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SHORTEST-DISTANCE AND MINIMUM-COST SELF-CHARGING PATH

PROBLEMS: FORMULATIONS AND APPLICATION

by

MARC MONROE TEETER

A THESIS

Presented to the Faculty of the Graduate School of the
MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

In Partial Fulfillment of the Requirements for the Degree
MASTER OF SCIENCE IN ENGINEERING MANAGEMENT

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Approved by

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ABSTRACT

In this study, self-charging paths for an electric bus are analyzed. Wireless-power-transfer technologies, when integrated on a road network, enable dynamic charging of electric vehicles. Roads implemented with a wireless-power-transfer technology are referred to as electric-roads in this study. Electric vehicles traversing on electric-roads, therefore, can be dynamically charged. This can further eliminate the need for static charging, i.e., the electric vehicle will not need to stop for charging.

This thesis analyzes the design of transit routes for an electric-bus so that the electric-bus is charged by only electric-roads. Specifically, the focus is on designing a path, which passes through a set of bus-stops, between an origin and a destination, such that the electric-bus travelling on this path does not need static charging. A path, on which the electric-bus does not need static charging, is referred to as a self-charging path.

First, the shortest-distance self-charging path problem with node visiting constraints, which represent the bus-stop requirements, is introduced. A network optimization model is formulated for the shortest-distance self-charging path problem with node-visiting constraints and a sequence-based solution approach is discussed. Next, the minimum-cost self-charging path problem with node visiting constraints is introduced. A network optimization model is formulated for the minimum-cost self-charging path problem with node-visiting constraints and a sequence-based solution approach is discussed. Both the shortest-distance and minimum-cost self-charging problems are illustrated using the electric-bus shuttling the Missouri University of Science and Technology campus. In solving these problems for this application, sequence-based solution approaches are used.

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ABBREVIATION LIST

E-road: Electric Road

E-Bus: Electric Bus

SD-SC-P-P: Shortest-Distance Self-Charging Path Problem

MC-SC-P-P: Minimum Cost Self Charging Path Problem

OLEV: Online Electric Vehicles

WPT: Wireless Power Transfer

EV: Electric Vehicle

VOT: Value of Time

1. INTRODUCTION

With the increasing concerns on climate change, the deployment of alternative fuel vehicles in personal, public, and commercial transportation is increasing. Transit agencies are noted to lead the way in using such clean transportation technologies [1]. For instance, American Public Transportation Association [2] notes that more than 40% of the transit buses use alternative fuels (other than diesel and gasoline) [3]. Specifically, among alternative fuel vehicles, all-electric buses (e-buses) are attractive options as electricity is readily available and e-buses have economical, safety, and environmental advantages. Furthermore, recent technological advancements such as longer-range batteries and fast-charging stations directly address transit needs, which will enhance the deployment of e-buses in transit.

Among these technologies, wireless-power-transfer (WPT) technology is a new advancement that enables dynamic charging, i.e., electric vehicles (EVs) being charged while being driven. Specifically, integrated on the roadways, WPT technologies can charge an EV traversing the roadway. A roadway integrated with a WPT technology is referred to as an electric-road (e-road) in this study. Particularly, we adopt the following definition of e-road from [3]: “The electric road is defined as a transportation infrastructure that is able to deliver the electric power to charge electric vehicles efficiently while stationary or in motion, using specific conductive or contactless charging systems.” Figure 1.1 illustrates the basics of WPT technology for charging an electric-bus (e-bus).

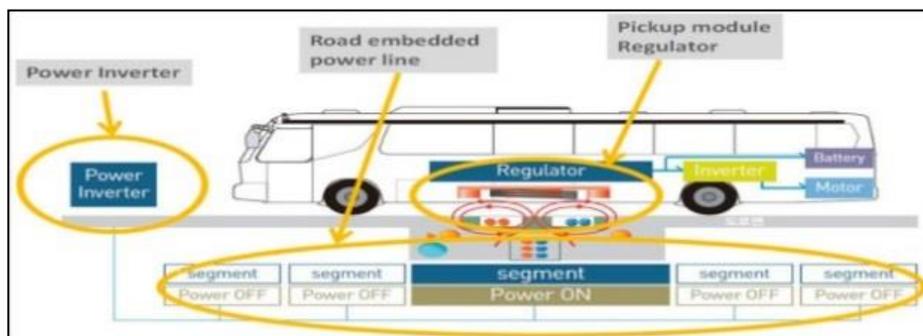


Figure 1.1 Wireless-Power-Transfer system of On-line Electric Vehicles (OLEV). Source [4].

WPT technology has been around since Nikola Tesla's initial work and the further research performed by Soljacic on WPT systems via magnetic resonances has helped bring more attention to the concept of dynamic charging [5]. With the recent advancements in WPT technology, the increasing use of EVs, and the concerns on rising prices in lithium batteries, the application WPT technologies in transportation is becoming more feasible.

There are two types of WPT: Static WPT and Dynamic WPT. Static WPT is a wireless charging system that can wirelessly charge an EV while the EV is stationed on a WPT unit such a wireless charging pad. On the other hand, dynamic WPT is a wireless charging system that can wirelessly charge an EV while the EV is in motion on a WPT unit such as a charging lane. Charging lanes are also known as an e-road as previously described. Both the static and the dynamic WPT systems are beneficial as there is no plugging in and the EV can be charged without the driver getting out of the vehicle. This eliminates the need to have a plug in for the EV and further eliminates the possibility of time loss due to having various plug styles. The main differences between static and dynamic WPT systems are the placement and timing needed to charge the EV. A static WPT system requires the EV to be stationary during charging, which adds time, whereas a

dynamic WPT system allows the EV to be in motion during charging, hence, does not add time. In addition, a static WPT system (such as a charging pad) should be placed in a specific spot on the transportation network, whereas a dynamic WPT system (such as a charging lane) can be placed on a road segment at a specified length considering the charging requirements. Figure 1.2 represent dynamic and static WPT systems for charging an e-bus.

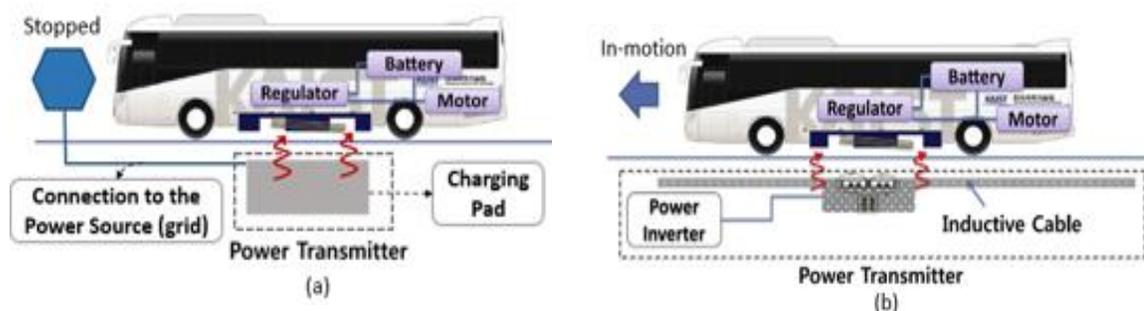


Figure 1.2 WPT For and E-Bus. (a) Static WPT and (b) Dynamic WPT. Source [6].

Currently, there are pilot studies that reflect and take into consideration of the static and dynamic WPT systems. The best documented pilot study for dynamic WPT testing is the On-Line Electric Vehicle (OLEV) system in South Korea designed by the researchers of the Korea Advanced Institute of Science and Technology [4-8]. Other studies investigating the use of WPT technologies in transportation applications are reviewed in the literature review. All these studies demonstrate the viability of using dynamic, as well as static, WPT technologies in transportation applications. Especially, considering the increasing deployment of e-buses in transit applications, analyzing the use of WPT technologies in transit applications is crucial.

This thesis addresses the integrated network design and operational planning decisions for an e-bus on a network that can be implemented with a dynamic WPT technology. In particular, routing decisions and e-road integration decisions are jointly determined for an e-bus that should visit a set of bus stops. In determining the routing decisions, it is considered that the e-bus should be charged completely by the e-roads; i.e., the e-bus will not require static charging. A path is referred to as a self-charging path if the e-bus continuously traveling on this path does not need to stop for battery charging. Two self-charging path problems are formulated: shortest-distance self-charging problem and minimum-cost self-charging problem. For each problem, a network optimization model is presented in Section 3 and an application along with a sequence-based solution approach are discussed in Section 4. Next section presents the review of the related literature.

2. LITERATURE REVIEW

To cut back on pollution, auto makers designed the hybrid vehicle, which is a combustion engine that can run off of an electric battery [7]. This design helps partially cut the pollution caused by combustible engines and, the next step to cut emissions completely from vehicles is to make all vehicles EVs. The main issues with EVs are their distance limitation due to battery capacities and the time required for charging the EVs. Both of these issues can be overcome with WPT technologies. Especially, dynamic WPT technologies, i.e., e-roads, can wirelessly charge EVs while EVs are in motion. Therefore, dynamic WPT eliminates the idle time required for charging. Furthermore, since the EVs can be charged while in motion, they can continuously operate, which eliminates the driving range limitation.

The most established work on using dynamic WPT technologies in transit applications is the Online Electric Vehicle (OLEV). An OLEV is an EV that is able to charge wirelessly via an e-road. The charging efficiency of an e-road depends on the output power and air gap, which are discussed in detail in [7]. These are to be taken into consideration when designing and developing a working OLEV system.

Another consideration that needs to be taken into consideration is the type of charging system the EV can use for wireless charging. As noted before, there are two types of WPT: static and dynamic. The way that the EV would be charged is dependent on what system is being used. In a static (or stationary) WPT system, charging is via a paddle transducer and has a higher efficiency than a dynamic WPT system [8]. While a dynamic WPT system charges an EV while it is moving over a power track (charging lane), i.e., an

e-road [8]. The first public transit system to put dynamic WPT into use is in Gumi City in South Korea [8].

The research in [8] accounts for how much an EV battery size can be reduced when e-road is used for the OLEV system. In another study, [9] looks at a commercialized OLEV system that is designed for shuttle buses looping in an amusement park. This study focuses on maintaining the battery level for the e-bus as it travels on its loop. The power comes from power transmitters and the e-bus either uses the power while it is driving or stores it in the battery for later use [9]. The paper takes a closer look at where and how to lay the power transmitter lines (e-roads) on the road system to maximize the active charging time while minimizing the overall length of the power transmitter line needed. For instance, they consider locations such as a bus stop where it is known that the e-bus will stop for a specific amount of time and will slow down as approaching to the stop as well as slowly speed up when leaving the stop. This allows a section of e-road integrated at the bus stop so as to utilize the e-road better by allowing more charging time for the e-bus.

In another study [10], authors investigate integrating not only a dynamic WPT system but also having a static WPT system and plug-in stations. They examine creating a way to help the government on installing the various types of re-charging stations (dynamic WPT, static WPT, and plug-in stations) while minimizing the total costs. In [10], the authors not only optimize the problem in a set example, but also account for the class of the vehicle and finding a price that would work with the consideration of the demand to use the specific charging system. The class of the vehicle is pertinent due to the need for higher torque on bigger vehicles, which require a higher amount of energy to move. The research in [10] also regards the development dynamic WPT systems (e-roads) and

people's willingness to pay a higher price to charge their vehicles while driving on the e-roads rather than paying a lower price at a static WPT system or a plug-in station.

In a recent study, Fuller [11] studies creating an EV road system that allows EVs to travel to and from popular cities in California. The paper focuses on the range of the EV, the amount of power used to recharge the vehicles, and the vehicle electrification. The study takes into consideration that EVs satisfy only 95% of the travel needs of people [11]. They focus on the other 5% of the travel needs for the users of the EVs. Given the range that a dynamic WPT system can emit power to the EVs and the range the EVs themselves can travel before needing to recharge were the factors on estimating a cost to implement e-road. The overall results showed that, with a dynamic WPT charging at 100kW and an EV capable of 200 miles on their battery, only 241 miles (4.9%) out of the 4891 miles of roadway considered would need to be integrated with dynamic WPT in the roadway. This is also considering that people would stop at least twice for 27 minutes on a trip from Sacramento to Los Angeles [11] and, at these stops, the EVs can be charged via a plug-in station or a static WPT system. With these numbers and the consideration of volume of EVs on the roadway, Fuller [11] was able to estimate costs for implementation and the price to be charged to the customers.

The other recent studies [12] and [13], in addition to considering cost of implementing the dynamic WPT charging systems and the static WPT charging systems, calculate what value those two systems as well as a third system known as battery swap have. In fact, Tesla has provided the solution for a battery swap via a network of battery switching stations [12]. In [12], the focus is solving an EV touring problem, which is a generalization of a traveling salesman problem, with a battery swap station plan. This

problem takes into consideration the starting point, the destination, and the battery capacity. The goal is to find the shortest path from origin to destination while making sure that the EV makes it to the battery swapping stations before the EV runs out of power. [13] builds on this concept with the thought of placing a dynamic WPT charging system and also having in place stationary charging systems (either static WPT system or plug-in charging station) along a specific corridor. The paper focuses on a corridor and has scenarios on what the cost would be for an integrated system that has both dynamic and static charging stations. It looks at what the cost would be if the infrastructure was provided by private or public funds. It also takes into consideration of the value of time for the vehicle. For example, a delivery truck has a higher value of time than a leisurely driver. So, the delivery truck would be willing to pay a higher price for electricity than the other driver for they have a higher value of time also known as VOT [13]. The VOT has an intricate way in dictating how much of the corridor will have e-roads integrated. The higher the VOT the more that customer would be willing to pay for the power from the e-road. This is because even if the cost to implement and deliver power via the e-road is higher than a static WPT charging system and the efficiency of an e-road is lower than a stationary WPT [8], e-roads save time by eliminating idle time spent for charging.

3. PROBLEM FORMULATION

In this section, two classes of self-charging path problems are introduced: shortest-distance self-charging problems and minimum-cost self-charging problems. At this point, it is important to define a self-charging path. A self-charging path on a network is a path integrated with e-roads such that the total energy needed by the EV travelling on the path can be charged by the implemented e-roads on the path. Therefore, an EV travelling on a self-charging path will not need to spend time for charging; the charging is achieved from the e-roads on the path while the EV travels on the path. As the applications of the models will be for an e-bus, the EV considered in this study is an e-bus.

First, in Section 3.2., the shortest-distance self-charging path problems are introduced and modeled. These problems aim to determine the shortest-distance path from an origin to a destination on a directed network (with and without node visiting constraints) and the road segments to be integrated with dynamic WPT technology (i.e., the arcs that will be integrated with e-roads) so that the e-bus can continuously travel on the path without a need for static charging. Then, in Section 3.3., the minimum-cost self-charging path problems are introduced and modeled. These problems aim to determine the minimum-cost path from an origin to a destination on a directed network (with and without node visiting constraints) and the road segments to be implemented with electric-roads so that the path is self-charging. Prior to formulation details, the problem settings that are common to both self-charging path problems are explained next in Section 3.1. Section 3.1. further gives the formal definition of a self-charging path.

3.1. PROBLEM SETTINGS AND SELF-CHARGING PATH

Consider a directed network with $|N|$ nodes and $|A|$ arcs, such that N is the set of nodes and A is the set of directed arcs. Let the nodes indexed by i such that $i \in N$ and let the arcs be defined as (i, j) such that $\text{arc } (i, j) \in A$. Here, it is assumed that an e-bus is being operated on the network. Furthermore, let $d_{(i,j)}$ be the length of arc (i, j) and $e_{(i,j)}$ be the energy consumed by the e-bus for traversing the arc (i, j) . An e-road technology (i.e., a dynamic WPT charging system) is available to be integrated on the network. It is simply assumed that r denotes the amount of energy charged to the e-bus per unit distance travelled on the e-road. For instance, if arc $(i, j) \in A$ is fully implemented with e-road, the total energy that can be charged to the e-bus traversing this arc will be $rd_{(i,j)}$. The cost of unit length of the e-road is considered to be w .

Suppose that a path is to be determined from an origin, node $o \in N$, to a destination, node $d \in N$, is to be determined. A path is defined by the set of arcs selected between the origin and destination. Therefore, let the path decisions be defined as $x_{(i,j)} = 1$ if arc $(i, j) \in A$ is selected to be on the path, and $x_{(i,j)} = 0$ otherwise. Note that the total energy consumed by the e-bus on a path defined by $x_{(i,j)}$ values will be $\sum_{(i,j) \in A} x_{(i,j)} e_{(i,j)}$. Furthermore, with the use of dynamic WPT technologies, the e-bus will be charged by e-roads to be integrated on the arcs of the network. Let $y_{(i,j)}$ be the length of e-road integrated on arc $(i, j) \in A$. Note that one should have $y_{(i,j)} \leq d_{(i,j)}$ because e-road on an arc cannot exceed the length of the arc. Furthermore, since e-roads will not be integrated on arcs that are not on the path, one can restrict $y_{(i,j)} \leq d_{(i,j)} x_{(i,j)}$. Given that $y_{(i,j)} \leq d_{(i,j)} x_{(i,j)}$, the

total energy that can be charged to the e-bus on the path defined by $x_{(i,j)}$ values will then be equal to $\sum_{(i,j) \in A} r y_{(i,j)}$.

Self-charging Path: Given $x_{(i,j)}$ and $y_{(i,j)}$ values $\forall (i,j) \in A$ such that $x_{(i,j)}$ values define a path from node $o \in N$ (origin) to node $d \in N$ (destination) and $y_{(i,j)} \leq d_{(i,j)} x_{(i,j)}$, the path defined by $x_{(i,j)}$ values is a self-charging path as long as $\sum_{(i,j) \in A} x_{(i,j)} e_{(i,j)} \leq \sum_{(i,j) \in A} r y_{(i,j)}$.

That is, a path is a self-charging path if the energy consumed by the e-bus while traversing the arcs on the path is less than or equal to the energy that can be charged by the e-roads integrated on the path arcs. Indeed, one can show that, given the battery capacity of the e-bus is sufficient to travel the path once without any charging (which is true in most practical cases as bus routes do not typically exceed 15 miles and e-buses have a range more than 150 miles), the e-bus will not need any stationary charging on a self-charging path.

Finally, before mathematically formulating the self-charging path problems, it is worthwhile to note two versions of path formulations: with and without node visiting restrictions. In some scenarios, it might be the case that the path should visit some specific nodes between the origin and destination. Let $P \subset N$ denote the set of nodes, other than the origin and destination nodes, that should be visited on the path from the origin to destination. Therefore, without node visiting constraints, the path can be any path from origin to destination, whereas, with node visiting constraints, the path should visit the nodes in P while going from the origin to destination.

3.2. SHORTEST-DISTANCE SELF-CHARGING PATH PROBLEMS

A shortest-distance self-charging path problem is to determine the shortest-distance path from an origin to a destination on a directed network and the arcs to be integrated with e-roads so that the path is a self-charging one. Two versions are considered: without and with node visiting constraints. As noted above, the node visiting constraints enforce the path to visit a set of nodes in the network, which represent the bus-stops that should be visited. First, the shortest-distance self-charging path problem without node visiting constraints is formulated. Then, the shortest-distance self-charging path problem with node visiting constraints is formulated.

3.2.1. Shortest-Distance Self-Charging Path Problem without Node Visiting.

The objective of a shortest-distance self-charging path problem without node visiting constraints (SD-SC-P-P-1) is to jointly determine the self-charging path from node $o \in N$ (origin) to node $d \in N$ and the e-road integration decisions on the path so that the total distance of the path is minimized. Recall that $x_{(i,j)}$ and $y_{(i,j)}$ are the path and e-road integration decision variables. Considering the definitions of $e_{(i,j)}$ and $d_{(i,j)}$, the shortest-distance self-charging path problem without node visiting constraints (SD-SC-P-P-1) from node $o \in N$ to node $d \in N$ can be formulated as follows:

$$\begin{aligned}
\text{(SD-SC-P-P-1):} \quad & \text{Min} \quad \sum_{(i,j) \in A} x_{(i,j)} d_{(i,j)} \\
& \text{s.t.} \quad \sum_{j:(o,j) \in A} x_{(o,j)} - \sum_{j:(j,o) \in A} x_{(j,o)} = 1 \quad (1) \\
& \quad \sum_{j:(i,j) \in A} x_{(i,j)} - \sum_{j:(j,i) \in A} x_{(j,i)} = 0 \quad \forall i \in N \setminus \{o, d\} \quad (2) \\
& \quad \sum_{j:(d,j) \in A} x_{(d,j)} - \sum_{j:(j,d) \in A} x_{(j,d)} = -1 \quad (3) \\
& \quad \sum_{(i,j) \in A} x_{(i,j)} e_{(i,j)} \leq \sum_{(i,j) \in A} r y_{(i,j)} \quad (4) \\
& \quad y_{(i,j)} \leq x_{(i,j)} d_{(i,j)} \quad \forall (i,j) \in A, \quad (5) \\
& \quad x_{(i,j)} \in \{0,1\} \quad \forall (i,j) \in A, \quad (6) \\
& \quad y_{(i,j)} \geq 0 \quad \forall (i,j) \in A. \quad (7)
\end{aligned}$$

In SD-SC-P-P-1, the objective function is to minimize the total distance of the path. Constraints (1)-(3) are the flow balance constraints for the origin, intermediate, and destination nodes, respectively, that guarantee the selected arcs form a path from the origin to destination. Constraint (4) ensures that the path is self-charging. Constraints (5) guarantees that the e-roads can be integrated only on the arcs within the path and the length of the e-road that can be implemented on an arc cannot be longer than the length of the arc. Constraints (6) are the binary definitions for the arc selection decisions and constraints (7) are the non-negativity constraints for e-road implementation decisions.

3.2.2. Shortest-Distance Self-Charging Path Problem with Node Visiting. A shortest-distance self-charging path problem with node visiting constraints (SD-SC-P-P-2) is defined similar to SD-SC-P-P-1 with the only difference is that, in SD-SC-P-P-2, the path to be determined should visit a set of nodes, P . Similar to SD-SC-P-P-1, the shortest-distance self-charging path problem with node visiting constraints (SD-SC-P-P-2) from node $o \in N$ to node $d \in N$ can be formulated as follows:

$$\begin{aligned}
\text{(SD-SC-P-P-2):} \quad & \text{Min} \quad \sum_{(i,j) \in A} x_{(i,j)} d_{(i,j)} \\
& \text{s.t.} \quad \sum_{j:(o,j) \in A} x_{(o,j)} - \sum_{j:(j,o) \in A} x_{(j,o)} = 1 \quad (8) \\
& \sum_{j:(i,j) \in A} x_{(i,j)} - \sum_{j:(j,i) \in A} x_{(j,i)} = 0 \quad \forall i \in N \setminus \{o, d\} \quad (9) \\
& \sum_{j:(d,j) \in A} x_{(d,j)} - \sum_{j:(j,d) \in A} x_{(j,d)} = -1 \quad (10) \\
& \sum_{(i,j) \in A} x_{(i,j)} e_{(i,j)} \leq \sum_{(i,j) \in A} r y_{(i,j)} \quad (11) \\
& \sum_{j:(i,j) \in A} x_{(i,j)} = 1 \quad \forall i \in P \quad (12) \\
& \sum_{(i,j) \in A(S)} x_{(i,j)} \leq |S| - 1 \quad \forall S \subset N: |S| \geq 2 \quad (13) \\
& y_{(i,j)} \leq x_{(i,j)} d_{(i,j)} \quad \forall (i,j) \in A, \quad (14) \\
& x_{(i,j)} \in \{0,1\} \quad \forall (i,j) \in A, \quad (15) \\
& y_{(i,j)} \geq 0 \quad \forall (i,j) \in A. \quad (16)
\end{aligned}$$

In SD-SC-P-P-2, similar to SD-SC-P-P-1, the objective function is to minimize the total distance of the path. Constraints (8)-(10), (11), and (14)-(16) are defined similar to constraints (1)-(3), (4), and (5)-(7) of SD-SC-P-P-1, respectively. The difference is in constraints (12) and (13). Constraints (12) assure that the path visits the nodes in set P that should be visited, i.e., the bus-stops. Constraints (13) are the sub-tour elimination constraints that avoid that the solution does not have unconnected sub-tours.

It is important to note that sub-tour elimination constraints are exponential and make SD-SC-P-P-2 more complex compared to SD-SC-P-P-1. In the application of the models, the solution approach proposed reduces SD-SC-P-P-2 model into multiple SD-SC-P-P-1 models.

3.3. MINIMUM-COST SELF-CHARGING PATH PROBLEMS

A minimum-cost self-charging path problem is to determine the path from an origin to a destination on a directed network and the arcs to be integrated with e-roads so that the path is a self-charging one and the total cost of integrating e-roads is minimized. Two versions are considered: without and with node visiting constraints. As aforementioned, the node visiting constraints enforce the path to visit a set of nodes in the network, which represent the bus-stops that should be visited. First, the minimum-cost self-charging path problem without node visiting constraints is formulated. Then, the minimum-cost self-charging path problem with node visiting constraints is formulated.

3.3.1. Minimum-Cost Self-Charging Path Problem without Node Visiting. The objective of a minimum-cost self-charging path problem without node visiting constraints (MC-SC-P-P-1) is to jointly determine the self-charging path from node $o \in N$ (origin) to node $d \in N$ and the e-road integration decisions on the path so that the total cost of e-road integration is minimized. Recall that $x_{(i,j)}$ and $y_{(i,j)}$ are the path and e-road integration decision variables. Considering the definitions of $e_{(i,j)}$ and $d_{(i,j)}$, the minimum-cost self-charging path problem without node visiting constraints (MC-SC-P-P-1) from node $o \in N$ to node $d \in N$ can be formulated as follows:

$$\begin{aligned}
\text{(MC-SC-P-P-1): } \quad & \text{Min} \quad \sum_{(i,j) \in A} w y_{(i,j)} \\
& \text{s.t.} \quad \sum_{j:(o,j) \in A} x_{(o,j)} - \sum_{j:(j,o) \in A} x_{(j,o)} = 1 \quad (17) \\
& \sum_{j:(i,j) \in A} x_{(i,j)} - \sum_{j:(j,i) \in A} x_{(j,i)} = 0 \quad \forall i \in N \setminus \{o, d\} \quad (18) \\
& \sum_{j:(d,j) \in A} x_{(d,j)} - \sum_{j:(j,d) \in A} x_{(j,d)} = -1 \quad (19) \\
& \sum_{(i,j) \in A} x_{(i,j)} e_{(i,j)} \leq \sum_{(i,j) \in A} r y_{(i,j)} \quad (20) \\
& y_{(i,j)} \leq x_{(i,j)} d_{(i,j)} \quad \forall (i,j) \in A, \quad (21) \\
& x_{(i,j)} \in \{0,1\} \quad \forall (i,j) \in A, \quad (22) \\
& y_{(i,j)} \geq 0 \quad \forall (i,j) \in A. \quad (23)
\end{aligned}$$

In MC-SC-P-P-1, the objective function is to minimize the total cost of the e-roads implemented on the network. Constraints (17)-(23) are defined similar to constraints (1)-(7) of SD-SC-P-P-1.

3.3.2. Minimum-Cost Self-Charging Path Problem with Node Visiting. A minimum-cost self-charging path problem with node visiting constraints (MC-SC-P-P-2) is defined similar to MC-SC-P-P-1 with the only difference is that, in MC-SC-P-P-2, the path to be determined should visit a set of nodes, P . Similar to MC-SC-P-P-1, the minimum-cost self-charging path problem with node visiting constraints (MC-SC-P-P-2) from node $o \in N$ to node $d \in N$ can be formulated as follows:

$$\begin{aligned}
\text{(MC-SC-P-P-2):} \quad & \text{Min} \quad \sum_{(i,j) \in A} w y_{(i,j)} \\
& \text{s. t} \quad \sum_{j:(o,j) \in A} x_{(o,j)} - \sum_{j:(j,o) \in A} x_{(j,o)} = 1 \quad (24) \\
& \sum_{j:(i,j) \in A} x_{(i,j)} - \sum_{j:(j,i) \in A} x_{(j,i)} = 0 \quad \forall i \in N \setminus \{o, d\} \quad (25) \\
& \sum_{j:(d,j) \in A} x_{(d,j)} - \sum_{j:(j,d) \in A} x_{(j,d)} = -1 \quad (26) \\
& \sum_{(i,j) \in A} x_{(i,j)} e_{(i,j)} \leq \sum_{(i,j) \in A} r y_{(i,j)} \quad (27) \\
& \sum_{j:(i,j) \in A} x_{(i,j)} = 1 \quad \forall i \in P \quad (28) \\
& \sum_{(i,j) \in A(S)} x_{(i,j)} \leq |S| - 1 \quad \forall S \subset N: |S| \geq 2 \quad (29) \\
& y_{(i,j)} \leq x_{(i,j)} d_{(i,j)} \quad \forall (i,j) \in A, \quad (30) \\
& x_{(i,j)} \in \{0,1\} \quad \forall (i,j) \in A, \quad (31) \\
& y_{(i,j)} \geq 0 \quad \forall (i,j) \in A. \quad (32)
\end{aligned}$$

In MC-SC-P-P-2, similar to MC-SC-P-P-1, the objective function is to minimize the total cost of the e-roads implemented on the network. Constraints (24)-(32) are defined similar to constraints (8)-(16) of SD-SC-P-P-2.

It is again important to note that sub-tour elimination constraints are exponential and make MC-SC-P-P-2 more complex compared to MC-SC-P-P-1. In the application of the models, the solution approach proposed reduces MC-SC-P-P-2 model into multiple MC-SC-P-P-1 models.

4. APPLICATION

In this section, an application of the self-charging path problems are presented for routing an e-bus. Specifically, the application scenario corresponds to a self-charging path problem with node visiting constraints. First, the application scenario is defined. Then, due to complexity of the self-charging path problems with node visiting constraints, a sequence-based solution approach is defined. The sequence-based solution approach uses the formulations for self-charging path problems without node visiting constraints for a given sequence of visited bus stops. After the solution approach is explained, the numerical results of the application scenario are presented.

4.1. APPLICATION SCENERIO

The application scenario is based on the Missouri University of Science and Technology (Missouri S&T) e-bus that has been shuttling the campus (see [14]). Particularly, in the forward direction, the e-bus should start from a specific point (miner village), then visit a set of bus-stops, and then reach a specific point (Havener Center). Similarly, in the backward direction, the e-bus should start from a specific point (Havener Center), then visit a set of bus-stops, and then reach a specific point (miner village). Currently, the e-bus is charged at a charging station at miner village periodically. In this application, we are trying to determine forward and backward routes for the e-bus.

Figure 4.1 illustrates the simple network representation of the campus-loop area, where the possible nodes are defined considering the turns and/or traffic-stops the e-bus must make. The arcs between the nodes are defined considering the road conditions. The blue nodes are the origin and destination nodes and the yellow nodes are the points for the

bus-stops (for passenger pick-up and drop). Therefore, this application corresponds to self-charging path problems with node visiting constraints.

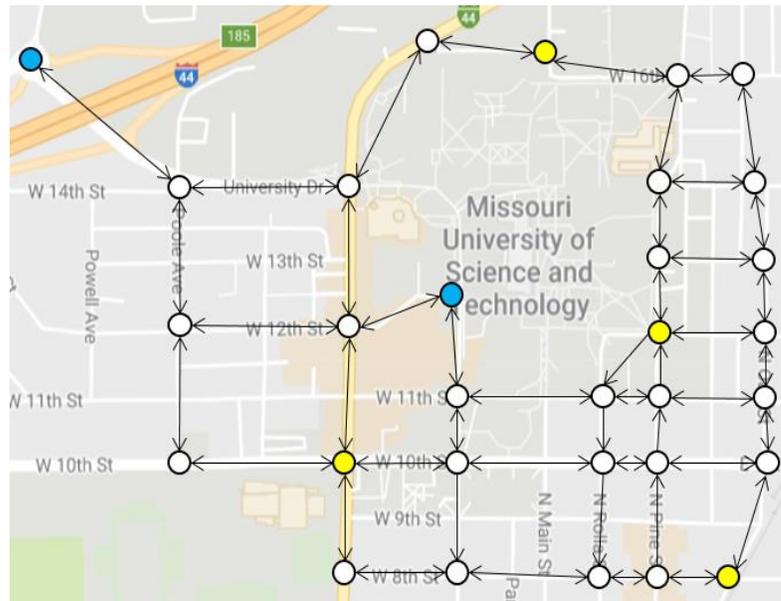


Figure 4.1 Network Representation of the campus-loop at Missouri S&T

In solving the self-charging problems with node visiting constraints for this application, it is assumed that the sequences of the bus-stops that the e-bus should visit on the tour in both directions are given. Therefore, the sequence of the bus-stops of a complete tour of the e-bus is known. Next, the solution approach is explained given the sequence of bus-stops.

4.2. SEQUENCE BASED SOLUTION APPROACH

The solution approach for both shortest-distance and minimum-cost self-charging path problems with node visiting constraints (i.e., SD-SC-P-P-2 and MC-SC-P-P-2) are similar. As noted previously, the difference is in the objective function of each problem: SD-SC-P-P-2 aims to minimize the total distance travelled and MC-SC-P-P-2 aims to minimize the total cost of e-road implementation.

Now, suppose that $S = \{s^1, s^2, \dots, s^k, \dots, s^n\}$ is the ordered set of nodes the e-bus should visit. Note that s^1 is the origin node (i.e., node $o \in N$), s^n is the destination node (i.e., node $d \in N$), and the other nodes are the bus-stops. Here, it is important to mention that as long as the sequence of the bus-stops is given, the origin and the destination can be the same nodes; and in such cases, the path corresponds to a tour. The solution approach discussed herein is therefore applicable to both finding a path from an origin to a destination, which visits a set of nodes, and finding a loop starting and ending at the same point, which visits a set of nodes.

Given the sequence $S = \{s^1, s^2, \dots, s^k, \dots, s^n\}$, the overall tour (a path or a loop) of the e-bus will consist of $n - 1$ sub-paths; from s^1 to s^2 , from s^2 to s^3 , and so on. Let $x_{(i,j)}^k = 1$ if arc $(i, j) \in A$ is on the path from s^k to s^{k+1} , and $x_{(i,j)}^k = 0$ otherwise. Therefore, if the sub-paths are determined, the overall path is determined and one does not need to consider the sub-tour elimination constraints. However, the sub-paths cannot be determined separately because the overall path should be self-charging. In what follows, SD-SC-P-P-2 and MC-SC-P-P-2 are reformulated considering these sub-paths given the sequence $S = \{s^1, s^2, \dots, s^k, \dots, s^n\}$. As defined previously, $y_{(i,j)}$ is the length of e-road integrated on arc $(i, j) \in A$.

Note that, once the self-charging path problems can be solved for a given sequence, one can compare the different solutions over all possible sequences and pick the best solution as the optimum solution.

4.2.1. Sequence-Based Formulation for SD-SC-P-P-2. Given the sequence of nodes to visit, $S = \{s^1, s^2, \dots, s^k, \dots, s^n\}$, the shortest-distance self-charging path problem with node visiting constraints (SD-SC-P-P-2) can be reformulated as follows:

$$\text{Min} \quad \sum_{k=1}^{n-1} \sum_{(i,j) \in A} x_{(i,j)}^k d_{(i,j)}$$

$$\text{s. t} \quad \sum_{j:(s^k,j) \in A} x_{(s^k,j)}^k - \sum_{j:(j,s^k) \in A} x_{(j,s^k)}^k = 1 \quad \forall k = 1, 2, \dots, n-1 \quad (33)$$

$$\sum_{j:(i,j) \in A} x_{(i,j)}^k - \sum_{j:(j,i) \in A} x_{(j,i)}^k = 0 \quad \forall k = 1, 2, \dots, n-1, \quad \forall i \in N \setminus \{s^k, s^{k+1}\} \quad (34)$$

$$\sum_{j:(s^{k+1},j) \in A} x_{(s^{k+1},j)}^k - \sum_{j:(j,s^{k+1}) \in A} x_{(j,s^{k+1})}^k = -1 \quad \forall k = 1, 2, \dots, n-1 \quad (35)$$

$$\sum_{k=1}^{n-1} \sum_{(i,j) \in A} x_{(i,j)}^k e_{(i,j)} \leq \sum_{(i,j) \in A} r y_{(i,j)} \quad (36)$$

$$y_{(i,j)} \leq \sum_{k=1}^{n-1} x_{(i,j)}^k d_{(i,j)} \quad \forall k = 1, 2, \dots, n-1, \quad \forall (i,j) \in A, \quad (37)$$

$$x_{(i,j)}^k \in \{0, 1\} \quad \forall (i,j) \in A, \quad (38)$$

$$y_{(i,j)} \geq 0 \quad \forall (i,j) \in A. \quad (39)$$

In the above model, the objective is to minimize the total distance of the overall tour, i.e., sum of the distances of the sub-paths of the given sequence. Constraints (33)-(35) are defined similar to constraints (1)-(3) of SD-SC-P-P-1. Particularly, they are the path constraints for each sub-path of the sequence. Constraint (36), similar to constraint (4) of SD-SC-P-P-1, ensures that the overall tour is self-charging. Note that, here, rather than enforcing each sub-path of the sequence to be self-charging, the overall path is enforced to be self-charging. Constraints (37) guarantees that the e-roads can be

integrated only on the arcs within the overall path. Note that, in constraint (36), the right-hand-side considers that the energy charged from an arc is $ry_{(i,j)}$ even though this arc might be travelled more than once during the whole tour. However, in constraint (37), we restrict $y_{(i,j)}$ to be less than the arc length times the number of times the arc is traversed during the whole tour. Therefore, constraints (36)-(37) satisfy the self-charging requirement. Constraints (38)-(39) are defined similar to constraints (6)-(7) of SD-SC-P-P-1.

4.2.2. Sequence-Based Reformulation for MC-SC-P-P-2. Given the sequence of nodes to visit, $S = \{s^1, s^2, \dots, s^k, \dots, s^n\}$, the minimum-cost self-charging path problem with node visiting constraints (MC-SC-P-P-2) can be reformulated as follows:

$$\begin{aligned} \text{Min} \quad & \sum_{(i,j) \in A} wy_{(i,j)} \\ \text{s. t} \quad & \sum_{j:(s^k,j) \in A} x_{(s^k,j)}^k - \sum_{j:(j,s^k) \in A} x_{(j,s^k)}^k = 1 \quad \forall k = 1, 2, \dots, n-1 \end{aligned} \quad (40)$$

$$\sum_{j:(i,j) \in A} x_{(i,j)}^k - \sum_{j:(j,i) \in A} x_{(j,i)}^k = 0 \quad \forall k = 1, 2, \dots, n-1, \quad \forall i \in N \setminus \{s^k, s^{k+1}\} \quad (41)$$

$$\sum_{j:(s^{k+1},j) \in A} x_{(s^{k+1},j)}^k - \sum_{j:(j,s^{k+1}) \in A} x_{(j,s^{k+1})}^k = -1 \quad \forall k = 1, 2, \dots, n-1 \quad (42)$$

$$\sum_{k=1}^{n-1} \sum_{(i,j) \in A} x_{(i,j)}^k e_{(i,j)} \leq \sum_{(i,j) \in A} ry_{(i,j)} \quad (43)$$

$$y_{(i,j)} \leq \sum_{k=1}^{n-1} x_{(i,j)}^k d_{(i,j)} \quad \forall k = 1, 2, \dots, n-1, \quad \forall (i,j) \in A, \quad (44)$$

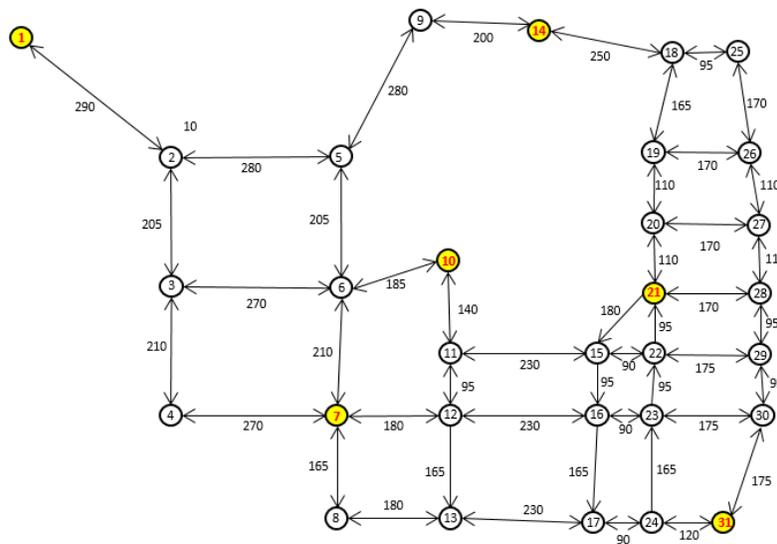
$$x_{(i,j)}^k \in \{0, 1\} \quad \forall (i,j) \in A, \quad (45)$$

$$y_{(i,j)} \geq 0 \quad \forall (i,j) \in A. \quad (46)$$

In the above model, similar to MC-SC-P-P-2, the objective is to minimize the total cost of e-road implementation. Constraints (40)-(46) are defined similar to constraints (33)-(39).

4.3. SCENARIO SOLUTIONS

Here, the Missouri S&T campus loop scenario is solved using the sequence-based solution approach for SD-SC-P-P-2 and MC-SC-P-P-2. Specifically, Figure 4.2 shows the details of the network representation given in Figure 4.1. The node numbers are noted in the circles and the links with two arrows represent two arcs (one in each direction). Note that some of the links have only one arrow as those are one-direction roads. Furthermore, the numbers next to the arcs are the length of the arcs in meters which is shown in Figure 4.2 (as gathered from Google maps). Finally, we randomly generate the energy consumption on the arcs by assuming that $e_{(i,j)} = vd_{(i,j)}$ where v is a uniformly distribution random variable between 0.5 and 1.5. We do this as the energy consumption is not linearly proportional to the distance traveled. Finally, we assume that $r = 1$ as the energy consumption is randomly generated and $w = 1$ as it does not affect the optimum solution because it is a constraint in the objective function of MC-SC-P-P-2.



4.3.1. Sequence Generation. In generating the sequences, we consider the bus-stops to be visited in the forward and backward directions. Specifically, each sequence should start at node 1, visit a set of other nodes in the forward direction, go to node 10, visit a set of other nodes in the backward direction, and go to node 1. The set of nodes to be visited in the forward direction are nodes 21 and 31 and one of the nodes 7 or 14. The set of nodes to be visited in the backward direction are nodes 21 and 31 and one of the nodes 7 or 14. If node 7 (node 14) is visited in the forward direction, then node 14 (node 7) should be visited in the backward direction. Based on these, we have the following 8 possible sequences as given in Table 4.1.

Table 4.1: Possible Sequences of Bus-Stops

Sequence	Stop 1	Stop 2	Stop 3	Stop 4	Stop 5	Stop 6	Stop 7	Stop 8	Stop 9
1	1	7	21	31	10	31	21	14	1
2	1	7	21	31	10	21	31	14	1
3	1	7	31	21	10	31	21	14	1
4	1	7	31	21	10	21	31	14	1
5	1	14	21	31	10	31	21	7	1
6	1	14	21	31	10	21	31	7	1
7	1	14	31	21	10	31	21	7	1
8	1	14	31	21	10	21	31	7	1

Given the flow of the e-bus and the sequence for the eight possible routes it is possible to find what the cost and distance would be with each sequence. Note, that the flow of these sequences are dependent upon the arcs directions, the cost is constant, and the energy consumption is a random variable. With each sequence the e-bus will go through specific nodes to get to the desired bus stops and this will dictate where the e-road

would be placed to maximize the charging while also focusing on minimizing the distance and the cost.

4.3.2. Results for SD-SC-P-P-2. Given the sequences, Matlab 2014 is used to solve the reformulated SD-SC-P-P-2 in Section 4.2.1. for each sequence. Then, the solutions of the sequences are compared to determine the final solution for SD-SC-P-P-2. Table 4.2 presents the details of the solutions achieved for each sequence.

Table 4.2: Shortest-Distance Self-Charging Tour For Each Sequence

Sequence	Total Distance	Tour
1	6040	1-2-3-6-7-12-16-23-22-21-28-29-30-31-24-23-22-15-11-10-11-15-16-17-24-31-24-23-22-21-20-19-18-14-9-5-2-1
2	6280	1-2-3-6-7-12-16-23-22-21-28-29-30-31-24-23-22-15-11-10-11-15-22-21-28-29-30-31-30-29-28-27-26-25-18-14-9-5-2-1
3	5785	1-2-3-6-7-12-13-17-24-31-24-23-22-21-15-11-10-11-15-16-17-24-31-24-23-22-21-20-19-18-14-9-5-2-1
4	6025	1-2-3-6-7-12-13-17-24-31-24-23-22-21-15-11-10-11-15-22-21-28-29-30-31-30-29-28-27-26-25-18-14-9-5-2-1
5	6045	1-2-5-9-14-18-19-20-21-28-29-30-31-24-23-22-15-11-10-11-15-16-17-24-31-24-23-22-21-15-16-12-7-6-5-2-1
6	5920	1-2-5-9-14-18-19-20-21-28-29-30-31-24-23-22-15-11-10-11-15-22-21-28-29-30-31-24-17-13-8-7-6-5-2-1
7	6160	1-2-5-9-14-18-25-26-27-28-29-30-31-24-23-22-21-15-11-10-11-15-16-17-24-31-24-23-22-21-15-16-12-7-6-5-2-1
8	6035	1-2-5-9-14-18-25-26-27-28-29-30-31-24-23-22-21-15-11-10-11-15-22-21-28-29-30-31-24-17-13-8-7-6-5-2-1

Comparing the solutions of the sequences, one can note that sequence 3 has the overall shortest distance; therefore, the tour corresponding to sequence 3 is accepted as the solution of SD-SC-P-P-2. This is based off of having the sequences put into Matlab and computing the distance with the constraints in mind and the arc flows.

In this scenario sequence 3 has the shortest distance. This means that the bus travels the least amount of distance when it starts at node 1 and ends at node 1 after visiting the necessary nodes during its tour. With this in mind it is possible to calculate the energy consumption. With the total energy used it is possible to determine how much the tour can be done via a battery capacity and how much energy the e-bus would need to gather via an

e-road. The sequence and the formulation is a building block to determine an even more complex problem when there are multiple buses being used and when there are different tours to satisfy the customer's needs.

4.3.3. Results for MC-SC-P-P-2. Given the sequences, Matlab 2014 is used to solve the reformulated MC-SC-P-P-2 in Section 4.2.2. for each sequence. Then, the solutions of the sequences are compared to determine the final solution for MC-SC-P-P-2. Table 4.3 presents the details of the solutions achieved for each sequence.

Table 4.3: Minimum Cost Self-Charging Tour For Each Sequence

Sequence	Total Cost	Tour
1	3228	1-2-3-6-7-12-16-23-22-21-28-29-30-31-24-23-22-15-11-10-11-15-16-17-24-31-24-23-22-21-20-19-18-14-9-5-2-1
2	3356	1-2-3-6-7-12-16-23-22-21-28-29-30-31-24-23-22-15-11-10-11-15-22-21-28-29-30-31-30-29-28-27-26-25-18-14-9-5-2-1
3	3092	1-2-3-6-7-12-13-17-24-31-24-23-22-21-15-11-10-11-15-16-17-24-31-24-23-22-21-20-19-18-14-9-5-2-1
4	3220	1-2-3-6-7-12-13-17-24-31-24-23-22-21-15-11-10-11-15-22-21-28-29-30-31-30-29-28-27-26-25-18-14-9-5-2-1
5	3231	1-2-5-9-14-18-19-20-21-28-29-30-31-24-23-22-15-11-10-11-15-16-17-24-31-24-23-22-21-15-16-12-7-6-5-2-1
6	3164	1-2-5-9-14-18-19-20-21-28-29-30-31-24-23-22-15-11-10-11-15-22-21-28-29-30-31-24-17-13-8-7-6-5-2-1
7	3292	1-2-5-9-14-18-25-26-27-28-29-30-31-24-23-22-21-15-11-10-11-15-16-17-24-31-24-23-22-21-15-16-12-7-6-5-2-1
8	3225	1-2-5-9-14-18-25-26-27-28-29-30-31-24-23-22-21-15-11-10-11-15-22-21-28-29-30-31-24-17-13-8-7-6-5-2-1

Comparing the solutions of the sequences, one can note that sequence 3 has the overall minimum cost; therefore, the tour corresponding to sequence 3 is accepted as the solution of MC-SC-P-P-2. This is due to the minimum cost and in this case is similar to the SD-SC-P-P-2 answer when finding the shortest distance traveled.

Given the nature of a singular e-bus the answer for both the SD-SC-P-P-2 and the MC-SC-P-P-2 are the same. For, the shorter the distance the less charge is needed to keep the bus going as it is completing the tour of its sequences. Since this is a simple sequence tour problem it is possible to determine the best sequence. For both the SD-SC-P-P-2 and

MC-SC-P-P-2 have sequence three as the optimal solution. But, as it was stated in the section above, this would change once more e-buses and tours are added to the problem. With knowing the constants such as cost and energy usage it can be determine that the shortest distance sequence will also be the minimal cost sequence based on these constancies.

5. CONCLUSIONS

In this thesis, two new types of network optimization problems are introduced: shortest-distance self-charging path problem and minimum-cost self-charging path problem. For each problem type, mathematical formulations for two versions are presented: without and with node visiting constraints. These problems are practical considering the potential use of WPT technologies in transportation, especially, in transit applications.

The first problem type, the shortest-distance self-charging path problem, aims to determine a self-charging path with the minimum total distance. The second problem type, the minimum-cost self-charging path problem, aims to determine a self-charging path with the minimum total cost of e-road implementations. The main contribution in both problems is introducing the self-charging path concept. A self-charging path utilized dynamic WPT technologies and completely eliminates idle time for charging.

Furthermore, a sequence-based solution approach is introduced for each problem type with node visiting constraints. The problems with node visiting constraints are applicable to many transit scenarios; hence, the proposed solution approach will be useful in adopting to WPT technologies in transit applications. Finally, an application using the Missouri S&T e-bus campus tour is presented. This application scenario demonstrates how to use the introduced models and developed solution approach.

This thesis pioneers the analyses of self-charging path problems. It is a building block for the future research that will be done. As the technology becomes more accessible and affordable the formulas can be used to solve simple tour problems or can be expanded to include multiple e-buses and tours. Something that will need to be taken into consideration when solving these more complex problems is items such as thickness of

roads, value of time, voltage used, source or power, number of OLEV using the system, and the possibility of having an integrated system that includes a static and dynamic WPT source.

The cost for these problems will become more in depth when considering the items mentioned in the previous paragraph. For example, the efficiency of the WPT via a transmitter in the road is dependent upon the gap between the transmitter line and the EV that is being charged. Also, if more EVs are being charged via the e-road the efficiency will decrease. With the decrease in the efficiency the solution to deter this loss is to increase the voltage the transmitter is emitting or to increase the length of the e-road itself.

With these additional consideration it must be seen that in the end this application can be very vital for the transit system particularly. With the ability of an e-bus being charged via an e-road dynamically it allows the transit system to cut back on many aspects. With the e-bus being charged as it drives it allows the e-bus to have a smaller battery size which would cut down on the cost of the e-bus and its weight. With a lower weight the e-bus can achieve a more efficient driving experience. Another thing that is a benefit is that if the static charging is decreased then that means that the number of buses can be reduced due to no down time for the transit buses.

This thesis will be used to expand the idea of having an OLEV system that reduces not only pollution but the necessity of having to stop and either refuel or recharge the vehicle. The potential of this specific formulation is endless and it will be exciting to see how the technology will incorporate optimization while improving the efficiency of the charging system and the improving the experience of the transient.

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