total Annual Cost}=\frac{A^c D}{Q}+\frac{h^c Q}{2}+p^c(Q)D \qquad (1)$$

Due to the non-linearity of the cost function, each cost term can be plotted against *Q* to distinguish the effects of costs at varying order sizes. Figure 1.1 describes the relationship between the size of the order, *Q,* and each specific cost, as well as the total cost function. From the figure, each cost function performs as addressed earlier. The ordering cost decreases with an increase in *Q* and holding cost increases with an increase in *Q*. The material cost has zero slope due to the cost being fixed in nature if there are no discounts. Together these cost terms combine to yield the total cost for a retailer [21]. From Figure 1.1, the total cost curve decreases until it reaches a minimum and then begins to increase with an increase in *Q*. The minimum point represents the optimal order quantity; it is the point that represents the optimal order size to achieve the lowest overall total cost [21].

1.1.2. Economic Order Quantity (EOQ) Model. Achieving lower cost profiles is not always the main objective of some supply chains. As supply chain networks increase, eventually cost will increase due to the expansion of operations performed in a supply chain. Because of this, supply chains will consider the trade-offs between ordering and inventory costs. Calculating total cost to achieve the optimal order size for each supply chain decision related to inventory management can become tedious and time consuming. By taking the derivative of the total cost function with respect to *Q*, and solving for *Q*, as shown in Equations 2 and 3, supply chains can easily find the optimal order size in several situations [21]. This optimal order size, *Q**, is known as the economic order quantity and plays an important role inventory and costs trade-offs [21]:

$$\text{TC'}\ (Q) = \frac{-A^c D}{Q^2} + \frac{h^c}{2} \tag{2}$$

$$Q^{*}=\sqrt{\frac{2A^{c}D}{h^{c}}} \tag{3}$$

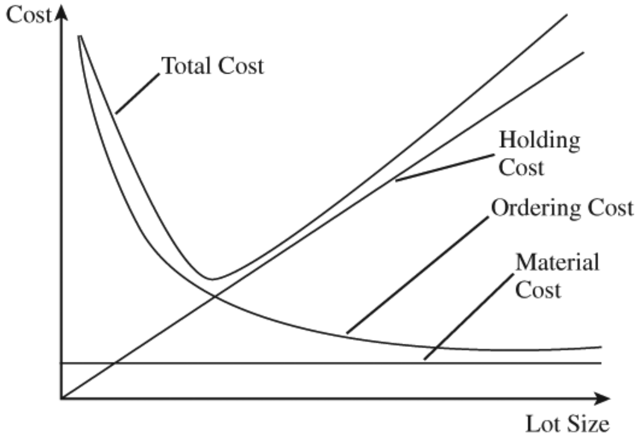


Figure 1.1. Economic Order Quantity Model [21].

The economic order quantity model is shown in Figure 1.1 and graphically represents the trade-offs between ordering and holding costs [21]. This model has been widely used in supply chain management to answer questions on how to replenish inventory at low costs and satisfy all demand. There are several advantages to using this model to consider inventory cost decisions. First, the EOQ model is easy to compute and does not require data that is hard to obtain. The EOQ model can answer questions such as when and how much to replenish inventory [21]. Due to the ease of use, several assumptions are implicated while using this model. These assumptions [21] will be true throughout this study and are listed below:

a. Demand is deterministic and constant, meaning demand is certain and is not random. Demand is constant over time.
b. All of the demand is satisfied, meaning there are no shortages.

c. The lead-time for the end user to receive the product is constant, meaning there is no lag time between production and the customer receiving the product.
d. The order quantity is received all at once.

The reader is referred to [1] on further discussion on the EOQ model setting and assumptions presented.

1.2. IMPORTANCE OF ENVIRONMENTAL CONSIDERATIONS

Supply chain sustainability is a growing topic due to the increasing demand of more environmentally friendly operations [1]. Their awareness forces businesses to execute operations using methods that are not harmful to the environment, do not deplete natural resources, and support the long-term objective to creating a friendly ecological balance. Regulations for environmentally friendly methods are adopted to hold businesses accountable in their operations and to teach businesses how to implement more environmentally safe procedures. With the increasing need to protect the environment, businesses are encouraged to decrease their impact on the environment [11]. Implementing environmentally friendly procedures allows businesses to take advantage of several benefits. Making the planet more environmentally friendly allows businesses to become more sustainable. Businesses can improve their quality by implementing more sustainable products and services [11]. With an increase in sustainable products and services, businesses can also take advantage of the influx of customers who value environmentally friendly products and operations. The list of benefits of becoming an environmentally friendly business increases as the demand for more environmentally friendly operations increase. Going green continues to benefit supply chain sustainability.

There are several ways supply chains can reduce their environmental impact. First, supply chains can shift to purchasing product from environmentally friendly suppliers [1]. This can easily help supply chain networks remain accountable in their operations by including environmentally friendly suppliers. This also may diversify the products and services supply chains offer, which in turn can create an influx of new environmentally friendly customers. Supply chains can reduce emissions and pollution by planning smarter transportation routes and shortening their distances [2-3]. Along with this concept, supply chains can rationalize sourcing, meaning implementing locations near business operations to decrease travel distance [21]. Lastly, recycling product and other forms of material can create a more environmentally friendly atmosphere [11]. The recycled product may also reduce some supply chain costs because of the network's ability to reuse the material. Overall, these ways help supply chains create a more sustainable network.

1.2.1. Environmental Impacts Related to Inventory Control. With hopes of decreasing their environmental impacts, supply chains use inventory control methods to execute eco-friendly operations. Inventory-related operations contribute to a supply chain's environmental impact [2-3]. For example, distribution, inventory holding, transporting product, and warehouse activities generate emissions [2-3]. Certain inventory levels may increase or decrease the amount of emissions generated based on operations to move product. It is important for supply chains to consider effectively managing inventory to reduce its impact on the environment. By implementing inventory control methods that are environmentally safe, supply chains can take advantage of the sustainability benefits mentioned above [1-3].

1.2.2. Environmental Impacts in Relation to the Economic Order Quantity Model. As mentioned above, environmental impacts are causing supply chains to strive for more sustainable operations. Due to the relation inventory management has with generating emissions, supply chains are encouraged to reduce their environmental impact [1]. This study investigates environmental considerations with the inventory control models. Using the EOQ model, as specified earlier, this study will analyze various aspects of inventory control and how emissions are affected at certain inventory levels of decision-making. Due to the wide use of the EOQ model, previous literature investigates using inventory control methods with environmental considerations [2-3]. Topics such as emissions cost [10], environmental regulations [5-9], and environmental objectives [11] all use methodology to relate inventory controlled environments with cost and profit objectives. These topics help to find a balance between cost and environmental impact with hopes of improving inventory-controlled operations. Other topics use the concept of inventory-controlled models to analyze how environmental impacts affect changes such as joint replenishment [12-13], lot sizing [1,14], and newsvendor systems [17-18].

This study also uses the EOQ model with environmental objectives to analyze a cost relationship. With the goal of minimizing both the cost and emissions functions, the basis of the EOQ extends to a bi-objective EOQ model [20]. Unlike the other studies mentioned earlier, a quantity discount environment is applied to our model to investigate cost and emissions from a retailer’s point of view. As mentioned earlier, distributors can offer a lower cost to buy product if the receiving end is buying larger quantities. In this study, a relationship is explored between the retailer and supplier in which the supplier offers quantity discounts to encourage the retailer to buy more, thus allowing the retailer to take advantage of economies of scale [19]. The discount environment will alter the decision the retailer has regarding the best quantity to buy, thus affecting the amount of emissions generated in an inventory control perspective. This study will be the first study to introduce quantity discounts in an inventory-controlled model with environmental considerations [20]. This study further implicates how discounts affect a retailer’s cost-minimizing-emissions environment. In addition, a solution set generated from the study characterizes Pareto efficient solutions that present options for a retailer to not only reduce cost, but also emissions within the discount environment. From this study, several cases present several solution sets that support the notion of a retailer being able to reduce cost and emissions. Other solution sets support the notion that a retailer may be able to decrease cost, but in doing so, increases emissions generated. The next section of this study presents the bi-objective EOQ model.

2. THE BI-OBJECTIVE ECONOMIC ORDER QUANTITY MODEL

2.1. PROBLEM FORMULATION

Earlier, the economic order quantity model was introduced as an inventory-controlled model to help retailers predict replenishment quantities that reduce cost. Section 1.1.2 lists the assumptions implicated by using the model. Such assumptions involving the EOQ model are demand is constant and deterministic, fixed lead times, and no shortages as well as all quantity orders being received at once. The retailer has specific costs that attribute to the total cost of inventory. Recall from Section 1.1.1, total inventory cost is the sum of the ordering cost, inventory-holding cost, and material cost. From Section 1.1.1, the total cost function is defined as Equation 1.To relate the retailer's total cost per unit time as a function of the order size, Q, *C (Q)* is denoted as the total retailer's cost per unit time [21]:

$$C\,(Q) = \frac{A^c D}{Q} + \frac{h^c Q}{2} + p^c(Q)D. \tag{4}$$

In this study, the retailer will take advantage of the economies of scale due to the supplier offering an all-units quantity discount schedule. From the retailer's cost per unit time, $p^c\,(Q)$ denotes the material cost per unit for any order size Q units. The discount schedule can be defined as follows:

$$p^C(Q) = \begin{cases} p_1 & 0 \le Q < Q_1 \\ p_2 & Q_1 \le Q < Q_2 \\ \vdots & \vdots \\ p_{n-1} & Q_{n-2} \le Q < Q_{n-1} \\ p_n & Q \ge Q_{n-1} \end{cases}$$

which is

$$p_1 > p_2 > \cdots > p_n.$$

From this mathematical formulation, the material cost per unit decrease with an increase of Q. This study focuses on investigating the environmental influence of the retailer in a quantity discount environment using an inventory-controlled mathematical model. From the investigation, a Pareto efficient order quantity will be emphasized to show the situations in which a retailer can minimize cost and emissions. Equation 5 [6] measures the environmental performance of the retailer expressed in units of emissions generated:

$$E(Q)=\frac{A^e D}{Q}+\frac{h^e Q}{2}+p^e(Q)D \text{ [6]} \tag{5}$$

From this equation, the parameters description is shown in Table 2.1. The emissions equation generated is similar to the cost, which is easier to relate in terms of inventory management. This equation is also in terms of Q, which is the size of the order dictated by the retailer. The goal of this study is to find solutions or order quantities that minimize both the emissions and cost functions related to the retailer. Using Equation 1 and 5, the bi-objective EOQ model formulation is denoted by **(P)**.

Table 2.1. Parameters for Total Emissions Equation.

Parameter	*Description*
Q	Size of order
D	Demand per unit time
A^e	Amount of emissions generated per order
h^e	Amount of emissions generated from inventory holding
p^c	Emissions generated from each unit purchased

$$\textbf{(P)} \qquad Minimize_{q\geq 0} \qquad C(Q) = \frac{A^c D}{Q} + \frac{h^c Q}{2} + p^c(Q)D \tag{6}$$

$$Minimize_{q\geq 0} \qquad E(Q) = \frac{A^e D}{Q} + \frac{h^e Q}{2} + p^e(Q)D. \tag{7}$$

This next section further explores the idea of Pareto efficient solutions given that the discount environment has been applied to **(P)**. This next section compares and contrasts the retailer’s emission function with and without the discount.

2.2. PARETO EFFICIENT SOLUTIONS

Pareto efficient solutions are solutions that represent the best possible outcomes of a problem formulation; thus, this represents the best solutions, which exist without changing other factors and forcing other factors to be in a worse state. A solution is not Pareto efficient if there is another solution that reflects an improvement made within for each objective. This study will reveal sets of solutions also known as a Pareto-front; thus, every solution in the set will be Pareto efficient. By restricting results to include only Pareto-efficient solutions, or the Pareto-front, conclusions can be drawn in regards to the most efficient order of quantity ranges, which minimize both the cost and emissions.

2.2.1. Effects of the Discount on the Retailer’s Cost and Emissions Functions. As stated earlier, the supplier offers the retailer a single discount to encourage the retailer to increase their purchases. The Pareto efficient solutions to **(P)** are characterized by several assumptions. First, the condition in which a purchase from the retailer does not include a discount from the supplier is defined as:

$$p^c(Q) = p_1. \tag{8}$$

Without a discount, the optimal order quantity is defined as the optimal lot size equation stated in Equation 3. Thus, Q^C , as shown in Equation 9 minimizes the retailer's cost function, *C (Q)* without a discount. From Figure 1.1, the retailer's total inventory cost represents a convex curve with respect to Q. In contrast, this study assumes the supplier offers a single discount to the retailer. Let Q^B or greater denote the order size in which the discount is applied. Thus, $p^c(Q) = p_1$ if $0 \leq Q < Q^B$, and $p^c(Q) = p_2$ if $Q \geq Q^B$, where $p_1 > p_2$. From the viewpoint of the retailer, the argument is logical because p_1 should be greater than p_2 because p_1 being the price of the order before the discount was applied. Thus, with the discount, *argmin* $\{C(Q^C), C(Q^B)\}$ minimizes *C (Q).* The relationships are also similar in the case of the emissions generated. From the emissions perspective, Equation 10 minimizes the *E (Q)*:

$$Q^C = \sqrt{\frac{2A^C D}{h^C}} \tag{9}$$

$$Q^E = \sqrt{\frac{2A^e D}{h^e}} \; . \tag{10}$$

Comparing *argmin* $\{C(Q^C), C(Q^B)\}$ vs Q^E and Q^C vs. Q^E, the effects of the discount can be seen on emissions. Next section shows the Pareto efficient solutions for **(P)** through several cases and discusses the effects of the discounts on both the cost and emissions.

2.2.2. Pareto Efficient Order Quantities. This study presents three difference cases to show how emissions and cost change with the discount environment. Each case specifies a range of Pareto efficient solutions that would satisfy those conditions. Recall that an order *Q'* is Pareto efficient if there is not a better solution that improves both the costs and emissions. For example, if *Q'* is Pareto efficient, there does not exist another

Q'' such that $C\ \ (Q\ \ '') \leq C\ \ (Q\ \ ')$, and for emissions, $E\ \ (Q\ \ '') \leq E\ \ (Q\ \ ')$. *PE* denotes the set of Pareto efficient solutions of **(P).**

2.2.2.1. Case 1. $\mathbf{Q^B} \leq \mathbf{Q^E}$. This case shows the retailer's optimal order size is Q^c with or without discount. Mathematically, Q^c=*argmin* $\{\boldsymbol{C}(\boldsymbol{Q^C}), \boldsymbol{C}(\boldsymbol{Q^B})\}$. All possibilities under Case 1 reflect the buying power of the retailer due to the emissions per unit time not relating with the discount environment. In these situations, the retailer can thus only minimize cost per unit time. The following subcases describe the *PE* of Case 1:

- Case 1.1. If $Q^c \leq\ Q^E$, then *PE* = $[Q^c, Q^E]$. Figure 2.1 (a) shows this result.
- Case 1.2. If $Q^B \leq\ Q^E < Q^c$, then *PE* = $[Q^E, Q^c]$. Figure 2.1 (b) shows this result.
- Case 1.3. If $Q^E \leq\ Q^B$, then *PE* = $[Q^E, Q^c]$. Figure 2.1 (c) shows this result.

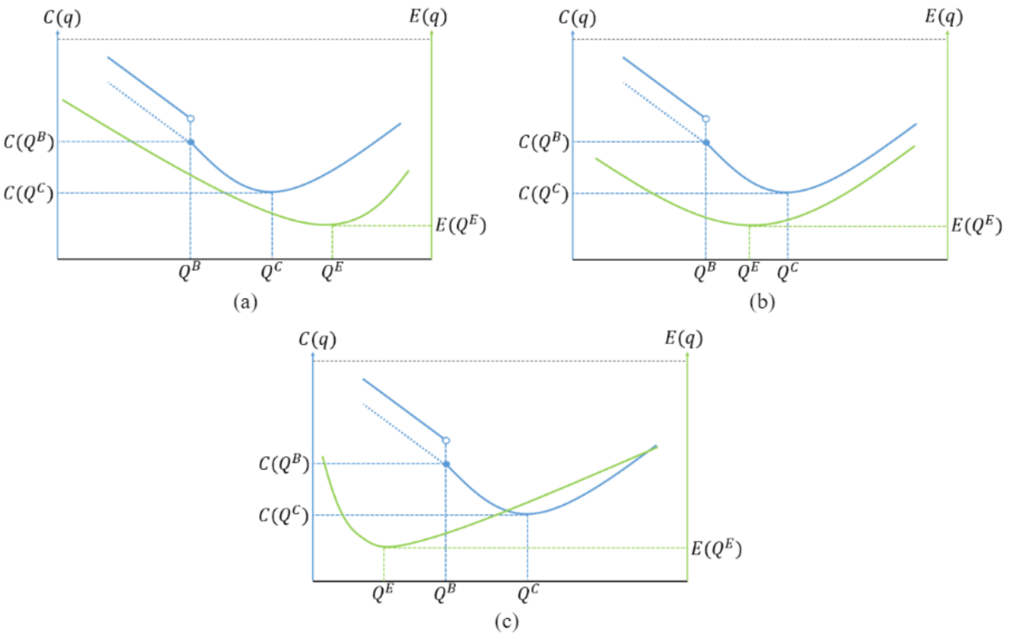


Figure 2.1. Total Cost and Emissions vs. Quantity for Cases 1.1-1.3 (a) (b) (c).

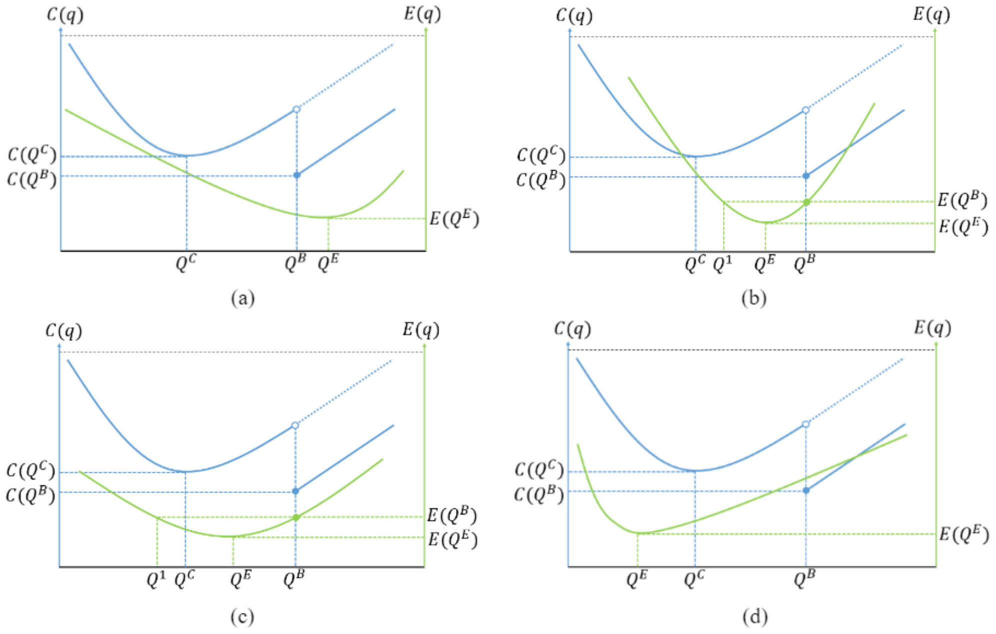
2.2.2.2. Case 2. $\mathbf{Q^C < Q^B}$ and $\mathbf{C(Q^B) < C(Q^C)}$**.** This case shows that the retailer's optimal order size without the discount Q^C does not produce the same results as with the discount Q^B. Mathematically, $Q^B = argmin\ \{\boldsymbol{C(Q^C)}, \boldsymbol{C(Q^B)}\}$. Thus, emissions will vary with the discount if the retailer only minimizes the cost per unit time. The following subcases describe the *PE* of Case 2:

- Case 2.1. If $Q^B \leq Q^E$, then *PE* = $[Q^B, Q^E]$. Figure 2.2 (a) shows this result.
- Case 2.2. If $Q^C \leq Q^E < Q^B$ and $Q^c \leq Q^1$, then

  PE = $(Q^1, Q^E] \cup \{Q^B\}$. Figure 2.2 (b) shows this result.
- Case 2.3. If $Q^C \leq Q^E < Q^B$ and $Q^1 < Q^C$, then

  PE = $[Q^C, Q^E] \cup \{Q^B\}$. Figure shows 2.2 (c) this result.
- Case 2.4. If $Q^E < Q^C$, then *PE* = $[Q^E, Q^C] \cup \{Q^B\}$.

  Figure 2.2 (d) shows this result.



(a) (b) (c) (d)

Figure 2.2. Total Cost and Emissions vs. Quantity for Cases 2.1-2.4 (a) (b) (c) (d).

2.2.2.3. Case 3. $\mathbf{Q^C < Q^B}$ and $\mathbf{C(Q^B) \geq C(Q^C)}$. This case shows the retailer's optimal order size with or without the discount, Q^C. Mathematically, Q^C=*argmin* $\{\boldsymbol{C(Q^C)}, \boldsymbol{C(Q^B)}\}$. Thus, emissions will not vary with the discount if the retailer only minimizes the cost per unit time. The following subcases describe the *PE* of Case 3:

- Case 3.1. If $Q^B \leq Q^E$, then *PE* = $[Q^C, Q^3) \cup [Q^B, Q^E]$. Figure 2.3 (a) shows this result.
- Case 3.2. If $Q^C \leq Q^E < Q^B$ and $Q^3 \leq Q^1$, then

  PE = $[Q^C, Q^3) \cup (Q^1, Q^E] \cup \{Q^B\}$. Figure 2.3 (b) shows this result.
- Case 3.3. If $Q^C \leq Q^E < Q^B$ and $Q^1 < Q^3$, then

  PE = $[Q^C, Q^E]$. Figure 2.3 (c) shows this result.
- Case 3.4. If $Q^E < Q^C$, then *PE* = $[Q^E, Q^C]$. Figure 2.3 (d) shows this result.

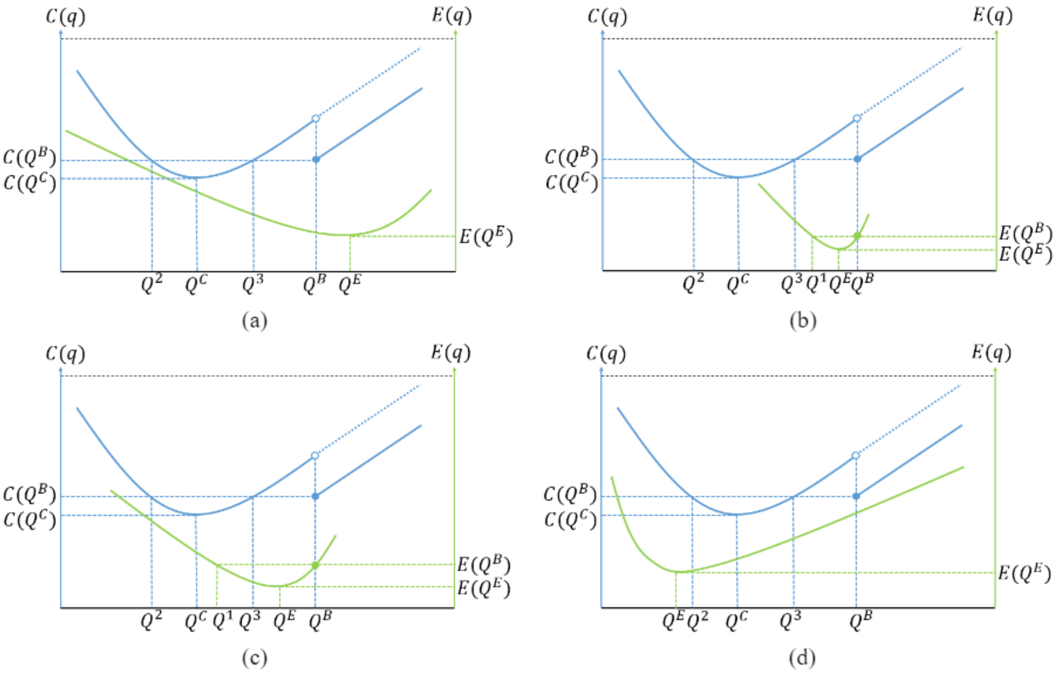


Figure 2.3. Total Cost and Emissions vs. Quantity for Cases 3.1-3.4 (a) (b) (c) (d).

3. NUMERICAL DATA

3.1. PROBLEM SETTINGS

This section revisits the three cases presented above to provide proof through numerical data and figures. This study presents each case again along with a brief explanation of the mathematical procedure. The numerical data gives further detail regarding how to achieve these data results as well as the solution sets for each result. The solution sets are Pareto efficient; therefore, the solution sets presented are the best conditions to satisfy each case.

Each parameter mentioned in Tables 3.1-3.11 is assigned a numerical value to analyze the relationship between them and Equations 6 and 7. The size of the order, Q, was given a domain anywhere from 1-250 units to show the gradual change of cost and emissions. Graphing Equations 6 and 7 show the linearity of cost and emissions functions. The size of the order, Q, is represented on the x-axis of each graph, while cost and emissions are represented on the y-axis. Graphing the cost (x-axis) against the emissions function (y-axis) yields the curve for the Pareto front solutions. Only the solutions ranging from the cost minimum point of the x-axis to the emissions minimum point of the y-axis are included in the Pareto front solutions. The Pareto optimal solutions represent different quantities that offer trade-offs between lowest cost and emissions.

3.1.1. Case 1. $\mathbf{Q^B \leq Q^E}$. This case shows the retailer’s optimal order size is Q^c with or without discount. Mathematically, $Q^c = argmin\ \{C(Q^C), C(Q^B)\}$. All possibilities under Case 1 reflect the buying power of the retailer because the emissions per unit time do not relate with the discount environment. In these situations, the retailer can thus only minimize cost per unit time.

- Case 1.1. If $Q^C \leq Q^E$, then PE = $[Q^C, Q^E]$. In order to achieve this condition, the following parameters and numerical data in Table 3.1 should be considered. From the data given in Table 3.1, Equations 6 and 7 can be graphed as shown in Figure 3.1. The Pareto optimal solutions represents the retailer’s optimal order size to achieve minimum cost and emissions as specified in the environment given. The solution set for this case is shown in Figure 3.2.
- Case 1.2. If $Q^B \leq Q^E < Q^C$, then PE = $[Q^E, Q^C]$. In order to achieve this condition, the following parameters and numerical data in Table 3.2 should be considered. From the data given in Table 3.2, Equations 6 and 7 can be graphed as shown in Figure 3.3. The Pareto optimal solutions represent the retailer’s optimal order size to achieve minimum cost and emissions as specified in the environment given. The solution set for this case is shown in Figure 3.4.
- Case 1.3. If $Q^E \leq Q^B$, then PE = $[Q^E, Q^C]$. In order to achieve this condition, the following parameters and numerical data in Table 3.3 should be considered. From the data given in Table 3.3, Equations 6 and 7 can be graphed as shown in Figure 3.5. The Pareto optimal solutions represent the retailer’s optimal order size to achieve minimum cost and emissions as specified in the environment given. The solution set for this case is shown in Figure 3.6.

Table 3.1. Numerical Data for Case 1.1.

Parameters	*Numerical Data*
q	1-120
D	600 units
A^c	120 ($/cycle)
h^c	50 ($/unit/year)
p^c	IF(*q*<30,5,3)
A^e	20 ($/kg em)
h^e	3 ($/kg em)
p^c	1 ($/kg em)

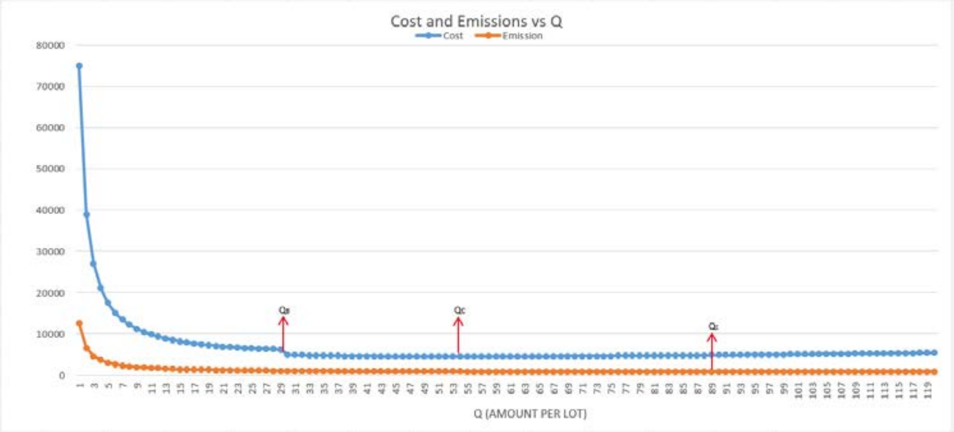


Figure 3.1. Numerical Cost and Emissions vs. Quantity for Case 1.1.

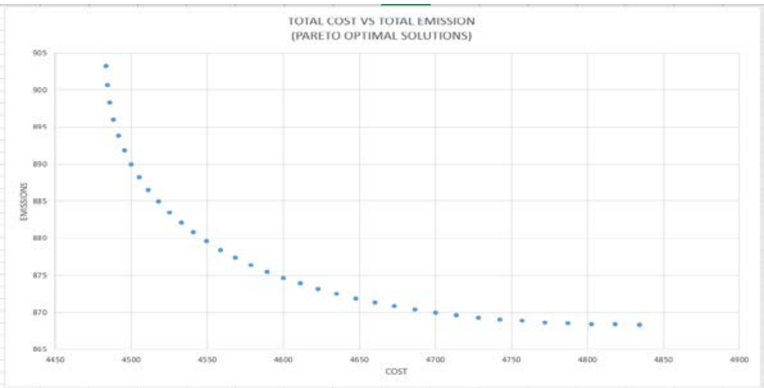


Figure 3.2. Numerical Cost vs. Emissions Pareto Front Solutions for Case 1.1.

Table 3.2. Numerical Data for Case 1.2.

Parameters	*Numerical Data*
q	1-250
D	600 units
A^c	50 ($/cycle)
h^c	2 ($/unit/year)
p^c	IF(q<75,5,3)
A^e	20 ($/kg em)
h^e	3 ($/kg em)
p^c	1 ($/kg em)

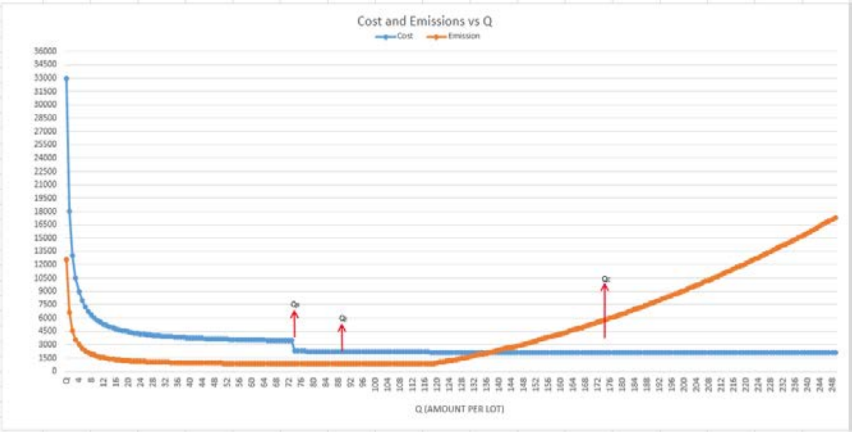


Figure 3.3. Numerical Cost and Emissions vs. Quantity for Case 1.2.

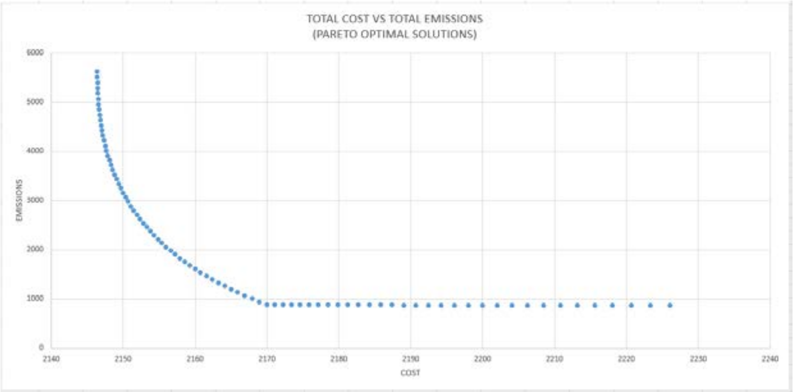


Figure 3.4. Numerical Cost vs. Emissions Pareto Front Solutions for Case 1.2.

Table 3.3. Numerical Data for Case 1.3.

Parameters	*Numerical Data*
q	1-200
D	600 units
A^c	50 ($/cycle)
h^c	2 ($/unit/year)
p^c	IF(*q*<100,5,3)
A^e	20 ($/kg em)
h^e	3 ($/kg em)
p^c	1 ($/kg em)

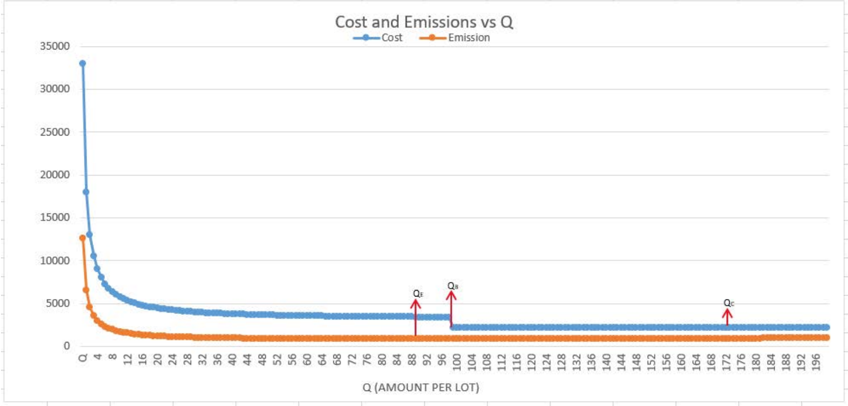


Figure 3.5. Numerical Cost and Emissions vs. Quantity for Case 1.3.

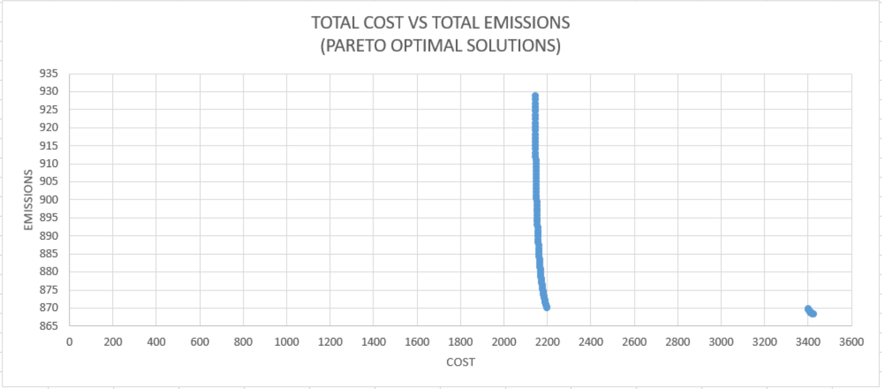


Figure 3.6. Numerical Cost vs. Emissions Pareto Front Solutions for Case 1.3.

In conclusion, the PE of Case 1 represents a continuous range of quantities in every subcase; however, the Pareto front is not a continuous curve for Case 1.3.

3.1.2. Case 2. $\mathbf{Q^C < Q^B}$ and $\mathbf{C(Q^B) < C(Q^C)}$. This case shows the retailer's optimal order size without the discount, where Q^C does not produce the same results as with the discount Q^B. Mathematically, $Q^B = argmin\ \{C(Q^C), C(Q^B)\}$. Thus, emissions will vary with the discount if the retailer only minimizes the cost per unit time.

- Case 2.1. If $Q^B \leq Q^E$, then $PE = [Q^B, Q^E]$. In order to achieve this condition, the following parameters and numerical data in Table 3.4 should be considered. From the data given in Table 3.4, Equations 6 and 7 can be graphed as shown in Figure 3.7. The Pareto optimal solutions represent the retailer's optimal order size to achieve minimum cost and emissions as specified in the environment given. The solution set for this case shown in Figure 3.8 proves a cost-minimizing retailer's emission per unit time will decrease with the discount.
- Case 2.2. If $Q^C \leq Q^E < Q^B$ and $Q^c \leq Q^1$, then $PE = (Q^1, Q^E] \cup \{Q^B\}$. Considering the conditions above, $Q^C \leq Q^E < Q^B$ a potential real-valued order quantity, $Q^1 \geq 0$ exists such that $Q^1 \leq Q^E$ and $C(Q^1) = C(Q^B)$. Case 2.2 shows a potential stance for Q^1 and a possible PE solution set. In order to achieve this condition, the following parameters and numerical data in Table 3.5 should be considered. From the data given in Table 3.5, Equations 6 and 7 can be graphed as shown in Figure 3.9. The Pareto optimal solutions represent the retailer's optimal order size to achieve minimum cost and

emissions as specified in the environment given. The solution set for this case below shown in Figure 3.10 proves a cost-minimizing retailer's emission per unit time will decrease with the discount.

- Case 2.3. If $Q^C \leq Q^E < Q^B$ and $Q^1 < Q^C$, then PE = $[Q^C, Q^E] \cup \{Q^B\}$. Considering the conditions above, $Q^C \leq Q^E < Q^B$ a potential real-valued order quantity, $Q^1 \geq 0$ exists such that $Q^1 \leq Q^E$ and C (Q^1) =C (Q^B). Case 2.3 shows a potential stance for Q^1 and a possible PE solution set. In order to achieve this condition, the following parameters and numerical data in Table 3.5 should be considered. From the data given in Table 3.6, Equations 6 and 7 can be graphed as shown in Figure 3.11. The Pareto optimal solutions represent the retailer's optimal order size to achieve minimum cost and emissions as specified in the environment given. The solution set for this case below shown in Figure 3.12 proves a cost-minimizing retailer's emission per unit time will increase with the discount.
- Case 2.4 If $Q^E < Q^C$, then PE = $[Q^E, Q^C] \cup \{Q^B\}$. In order to achieve this condition, the following parameters and numerical data in Table 3.7 should be considered. From the data given in Table 3.7, Equations 6 and 7 can be graphed as shown in Figure 3.13. The Pareto optimal solutions represent the retailer's optimal order size to achieve minimum cost and emissions as specified in the environment given. The solution set for this case shown in Figure 3.14 proves a cost-minimizing retailer's emission per unit time will increase with the discount.

Table 3.4. Numerical Data for Case 2.1.

Parameters	*Numerical Data*
q	1-150
D	600 units
A^c	5 ($/cycle)
h^c	2 ($/unit/year)
p^c	IF(*q*<75,6,3)
A^e	20 ($/kg em)
h^e	3 ($/kg em)
p^c	1 ($/kg em)

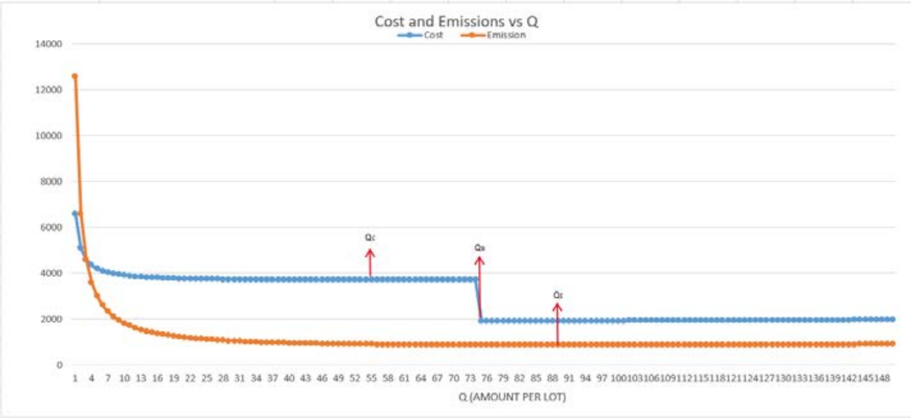


Figure 3.7. Numerical Cost and Emissions vs. Quantity for Case 2.1.

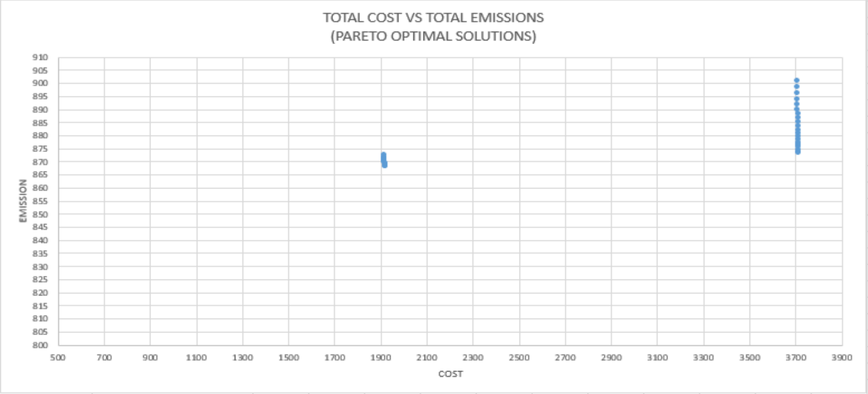


Figure 3.8. Numerical Cost vs. Emissions Pareto Front Solutions for Case 2.1.

Table 3.5. Numerical Data for Case 2.2.

Parameters	*Numerical Data*
q	1-150
D	600 units
A^c	2 ($/cycle)
h^c	200 ($/unit/year)
p^c	IF(*q*<100,6,3)
A^e	50 ($/kg em)
h^e	20 ($/kg em)
p^c	1 ($/kg em)

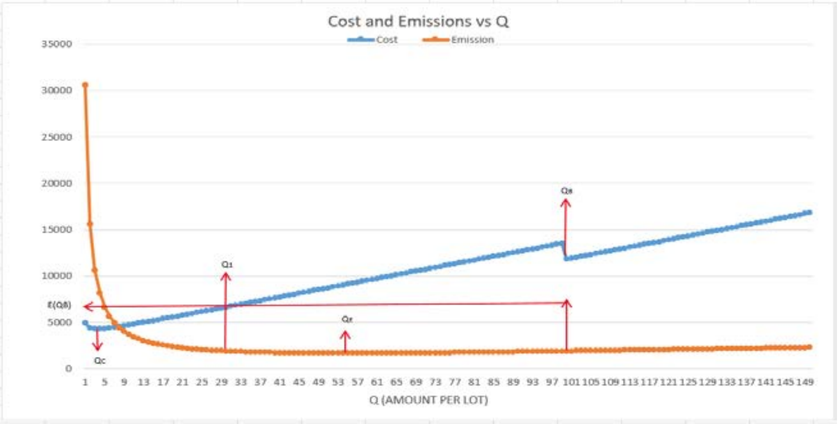


Figure 3.9. Numerical Cost and Emissions vs. Quantity for Case 2.2.

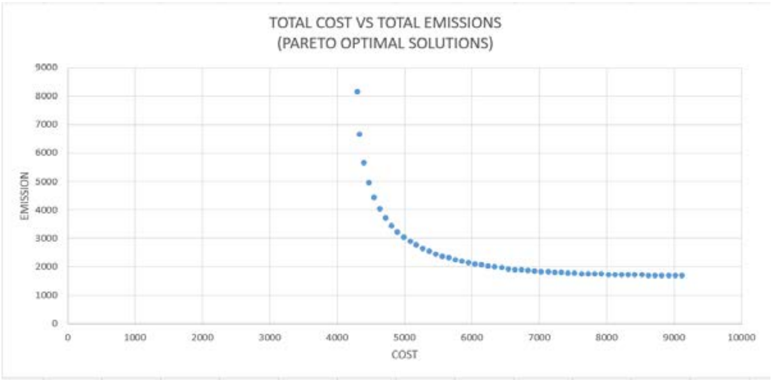


Figure 3.10. Numerical Cost vs. Emissions Pareto Front Solutions for Case 2.2.

Table 3.6. Numerical Data for Case 2.3.

Parameters	*Numerical Data*
q	1-150
D	600 units
A^c	45 ($/cycle)
h^c	75 ($/unit/year)
p^c	IF(*q*<100,6,3)
A^e	75 ($/kg em)
h^e	50 ($/kg em)
p^c	1 ($/kg em)

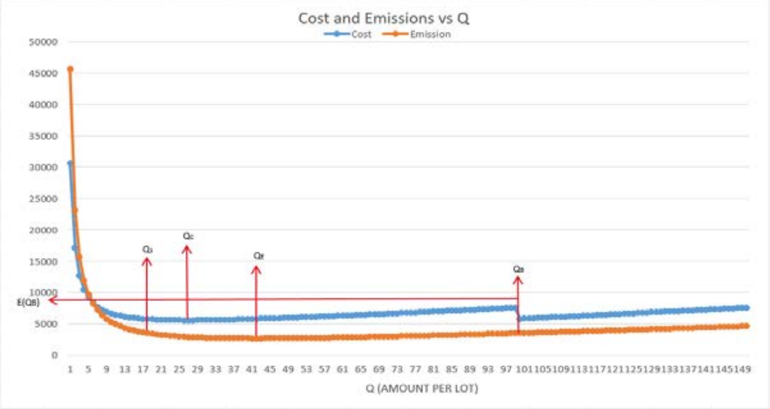


Figure 3.11. Numerical Cost and Emissions vs. Quantity for Case 2.3.

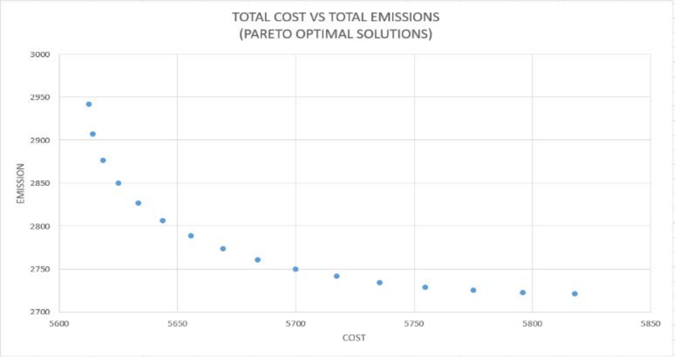


Figure 3.12. Numerical Cost vs. Emissions Pareto Front Solutions for Case 2.3.

Table 3.7. Numerical Data for Case 2.4.

Parameters	*Numerical Data*
q	1-150
D	600 units
A^c	300 ($/cycle)
h^c	500 ($/unit/year)
p^c	IF(q<100,6,3)
A^e	40 ($/kg em)
h^e	25 ($/kg em)
p^c	1 ($/kg em)

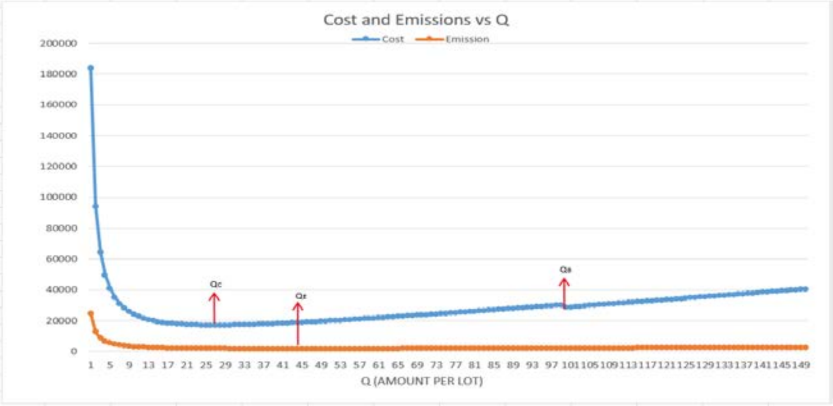


Figure 3.13. Numerical Cost and Emissions vs. Quantity for Case 2.4.

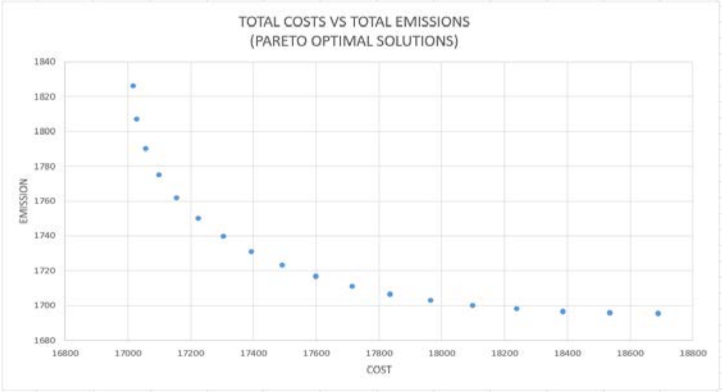


Figure 3.14. Numerical Cost vs. Emissions Pareto Front Solutions for Case 2.4.

In conclusion, under Case 2, the entire Pareto front is a continuous curve except for Case 2.1.

3.1.3. Case 3. $\mathbf{Q^{C} < Q^{B}}$ and $\mathbf{C(Q^{B}) \geq C(Q^{C})}$. This case shows the retailer's optimal order size with or without the discount, Q^C. Mathematically, Q^C=*argmin* $\{C(Q), C(Q)\}$. Thus, emissions will not vary with the discount if the retailer only minimizes the cost per unit time.

- Case 3.1. If $Q^B \leq Q^E$, then *PE* = $[Q^C, Q^3) \cup [Q^B, Q^E]$. In order to achieve this condition, the following parameters and numerical data in Table 3.8 should be considered. From the data given in Table 3.8, Equations 6 and 7 can be graphed as shown in Figure 3.15. The Pareto optimal solutions represent the retailer's optimal order size to achieve minimum cost and emissions as specified in the environment given. The solution set for this case shown in Figure 3.16 proves a cost-minimizing retailer's emission per unit time does not change with Q^C with or without the discount.
- Case 3.2. If $Q^C \leq Q^E < Q^B$ and $Q^3 \leq Q^1$, then *PE* = $[Q^C, Q^3) \cup (Q^1, Q^E] \cup \{Q^B\}$. Considering, $Q^C \leq Q^E < Q^B$, and recalling from Case 2, there is a potential real-valued nonnegative Q^2 and Q^3 that exists, such that C(Q^2)=C(Q^3)=C(Q^B) and $Q^2 \leq Q^C \leq Q^3$. This case presents one stance for Q^1. In order to achieve this condition, the following parameters and numerical data in Table 3.9 should be considered. From the data given in Table 3.9, Equations 6 and 7 can be graphed as shown in Figure 3.17. The Pareto optimal solutions represent the retailer's optimal order size to achieve minimum cost and emissions as specified in the environment given. The

solution set for this case shown in Figure 3.18 proves a cost-minimizing retailer's emission per unit time does not change with Q^{C} with or without the discount.

- Case 3.3. If $Q^C \leq Q^E < Q^B$ and $Q^1 < Q^3$, then PE = $[Q^C, Q^E]$. Considering, $Q^C \leq Q^E < Q^B$, and recalling from Case 2, there is a potential real-valued nonnegative Q^2 and Q^3 that exists, such that C (Q^2)=C (Q^3)=C (Q^B) and $Q^2 \leq Q^C \leq Q^3$. This case presents another stance for Q^1. In order to achieve this condition, the following parameters and numerical data in Table 3.9 should be considered. From the data given in Table 3.10, Equations 6 and 7 can be graphed as shown in Figure 3.19. The Pareto optimal solutions represent the retailer's optimal order size to achieve minimum cost and emissions as specified in the environment given. The solution set for this case shown in Figure 3.20 proves a cost-minimizing retailer's emission per unit time does not change with Q^C with or without the discount.
- Case 3.4. If $Q^E < Q^C$ then, PE = $[Q^E, Q^C]$. In order to achieve this condition, the following parameters and numerical data in Table 3.11 should be considered. From the data given in Table 3.11, Equations 6 and 7 can be graphed as shown in Figure 3.21. The Pareto optimal solutions represent the retailer's optimal order size to achieve minimum cost and emissions as specified in the environment given. The solution set for this case shown in Figure 3.22 proves a cost-minimizing retailer's emission per unit time does not change with Q^C with or without the discount.

Table 3.8. Numerical Data for Case 3.1.

Parameters	*Numerical Data*
q	1-150
D	400 units
A^c	35 (\$/cycle)
h^c	700 (\$/unit/year)
p^c	IF(*q*<40,5,3)
A^e	7 (\$/kg em)
h^e	2.5 (\$/kg em)
p^c	1 (\$/kg em)

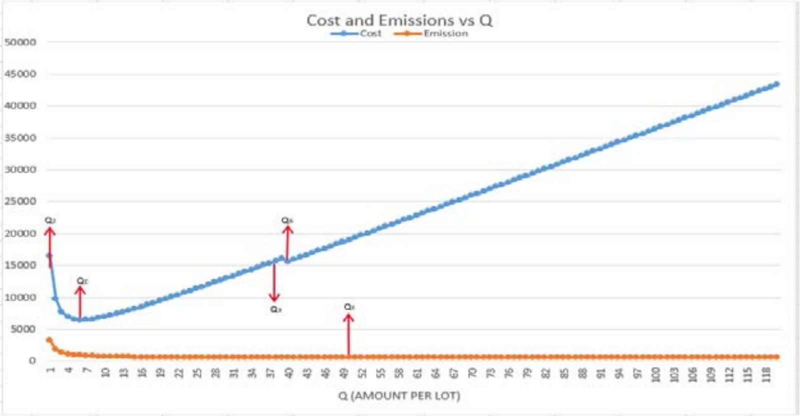


Figure 3.15. Numerical Cost and Emissions vs. Quantity for Case 3.1.

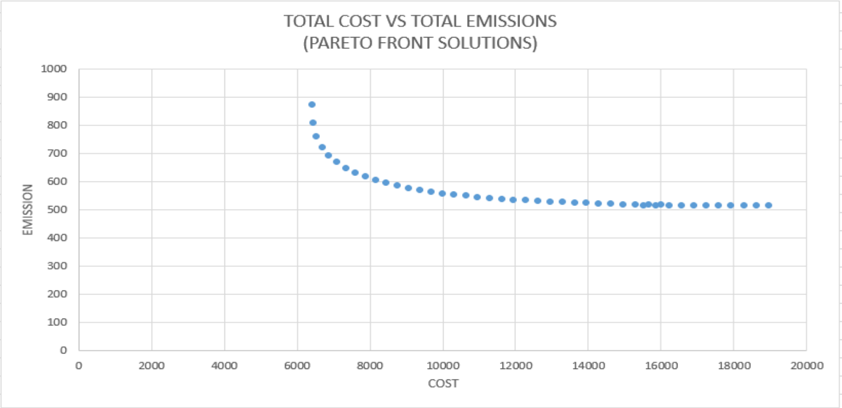


Figure 3.16. Numerical Cost vs. Emissions Pareto Front Solutions for Case 3.1.

Table 3.9. Numerical Data for Case 3.2.

Parameters	*Numerical Data*
q	1-120
D	400 units
A^c	35 ($/cycle)
h^c	700 ($/unit/year)
p^c	IF(*q*<40,6,3)
A^e	4.25 ($/kg em)
h^e	2.25 ($/kg em)
p^c	1 ($/kg em)

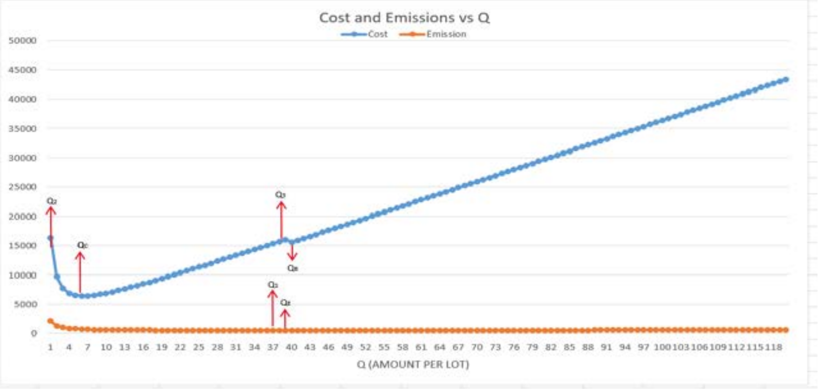


Figure 3.17. Numerical Cost and Emissions vs. Quantity for Case 3.2.

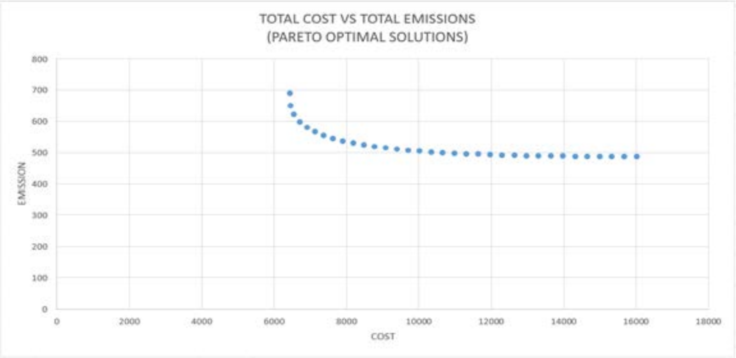


Figure 3.18. Numerical Cost vs. Emissions Pareto Front Solutions for Case 3.2.

Table 3.10. Numerical Data for Case 3.3.

Parameters	*Numerical Data*
q	1-120
D	400 units
A^c	35 ($/cycle)
h^c	700 ($/unit/year)
p^c	IF(q<40,5,3)
A^e	4 ($/kg em)
h^e	2.5 ($/kg em)
p^c	1 ($/kg em)

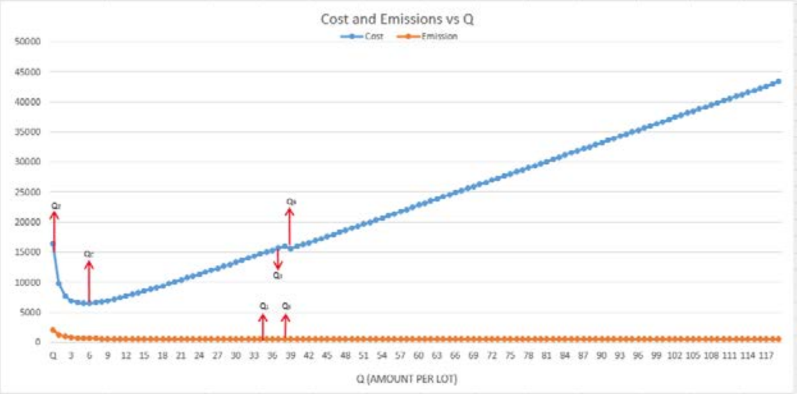


Figure 3.19. Numerical Cost and Emissions vs. Quantity for Case 3.3.

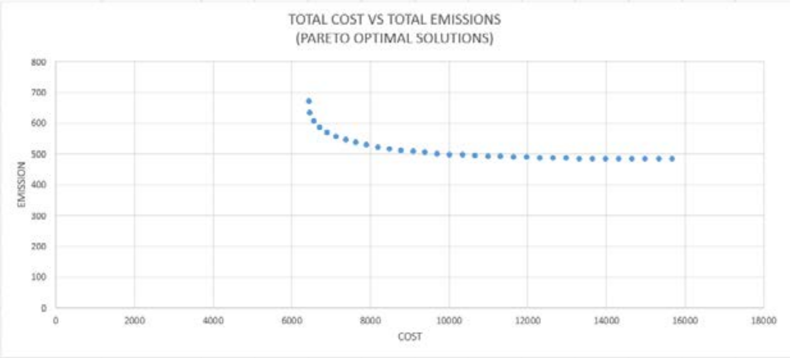


Figure 3.20. Numerical Cost vs. Emissions Pareto Front Solutions for Case 3.3.

Table 3.11. Numerical Data for Case 3.4.

Parameters	*Numerical Data*
q	1-120
D	400 units
A^c	100 ($/cycle)
h^c	400 ($/unit/year)
p^c	IF(*q*<40,5,3)
A^e	0.25 ($/kg em)
h^e	15 ($/kg em)
p^c	1 ($/kg em)

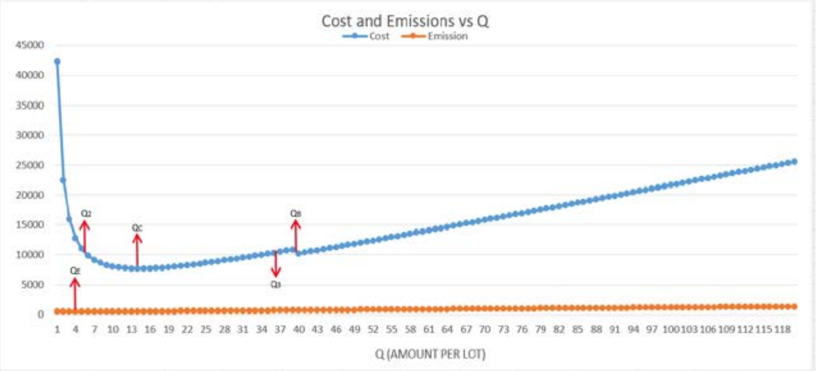


Figure 3.21. Numerical Cost and Emissions vs. Quantity for Case 3.4.

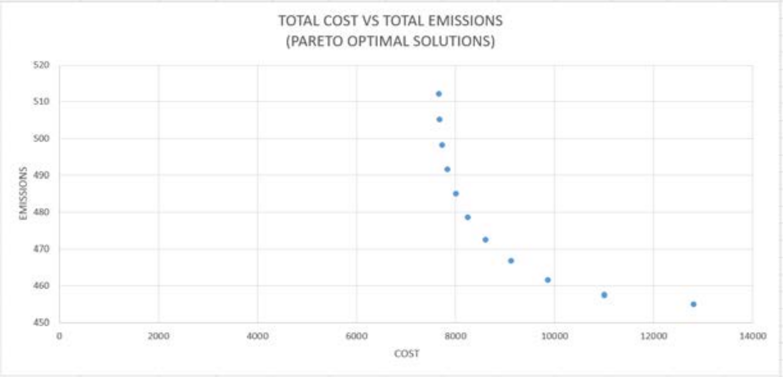


Figure 3.22. Numerical Cost vs. Emissions Pareto Front Solutions for Case 3.4.

In conclusion, the Pareto front is not a continuous curve for Case 3.1, although Cases 3.2-3.4 present a continuous curve for their *PE* solutions. The discount offered did not change the retailer’s cost-minimizing order quantities. Overall implications of the case are discussed within the next section.

4. CONCLUDING REMARKS

4.1. IMPLICATIONS OF THE CASE

This study investigated the retailer’s buying options in the presence of quantity discounts with environmental considerations. The ultimate goal of this study is to determine whether retailers can decrease their cost and emissions using quantity discounts from a supplier. Using a bi-objective EOQ model, the objective of the study yielded several cases that characterized the cost and emissions minimizing objectives when an all-units quantity discount was applied. A solution set of Pareto efficient order quantities explicitly state the retailer’s buying power in each case in the presence of a single discount. The results yield 11 different cases (Cases 1.1-3.4), which were analyzed to present the best buying option. Furthermore, the effects of a discount on a cost-minimizing retailer’s emission was shown through the solution set provided. The following implications are drawn from the results of this study.

4.1.1. The Discount Does Not Affect the Environment. In several instances, the discount did not affect the retailer’s buying power. For example, in Case 1 and Case 3, the discount did not change the cost-minimizing retailer’s order quantity, meaning the cost-minimizing retailer’s optimum order quantities are the same with and without the discount as expected. Along the same trend, the retailer’s cost-minimizing emissions did not change as well. This occurs when the retailer’s optimum order quantity is large enough to qualify for the discount, thereby minimizing cost as in Case 1. This trend is also seen in the reverse, such that the retailer’s optimum order quantity is very small, so small that prices do not reduce because the quantity does not qualify for the discount. The order quantity here thus serves as the smallest price point as seen in Case 3.

4.1.2. The Changing Discount Environment. The discount changes the retailer's cost-minimizing order quantity in several instances. Furthermore, the results showed that when the discount changed the cost-minimizing order quantity, the emissions changed as well. More specifically, the retailer's optimum order quantity before the discount will result in higher cost premiums. The cost-minimizing order quantity is achieved when the retailer's order quantity qualifies for a discount, thus reducing the cost at the discount breakpoint, as in Case 2. Furthermore, this change can either decrease emissions (Cases 2.1 and Cases 2.2) or increase emissions (Cases 2.3 and 2.4). In some instances, when the cost is minimized and the retailer takes advantage of quantity discounts, the emissions will not change. More specifically, when E $(\boldsymbol{Q^C})$=C $(\boldsymbol{Q^B})$ and $\boldsymbol{Q^C = Q^1}$, the emissions will not change even if the cost is minimized.

BIBLIOGRAPHY

[1] Benjaafar, S., Li Y., and Daskin, M., 2013, “Carbon Footprint and the Management of Supply Chains: Insights from Simple Models,” IEEE Transcations on Automation Science and Engineering, 10 (1), 99-116.

[2] Toptal, A., Ozlu, H., and Konur, D., 2014, “Joint Decisions on Inventory Replenishment and Emission Reduction Investment Under Different Emission Regulations,” International Journal of Production Research, 52(1), 243-269.

[3] Konur, D., Campbell, J.F., and Monfared, S.A., 2017, “Economic and Environmental Considerations In A Stochastic Inventory Control Model With Order Splitting Under Different Delivery Schedules Among Supplier,” Omega, 71, 46-65.

[4] Keskin, B.B., and Capar, I., 2014, “The Utility of EOQ in Supply Chain Design and Operation,” In Handbook of EOQ Inventory Control Problems: Stochastic and Deterministic Models and Applications, International Series in Operations Research and Management Science, Vol. 197, 221-245. Berlin: Springer.

[5] Arslan, M.C., and Turkay, M., 2013, “EOQ Revisited with Sustainability Considerations,” Foundations of Computing and Decision Sciences, 38(4), 223-49.

[6] Chen, X., Benjaafar S., and Elomri, A., 2013, “The Carbon Constrained EOQ,” Operations Research Letters, 41(2), 172-9.

[7] Hua G., Cheng, T., and Wang, S., 2011, “Managing Carbon Footprints in Inventory Management,” International Journal of Production Economics, 132(2), 178-85.

[8] Konur, D., 2014, “Carbon Constrained Integrated Inventory Control ad Truckload Transportation with Heterogeneous Freight Trucks,” International Journal of Production Economics, 153, 268-79.

[9] Konur, D., and Schaefer, B., 2014, “Integrated Inventory Control and Transportation Decisions Under Carbon Emissions Regulations: LTL vs. TL carriers,” Transportation Research Part E: Logistics and Transportation, 68, 14-38.

[10] Bonney, M., and Jaber M.Y., 2011, “Environmentally Responsible Inventory Models: Non-Classical Models for a Non-Classical Era,” International Journal of Production Economics, 133(1), 45-53.

[11] Bouchery, Y., Ghaffari, A., Jemai, Z., and Dallery, Y., 2012, “Including Sustainability Criteria Into Inventory Models,” European Journal of Operational Research, 222 (2), 229-40.

[12] Schaefer, B., and Konur, D., 2014, “Joint Replenishment Problem with Carbon Emissions Constraint,” Proc. Of the IIE Annual Conference, May 31-June 3, Montreal, QC, Canada, 1950-1958.

[13] Konur, D., Schaefer, B., 2016, “Economic and Environmental Comparison of Grouping Strategies in Coordinated Multi-Item Inventory Systems,” Journal of the Operational Research Society, 67, 421-436.

[14] Absi, N., Dauzere-Peres, S., Kedad-Sidhourn, S., Penz B., and Rapine, C., 2013, “Lot Sizing with Carbon Emission Constraints,” European Journal of Operational Research, 227(1), 55-61.

[15] Schaefer, B., Konur, D., 2015, “Economic and Environmental Considerations in a Continuous Review Inventory Control System with Integrated Transportation Decisions,” Transportation Research Part E: Logistics and Transportation Review, 80,142-165.

[16] Konur, D., Campbell, J.F., and Monfared, S.A., 2017, “Effects of Sourcing Decisions in a (Q, R) Model Under Carbon Trading,” Proc. Of the IIE Annual Conference, May 20-23, Pittsburg, Pennsylvania, 585-590.

[17] Hoen, K.M.R, TanT., Fransoo, J.C., and van Houtum, G.J., 2014, “Effect of Carbon Emission Regulations on Transport Mode Selection Under Stochastic Demand,” Flexible Services and Manufacturing Journal, 26(1),170-195.

[18] Manikas, A.S., and Kroes, J.R., 2015, “A Newsvendor Approach to Compliance and Production Under Cap and Trade Emissions Regulation,” International Journal of Production Economics, 159, 274-284.

[19] Konur, D., and Toptal, A., 2012, “Analysis and Applications of Replenishment Problems Under Stepwise Transportation Costs and Generalized Wholesale Prices,” International Journal of Production Economics, 140(1), 521-529.

[20] Konur, D., 2018, “An EOQ Model with Quantity Discounts and Environmental Objective,” Proc. Of the IISE Annual Conference, May 19-22, Orlando, Florida.

[21] Chopra, S., and Meindl, P., 2016, “Supply Chain Management-Strategy, Planning, and Operation”, Sixth Edition Textbook, 1-40, 267-370.

VITA

Tiffanie Marie Toles was born in Saint Louis, MO. As a young child, she was interested in science and mathematics. Earning the highest grades of her class and showing her eagerness to learn allowed her to join the REACH program of Saint Louis. Through this program, she was offered an opportunity to extend her learning in other academic applications such as critical thinking and pre-engineering courses. In high school, she knew she wanted to become an engineer and continued to take rigorous math and science courses. She joined engineering clubs to maintain her skills and enrolled in engineering courses that qualified for college credit through the PLTW (Project Lead the Way) Program. Due to her extensive background and hard work, she earned the Gates Millennium Scholarship in 2011, which allowed her to attend any college she wanted to attain graduate and postgraduate degrees. She chose to further her scholastic journey at Missouri University of Science and Technology. Maintaining over a 3.0 GPA, she earned her bachelor's degree in petroleum engineering in May 2016 from Missouri S&T. After several internships and co-ops, she realized her passion in engineering relates to supply chain management. She decided to continue her education and received a Master of Science in engineering management, as well as a certificate in project management from Missouri S&T in May 2018.