1970

An analysis of a quadrature double-sideband/frequency modulated communication system

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AN ANALYSIS OF A QUADRATURE DOUBLE-SIDEBAND/FREQUENCY MODULATED COMMUNICATION SYSTEM

BY

DENNY RAY TOWNSON, 1947-

A THESIS

Presented to the Faculty of the Graduate School of the

UNIVERSITY OF MISSOURI - ROLLA

In Partial Fulfillment of the Requirements for the Degree

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

1970

Approved by

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ABSTRACT

A QDSB/FM communication system is analyzed with emphasis placed on the QDSB demodulation process and the AGC action in the FM transmitter. The effect of noise in both the pilot and message signals is investigated. The detection gain and mean square error is calculated for the QDSB baseband demodulation process. The mean square error is also evaluated for the QDSB/FM system. The AGC circuit is simulated on a digital computer. Errors introduced into the AGC system are analyzed with emphasis placed on nonlinear gain functions for the voltage controlled amplifier.

ACKNOWLEDGEMENT

The author expresses his appreciation to Dr. William H. Tranter for the suggestion of this thesis topic and for the many helpful discussions concerning this work.

The author also acknowledges the patience and encouragement offered by his wife, Carol, throughout this work.

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I. INTRODUCTION

Present day communication systems involve many different combinations of subsystems in order to transfer information from a source to a destination. The block diagram of the communication system in Figure 1-1 indicates that one such combination consists of a baseband modulator, a transmitter, a transmission link, a receiver, and a baseband demodulator. Many types of modulation methods, transmitters, and transmission links exist, from which one combination must be chosen which best meets system requirements.

The following work investigates a method of modulation known as quadrature double-sideband (QDSB) modulation. QDSB modulation possesses several desirable characteristics. When transmitting several message channels, the QDSB bandwidth requirement is equivalent to the bandwidth required for single-sideband (SSB) modulation; however, QDSB is less complex to implement than SSB. In addition, QDSB allows the transmission of low frequency and dc messages, while SSB does not possess this capability. Double-sideband (DSB) modulation requires twice the bandwidth used by SSB or QDSB and is therefore inferior to SSB and QDSB in this respect. From a bandwidth requirement, degree of complexity, and message capability

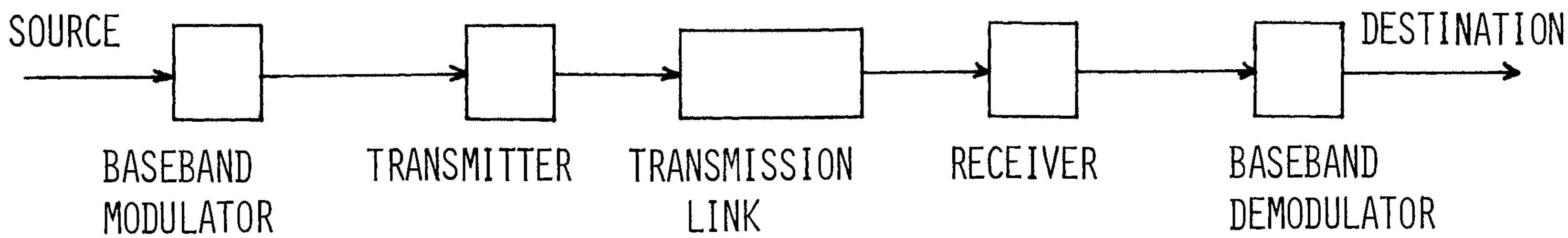

Figure 1-1. Block Diagram of a Communication System

consideration, it appears that QDSB could offer a practical method of modulation.

The analysis of a QDSB system is more general than is the analysis of a DSB or SSB system. Once calculations have been performed for the QDSB system, the DSB or SSB results may be obtained by letting the messages take on specific values. The time domain representation for a QDSB signal is

$$e(t) = m_1(t) \text{ Cos } \omega_c t + m_2(t) \text{ Sin } \omega_c t \qquad (1\text{-}1)$$

where $m_1(t)$ = channel 1 message

and $m_2(t)$ = channel 2 message .

If $m_2(t) = 0$, the resulting signal is that signal obtained for DSB modulation. If $m_2(t) = \hat{m}_1(t)$, where $\hat{m}_1(t)$ is the Hilbert transform of $m_1(t)$, the resulting signal is that signal obtained for SSB lower sideband modulation. Thus, the DSB and SSB results can be easily obtained from the QDSB results.

With this in mind, the QDSB system will be further analyzed. Mean square error in the demodulated output and signal-to-noise ratios will provide performance criteria for evaluating the QDSB baseband modulation system operating in the presence of noise. The results will be used to obtain similar results for SSB and DSB and comparisons will be made to QDSB.

A frequency modulated (FM) transmitter will be chosen for this system, due to its frequent use in telemetry systems of the present day and its desirable characteristics over amplitude modulated (AM) type transmission systems. Signal-to-noise ratio improvement due to automatic-gain-control (AGC) circuits will be discussed and the AGC circuit will be simulated on the digital computer. The computer simulation will allow the study of system errors due to AGC and errors introduced due to nonlinearities in the AGC circuit.

II. REVIEW OF THE LITERATURE

A. V. T. Day and R. V. L. Hartley, working separately, at approximately the same time, proposed a modulation system employing the use of quadrature carriers[1]. A. V. T. Day filed for a patent described as:

> *"A method of multiplexing carrier wave signals which consists of superposing in a common transmission medium, two phase differentiated synchronous carrier waves. . ."*

on July 24, 1923[2]. The system was consequently known as the "Day system".

The "Day system" today is more commonly known as phase-discrimination multiplexing, which indicates the system may have many different sets of carrier phases rather than the two orthogonal functions implied by the term "quadrature"[3]. Nyquist examined the problem of phase discrimination multiplexing in a general manner with specific interest in the quadrature arrangement[4]. In a discussion for overcoming frequency spectrum inefficiency of carrier telegraphy by quadrature carrier techniques, Nyquist indicates the double signal capability and the fact that there is "substantially no mutual interference",

[1]Superscripts refer to numbered references.

yielding an ac telegraph system equal in frequency conservation to a dc system.

Tranter considers the coherent demodulation of a QDSB signal with phase error in the demodulation carrier. It is shown that the mean-square error resulting from the phase error in SSB and QDSB systems are equal if all modulating signals have the same mean-square value, and that this mean-square error is greater than for a DSB system[5].

The AGC circuit which will be used at the input to the FM transmitter has been analyzed theoretically by several authors. Gill and Leong investigate the response of AGC to two narrow-band input signals[6]. A linear control characteristic is assumed. Schachter and Bergstein analyze an AGC circuit for the effects of white Gaussian noise on the gain of the system[7]. Again, a linear control characteristic is assumed. Banta analyzes AGC to determine if the output signal can be maintained at a fixed level while retaining the modulation terms[8]. This work also assumes a linear gain control characteristic. Oliver treats AGC as a feedback problem, and theoretically investigates the AGC characteristics[9]. He also uses a linear gain control characteristic function. Victor and Brockman develop an analytic technique for the design of AGC circuits[10]. This work assumes a nonlinear function, F(b), for the voltage controlled amplifier of the AGC circuit. The nonlinear function is chosen such that the log F(b) is a linear function. The analytic calculations then proceed using linear functions. Tranter has studied the tracking errors present in a linear, noiseless, system using two AGC circuits in cascade[11]. Here a linear gain control function is also assumed.

Many other authors have analyzed AGC circuits; however, most assume a linear gain function for the voltage controlled amplifier. This assumption allows an analytic analysis of AGC, whereas a nonlinear gain assumption causes the calculations to become extremely involved.

III. SYSTEM DESCRIPTION

A. QDSB Modulator

A QDSB modulator can be modeled as shown in Figure 3-1. One of the multiplier units uses Cos $\omega_c t$ as the carrier frequency, while the other uses Sin $\omega_c t$. The resulting suppressed carrier DSB signals are summed together to form the QDSB signal. The assumed frequency spectrum of the messages, $m_1(t)$ and $m_2(t)$, and the resulting QDSB signal spectrum are also shown in Figure 3-1. From this figure it can be observed that for a QDSB signal the messages are not separated in frequency. It now becomes apparent as to why QDSB has a lower bandwidth requirement than does DSB, when both messages must be transmitted at the same time.

A pilot signal (Cos $\omega_p(t)$) is also summed with the two DSB signals in the QDSB modulator. This pilot signal is used to synthesize the demodulation carrier signal at the receiver in order to perform coherent demodulation.

B. FM Transmitter - Receiver

For this work, an FM transmission system has been chosen and can be modeled as shown in Figure 3-2. AGC is used here as a controlling device for regulating the power of the FM transmitter input to a predetermined

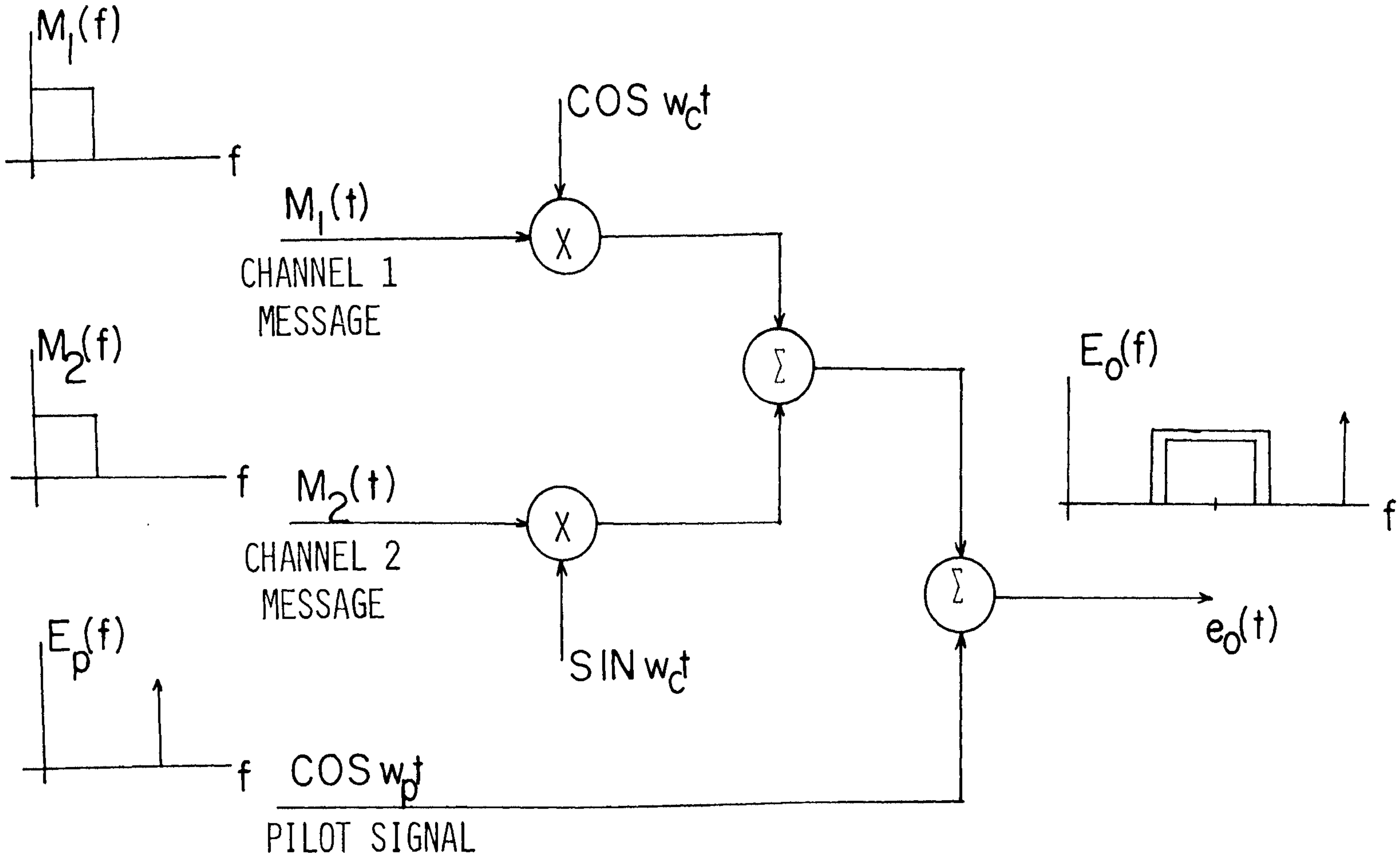

Figure 3-1. Model of a QDSB Modulator Unit

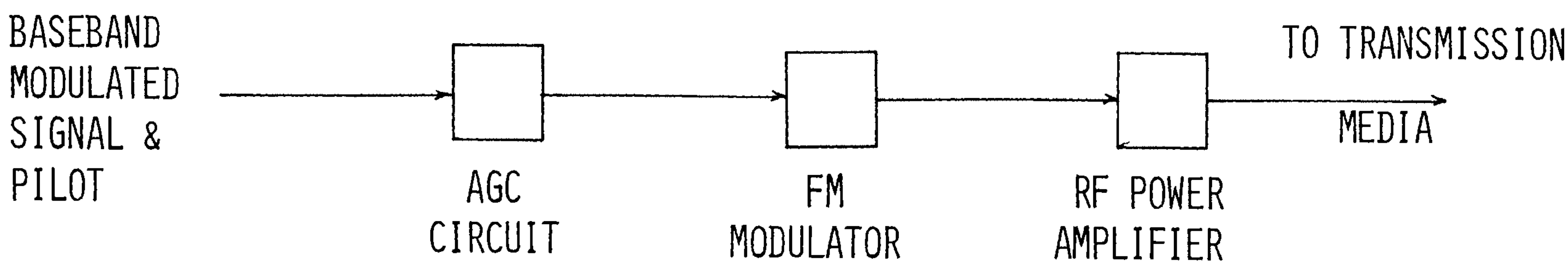

Figure 3-2. Block Diagram of an FM Transmitter

level. Therefore, by use of an AGC circuit, the FM transmitter deviation can be kept near its maximum value, independent of the power in the unregulated message channels. This will allow a higher signal-to-noise ratio when the power in the baseband channel signal is lower than would be possible without the AGC circuit, while at the same time possessing the capability to avoid nonlinear distortion from too large a derivation ratio when the power in the baseband channel signals is high.

The AGC circuit can be modeled as shown in Figure 3-3[12]. Here, the pilot signal has been translated to dc for simplification of the AGC model. Thus, EP is the amplitude of the pilot signal and represents a dc value during this analysis. The output, EO, of the circuit is dependent on the error signal, G(t), and K(t), the gain of the voltaged controlled amplifier (VCA). In previous analyses, the gain K(t) is assumed to be a linear function of G(t) such that

$$K(t) = 1 + G(t) \quad . \tag{3-1}$$

This allows a theoretical analysis of the errors introduced due to the AGC action. For this work, K(t) will be assumed nonlinear and the computer simulation will be used to evaluate the AGC circuit when operating in the presence of noise.

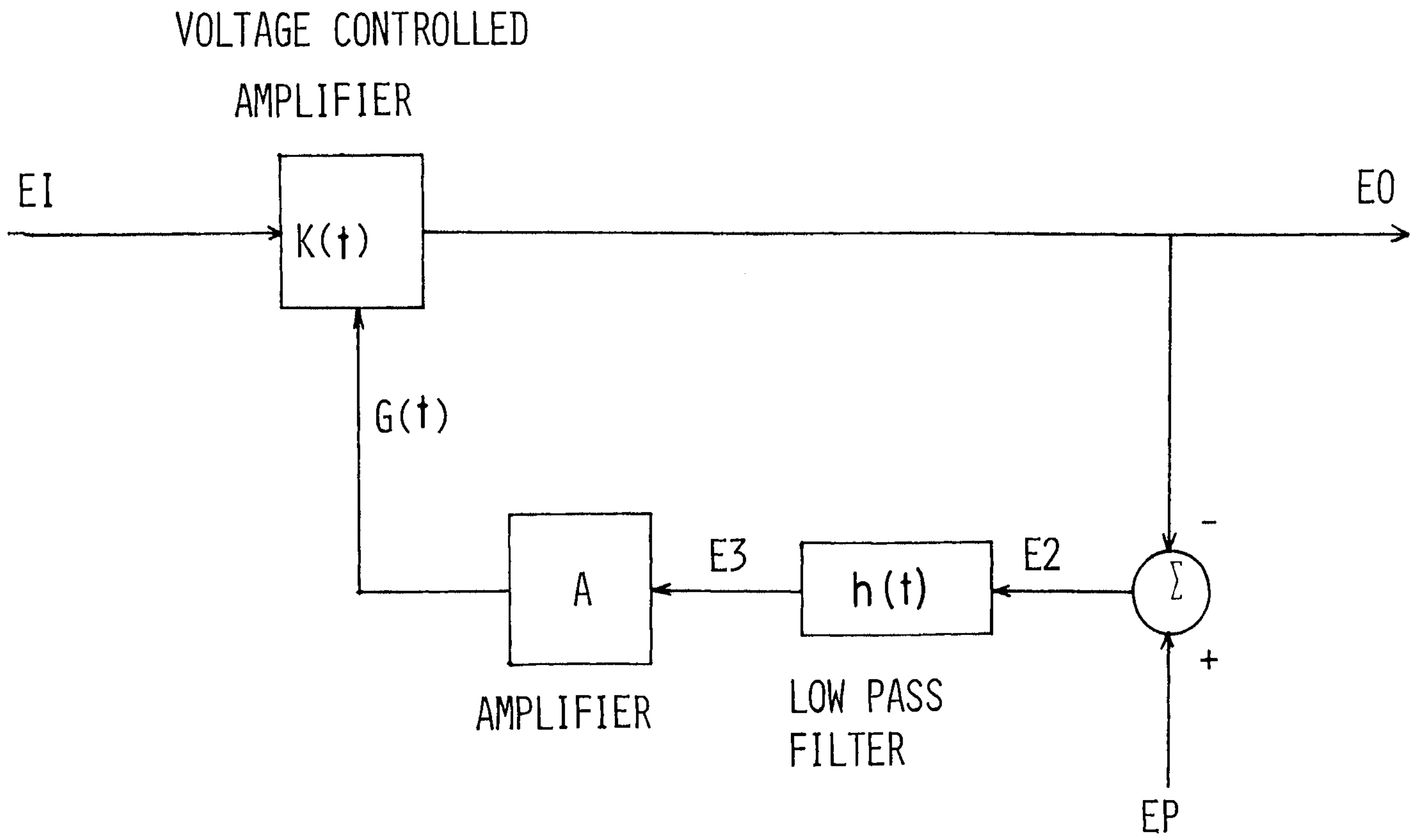

Figure 3-3. Model of an AGC Circuit

C. Transmission Media

The channel between the transmitter and receiver is extremely difficult to model, so that the model is correct for all time and all conditions. As discussed by Hancock and Wintz, those channels which utilize electromagnetic radiation are included in a large class of channels whose characteristics are nondeterministic and must be specified in terms of their statistical properties[13]. These channels have two major types of perturbation effects on the signal being transferred through it. One is termed "additive noise" and the other "multiplicative disturbances".

"Additive noise" includes all types of "noise" which interact by a summation process with the message signal, yielding an output signal from the transmission media of the original signal plus "noise". A useful statistical description of additive noise for QDSB calculations is that where the noise is expressed in terms of the direct and quadrature components as[14]

$$n(t) = n_c(t) \text{ Cos } \omega_c t - n_s(t) \text{ Sin } \omega_c t \,. \qquad (3\text{-}2)$$

The terms $n_c(t)$ and $n_s(t)$ may be formed by the summation of a number of cosine and sine waves, respectively[15].

$$n_c(t) = \sum_{m=1}^{n} A_m \text{ Cos}(2\Pi \; m \; \Delta ft + \Theta_m) \qquad (3\text{-}3)$$

$$n_s(t) = \sum_{m=1}^{n} A_m \, \mathrm{Sin}(2\Pi \, m \, \Delta ft + \Theta_m) \quad (3\text{-}4)$$

Δf = frequency separating each sine or cosine term in the summation (3-5)

n = the number of terms used (3-6)

Θ_m = arbitrary phase term (3-7)

The amplitude frequency spectrum of $n_c(t)$ and $n_s(t)$ is shown in Figure 3-4. The statistical properties of $n_s(t)$ and $n_c(t)$ have been investigated, and it has been found that $n_s(t)$ and $n_c(t)$ are statistically independent, Gaussian-distributed, with zero mean and variance of $n(A_m^2/2)$ [16].

Multiplicative disturbances consist of such perturbations as fading, phase distortion, and nonlinear effects. These disturbances vary widely, depending on the transmission frequency, transmitter and receiver location, weather, and other such factors. These types of disturbances are very difficult to accurately represent mathematically, or to describe statistically. For this reason, the transmission media in this work will be that described by use of the "additive noise" type perturbations.

Noise may also enter the message signal during signal processing in the transmitter and receiver. This noise will also be assumed additive, and for analysis purposes will be included as if the perturbation occurred in the transmission channel. This is equivalent to considering

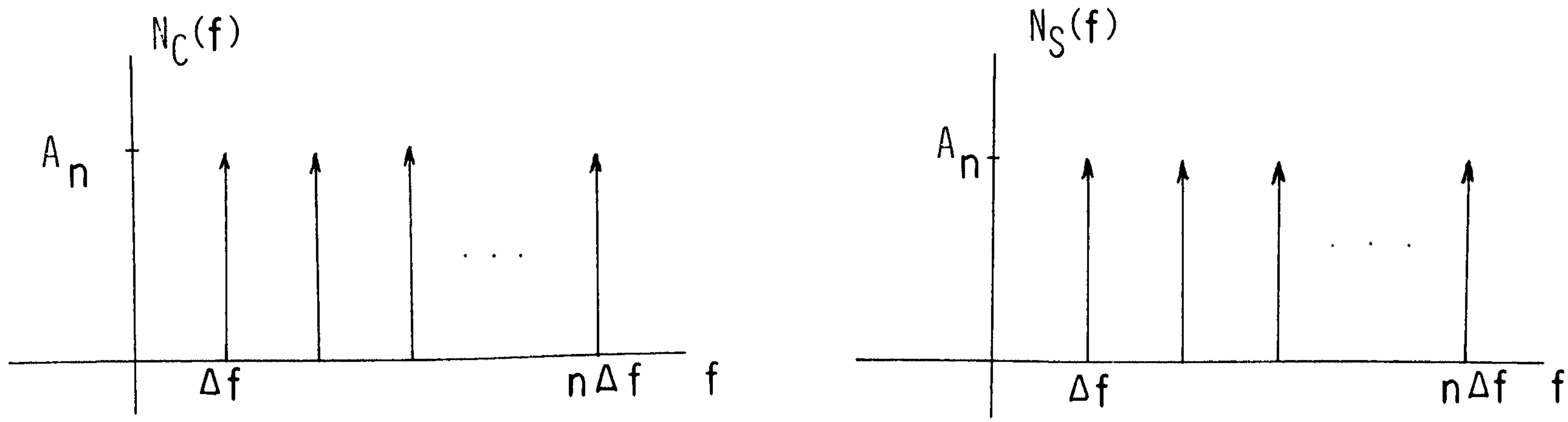

Figure 3-4. Amplitude Frequency Spectrum Plot of $n_c(t)$ and $n_s(t)$

the receiver RF and IF stages as a part of the transmission channel.

D. QDSB Demodulator

The output of the FM receiver, $e_b(t)$, consists of the original baseband signal and pilot plus signal perturbations resulting from the transmission process as previously discussed. Thus,

$$e_b(t) = m_1(t) \text{ Cos } \omega_c t + n_c(t) \text{ Cos } \omega_c t + m_2(t) \text{ Sin } \omega_c t - n_s(t) \text{ Sin } \omega_c t + R(t) \text{ Cos } (\omega_p t + \Theta_p(t)) \tag{3-8}$$

where $R(t) \text{ Cos } (\omega_p t + \Theta_p(t))$ is a general expression for a sinewave plus additive Gaussian noise. As shown by S. O. Rice, $\Theta_p(t)$ is a slowly varying phase function[17]. The statistics of $\Theta_p(t)$ will be discussed during the analysis of the effect of pilot phase error on the QDSB signal. A block diagram of a QDSB demodulator is shown in Figure 3-5. The pilot filter, a bandpass filter centered at ω_p radians per second, passes the pilot signal to the carrier synthesis section of the demodulator unit. The carrier synthesis process is a frequency division where $\omega_p + \Theta_p(t)$ is divided by ω_p/ω_c to yield the demodulation carrier of $\text{Cos } (\omega_c t + \Theta(t))$ and $\text{Sin } (\omega_c t + \Theta(t))$. The channel filter, a bandpass filter centered at ω_c radians per second, passes the message channels to the DSB modulator units for demodulation. A low pass filter is

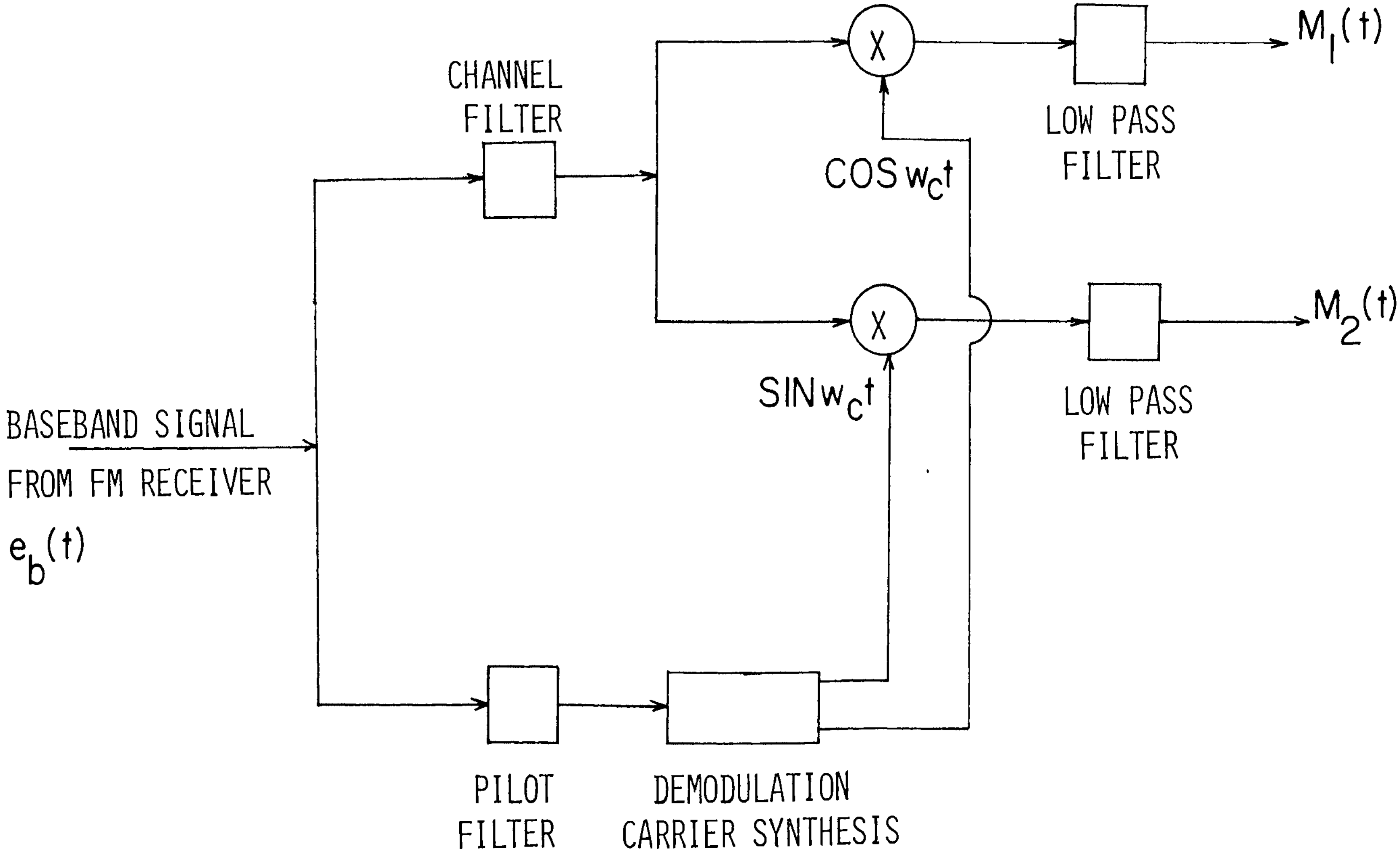

Figure 3-5. Block Diagram of a QDSB Demodulator

included at the output of the DSB modulators to remove the double frequency terms from the messages $m_1(t)$ and $m_2(t)$.

IV. THEORETICAL ANALYSIS

A. Effect of Pilot Phase Error in the Baseband System

Perturbation of the signal due to noise has been shown to occur during the transmission of the electromagnetic wave through the transmission channel. Therefore, an analysis of the demodulation of the perturbed signal will establish the effects of the perturbation on the output signal and allow the establishment of a performance index for the QDSB/FM system. Analysis will be performed for the direct channel, $m_1(t)$, since a calculation for the quadrature channel, $m_2(t)$, would be analagous.

For n a finite value with $n\Delta f << f_c$, (3-2) becomes the representation for narrowband noise which will be used to study the effect of noise perturbing the pilot signal. The perturbed pilot signal, after passing through the pilot filter in the demodulation unit of Figure 3-5 is represented by

$$e_p(t) = R(t) \text{ Cos } (\omega_p t + \Theta_p(t)) \text{ .} \tag{4-1}$$

The term R(t) is of little interest since it can be removed during the carrier synthesis process. After the carrier synthesis process, the resulting demodulation carriers can be represented by

$$e_1(t) = 2K \text{ Cos } (\omega_c t + \Theta(t)) \tag{4-2}$$

and

$$e_2(t) = 2K \text{ Sin } (\omega_c t + \Theta(t)) \quad (4\text{-}3)$$

where 2K is a constant of the synthesis process, and

$$\Theta(t) = (\Theta_p(t))(\omega_p/\omega_c) \ . \quad (4\text{-}4)$$

To examine the result of the phase perturbation of the pilot, the noise-free channel signal

$$e_c(t) = m_1(t) \text{ Cos } \omega_c t + m_2(t) \text{ Sin } \omega_c t \quad (4\text{-}5)$$

can be demodulated using (4-2). The demodulation process, after filtering the double frequency terms yields

$$e_{1D}(t) = K[m_1(t) \text{ Cos } \Theta - m_2(t) \text{ Sin } \Theta] \quad (4\text{-}6)$$

Since K is a constant value determined by the demodulation synthesis process, the above result may be normalized with respect to K by letting K = 1, yielding

$$e_D(t) = m_1(t) \text{ Cos } \Theta - m_2(t) \text{ Sin } \Theta \ . \quad (4\text{-}7)$$

Since the desired output is $m_1(t)$, it is observed that a portion of the message in the quadrature channel, $m_2(t)$, is included in the output of the direct channel, $m_1(t)$. This can be categorized as channel crosstalk. In addition to crosstalk, the direct channel is also attenuated by a factor of Cos Θ. Black has generalized crosstalk in the "Day system" by normalizing $e_D(t)$ with respect to Cos $\Theta(t)$ and thus defining crosstalk proportional to Tan Θ [18]. Black does therefore not consider the attenuation of $m_1(t)$ separately. However, it is very informative to investigate the magnitude of errors introduced by the two factors independently and to determine which of the two is most significant.

Channel crosstalk will be defined as $m_2(t)\ \mathrm{Sin}(\Theta)$ and is therefore proportional to $|\mathrm{Sin}\ \Theta|$. To achieve a specified degree of channel isolation requires that the pilot phase error be held to some minimum value. The phase error in the pilot is related to the transmitted pilot signal-to-noise ratio by the following phase probability density function obtained from the general expression for the pilot after having been perturbed by noise[19].

$$q(\Theta) = \sqrt{p/\Pi}\ \ \mathrm{Cos}\ \Theta\ e^{-p\ \mathrm{Sin}^2\ \Theta} \tag{4-8}$$

where p is the pilot signal-to-noise ratio.

For large signal-to-noise ratios, $\mathrm{Cos}\ \phi \cong 1$ and $\mathrm{Sin}^2\ \phi \cong \phi^2$, therefore, Equation (4-8) becomes:

$$q(\Theta) = \sqrt{p/\Pi}\ e^{-p^2\Theta^2} \tag{4-9}$$

which is a Gaussian density function with zero mean and variance (1/2p). The square root of the variance ($\sqrt{1/2p}$) is the RMS phase error as a function of pilot signal-to-noise ratio. Therefore, the degree of channel isolation in decibels versus pilot, signal-to-noise ratio in decibels is given by (4-10) and is plotted in Figure 4-1.

$$X_{cc} = \left| 20\ \log_{10} \frac{1}{\sqrt{(2)(10^{P_1/10})}} \right| \tag{4-10}$$

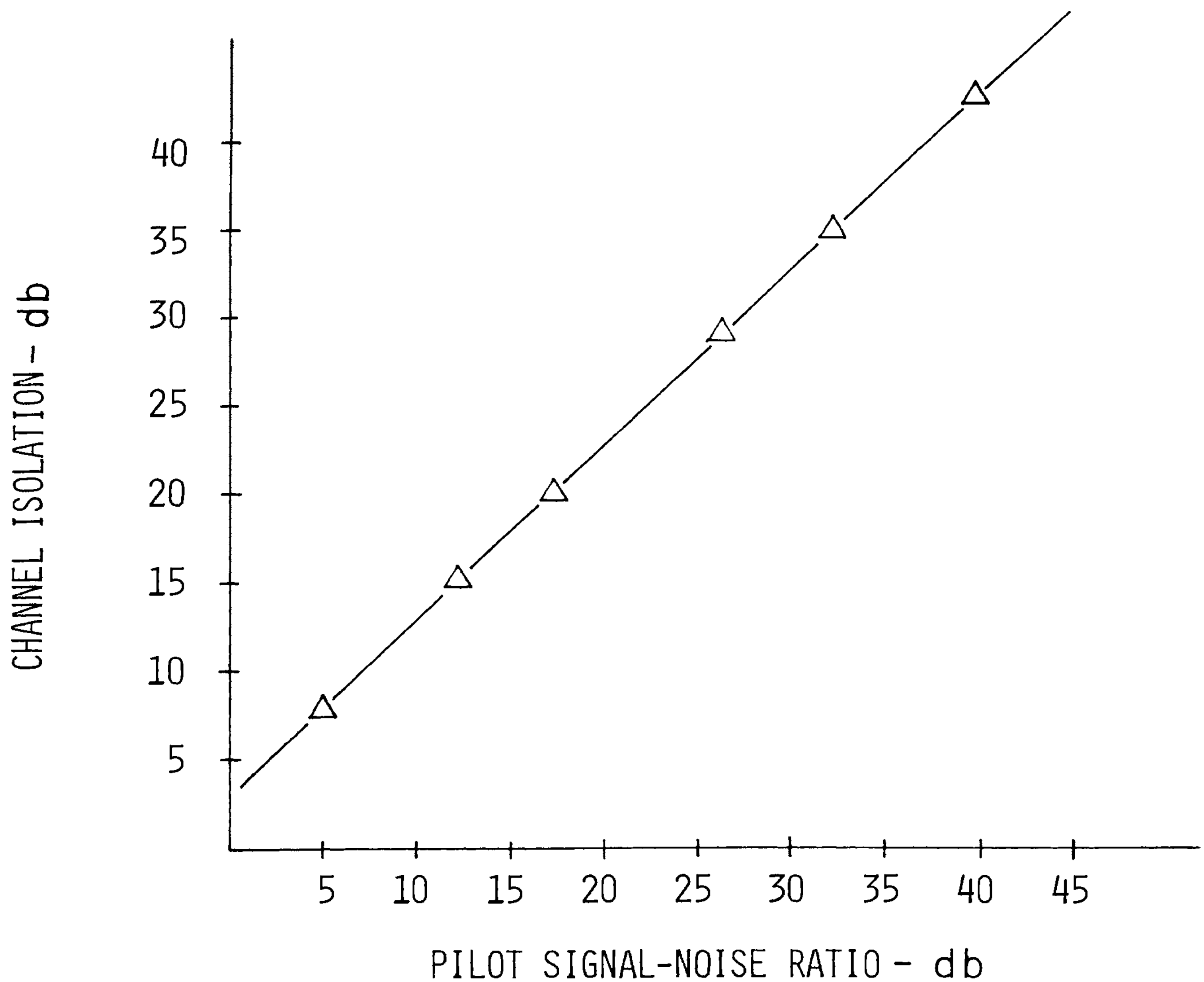

Figure 4-1. Pilot Signal-to-Noise Ratio versus Channel Isolation

where x_{cc} is the channel isolation in db normalized with resepct to $m_2(t)$ and p_1 is pilot signal-to-noise ratio in db.

The error in the attenuation of $m_1(t)$ due to Cos $\Theta(t)$ may be expressed by

$$ER = m_1(t) - m_1(t) \text{ Cos } \Theta(t) \; . \tag{4-11}$$

The mean square error is

$$\overline{ER^2} = \overline{m_1{}^2(t) - 2m_1{}^2(t) \text{ Cos } \Theta(t) + m_1{}^2 \text{ Cos}^2 \; \Theta(t)} \tag{4-12}$$

For general types of messages, let $m_1(t)$ possess a Gaussian density function with zero mean and variance $\sigma_{m_1}{}^2$. Representing Cos Θ by its series expansion, where all terms higher than order four may be ignored since $|\phi|$ must be small for channel isolation, yields

$$\begin{aligned}\overline{ER^2} &= \overline{m_1{}^2(t)} - \overline{2m_1{}^2(t)[1 - \tfrac{1}{2}\,\Theta^2]} + \overline{m_1{}^2(t)[1 - \Theta + \tfrac{1}{4}\,\Theta^4]} \\ &= \tfrac{1}{4}\,\overline{m_1{}^2(t)}\;\overline{\Theta^4} \\ &= \tfrac{3}{4}\,\sigma_{m_1}{}^2\;\sigma_\Theta{}^4\end{aligned} \tag{4-13}$$

where[20]

$$\overline{\Theta^4} = 3\,\sigma_\Theta{}^4 \tag{4-14}$$

Normalizing the mean square error with respect to $\sigma_{m_1}{}^2$ and expressing $\sigma_\Theta{}^4$ in terms of pilot signal-to-noise ratio gives

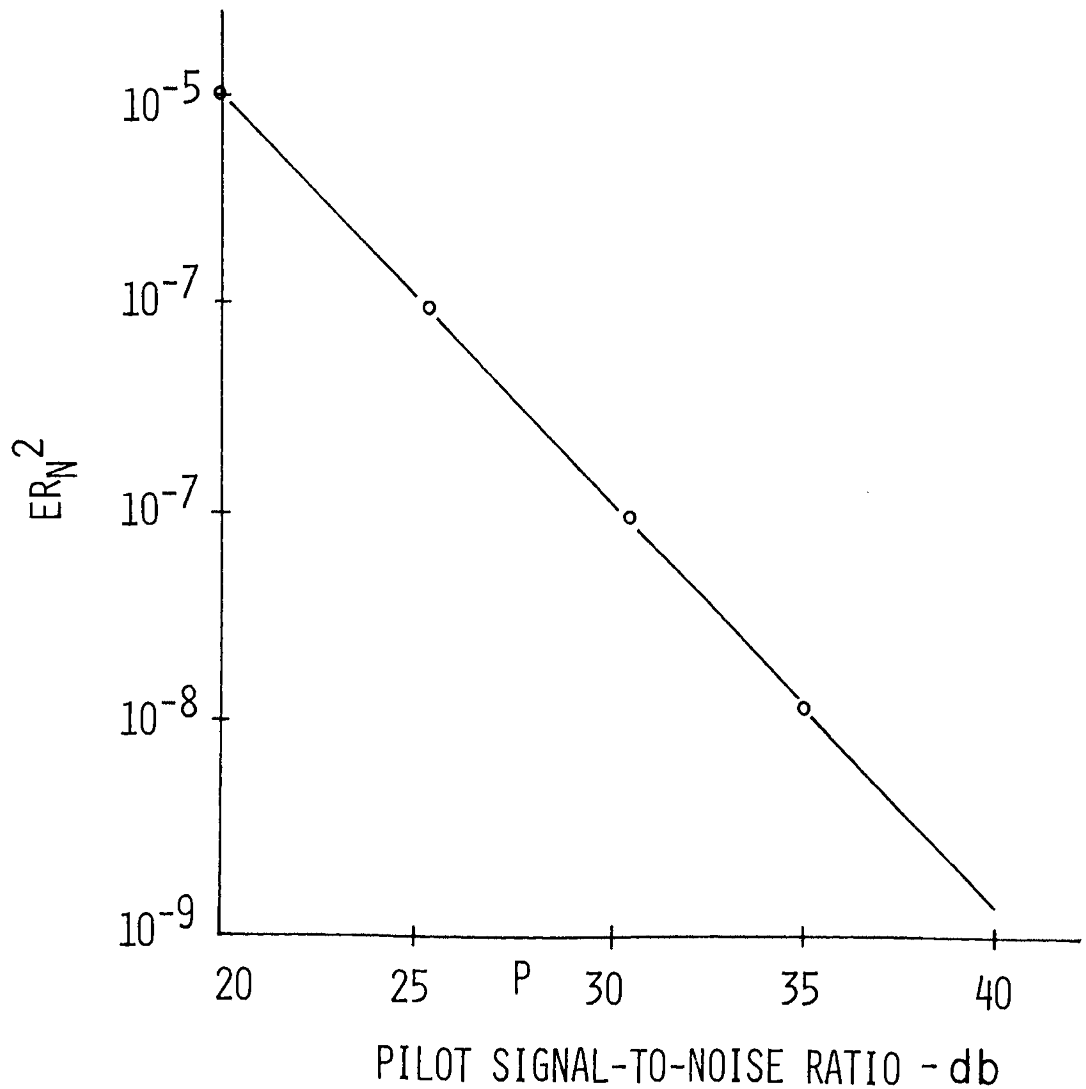

Figure 4-2. Normalized Mean Square Error Due to Attenuation of Message

$$\overline{ER_n^{\ 2}} = \frac{3}{4} \frac{1}{4(10^{p_1/10})^2}$$

$$= (3/16)10^{-p_1/5} \qquad (4\text{-}15)$$

where p_1 is the pilot signal-to-noise ratio in db.

Equation (4-15) is plotted in Figure 4-2. It can be observed that the normalized mean square error due to the attenuation of $m_1(t)$ by Cos $\phi(t)$ is extremely small once desirable channel isolation requirements are met.

B. The Effect of Noise in the Pilot and Message in the Baseband System

In a practical system, noise will perturb both the pilot and message signals. One criterion of interest for the QDSB baseband modulation system is the signal-to-noise ratio of the detected output and the signal-to-noise ratio of the predetected transmitted signal. From these quantities, the detection gain may then be obtained for QDSB and compared to the detection gain for SSB and DSB.

To investigate the detection gain of a QDSB system, the system model shown in Figure 4-3 is used. As in previous calculations, the messages $m_1(t)$ and $m_2(t)$ are assumed to be bandlimited, zero mean, Gaussian distributed, with variances $\sigma_{m_1}^{\ 2}$ and $\sigma_{m_2}^{\ 2}$ respectively. The additive noise, n(t), is assumed white, with a two sided power spectral density of n/2 watts per Hertz.

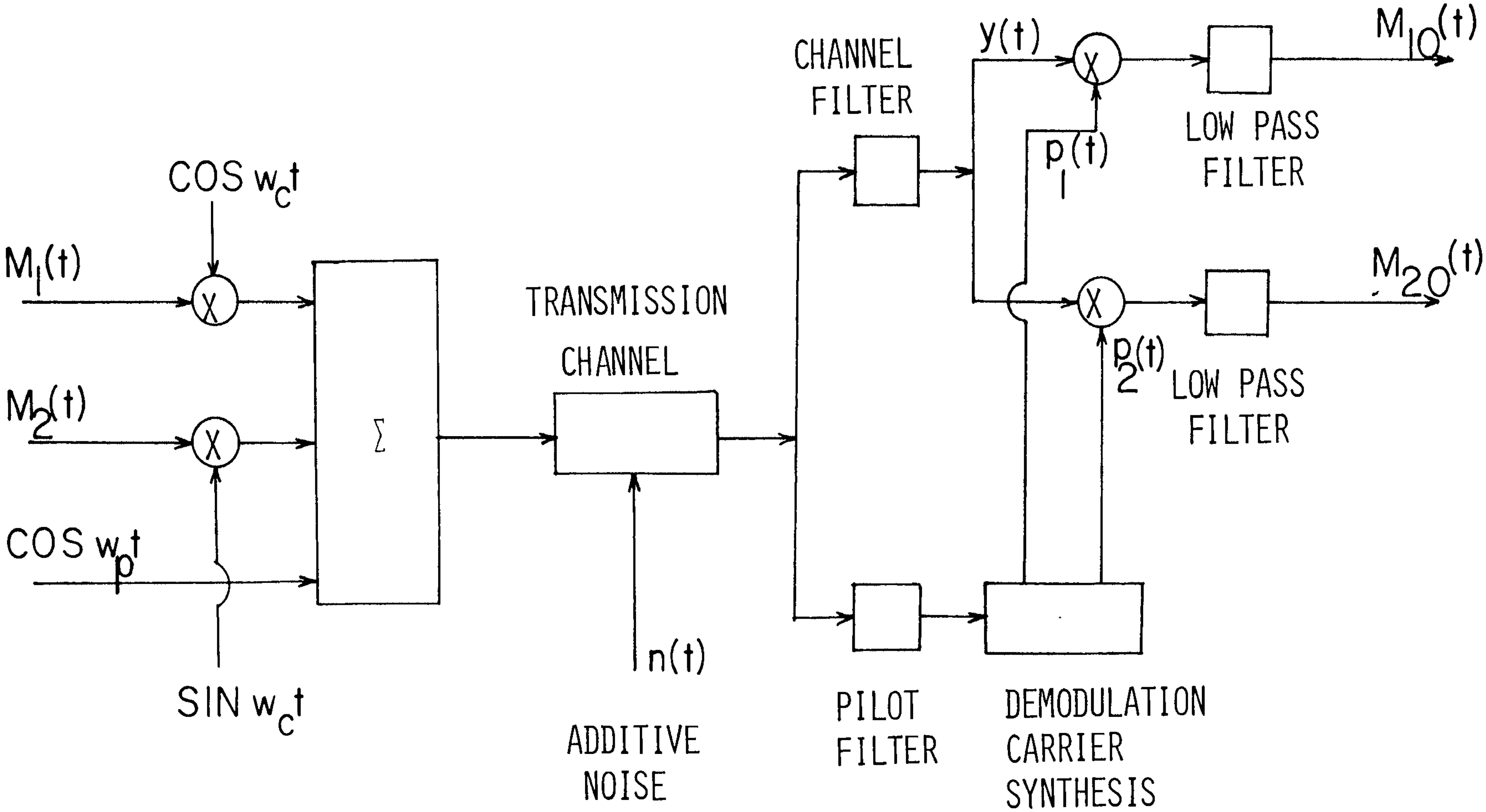

Figure 4-3. QDSB System Model for the Calculation of Detection Gain

The output of the channel filter is

$$y(t) = [m_1(t) + n_c(t)]\text{Cos}\ \omega_c t + [m_2(t) - n_s(t)]\,\text{Sin}\ \omega_c t\ , \tag{4-16}$$

and the synthesized demodulation carrier for the direct channel is

$$p_1(t) = 2\ \text{Cos}(\omega_c t + \Theta(t))\ . \tag{4-17}$$

The demodulated output for the direct channel is

$$m_{10}(t) = y(t)\ p_1(t) - \text{All Double Freq. Terms} \tag{4-18}$$

$$= [m_1(t) + n_c(t)][\text{Cos}\ (\Theta(t))]$$

$$- [m_2(t) - n_s(t)]\text{Sin}\ \Theta\ .$$

For good system performance, Chapter IV, Section A, has shown $|\Theta| << 1$ which allows Cos $\Theta(t)$ to be replaced by $1 - (1/2)\Theta^2(t)$ and Sin $\Theta(t)$ to be replaced by $\Theta(t)$. Performing these substitutions yields

$$m_{10}(t) = m_1(t) - \frac{m_1(t)\ \Theta^2(t)}{2} + n_c(t) - \frac{n_c(t)\ \Theta^2(t)}{2}$$

$$- m_2(t)\ \Theta(t) + n_s(t)\ \Theta(t)\ . \tag{4-19}$$

The signal and noise power in $m_{10}(t)$ can be determined from the mean square value of $m_{10}(t)$. Thus

$$\overline{m_{10}^2(t)} = \overline{m_1^2(t)} - \overline{m_1^2(t)}\ \overline{\Theta^2(t)} + \frac{1}{4}\overline{m_1^2(t)}\ \overline{\Theta^4(t)}$$

$$+ \overline{n_c^2(t)} - \overline{n_c^2(t)}\ \overline{\Theta^2(t)} + \frac{1}{4}\overline{n_c^2(t)}\ \overline{\Theta^4(t)}$$

$$+ \overline{m_2^2(t)}\ \overline{\Theta^2(t)} + \overline{n_s^2(t)}\ \overline{\Theta^2(t)} \qquad (4\text{-}20)$$

which becomes

$$\overline{m_{10}^2(t)} = \sigma_{m_1}^2 - \sigma_{m_1}^2 \sigma_\Theta^2 + \frac{3}{4}\sigma_{m_1}^2 \sigma_\Theta^4 + \sigma_n^2$$

$$+ \frac{3}{4}\sigma_n^2 \sigma_\Theta^4 + \sigma_{m_2}^2 \sigma_\Theta^2 \qquad (4\text{-}21)$$

where

$$\sigma_\Theta^2 = \overline{\Theta^2(t)} .$$

Since this analysis is for the direct channel, the signal will be defined as $m_1(t)$ and the signal power is therefore $\sigma_{m_1}^2$. All other terms in the demodulated output perturb the desired signal and are therefore considered noise. The detected signal power, S_D, and the detected noise power, N_D, are

$$S_D = \sigma_{m_1}^2 \qquad (4\text{-}22)$$

and

$$N_D = -\sigma_{m_1}^2 \sigma_\Theta^2 + \frac{3}{4}\sigma_{m_1}^2 \sigma_\Theta^4 + \sigma_n^2$$

$$+ \frac{3}{4}\sigma_n^2 \sigma_\Theta^4 + \sigma_{m_2}^2 \sigma_\Theta^2 \qquad (4\text{-}23)$$

respectively.

The post detection signal-to-noise ratio, given by $(S/N)_D$, is

$$(S/N)_D = \frac{\sigma_{m_1}^{\;2}}{-\sigma_{m_1}^{\;2}\,\sigma_\Theta^{\;2} + \frac{3}{4}\,\sigma_{m_1}^{\;2}\,\sigma_\Theta^{\;4} + \sigma_n^{\;2} + \frac{3}{4}\,\sigma_n^{\;2}\,\sigma_\Theta^{\;4} + \sigma_{m_2}^{\;2}\,\sigma_\Theta^{\;2}} . \tag{4-24}$$

The predetection signal-to-noise ratio will be determined by finding the signal and noise power in the output of the channel filter, y(t), by

$$y(t) = m_1(t)\ \mathrm{Cos}\ \omega_c t + n_c(t)\ \mathrm{Cos}\ \omega_c t + m_2(t)\ \mathrm{Sin}\ \omega_c t - n_s(t)\ \mathrm{Sin}\ \omega_c t \tag{4-25}$$

$$\overline{y^2(t)} = \frac{1}{2}\,\overline{m_1^{\;2}(t)} + \frac{1}{2}\,\overline{n_c^{\;2}(t)} + \frac{1}{2}\,\overline{m_2^{\;2}(t)} + \frac{1}{2}\,\overline{n_s^{\;2}(t)} = \frac{1}{2}\,\sigma_{m_1}^{\;2} + \frac{1}{2}\,\sigma_{m_2}^{\;2} + \sigma_n^{\;2} . \tag{4-26}$$

The predetection signal power, S_T, and the predetection noise power, N_T, can now be defined as

$$S_T = \frac{1}{2}\,\sigma_{m_1}^{\;2} + \frac{1}{2}\,\sigma_{m_2}^{\;2} \tag{4-27}$$

and

$$N_T = \sigma_n^{\;2} , \tag{4-28}$$

respectively.

The signal power is chosen as the sum of the power in the direct and quadrature channels for two reasons. First, at this point in the detection process, both $m_1(t)$ and $m_2(t)$ are message signals and nothing has been said as to which channel output is of interest; hence, both are of interest, and must therefore be considered as contributing to the total signal power. Second, the above definition allows consistent results with other authors when using the QDSB representation to simulate SSB or DSB, as will be demonstrated shortly. Thus, the predetection signal-to-noise ratio is

$$(S/N)_T = \frac{\sigma_{m_1}^2 + \sigma_{m_2}^2}{2\sigma_n^2} \qquad (4\text{-}29)$$

The signal-to-noise ratio detection gain is given by $(S/N)_G$ where

$(S/N)_G =$

$$\frac{2\,\sigma_n^2\,\sigma_m^2}{(\sigma_{m_1}^2 + \sigma_{m_2}^2)(-\sigma_{m_1}^2\sigma_\Theta^2 + \frac{3}{4}\sigma_{m_1}^2\sigma_\Theta^4 + \sigma_n^2 + \frac{3}{4}\sigma_n^2\sigma_\Theta^4 + \sigma_{m_2}^2\sigma_\Theta^2)} \qquad (4\text{-}30)$$

If $\sigma_{m_1}^2 = \sigma_{m_2}^2 = \sigma_m^2$

$(S/N)_G =$

$$\frac{\sigma_n^2}{-\sigma_m^2\sigma_\Theta^2 + \frac{3}{4}\sigma_m^2\sigma_\Theta^4 + \sigma_n^2 + \frac{3}{4}\sigma_n^2\sigma_\Theta^4 + \sigma_m^2\sigma_\Theta^2} \cdot \qquad (4\text{-}31)$$

With a perfectly coherent demodulation carrier, $\sigma_\Theta^2 = 0$, the above result yields the maximum detection gain

$$(S/N)_{G_{max}} = 1 \ . \qquad (4\text{-}32)$$

As stated previously, the QDSB system can be used to simulate SSB or DSB. The signal-to-noise ratio detection gain for DSB calculated from the QDSB results is given by

$$(S/N)_{G_{DSB}} = \frac{2\sigma_n^2}{-\sigma_m^2\sigma_\Theta^2 + \frac{3}{4}\sigma_m^2\sigma_\Theta^4 + \sigma_n^2 + \frac{3}{4}\sigma_n^2\sigma_\Theta^4} \qquad (4\text{-}33)$$

The maximum detection gain, occurring for the perfectly coherent demodulation carrier, $\sigma_\Theta^2 = 0$, is

$$(S/N)_{G_{DSB_{max}}} = 2 \ . \qquad (4\text{-}34)$$

The signal-to-noise ratio detection gain for SSB calculated from the QDSB results is given by

$$(S/N)_{G_{SSB}} = \frac{\sigma_n^2}{\frac{3}{4}\sigma_m^2\sigma_\Theta^4 + \sigma_n^2 + \frac{3}{4}\sigma_n^2\sigma_\Theta^4} \qquad (4\text{-}35)$$

where[21]

$$\sigma_{\hat{m}_1}^2 = \sigma_{m_1}^2 \ .$$

The maximum detection gain, occurring for the perfectly coherent demodulation carrier, for the SSB system is

$$(S/N)_{G_{SSB_{max}}} = 1 \qquad (4\text{-}36)$$

The above results are consistent with other authors' calculations using DSB and SSB systems to obtain the results[22].

The above calculations using the QDSB system have thus yielded the signal-to-noise ratio detection gain for the QDSB, DSB, and SSB systems. The results were obtained for noise in both the pilot and message signals. Maximum detection gain was calculated for each system from which it was found that QDSB and SSB were equivalent, with a maximum gain of 1 while DSB has a maximum gain of 2.

Since noise has been shown to cause a resulting error in the demodulated output, an error comparison of QDSB to SSB and DSB will give results by which to evaluate the QDSB modulation system. The message signal perturbed by noise is given by (4-16) and the pilot signal perturbed by noise is given by (4-17). The demodulated output after filtering the double frequency terms becomes

$$e_0(t) = [m_1(t) + n_c(t)]\text{Cos}\ \Theta(t) - [m_2(t) - n_s(t)]\text{Sin}\ \Theta(t)\ . \qquad (4\text{-}37)$$

The resulting error is

$$ER = m_1(t) - e_0(t)$$

$$= m_1(t) - [m_1(t) + n_c(t)]\text{Cos }\Theta(t)$$

$$+ [m_2(t) - n_s(t)]\text{Sin }\Theta(t) \quad . \qquad (4\text{-}38)$$

The mean square error can thus be calculated, yielding

$$\overline{ER^2} = \overline{m_1^{\,2}(t)} - \overline{2m_1^{\,2}(t)\ \text{Cos }\Theta(t)} - \overline{2m_1(t)\ n_c(t)\ \text{Cos }\Theta(t)}$$

$$+ \overline{2m_1(t)\ m_2(t)\ \text{Sin }\Theta(t)} - \overline{2m_1(t)\ n_s(t)\ \text{Sin }\Theta(t)}$$

$$+ \overline{m_1^{\,2}(t)\ \text{Cos}^2\ \Theta(t)} + \overline{2m_1(t)\ n_c(t)\ \text{Cos}^2\ \Theta(t)}$$

$$+ \overline{n_c^{\,2}(t)\ \text{Cos}^2\ \Theta(t)} - \overline{2m_1(t)\ m_2(t)\ \text{Sin }\Theta(t)\ \text{Cos }\Theta(t)}$$

$$+ \overline{2m_1(t)\ n_s(t)\ \text{Sin }\Theta(t)\ \text{Cos }\Theta(t)}$$

$$- \overline{2n_c(t)\ m_2(t)}\ \overline{\text{Sin }\Theta(t)\ \text{Cos }\Theta(t)} + \overline{[2n_c(t)\ n_s(t)]}$$

$$\times\ \overline{[\text{Sin }\Theta(t)\ \text{Cos }\Theta(t)]} + \overline{m_2^{\,2}(t)\ \text{Sin}^2\ \Theta(t)}$$

$$- \overline{2m_2(t)\ n_s(t)\ \text{Sin}^2\ \Theta(t)} + \overline{n_s^{\,2}(t)\ \text{Sin}^2\ \Theta(t)} \ .$$

(4-39)

Since $m_1(t)$, $m_2(t)$, $\Theta(t)$, $n_s(t)$, and $n_c(t)$ are all statistically independent and have Gaussian amplitude density functions, with zero means and variances $\sigma_{m_1}{}^2$, $\sigma_{m_2}{}^2$, $\sigma_\Theta{}^2$, $\sigma_{n_s}{}^2 = \sigma_{n_c}{}^2$, respectively, the mean square error becomes

$$\overline{ER^2_{QDSB}} = \overline{m_1^{\,2}(t)} - \overline{2m_1^{\,2}(t)}\ \overline{\mathrm{Cos}\ \Theta(t)} + \overline{m_1^{\,2}(t)}\ \overline{\mathrm{Cos}^2\ \Theta(t)}$$

$$+ \overline{m_2^{\,2}(t)}\ \overline{\mathrm{Sin}^2\ \Theta(t)} + \overline{n_s^{\,2}(t)}\ \overline{\mathrm{Sin}^2\ \Theta(t)}$$

$$+ \overline{n_c^{\,2}(t)}\ \overline{\mathrm{Cos}^2\ \Theta(t)}\ . \qquad (4\text{-}40)$$

If $\sigma_m^{\,2} = \sigma_{m_1}^{\,2} = \sigma_{m_2}^{\,2}$, the above equation becomes:

$$\overline{ER^2_{QDSB}} = \sigma_{m_1}^{\,2} - 2\sigma_{m_1}^{\,2}\ \overline{\mathrm{Cos}\ \Theta(t)} + \sigma_m^{\,2}\ [\overline{\mathrm{Cos}^2\ \Theta(t)}$$

$$+ \overline{\mathrm{Sin}^2\ \Theta(t)}] + \sigma_n^{\,2}\ [\overline{\mathrm{Cos}^2\ \Theta(t)} + \overline{\mathrm{Sin}^2\ \Theta(t)}]$$

$$(4\text{-}41)$$

$$= \sigma_m^{\,2} - 2\sigma_m^{\,2}\ \overline{\mathrm{Cos}\ \Theta(t)} + (\sigma_m^{\,2})(1) + (\sigma_n^{\,2})(1)$$

$$(4\text{-}42)$$

$$= 2\sigma_m^{\,2} - 2\sigma_m^{\,2}\ \overline{\mathrm{Cos}\ \Theta(t)} + \sigma_n^{\,2} \qquad (4\text{-}43)$$

Since good system performance requires $|\Theta(t)| \ll 1$, Cos $\Theta(t)$ can be approximated by the first two terms of its series expansion. Therefore, the mean square error becomes:

$$\overline{ER^2_{QDSB}} = 2\sigma_m^{\,2} - 2\sigma_m^{\,2}\ (1 - \tfrac{1}{2}\ \overline{\Theta^2(t)}) + \sigma_n^{\,2}$$

$$= \sigma_m^{\,2}\ \sigma_\Theta^{\,2} + \sigma_n^{\,2} \qquad (4\text{-}44)$$

Normalizing the mean square error with respect to the power in the message, $\sigma_m^{\,2}$, gives

$$\overline{ER^2_{N_{QDSB}}} = \sigma_\Theta^{\,2} + \frac{\sigma_n^{\,2}}{\sigma_m^{\,2}} \,. \tag{4-45}$$

The quantity $\sigma_n^{\,2}/\sigma_m^{\,2}$ is the reciprocal of the message signal-to-noise ratio. The quantity $\sigma_\Theta^{\,2}$ is related to the pilot signal-to-noise ratio as previously shown. Therefore, the normalized mean square error is expressed in terms of the transmitted signal-to-noise ratios by (4-45). The normalized mean square error is plotted in Figure 4-4.

Equation (4-40) can be used to evaluate the mean square error for DSB and SSB from the QDSB results. Approximating Sin $\Theta(t)$ and Cos $\Theta(t)$ by the first two terms of their series expansion in (4-40) yields

$$\overline{ER^2_{QDSB}} = \overline{\frac{1}{4}\, m_1^{\,2}(t)\ \Theta^4(t)} + \overline{m_2^{\,2}(t)\ \Theta^2(t)} + \overline{n_c^{\,2}(t)} + \overline{\frac{1}{4}\, n_c^{\,2}(t)\ \Theta^4(t)} \,. \tag{4-46}$$

If $m_2(t) = 0$, the above equation yields the DSB results of

$$\overline{ER^2_{DSB}} = \frac{1}{4}\, \overline{m_1^{\,2}(t)}\ \overline{\Theta^4(t)} + \overline{n_c^{\,2}(t)} + \frac{1}{4}\, \overline{n_c^{\,2}(t)}\ \overline{\Theta^4(t)}$$

$$= \frac{3}{4}\, \sigma_m^{\,2}\, \sigma_\Theta^{\,4} + \frac{3}{4}\, \sigma_n^{\,2}\, \sigma_\Theta^{\,4} + \sigma_n^{\,2} \tag{4-47}$$

Normalizing the above error with respect to the power in the message gives

$$\overline{ER^2_{N_{DSB}}} = \frac{3}{4}\, \sigma_\Theta^{\,4} + \frac{3}{4}\, \sigma_\Theta^{\,4}\, \frac{\sigma_n^{\,2}}{\sigma_m^{\,2}} + \frac{\sigma_n^{\,2}}{\sigma_m^{\,2}} \tag{4-48}$$

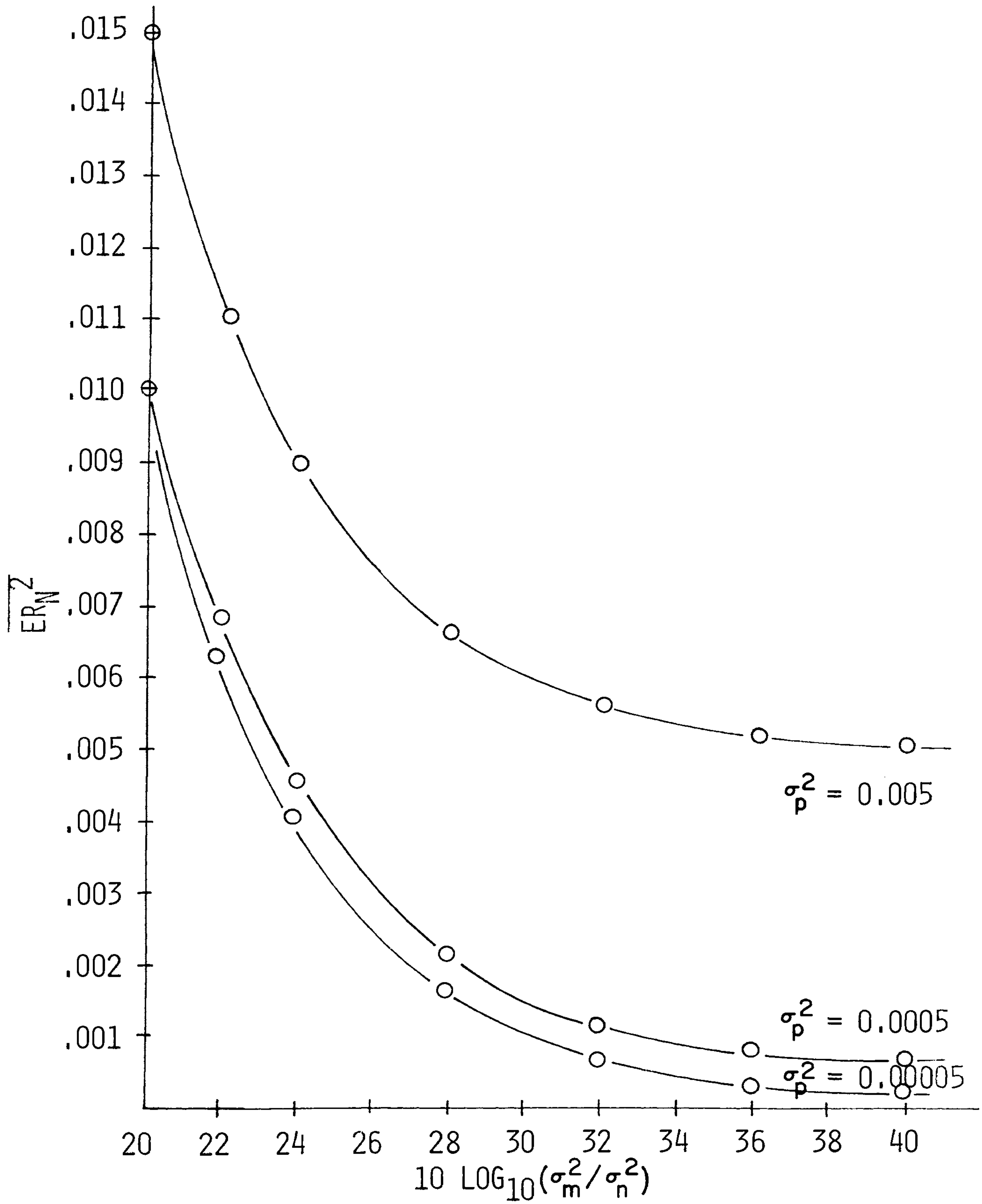

Figure 4-4. Mean Square Error versus Signal-to-Noise Ratio for QDSB System

If $m_2(t) = \hat{m}_1(t)$, Equation (4-46) yields the result for an SSB system, which is:

$$\overline{ER^2_{N_{SSB}}} = \sigma_\Theta^2 + \frac{\sigma_n^2}{\sigma_m^2} . \tag{4-49}$$

The mean square error for the QDSB and SSB systems are thus seen to be the same.

C. <u>Mean Square Error for the QDSB/FM System</u>

The normalized mean square error for QDSB baseband modulation was found to be

$$\overline{ER^2_{N_{QDSB}}} = \sigma_\Theta^2 + \frac{\sigma_n^2}{\sigma_m^2} . \tag{4-50}$$

When the baseband modulated signal is transmitted using the FM transmitter, the mean square error can be evaluated using (4-50); however, the calculation of σ_Θ^2 and σ_n^2/σ_m^2 must now include the effect of the parabolic noise spectrum resulting from the demodulation of the FM signal. The output noise power from the FM demodulator is

$$N_D = \int_{f_c-B/2}^{f_c+B/2} \frac{nf^2}{A_c^2}\, df \tag{4-51}$$

$$= n/A_c^2 \left[3\, f_c^2 B + \frac{1}{4} B^3\right] \tag{4-52}$$

where n is the power spectral density of the additive noise, f_c the center frequency of a predetection filter of bandwidth B, and A_c the amplitude of the FM carrier[23].

The variance of the pilot phase error is therefore going to depend on the frequency of the transmitted pilot and the bandwidth of the pilot filter in the QDSB demodulator. The synthesized demodulation carrier for the direct channel in terms of the transmitted pilot frequency is

$$P_1(t) = f_d \, E_p \, K \, \mathrm{Cos}[(\omega_p)(\omega_c/\omega_p)t + (\omega_c/\omega_p) \, \Theta_p(t)] \tag{4-53}$$

$$= f_d \, E_p \, K \, \mathrm{Cos}[\omega_c t + \Theta(t)] \tag{4-54}$$

where ω_c and ω_p is the carrier frequency and pilot frequency in the baseband channel in radians per second respectively, f_d is the FM transmitter deviation, E_p the amplitude of the transmitted pilot signal, and K the constant of the synthesis process. The value of K will be assumed to be unity in the following calculations. To compute the value of $\sigma_{\Theta_{FM}}^2$, $\Theta(t)$ is expressed as

$$\Theta(t) = \frac{\omega_c}{\omega_p} \Theta_p(t)$$

$$= \frac{f_c}{f_p} \Theta_p(t) \ . \tag{4-55}$$

The mean square value, $\overline{\Theta^2(t)}$, is then given by

$$\overline{\Theta^2(t)} = \left(\frac{f_c}{f_p}\right)^2 \overline{\Theta_p^{\,2}(t)} \qquad (4\text{-}56)$$

and

$$\sigma_\Theta^{\,2} = \left(\frac{f_c}{f_p}\right)^2 \sigma_{\Theta_p}^{\,2} \, . \qquad (4\text{-}57)$$

Expressing $\sigma_{\Theta_p}^{\,2}$ in terms of signal-to-noise ratio by

$$\sigma_{\Theta_p}^{\,2} = \frac{1}{2p} \qquad (4\text{-}58)$$

where p is the pilot signal-to-noise ratio, allows $\sigma_{\Theta_p}^{\,2}$ to be expressed in terms of the pilot frequency and pilot filter bandwidth. Using (4-52) and (4-58) gives

$$p = \frac{f_d^{\,2} E_p^{\,2}}{2} \left[\frac{1}{n/A_c^{\,2}(3f_p^{\,2}\, B_p + \frac{1}{4}\, B_p^{\,3})}\right] \qquad (4\text{-}59)$$

where f_p is the pilot frequency and B_p the bandwidth of the pilot filter. Therefore, $\sigma_{\Theta_p}^{\,2}$ is

$$\sigma_{\Theta_p}^{\,2} = \frac{n(3f_p^{\,2}\, B_p + \frac{1}{4}\, B_p^{\,3})}{f_d^{\,2}\, E_p^{\,2}\, A_c^{\,2}} \, . \qquad (4\text{-}60)$$

Thus, $\sigma_{\Theta_{FM}}^{\,2}$ can now be expressed as

$$\sigma_{\Theta_{FM}}^{\,2} = \left(\frac{f_c^{\,2}}{f_p^{\,2}}\right) \frac{n(3f_p^{\,2}\, B_p + \frac{1}{4}\, B_p^{\,3})}{f_d^{\,2}\, E_p^{\,2}\, A_c^{\,2}} \, . \qquad (4\text{-}61)$$

To evaluate the quantity σ_n^2/σ_m^2, the term σ_m^2 becomes the message power after FM transmission and σ_n^2 the noise power after FM transmission. The message power is

$$\sigma_{m_{FM}}^2 = f_d^2 \sigma_m^2 . \tag{4-62}$$

The noise power can be determined using (4-52) and is

$$\sigma_{n_{FM}}^2 = \frac{n}{A_c} [3f_c^2 B_c + \frac{1}{4} B_c^3] \tag{4-63}$$

where f_c is the center frequency of the message channel filter of bandwidth B_c in the QDSB demodulator.

The normalized mean square error for the QDSB/FM system is therefore given by

$$\overline{ER_{N_{QDSB/FM}}^2} = \sigma_{\Theta_{FM}}^2 + \frac{\sigma_{n_{FM}}^2}{\sigma_{m_{FM}}^2}$$

$$= \frac{n}{f_d^2 A_c^2} \{\frac{f_c^2}{E_p^2} [3B_p + \frac{1}{4} \frac{B_p^3}{f_p^2}]$$

$$+ \frac{f_c^2}{\sigma_m^2} [3B_c + \frac{1}{4} \frac{B_c^3}{f_c^2}]\} . \tag{4-64}$$

If the system is designed such that B_p^3/f_p^2 is small compared to B_p and B_c^3/f_c^2 is small compared to B_C, the mean square error can be approximated by

$$\overline{ER_{N_{QDSB/FM}}^2} = \frac{n}{f_d^2 A_c^2} [\frac{f_c^2}{E_p^2} (3B_p) + \frac{f_c^2}{\sigma_m^2} (3B_c)] . \tag{4-65}$$

The power in the baseband pilot signal before transmission is

$$S_p = \frac{E_p^2}{2} , \tag{4-66}$$

and the power in the baseband message before transmission is

$$S_c = \sigma_m^2 . \tag{4-67}$$

Expressing (4-65) in terms of S_p and S_c gives

$$\overline{ER_{N_{QDSB/FM}}^2} = \frac{nf_c^2}{f_d^2 \ A_c^2} \left[\frac{3B_p}{2S_p} + \frac{3B_c}{S_c}\right] \tag{4-68}$$

$$= K \ f_c^2 \left[\frac{B_p}{2S_p} + \frac{B_c}{S_c}\right] \tag{4-69}$$

where K is $3n/f_d^2 A_c^2$.

If the pilot signal power and the pilot filter bandwidth are held constant, the effect of the message channel on the mean square error is

$$\overline{ER_m^2} = K_1 \ f_c^2 \left[\frac{1}{2} + \frac{S_p}{B_p} \frac{B_c}{S_c}\right] \tag{4-70}$$

$$= K_1 \ f_c^2 \left[\frac{1}{2} + y_m\right] \tag{4-71}$$

where $$K_1 = K \frac{B_p}{S_p} \tag{4-72}$$

and $$y_m = \frac{S_p}{B_p} \frac{B_c}{S_c} . \tag{4-73}$$

If the message signal power and the message filter bandwidth are held constant, the effect of the pilot channel on the mean square error is

$$\overline{ER_p^{\,2}} = K_2 \, f_c^{\,2} \, [\frac{1}{2} \frac{B_p}{S_p} \frac{S_c}{B_c} + 1] \qquad (4\text{-}74)$$

$$= K_2 \, f_c^{\,2} \, [\frac{1}{2} y_p + 1] \qquad (4\text{-}75)$$

where $$K_2 = K \frac{B_c}{S_c} \qquad (4\text{-}76)$$

and $$y_p = \frac{B_p}{S_p} \frac{S_c}{B_c} \, . \qquad (4\text{-}77)$$

Equation (4-71) is plotted in Figure 4-5 and (4-75) is plotted in Figure 4-6. From these figures it is possible to obtain a measure of the effect of varying the pilot parameter as compared to the effect of varying the message channel parameters by an equal amount. The frequency axis is expressed as $f_c \times 10^k$ where k is an integer (0, 1, 2, ···).

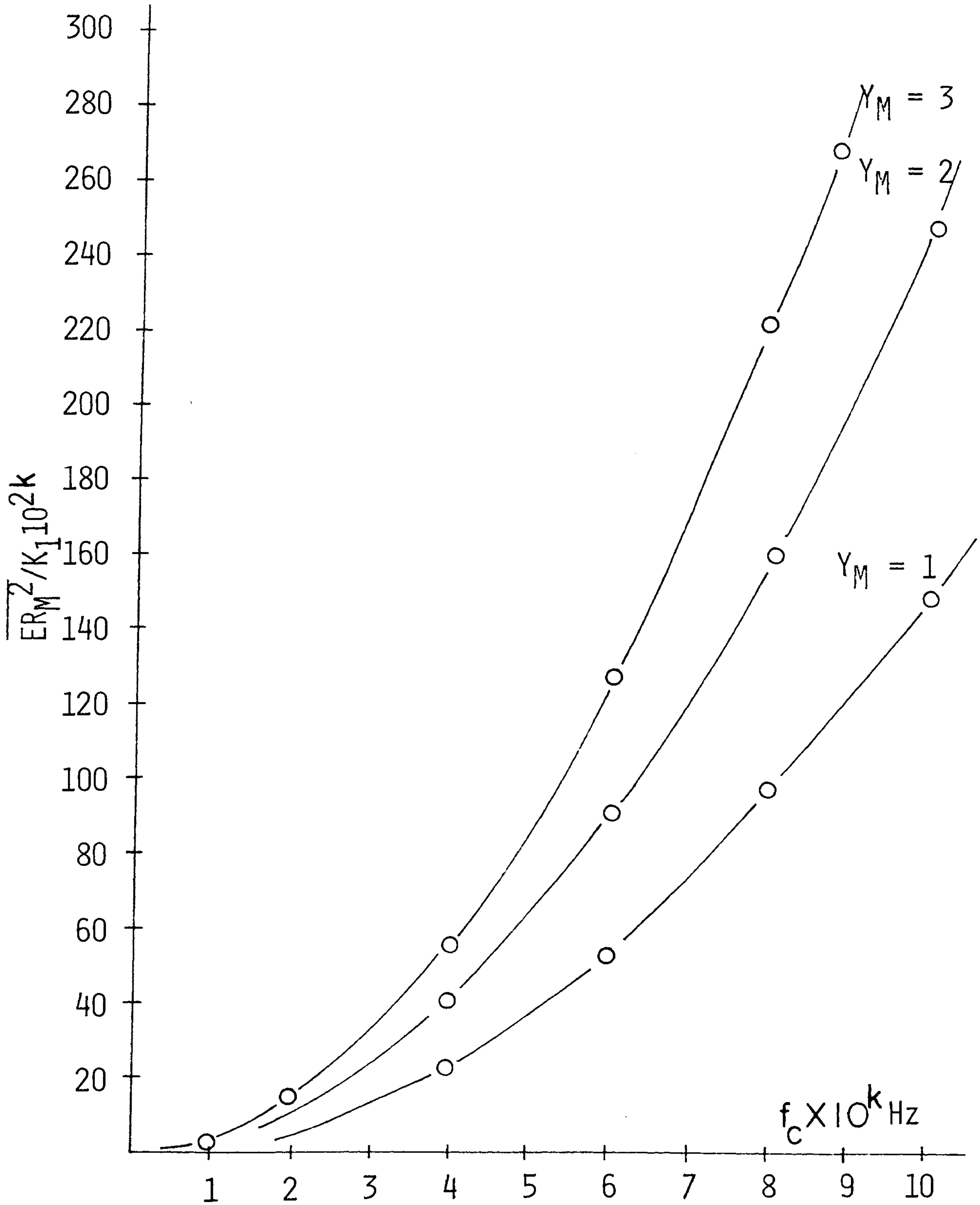

Figure 4-5. Mean Square Error for QDSB/FM with Constant Pilot Channel Parameters

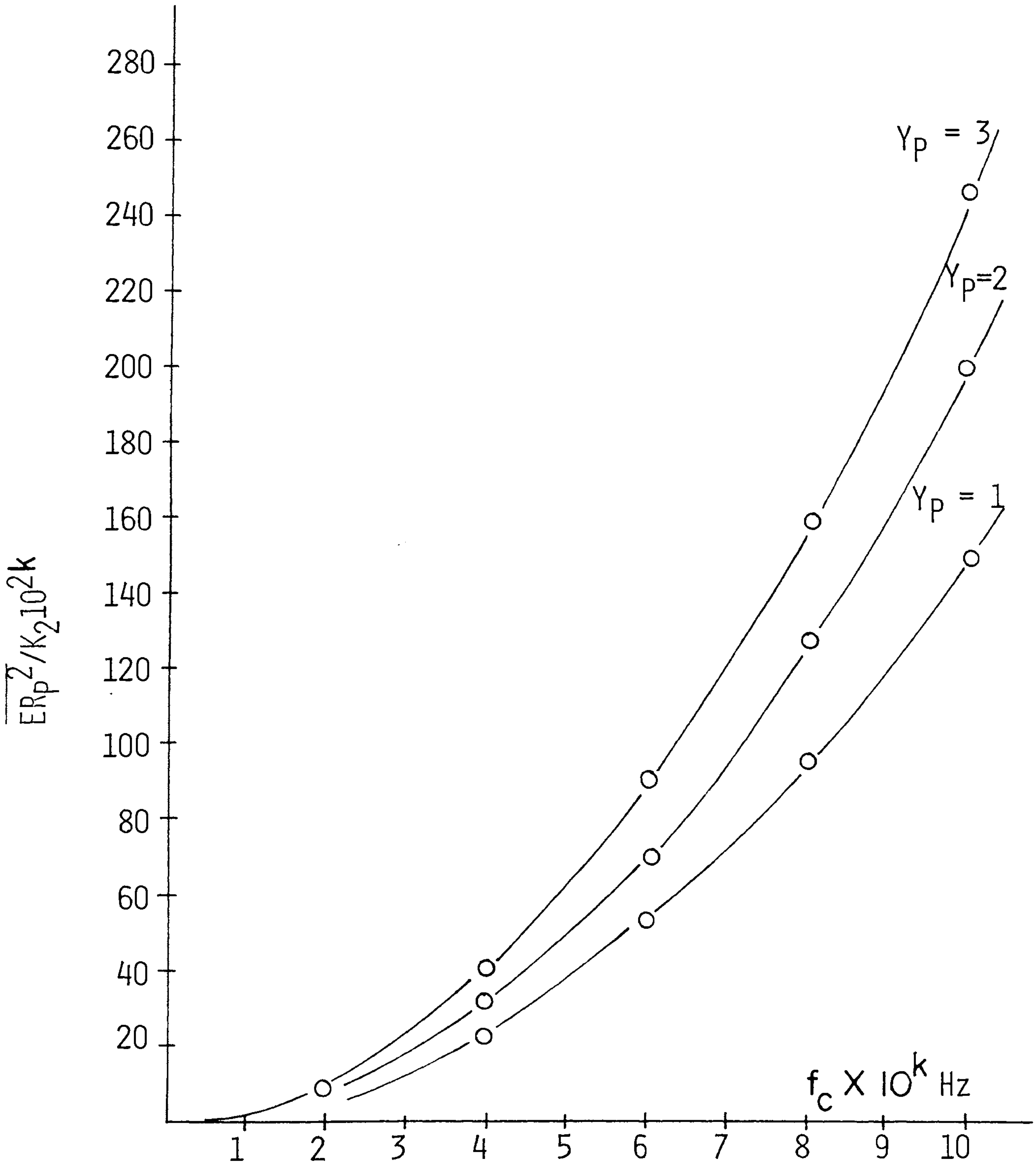

Figure 4-6. Mean Square Error for QDSB/FM with Constant Message Channel Parameters

V. COMPUTER SIMULATION OF AGC

The calculation of predetection and postdetection signal-to-noise ratios can be performed for the FM transmission system. The results of these calculations can be found in most textbooks concerning communications systems. These results are listed below[24]:

$$(S/N)_T = \frac{A_c^2}{2\,\overline{n^2(t)}} \tag{5-1}$$

A_c = the amplitude of the carrier frequency

$\overline{n^2(t)}$ = the additive noise power in the predetected noise.

$$(S/N)_D = \left(\frac{3}{2}\,\frac{A_c^2}{\overline{n^2(t)}}\,\frac{f_d^2}{w^2}\right)\,\overline{m^2(t)} \tag{5-2}$$

f_d = the frequency-deviation constant

w = the bandwidth of the message and ideally the bandwidth of the lowpass filter following the FM demodulator.

The detected signal-to-noise ratio and the signal-to-noise ratio detection gain for an FM signal is thus seen to be proportional to the mean square value of the message signal. In many cases, the message signal will be composed of nonstationary data which, if transmitted in this form, would result in a varying detected signal-to-noise ratio. By regulating the mean square value of the nonstationary data to a predetermined level, the signal-to-noise ratio could be improved and maintained at a higher value. By use of an AGC circuit at the input to the FM transmitter, the mean square value of the message signal can be regulated to this predetermined level, allowing such an improvement in the signal-to-noise ratio at the output of the FM receiver. An AGC circuit at the input to the FM transmitter requires an AGC circuit to be used at the output of the FM receiver to restore the proper amplitude to the baseband spectrum. The FM link is shown in Figure 5-1.

The AGC circuit will introduce errors into the system due to nonideal response characteristics. As stated previously, AGC circuits have been analyzed for the linear gain control case; therefore, this work will concentrate on the AGC circuit when the gain function of the voltage controlled amplifier (VCA) is nonlinear. The nonlinear function, being extremely difficult to handle mathematically, may be dealt with much easier by using a computer simulation to evaluate the AGC response characteristics.

The AGC circuit can be modeled as described in Chapter III, Section B, where K(t) is the gain function of the VCA, A the gain of the feedback loop, h(t) the impulse response of the loop low pass filter, and E_p the desired dc value of the output of the AGC circuit. The

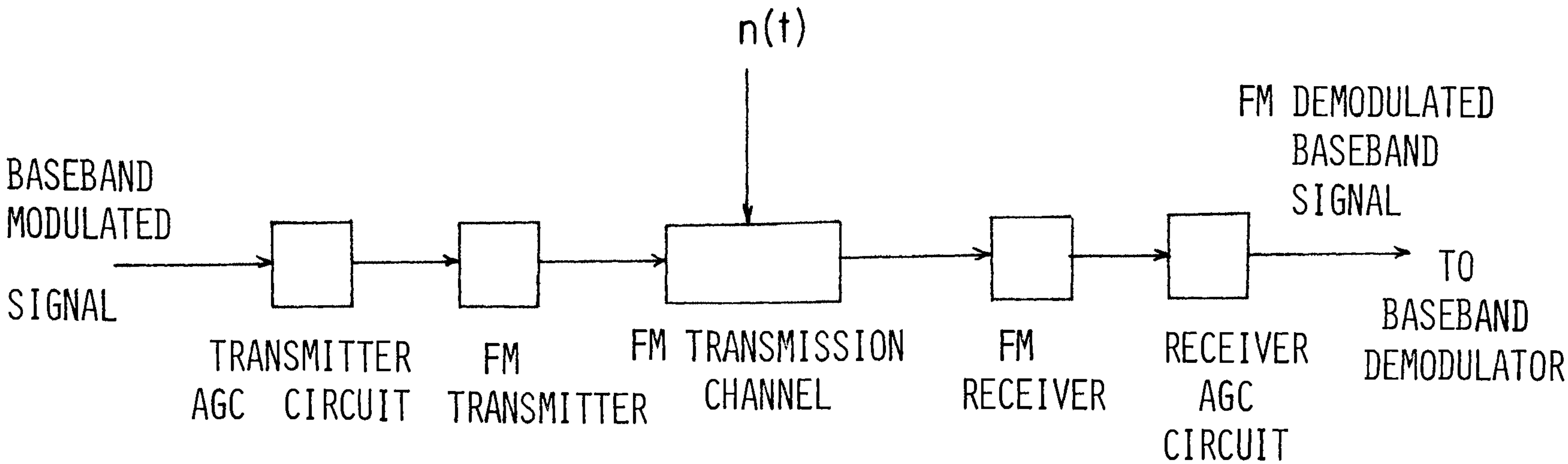

Figure 5-1. FM Portion of a Communication System Using AGC

impulse response, h(t), for the low pass filter with bandwidth B radians per second is

$$h(t) = Be^{-Bt} . \quad (5\text{-}3)$$

If the AGC circuit is to introduce no error, then it must respond instantaneously to a step input. This instantaneous response requires infinite bandwidth and is thus impractical. The response time of the AGC circuit was evaluated for various nonlinear gain functions and compared to that of the linear case. The data was obtained by use of a computer simulation using the System/360 Continuous System Modeling Program (CSMP). Referring to Figure 3-4, the following equations can be written to describe the AGC circuit.

$$EO = KEI \quad (5\text{-}4)$$

$$E2 = EP - EO \quad (5\text{-}5)$$

$$\frac{dE3}{dt} = BE2 - BE3 \quad (5\text{-}6)$$

$$G = AE3 \quad (5\text{-}7)$$

$$K = \text{gain function chosen for the VCA} . \quad (5\text{-}8)$$

Equation (5-6), obtained as the time domain representation of the transfer function of the low pass filter, may be solved when K is a linear function. When K becomes nonlinear, the solution of Equation (5-6) becomes difficult. The computer simulation simplifies this task. For the time response analysis, EI = 8.0 v step function at time t = 0. The output, EO, was plotted and the time at which the output was within 36.8% of being at its steady state value is defined as the settling time. Figure 5-2 is EO obtained from an actual computer simulation. It can be noted that there is also an error in the steady-state value of the output signal. Table I tabulates the data received from the time response analysis of the AGC loop. Figure 5-3 is a sample CSMP program used to obtain this data.

In the above analysis, the input to the AGC circuit was assumed to be noise free. In actual conditions, noise will perturb the input signal and thus errors will be introduced at the output of the AGC loop due to the tracking of the input noise. This tracking error can be defined as

$$E_T = KEI(t) - E_p \; . \tag{5-9}$$

The noise will be the additive noise used previously in this work. If

$$EI = 4.0 + n(t) \tag{5-10}$$

and

$$EP = 4.0 \tag{5-11}$$

then the resulting error is the tracking error caused by the noise in the input. Since $\overline{n(t)} = 0$, the average value of the tracking error will also be zero and the power in the error becomes the variance of the tracking error. The variance of the tracking error as a function of signal-to-noise ratio of the input signal for various VCA gain

```
                                                                                  PAGE 1

                                    MINIMUM          Z      VERSUS TIME                 MAXIMUM
                                  5.0682E-01                                           1.000E 00
 TIME               7                I                                                        I
0.0               1.0000E 00         --------------------------------------------------+
1.0000E-01        9.5305E-01         ----------------------------------------------+
2.0000E-01        9.1057E-01         ----------------------------------------+
3.0000E-01        8.7214E-01         -------------------------------------+
4.0000E-01        8.3736E-01         ---------------------------------+
5.0000E-01        8.0590E-01         ------------------------------+
6.0000E-01        7.7743E-01         ---------------------------+
7.0000E-01        7.516E-01          ------------------------+
8.0000E-01        7.2836E-01         ---------------------+
9.0000E-01        7.0727E-01         -------------------+
1.0000E 00        6.8819E-01         -----------------+
1.1000E 00        6.7092E-01         ---------------+
1.2000E 00        6.5530E-01         --------------+
1.3000E 00        6.4117E-01         ------------+
1.4000E 00        6.2838E-01         -----------+
1.5000E 00        6.1681E-01         ----------+
1.6000E 00        6.0634E-01         ---------+
1.7000E 00        5.9686E-01         --------+
1.8000E 00        5.8829E-01         -------+
1.9000E 00        5.8054E-01         ------+
2.0000E 00        5.7352E-01         -----+
2.1000E 00        5.6717E-01         -----+
2.2000E 00        5.6143E-01         ----+
2.3000E 00        5.5623E-01         ----+
2.4000E 00        5.5153E-01         ---+
2.5000E 00        5.4727E-01         ---+
2.6000E 00        5.4342E-01         --+
```

Figure 5-2. Computer Output Used to Determine Settling Time of AGC Circuit

Table I. Time Response Analysis Data for the Noise Free AGC Circuit

K(G)	Settling Time Sec	EO Steady State Volts
$1 + G$	1.000	4.049
$1 + G + .01G^2$	1.021	4.049
$1 + G + .03G^2$	1.031	4.050
$1 + G + .05G^2$	1.038	4.051
$1 + G + .07G^2$	1.050	4.052
$1 + G + .1G^2$	1.061	4.052
$1 + G + .2G^2$	1.100	4.057
Deap Sp(-.1,+.1)	1.228	4.059
e^G	1.370	4.067

```
            ****CONTINUOUS SYSTEM MODELING PROGRAM****

      ***PROBLEM INPUT STATEMENTS***

       EI=8.0
       EP=4.0
       BETA=0.01235
       A=10.0
       E0=EI*K
       E2=EP-E0
       E3D=BETA*E2-BETA*E3
       E3=INTGRL(0.0,E3D)
       G=A*E3
       K=1+G
TIMER DELT=0.001,FINTIM=50.0,OUTDEL=0.10
PRINT E0
PRTPLT E0
END
STOP
```

Figure 5-3. Sample CSMP Program to Determine Transient Response of AGC Circuit characteristics was simulated using CSMP. A sample program for these simulations is shown in Figure 5-4. The results of these simulations are presented in Table II.

The computer simulation of the AGC circuit used to obtain the above results thus provides a method to evaluate AGC performance using actual circuit conditions rather than ideal conditions. The simulation allows the investigation of many aspects of the AGC circuit with the ease of varying circuit parameters, input signals, filter functions, and obtaining data to a very high degree of accuracy. During this work, the gain function of the VCA was varied using several nonlinear functions. The resulting errors increased only slightly from those existing when the normal linear gain function is used.

```
          ****CONTINUOUS SYSTEM MODELING PROGRAM****

     ***PROBLEM INPUT STATEMENTS***

PARAMETER C=(0.3,0.7,0.9)
B=6.283
      A0=SIN( B*(0.1+0.0*0.2)*TIME+2.23)
      A1=SIN( B*(0.1+1.0*0.2)*TIME+5.76)
      A2=SIN( B*(0.1+2.0*0.2)*TIME+1.51)
      A3=SIN( B*(0.1+3.0*0.2)*TIME+4.46)
      A4=SIN( B*(0.1+4.0*0.2)*TIME+2.58)
      A5=SIN( B*(0.1+5.0*0.2)*TIME+4.32)
      A6=SIN( B*(0.1+6.0*0.2)*TIME+3.39)
      A7=SIN( B*(0.1+7.0*0.2)*TIME+1.19)
      A8=SIN( B*(0.1+8.0*0.2)*TIME+1.00)
      A9=SIN( B*(0.1+9.0*0.2)*TIME+1.13)
      A10=SIN( B*(0.1+10.0*0.2)*TIME+4.58)
      A11=SIN( B*(0.1+11.0*0.2)*TIME+3.58)
      A12=SIN( B*(0.1+12.0*0.2)*TIME+1.88)
      A13=SIN( B*(0.1+13.0*0.2)*TIME+1.00)
      A14=SIN( B*(0.1+14.0*0.2)*TIME+6.02)
      A15=SIN( B*(0.1+15.0*0.2)*TIME+2.38)
      A16=SIN( B*(0.1+16.0*0.2)*TIME+1.94)
      A17=SIN( B*(0.1+17.0*0.2)*TIME+4.90)
      A18=SIN(B*3.7*TIME+1.61)
      A19=SIN(B*3.9*TIME+2.26)
      A20=SIN(B*4.1*TIME+1.39)
      EI=4.0+C*(A1+A2+A3+A4+A5+A6+A7+A8+A9+A10+A11+A12+A13..
      +A14+A15+A16+A17+A18+A19+A20)
      EP=4.0
      BETA=0.01235
      A=10.0
      E0=EI*K
      E2=EP-E0
      E3D=BETA*E2-BETA*E3
      E3=INTGRL(0.0,E3D)
      G=A*E3
      K=1.0+G+0.05*(G**2)
      TE=4.0-E0
      TESQ=TE**2
      ITE=10.0**(-10.0)+INTGRL(0.0,TE)
      ITESQ=10.0**(-10.0)+INTGRL(0.0,TESQ)
      D=TIME+10.0**(-10.0)
      MS=(1.0/D)*ITESQ
      AVSQ=(1.0/D*ITE)**2
      VARTE=MS-AVSQ
TIMER DELT=0.001,OUTDEL=0.1,FINTIM=15.0
PRINT VARTE
END
STOP
```

Figure 5.4. Sample CSMP Program to Determine Variance in Tracking Error

Table II. Tracking Error Data for AGC Circuit Operating in the Presence of Noise

K(G)	Variance in Tracking Error	Amplitude of Noise Terms
$1 + G$	0.9223	0.3
	5.107	0.7
	8.562	0.9
$1 + G + .03G^2$	0.9223	0.3
	5.108	0.7
	8.564	0.9
$1 + G + .05G^2$	0.9223	0.3
	5.108	0.7
	8.564	0.9
$1 + G + 0.2G^2$	0.9223	0.3
	5.108	0.7
	8.564	0.9

VI. RESULTS AND CONCLUSIONS

The analysis of the QDSB demodulator began by examining the effect of pilot phase error. It was found that channel crosstalk and message attenuation were caused by pilot phase error. Channel isolation was determined as a function of pilot signal-to-noise ratio and it was determined that once desirable channel isolation is achieved, the pilot signal-to-noise ratio is high and thus the pilot phase error is extremely small. The mean square error due to the attenuation of the message signal is insignificant once desirable channel isolation requirements are achieved.

The effect of noise in both the pilot and message was examined. The signal-to-noise ratio detection gain was calculated for the QDSB demodulation. The maximum detection gain, found to occur for the case of perfect coherent demodulation, is equal to unity. This detection gain was compared to results for DSB and SSB. The maximum detection gain for DSB was found to be twice that for QDSB and that for SSB was found equal to that for QDSB.

The normalized mean square error was calculated for QDSB, DSB, and SSB. For the special case of perfect coherent demodulation, the mean square error is equal for the three types of modulation. When including the effect of pilot phase error, the magnitudes of the mean square error can be compared by observing that the value of σ_Θ^2 will be larger than the value of σ_Θ^4. It was previously determined that the pilot phase error will be extremely small once desirable channel isolation requirements are achieved. If the phase error is less than unity, $\sigma_\Theta^2 >> \sigma_\Theta^4$. The normalized mean square error for QDSB and SSB is equal and contains terms of σ_Θ^2. The mean square error for DSB contains terms of σ_Θ^4. Therefore, the mean square error for DSB will be smaller than that for QDSB or SSB.

The mean square error for the QDSB/FM system was calculated as a function of the baseband modulation frequency, the pilot frequency, and the filter bandwidths in the QDSB demodulator. It was found that if $B_p^3/f_p^2 << B_p$ and $B_c^3/f_c^2 << B_c$, the mean square error could be expressed in terms of the power in the pilot, the power in the message, the bandwidth of the message channel filter and the bandwidth of the pilot channel filter. For the pilot power and pilot filter bandwidth held constant, the mean square error was expressed as a function of the signal power and message filter bandwidth. For the signal power and message filter bandwidth held constant, the mean square error was expressed as a function of the pilot power and pilot filter bandwidth. The graph of the mean square error for these two conditions shows that the magnitude of the mean square error increases by a larger amount when the ratio of power to filter bandwidth for the message is decreased than when the ratio for the pilot is decreased by a corresponding amount. Thus the message channel parameters have a larger effect on the mean square error than the pilot channel parameters. This does not imply that the pilot is not important. The pilot must be present with small phase error to achieve demodulation of the message signals.

A digital computer simulation of the QDSB demodulation was attempted. It was possible to simulate crosstalk error and message channel attenuation; however, a simulation for the calculation of mean square error was not successful. Results were obtained when the demodulator was operating with a large mean square error. As the magnitude of the mean square error decreased, it was found that the computer run time must increase in order to get a reasonable average of the channel statistics to compute the mean square error. It was concluded that the lengthy computer time would not justify the continuation of the simulation. Frank and Kurland recommended the use of an analog/hybrid computer for such simulations to decrease computer run time[25].

The AGC circuit for the FM transmission system was simulated using a digital computer. Various nonlinear gain control functions were used for the VCA and the results compared to those when a linear gain control function is used. It was evident that the errors obtained for the nonlinear case were only slightly larger than those for the linear case. Therefore, the assumption of a linear gain control characteristic is justified. The linear case has been analyzed by many authors; therefore, no further analysis was attempted in this work.

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