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# Generation and validation of optimal topologies for solid freeform fabrication

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**Department:**

# GENERATION AND VALIDATION OF OPTIMAL TOPOLOGIES FOR SOLID FREEFORM FABRICATION

by

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## ABSTRACT

The study of fabricating topologically optimized parts is presented hereafter. The mapping of topology optimization results for Standard Tessellation Language (STL) writing would enable the solid freeform fabrication of lightweight mechanisms. Aerospace leaders such as NASA, Boeing, Airbus, European Aeronautic Defense And Space Company (EADS), and GE Aero invest in topology optimization research for the production of lightweight materials. Certain concepts such as microstructural homogenization, discretization , and mapping are reviewed and presented in the context of topology optimization . Future biomedical applications of solid freeform fabrication such as organ printing stand to save millions of lives through the robust development of optimized technology. The ability of topologically optimized parts to perform mechanically is presented using FEA and compression testing. A comprehensive user input/output topology optimization software results from the investigation. Functions such as accepting any user design volume, loading, constraining, performing optimization, scaling, and writing an STL file are coalesced into one program named `optstl`. The pre-existing publicly available software packages have been primarily for graphical use, such as 3D plots, and thus cannot be directly interfaced with solid freeform fabrication technology. The reduction of multiple software interfaces into a simplified MATLAB program and the ability to write STL files of topologically optimized models provides scientists and engineers this interfacing ability. The results of this study are evaluated using finite element analysis (FEA), compression testing, and statistical testing.

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# TABLE OF CONTENTS

# LIST OF ILLUSTRATIONS

# LIST OF TABLES

# 1. INTRODUCTION

Design engineering has always been mankind's segue from the status quo. The competition for natural resources necessitates this study of structural design [8] and topology optimization. The result has been several evolutions of product lifecycle and database management interfaces [6]. The industrial revolution catapulted the process of design into a major business, and the advent of computer and internet technology brought computational methods, AI, and data sharing to light [14]. The main goal of design research is the resolution of shortcomings and obstacles. Preprocessors such as the one developed in this study introduce the phenomenon of automated engineering design.

This study presents a computational method for mapping the solution of 3D topology optimization for STL writing on a standard PC equipped with MATLAB. The convergence of the optimization is illustrated. Studies have proven the regulated Solid Isotropic Material Penalization (SIMP) gradient based descent used in this study is the best method for topology optimization [1, 19, 20]. A MATLAB implementation of the regulated SIMP is studied, developed, and supplemented using additional MATLAB functions. The user is responsible for inputting the design volume, loads, simply supported constraints, scaling, and mesh fineness. All computations in this study have been computed using an Intel dual core i7 processor at 2.10 GHz and 2.70 GHz, 16 GB RAM, 1 TB ROM, a 64 bit Windows 7 Pro OS, and 64 bit CAE Linux OS. STL photos and triangulated volumes are from XYZware and GMesh. Validation FEA is provided using ImpactFEA, Salome Meca, and Code Aster. All solid freeform fabrication in this study is presented from fused deposition modeling using a DaVinci 1.0 machine.

Structures optimization advances from the use of topology optimization where excess material in limited space can create negative effects. An example is the study of poroelastic materials for the actuation of linear motors. Research in the study material moduli filtered an iterative gradient based descent of elasticity, so the solution's convergence achieved the desired vertical and torsional deflections [3]. Another example is the application of topology optimization in determining the bounds of viscoelastic microstructures [4]. The negative stiffness of these dampers absorbs vibrations and shifts the frequencies of an unconstrained beam [17]. Adequately mapping these results of topology optimization for STL file writing is required for the application of solid freeform fabrication methods [26].

The use of multiple software interfaces and manual re-renderings have hampered the ability of businesses to physically manufacture topologically optimized parts. Aerospace firms such as NASA, Boeing, Airbus, EADS, and GE Aero invest in topology optimization research for the production of lightweight materials [10,11,24,25]. GE predictions report an aircraft engine's weight, assembled from subtractively machined parts, can be reduced by potentially 1,000 pounds using additive manufacturing after the year 2020 [12]. An estimate of the presidentially appointed U.S. Digital Manufacturing and Design Innovation Institute concurs that the use of digital manufacturing technology can save the aviation industry $30 billion by 2030 [5].

EADS, an aerostructure manufacturing company published a case study [25], and the study explained that fabrication of optimal topologies required iterative cycles of

design in CATIA, meshing in HyperWorks, FEA in Obtistruct, topology optimization, and finally STL smoothing in 3 Matic [25]. The results of EADS' testing have been an impetus in the mapping of optimal topologies for directly writing STL files.

This study focuses on unifying three of the five different functions demanded in industry in one package named `optstl`: mesh, load, and constrain any user defined volume, topologically optimize the volume, and then write an STL of the topologically optimized volume. The meshing function can create a uniform mesh of user defined density. The density is determined when the user inputs the discretization factor. Discretization factors can range from one to any integer greater than one. Voxel cubes are used in `top3d` which is the optimization engine modified in this study, so voxelization was chosen as the method of discretization for this study. Voxelization required discretization of the user defined system in this study via trilinear mapping to split the model into a mesh of cubes or voxels. User defined loads and constraints are mapped to the mesh using Booleans and nested loops. Loads and constraints can be distributed over entire surfaces, mapped to a specific point, or a combination of each. These loads and constraints are input via argument into the `top3dFlex` algorithm. The `optstl` code made in this study improves upon the `top3d` code by allowing the use of Cartesian coordinates instead of nodal indices. A simple example of the difference between Cartesian coordinates and nodal indices comes from a 2 mm x 2 mm mesh example. The node at x = 2 mm and y = 2 mm has a nodal index of 4. Most design volumes in this study are significantly more complex, so calculation of the nodal index is pre-programmed into the `optstl` package. All the user must do is correctly input Cartesian coordinates of each

load and constraint into the MATLAB command window after having loaded either an STL model or a set of design volume parameters. Distributed loads can only be made perpendicular to the *xy*, *yz*, and *zx* plane, and constraints are always simply supported constraints for the optimization engine `top3dFlex`. Finite element analysis of the voxelized system solves the displacement value of each node and the optimization function optimizes the elasticity of each voxel. Displacement is controlled using weighted filtering before each search for the set of optimal voxel densities. Users get a 3D MATLAB plot and three choices: 1) to scale the result, 2) to make a point cloud from the result, and 3) to write an STL file from the result.

A company requiring all these functions can save money otherwise spent on purchasing separate software for each function. Scientists have considered the economics of solid freeform fabrication [23], and a 2006 study has shown the benefits of this technology exceedingly outperform subtractive, casting, and molding methods at low volumes of production [23]. Even the nesting of multiple parts during solid freeform fabrication means one machine can produce an entire assembly after just one iteration of lowering the machine bed [23]. The use of optimal topology STL writing enables shifting cost estimator variables of production time and material cost further in favor of solid freeform fabrication.

The pre-existing publicly available software packages have been primarily for .obj files and graphical displays and thus cannot directly interface with solid freeform fabrication technology. The mapping of topology optimization results for STL writing

enables the solid freeform fabrication of lightweight mechanisms. Epistemic errors are systemic and random in nature [18], and `optstl` can eliminate epistemic errors that are encountered during manual mappings and re-renderings. One concern respecting these computationally optimized parts pertains sufficient load bearing behavior, so validation is required. FEA plots and compression testing from this study prove whether the optimized parts exhibit acceptable mechanical behavior.

## 2. BACKGROUND

The objective of topology optimization is the minimization of the model's volume given loads and constraints. The use of explicit functional parameters leads to an impracticable state space solution [20]. Therefore, the structure should be expressed implicitly non-parametrically for optimal results. The objective is minimization of the design model's volume:

$$V(\rho) = \oint \rho dV \leq V_s \quad \text{(Eq 1)}$$

where $V_s$ is the maximum user defined volume, and $\rho \in [0\ ,1]$ is the normalized density of each element in the final volume $V$. The normalized density, $\rho$, relates to the compliance of the model:

$$K(\rho)u = f \quad \text{(Eq 2)}$$

where $K(\rho)$ is the stiffness of an element, $u$ is the displacement of this element, and $f$ is the force acting on this element. The value of $\rho$ for each element can be computed using gradient-based descent [7, 20].

The `optstl` optimal topology STL writing program, which is developed in this study, is based on Liu and Tovar's `top3D` algorithm [19]. The regulated SIMP based gradient descent, stiffness matrix, and display functions of `top3d` were used in `optstl` via a modified version named `top3dFlex`:

```
function xPhys = top3dFlex(nelx,nely,nelz,volfrac,penal,rmin, loadnid,
fixednid, passive, Young, load_mag)
```

where `loadnid` and `fixednid` are the node indices for the loads and constraints. A node is a vertex of a voxel, and `nelx`, `nely`, and `nelz` are the number of voxel elements in the *x*, *y*, and *z* directions, respectively. Voxels in the design volume which should be void are voided using `passive`. Voids represent areas such as holes for fasteners or other features defined in the input STL file. All voids are determined computationally beforehand using the results of the `VOXELISE_FLEX` function as discussed in later sections. The product of `nelx`, `nely`, and `nelz` creates the design volume, and the variable `volfrac` ∈ [0,1] is the desired fraction of the design volume for the final structure. Both the initial value of the design volume and the midsection search Boolean employ `volfrac`:

```
x = repmat(volfrac,[nely,nelx,nelz]); xPhys = x;
```

Where the 3D array `xPhys` contains the current normalized density of each voxel, and the initial value definition sets all element volumes in `xPhys` equal to `volfrac`.

```
if sum(xPhys(:)) > volfrac*nele, l1 = lmid; else l2 = lmid; end
```

Where the volume `nele` is the product of number of elements in the *x*, *y*, and *z* along the *x*, *y*, and *z* axes, and `xPhys` has been subjected to the regulated Solid Isotropic Material Penalization method:

$$K(\tilde{x}) = \sum_{i=1}^{n}[E_{min} + \tilde{x}^p(E_0 - E_{min})]K_i^0 \; \tilde{x}_i \epsilon [0,1] \qquad \text{(Eq 3)}$$

Where $E_0$ and $E_{min}$ are the Young's and minimum moduli of the material respectively, and the MATLAB code for Equation 3 requires one line:

```
sK = KE(:)*(Emin+xPhys(:)'.^penal*(E0-Emin));
```

`KE(:)` is the global stiffness matrix $K_i^0$ computed using the function `lk_h8` as explained in detail per [19]. Each element *i* of `xPhys`, is a voxel element's normalized density where $i \, \epsilon [0, number\ of\ elements]$, and $\tilde{x}_i$ is a multiresolutional or regulated

density as a function of the neighboring normalized densities subject to the user defined exponent `penal` > 1 for convergence. The variable `xPhys` equals the variable $\tilde{x}$ from Equation 3. Researchers find maintaining a coarse FEA mesh while finely computing the SIMP requires factoring in weighted contributions of neighboring elements to the deformation of any single element [19, 20]. The neighboring normalized element densities contribute weight as a function of the user input `rmin`:

```
sH(k) = max(0,rmin-sqrt((i1-i2)^2+(j1-j2)^2+(k1-k2)^2));
```

$$H_{ij} = R - dist(element1, element2) \quad \text{(Eq 4)}$$

Where element2 is nearest element to element1 within the distance $R$ = `rmin`, and *(i1-i2)*, *(j1-j2)*, and *(k1-k2)* are the differences between the *x*, *y*, and *z* coordinates of *element1* and *element2*.

$$\tilde{x}_i = \frac{\sum_{j \in N_j} H_{ij} v_j x_j}{\sum_{j \in N_j} H_{ij_0} v_{j0}} \quad \text{(Eq 5)}$$

```
xPhys(:) = (H*xnew(:))./Hs);
```

Where the denominator `Hs` is the initial weighted contribution of the neighboring elements of all the normalized element densities initially set to 0.5, and `H*xnew(:)` is the updated contribution of the neighboring normalized element densities after calculating each `xnew` via gradient based descent, and the convergence is checked using a midsection search method:

```
    l1 = 0; l2 = 1e9; move = 0.2;
    while (l2-l1)/(l1+l2) > 1e-3
        lmid = 0.5*(l2+l1);
        xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-
dc./dv/lmid)))));
        xPhys(:) = (H*xnew(:))./Hs;
        if sum(xPhys(:)) > volfrac*nele, l1 = lmid; else l2 = lmid; end
    end
```

where the variables `dc` and `dv` are defined,:

```
dc = -penal*(E0-Emin)*xPhys.^(penal-1).*ce;
dv = ones(nely,nelx,nelz);
```

where `dc` is the first derivative of the SIMP computation and where `ce` is the constitutive matrix, and the product of the constitutive matrix, the normalized elements' moduli, and the $E_0$ is the system's stiffness, *k*, as defined in Hooke's Law:

$$F = -kx \quad \text{(Eq 6)}$$

where *k* is the system's stiffness and can be related to the modulus in terms of axial stress and strain:

$$E = \frac{\sigma}{\varepsilon} \quad \text{(Eq 7)}$$

where *E* is Young's modulus, $\sigma$ is stress on the area, and $\epsilon$ is strain in the direction of the stress.

$$E = \frac{F/A}{\Delta l/l} \quad \text{(Eq 8)}$$

where *F* is the force exerted on area *A* and $\Delta l$ is the change of l in the direction of the force *f*. Equation 6 can be rearranged to resemble equation 2:

$$F = \frac{EA\Delta l}{l} \quad \text{(Eq 9)}$$

where *EA/l* equals the stiffness *k,* $\Delta l$ equals the displacement *x,* and Eq 9 now resembles the function in Eq 2 for meshing discretization of *i* elements with modulus *E* as defined in Eq 3. Varying the elasticity value inversely varies the displacement. Locations where the model is not strained signify areas of little to no force transmission, so the modulus can be revised to zero in these locations only. The SIMP model from equations 1 – 3 allows for penalizing such locations until there are only sufficient voxels left for mechanical compliance. If the voxel's normalized density is less than unity, then subjecting this density to any exponent greater than one causes the modulus to approach zero. Only

voxels having a density equal to one can remain unchanged after SIMP. Penalization converges through each iteration and the updated values for $\Delta l$ are inputs into the next iteration of the constitutive matrix, `ce`.

Several supplemental functions are added in `optstl` for `top3dFlex` such as the ability to read in any model space via STL, adjust mesh density, scale the result, make a point cloud, and write an STL. Reading in any STL file is the function of `VOXELISE_FLEX` which returns a binary 3D array where the any element in the array can have either a 0 or 1 value. Voxels on the inside of the model are given a value of 1, and voxels outside the model are given a value of 0. Such valuation is known as binary homogenization. Voxel size is determined during this homogenization, so the user is first asked for the discretization factor before proceeding. The minimum discretization factor is one voxel per millimeter. Scaling is permitted after `top3dFLEX` produces an optimal result, so the user can choose to optimize a small scale model of his or her system and then scale the result. An option is given, so the user can then generate a point cloud in the dxf format for inspection in AutoCAD. A final option is given for the user to write an STL file for solid freeform fabrication, so the user can fabricate the optimized topology. If the user decides not to use any of these supplements such as input an STL file, then the user can still use `top3dFlex` via a secondary set of requests for the user to define only the length , width, and height of the volume. The iterative solver as recommended in Liu and Tovar's paper [17]for optimizing a large volume is fully implemented in `optstl`, so no restriction is placed on the size of the user defined design volume or input STL model.

# 3. PURPOSE

The purpose of this study is to advance the fabrication of light weight or spatially optimized mechanisms for solid freeform fabrication that can be applicable to vehicle development, bionics, consumer electronics, and civil structures. Scientists have studied optimal topologies for force inverters [10], interiors of sandwich panels [26], and building infrastructure [15, 21]. Optimizing these mechanisms involves minimizing volume while maintaining mechanical performance. Any volume eliminated during this process reduces the amount of material for fabrication and energy required for work and heat transfer, and the eliminated volume presents space for embedding hardware.

A major constraint of the existing `top3d` is the definition of the user's design volume as just a rectangular block specified as a length, width, and height. Liu and Tovar do describe a method for adding features using active and passive voxels [19], yet the user would have to explicitly parameterize each active and passive voxel. Active voxels represent voxels within the model while passive voxels represent voxels inside the design volume yet outside the model. Defining these active and passive voxels parametrically requires formulation of feature geometry into functional notation. Used in this study is a simplified means where the user can save any solid CAD model into an STL format. Any structure having already been saved in the STL format can be voxelized via use of the `VOXELISE_FLEX` function in `optstl` for `top3dFLEX`, so the voxel format of the original `top3d` algorithm is retained in `optstl`. Passive voxel assignments of 0 are then assigned to any voxel outside the solid, and active voxel assignments of 1 are assigned to

any voxel within the solid. Voxelization of the input STL file produces this binary array for input into `top3dFlex`, yet these binary voxels are returned having any value in the continuous distribution [0, $E_0$]. Fabrication of this continuous distribution is highly technical and requires machinery capable of depositing or binding materials of varying moduli. A DaVinci 1.0 printer which is employed in this study is capable of extruding only a single filament of material, so `optstl` is made to homogenize the continuous moduli distribution back to a binary voxel format via the `CONVERT_voxels_to_stl` function. Researchers have studied the voxels as the base units of structural homogenization [16]. Voxels can tessellate readily, so larger structures can be made from a voxel microstructure. Using voxels as homogenous building blocks this way is known as microstructural homogenization much like a brick wall is made from the homogenous assembly of bricks. Making one load bearing microstructure can scale to that of a larger system of homogeneous microstructures. The user can now decide whether to optimize and fabricate any system of components or any component within the system.

# 4. RESULTS

One primary result of this study was the development of one software package, `optstl` for which a process diagram is shown in Figure 4.1.

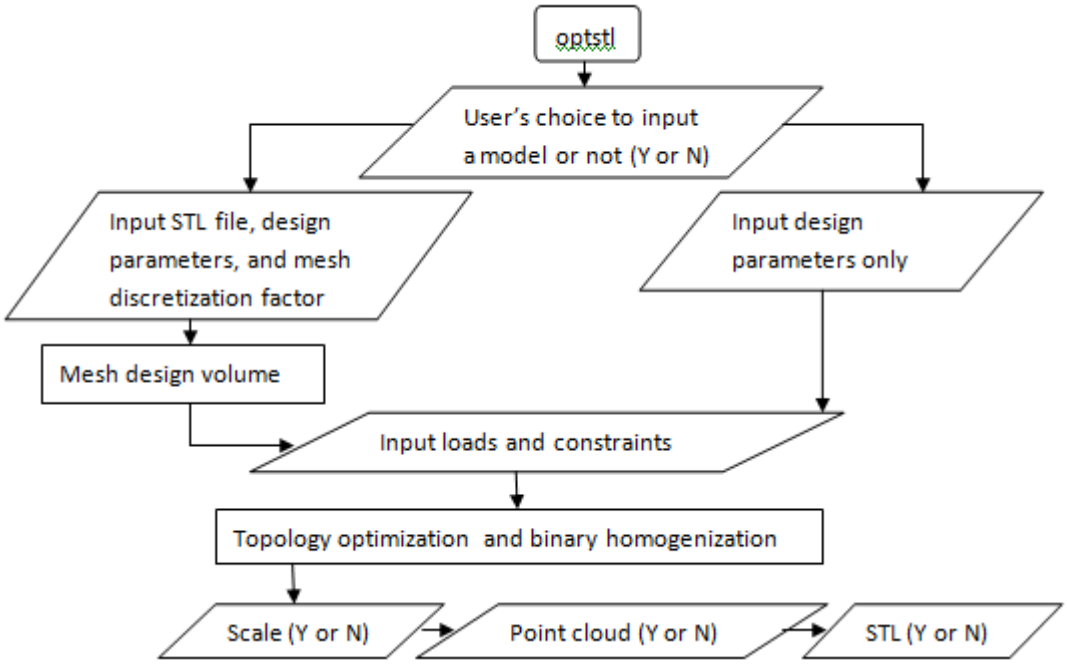
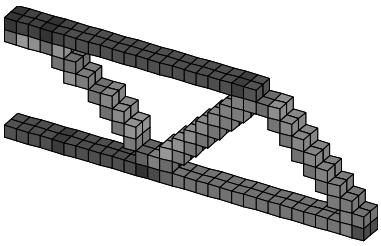



Figure 4.1: `optstl` Program Flowchart

The resulting program requires the user to input `optstl` into the MATLAB command window. There are sixteen .m MATLAB files which contain function scripts that the user must have in the current working MATLAB directory. A series of questions follow the command line function call to proceed to determine the design volume. Either an STL or just the length, width, and height parameters are acquired. If the user does

input an STL, then the user is still responsible for inputting the model length, width, and height of model as well as a discretization factor for meshing. Load and constraint inputs are required following the determination of the design volume. Topology optimization can then proceed and then binary homogenization. The resulting 3D array contains only 0's and 1's. All of the 0's represent space outside the optimal model, and all of the 1's represent space within the optimal model. Should the user prefer to scale these results before writing a point cloud or an STL file, the user asked for a scaling factor. Appendix E contains a user's training manual for practicing three examples studied here.

The topology optimization engine named `top3DFlex` here was developed from Liu and Tovar's `top3D` script [19]. Liu and Tovar presented several examples of how to use `top3d`. Figures 4.2 and 4.3 here show adapted results from these examples:



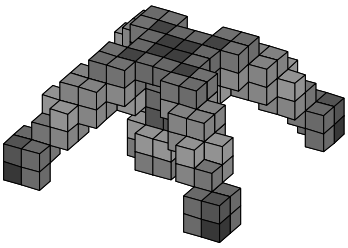


Figure 4.2: (Left) Optimal Cantilever Topology Under a -1N Distributed Force at the Cantilever's Tip Produced in `top3D`. Figure 4.3: (Right) Optimal Platform Topology Under a -1N Point Force Along the Central Vertical Axis Produced in `top3D`.

Figures 4.2 and 4.3 are MATLAB figures shaded as functions of each voxel's moduli [19]. Figure 4.2 contains a model which has overall dimensions of 30 mm x 10

mm x 2 mm, and Figure 4.3 has overall dimensions of 12 mm x 6 mm x 12 mm. Each voxel in both figures represents a 1 mm x 1 mm x 1 mm volume. The modulus of each voxel is stored in the variable `xPhys` of `top3d`. Locating any single voxel in `xPhys` is outlined in [19], and the location of a voxel is known as its index. Mechanical loading and constraint functions require the computer to have the index for each voxel and each voxel's vertices. A result of this study is automated mapping based on user defined coordinate information. Every possible vertex coordinate in the design volume is generated using `generate_cube_M`. Mapping the voxels of these vertices is dependent of the type of discretization found. Trilinear discretization connects vertices using cubes while cubic discretization connects vertices using triangular pyramids. Connectivity within cube voxel elements correlates with the trilinear discretization, so each voxel is assigned eight rows in the connectivity list generated from `generate_cube_M`.

```
function [M,T] =generate_cube_M(left, right, bottom, top, back, front,
h_partition,scale)
```

where the variables `left`, `right`, `bottom`, `top`, `back` and `front` together define the width, depth, and height of the overall design volume from the user defined inputs of `optstl`. The variable `h_partition` is a 3x1 array for defining the fineness or coarseness of nodal map, and the variable `scale` applies when the user wishes to scale the model. An `h_partition` value of [1,1,1] means the element voxels of the system will have dimensions of 1 mm x 1 mm x 1 mm. If a finer mesh is required, then the user must decrease the value for each element of `h_partition`. Two examples are shown below. Reducing the `h_partition` value in half increases the point cloud fineness eight times for the stl. There are 1000 elements in Figure 4.4 and 8000 elements in Figure 4.5. The

design volume is initially appearing made from an `h_partition` value of `[10,10,10]` meaning each voxel has dimensions of 10 mm x 10 mm x 10 mm. Decreasing the coarseness of the design volume means each resulting voxel should occupy less space thereby making more voxels necessary for meshing. Decreasing the value of `h_partition` causes the indirectly proportional change in the quantity of elements without changing the overall scale of the design volume:

```
h = h_partition;

n_hor = scale*(right - left)/h(1); %parallel to the x-axis
n_vert = scale*(top - bottom)/h(3); %parallel to the y-axis
n_depth = scale*(front - back)/h(2); %parallel to the z axis
```

where `n_hor`, `n_vert`, and `n_depth` are the number of elements along the *x*, *y*, and *z* axes respectively. The changed coarseness seen in Figures 4.3 and 4.4 below is the result of decreasing the value of `h_partition` to `[5,5,5]`. An even finer point cloud for stl has been computed in this study using a value of `h_partition [4,4,4]`. Further decreasing each value of `h_partition` increases the amount time required in computing the point cloud, triangulations, and normal vectors for these binary stl files. Each value of `h_partition` can be different, so the mesh has a unique density in each axial direction.

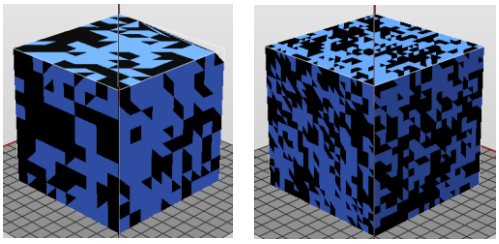


Figure 4.4: (Left) Mesh Made from `generate_cube_m` Function with an `h_partition` Value of [10,10,10]. Figure 4.5: (Right) Mesh Made from the `generate_cube_m` Function with an `h_partition` Value of [5,5,5].

The dimensions of the cube in Figure 4.4 and Figure 4.5 are equal: width = 100 mm, depth = 100 mm, and height = 100 mm, yet the number of elements and ensuing computations are different. Further decreasing the value of `h_partition` to [4, 4, 4] results in 15,625 voxel elements for the same100 mm x 100 mm x 100 mm design volume. All of the vertex information for each voxel is stored a nodal index matrix `M`, and the trilinear discretization connectivity of each vertex composing each voxel is stored in an element index matrix `T`. The size of matrix `M` for the cube shown in Figures 4.4 – 4.5 is $(s+1)^3/(h+1)^3$ x 3, and the size of matrix `T` for the same figures is $(s/h)^3$ x 8 where s is the length of one side and h equals `h_partition`. Variation in the number of voxels of a given design volume due to varying `h_partition` is shown in Table 4.1.

Table 4.1: Variation in the Design Volume as a Result of Varying `h_partition`

| h_partition | Length, width, and height of design volume (mm) | Number of Elements |
|---|---|---|
| 10 | 100 x 100 x 100 | 1000 |
| 5 | 100 x 100 x 100 | 8000 |
| 4 | 100 x 100 x 100 | 15625 |

The amount of time required for writing an STL file without voxelization varies proportionally with the number of elements. A regression analysis is presented below in Figure 4.6 for estimating the time of computation. Time for writing the STL's corresponding with the values of `h_partition` in Table 1has been calculated using the

MATLAB `cputime` variable. The variable `cputime` is reserved for recording the running time of the MATLAB application. Solving for the difference between the value of `cputime` before starting the meshing and stl writing scripts and the value of `cputime` after running these scripts is the running time required. The time study here is the result of timing only with the `generate_cube_m` and `xyzstlwrite` functions. The $R^2$ regression coefficient equals 1 for measuring the squared residuals of a second order polynomial best fit to this data, so the correlation between the computer's behavior and expected behavior is predictable.

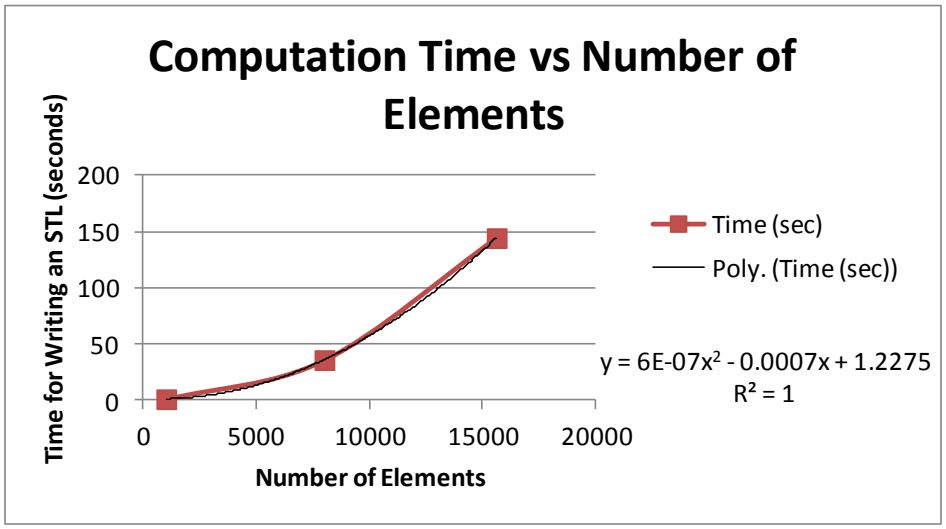



Figure 4.6: Varying Computation Time as a Result of Increasing the Number of Elements in the Design Volume

If the `h_partition` is further reduced to `[2,2,2]` in the hopes of increasing the fineness of the design volume, then the resulting number of elements becomes 125,000.

The estimated computation time then becomes 9288.7 seconds or 2 hours and 35 minutes for running only the `generate_cube_m` and `xyzstlwrite` functions.

Proving these discretization and mapping algorithms worked was necessary for saving time and materials to be invested in the fabrication of the results. Mapping and discretization directly influenced the storage and application of user defined constraints and loads, so these numerical models of the system had to represent the real system accurately. Discretization and mapping were hence tested using the results of `top3d` shown in Figure 4.2 and Figure 4.3. The test consisted of creating and identifying the vertices of each voxel element in the system and then removing those vertices that had been removed during the topology optimization. The required alogorithm for this process was written during this study and named `optcoordinates`:

```
function Mopt = optcoordinates (M,T, scaled_weight_mat)
zero_weights= find(scaled_weight_mat<=0.95);
```

where the input arguments are the nodal coordinates `M`, the node to element connectivity list `T`, and the 3D array of voxel moduli with any scaling named `scaled_weight_mat`, and the output is the array `Mopt`. Creating this output array requires the MATLAB function `find` which returns only the indices of elements in `scaled_weight_mat` greater than or equal to the set threshold value. A threshold value of 0.95 appears in the example above, so only elements with a density greater than 0.95 would remain for the STL. Using a single threshold to filter data is known as binary homogenization. Varying the binary homogenization threshold varies the amount of data passed through this type filter. If more points are required after filtering, then the filter should be re-run using a lower

threshold value. The indices of `scaled_weight_mat` and the column indices of the `T` matrix represent the same elements, so identifying the index of voxel element in the `scaled_weight_mat` correlates with a column of the same index in the `T` matrix. Vertex information is additionally available in the `T` matrix, so if the voxel element must be removed after filtering then removing the entire corresponding column from `T` removes the voxel and its vertices from the system. All the elements that do not meet the threshold value require only MATLAB empty brackets [] for removal:

```
T(:,zero_weights') = [];
```

The remaining columns of `T` represent elements that meet the threshold. Many of the resulting columns can contain repeating values because a vertex can be shared amongst eight voxel elements. Eliminating any repeating values requires the standard MATLAB `unique` function:

```
non0_elnodes = unique(T)';
```

where `non0_elnodes` is the output of the unique function applied to `T` and contains the index of every node for each element of a density meeting the threshold set in the `find` function. Finally, `Mopt` is the return argument and contains the point cloud of all the nodes for elements meeting the density threshold.

```
Mopt = M(:,non0_elnodes);
```

where the number of elements in `Mopt` can be varied using varied the is top3d's result and the MATLAB find functions imposed the homogenization threshold. The threshold used was 0.5, so any voxel existing under the threshold was assigned a 0 value. Voxels above this threshold were assigned a 1 value.

Examples of point clouds using `optcoordinates` from this study before and after optimization are shown in Figure 4.7-4.10. Each point in these point clouds is a vertex of a voxel in the original design volume.

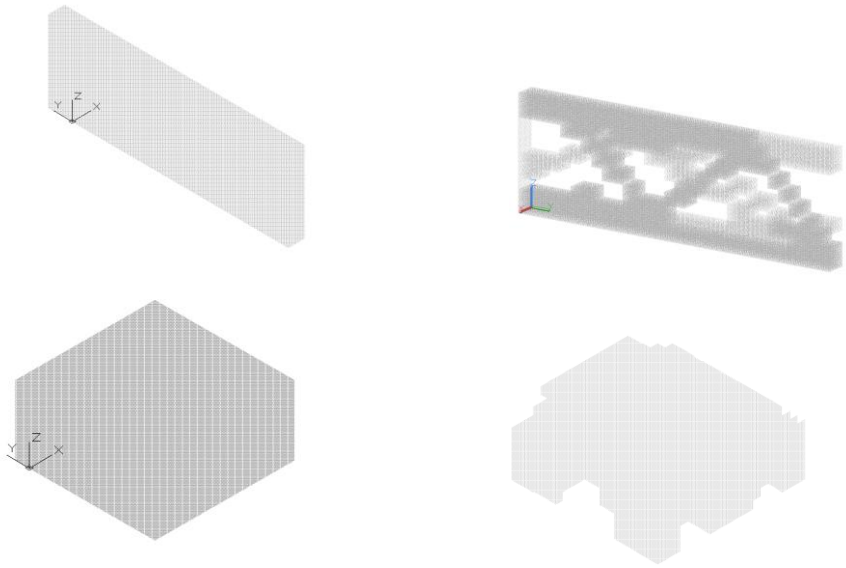


Figure 4.7: (Top Left) The Design Volume Point Cloud for the Cantilever. Figure 4.8: (Top Right) The Optimized Design Volume Point Cloud for the Cantilever. Figure 4.9: (Bottom Left) The Design Volume for the Platform. Figure 4.10: (Bottom Right) The Optimized Design Volume for the Platform.

The optimized point clouds appear to the right of their respective original design spaces. The original design volume for the cantilever is 600 $cm^3$. The optimized design volume for the cantilever is 250 $cm^3$ as a result of -1N loads distributed at the tip and simply supported as shown in Figure 4.8. The platform's original design volume is 4000 $cm^3$. The platform's optimized design volume is 1412 $cm^3$ as the result of a -1N point force placed at the top dead center and simply supported as shown in Figure 4.10. The

time required for generating these point clouds through the sequential use of `top3d`, `generate_cube_M`, and `optcoordinates` is shared in Table 4.2. Each computation time is the total time between inputting the design parameters and outputting the point cloud file. Computation time increases expectedly with the number of voxel elements involved. Topology optimization is found to increase the computation time as well:

Table 4.2: Results of the Original and Optimized Point Clouds

| Mechanism | Original Volume ($cm^3$) | Computing Time for the Original Point Cloud (sec) | Optimized Volume ($cm^3$) | Computing Time for the Optimized Point Cloud (sec) |
|---|---|---|---|---|
| Cantilever | 600 | 0.1872 | 250 | 31.8242 |
| Platform | 4000 | 0.6396 | 1412 | 123.5216 |

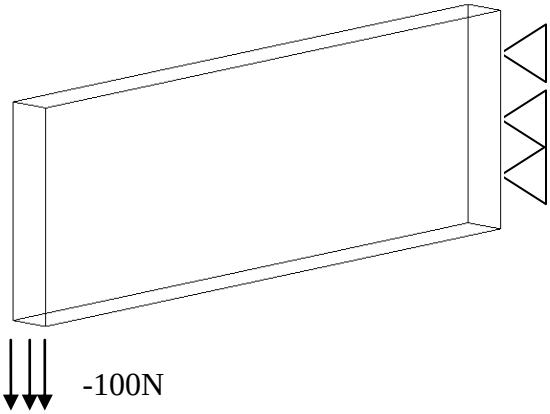
Times for the original volume hence are shorter than the times for the optimized volumes because these latter volumes required running the topology optimization function. The difference in computing times between the original and optimized cantilever is 31.637 seconds. The difference in computing times between the original and optimized platform is 122.882 seconds. The cantilever's optimization achieved a 58.33% reduction in volume and consequently required 170 times longer than the computing time for the original cantilever point cloud. The platform's optimization has achieved a 64.7%

reduction and consequently required 193 times longer than the computing time for the original platform's point cloud.

The tradeoff between topology optimization and computation time is meaningful only in the event that these topologically optimized light weight models are as stiff as the original models. If these optimal models are not compliant in terms of sustaining the user defined loads, then the original models are sufficient. Mechanical stiffness or the ability of a mechanism to sustain a load given constraints is critical to the quality of the end user's safety and experience. Each optimized model is thus subjected to validation using FEA to compute the strained displacements incurred under the given loading and constraint conditions.

A 30 mm x 10 mm x 2 mm simply supported cantilever was loaded with 100N uniformly distributed as shown in Figure 4.11. The coordinates of the loads and constraints are shared in Appendix K. The maximum volume of $V_s$ for the objective function in equation 1 was set to 0.3 meaning 30% of the entire 30 mm x 10 mm x 2 mm original volume. Therefore, the objective was to find at most a 180 $mm^3$ design which supported the 100N distributed load while simply supported. The value of $V_s$ is the determining factor in the optimization, so too low of a $V_s$ could produce a design that does not support its load. Too high of a $V_s$ may not decrease the volume sufficiently. The FEA displacement plot of this system shows displacement existing primarily near the loading point in the green and red regions of Figure 4.12. The majority of the cantilever

was left un-strained as shown in the large blue region. The maximum displacement in the red region was 0.000480 m, so the overall design envelop did not show necking.



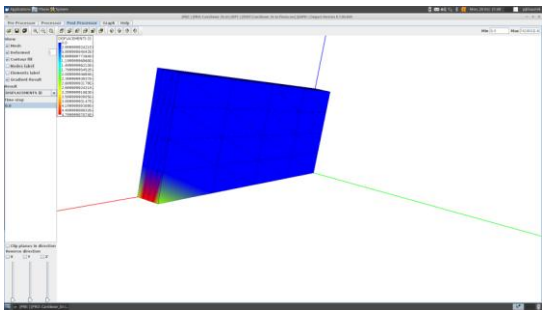



Figure 4.11: (Left) The Loaded and Constrained Cantilever. Figure 4.12: (Right) FEA Displacement Plot of This Cantilever System Made in This Study Using Linux Based Software Named ImpactFEA

Regulated SIMP based topology optimization of the cantilever subject to the loads, constraints, and objective which were discussed immediately before Figure 4.11 removed 286 $mm^3$ from the original 600 $mm^3$ design. The resulting 314 $mm^3$ STL model shown in Figure 4.13 was printed during this study using a DaVinci 1.0 printer to make the prototype shown in Figure 4.14. The ruler shown in juxtaposition with the prototype of the optimal cantilever proves the topology optimization did not adversely alter the scale of the 30 mm length dimension. The width and height dimensions were preserved in the optimization as well. All of the removed material was only removed from within the original design envelop, so if this prototype were part of a larger assembly of components, then assembly fit would not be affected.

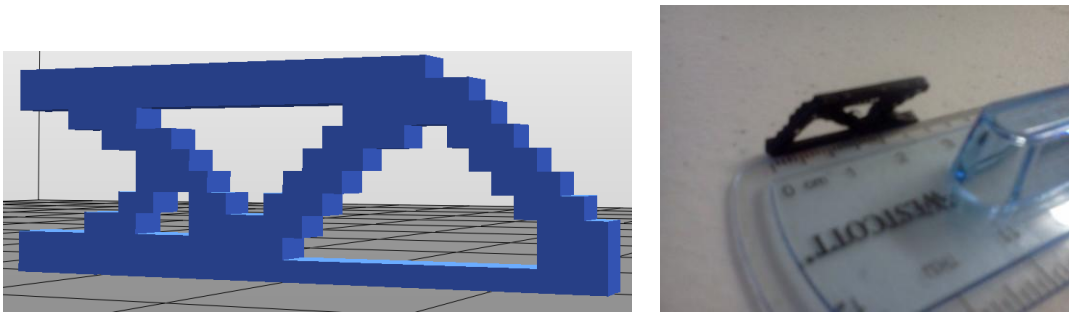


Figure 4.13: (Left) STL File Made in This Study as Viewed Using XYZware. Figure 4.14: (Right) Printed Optimal ABS Cantilever Prototyped in This Study Using a DaVinci 1.0 FDM Machine

The print time was under 30 minutes. The 314 $mm^3$ volume of the STL file matched the volume as computed in top3D. If any discrepancy had occurred between these two volumes, then an issue would have been revealed in the discretization and mapping functions discussed earlier. Tetrahedral meshing was imposed on a solid step file converted from the STL file shown in Figure 4.13 for testing the optimal cantilever model using FEA. File conversion from the STL file to step file (.stp) in this study was executed using Linux based FreeCAD. Meshing and FEA in this case were executed in Linux based Salome Meca and Code Aster plug-ins respectively. The maximum displacement in the red region of Figure 4.15 is 0.00406 mm which is an order of magnitude larger than the original model. The strain in this red region is only 0.041% of the original 10 mm cantilever height.

Therefore, topology optimization, using equations 1-3, the loads, constraints, and objectives as defined immediately before Figure 4.11, decreased the volume 52.3%, yet strain has not even passed 0.1%. There is a way to reduce the volume here even further.

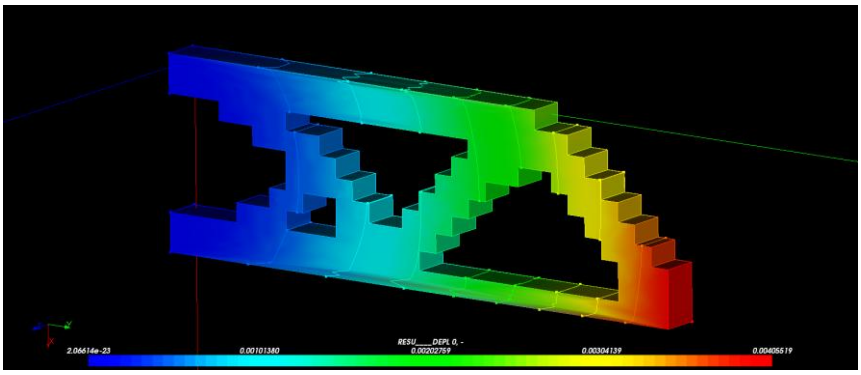
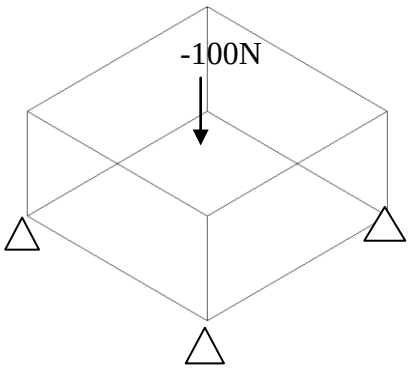



Figure 4.15: FEA Made in This Study of the Optimal Cantilever Prototype Model Using Linux Based Software Salome Meca and Code Aster

Recalling the topology optimization results in a 3D array of moduli for each voxel, and the moduli belong to a continuous range [0, $E_0$]. The only means of writing the STL file in Figure 4.13 was setting a binary threshold, so the moduli under the threshold were eliminated leaving only moduli above the threshold in the model. Increasing the threshold value slightly should eliminate slightly more material and cause a slightly further reduction in the prototype's volume without increasing the strain much.

A 40 mm x 20 mm x 40 mm simply supported platform was loaded with a 100N point force in the top dead center as shown in Figure 4.16. The volume constraint $V_s$ for equation 1 was set to 0.5 or 50% of the 40 mm x 20 mm x 40 mm original volume. Therefore, the objective was to find at most a 16000 $mm^3$ design which supported the 100N distributed load while simply supported. The coordinates of the loads and constraints are shared in Appendix K. The FEA displacement plot of this system shows displacement existing primarily near the loading point in the red region of Figure 4.17. Most of the platform was left un-strained as shown in the blue region. The maximum

displacement in the red region was 0.00000399 m in the z-direction only, and no buckling was observable.



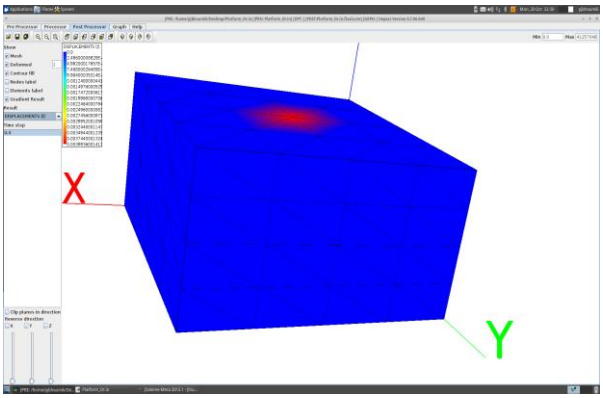




Figure 4.16: (Left) The Force Diagram for the Platform. Figure 4.17: (Right) The FEA Plot of This Platform System Made in This Study Using Linux Based ImpactFEA

Regulated SIMP based topology optimization of the platform subject to the load, constraints, and the objective volume constraint as defined immediately before Figure 4.16 removed 25616 $mm^3$ from the original 32000 $mm^3$ design. The resulting 6384 $mm^3$ STL file shown in Figure 4.18 was printed during this study using a DaVinci 1.0 printer to make the prototype shown in Figure 4.19. The ruler shown in juxtaposition with the prototype of the optimal cantilever proves the topology optimization did not adversely alter the scale of the 40 mm length dimension. The width and height dimensions are preserved after the optimization as well, so any assembly fit requirements for this part are still preserved.

The print time during this study was under 60 minutes for printing the prototype in Figure 4.19 from the STL file in Figure 4.18 using a DaVinci 1.0 FDM machine. The

6384 $mm^3$ volume of the STL file matched the volume as computed in `top3D`, so again, the discretization and mapping functions which were made in this study and discussed earlier were accurate.

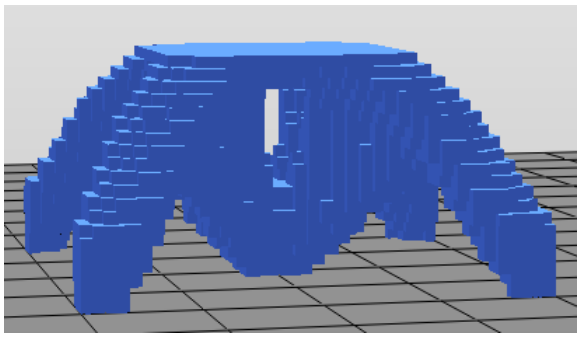


Figure 4.18: (Left) STL File Made in This Study as Viewed Using XYZware. Figure 4.19: (Right) Printed Optimal ABS Platform Prototyped in This Study Using a DaVinci 1.0 FDM Machine.

Tetrahedral meshing was imposed on a solid step file converted from the STL file shown in Figure 4.18 for testing the optimal cantilever model using FEA. File conversion from STL to stp in this study was executed using Linux based FreeCAD. Meshing and FEA in this case were executed in Linux based Salome Meca and Code Aster plug-ins respectively. The maximum displacement in the red region is 0.0341 mm which is an order of magnitude larger than the original model. The strain in this red region, shown in Figure 4.20, is only 0.171% of the original 20 mm platform height.

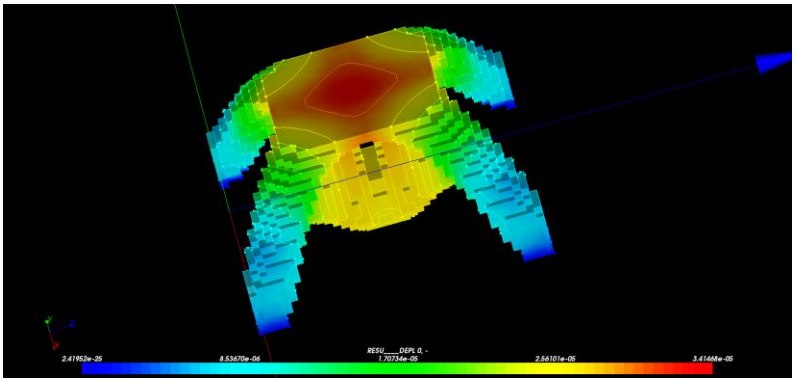


Figure 4.20: FEA of the Optimal Platform Prototype Model Using Linux Based Software Salome Meca and Code Aster

Therefore, topology optimization, using equations 1-3, the loads, constraints, and objective function volume constraint as defined immediately before Figure 4.16, decreased the volume 80%, yet strain had not even passed 0.1%. An in depth study of the strain behavior required physical compression testing of this optimal 40 mm x 20 mm x 40 mm platform. Five platforms were printed using a DaVinci 1.0 FDM printer. The DaVinci 1.0 prints quasi-hollow models using a honey comb lattice. Lattice density can be varied using the XYZware software of the DaVinci 1.0. Density can range from 30% to 90%. The 90% density setting was used in fabricating the platforms tested in this study.

Each platform was placed on a level plane and compressive forces were added using free weights to a top surface placed over the platform. An example compression test setup was photographed and shown in Figure 4.21. The photographed platform was optimized for a 100 N load, and the compression test range was 0 N to 178 N.

Compression occurred on the vertical axis of the platform. The initial height was 20 mm, and initial load was 0 N.

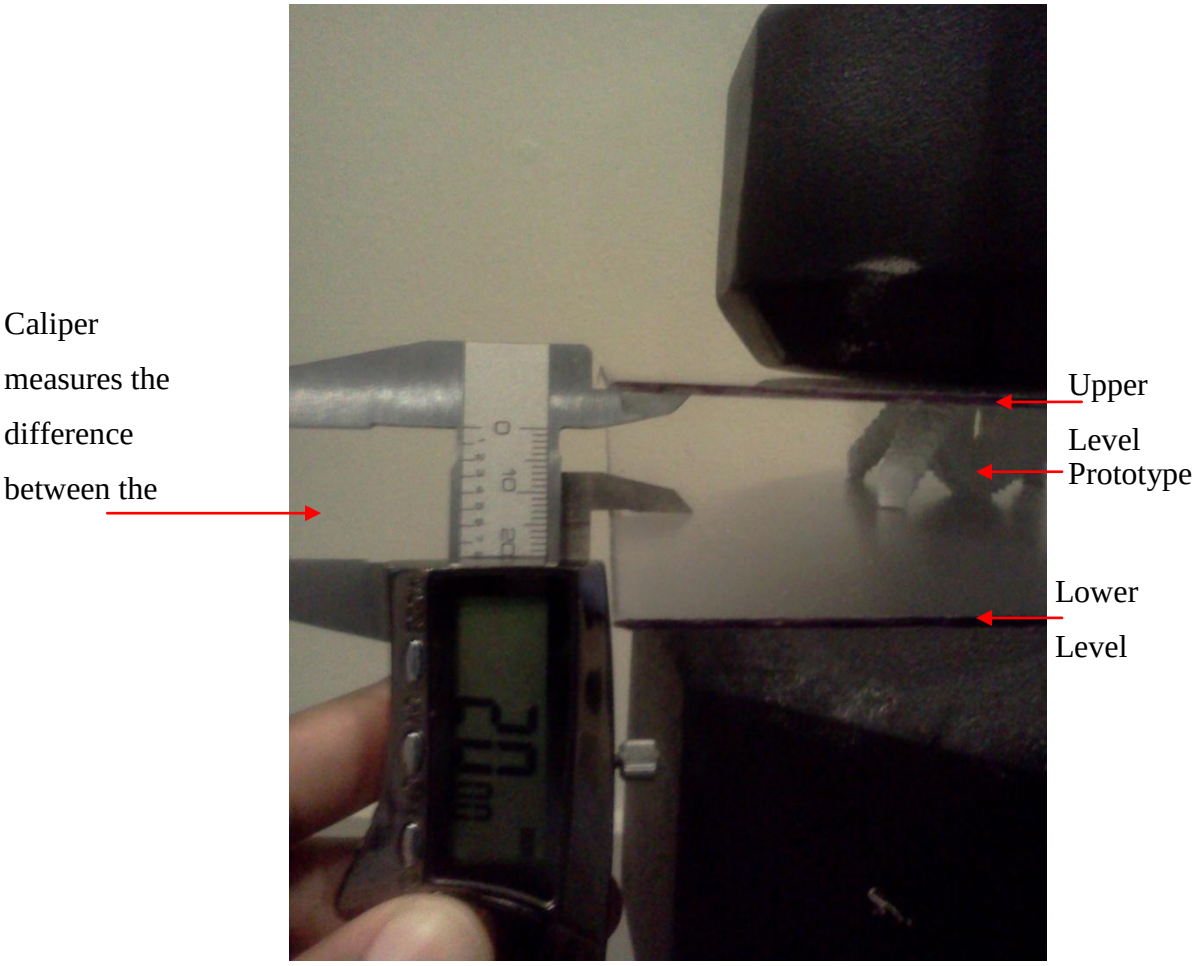



Figure 4.21: An Example of the Compression Test Setup Employed in This Study. The 40 mm x 20 mm x 40 mm optimal platform sits on a level surface under 44,497 N of compressive weight.

A General UltraTech digital caliper with a resolution of 0.01 mm was used for measuring the height of each platform three times for each compressive load. One inside jaw was placed against the top level and the other inside jaw was rested against the

bottom level. The resulting measurement equals the height of the prototype under compression. The average of three such height measurements for each prototype per compressive load is shown in Table 4.3 and all the individual measurements are shared in Appendix J.

Table 4.3: Compression Test Results

| Compressive Force (N) | Average Height of Prototype 1 after compression | Average Height of Prototype 2 after compression | Average Height of Prototype 3 after compression | Average Height of Prototype 4 after compression | Average Height of Prototype 5 after compression |
|---|---|---|---|---|---|
| 0 | 20.577 | 20.000 | 20.013 | 20.000 | 19.997 |
| 44.5 | 20.557 | 19.987 | 20.007 | 19.997 | 19.987 |
| 66.75 | 20.550 | 19.983 | 20.003 | 19.990 | 19.977 |
| 89 | 20.543 | 19.973 | 20.000 | 19.983 | 19.973 |
| 111.25 | 20.537 | 19.970 | 19.990 | 19.980 | 19.963 |
| 133.49 | 20.523 | 19.960 | 19.983 | 19.970 | 19.963 |
| 177.99 | 20.513 | 19.953 | 19.973 | 19.967 | 19.953 |

The results of compression testing in Table 4.3 are discussed in terms of stress-strain behavior in Section 5.6.

The results of 40 mm x 20 mm x 40 mm optimization were scaled using a scale factor of 5 to assess practicality of fabricating larger optimal objects. Scaling 5 times in each direction made the 20 mm x 10 mm x 20 mm original design volume into a 200 mm x 100 mm x 200 mm envelope. Meshing this new volume meant increasing the number of

voxels 125 times, yet the computational time required for optimization on this scale would require 34 hours extrapolating from the results of Table 2. Scaling directly the results of optimizing these small volumes required the development of `scaletop3D` which scales the voxel moduli *n* x *n* x *n* array and returns a scaled *sn/h* x *sn/h* x *sn/h* array where *s* is the scaling factor scale and *h* is the mesh discretization factor `h_partition`. The scale is user defined in the function call for `scaletop3D`:

```
optmodel = scaletop3D(optmodel,scale, h_partition)
```

where the argument weight represents the modulus of each voxel element in the design volume and the argument scale is a 1x3 vector representing the user's desired 3D scale factor for each direction. If the user enters a scale factor less than unity for any direction, then `scaletop3D` does not work. The scale factor is used in this study for the purpose of magnification only, so the expected value of each scale factor is greater than or equal to unity. MATLAB does not directly provide a means for scaling 3D arrays like weight, so the `scaletop3d` function replaces each voxel with its own *s/h* x *s/h* x *s/h* array named `del_V`.

```
del_V = zeros(scale/h_partition(1),scale/h_partition(2),scale/h_partition(3));

for z=1:size(weight, 3)
    for y=1:size(weight, 2)
        for x = 1:size(weight, 1)
            for z_scale = 1:scale/h_partition(3)
                for y_scale = 1:scale/h_partition(2)
                    for x_scale = 1:scale/h_partition(1)
                       del_V(x_scale, y_scale, z_scale) =weight(x,y,z);
                    end
                end
            end
            scaled_weight_arr{x,y,z}=del_V;
        end
    end
end
```

Every element of `del_V` equals the modulus of the previously single element from the weight array. The single element has thus been successfully scaled. Each `del_V` array is then stored in a structure, and then a new `del_V` array is made for the next voxel element in weight. The resulting structure must be concatenated along all three dimensions, so the result is transformation of the original 3D array into a structure and finally into a scaled 3D array. Appendix C contains the full code required in this transformation.

The number of elements in the scaled array is inversely proportional to the discretization factor `h_partition` and directly proportional to the scaling factor scale. The 704000 $mm^3$ optimized platform occupies only 17.6% of the overall design envelope's volume as shown in Figure 4.22.

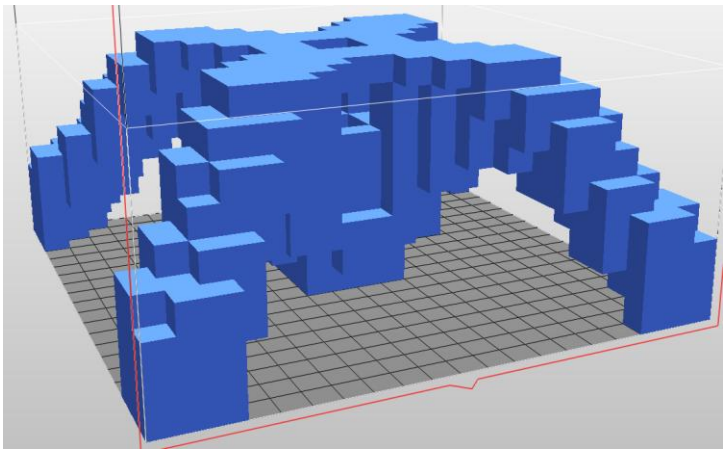


Figure 4.22: A 3D Scaled STL Made in This Study of the Optimal Platform Prototype

Finally, the capability of `optstl` to read in and optimize a pre-existing STL file was tested. Figure 4.23 shows an example of a model FDM tool:

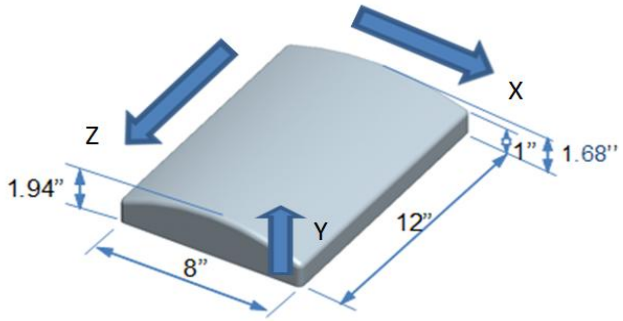



Figure 4.23: Example FDM Tool

An stl file was made after rendering a model of the tool shown in Figure 4.23. The stl file image is shown in Figure 4.24. The length and width of the tool were modified to 4 inches by 6 inches, so the tool would fit inside the FDM platform of the DaVinci 1.0 printer used in this study.

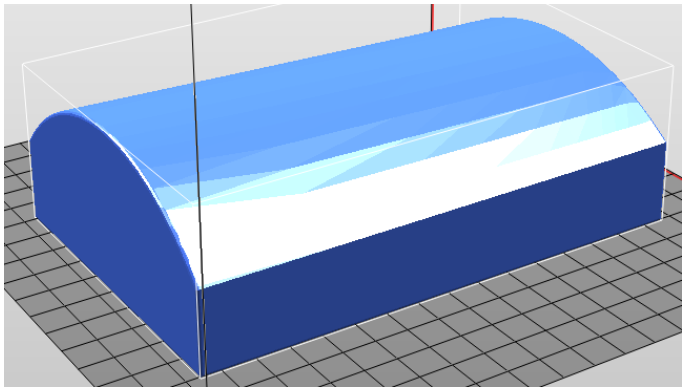


Figure 4.24: STL File of the FDM Tool as Viewed in XYZware

The stl file shown in Figure 4.24 was loaded into the optimization engine using a modified version of the publicly available `VOXELISE` MATLAB function. The modified version of this function made as a result of this study is `VOXELISE_FLEX`:

```
function [gridOUTPUT,varargout] = VOXELISE_FLEX(gridX,gridY,gridZ,
discretization, filename)
```

where `gridX`, `gridY`, and `gridZ` are the overall x, y, and z dimensions of the model rounded up to the nearest millimeter, `filename` is the file path of the STL model, and `discretization` is the number of voxel lengths per millimeter. A `discretization` factor of 1 would produce a mesh of 1 voxel per $mm^3$. Voxelising the stl shown in Figure 4.24 using a discretization factor of 1 yielded 875000 voxels. The stl file was then scaled down by a factor of 10 on each side, so the discretization factor could be increased and computation time decreased. Figure 4.25 shows an image of the voxelised model before optimization.

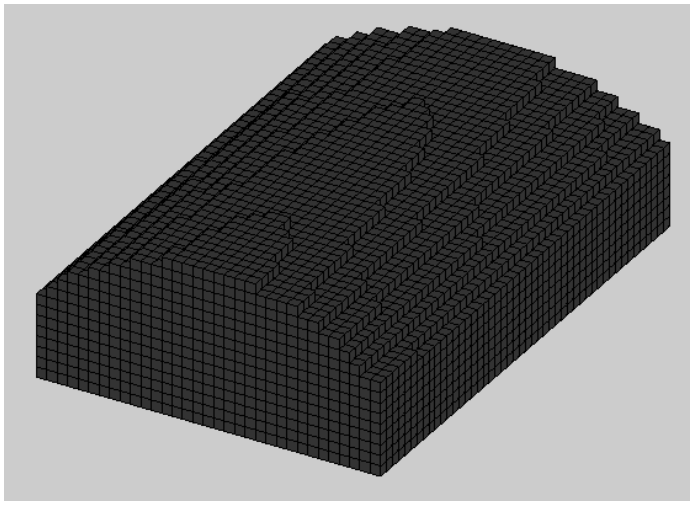


Figure 4.25: Voxelised FDM Tool as Viewed in the MATLAB Figure Window

A 100N point force and simply supported constraints were placed on the model as shown in Figure 4.26, and the volume constraint, $V_s$, for the objective function in equation 1 was set 0.5 meaning 50% of 30 mm x 48 mm x 15 mm envelope of the design. The locations of the loads and constraints are shared in Appendix K :

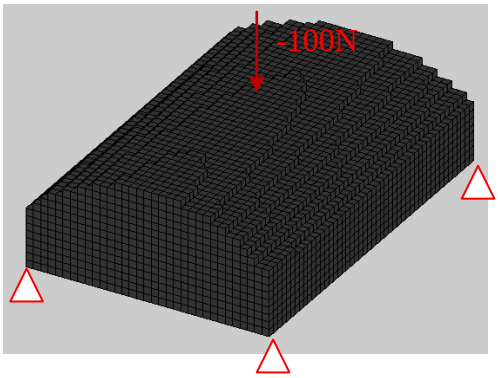



Figure 4.26: FDM Tool Load and Constraint Diagram

Figure 4.13 and 4.18 show the optimized model to have significantly different surface topology from the original model of each. Surfaces are often key components in a products function, so preserving the surface of the top and sides of the tool was studied. A `boundaryelements` function was written to find all the voxels on the top and sides of a model:

```
function boundaryelements = findboundary(gridOUTPUT, nelx, nely, nelz)
```

where `gridOUTPUT, nelx, nely,` and `nelz` are the original voxelised model, the number of elements in the x direction, the number of elements in the y direction ,and the number of elements in the z direction. Three Booleans are used to determine whether a

voxel is on the boundary of the model. First, the voxels at the bounds of the design volume can be found using this Boolean:

```
(gridOUTPUT(i,j,k)==1)&& ((i~=length(nelx) || (i==length(nelx) &&
((j==1) || (j==length(nely)) || (k==1) || (k==length(nelz))))))
```

Voxels not at the boundary of the design volume yet at the boundary of the model can be found using these two Booleans in order:

```
gridOUTPUT(i,j,k)==1) && (i~=1) && (i~=length(nelx)) && (j~=1) &&
(j~=length(nely)) && (k~=1) && (k~=length(nelz))

gridOUTPUT(i+1,j,k)==0 || gridOUTPUT(i-1,j,k)==0 ||
gridOUTPUT(i,j+1,k)==0 || gridOUTPUT(i,j,k+1)==0 ||gridOUTPUT(i,j,k-
1)==0
```

Figure 4.27 shows the resulting optimized model which from the top and sides is identical to topology of the original voxelised model in Figure 4.26. The resulting STL file was scaled up by a factor of 10 in each direction to produce the topologically optimized STL of the original STL shown in Figure 4.26. Figure 4.28 reveals the optimal interior of the model.

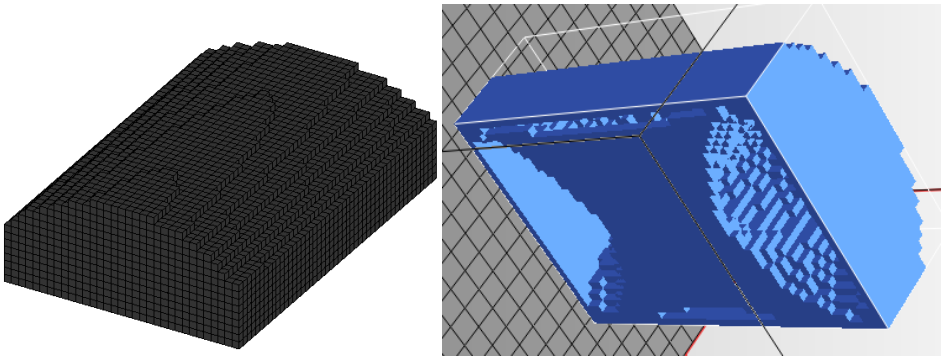


Figure 4.27: (Left) Optimized FDM Tool as Viewed in a MATLAB Figure Window.
Figure 4.28: (right) Optimized FDM Tool as Viewed at an Angle in XYZware.

# 5. DISCUSSION

## 5.1 CAPABILITY OF A SINGLE INTERFACE

The `optstl` interface provides a vast array of STL modification functions: meshing via `VOXELISE_FLEX`, mapping via `generate_cube_m`, topology optimization via `top3dFlex`, scaling transformation via `scaletop3D`, binary homogenization via `optcoordinates`, user input/output, point cloud generation, and stl writing. The `VOXELISE_Flex` function was adapted and modified from the publicly available `VOXELISE` Matlab function. Modifications to this function include the ability for the user to change the discretization factor and mesh density in the *x*, *y*, and *z* direction individually. The `generate_cube_m` was made originally in this study for trilinear discretization of the design volume for mapping into the optimization engine. The `top3dFlex` function was modified from the `top3d` function, so user no longer needs to know parametric functions to describe the surface of the input design volume. The original `top3d` function required explicit parameterized representation of the design volume's surface to map loads and constraints. Explicit parameterization of more complex design volume may not be feasible, so the modified `top3dFlex` function was developed in this study to allow a user to input just the Cartesian coordinates of the loads and constraints. Optimization increased the point cloud computation time by a minimum factor of 170 times, so a `scaletop3D` function was developed in this study to allows a user to optimize a small scale system and the scale up the results. Getting the results of the topology optimization required homogenization from the continuous distribution of moduli to a binary distribution, so the `optcoordinates` function was made originally in

this study to find all the vertices of voxels which met and did not meet a threshold value. Voxels which met the threshold value were assigned a 1, and voxels which did not meet the threshold value were assigned a 0. The `optstl` script passes this binary homogenized data into a copy of the publicly available `dxfpoint` function for generation of a point. A copy of the publicly available `CONVERT_voxels_to_stl` MATLAB function similarly interprets the binary distribution of voxels as those voxels with a 0 value were outside the stl while voxels with a 1 value were inside the stl.

### 5.2 TOP3DFLEX

Two of the three examples accompanying the `top3D` software were tested and worked successfully for this study. One example was the cantilever beam and the second was a platform [18]. The loads were changed in this study for testing the practicality of the software in fabricating ABS prototypes. The cantilever load in this study was a -100 N/mm distributed load putting the tip in shear, and the platform load in this study with a -100 N point force in the center of the platform. The Young's modulus used for ABS was 2150 MPa, and a table of the material properties appears in Appendix I. Liu and Tovar used a Young's modulus of 1 MPa and load magnitudes of -1N/mm and -1N for the cantilever and platform respectively [18]. The topology optimization produced a 58.3% reduction in cantilever's volume and a 92% reduction in the platform's volume. Liu and Tovar's software required the user to adjust values directly within the script [19]. The `top3dFlex` script operates within the `optstl` main script, so the user is allowed to input values for the Young's modulus, loads, and constraints on a case by case basis without risking corruption of the optimization engine. The loads and constraints of

Liu and Tovar's script required surface parameterization [19]. The `optstl` main script has user input/output which allows the user to input only the Cartesian coordinates of the loads and constraints, and then `optstl` maps these coordinates to voxel vertices using the trilinear discretization information of the `generate_cube_m` function. The `optstl` main script then passes these arguments directly into the `top3dFlex` script. The potential for the user to input large stl files for topology optimization meant a large number of voxel elements could be involved in the computation. Liu and Tovar discussed a fast iterative solver option for `top3d` [19], so `top3dFlex` included this iterative solver as a default solver.

### 5.3 MESHING, DISCRETIZATION, AND MAPPING

A user can now directly input any design volume via stl such as FDM tool model shown in Figure 4.23 to voxelized as shown in Figure 4.24 for input into the optimization engine. The `VOXELISE_Flex` function reads and meshes the input stl file as a 3D array of elements. The original `VOXELISE` function assigns 0's to all voxels in the design volume yet outside the stl and 1's to all the voxels inside the stl. Each voxel from the `VOXELISE` function represented 1 $mm^3$ of the model. Modifying the $1mm^3$ mesh density may be a user priority especially when working with complex surface geometry, so the `VOXELISE_Flex` function was developed in this study. Mesh density in the `VOXELISE_Flex` function is determined using a user defined discretization factor where the user is allowed to input the number of voxels required per millimeter.

The meshed design volume from the VOXELISE_Flex function is passed through optstl for mapping the user defined loading and constraints. User defined Cartesian coordinates of the loads and constraints are mapped to mesh indices using the generate_cube_m function. The first voxel in the system is located in the top back left and the last voxel is location in the bottom front left. Indexing first traverses top to bottom, then left to right, and finally back to front. Trilinear discretization in the generate_cube_m mapping algorithm was tested for two abilities. Simply mapping indices of all the voxel vertices in the design volume and connecting the vertex indices in order to create each associated voxel was one test. Figure 4.4 and Figure 4.5 proved the generate_cube_m function with optcoordinates can correctly map the topology optimization results from MATLAB figures to voxels for the stl files. The second feature tested was the capacity for adjusting the mesh fineness. Figures 4.4 and 4.5 illustrated the different discretization and mesh density possible through manipulation of the h_partition argument in generate_cube_m. Varying the values of h_partition indirectly varies the mesh density, so low values of h_partition create the most dense voxel meshes.

## 5.4 SCALING

Figure 4.22 illustrated the ability of optstl to scale an optimized model. Successfully overcoming the problem of scale here required the implementation of a scaling algorithm. MATLAB only has a scaling algorithm for 2D arrays, yet users can require scaling length, width, and height dimensions simultaneously. The scaling of each dimension was accomplished using scaletop3D. Any 3D array can be scaled using this

function, and the output mesh density can be adjusted using the user defined `h_partition`. Adjusting the value of h_partition indirectly changes the mesh density just the like in the case of generate_cube_m.

## 5.5 STL WRITING

Figures 4.13, 4.18, and 4.28 illustrated `optstl`'s ability to write stl files. A 3D array of 1's and 0's is passed via `optstl` to the publicly available `CONVERT_voxels_to_stl` function. The 1's indicate a voxel element is located within the model while the 0's indicate the voxel element is outside the model. All of the voxel elements are processed through a set of binary homogenization instructions from the optimization engine. Binary homogenization is the process of preparing the data for the stl writing function. Processing the data is a matter of having a set binary homogenization threshold value as discussed in paragraphs preceding Figure 4.7-4.10. Images for Figures 4.13, Figure 4.18, and Figure 4.28 use a binary homogenization threshold of 0.5. Topology optimization produced a 3D array with a continuous distribution of moduli from [0, $E_0$] where $E_0$ is the Young's modulus of the material used in fabrication. Finding all of the resulting values above the threshold and setting these values equal to 1 determined the interior of the stl models. Values below the threshold were set to 0 determining the outside of the stl models. Completing the binary homogenization was the last step required before writing the stl files.

## 5.6 TOPOLOGICALLY OPTIMIZED SOLID FREEFORM FABRICATION

The STL for each optimal model example was printed successfully during this study as illustrated in Figure 4.14 and Figure 4.19. A DaVinci 1.0 printer extruding ABS

filament was used in fused deposition modeling of each print in this study. None of these prototypes required over 1 hour to print. Volumes for each printed model matched the volumes of the computed models revealing no error in the discretization, meshing, and mapping functions discussed earlier. Testing the integrity of the printed STL was paramount after analyzing the FEA plots of the optimized models. Each model showed less than 0.1% strain after optimization as illustrated in Figure 4.15 and Figure 4.20. Only physical testing could validate the true mechanical compliance evident in these results, so the printed optimal ABS platform was subjected to compression testing. Data from this compression test was shared in Table 4.3, and the raw data is shared in Appendix J. Table 5.1 and Figure 5.1 display the prototype's behavior under compression, the theoretical behavior based on the Young's modulus of ABS, and a light blue line indicates the design load of 100 N.

Strains were calculated for each data point from Table 4.3 was calculated using Eq 10:

$$\varepsilon_i = \frac{h_0 - h_i}{h_0} \quad \text{(Eq 10)}$$

where the *i*th strain value $\varepsilon_i$ is the difference between the prototype's initial height $h_0$ and the height after the *i*th load divided by the prototype's initial height $h_0$. Forces from Table 5.1 are the numerator for the stress calculated in Pascals for Figure 5.1 using Eq 11:

$$\sigma_i = \frac{F_i}{A} \quad \text{(Eq 11)}$$

where the *i*th stress value $\sigma_i$ is the ratio of the ith compressive force $F_i$ to the prototype's constant load bearing surface area, A = 4.75 x $10^{-4}$ $m^3$. Substituting values for each load

into Eq 5 produces the control strain distribution shown in the second column of Table 5.1:

$$\varepsilon_{ci} = \frac{\sigma_i}{E} \quad \text{(Eq 12)}$$

where the *i*th control strain $\varepsilon_{ci}$ is the ratio of the *i*th strain to the constant Young's modulus of ABS, E = 2150 MPa. The results were tabulated in Table 5.1.

Table 5.1 Prototype Compression Test Stress and Strain Results

| Compressive force (N) | Control Strain | Strain 1 | Strain 2 | Strain 3 | Strain 4 | Strain 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4.450E+01 | 2.069E-08 | 0.001 | 6.667E-04 | 3.331E-04 | 1.667E-04 | 5.001E-04 |
| 6.675E+01 | 3.104E-08 | 1.296E-03 | 8.333E-04 | 4.997E-04 | 5.000E-04 | 1.000E-03 |
| 8.900E+01 | 4.139E-08 | 0.002 | 1.333E-03 | 6.662E-04 | 8.333E-04 | 1.167E-03 |
| 1.112E+02 | 5.174E-08 | 1.944E-03 | 1.500E-03 | 1.166E-03 | 1.000E-03 | 1.667E-03 |
| 1.335E+02 | 6.209E-08 | 2.592E-03 | 2.000E-03 | 1.499E-03 | 1.500E-03 | 1.667E-03 |
| 1.780E+02 | 8.278E-08 | 3.078E-03 | 2.333E-03 | 1.999E-03 | 1.667E-03 | 2.167E-03 |

The calculated values from Table 5.1 are plotted in Figure 5.1, so the typical compression behavior can be observed. Compression testing occurred in the elastic region of ABS, so the Young's modulus could be chosen a control. The plot in Figure 5.1 shows the compression testing stress strain behavior of the five prototypes. One line in Figure 5.1 shows the theoritical stress-strain behavior as modelled using Young's modulus, and another line shows the stress-strain behavior as predicted using the FEA in

Figure 4.20. No fractures were observed after the final load step. Wave like behavior in Figure 5.1 is attributed to the experiment setup and instrument resolution in Figure 4.21.

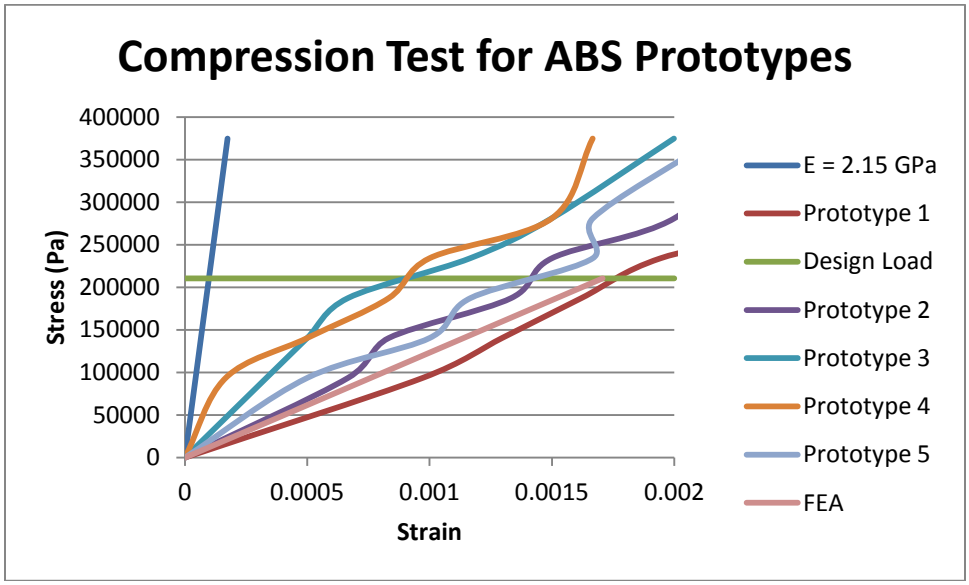



Figure 5.1: The ABS Prototype's Performance vs the Young's Modulus of ABS and the Computed FEA Result

The FEA shown in Figure 4.20 computed that the maximum strain should be 0.171%. A one sample t-test of the compression test data near the design load at 234,082.6 Pa yielded a $t$ value of -2.615. If $t < t_{critical,\alpha}$ for the one-sided one sample t-test, then there is a statistical directional difference from the expected mean value. A one-tailed 95% confidence $t_{critical,\alpha}$ one tailed value computed in MS Excel was -2.132, so the compression testing results were significantly less than the FEA's result with 95% confidence. However, the one tailed 99% confidence $t_{ciritical,\alpha}$ value computed in MS

Excel was -3.747, so the differences between the prototypes and FEA are not signficantly different at the 99% confindence level.

The $\chi^2$ test was selected for testing the difference between the observed prototypes' behavior under compression and the Young's modulus as shown in Table 5.2. If the p-value is less than 0.01, then the two behaviors are different with 99.99% confidence.

Table 5.2: *p*-values for χ^2 Test Comparison of Each Prototype Sample Against the Expected Behavior from Young's Modulus

| Prototype # | p-Value of χ^2 Comparison Prototype and Control |
|---|---|
| 1 | 6.24947E-97 |
| 2 | 3.15124E-54 |
| 3 | 2.24923E-27 |
| 4 | 3.13694E-23 |
| 5 | 3.33495E-48 |

All of the *p*-values are less than 0.01, so the prototypes'stress-strain behavior under compression was signfiantly different from the Young's modulus. The reason for this difference is the change in cross sectional area. Cross sections of the control modelled using Young's modulus were uniform while the cross sections of the prototype shown in Figure 4.18 are non-uniform. The second area moment of inertia is directly proportional to cross sectional area and measures the amount of resistance an object may have to a given static load. Therefore, decreasing cross sectional areas from the model in

the prototype reduced the second area moment of inertia in the prototype and caused the decrease in the stress-strain relationship from the Young's modulus behavior.

## 5.7 ADAPTIVE PLACEMENT OF USER DEFINED LOAD(S) AND CONSTRAINT(S) ON USER DEFINED VOLUMES

Images shown in Figures 4.2-4.5, 4.7-4.10, 4.13-4.14, 4.18-4.19, 4.22 and 4.27-4.28 were all made in this study using `optstl`. The `optstl` loading and constraint instructions allow the user to input Cartesian coordinates of loads and constraints. Loading this information using coordinates is not permitted in the original `top3d`. Load and constraint information in `top3d` required explicit surface parameterization meaning the user had to know how to calculate the index of a voxel at an (*x*,*y*,*z*) coordinate. Using `optstl` the user can input just (*x*,*y*,*z*) coordinate of a point load or constraint, and the optstl alogorithm uses the `generate_cube_m` function made in this study. Use of the `generate_cube_m` function generates both a list of all the (*x*,*y*,*z*) voxel vertice coordinates in the design volume and a list of the order in which to connect these vertices. Trilinear discretization dictates the programmed order in which these vertices are connected to produce the voxelized space. Surface constraints and uniformly distributed loads can be applied as well. If the user wishes, then `optstl` holds a user defined dimension constant in order to load or constrain an entire 2D plane of active voxels. Active voxels were stored in the memory as voxel elements with a value of 1using the binary homogenization instructions of optcoordinates discussed earlier. Passive voxels would have a value of 0. Setting loads and constraints based on Cartesian coordinates simplifies the surface parameterization step. A resulting list of loaded and constrained vertices are passed to `top3dFlex` along with the Young's modulus and an

array of passive elements. Passing the passive elements into the topology optimization engine forces the computation to retain any features such as holes [19]. Large quantities of passive or active elements could cause long computation times [19], so the `top3dFlex` algorithm was adapted to implement the MATLAB `pcg` solver. Liu and Tovar recommended the `pcg` solver for the fastest computation speed [19]. Implementing the `pcg` solver allows MATLAB to determine the best solution for system of linear equations at hand. Adapting `top3d` into `top3dFlex` and supplementing this function using the `optstl` main script decreases the amount of hard coding the user must do in order to modify the program to differenct design requirements. Supplementing the script further with an STL reading function allows the user to load and constrain any design volume. Figure 4.17 demonstrates the ability of any STL to be voxelized for input into the optimization engine. If the model's function requires that certain voxels be preserved such as the outermost layer of voxels, then the `boundaryelements` function can be used as demonstrated in Figure 4.28.

# 6. CONCLUSION

The fabrication of topologically optimized parts is realizable using `optstl` . Topology optimization was shown in this study to produce parts of significantly less elastic modulus than unoptimized parts of the same load and constraint parameters. ABS parts were fabricated in this study using `optstl` and a DaVinci 1.0 FDM machine. Fabrication required the adaption of Liu and Tovar's `top3d` algorithm and several supplemental functions. Adapting the `top3d` algorithm resulted in a user input/output interface for optimizing any stl input subject to user defined load and constraint coordinates. Output from this adapted program named `optstl` can be a scaled model, point cloud, and/or an stl file. Each optimized prototype was analyzed using FEA, and one optimized prototype was subjected to compression testing. Supplemental FEA's of the original models are shared in Appendix H. An order of magnitude increase in the strain was observed in the optimized prototypes when compared with the original models. Statistical testing of the compression test results did reveal a significant statistical difference between a theoretical solid ABS volume's behavior and the optimized prototype's behavior. The actual stress-strain behavior resembles that as predicted in the FEA result in Figure 4.20.

Solid freeform fabrication users now have a MATLAB preprocessor for loading, optimizing, and writing the STL's of design volumes. Unification of these engineering design processes is provided in `optstl`. The publicly available `top3D`, `VOXELISE`, and `CONVERT_voxels_to_stl` algorithms were found, modified, and coalesced

algorithmically in `optstl`. Added functionality such as scaling, discretization, binary homogenization, and user defined volumes required supplemental functions and modifications of the pre-existing function arguments. Adjusting a 3D array's scale was a matter of applying fundamentals from the 1D and 2D mathematical transformations for the `scaletop3D` function. All of the results produced in this study can be scaled using a scaling algorithm for 3D MATLAB arrays developed successfully during this study. Optimizing the smaller design envelope and then scaling the result can save computation time of a large volume. MATLAB did not previously have a 3D scaling function in the MATLAB library or on the internet. Using MATLAB structures, arrays, and concatenation in a combination of nested for loops yielded the sufficient scaling transformation for this study. Mapping required knowledge of FEA techniques for solving partial differential equations in a trilinear discretized system for the `generate_cube_M function`. Trilinear discretization produces finite element mesh comprised of cubes which can map 1 to 1 with voxels. Indexing the trilinear discretized mesh for this study was already discussed in Liu and Tovar's paper, so the indexing process was simply automated for user's of `optstl`. Automating and storing the index data mapping of the voxelized system enables the user to input an (x,y,z) coordinate and return an array element index for the optimization functions. Allowing the user to communicate using the Cartesian coordinate system simplifies the process of translating the real data into the computer, for example a user could use CMM or point cloud data. All of `optstl` is written in MATLAB. Core components of `optstl` are shared in Appendix A- Appendix D, and converting these scripts to a C based programming language or parallel computing algorithms can increase computation speed.

The `optstl` package coalesced the functions of model loading, constraining, topology optimization, scaling, mapping, and stl writing. Using `optstl` for solving equations 1-3 for design volumes did reduce the amount of raw material required for the fabrication of load bearing structures. Material costs and fabrication times were in turn reduced because of the decrease in the volume and amount of material required. Compression testing showed that the optimized parts deformed significantly more than unoptimized parts, yet each prototype in this study did support its design load. Any user defined stl could be optimized using equations 1-3, loads, and simply supported constraints in `optstl`, and the function of the model could be preserved using the `boundaryelements` function of `optstl`.

# APPENDIX A:

# OPTSTL MAIN SCRIPT

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
%%Purnajyoti Bhaumik wrote this for topology optimization of any 3D
model

function optstl

prompt = 'Would you like to input an STL file or optimize a rectangular
prism? Enter Y or N. ';
source_type = input(prompt,'s');
if isempty(source_type)
    return
end
if source_type == 'Y'
    prompt = 'What is the source STL filename (include file path and
extension ex: C:\test.stl)? ';
    STLin = input(prompt,'s');
    if isempty(STLin)
        return
    end
    gridX = input('What is the overall height (in mm)?');
if isempty(gridX)
    return
end

gridY = input('What is the overall width (in mm)?');
if isempty(gridY)
    return
end

gridZ = input('What is the overall depth (in mm)?');
if isempty(gridZ)
    return
end
end



if source_type == 'N'
        gridX = input('What is the overall width (in mm)?');
if isempty(gridX)
    return
end

gridY = input('What is the overall height (in mm)?');
if isempty(gridY)
    return
end

gridZ = input('What is the overall depth (in mm)?');
if isempty(gridZ)
    return
end
```

```
    nelx = 1:1:gridX;
    nely = 1:1:gridY;
    nelz = 1:1:gridZ;

    for k = 1:gridZ
        for i = 1:gridX
            for j = 1:gridY
                gridOUTPUT(i,j,k) = 1;
            end
        end
    end
    %gridOUTPUT(:,:,:)=1;
    gridOUTPUT= permute(gridOUTPUT, [2,1,3]);
    display_3D(gridOUTPUT)
end

if source_type == 'Y'
    prompt = 'How many voxels per millimeter? ';
    discretization = input(prompt);
    if isempty(discretization)
        returns
    end
[gridOUTPUT,nely,nelx,nelz] = VOXELISE_FLEX(gridX, gridY, gridZ,
discretization,STLin);
gridX = length(nelx);
gridY = length(nely);
gridZ = length(nelz);
display_3D(gridOUTPUT)
end

passive = find(~gridOUTPUT);
active = find(gridOUTPUT);

[M, T] = generate_cube_M(0, length(nelx), 0, length(nely), 0,
length(nelz), [1,1,1],1);

load_answer = 'Y';
i=0;
while load_answer == 'Y'
prompt = 'Would you like a distributed load or a point force? Enter D
or P: ';
load_type = input(prompt, 's');
if isempty(load_type)
    return
end
i = i+1;
if load_type == 'D'
    prompt = 'Would you like this load distributed in a perpindicular
to the width, depth, or height of the model? Enter w, d, or h: ';
        load_plane = input(prompt, 's');
        if load_plane == 'h'
            prompt = ('At what distance from the bottom would you like
this distributed load? Enter this distance in whole millimeters.');
            load_plane_width = input(prompt);
```

```
            if isempty(load_plane_width)
                return
            end
             for z = 1:gridZ
                for x = 1:gridX
                     for m = 1:length(active)
                         if active(m)== load_plane_width+(x-
1)*(gridY)+(z-1)*(gridY*gridX)
                             loadnid{i} =
unique(T([5,6,7,8],active(m)));
                             i = i+1;
                         end
                     end
                end
              end
              i = i-1;
        elseif load_plane == 'd'
            prompt = ('At what distance from the back would you like
this surface constraint? Enter this distance in whole millimeters.');
            load_plane_width = input(prompt);
            if isempty(load_plane_width)
                return
            end
              for x = 1:gridX
                  for y = 1:gridY
                     for m = 1:length(active)
                         if active(m)==(x-
1)*gridY+y+load_plane_width*(gridY*gridZ)
                             loadnid{i} =
unique(T([1,2,7,8],active(m)));
                             i = i+1;
                         end
                     end
                  end
              end
              i = i-1;
        elseif load_plane == 'w'
            prompt = ('At what distance from the left would you like
this surface constraint? Enter this distance in whole millimeters.');
            load_plane_width = input(prompt);
            if isempty(load_plane_width)
                return
            end
              for z = 1:gridZ
                  for y = 1:gridY
                     for m = 1:length(active)
                         if active(m)==(z-
1)*(gridX*gridY)+y+load_plane_width*gridY
                             loadnid{i} =
unique(T([1,4,5,8],active(m)));
                             i = i+1;
                         end
                     end
                  end
              end
              i = i-1;
```

```
        end
elseif load_type == 'P'
    prompt = ('You will be asked for the 3D coordinates of this point
force. What is the x-coordinate in millimeters?');
    pforcex = input(prompt);
    prompt = ('What is the y-coordinate in millimeters?');
    pforcey = input(prompt);
    prompt = ('What is the z-coordinate in millimeters?');
    pforcez = input(prompt);
    for k = 1:size(M,2)
        if M(1,k)==pforcex && M(2,k)==pforcey && M(3,k)==pforcez
            loadnid{i} = k;
        end
    end
end
prompt = ('Would you like to enter another load? Enter Y or N');
load_answer = input(prompt, 's');
if isempty(load_answer)
    return
end
end

final_load = loadnid{1};
for j = 2:i
final_load = cat(1, final_load, loadnid{j});
end

prompt = 'What is the magnitude of the load?';
load_mag = input(prompt);
if isempty(load_mag)
    return
end

constraint_answer = 'Y';
i=0;
while constraint_answer == 'Y'
prompt = 'Would you like a surface or a point constraint? Enter S or P:
';
constraint_type = input(prompt, 's');
if isempty(constraint_type)
    return
end
i = i+1;
if constraint_type == 'S'
    prompt = 'Would you like this load distributed perpindicular the
width, depth, or height of the model? Enter w, d, or h: ';
        constraint_plane = input(prompt, 's');
        if constraint_plane == 'h'
            prompt = ('At what distance from the bottom would you like
this surface constraint? Enter this distance in whole millimeters.');
            constraint_plane_width = input(prompt);
            if isempty(constraint_plane_width)
                return
            end
             for z = 1:gridZ
                for x = 1:gridX
```

```
                    for m = 1:length(active)
                        if active(m)== constraint_plane_width+(x-
1)*(gridY)+(z-1)*(gridY*gridX)
                            constraintnid{i} =
unique(T([5,6,7,8],active(m)));
                            i = i+1;
                        end
                    end
                end
            end
            i = i-1;
        elseif constraint_plane == 'd'
            prompt = ('At what distance from the back would you like
this surface constraint? Enter this distance in whole millimeters.');
            constraint_plane_width = input(prompt);
            if isempty(constraint_plane_width)
                return
            end
              for x = 1:gridX
                  for y = 1:gridY
                     for m = 1:length(active)
                        if active(m)==(x-
1)*gridY+y+constraint_plane_width*(gridY*gridZ)
                            constraintnid{i} =
unique(T([1,2,7,8],active(m)));
                            i = i+1;
                        end
                    end
                end
            end
            i = i-1;
        elseif constraint_plane == 'w'
            prompt = ('At what distance from the left would you like
this surface constraint? Enter this distance in whole millimeters.');
            constraint_plane_width = input(prompt);
            if isempty(constraint_plane_width)
                return
            end
              for z = 1:gridZ
                  for y = 1:gridY
                     for m = 1:length(active)
                        if active(m)==(z-
1)*(gridX*gridY)+y+constraint_plane_width*gridY
                            constraintnid{i} =
unique(T([1,4,5,8],active(m)));
                            i = i+1;
                        end
                    end
                end
            end
            i = i-1;
        end
elseif constraint_type == 'P'
    prompt = ('You will be asked for the 3D coordinates of this point
force. What is the x-coordinate in millimeters?');
    pconstraintx = input(prompt);
```

```
    prompt = ('What is the y-coordinate in millimeters?');
    pconstrainty = input(prompt);
    prompt = ('What is the z-coordinate in millimeters?');
    pconstraintz = input(prompt);
    for k = 1:size(M,2)
        if M(1,k)==pconstraintx && M(2,k)==pconstrainty &&
M(3,k)==pconstraintz
            constraintnid{i} = k;
        end
    end;
end
prompt = ('Would you like to enter another constraint? Enter Y or N');
constraint_answer = input(prompt, 's');
if isempty(constraint_answer)
    return
end
end

final_constraint = constraintnid{1};
for j = 2:i
final_constraint = cat(1, final_constraint, constraintnid{j});
end
final_constraint = unique(final_constraint);


t = cputime


prompt = ('What is the modulus of elasticity in MPa?');
Young_answer = input(prompt);
if isempty(Young_answer)
    return
end

active = findboundary(gridOUTPUT, nely, nelx, nelz);
test = gridOUTPUT;
test(~active) = 0;
test(active) = 1;
clf;
display_3D(test)
optmodel = top3dFlex(length(nelx),length(nely),length(nelz), 0.3, 3,
1.5, final_load, final_constraint, passive, Young_answer,
load_mag,active);


prompt = ('Would you like to scale this model? Please enter Y or N');
scale_answer = input(prompt, 's');
if isempty(scale_answer)
    return
end

if scale_answer == 'Y'
```

```
    prompt = 'How many voxels per millimeter? (recommended at least 1
voxel per millimeter) ';
    discretization = input(prompt);
    if isempty(discretization)
        return
    end

    prompt = ('Please enter a scale factor for the x-axis: Only whole
numbers greater than or equal to 1');
    x_scale = input(prompt);
    if isempty(x_scale)
        return
    end
    for i = 1:x_scale*length(nelx)/discretization
        nelx(i) = min(nelx)+(i-1)*(discretization);
    end
    prompt = ('Please enter a scale factor for the y-axis: Only whole
numbers greater than or equal to 1');
    y_scale = input(prompt);
    if isempty(y_scale)
        return
    end
    for i = 1:y_scale*length(nely)/discretization
        nely(i) = min(nely)+(i-1)*(discretization);
    end
    prompt = ('Please enter a scale factor for the z-axis: Only whole
numbers greater than or equal to 1');
    z_scale = input(prompt);
    if isempty(z_scale)
        return
    end
    for i = 1:z_scale*length(nelz)/discretization
        nelz(i) = min(nelz)+(i-1)*(discretization);
    end
    scale = [x_scale, y_scale, z_scale];
    h_partition = [discretization, discretization, discretization];
    optmodel = scaletop3D(optmodel,scale, h_partition)
end

opt_passive = find(optmodel<=0.5);
opt_active = find(optmodel>0.5);
optmodel(opt_passive) = 0;
optmodel(opt_active) = 1;
optmodel(active) = 1;
clf;
display_3D(optmodel)

prompt = ('Would you like to make a point cloud? Please enter Y or N');
cloud_answer = input(prompt, 's');
if isempty(cloud_answer)
    return
elseif cloud_answer == 'Y'
    point_cloud = optcoordinates(M,T, optmodel)
    prompt = ('What is the destination dxf filename (include file path
and extension ex: C:\test.dxf)? ');
    cloud_answer = input(prompt, 's');
```

```
    if isempty(cloud_answer)
    return
    end
    FID = dxf_open(cloud_name);
    dxf_point(FID,point_cloud(3,:), point_cloud(1,:),
point_cloud(2,:));
    dxf_close(FID);
end


prompt = ('Would you like to make an STL file? Please enter Y or N');
stl_answer = input(prompt, 's');
if isempty(stl_answer)
    return
elseif stl_answer == 'Y'
prompt = ('What is the destination STL filename (include file path and
extension ex: C:\test.stl)? ');
STLout = input(prompt,'s');
CONVERT_voxels_to_stl(STLout, optmodel, nely, nelx, nelz,'ascii');
end
cputime - t
end
% DISPLAY 3D TOPOLOGY (ISO-VIEW)is copied from Liu and Tovar's top3d
function display_3D(rho)
[nely,nelx,nelz] = size(rho);
hx = 1; hy = 1; hz = 1;            % User-defined unit element size
face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
set(gcf,'Name','ISO display','NumberTitle','off');
for k = 1:nelz
    z = (k-1)*hz;
    for i = 1:nelx
        x = (i-1)*hx;
        for j = 1:nely
            y = nely*hy - (j-1)*hy;
            if (rho(j,i,k) > 0.5)  % User-defined display density
threshold
                vert = [x y z; x y-hx z; x+hx y-hx z; x+hx y z; x y
z+hx;x y-hx z+hx; x+hx y-hx z+hx;x+hx y z+hx];
                vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -
vert(:,2,:);

patch('Faces',face,'Vertices',vert,'FaceColor',[0.2+0.8*(1-
rho(j,i,k)),0.2+0.8*(1-rho(j,i,k)),0.2+0.8*(1-rho(j,i,k))]);
                hold on;
            end
        end
    end
end
axis equal; axis tight; axis off; box on; view([30,30]); pause(1e-6);
end
```

# APPENDIX B:

# TOP3DFLEX FUNCTION

```
%P. Bhaumik's Oct 2014 optimization code based on code by LIU AND TOVAR
(JUL 2013)
function xPhys = top3dFlex(nelx,nely,nelz,volfrac,penal,rmin, loadnid,
fixednid,passive, Young, load_mag,active)
% USER-DEFINED LOOP PARAMETERS
maxloop = 200;    % Maximum number of iterations
tolx = 0.01;      % Termination criterion
displayflag = 1;  % Display structure flag
% USER-DEFINED MATERIAL PROPERTIES
E0 = Young;           % Young's modulus of solid material titanium
alloy
Emin = 1e-9;      % Young's modulus of void-like material
nu = 0.3;         % Poisson's ratio
% USER-DEFINED LOAD DOFs
loaddof = [3*loadnid(:) - 1];                              % DOFs
% USER-DEFINED SUPPORT FIXED DOFs
fixeddof = [3*fixednid(:); 3*fixednid(:)-1; 3*fixednid(:)-2]; % DOFs
% PREPARE FINITE ELEMENT ANALYSIS
nele = nelx*nely*nelz;
ndof = 3*(nelx+1)*(nely+1)*(nelz+1);
F = sparse(loaddof,1,load_mag/size(loadnid,1),ndof,1);
U = zeros(ndof,1);
freedofs = setdiff(1:ndof,fixeddof);
KE = lk_H8(nu);
nodegrd = reshape(1:(nely+1)*(nelx+1),nely+1,nelx+1);
nodeids = reshape(nodegrd(1:end-1,1:end-1),nely*nelx,1);
nodeidz = 0:(nely+1)*(nelx+1):(nelz-1)*(nely+1)*(nelx+1);
nodeids = repmat(nodeids,size(nodeidz))+repmat(nodeidz,size(nodeids));
edofVec = 3*nodeids(:)+1;
edofMat = repmat(edofVec,1,24)+ ...
    repmat([0 1 2 3*nely + [3 4 5 0 1 2] -3 -2 -1 ...
    3*(nely+1)*(nelx+1)+[0 1 2 3*nely + [3 4 5 0 1 2] -3 -2 -
1]],nele,1);
iK = kron(edofMat,ones(24,1))';
jK = kron(edofMat,ones(1,24))';
% PREPARE FILTER
iH = ones(nele*(2*(ceil(rmin)-1)+1)^2,1);
jH = ones(size(iH));
sH = zeros(size(iH));
k = 0;
for k1 = 1:nelz
    for i1 = 1:nelx
        for j1 = 1:nely
            e1 = (k1-1)*nelx*nely + (i1-1)*nely+j1;
            for k2 = max(k1-(ceil(rmin)-1),1):min(k1+(ceil(rmin)-
1),nelz)
                for i2 = max(i1-(ceil(rmin)-1),1):min(i1+(ceil(rmin)-
1),nelx)
                    for j2 = max(j1-(ceil(rmin)-
1),1):min(j1+(ceil(rmin)-1),nely)
                        e2 = (k2-1)*nelx*nely + (i2-1)*nely+j2;
                        k = k+1;
                        iH(k) = e1;
                        jH(k) = e2;
```

```
                        sH(k) = max(0,rmin-sqrt((i1-i2)^2+(j1-
j2)^2+(k1-k2)^2));
                    end
                end
            end
        end
    end
end
H = sparse(iH,jH,sH);
Hs = sum(H,2);
% INITIALIZE ITERATION
x = repmat(volfrac,[nely,nelx,nelz]);
x(passive)=0;
xPhys = x;
loop = 0;
change = 1;
% START ITERATION
while change > tolx && loop < maxloop
    loop = loop+1;
    % FE-ANALYSIS
    sK = KE(:)*(Emin+xPhys(:)'.^penal*(E0-Emin));
    K = sparse(iK(:),jK(:),sK(:)); K = (K+K')/2;
    tolit = 1e-8;
    maxit = 8000;
    %M = diag(K);
    M = diag(diag(K(freedofs, freedofs)));
    U(freedofs,:)=pcg(K(freedofs, freedofs),F(freedofs,:), tolit, 1000,
M);
    %[num_nodes, num_loads] = size(U);
%     for i = 1:num_loads
%     U(freedofs,i)=pcg(K(freedofs, freedofs),F(freedofs,i), tolit,
1000, M);
%     end
    % OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
    ce = reshape(sum((U(edofMat)*KE).*U(edofMat),2),[nely,nelx,nelz]);
    c = sum(sum(sum((Emin+xPhys.^penal*(E0-Emin)).*ce)));
    dc = -penal*(E0-Emin)*xPhys.^(penal-1).*ce;
    dv = ones(nely,nelx,nelz);
    % FILTERING AND MODIFICATION OF SENSITIVITIES
    dc(:) = H*(dc(:)./Hs);
    dv(:) = H*(dv(:)./Hs);
    % OPTIMALITY CRITERIA UPDATE
    l1 = 0; l2 = 1e9; move = 0.2;
    while (l2-l1)/(l1+l2) > 1e-3
        lmid = 0.5*(l2+l1);
        xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-
dc./dv/lmid)))));
        xnew(passive) = 0;
        xPhys(:) = (H*xnew(:))./Hs;
        if sum(xPhys(:)) > volfrac*nele, l1 = lmid; else l2 = lmid; end
    end
    change = max(abs(xnew(:)-x(:)));
    x = xnew;
    % PRINT RESULTS
    fprintf(' It.:%5i Obj.:%11.4f Vol.:%7.3f
ch.:%7.3f\n',loop,c,mean(xPhys(:)),change);
```

```
    % PLOT DENSITIES
    if displayflag, clf;
        %display_3D(xPhys);
    end
end
clf; display_3D(xPhys);
end
% ==================== AUXILIARY FUNCTIONS
================================
% GENERATE ELEMENT STIFFNESS MATRIX
function [KE] = lk_H8(nu)
A = [32 6 -8 6 -6 4 3 -6 -10 3 -3 -3 -4 -8;
    -48 0 0 -24 24 0 0 0 12 -12 0 12 12 12];
k = 1/72*A'*[1; nu];
% GENERATE SIX SUB-MATRICES AND THEN GET KE MATRIX
K1 = [k(1) k(2) k(2) k(3) k(5) k(5);
    k(2) k(1) k(2) k(4) k(6) k(7);
    k(2) k(2) k(1) k(4) k(7) k(6);
    k(3) k(4) k(4) k(1) k(8) k(8);
    k(5) k(6) k(7) k(8) k(1) k(2);
    k(5) k(7) k(6) k(8) k(2) k(1)];
K2 = [k(9)  k(8)  k(12) k(6)  k(4)  k(7);
    k(8)  k(9)  k(12) k(5)  k(3)  k(5);
    k(10) k(10) k(13) k(7)  k(4)  k(6);
    k(6)  k(5)  k(11) k(9)  k(2)  k(10);
    k(4)  k(3)  k(5)  k(2)  k(9)  k(12)
    k(11) k(4)  k(6)  k(12) k(10) k(13)];
K3 = [k(6)  k(7)  k(4)  k(9)  k(12) k(8);
    k(7)  k(6)  k(4)  k(10) k(13) k(10);
    k(5)  k(5)  k(3)  k(8)  k(12) k(9);
    k(9)  k(10) k(2)  k(6)  k(11) k(5);
    k(12) k(13) k(10) k(11) k(6)  k(4);
    k(2)  k(12) k(9)  k(4)  k(5)  k(3)];
K4 = [k(14) k(11) k(11) k(13) k(10) k(10);
    k(11) k(14) k(11) k(12) k(9)  k(8);
    k(11) k(11) k(14) k(12) k(8)  k(9);
    k(13) k(12) k(12) k(14) k(7)  k(7);
    k(10) k(9)  k(8)  k(7)  k(14) k(11);
    k(10) k(8)  k(9)  k(7)  k(11) k(14)];
K5 = [k(1) k(2)  k(8)  k(3) k(5)  k(4);
    k(2) k(1)  k(8)  k(4) k(6)  k(11);
    k(8) k(8)  k(1)  k(5) k(11) k(6);
    k(3) k(4)  k(5)  k(1) k(8)  k(2);
    k(5) k(6)  k(11) k(8) k(1)  k(8);
    k(4) k(11) k(6)  k(2) k(8)  k(1)];
K6 = [k(14) k(11) k(7)  k(13) k(10) k(12);
    k(11) k(14) k(7)  k(12) k(9)  k(2);
    k(7)  k(7)  k(14) k(10) k(2)  k(9);
    k(13) k(12) k(10) k(14) k(7)  k(11);
    k(10) k(9)  k(2)  k(7)  k(14) k(7);
    k(12) k(2)  k(9)  k(11) k(7)  k(14)];
KE = 1/((nu+1)*(1-2*nu))*...
    [ K1  K2  K3  K4;
    K2'  K5  K6  K3';
    K3' K6  K5' K2';
    K4  K3  K2  K1'];
```

```
end
% DISPLAY 3D TOPOLOGY (ISO-VIEW)
function display_3D(rho)
[nely,nelx,nelz] = size(rho);
hx = 1; hy = 1; hz = 1;            % User-defined unit element size
face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
set(gcf,'Name','ISO display','NumberTitle','off');
for k = 1:nelz
    z = (k-1)*hz;
    for i = 1:nelx
        x = (i-1)*hx;
        for j = 1:nely
            y = nely*hy - (j-1)*hy;
            if (rho(j,i,k) > 0.5)  % User-defined display density
threshold
                vert = [x y z; x y-hx z; x+hx y-hx z; x+hx y z; x y
z+hx;x y-hx z+hx; x+hx y-hx z+hx;x+hx y z+hx];
                vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -
vert(:,2,:);

patch('Faces',face,'Vertices',vert,'FaceColor',[0.2+0.8*(1-
rho(j,i,k)),0.2+0.8*(1-rho(j,i,k)),0.2+0.8*(1-rho(j,i,k))]);
                hold on;
            end
        end
    end
end
axis equal; axis tight; axis off; box on; view([30,30]); pause(1e-6);
end
```

# APPENDIX C:

# 3D ARRAY SCALING FUNCTION

```
%%Author: Purnajyoti Bhaumik
%%This code outputs a scaled version of the top3D results
%%****************************** Thesis
Requirement****************************


function scaled_weight_mat = scaletop3D(weight,scale)

scaled_weight_arr = {zeros(size(weight,1)), zeros(size(weight, 2)),
zeros(size(weight, 3))};
del_V = zeros(scale,scale,scale);

for z=1:size(weight, 3)
    for y=1:size(weight, 2)
        for x = 1:size(weight, 1)
            for z_scale = 1:scale
                for y_scale = 1:scale
                    for x_scale = 1:scale
                       del_V(x_scale, y_scale, z_scale) =weight(x,y,z);
                    end
                end
            end
            scaled_weight_arr{x,y,z}=del_V;
        end
    end
end

for z = 1:size(weight, 3)
    for y = 1:size(weight, 2)
        scaled_weight_mat = scaled_weight_arr{1,y,1};
            for x=1:size(weight, 1)-1
                scaled_weight_mat =
cat(1,scaled_weight_mat,scaled_weight_arr{x+1,y,z});
            end
        smat{z,y}=scaled_weight_mat;
    end
end

for z_cat = 1:z
    scaled_weight_mat2 = smat{z_cat,1};
    for y_cat = 1:y-1
        scaled_weight_mat2 =
cat(2,scaled_weight_mat2,smat{z_cat,y_cat+1});
    end
    smat2{z_cat}=scaled_weight_mat2;
end

scaled_weight_mat3 = smat2{1};

for z_cat = 1:z-1
    scaled_weight_mat3 = cat(3, scaled_weight_mat3, smat2{z_cat+1});
end
 scaled_weight_mat = scaled_weight_mat3;
```

# APPENDIX D:

# MODEL SPACE NODE AND INDEX CODE

```
%%Author: Purnajyoti Bhaumik
%%This code outputs the node and index matrices for 3D linear FEA
%%****************************** Thesis
Requirement*****************************


function [M,T] =generate_cube_M(left, right, bottom, top, back, front,
h_partition,scale)

h = h_partition;

n_hor = scale*(right - left)/h(1); %parallel to the x-axis
n_vert = scale*(top - bottom)/h(3); %parallel to the y-axis
n_depth = scale*(front - back)/h(2); %parallel to the z axis



total_nodes = (n_hor+1)*(n_vert+1)*(n_depth+1);
total_elements = (n_hor)*(n_vert)*(n_depth);

M = zeros(3, total_nodes);
T = zeros(8, total_elements);

count = 1;
count2 = 1;

%while count <= total_nodes
    for k = 1: n_depth+1 %transverses z-axis (depth)
        for j = 1:n_hor+1 %transverses x-axis (horizontal)
            for i = 1:n_vert+1 %transverses y-axis (vertical)
                M(1,count)= (j-1)*h(1);
                M(2,count)= n_vert-(i-1)*h(2);
                M(3,count) = (k-1)*h(3);
                count = count+1;
            end
        end
    end
%end

%while count2 <= total_elements

    for k = 1: n_depth %transverses z-axis (depth)
        for j = 1:n_hor %transverses x-axis (horizontal)
            for i = 1:n_vert %transverses y-axis (vertical)
                T(1,count2)= i+(j-1)*(n_vert+1)+(k-
1)*(n_vert+1)*(n_hor+1);
                T(2,count2) = i+(j-1)*(n_vert+1)+(k-
1)*(n_vert+1)*(n_hor+1)+(n_vert+1)*(j);
                T(3, count2) =
i+(k)*(n_vert+1)*(n_hor+1)+(n_vert+1)*(j);
                T(4, count2) = i+(k)*(n_vert+1)*(n_hor+1);
                T(5, count2) = i+(k)*(n_vert+1)*(n_hor+1)+1;
                T(6, count2) =
i+(k)*(n_vert+1)*(n_hor+1)+(n_vert+1)*(j)+1;
```

```
                T(7, count2) = i+(j-1)*(n_vert+1)+(k-
1)*(n_vert+1)*(n_hor+1)+(n_vert+1)*(j)+1;
                T(8, count2) = i+(j-1)*(n_vert+1)+(k-
1)*(n_vert+1)*(n_hor+1)+1;
                count2 = count2+1;
            end
        end
    end
```

# APPENDIX E:

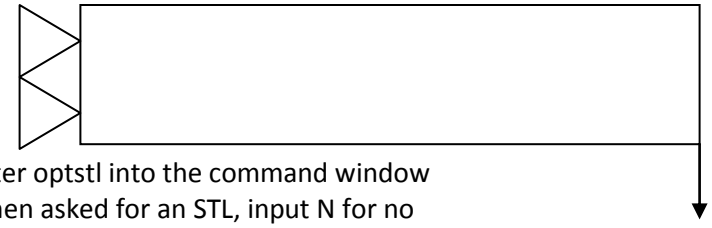
# FIND THE TOP AND SIDE BOUNDARY ELEMENTS

```
function boundaryelements = findboundary(gridOUTPUT, nelx, nely, nelz)


a=1;


for i = 1:length(nelx)
    for j = 1:length(nely)
        for k = 1:length(nelz)
            if (gridOUTPUT(i,j,k)==1) && (i~=1) && (i~=length(nelx)) &&
(j~=1) && (j~=length(nely)) && (k~=1) && (k~=length(nelz))
                if gridOUTPUT(i+1,j,k)==0 || gridOUTPUT(i-1,j,k)==0 ||
gridOUTPUT(i,j+1,k)==0 || gridOUTPUT(i,j,k+1)==0 ||gridOUTPUT(i,j,k-
1)==0
                   boundaryelement{a} = i+(j-1)*length(nelx)+(k-
1)*length(nelx)*length(nely);
                   a = a+1;
                end
            elseif (gridOUTPUT(i,j,k)==1)&& ((i~=length(nelx) ||
(i==length(nelx) && ((j==1) || (j==length(nely)) || (k==1) ||
(k==length(nelz))))))
                boundaryelement{a} = i+(j-1)*length(nelx)+(k-
1)*length(nelx)*length(nely);
                a = a+1;
            end
        end
    end
end

boundaryelements = boundaryelement{1};
for j = 2:a-1
boundaryelements = cat(1, boundaryelements, boundaryelement{j});
end
```

# APPENDIX F:

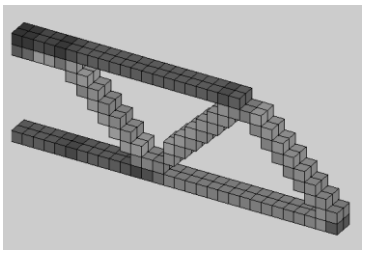
# OPTSTL TRAINING MANUAL

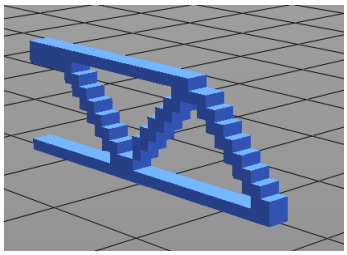
Please note: all dimensions are millimeters, all forces are Newtons, so all moduli are MPa.
EVERYTHING HERE IS CASE SENSITIVE. MAKE SURE ALL OF THE FILES ARE IN THE CORRECT FOLDER.
Example 1: Cantilever



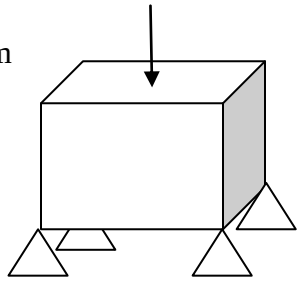
1. Enter optstl into the command window
2. When asked for an STL, input N for no
3. When asked for the overall width, input 30
4. When asked for the overall height, input 10
5. When asked for the overall depth, input 2
6. When asked for a distributed or point force, enter P
7. When asked for a x coordinate, input 30
8. When asked for a y coordinate, input 0
9. When asked for a z coordinate, input 0
10. When asked for another load, input Y
11. When asked for a distributed or point force, input P
12. When asked for a x coordinate, input 30
13. When asked for a y coordinate, input 0
14. When asked for a z coordinate, input 1
15. When asked for another load, input Y
16. When asked for a distributed or point force, input P
17. When asked for a x coordinate, input 30
18. When asked for a y coordinate, input 0
19. When asked for a z coordinate, input 2
20. When asked for another load, input N
21. When asked for the magnitude of these loads, input -1
22. When asked for a surface or point constraint, input S
23. When asked to perpendicular to which axis, input w
24. When asked for distance from the left, input 0
25. When asked for another constraint, input N
26. When asked for a modulus, input 2150 (Young's modulus of ABS)
27. When asked to scale the model, input N
28. When asked respecting boundary elements, enter N
29. When asked to create a point cloud, input N
30. When asked to write an STL, input Y
31. When asked for a filename, enter a full file path ex: C:\example.stl

Example output: (Left) matlab figure and (Right) STL file





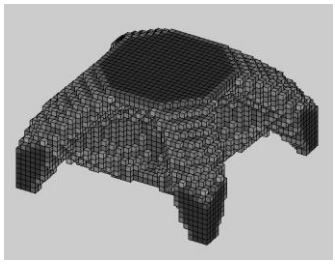
Example 2: Platform

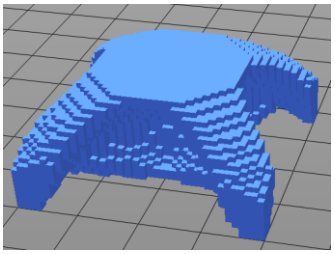


1. Enter optstl into the command window
2. When asked for an STL, input N
3. When asked for the overall width, input 40
4. When asked for the overall height, input 20
5. When asked for the overall depth, input 40
6. When asked for a distributed or point force, enter P
7. When asked for a x coordinate, input 20
8. When asked for a y coordinate, input 20
9. When asked for a z coordinate, input 20
10. When asked for another load, input Y
11. When asked for the magnitude of these loads, input -1
12. When asked for a surface or point constraint, input P
13. When asked for a x coordinate, input 0
14. When asked for a y coordinate, input 0
15. When asked for a z coordinate, input 0
16. When asked for another constraint, input Y
17. When asked for a surface or point constraint, input P
18. When asked for a x coordinate, input 40
19. When asked for a y coordinate, input 0
20. When asked for a z coordinate, input 0
21. When asked for another constraint, input Y
22. When asked for a surface or point constraint, input P
23. When asked for a x coordinate, input 40
24. When asked for a y coordinate, input 0
25. When asked for a z coordinate, input 40
26. When asked for another constraint, input Y
27. When asked for a surface or point constraint, input P
28. When asked for a x coordinate, input 0
29. When asked for a y coordinate, input 0

30. When asked for a z coordinate, input 40
31. When asked for another constraint, input N
32. When asked for a modulus, input 2150 (Young's modulus of ABS)
33. When asked to scale the model, input N
34. When asked respecting the boundary, enter N
35. When asked to create a point cloud, input N
36. When asked to write an STL, input Y
37. When asked for a filename, enter a full file path ex: C:\example1.stl

Example output: (Left) matlab figure and (right) STL file



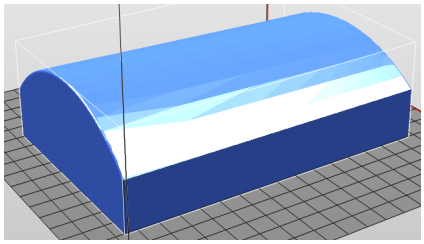


Example 3: Input Any STL File in this case an FDM tool

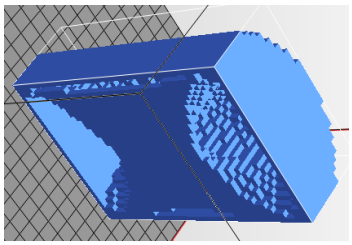
1. Enter optstl into the command window
2. When asked to input an STL, enter Y
3. When asked the STL filepath use FDMtool2.stl, for example C:\FDMtool2.stl. FDMtool2.stl is scaled down for decreasing the number of elements and computation time.
4. When asked the height, enter 5
5. When asked the width, enter 11
6. When asked the depth, enter 16
7. When asked for a discretization factor, enter 3. You should get a MATLAB figure of the voxelised model.
8. When asked for a load, enter P
9. When asked for the x-coordinate, enter 16
10. When asked for the y-coordinate, enter 14
11. When asked for the z-coordinate, enter 24
12. When asked for another load, enter N.
13. When asked for the load's magnitude, enter -100
14. When asked for constraint, enter P
15. When asked for the x-coordinate, enter 0
16. When asked for the y-coordinate, enter 0
17. When asked for the z-coordinate, enter 0
18. When asked for another constraint, enter Y
19. When asked for constraint, enter P
20. When asked for the x-coordinate, enter 33
21. When asked for the y-coordinate, enter 0
22. When asked for the z-coordinate, enter 0
23. When asked for another constraint, enter Y
24. When asked for constraint, enter P
25. When asked for the x-coordinate, enter 33

26. When asked for the y-coordinate, enter 0
27. When asked for the z-coordinate, enter 48
28. When asked for another constraint, enter Y
29. When asked for constraint, enter P
30. When asked for the x-coordinate, enter 0
31. When asked for the y-coordinate, enter 0
32. When asked for the z-coordinate, enter 48
33. When asked for another constraint, enter N
34. When asked for the modulus, enter 2150 for ABS (e.g. 2150 MPa) . You should then get an optimized model.
35. When asked to scale the model, enter N.
36. When asked respecting the boundary, enter Y. You should then get the optimized model plus the boundary elements.
37. When asked to write an STL file, enter Y.
38. When asked for the file path, enter a full path for example C:\optFDMtool.st

Original STL

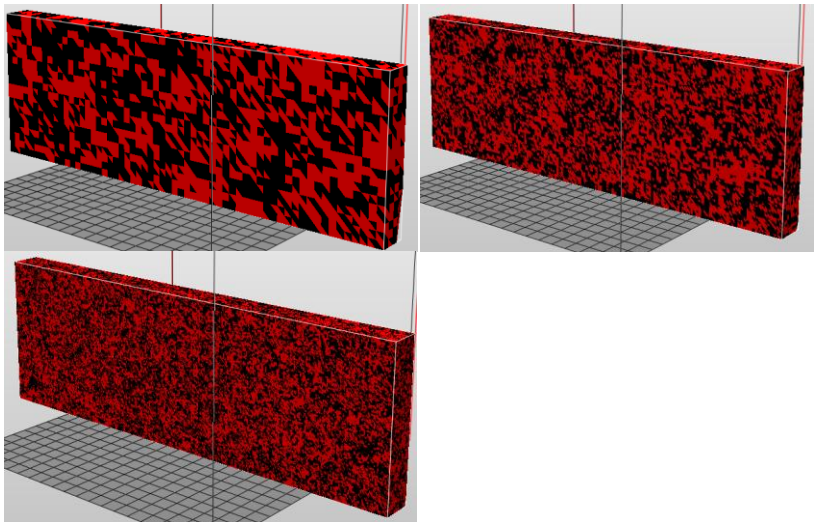


Topologically Optimized STL

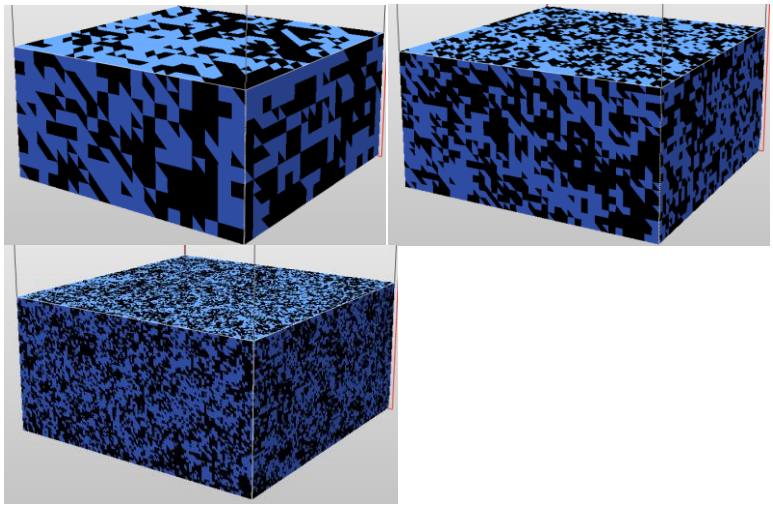


# APPENDIX G:

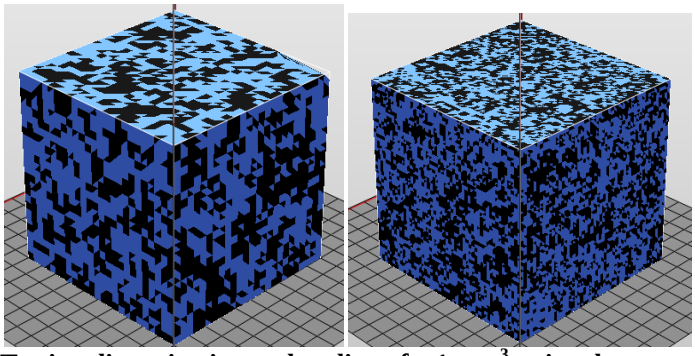
# STL IMAGES



**Increasing Cantilever Discretization**



**Increasing platform discretization**



**Testing discretization and scaling of a 1 mm$^3$ unit cube**

# APPENDIX H:

# SUPPLEMENTAL FEA

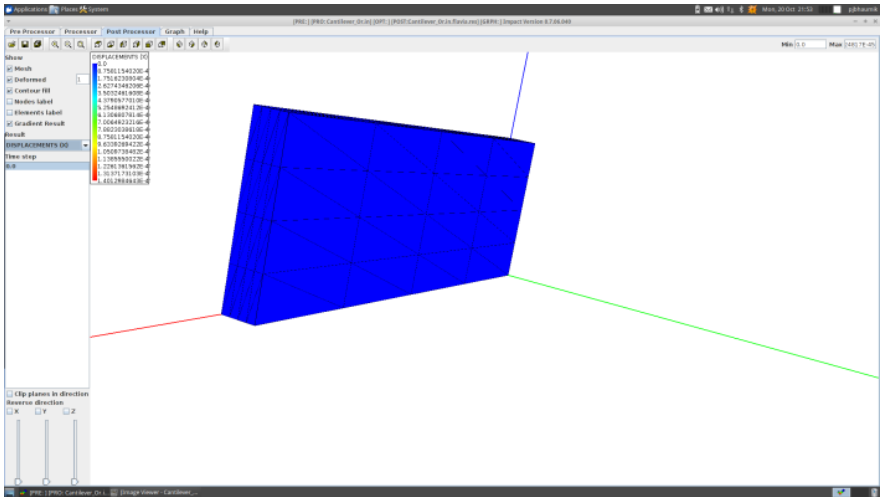


**Figure H.1: The original cantilever's plot for displacement in the X direction**

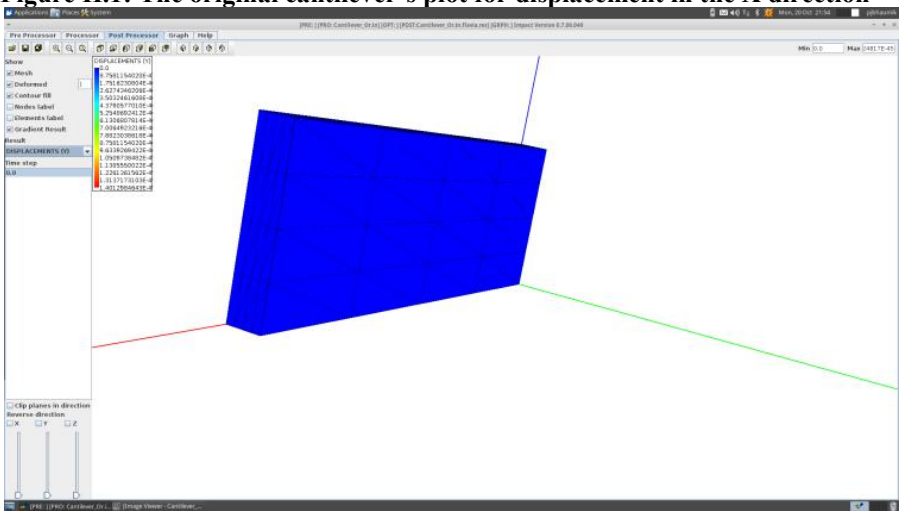


**Figure H.2: The original cantilever's plot for displacement in the Y direction**

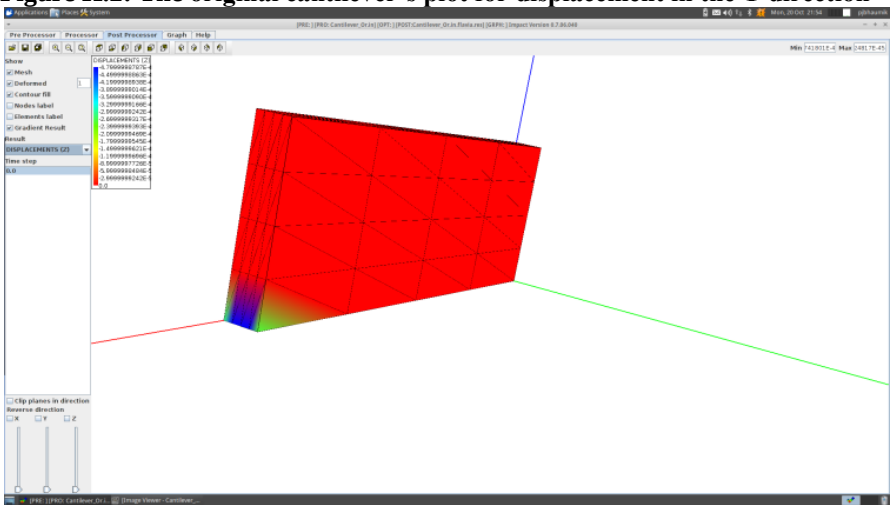


**Figure H.3: The original cantilever's plot for displacement in the Z direction**

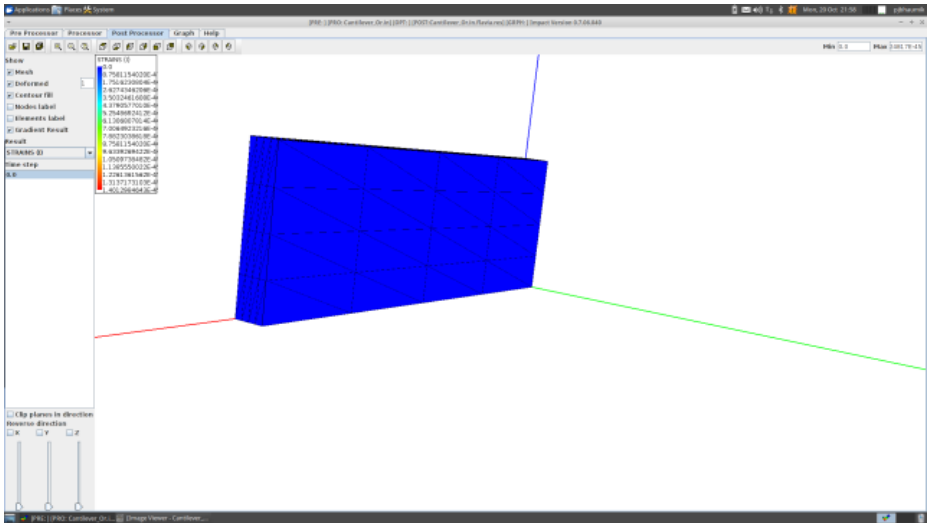


**Figure H.4: The original cantilever's plot for resultant 3D strain**

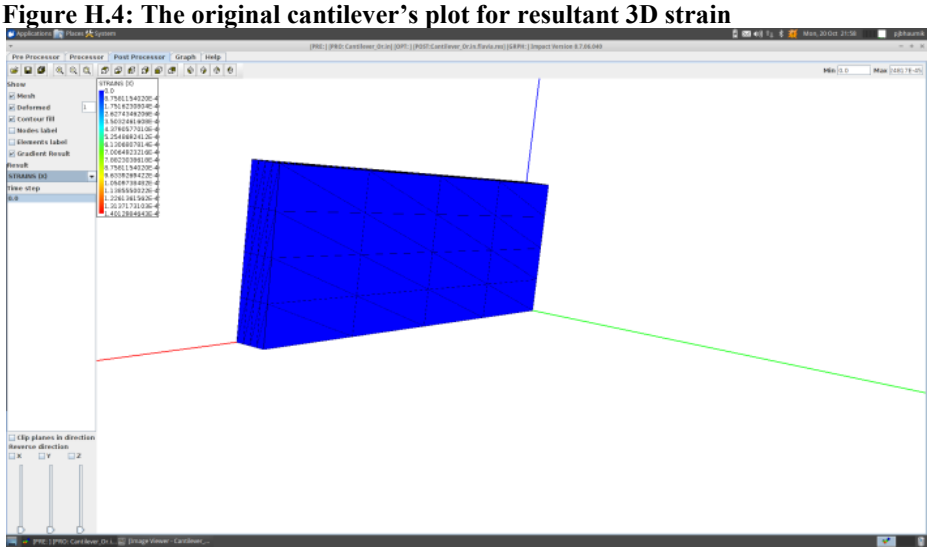


**Figure H.5: The original cantilever's plot for strain in the X direction**

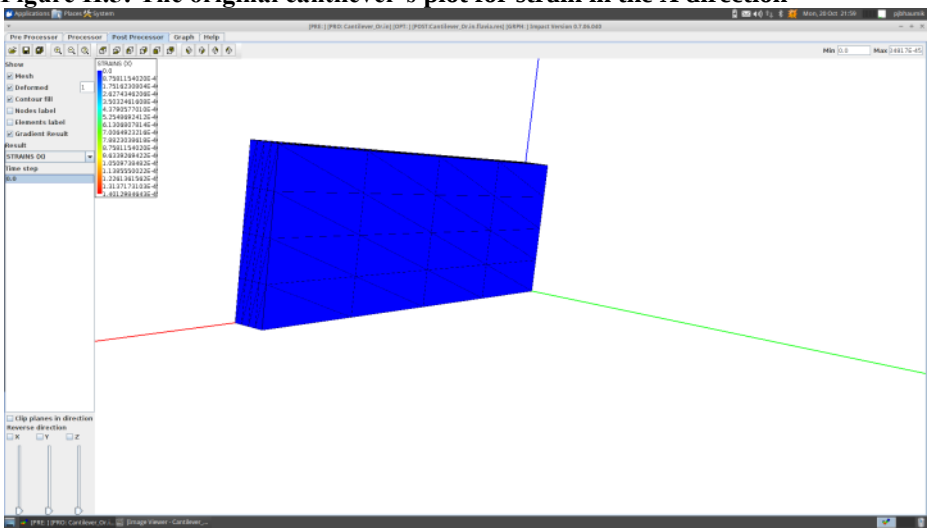


**Figure H.6: The original cantilever's plot for strain in the Y direction**

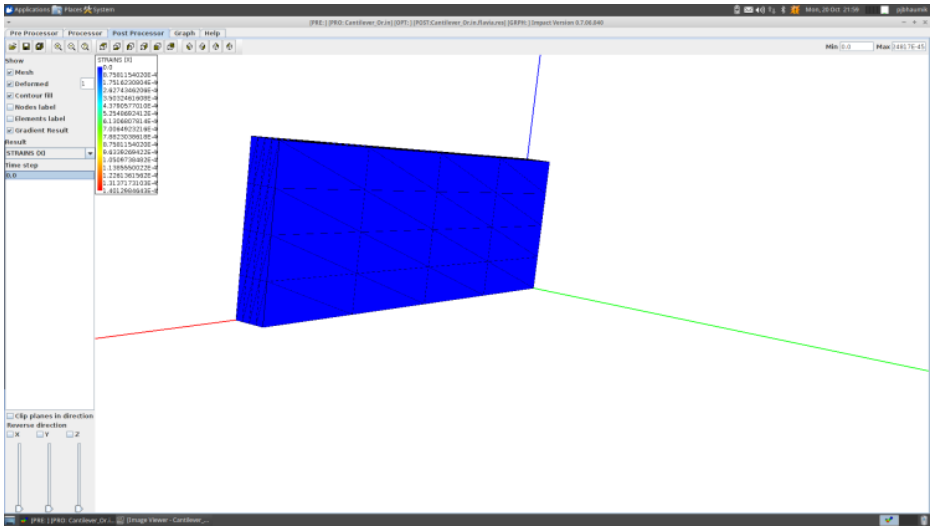


**Figure H.7: The original cantilever's plot for strain in the Z direction**

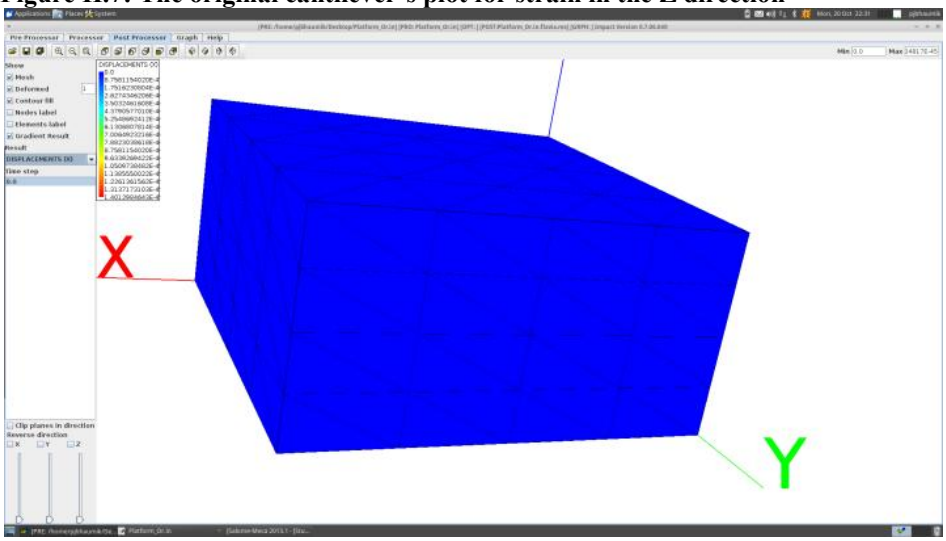



**Figure K.8: The original platform's plot for displacement in the X direction**

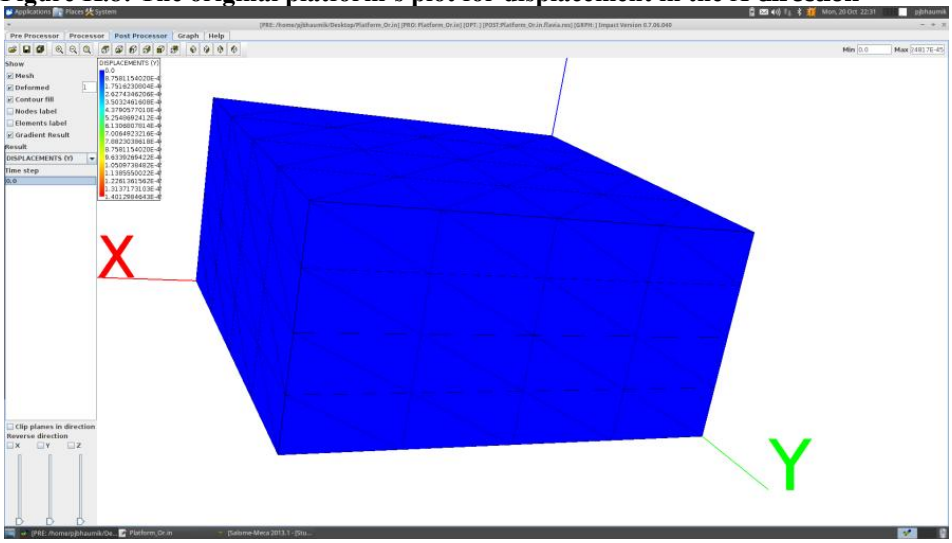


**Figure K.9: The original platform's plot for displacement in the Y direction**

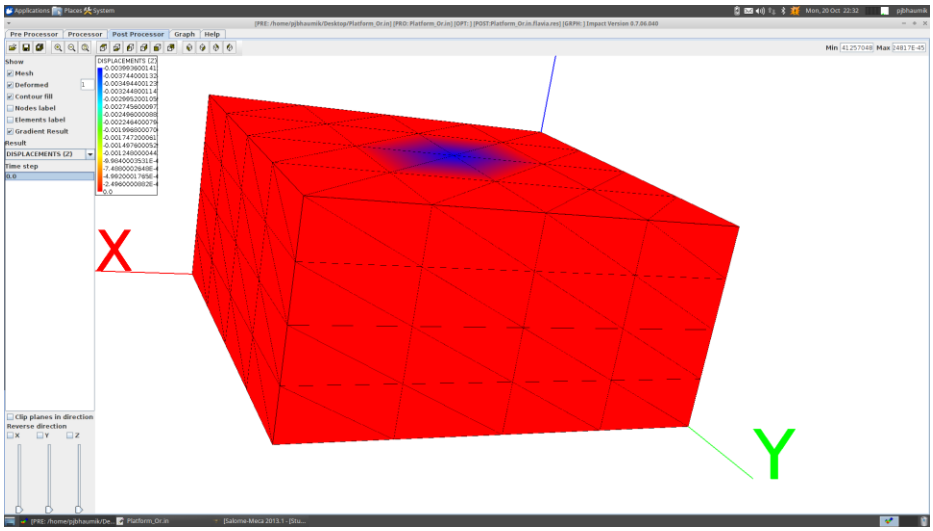



**Figure K.10: The original platform's plot for displacement in the Z direction**

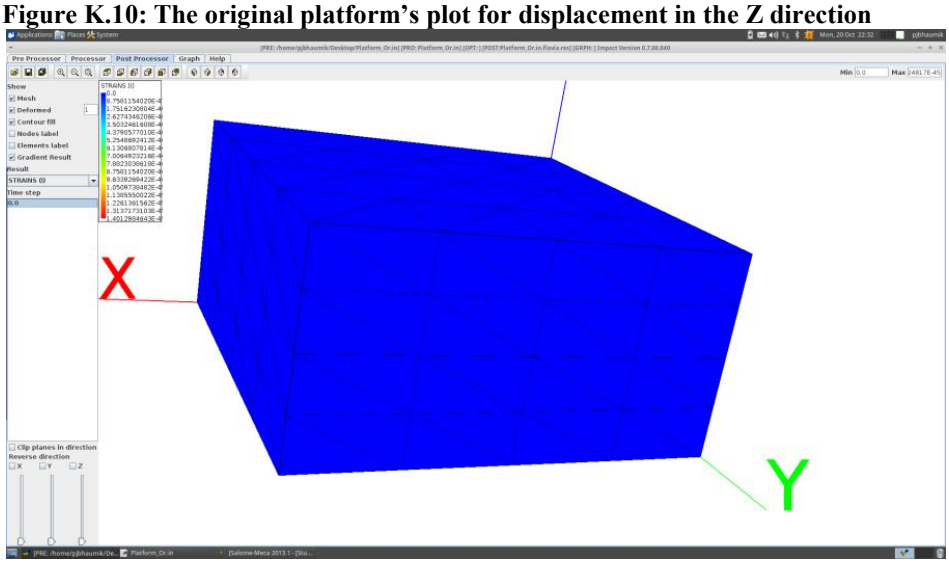



**Figure K.11: The original platform's plot for resultant 3D strain**

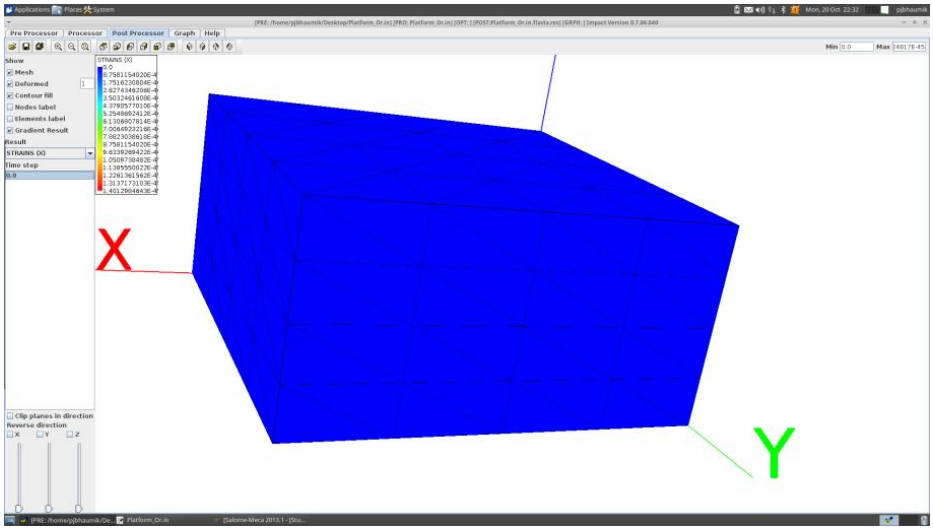



**Figure K.12: The original cantilever's plot for displacement in the X direction**

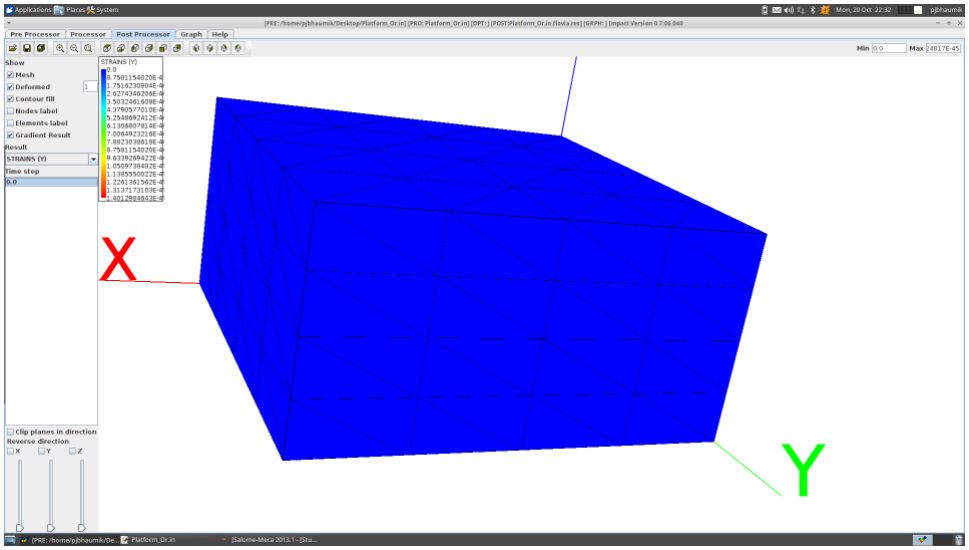



**Figure K.13: The original platofrm's plot for displacement in the Y direction**

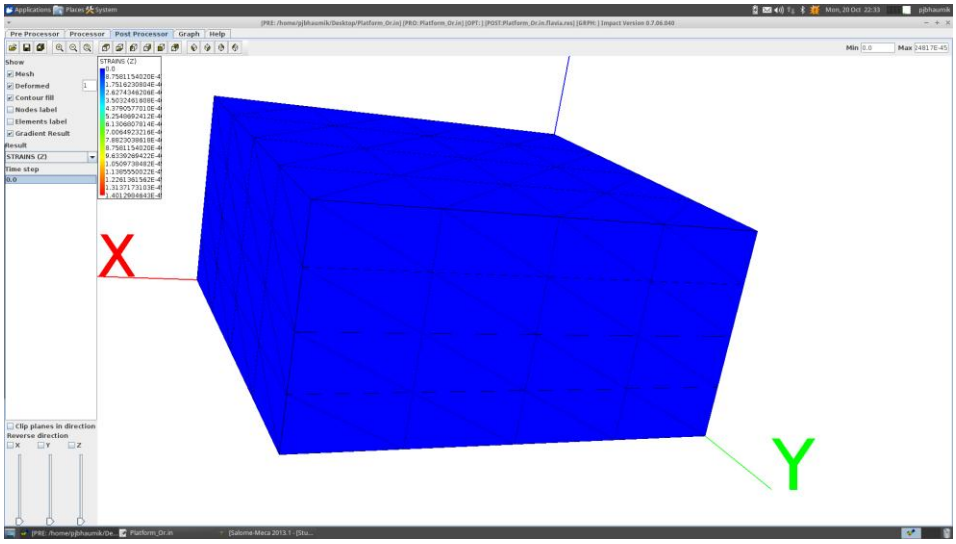



**Figure K.14: The original platform's plot for displacement in the Z direction**

# APPENDIX I:

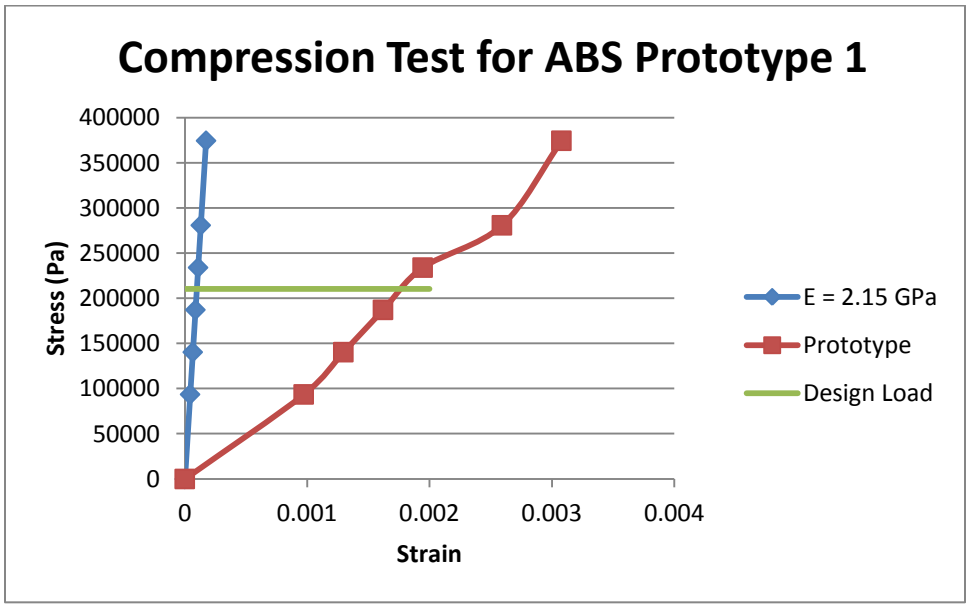
# MATERIAL PROPERTIES

| Material | Young's Modulus (GPa) | Density (g/cc) | Poisson's Ratio |
|---|---|---|---|
| ABS | 2.15 Gpa | 1.07 | 0.3 |
| Titanium Alloy | 115 GPa | 4.03 | 0.3 |

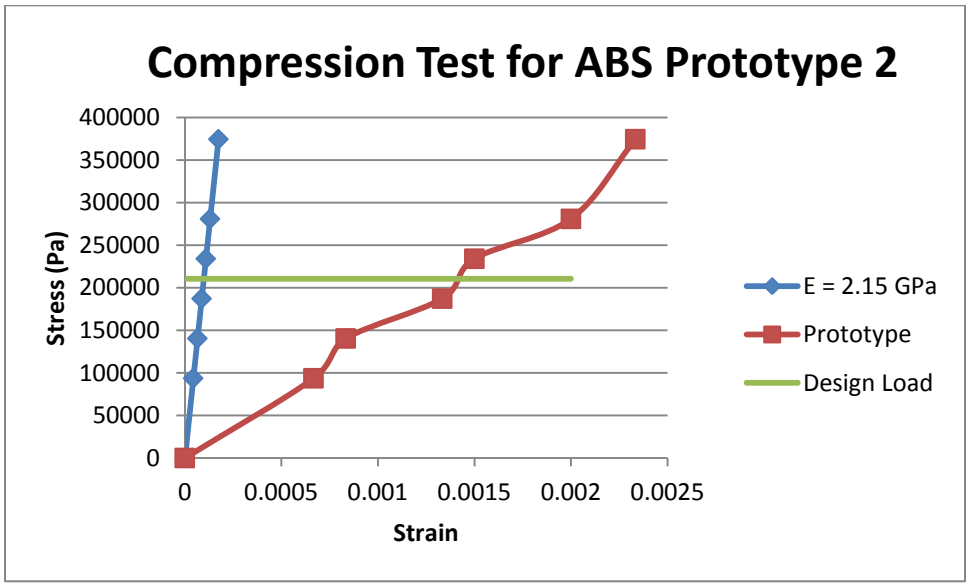
# APPENDIX J:

# COMPRESSION TESTING RESULTS

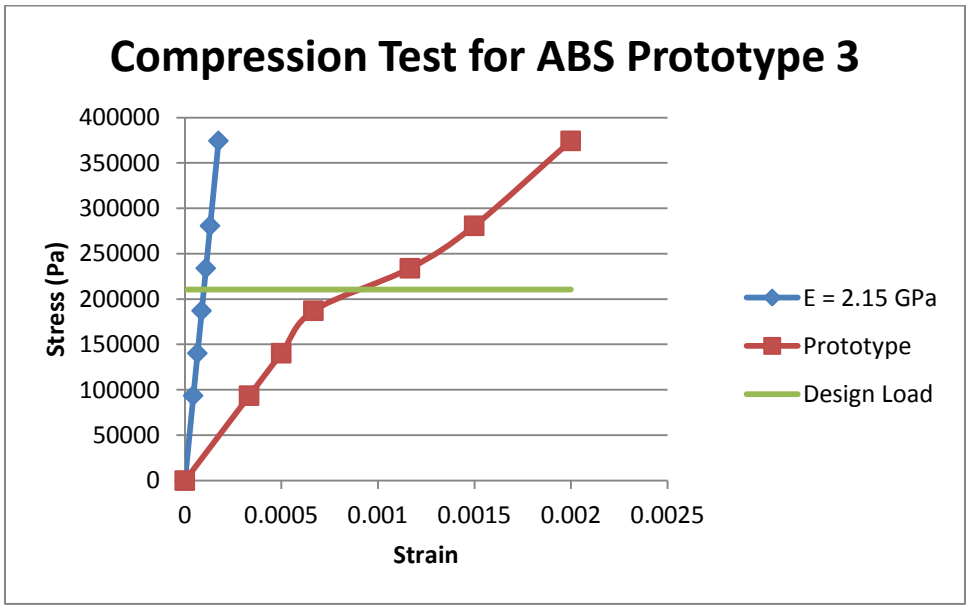
| Mass for Compressing Prototype | Compressive force (N) | height (mm) 1st Measurement | height (mm) 2nd Measurement | height (mm) 3rd Measurement | height (mm) Average | Std. Dev (mm) | Prototype's Strain (ε) | Stress, Pa | Control Strain (ε) | Design Load |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 20.53 | 20.76 | 20.44 | 20.57667 | 0.165025 | 0 | 0 | 0 | 210420 |
| 10 | 44.49816 | 20.53 | 20.54 | 20.6 | 20.55667 | 0.037859 | 0.000972 | 93633.03 | 4.36E-05 | 210420 |
| 15 | 66.74724 | 20.58 | 20.51 | 20.56 | 20.55 | 0.036056 | 0.001296 | 140449.5 | 6.53E-05 | 210420 |
| 20 | 88.99632 | 20.54 | 20.52 | 20.57 | 20.54333 | 0.025166 | 0.00162 | 187266.1 | 8.71E-05 | 210420 |
| 25 | 111.2454 | 20.56 | 20.52 | 20.53 | 20.53667 | 0.020817 | 0.001944 | 234082.6 | 0.000109 | 210420 |
| 30 | 133.4945 | 20.52 | 20.52 | 20.53 | 20.52333 | 0.005774 | 0.002592 | 280899.1 | 0.000131 | 210420 |
| 40 | 177.9926 | 20.51 | 20.5 | 20.53 | 20.51333 | 0.015275 | 0.003078 | 374532.1 | 0.000174 | 210420 |



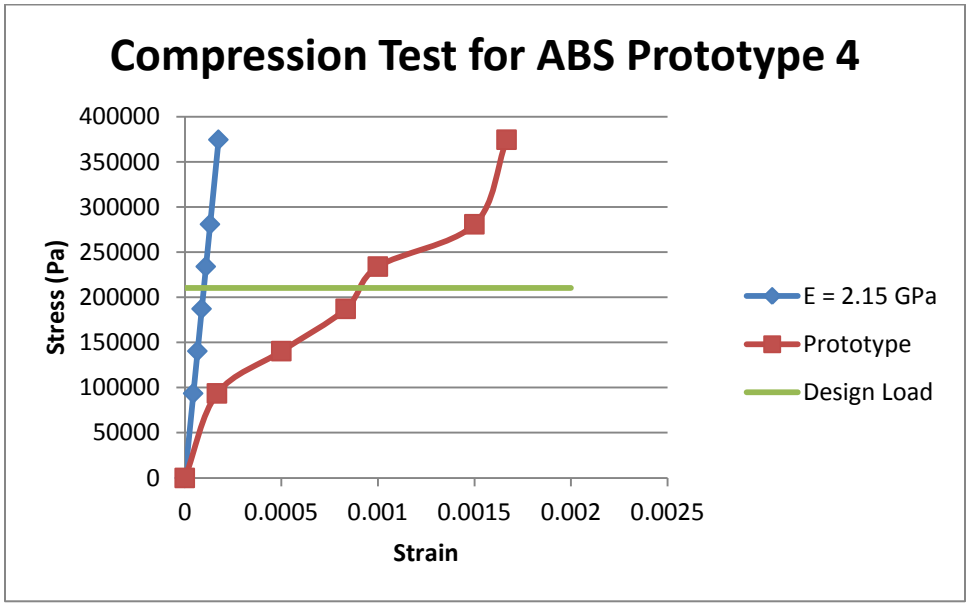
| Mass for Compressing Prototype | Compressive force (N) | height (mm) 1st Measurement | height (mm) 2nd Measurement | height (mm) 3rd Measurement | height (mm) Average | Std. Dev (mm) | Prototype's Strain (ε) | Stress, Pa | Control Strain (ε) |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 20 | 19.99 | 20.01 | 20 | 0.01 | 0 | 0 | 0 |
| 10 | 44.49816 | 19.99 | 19.98 | 19.99 | 19.98667 | 0.005774 | 0.000667 | 93633.03 | 4.36E-05 |
| 15 | 66.74724 | 19.98 | 19.99 | 19.98 | 19.98333 | 0.005774 | 0.000833 | 140449.5 | 6.53E-05 |
| 20 | 88.99632 | 19.96 | 19.98 | 19.98 | 19.97333 | 0.011547 | 0.001333 | 187266.1 | 8.71E-05 |
| 25 | 111.2454 | 19.97 | 19.97 | 19.97 | 19.97 | 0 | 0.0015 | 234082.6 | 0.000109 |
| 30 | 133.4945 | 19.95 | 19.96 | 19.97 | 19.96 | 0.01 | 0.002 | 280899.1 | 0.000131 |
| 40 | 177.9926 | 19.94 | 19.96 | 19.96 | 19.95333 | 0.011547 | 0.002333 | 374532.1 | 0.000174 |



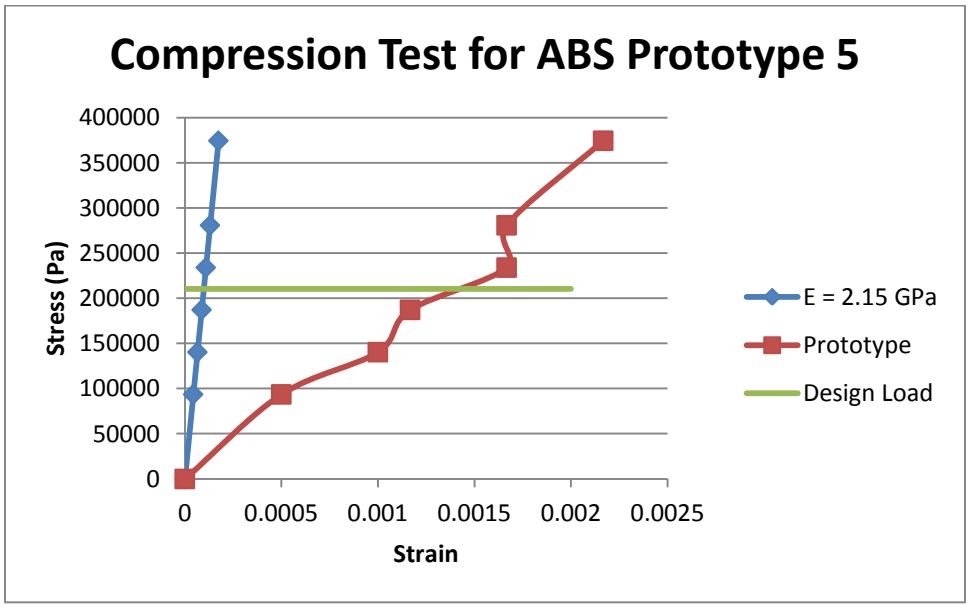
| Mass for Compressing Prototype | Compressive force (N) | height (mm) 1st Measurement | height (mm) 2nd Measurement | height (mm) 3rd Measurement | height (mm) Average | Std. Dev (mm) | Prototype's Strain (ε) | Stress, Pa | Control Strain (ε) | Design Load |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 20 | 20.02 | 20.02 | 20.01333 | 0.011547 | 0 | 0 | 0 | 210420 |
| 10 | 44.49816 | 20.01 | 20.01 | 20 | 20.00667 | 0.005774 | 0.000333 | 93633.03 | 4.36E-05 | 210420 |
| 15 | 66.74724 | 20.01 | 20 | 20 | 20.00333 | 0.005774 | 0.0005 | 140449.5 | 6.53E-05 | 210420 |
| 20 | 88.99632 | 20.01 | 19.99 | 20 | 20 | 0.01 | 0.000666 | 187266.1 | 8.71E-05 | 210420 |
| 25 | 111.2454 | 19.99 | 19.99 | 19.99 | 19.99 | 0 | 0.001166 | 234082.6 | 0.000109 | 210420 |
| 30 | 133.4945 | 19.99 | 19.98 | 19.98 | 19.98333 | 0.005774 | 0.001499 | 280899.1 | 0.000131 | 210420 |
| 40 | 177.9926 | 19.97 | 19.98 | 19.97 | 19.97333 | 0.005774 | 0.001999 | 374532.1 | 0.000174 | 210420 |



| Mass for Compressing Prototype | Compressive force (N) | height (mm) 1st Measurement | height (mm) 2nd Measurement | height (mm) 3rd Measurement | height (mm) Average | Std. Dev (mm) | Prototype's Strain (ε) | Stress, Pa | Control Strain (ε) | Design Load |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 20 | 20 | 20 | 20 | 0 | 0 | 0 | 0 | 210420 |
| 10 | 44.49816 | 19.99 | 20 | 20 | 19.99667 | 0.005774 | 0.000167 | 93633.03 | 4.36E-05 | 210420 |
| 15 | 66.74724 | 20 | 19.98 | 19.99 | 19.99 | 0.01 | 0.0005 | 140449.5 | 6.53E-05 | 210420 |
| 20 | 88.99632 | 20 | 19.98 | 19.97 | 19.98333 | 0.015275 | 0.000833 | 187266.1 | 8.71E-05 | 210420 |
| 25 | 111.2454 | 19.99 | 19.98 | 19.97 | 19.98 | 0.01 | 0.001 | 234082.6 | 0.000109 | 210420 |
| 30 | 133.4945 | 19.98 | 19.97 | 19.96 | 19.97 | 0.01 | 0.0015 | 280899.1 | 0.000131 | 210420 |
| 40 | 177.9926 | 19.97 | 19.97 | 19.96 | 19.96667 | 0.005774 | 0.001667 | 374532.1 | 0.000174 | 210420 |



| Mass for Compressing Prototype | Compressive force (N) | height (mm) 1st Measurement | height (mm) 2nd Measurement | height (mm) 3rd Measurement | height (mm) Average | Std. Dev (mm) | Prototype's Strain (ε) | Stress, Pa | Control Strain (ε) | Design Load |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 19.99 | 20 | 20 | 19.99667 | 0.005774 | 0 | 0 | 0 | 210420 |
| 10 | 44.49816 | 19.99 | 19.98 | 19.99 | 19.98667 | 0.005774 | 0.0005 | 93633.03 | 4.36E-05 | 210420 |
| 15 | 66.74724 | 19.97 | 19.98 | 19.98 | 19.97667 | 0.005774 | 0.001 | 140449.5 | 6.53E-05 | 210420 |
| 20 | 88.99632 | 19.97 | 19.97 | 19.98 | 19.97333 | 0.005774 | 0.001167 | 187266.1 | 8.71E-05 | 210420 |
| 25 | 111.2454 | 19.97 | 19.96 | 19.96 | 19.96333 | 0.005774 | 0.001667 | 234082.6 | 0.000109 | 210420 |
| 30 | 133.4945 | 19.96 | 19.96 | 19.97 | 19.96333 | 0.005774 | 0.001667 | 280899.1 | 0.000131 | 210420 |
| 40 | 177.9926 | 19.95 | 19.95 | 19.96 | 19.95333 | 0.005774 | 0.002167 | 374532.1 | 0.000174 | 210420 |



# APPENDIX K:

# LOAD AND CONSTRAINT DATA

Cantilever loads and constraints

Load Location: point force at (x = 30 mm , y = 0 mm, and z = 0 mm)

Load Location: point force at (x = 30 mm , y = 0 mm, and z = 1 mm)

Load Location: point force at (x = 30 mm , y = 0 mm, and z = 2 mm)

Load magnitude = -100 N (program will distribute this load across the thre nodes above which represent the tip of the cantilever, so each load is -33.333 N)

Simply Supported Constraints at $x = 0$ mm, $0 \text{ mm} \leq y \leq 10$ mm, and $0 \text{ mm} \leq z \leq 2$ mm

Platform loads and constraints

Load Location: point force at (x = 20 mm , y =20 mm, and z = 20 mm)

Simply Supported Constraints at (x=0mm, y = 0mm, z = 0mm), (x=40mm, y = 0mm, z = 0mm), (x=40 mm, y = 0 mm, z = 40 mm), and (x=0mm, y = 0mm, z =40 mm)

FDM Tool loads and constraints

Load Location: point force at (x = 15mm , y = 14 mm, and z = 24 mm)

Load Magnitude: -100N

Simply Supported Constraints at (x=0mm, y = 0mm, z = 0mm), (x=33mm, y = 0mm, z = 0mm), (x=33mm, y = 0mm, z = 48mm), and (x=0mm, y = 0mm, z =48 mm)

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# VITA

Purnajyoti Bhaumik is 28 year old full time Mechanical Project Engineer for Vanderlande Industries. He has 2.5 years of experience as an engineer and served in the United States Marine Corps Reserve for 6 years. He has a Bachelor of Science degree in Mechanical Engineering from the Georgia Institute of Technology, an Engineer Intern/Engineer in Training Certificate in Mechanical Engineering, a Lean Six Sigma Black Belt, and a Graduate Certificate in CAD/CAM and Rapid Prototyping.

He started school at the Missouri University of Science and Technology in the Fall of 2012. He completed my graduate certificate work in the Spring of 2013 with a 3.25 GPA. He decided to try the Master of Science in Mechanical Engineering degree program and maintained a 3.28 GPA. He studied state space controls, optimization, DFMA, solid freeform modeling, solid freeform fabrication, and FEA.

He shall begin studying for his PE exam in 2015. At work, he shall be responsible for optimizing engineering design specifications based on functional requirements, quality, cost, and time. He shall look forward to traveling and visiting my extended family across the globe. He shall consider undertaking a PhD in Mechanical Engineering.