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Development of an optimization model to determine sampling levels

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DEVELOPMENT OF AN OPTIMIZATION MODEL TO DETERMINE SAMPLING

LEVELS

by

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ABSTRACT

As the complexity of multi-component products increases the quality of these products becomes increasingly difficult to control. The first step to manufacturing a quality product is making sure that the components of the product meet specifications. Product quality can be controlled through sampling inspection of the components. Two models were developed in this research to determine the optimal sampling levels for incoming lots containing parts for production and assembly of multi-component systems. The main objective of the first model is to minimize the expected cost that is associated with a nonconforming item reaching assembly. In this model the time available for inspection is limited. The main objective in the second model is to minimize total cost, which includes the appraisal cost (inspection cost) and the cost associated with nonconformance reaching assembly. In this model the time available is not a constraint. The distribution of defects is assumed to follow the binomial distribution, and the distribution of accepting the lot with defects follows the hypergeometric distribution. In addition, the inspection is considered to be accurate and, if a nonconforming item is found in the inspected sample, the entire lot is rejected. An example is given with real world data and the results are discussed.

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NOMENCLATURE

Symbol	Description
$I = \{i \mid i = 1, 2, \ldots, M\}$	index set of parts considered by inspections
t_i	units of time needed to inspect a single item of part i
N_i	total number of items in the lot for part i (lot size)
d_i	probability of a defective item in the lot for part i (defect rate)
D_i	total number of defective items in the lot i
C_i	cost of a nonconforming item reaching assembly for part i
C_L	cost of labor per unit of time
T	total time available for inspection
$P(D_i)$	probability of having D_i number of defect following binomial distribution
$P(N_i,D_i,n_i)$	probability of accepting the lot with D_i number of defects after inspecting n_i number of items following hypergeometric distribution
n_i	the number of items to be inspected for part i

1. INTRODUCTION

Multi-component systems have become an everyday life occurrence and many depend on them for the simplest things in their lives. Therefore, the quality of these systems is very important to the customer and to the manufacturer. One of the ways that the companies can control the quality of their product is to perform sampling inspection on incoming lots on the parts that make the multi-component system.

In order to learn what has already been researched in the field of sampling inspection Paper I covers the current literature review in sampling inspection. It looks into allocation of inspection stations in multi-stage manufacturing process, which provides certain quality control, as well as sampling inspection of multi-component systems. It also gives several different approaches of solving the problem with various different assumptions.

This research focuses on sampling inspection of incoming lots. Some companies do not have the resources to perform inspection that would guarantee that the nonconforming items are reaching assembly; therefore, they must balance inspection with the provided resources. Paper II covers the model that assumes that the time to inspect the incoming lots is limited. Meaning that the company has to determine the optimal sampling strategy within the time frame (available resources) they have designated for inspection. The model developed in this paper is set to minimize the total expected cost associated with a nonconforming item reaching assembly by creating an optimal sampling level.

However, some companies are able to expand their resources, for instance hiring more people or outside services to inspect the incoming lots. Paper III visits this assumption and the model developed in this paper minimizes the total cost of quality control. In this model, the cost of inspection is included and the cost associated with a nonconforming item reaching assembly. In order to achieve this, the model suggests the optimal sampling levels.

PAPER

I. A REVIEW OF THE CURRENT LITERATURE IN INSPECTION SAMPLING OPTIMIZATION

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Abstract

This paper reviews the literature on the optimization of inspection sampling. Inspection sampling is critical to the prevention of nonconformance from reaching production. Optimization can be used to determine a sampling strategy yielding the best tradeoff between the risks of nonconformance and the sampling costs for avoiding the risks. This paper performs a literature review on the research that contributes to this problem and, accordingly, recommends a few research directions to the solution. Areas of research reviewed in this paper include economics of quality inspection, probabilistic risk models, inspection sampling, statistical models of inspection sampling, cost optimization, and inspection versus reliability.

Keywords
Sampling inspection, inspection optimization, quality engineering, cost optimization, quality control

1. INTRODUCTION

The products that are manufactured today have become more complex than those in the past. In industries, such as aerospace, electronic, automotive, heavy equipment industry, and off highway vehicle the number of parts that are included in the final product has increased and the number of process steps has also increased dramatically. Therefore, the quality in modern industry has become increasingly difficult to control.

The quality of a product corresponds to the durability and reliability of it, and impacts the safety of the customers using the product. Therefore, quality is a very important aspect in today's manufacturing. Competition is a reason for maintaining the high quality of the product. If the customer cannot choose a different product, the manufacturer can easily disregard the quality control of the product. Therefore, the manufacturer can save on quality control [1]. In order to know how much the quality of the product plays a part in consumer interest, it is important to quantify how customers respond and value quality improvements. These measures can then help price the products [2].

If quality concerns the consumers, quality inspection needs to be performed. But how much inspection is needed? Too little inspection could result in a nonconformance reaching the customer. This might result in penalty costs such as shipping charges, loss of faith in the product, or even lawsuits. All of these costs will drive up the total cost of the product. If the company performs a 100% inspection, it would cause the product to, again, have a high total cost [3]. Therefore, an optimal inspection strategy is needed in order to minimize the total cost while being able to guarantee a certain level of quality. In order to minimize the total cost, an optimal trade-off between the appraisal cost, which is the cost that is generated from doing quality control, and the prevention cost, which is the cost that is generated from preventing the defects from reaching the consumer, must be established to lower the failure cost and, therefore, the total cost. One of the main issues is the attitude of management and what they perceive as cost of quality and if it is relevant at all [4]. Researchers then turned to determine the optimal tradeoff between inspection cost and penalty cost such that the total cost is minimized.

This paper aims at defining the current status of the optimization of inspection sampling through reviewing relevant literature. The paper will address the different production phases (from assembly to usage) and the methods researchers have used in order to optimize the costs associated with quality control. Based on the review, research directions to fill the gap between current research and emerging needs will be determined. The remainder of the paper is organized as follows. In Section 2, inspection sampling methods for multi-state manufacturing processes are described. In Section 3, sampling inspection and system maintenance techniques for multi-component systems are presented. In Section 4, the current work and propose possible directions for future research are summarized.

2. MULTI-STAGE MANUFACTURING PROCESS

A multi-stage manufacturing process is a system of subsequent stations or stages that are necessary for the products in order for the product to be finalized. Most products today are processed through a multi-stage manufacturing process in order to meet the growing demand in the market. In order to guarantee that the product conforms to specifications and customer requirements, companies must inspect the product throughout the process (see Figure 1). These inspections then determine whether the product satisfies the quality requirements for that stage or if it needs to be reworked or scrapped.

Figure 2.1. Multi-Stage Manufacturing Process with Inspection Stations

The goal is to catch the defects when they happen, which would make it easier to determine what inspection is required such that the final product meets the quality requirements. If the defect were caught at a later stage in the production process, the detection and cost of the defect is more time consuming and more costly. On the other hand, if excessive inspection is performed during or after every stage of the multi-stage process it might result in a greater cost than if the nonconformance product was reworked.

Therefore, extensive research exists on determining the optimal allocation of inspection stations in multistage manufacturing systems. Shetwan et al. [5] researched methods for determining the distribution of quality control stations in multistage processes. They provided historic input on how previous researchers used different ways to solve the problem. Dynamic programing and nonlinear programing were shown to be the most commonly used methods for a small number of workstations. Currently, the heuristic approach has been most widely used in finding the solution for the problem.

2.1. DYNAMIC PROGRAMMING

Dynamic programming was developed by Richard Bellman and it was an improvement in decision making for multistage systems. Bellman [6] argued that previous methods of solving these problems, such as linear and non-linear programming, even in the simplest form were very time consuming and difficult to calculate and even became unsolvable. Dynamic programming was easier to compute and it gave a unique solution to the problem [6]. Dynamic programming breaks down a multistage problem into small problems that can be solved easily. The popularity of dynamic programming grew greatly. Bellman used dynamic programming to minimize the total cost of the system in some instances or maximize the total output of the system in others. Dynamic programming has a wide range of application, not just in allocation problems for multistage systems. Bellman was the first to make variations to the original theorem in order to fit new problems [7-9].

White [10] included limited inspection stations in the problem. These stations are only able to perform 100% or 0% inspection. When the defects are found they can be either reworked or scrapped. Using dynamic programming White solved the problem while considering cost of inspection, cost of repair, cost of disposing the nonconforming item, and cost of a nonconforming item going through the process. White also acknowledged that if the number of stages exceeds 20 the computation would be very time consuming.

Knowing that perfect inspection in many cases is not possible; researchers included imperfect inspection while working on optimal allocation of inspection stations [11, 12]. This means that during inspection a conforming item might be rejected (type I error) or a nonconforming item might be accepted (type II error). Eppen and Hurst [12] made the assumption that nonconforming items stay nonconforming while a conforming item might become nonconforming during the multi-stage process. In their research they also included the cost of the nonconforming item reaching the customer, where the company is responsible for replacing the item and shipping costs.

Dynamic programming was used to allocate the inspection station in a multistage system that would minimize the cost of inspection for a set quality level of the final product [13]. Oppermann et al. [3] studied the optimal quality control in electronic production. The authors used dynamic programming in order to determine the most cost effective solution for the problem. The authors extended their work in 2003 by including the cost of quality [14, 15]. The problem considered was whether to perform 100% inspection, no inspection, or a statistically controlled inspection. The paper concluded that different approaches of inspection are more desirable than others for different defect rates.

More recently, due to the increased complexity of the multistage systems, dynamic programming has been increasingly difficult to calculate. Optimizing the allocation of inspection stations of a multi-stage process where cost depends on the whole system and not just on the two consecutive stages has shown to be very hard or impossible to calculate with the increase in the stages of the system. Therefore, new methods of calculating solutions for such problems had to be found. Recent research indicates that heuristic methods may be the solution.

2.2. HEURISTIC METHODS

Heuristic methods have become increasingly popular in solving the problem of allocating inspection stages in multistage systems. The most commonly used heuristic methods are genetic algorithms and evolutionary algorithms. These methods have made it possible to calculate solutions within a fraction of the time of dynamic programming. The weakness of these methods is that they do not give the unique optimal solution, but rather an approximate of the optimal solution. Another drawback is that the more complex the system, the harder it is to know how far away the provided solution is from the optimal solution.

Evolutionary algorithms are designed to use the Darwinian principle of evolution to find the solution to complex problems. The principle is to generate a solution by using previous parent solutions. These parent solutions are ranked by the effectiveness of their solutions. Then the children solutions are generated by mutation of the parent solutions in order to find a better solution. The process is repeated until there is a solution that fits the best [16]. Genetic algorithms, on the other hand, are developed to find the best parent solution in the population that is then “cross bred” with a random solution from another population of solutions. From these solutions, a child solution is generated such that the child solution would have the best or dominant "genes" (parts of the algorithm that are generating a good solution) from both parents that would generate the better solution. In both cases the objective is to generate a solution by using the previous solutions. This sometimes means that the solution generated is not going to be optimal but just better than the previous solutions.

Many researchers have considered using heuristic methods to find the solutions for problems in various fields. Taneja and Viswanadham [17] studied the problem of allocation of inspection stations in multistage manufacturing systems. They determined the number of inspection stations needed in order to prevent a nonconformance from reaching the customer while minimizing the cost of production. Taneja and Viswanadham developed a genetic algorithm that incorporated the probability of type I and type II errors, number of stages, and probability of conformance at the inspection stage. They showed three cases where different assumptions are made in order to find the minimum cost. These assumptions are whether repetitive inspections are allowed or not allowed and whether rejected items are reworked or scrapped. Their work also shows how the complexity of finding the solutions increases as the number of stages increases. The number of generations needed to find the solution increases with increased number of stages. The solution for the problem is presented as a series of 0's and 1's (termed a chromosome) where 0 represents no inspection station after that stage and 1 represents that there should be an inspection station after that stage.

Van Volsem et al. [18] considered the same problem of inspection stations allocation. The work mainly focused on the trade-off between the cost of inspection and the penalty of a nonconformance reaching the customer. The solutions are, again, presented as a series of 0's and 1's. Van Volsem et al. [18] used the evolutionary algorithm to solve the problem. The algorithm considers a wide range of factors such as cost of inspection, upper and lower inspection limits for the item, batch size, and sample size. All of these are used in order to determine the minimum total cost of production. Van Volsem [19] later showed that the number of the inspection stations and the allocation of these stations changes as the factors in the problem changes. The paper showed that, with an increase of penalty and standard deviation of the nonconformance, the number of inspection stations increases and the location varies for the better solution. Van Volsem also stated that the solution to the problem is reached much faster than with dynamic programming but there is the drawback of not having the optimal solution but rather a better solution to the problem [19].

These heuristic methods have found their way into different fields. Leung [20] studied the approximation for determining near optimal inspection of the intervals in deteriorating production systems. The problem focuses on the optimal interval of inspection for the system that is in use. The factors considered are the profitability of the system if it functions well and a reduced profitability of the system that has experienced a certain failure. The author modified an existing model and developed two heuristic models that would solve the same problem with a better approximation and would be easier to compute. Farmani et al. [21] investigated the trade-off between resilience and total cost of the water distribution system. Zhou and Zhao [22] focused on planning quality control. Their main idea was to match different values of the factors involved in the problem with best fitting values of other factors in order to find the optimal solution.

Rajagopalan and Rajagopalan [23] described how another heuristic approach called neural networks can be trained in order to find solutions in manufacturing systems. Kakade et al. [24] used another heuristic method to find the best location of quality control inspection. They advocated the simulation approach to solve the problem. The simulated annealing approach was shown to be an efficient way of solving the problem on a small scale and was able to measure the variation to the optimal solution. However, on a large scale, the research concluded that there is no way of determining the optimal solution and that the simulated annealing method, while giving a solution, is not able to estimate how far away it is from the optimal solution.

3. MULTI-COMPONENT SYSTEMS

Multi-component systems are systems that are built from components of different and same functions. The reliability and quality of the components in the system is correlated with the reliability of the system. These components may or may not have mutual dependency on each other; meaning if one component fails the dependent component fails. In these multi-component systems, failure of the component may cause the system to fail; therefore, inspection of these components is needed in order to guarantee the quality and reliability of the system.

When solving optimization problems in multi-component systems, researchers usually turn to statistical and mathematical models and methods. Researchers are usually looking for the optimal solution and these methods are able to provide these solutions. Shi and Zhou [25] gave a brief survey of the various techniques for quality control improvement in multiple stage and component processes. Among the discussed methods are the physical method, data-driven model, and statistical process control. Physical methods require previous knowledge about the process. Data-driven models need sufficient knowledge in mathematics and statistics. It also requires a vast historical database in order to provide reasonable estimates. Data-driven models are appealing because they do not require the previous knowledge of the process in order to be applied. Statistical process control has a high “false alarm” probability and, according to Shi and Zhou “lacks the capability to discriminate among changes at different stages”. The research concluded that the most attractive methods for solving these problems would typically be data-driven models and other quantitative models because they can be applied to various systems in the market. In order to guarantee the quality of the final product, companies use sampling inspection plans for the system components. In addition, in order for these multi-component systems to stay operational, certain maintenance plans have to be developed.

3.1. SAMPLING INSPECTION

In order to have a high quality product the components in that product also have to meet their respective quality requirements. Therefore, the company should perform inspection of the components based on a specified frequency prior to assembly in order to guarantee that conforming components are being used in the assembled product. If the nonconforming component reaches the final product that system might not function as intended. This would cause the company to either rework or scrap the product which would drive up the cost and cause customer dissatisfaction. In order to minimize this cost companies have sampling inspection plans that are aimed to prevent the nonconforming components from reaching the final product. 100% inspection is time-consuming and drives up the cost of the final product. 0% inspection does not guarantee the quality of the product. Therefore, researchers have studied methods for optimizing sampling inspection plans in order to minimize product cost.

The quality of the final product starts with quality components. In other words, the product has a good chance of meeting the quality specification if the components in the product also meet specifications. However, every lot of components that are received in a factory has a probability that some items do not meet the requirements, which would diminish the quality of the final product or cause it to fail. Standards such as Military Standard MIL-STD-1916 are used in order to determine whether the batch should be accepted or rejected. The MIL-STD-1916 sampling plan works under "zero accept one reject" premises, meaning that if there is a nonconformance in the sample of the population then the whole population is rejected [26]. Li et al. acknowledge that just because there are no nonconformances in the sample it does not mean that the population meets conformance requirements.

Hamaker [27] described three different approaches to sampling inspection: sampling tables, collecting data, and constructing inspection plans. He also modeled a plan of using economic theories where he concluded that it might be more economical not to inspect the lots with a small probability of nonconforming items. While all the methods have been implemented in the real world the author warned that the data collection and sampling tables might lead to over sampling while using economic theories might not always be possible because certain factors might not be obtainable. Hamaker then suggested that a sampling plan should be selected and monitored for its performance and then later if needed adjusted for the new data. Calvin [28] made similar remarks when considering the zero defect philosophy. He pointed out that many managers are looking only for the ways to reach the zero defects but not to stay at the zero defect level. The author argued that there are different statistical methods such as control charts and acceptance sampling plans that managers can use in order for the product to stay at zero defects. The author suggested that the managers should consider how many good parts are in between two bad parts and in this way could control the zero defect level. If there are a smaller number of good parts between two bad parts then the lower limit suggests that the batch is rejected. Calvin also urged that the data collection has to be thorough because statistical importance may be lost in the process. The limits of keeping or discarding the batch should be challenged in a way that zero defects are still achieved for a lower cost.

If the population is rejected then the production might slow down or stop because of the lack of components that are necessary to complete the final product. Therefore, Salameh and Jaber [29] focused on the optimal inventory of the items that might contain items of imperfect quality. They found that the quantity of the items per order increases as the probability of defective item increase.

Maddah and Jabber [30], on the other hand, observed that the order of large quantity of imperfect quality items is not always very profitable; therefore, a proper trade-off between the shipping cost (of small order size) and inventory holding cost (of large order size) is needed in order to determine the most profitable solution. The findings by Maddah and Jabber show that "the optimal order quantity is increasing in the screening rate and in the variability of the fraction of imperfect items". Also, it should be taken into consideration whether to discard the item of lower quality or sell in the market as a product of lower quality [31].

After the conforming product reaches the customer the components are still at risk of failing due to wear while the system is working. Therefore, a certain maintenance policy is needed in order for the product to remain in working condition.

3.2. SYSTEM MAINTENANCE

Optimal system maintenance has been researched in detail and the most common approach is optimal inspection intervals. Multi-component systems usually go through two types of maintenance: corrective maintenance (when the component fails) and preventive maintenance (other components are inspected for possible future failure).

Tian and Liao [32] devised a method of finding the optimal maintenance policy. They consider an optimal economic decision while performing maintenance on the system. The question is whether to replace the working parts on the multi-component system when one of the components already fails in order to prevent future failures. The replacement decision is made based on the age of the product's component and its hazard value. If these values are greater than the risk thresholds, the component is replaced. This works only if the components have economic dependency. They concluded that it is cost effective to perform preventive replacement by keeping those components from failing if there is an economic dependency on the component. The reasoning is that the cost of maintaining the system in the long run should be minimized.

Many researchers focused on finding the optimal inspection intervals in order to prevent the component failures that would cause the system failure [33-40]. Many of these made an assumption that the defects follow a Poisson distribution, which is a stochastic interval that these failures occur.

Taghipour and Banjevic [33] and Taghipuor et al. [34] also considered an economic aspect of the problem. They devised two different types of failures in the system: the "hard" failures that would cause a system failure and the "soft" failures that would not cause system failure but would diminish the effectiveness of the system; therefore, this system will not run as efficiently as if there were no "soft" failures causing the cost of running the system to be greater. The issue with a "soft" failure is that it is a hidden failure and could only be fixed upon inspection. The system would be inspected for the "soft" failures when the "hard" failure occurs. This is what the authors call opportunistic inspection. However, the research was more concerned with the optimal inspection interval for the "soft" failures in order to minimize the cost of inspection. The problem requires solving for the failure probabilities and expected time of failure. They also concluded that the calculations are very intensive in order to reach an optimal solution.

Zhao et al. [35] made the assumptions that the defects follow the non-homogeneous Poisson distribution and that the inspection of the component is imperfect.

The main research goal was to find the expected number of failures under the inspection. Zhao et al. found that if the defect rate is increasing or decreasing then the optimal inspection interval is no longer optimal. Therefore, this factor has to be considered as well. The authors also mentioned that due to imperfect inspection, detection rate is a very important factor in determining the optimal inspection interval.

Anisimov [36] assumed that the system has fast Markov switches. The focus of the research was to approximate the long run cost of maintaining the multicomponent system and the optimal interval of the inspection by minimizing the long run cost of maintenance. Anisimov argued that for a system with a great number of components the optimal interval of inspection is difficult to calculate. He also considered that the inspection is not optimal and that there is a possibility that a component that has suffered a failure can still be in the system unnoticed. The author later proved that there has to be an optimal interval of maintenance that minimizes the long-term cost. Anisimov and other researchers [33, 37-40] proved that the optimal interval for the inspection exists and that the problem occurs as the system becomes more complex. Table 3.1 provides a summary of the literature review.

Table 3.1. Summary Table

Paper	Technique	System	Inspection/ Maintenance	Error
Bellman (1952)	DP	Multi-Stage		
Bellman (1953)	DP	Multi-Stage		
Bellman (1956)	DP	Multi-Stage		
Hamaker (1958)	SP	Multi-Component		
Lindsay & Bishop (1964)	DP	Multi-Stage		
White (1969)	DP	Multi-Stage		
Hurst (1973)	DP	Multi-Stage	Imperfect	Type I/Type II
Eppen & Hurst (1974)	DP	Multi-Stage	Imperfect	Type I/Type II
Calvin (1983)	SP	Multi-Component		
Taneja & Viswanadham (1994)	GA	Multi-Stage		Type I/Type II
Rajagoplan & Rajagoplan (1996)	NN	Application		
Jones (1998)	GA/EA	Application		

Table 3.2. Summary Table (cont.)

Paper	Technique	System	Inspection/ Maintenance	Error
Salameh & Jaber (2000)	EOQ	Multi-Component	Imperfect	
Opperman et al. (2001)	DP	Multi-Stage		
Zhou & Zhao (2002)	GA	Multi-Stage		
Opperman et al. (2003)	DP	Multi-Stage		
Wang & Christer (2003)	NHPP	Multi-Component	Perfect	
Kakade et al. (2004)	GA	Multi-Stage		
Anisimov (2005)	MP	Multi-Component		
Farmani et al. (2005)	GA	Multi-Stage		
Zhao et al. (2005)	NHPP	Multi-Component	Imperfect	
Van Volsem et al. (2007)	EA	Multi-Stage	Perfect	
Maddah & Jabber (2008)	EOQ	Multi-Component	Imperfect	
Leung (2009)	GA	Multi-Component	Imperfect	
Shi & Zhou (2009)	DD/SPC/ PM	Multi-Component/ Multi-Stage		
Sung & Scharge (2009)	MCS	Multi-Component	Perfect	
Maddah et al. (2010)	EOQ	Multi-Component	Imperfect	
Taghipour et al. (2010)	NHPP	Multi-Component		Type I/Type II
Van Volsem (2010)	EA	Multi-Stage	Perfect	
Cheng et al. (2011)	MP	Multi-Component		
Li et al. (2011)	MIL	Multi-Component		
Taghipour & Banjevic (2011)	NHPP	Multi-Component	Perfect	
Tian & Liao (2011)	CBM	Multi-Component		
Van der Weide & Pandey (2011)	NHPP	Multi-Component		

Dynamic Programming (DP), Sampling Plan (SP), Genetic Algorithm (GA), Evolutionary Algorithm (EA), Neural Network (NN), Military Standard (MIL), Data Driven (DD), Statistical Process Control (SPC), Economic Order Quantity (EOQ), Condition Based Maintenance (CBM), Non-homogeneous Poisson Process (NHPP), Markov Process (MP), Monte Carlo Simulation (MCS)

4. CONCLUSIONS AND FUTURE WORK

As products are becoming more and more complex, the quality of these products has shown to be increasingly difficult to control. However, there has been a great deal of research conducted in order to control the quality of the products. The main issues are how much does the quality cost, how much attention should managers give to quality inspection, and are the quality inspections profitable. As we can see, a certain trade-off between inspection cost and penalty cost needs to be established in order to lower the total cost while controlling the quality of the product.

From the current literature we can see that numerous researchers have studied the existing problems in great detail and have included the factors that are found in real world problems. Researchers have also shown that they are able to adapt to the increasing difficulty of the problems. In the problem of allocating inspection stations in multi-stage manufacturing systems, the focus has changed from dynamic programming, which is an accurate optimal but a difficult method of solving the problem, to heuristic methods, which give a close to optimal solution but with a significantly smaller amount of time. Practically speaking in industry, this might be a better solution considering that the information for optimizing systems is needed quickly.

Researchers that investigated multi-component systems approached the problem statistically while searching for the optimal solution. The possible drawback of this method is the intensity of statistical knowledge needed in order to find the optimal solution.

Implementing these methods to the problems in industry might be difficult. While the researchers have considered certain factors, not all of the researchers included all of the factors. Assumptions such as perfect inspection and no error were made while devising the models in certain studies [17-19, 24, 33, 34, 38, 40]. These assumptions cannot be made in the real world; therefore, the calculations would not be optimal.

In order to find the optimal solution in every aspect of manufacturing further research is needed. Applying different approaches that seem to dominate certain aspects of manufacturing and provide reasonable results should be applied to other aspects. One of the directions that should be considered would be to implement heuristic methods in order to find the solution for sampling inspection and maintenance of multi-component systems. This method might prove useful when the solutions are needed in a short period of time and management does not want an optimal solution but rather a better one to what is implemented at the moment. Another would be to apply dynamic programming into sampling inspection and maintenance for the problems that would be solvable for this approach. Also, research should test whether the statistical approaches would be as efficient in finding the solutions in the multi-stage processes as they were in the multi-component systems. It is also important to determine how well these models fit in the real world applications to see whether theory and practice meet.

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II. DEVELOPMENT OF AN OPTIMIZATION MODEL TO DETERMINE SAMPLING LEVELS

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Abstract

As the complexity of the multi-component products increases the quality of these products becomes increasingly difficult to control. The first step to manufacturing a quality product is making sure that the components of the product meet specifications. Product quality can be controlled through sampling inspection of the components. The model presented in this paper was developed to determine the optimal sampling levels for incoming lots containing parts for production and assembly of multi-component systems. The main objective of the model is to minimize the expected cost that is associated with a nonconforming item reaching assembly. In this model the time available for inspection is limited, the distribution of defects is assumed to follow the binomial distribution, and the distribution of accepting the lot with defects follows the hypergeometric distribution. In addition, the inspection is considered to be accurate and, if a nonconforming item is found in the inspected sample, the entire lot is rejected. An example is given with real world data and the results are discussed.

Keywords
Sampling inspection, inspection optimization, quality engineering, cost optimization, quality control

1. INTRODUCTION

The purpose of this research is to determine the optimal sampling inspection plan of incoming lots. These lots contain a specific number of individual items for manufacturing and assembly of multi-component systems. These systems are common in the automotive, aerospace, heavy equipment, off highway vehicle, and electronic industry. The complexity and demand for these products have increased dramatically. Therefore, the number of incoming lots and parts used in production has also increased dramatically. Since the quality of the product corresponds to the durability, reliability, and customer's safety and satisfaction, quality controls are necessary to improve the quality of the final product. Competition in the market and quality appreciation by consumers has driven manufacturers to pay more attention to the quality of their products (Marttinen, 2002; Setijono and Dahlgaard, 2008).

One method to improve the product quality is to perform sampling inspection on the incoming lots. In order to do this, it is then necessary to determine the appropriate level of inspection. If the company is not inspecting enough, there is a risk of a nonconforming item reaching the assembly line and possibly remaining in the system as a finished product. This would result in a final product that does not meet the customer's specifications and possible penalty costs such as shipping charges, loss of faith in the product and manufacturer, or even lawsuits. Since these costs affect the company, they increase the cost of the final product and reduce the profit from the product. On the other hand, if the company performs 100% inspection, the risk of nonconforming items reaching assembly would be minimized. The cost associated with 100% inspection (manpower, equipment, etc.) would, again, drive up the production cost of the final product and even possibly delay production (Oppermann et al., 2001). Therefore, an optimal inspection strategy is needed in order to minimize the total cost while providing a certain level of quality. In order to minimize the total cost, an optimal trade-off between the appraisal cost, which is the cost that is generated from performing quality inspection, and the prevention cost, which is the cost that is generated from preventing the defects from reaching the consumer, must be established to lower the failure cost and, therefore, the total cost (Keogh et al., 2000).

Companies typically follow some type of sampling inspection procedure in their facilities. A common practice of companies is to follow the “trust the supplier” ideology where only a few items in the first lot are inspected. If these items meet the specifications, that lot and consecutive lots are sent to the assembly line without further inspection. It should be also noted that some companies do not have the ability to inspect certain features of the items in the lot, which forces them to trust the supplier.

This research considers sampling inspection optimization and provides a model that determines the inspection levels. The research focuses on determining the inspection levels that would minimize the expected total cost of nonconforming items in the time available. The paper is organized as follows. Section 2 covers the literature review. Section 3 proposes and describes the model. Section 4 describes the solution approach. Section 5 covers the analysis and the results. Lastly, section 6 discusses future work and provides conclusions.

2. LITERATURE REVIEW

2.1. SAMPLING INSPECTION OF LOTS

Research and publications on sampling inspection of lots increased during and after World War II. Demand for military products has increased greatly and tolerance for faulty equipment was low during this period. Since production increased dramatically, unit-by-unit, or 100%, inspection was not practical. Therefore, quality control has shifted from unit-to-unit inspection to statistically controlled sampling inspection. Various military standards schemes were created in order to control the quality of the incoming lots (Champernowne, 1953; Barnard, 1954). Military standards first inspect a large sample size to determine the distribution of defects. If the lots are found to meet the specifications, the inspection on the consecutive lots is then relaxed.

Li et al. (2011) examined Military Standard MIL-STD-1916. This standard works under “zero accept one reject” premises; meaning that if there is a nonconformance in the sample of the population then the entire population is rejected. Lie et al. revised MIL-STD-1916 by expanding the current standard from 11 to 18 groups of inspection in order to separate the sampling plans from 100% inspection. Li et al. acknowledge that just because there are no nonconforming items in the sample it does not mean that the population meets conformance requirements. Meaning that the lots can still carry a risk of a defect reaching the final product.

The research of Champernowne (1953) focused on the economic success of the problem by using the sampling inspection as a tool in the process. For the purpose of the study Champernowne assumed that several variables in the problem are known:

> “(i) the average quality of the batches to be tested and the variation between batches of quality about that average, (ii) the cost of inspection and its dependence on the amount of inspection undertaken, and (iii) the cost involved by deciding wrongly to accept or wrongly to reject a batch, and the way this cost depends on the quality of the batch.”

Using this information, Champernowne developed an economical boundaries model that uses sampling inspection results (number of effective and defective items) to determine whether the lot should be accepted or rejected. Champernowne mainly focused on satisfying the economical aspect of the problem. Meaning that as long as the result is within the economical boundaries the lot would be accepted even if the defects were found in the sample On the other hand, Barnard (1954) argued that the information, which Champernowne assumes are given, are not readily available in the real world. Barnard argues that assigning a distribution for defects is needed in order to solve the problem. Barnard also argues that a considerable amount of information of each lot is needed to make an optimal decision for the problem.

Hamaker (1958) described three different approaches to sampling inspection: sampling tables, collecting data, and constructing inspection plans. Hamaker also modeled a plan of using economic theories where the research concluded that it might be more economical not to inspect the lots with a small probability of nonconforming items. While all the methods have been implemented in the real world, Hamaker warned that the data collection and sampling tables might lead to over sampling while using economic theories might not always be possible because certain factors might not be obtainable. Hamaker then suggested that a sampling plan should be selected and monitored for its performance and then later adjusted for the new data if needed.

2.2. SAMPLING INSPECTION IN MULTI-STAGE PROCESS SYSTEMS

Research performed in this field has mainly focused on the allocation of inspection stations within multi-stage process systems (MSPS). These inspection stations are supposed to catch the possible defects that might be experienced during production. The solutions have mainly been developed using dynamic programing or heuristic methods. The published research has commonly considered the economical aspect of the problem, trading off the risk and cost of inspection.

Dynamic programming has widely been considered while searching for the problem solution. It managed to break down the multi-stage problem into smaller, more manageable problems, which are then easier to solve (Bellman, 1952; Bellman, 1953a Bellman, 1953b; Bellman, 1956). Other researchers have expanded the problem considering among others that only no inspection or 100% inspection is available (White, 1968), imperfect inspection where inspection stations may label a nonconforming item conforming and vice versa (Hurst, 1973; Eppen, 1975), and statistically controlled inspection (Oppermann et al., 2001; Oppermann et al., 2003). Dynamic programming was able to determine an optimal solution to the problem and it was very effective for MSPS with a small number of stations. An increase in the number of stations in the MSPS dynamic programming took longer than desired to find a solution. New methods, such as heuristic methods, have been found for calculating solutions for the problem.

Heuristic methods such as evolutionary and genetic algorithms are the two most popular methods in finding the solution to the inspection stations allocation problem. Researchers have, again, considered imperfect inspection (Taneja and Viswanadham, 1994), and economical trade-offs (Van Volsem et al., 2007; Van Volsem, 2010). While providing a fairly quick solution, heuristic methods are not guaranteeing optimal, but rather a close to optimal solution.

3. THE MODEL

Consider an assembly line that has M different parts coming in. These parts have different lot size, defect rate, and repair cost if a defective item enters the assembly line. They also have a specific time interval needed to inspect a single item. An incoming inspection is performed on these parts in order to control the quality of the final product. The problem facing management is to determine the appropriate inspection sample size for each part considering the variability of risks associated with the M parts and the limited resource of labor hours the assembly line can spend on inspection. The problem can be modeled as a Nonlinear Integer Programming (NIP) problem as follows.

Index sets:

$I = \{i \mid i = 1, 2, \ldots, M\}$ = index set of parts considered by inspections

Parameters:

T = Total labor hours available

t_i = time needed to inspect a single item of part i

N_i = total number of items in the lot for part i (lot size)

d_i = probability of a defective item in the lot for part i (defect rate)

D_i = total number of defective items in the lot i

C_i = cost of a nonconforming item reaching assembly for part i

Decision variables:

n_i = the number of items to be inspected for part i

Minimize:

$$\sum_{i=1}^{M} \sum_{D_i=0}^{N_i} P(D_i)\, D_i\, C_i\, P(N_i, D_i, n_i) \tag{1}$$

Subject to:

$$P(D_i) = \binom{N_i}{D_i} {d_i}^{D_i} (1 - d_i)^{N_i - D_i} \tag{2}$$

$$P(N_i, D_i, n_i) = \frac{\binom{N_i - D_i}{n_i}}{\binom{N_i}{n_i}} \tag{3}$$

$$\sum_{i=1}^{M} t_i n_i \leq T \tag{4}$$

$$0 \leq n_i \leq N_i, \quad n_i \text{ are integers} \tag{5}$$

It is assumed that the parts that are in the lot can either pass (conforming items) or fail inspection (nonconforming items). Since there are only two possible outcomes (pass, fail), it is assumed that the probability of having D_i number of defects of part i in the lot follows the binomial distribution. Therefore, calculating the probability of having an exact number of nonconforming items ($P(D_i)$) in the lot is possible as long as the defect rate and the lot size for part i is available. The cost of the exact number of nonconforming items reaching assembly is calculated by multiplying the number of defects in the lot with the cost of a nonconforming item reaching assembly for part i (C_i). Using this cost and the probability of having a specific number of defects is multiplied to obtain an expected cost of nonconformance for the specific number of defects. In order to cover all the possible values of D_i ($0 \leq D_i \leq N_i$) and to calculate the total expected cost of nonconforming items in the lot for part i, all possible outcomes are summarized ($\sum_{D_i=0}^{N_i} P(D_i)\, D_i\, C_i$). This also represents the total expected cost of nonconformance for part i if there is no inspection performed and the lot is sent directly to the assembly line.

With the inspection of a certain number of items (n_i), it is expected that the probability of a nonconforming item reaching assembly for that particular part number will be reduced. The number of defects found in the sample size that would be tolerated is zero, meaning that if a nonconformance is found in the sample size the entire lot is rejected. It is assumed that the inspection is performed without replacement. Since two mutually exclusive categories (pass/fail) are considered, it is assumed that the probability of accepting the lot with a defect follows the hypergeometric distribution shown in Equation 3.

The sample size n_i can be any number between zero and lot size N_i (Equation 5). Also, n_i must be an integer (Equation 5). If the sample size is zero, then no inspection performed. This means that the risk of accepting the lot with D_i defects is large. However, if the sample size is N_i, then 100% inspection is performed and the risk of accepting the lot with D_i defects is zero; however, the inspection cost would be high. The decision variable is, therefore, the sample size, n_i. With the increase of the sample size, the probability of accepting the lot with D_i defects decreases. Therefore, the bigger the sample size n, the smaller the expected cost of a nonconforming item reaching assembly for a specific number of defects D_i:

$$P(D_i)\ D_i\ C_i\ P(N_i, D_i, n_i) \tag{6}$$

In order to find the total expected cost for the specific part with all possible values of D_i, the summation of these equations is needed:

$$\sum_{D_i=0}^{N_i} P(D_i)\ D_i\ C_i\ P(N_i, D_i, n_i) \tag{7}$$

Finally, the research goal is to minimize the expected total cost of the nonconforming items for all the parts M in the system as shown in the Equation 1.

Since the time for inspection (T) is limited and there is large number of different parts (M) with various lot sizes, 100% inspection is time consuming, expensive, and unpractical. Each part i has a specific time interval (t_i) it takes the operator to inspect one item of part i. Therefore, the time it takes to inspect sample size n_i, for all parts M, must be less than or equal to the total time available for the inspection, which is the constraint show in Equation 4.

It is known from the problem statement and the objective that the purpose of the model is to find an optimal sampling inspection plan that would minimize the expected cost of a nonconforming item reaching the assembly line in the limited time available. If the sample size n_i is equal to zero then the probability of a lot with defectives being accepted would be equal to one. This would then result in the maximum expected cost of the nonconforming item. However, if inspection is performed and the sample size increases then the probability of accepting the lot with D_i defects decreases. The model, therefore, provides a sample size n_i for all parts M in the system.

4. ANALYSIS AND RESULTS

Since time to calculate these inspection plans is limited and the size of the problem is usually large, it was decided to use an evolutionary algorithm to solve the problem. Industry is typically interested in a better solution than the one they currently have and not the optimal solution, particularly if the solution is fast and easy to obtain. In the testing phase Excel was used to program the model. The model was built using the Solver program and its built in evolutionary algorithm. The advantage of this algorithm is that it gives a fast solution. However, the disadvantage of the algorithm is that the generated solution might not be the optimal solution, but rather a better solution than the previous one. Another disadvantage of the evolutionary algorithm is that it may show some inconsistencies in generating the solutions.

The model was initially tested for two parts. The data used for the two-part problem was provided by the automotive industry. The two parts in question are a tube and a harness. The tube has a historic defect rate of 1.93%, lot size of 125, time needed to inspect is 30 minutes, and cost of nonconformance of $17. The harness has a historic defect rate of 3.13%, lot size of 300, time needed to inspect is 5 minutes, and cost of nonconformance of $235. The time available is one workday of 8 hours or 480 minutes and the wage for the inspectors was set to $40.

The small problem analysis was set up for the user to input following data: lot size (N_i) for each part, defect rate (d_i) for each part, time needed to inspect (t_i) for each part, cost of nonconformance reaching assembly (C_i) for each part, time available for inspection (T), and the employee's salary (C_S). All of the constraints were set up as the model suggests and the program was set to determine the solution using the evolutionary algorithm. While using evolutionary algorithm it is expected to see some inconsistencies in the results.

The results that were found were promising for the real world application. In the two-part example, the expected cost of nonconformance was decreased by 83% and the total cost was decreased by 63% as shown in Table 4.1 In addition, the defect rate was reduced with inspection as shown in Figure 4.1 The expected cost of nonconformance also reduced with inspection as shown in Figure 4.2 The model comes back with a sample size of 56 for the harness (44.8% of the lot size) and a sample size of 6 for the tube (2% of the lot size). After inspection performed the defect rate of the lot 0.29% for the harness and 1.68% for the tube.

Table 4.1. Comparison of Costs With and Without Inspection for a Two-Part Problem

	No Inspection	Optimized Inspection	Change in Cost	% Change in Cost
Cost of Work Force	$320.00	$320.00	$0.00	0%
Expected Cost of N-C	$1,019.03	$172.34	$846.69	83%
Total Cost	$1,339.03	$492.34	$846.69	63%

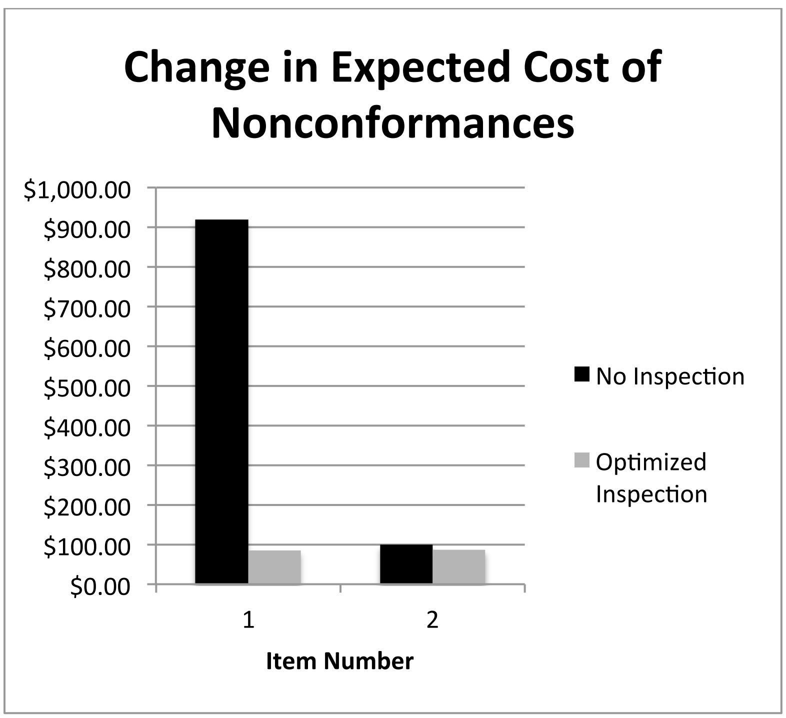

Figure 4.1. Change in defect rate for the two-part problem

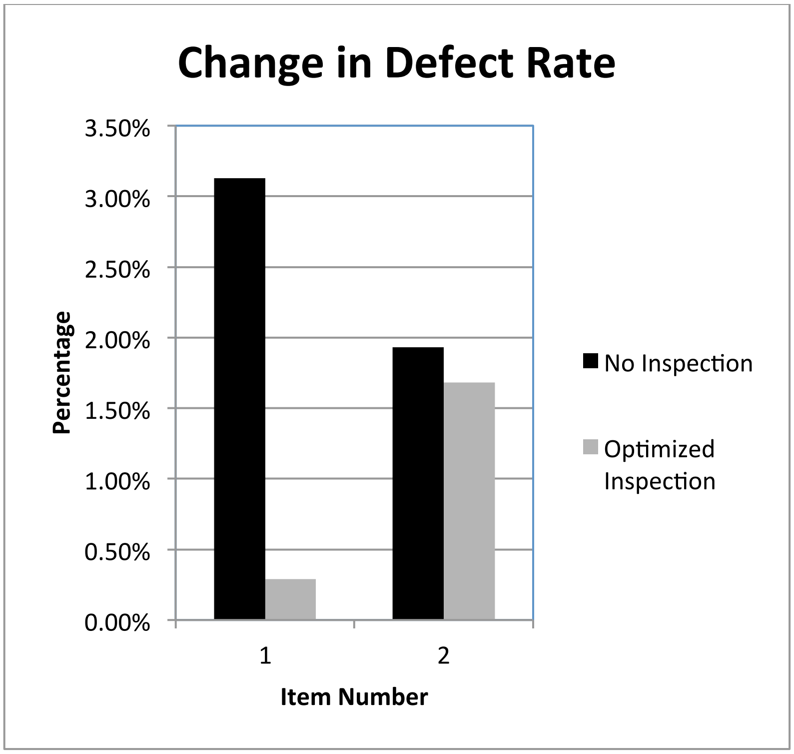

Figure 4.2. Change in expected cost for the two-part problem

The model was then tested for a 20-part problem. The data was randomly generated where the defect rate had a range from 1% to 10%, lot size had a range from 10 to 500, time needed to inspect certain item ranged from 1 to 30 minutes, and the cost of a nonconforming item ranged from \$10 to \$300 as shown in Table 4.2 In order to compare the 20-part problem to the 2-part problem the time to inspect remained the same at 480 minutes.

Table 4.3 shows the output provided by the program. It calculates the defect rate after inspection in order to see what type of risk the lot is still carrying as we expected cost of nonconformance without and with inspection. It also provides the sample size for inspection.

Table 4.2. Inputs for the 20-part problem

INPUTS				
Total Time Available (min)	2400			
Hourly Wage =	$40.00			
	Inspection Time per Piece *i* (min)	Defect Cost per Piece *i* ($)	Lot Size *i*	Defect Rate for Part *i* (%)
Part Number	t_i	c_i	N_i	d_i
1	10	$86.00	450	8.00%
2	5	$129.00	35	3.00%
3	11	$121.00	165	7.00%
4	3	$182.00	425	10.00%
5	20	$60.00	100	8.00%
6	10	$61.00	175	10.00%
7	18	$40.00	350	2.00%
8	3	$76.00	15	7.00%
9	20	$111.00	60	1.00%
10	16	$74.00	90	9.00%
11	19	$182.00	120	4.00%
12	23	$189.00	500	5.00%
13	20	$28.00	100	7.00%
14	3	$69.00	300	4.00%
15	17	$67.00	465	5.00%
16	8	$104.00	160	4.00%
17	20	$45.00	120	6.00%
18	3	$129.00	455	6.00%
19	2	$82.00	255	10.00%
20	17	$49.00	190	3.00%

Table 4.3. Outputs for a 20-part problem with available time of 2400 minutes.

Defect Rate After Inspection (%)	Change in Defect Rate (%)	Expected Cost of Nonconformance Without Inspection ($)	Expected Cost of Nonconformance With Inspection ($)	Change in Expected Cost of Nonconformance (%)	Inspection Size
					n_i
2.41%	5.59%	$3,096.00	$933.48	70%	14
2.83%	0.17%	$135.45	$127.63	6%	1
1.83%	5.17%	$1,397.55	$365.05	74%	17
0.21%	9.79%	$7,735.00	$159.50	98%	36
6.04%	1.96%	$480.00	$362.56	24%	3
2.35%	7.65%	$1,067.50	$251.19	76%	13
1.45%	0.55%	$280.00	$202.58	28%	14
6.08%	0.92%	$79.80	$69.27	13%	1
0.92%	0.08%	$66.60	$61.39	8%	3
5.90%	3.10%	$599.40	$392.77	34%	4
3.45%	0.55%	$873.60	$753.58	14%	3
2.25%	2.75%	$4,725.00	$2,123.38	55%	15
5.03%	1.97%	$196.00	$140.75	28%	4
2.46%	1.54%	$828.00	$509.09	39%	11
1.31%	3.69%	$1,557.75	$408.87	74%	26
2.27%	1.73%	$665.60	$377.23	43%	12
4.22%	1.78%	$324.00	$227.88	30%	5
2.45%	3.55%	$3,521.70	$1,435.39	59%	14
2.16%	7.84%	$2,091.00	$452.09	78%	14
2.51%	0.49%	$279.30	$233.53	16%	5
		Expected Total Cost of No Inspection	Expected Total Cost of a Nonconforming		
		$29,999.25	$9,587.21		

After running the program the model came to a solution where the expected cost of nonconformance decreased by 18% and the total cost (the expected cost of nonconformance and the cost of labor) decreased by 18% as shown in Table 4.4.

Table 4.4. Comparison of costs with and without inspection for a 20-part problem with available time of 480 minutes.

	No Inspection	Optimized Inspection	Change in Cost	% Change in Cost
Cost of Work Force	$320.00	$320.00	$0.00	0%
Expected Cost of N-C	$29,999.25	$24,490.07	$5,509.18	18%
Total Cost	$30,319.25	$24,810.07	$5,509.18	18%

It can be seen from Table 4.4, the change in expected cost of nonconformance without and with inspection is significantly smaller than the cost of workforce; therefore, performing sampling inspection on all incoming lots would be recommended from an economical viewpoint.

The problem was then run for 5 work days or 2400 minutes with the same data as the one-day 20-part problem. The model lowered the total expected cost of nonconformance by 68% and the total cost by 65% (Table 4.5). The changes in the defect rate and the expected cost of nonconformance without inspection and after suggested inspection are also shown in Figure 4.3 and Figure 4.4, respectively.

Table 4.5. Comparison of costs with and without inspection for a 20-part problem with available time of 2400 minutes.

	No Inspection	Optimized Inspection	Change in Cost	% Change in Cost
Cost of Work Force	$1,600.00	$1,600.00	$0.00	0%
Expected Cost of N-C	$29,999.25	$9,587.21	$20,412.04	68%
Total Cost	$31,599.25	$11,187.21	$20,412.04	65%

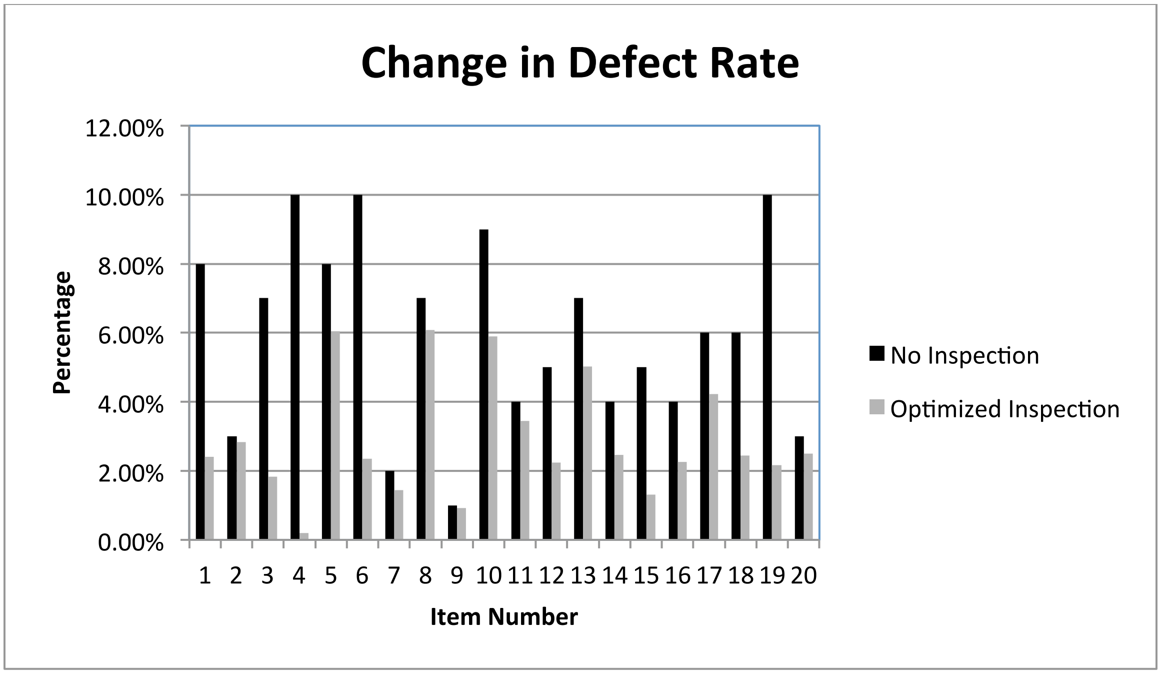

Figure 4.3. Change in defect rate for the 20-part problem with available time of 1600 minutes

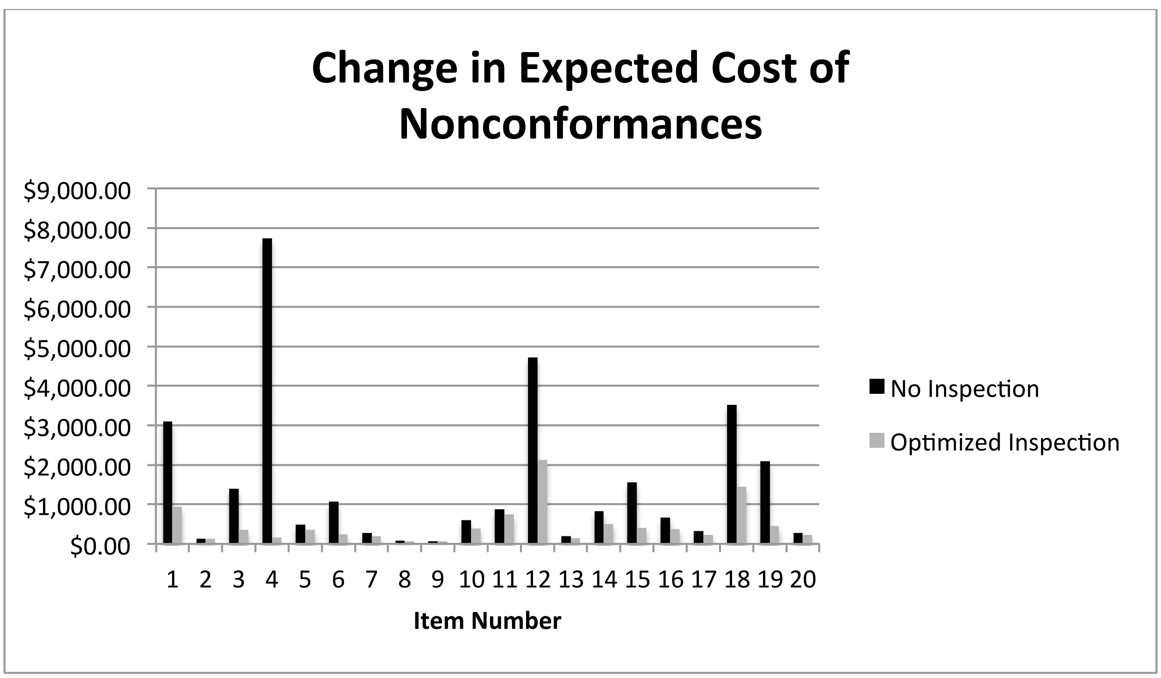

Figure 4.4. Change in expected cost of nonconformance for the 20-part with available time of 1600 minutes

5. CONCLUSIONS AND FUTURE WORK

The proposed model has a potential of solving the problem if the necessary inputs are available. In this research, the results showed that even with limited time available for inspection, performing sampling inspection significantly reduced the expected cost of a nonconforming item reaching assembly. The model was able to provide a meaningful solution to the problem although not necessarily an optimal solution as expected from using the evolutionary algorithm given that the algorithm provides a better, but not an optimal solution. Programming the model in a different programming language might provide a more consistent and more accurate solutions.

Future work includes developing a model that would not just look into the number of items that need to be inspected but also the specific characteristic of the item that is proven to have a possible issue. This would increase the efficiency of inspection, which means that operators could inspect more items if they know which particular characteristic needs more attention.

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III. OPTIMIZING SAMPLING INSPECTION TO REDUCE THE TOTAL COST OF QUALITY

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Abstract

Multi-component products have become prevalent in manufacturing and consumer products. Therefore, the quality of these products is very important to the customers. In order to manufacture a quality product companies have to make certain high quality components are assembled for the product. To ensure that the components meet the specifications companies can perform sampling inspection on the incoming lots consistent of these components. The focus of this paper was to develop a model that would determine the optimal sampling levels for incoming lots containing parts for production and assembly of multi-component systems such that the total cost of quality control is minimized. This cost includes the inspection cost and the cost associated with a nonconforming item reaching assembly. Assumptions made in the study are that the inspection is accurate, if one item is found to be defective in the sample size the entire lot is rejected, distribution of defects follow binomial distribution, and the probability of accepting the lot with defects after inspection follows the hypergeometric distribution. An example is given with randomly generated data and the results are discussed.

Keywords
Sampling inspection, inspection optimization, quality engineering, cost optimization, quality control

1. INTRODUCTION

Dependency on multi-component systems have found their way in everyday life due to an increase in product complexity. Since customers depend on these products they expect a certain level of quality from these multi-component systems. Because of the quality appreciation by customers and the competition in the market, manufacturers are paying increased attention to the quality of their products (Marttinen, 2002; Setijono and Dahlgaard, 2008). The cost of quality is not always easy to measure. However, being associated with reliability, durability, and customer's safety and satisfaction, quality is a very important aspect of modern industry.

In order to improve the quality of their products companies can perform sampling inspection on the incoming lots. Every incoming lot carries a risk that a certain amount of nonconforming items may be in the lot. However, the question that arises is how much inspection is necessary? If the company does not inspect at all, the risk of sending the lot with defects to assembly is maximized. These defective items can then be assembled in the final product. The problem occurs when the defective final product reaches the customer, which may lead to customer's dissatisfaction and different types of costs (shipping, repair, loss of faith, lawsuits). This, ultimately, drives up the cost of production. If the company performs 100% inspection the risk of accepting the lot with defects would be minimized. However, 100% inspection might not be desirable since the cost of manpower and equipment usage can drive up the cost of production and, in some cases, slow down production (Oppermann et al., 2001). In some cases 100% inspection is not possible, either because of lack of manpower or the lack of equipment necessary for inspection where companies are forced to trust the supplier. Therefore, an optimal sampling strategy is needed that would minimize the total cost of quality control. In order to minimize the total cost of quality control there has to be an optimal tradeoff between inspection cost and penalty cost associated with a nonconforming item reaching assembly (Keogh et al., 2000).

Figure 1.1 shows the tradeoff between the cost of inspection and the cost of nonconformance. Based on this tradeoff, it is possible to minimize the total cost of quality control. As the company increases inspection of the incoming lots the probability of accepting the lot that contains defects exponentially decreases. Therefore, the cost of nonconformance reaching assembly decreases exponentially as well. The inspection cost, on the other hand, is an increasing linear function. Meaning that when the company increases inspection of incoming lots the cost of inspection increases in a linear trend. The point where these two lines intersect represents the optimal inspection that would minimize the total cost of quality control.

The purpose of this research is to determine the optimal inspection plan that would minimize the total cost of quality control. This cost includes the inspection (appraisal) cost and the cost associated with a nonconforming item reaching assembly (failure cost). A tradeoff between these two costs needs to be found in order to minimize the total cost of the quality control. The paper is organized as follows. Section 2 covers the literature review. Section 3 presents and describes the model. Section 4 presents examples and discusses the results and major findings. Section 5 covers conclusion and future work.

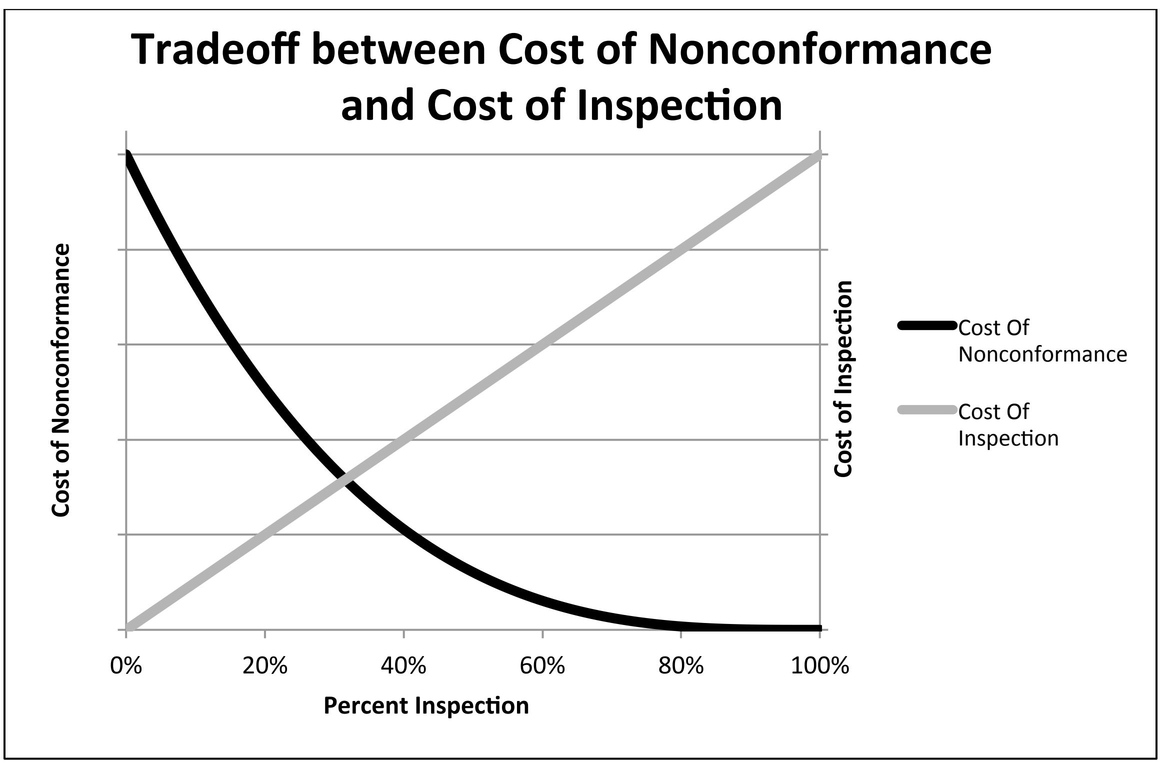

Figure 1.1. Tradeoff between cost of nonconformance and cost of inspection

2. LITERATURE REVIEW

Research and publications on sampling inspection of lots increased during and after World War II. Demand for military products has increased greatly and tolerance for faulty equipment was low during this period. Since production increased dramatically, unit-by-unit, or 100%, inspection was not practical. Therefore, quality control has shifted from unit-to-unit inspection to statistically controlled sampling inspection. Various military standards schemes were created in order to control the quality of the incoming lots (Champernowne, 1953; Barnard, 1954). Military standards first inspect a large sample size to determine the distribution of defects. If the lots are found to meet the specifications, the inspection on the consecutive lots is then relaxed.

Li et al. (2011) examined Military Standard MIL-STD-1916. This standard works under "zero accept one reject" premises; meaning that if there is a nonconformance in the sample of the population then the entire population is rejected. Lie et al. revised MIL-STD-1916 by expanding the current standard from 11 to 18 groups of inspection in order to separate the sampling plans from 100% inspection. Li et al. acknowledge that just because there are no nonconforming items in the sample it does not mean that the population meets conformance requirements. Meaning that the lots can still carry a risk of a defect reaching the final product. Military standards also require a large workforce in order to inspect the proposed sample sizes which is not always available in the real world application.

The research of Champernowne (1953) focused on the economic success of the problem by using the sampling inspection as a tool in the process. For the purpose of the study Champernowne assumed that several variables in the problem are known:

> "(i) the average quality of the batches to be tested and the variation between batches of quality about that average, (ii) the cost of inspection and its dependence on the amount of inspection undertaken, and (iii) the cost involved by deciding wrongly to accept or wrongly to reject a batch, and the way this cost depends on the quality of the batch."

Using this information, Champernowne developed an economical boundaries model that uses sampling inspection results (number of effective and defective items) to determine whether the lot should be accepted or rejected. Champernowne mainly focused on satisfying the economical aspect of the problem. Meaning that as long as the result is within the economical boundaries the lot would be accepted even if the defects were found in the sample On the other hand, Barnard (1954) argued that the information, which Champernowne assumes are given, are not readily available in the real world. Barnard argues that assigning a distribution for defects is needed in order to solve the problem. Barnard also argues that a considerable amount of information of each lot is needed to make an optimal decision for the problem.

Hamaker (1958) described three different approaches to sampling inspection: sampling tables, collecting data, and constructing inspection plans. Hamaker also modeled a plan of using economic theories where the research concluded that it might be more economical not to inspect the lots with a small probability of nonconforming items. While all the methods have been implemented in the real world, Hamaker warned that the data collection and sampling tables might lead to over sampling while using economic theories might not always be possible because certain factors might not be obtainable. Hamaker then suggested that a sampling plan should be selected and monitored for its performance and then later adjusted for the new data if needed.

Calvin (1983) made similar remarks when considering the zero defect philosophy. He pointed out that many managers are looking only for the ways to reach the zero defects but not to stay at the zero defect level. Calvin argued that there are different statistical methods such as control charts and acceptance sampling plans that managers can use in order for the product to stay at zero defects. The research suggested that the managers should consider how many good parts are in between two bad parts and in this way could control the zero defect level. If there are a smaller number of good parts between two bad parts then the lower limit suggests that the batch is rejected. Calvin also argued that the data collection has to be thorough because statistical importance may be lost in the process. The limits of keeping or discarding the batch should be challenged in a way that zero defects are still achieved for a lower cost.

Shi and Zhou (2009) gave a brief survey of the various techniques for quality control improvement in multiple stage and component processes. Among the discussed methods are the physical method, data-driven model, and statistical process control.

Physical methods require previous knowledge about the process. Data-driven models need sufficient knowledge in mathematics and statistics. It also requires a vast historical database in order to provide reasonable estimates. Data-driven models are appealing because they do not require the previous knowledge of the process in order to be applied. Statistical process control has a high “false alarm” probability and, according to Shi and Zhou “lacks the capability to discriminate among changes at different stages”. The research concluded that the most attractive methods for solving these problems would typically be data-driven models and other quantitative models because they can be applied to various systems in the market. In order to guarantee the quality of the final product, companies use sampling inspection plans for the system components. In addition, in order for these multi-component systems to stay operational, certain maintenance plans have to be developed.

One of the risks that researchers have noticed is that rejecting lots might slow down or even stop the production due to limited components needed for the assembly. Therefore, Salameh and Jaber (2000) focused on the optimal inventory of the items that might contain items of imperfect quality. They found that the quantity of the items per order increases as the probability of defective item increase.

3. MODEL

Consider an assembly line that has M different parts coming in. These parts have different lot size, defect rate, and repair cost if a defective item enters the assembly line. They also have a specific time interval needed to inspect a single item. An incoming inspection is performed on these parts in order to control the quality of the final product. The problem facing the management is to determine the right inspection sample size for each part considering the variability of risks associated with the M parts and the cost of labor needed for inspection. The problem can be modeled as a Nonlinear Integer Programming (NIP) problem as follows.

Index sets:

$I = \{i |\ i = 1, 2, \ldots, M\}$ = index set of parts considered by inspections

Parameters:

t_i = units of time needed to inspect a single item of part i

N_i = total number of items in the lot for part i (lot size)

d_i = probability of a defective item in the lot for part i (defect rate)

D_i = total number of defective items in the lot i

C_i = cost of a nonconforming item reaching assembly for part i

C_L = cost of labor per unit of time

Decision variables:

n_i = the number of items to be inspected for part i

Minimize:

$$\sum_{i=1}^{M}(C_L)(t_i)(n_i) + \sum_{i=1}^{M}\sum_{D_i=0}^{N_i} P(D_i)\, D_i\, C_i\, P(N_i, D_i, n_i) \tag{1}$$

Subject to:

$$P(D_i) = \binom{N_i}{D_i} {d_i}^{D_i}(1-d_i)^{N_i-D_i} \tag{2}$$

$$P(N_i, D_i, n_i) = \frac{\binom{N_i - D_i}{n_i}}{\binom{N_i}{n_i}} \tag{3}$$

$$0 \leq n_i \leq N_i, \qquad n_i \; are \; integers \tag{4}$$

The first part of Equation 1 represents the total cost of inspection. The number of items that are to be inspected for part i (n_i) is multiplied by the units of time needed to inspect a single item of part i (t_i). This provides the time needed to inspect the sample size for the part i. This value is then multiplied by the cost of labor per unit of time (C_L) in order to find the cost of inspection for part i. This function is an increasing function, meaning that with an increase of sample size (n_i) the cost of inspection will increase. This is then repeated for all M parts, which are then summed up in order to find the total cost of inspection for the system.

The second part of Equation 1 represents the expected total cost of nonconformance reaching assembly. It is assumed that the parts that are in the lot can either pass (conforming items) or fail inspection (nonconforming items) and that the inspection is performed without error. Since there are only two possible outcomes (pass, fail) it is assumed that the probability of having D_i number of defects of part i in the lot follows the binomial distribution (Equation 2). Therefore, calculating the probability of having an exact number of nonconforming items ($P(D_i)$) in the lot is possible as long as the defect rate and the lot size for part i is available. The cost of the exact number of nonconforming items reaching assembly is calculated by multiplying the number of defects in the lot with the cost of a nonconforming item reaching assembly for part i (C_i). Using this cost and the probability of having a specific number of defects is multiplied to get an expected cost of nonconformance for the specific number of defects. In order to determine what is the total cost of nonconformance for part i, the expected costs of nonconformance are calculated for all possible values of D_i ($0 \leq D_i \leq N_i$). These values are then added together to find what is the total expected cost of nonconformance for part i. This also represents the total expected cost of nonconformance for part i if there is no inspection performed and the lot is sent directly to the assembly line.

With the inspection of a certain number of items (n_i), it is expected that the probability of a nonconforming item reaching assembly for that particular part number will be reduced. The number of defects found in the sample size that would be tolerated is zero, meaning that if a nonconformance is found in the sample size the entire lot is rejected. It is assumed that the inspection is performed without replacement. Since two mutually exclusive categories (pass/fail) are considered, it is assumed that the probability of accepting the lot with a defect follows the hypergeometric distribution as shown in Equation 3.

The sample size n_i can be any number between zero and lot size N_i and n_i must be an integer (Equation 4). If the sample size is zero, no inspection is performed. This means that the risk of accepting the lot with D_i defects is large and the expected cost of defects reaching the customer is high. However, if sample size is N_i, 100% inspection is performed, the risk of accepting the lot with D_i defects is zero, but the inspection cost would be high. This function is therefore decreasing where the decision variable is the sample size, n_i. With the increase of the sample size, the probability of accepting the lot with D_i defects decreases. Therefore, the bigger the sample size n the smaller the expected cost of a nonconforming item reaching the assembly for a specific number of defects D_i:

$$P(D_i)\ D_i\ C_i\ P(N_i, D_i, n_i) \tag{5}$$

And, in order to calculate the total expected cost for the specific part with all the possible values of D_i, the summation of these equations is needed:

$$\sum_{D_i=0}^{N_i} P(D_i)\ D_i\ C_i\ P(N_i, D_i, n_i) \tag{6}$$

Finally, in order to find the expected total cost of the nonconforming items for all the parts M in the system, the values are summed for all the parts M in the system.

Since the cost of inspection is an increasing function and the expected total cost of nonconformance reaching assembly is a decreasing function with both functions having a sample size n_i as a decision variable, it is possible to calculate the specific sample size for all the parts M in the system, which would minimize the total cost of quality control (Equation 1). The model works under the assumption that the time needed to inspect all the sample sizes is flexible, meaning that it can be increased or decreased as long as the economical aspect of the model is satisfied and the total cost of quality control is minimized.

In some cases the model is equivalent to a set of smaller, part by part, problems. It is therefore possible to solve the problem by breaking it down into independent problems and solving it for each part. However, this is not universally true.

$$\sum_{i=1}^{M}(\min\,[(C_L)(t_i)(n_i)+\sum_{D_i=0}^{N_i}P(D_i)\,D_i\,C_i\,P(N_i,D_i,n_i)]) \tag{7}$$

4. ANALYSIS AND RESULTS

The model was programed in Excel where it was set to find the global minimum for each part in the system. These values are then added in order to calculate the minimum total cost of quality control for the entire system. The program was initially tested for three parts. The data was randomly generated and the inputs are show in Table 4.1.

Table 4.1. Inputs for the 3-part problem

INPUTS				
Hourly Wage =	$40.00			
	Inspection Time per Piece i (min)	Defect Cost per Piece i ($)	Lot Size i	Defect Rate for Part i (%)
Part Number	t_i	c_i	N_i	d_i
1	20	$400.00	30	2.90%
2	30	$17.20	20	1.93%
3	5	$235.00	10	3.13%

After running the 3-part program the results were obtained and the outputs are shown in Tables 4.2 and 4.3. It was found that there would be three different possibilities of inspection in the system. First, the function for part i is strictly increasing. This means that the cost of inspecting one item is higher than the expected cost of nonconformance if no inspection is performed; therefore, the model recommends no inspection. Second, the function for part i is strictly decreasing. This means that the cost of inspecting N_i items in the lot is lower than the expected cost of nonconformance if no inspection is performed and it is lower than the total cost if N_i -1 items are inspected; therefore, the model recommends 100% inspection. Lastly, the function for part i is convex. This means that there is a specific sample size that would lower the total cost of quality control and it has a global minimum. In this third possibility, the model recommends a sampling inspection with a specific sample size as a solution.

Table 4.3 shows the changes in costs if the inspection is performed or not. It can be seen that the model is able to lower the total cost of quality control by 19% while lowering the total expected cost of nonconformance by 61%. The calculations were also performed by hand in order to verify the performance of the program and the model.

Table 4.2. Outputs for the 3-part problem

OUTPUTS						
Part Number	Expected Cost of Nonconformance Without Inspection	Expected Cost of Nonconformance With Inspection	Cost of Inspection per Part	Cost of Quality Control per Part	Sample Size	Percent Inspection
					n_i	
1	$348.00	$159.45	$146.67	$306.12	11	36.67%
2	$6.56	$6.56	$0	$6.56	0	0%
3	$73.56	$0.00	$33.33	$33.33	10	100%
	Total No Inspection	Total With Inspection	Total Inspectio n Cost	Total Cost of Quality Control		
	$428.12	$166.01	$180.00	$346.01		

Table 4.3. Change in costs if no inspection is performed and if inspection is performed

	No Inspection	Optimized Inspection	Change in Cost	% Change in Cost
Cost of Work Force	$0.00	$180.00		
Expected Cost of N-C	$428.12	$166.01	$262.11	61%
Total Cost	$428.12	$346.01	$82.11	19%
Time Needed	0	270 min		

The problem was then expanded to a 20-part problem in order to bring the model closer to a real world application. The data was randomly generated for the problem where the time needed to inspect one item of part *i* was given an interval between 0 and 30 minutes, cost of a nonconforming item reaching assembly for part *i* between \$10 and \$300, lot size between 10 and 500, defect rate between 1% and 10%, and the hourly wage was set for \$25.

After running the program it was successful in obtaining the optimal solution. All the results were checked and verified. Figure 4.1 shows the difference in the defect rate without inspection versus with inspection that is suggested by the model. Figure 4.2 shows the difference in expected cost of nonconformance for each part without inspection performed versus with inspection suggested by the model.

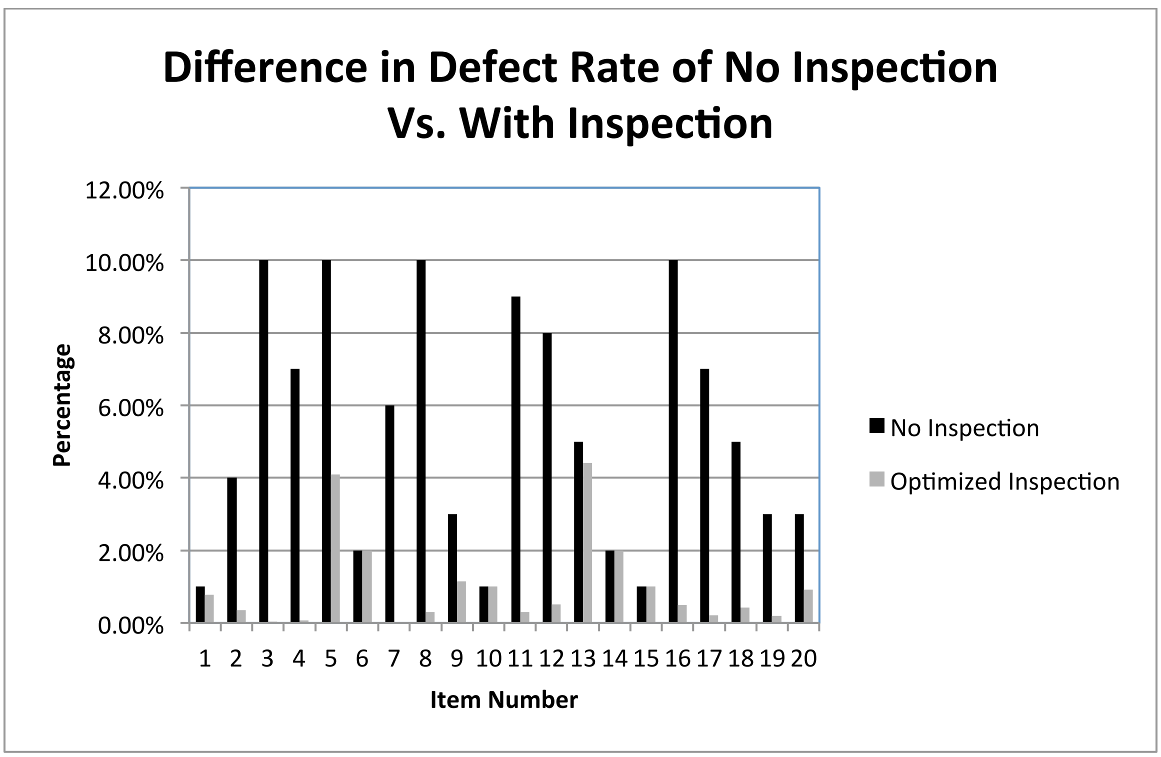

Figure 4.1. Difference in defect rate between no inspection versus with inspection for the 20-part problem.

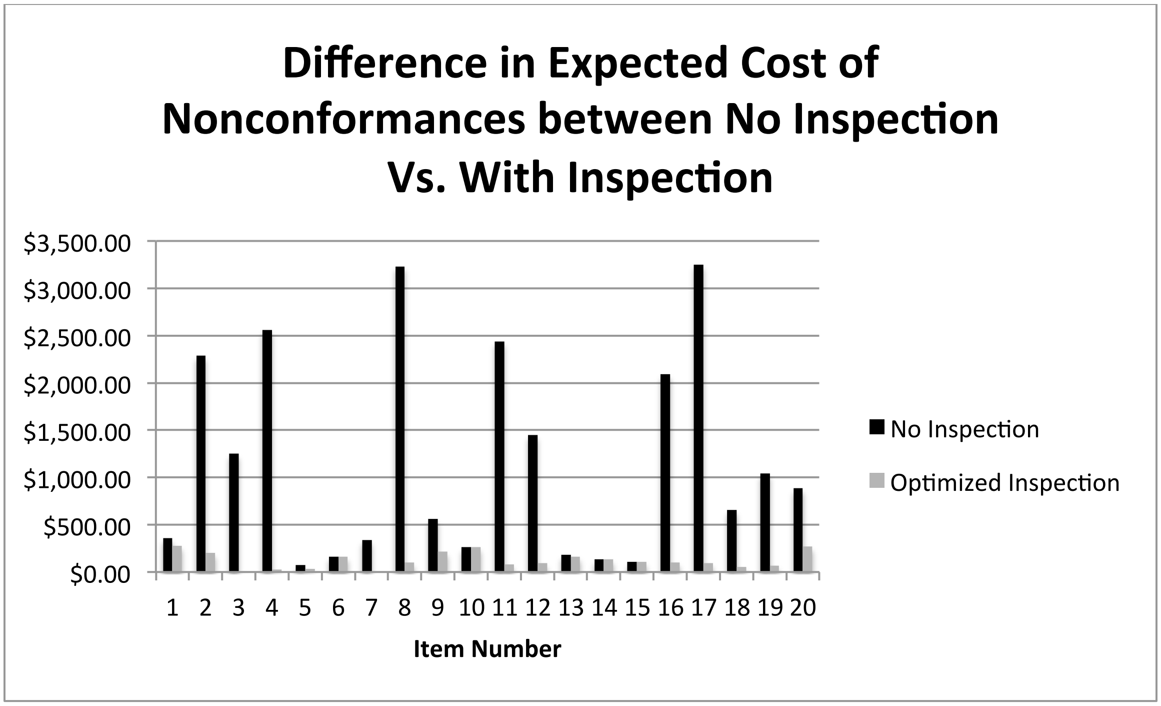

Figure 4.2. Difference in expected cost of nonconformance between no inspection versus with inspection for the twenty-part problem.

Figure 4.2 shows that the expected cost of nonconformance was significantly lowered for all parts. This leads to a considerably lower total expected cost of nonconformance as shown in Table 4.4. The results show that the total expected cost of nonconformance is reduced by 89% and that the total cost is reduced by 76%.

Table 4.4. Change in costs if no inspection is performed and if inspection is performed for 20-part problem

	No Inspection	Optimized Inspection	Change in Cost	% Change in Cost
Cost of Work Force	$0.00	$3,218.33		
Expected Cost of N-C	$23,339.03	$2,451.60	$20,887.43	89%
Total Cost	$23,339.03	$5,669.93	$17,669.10	76%
Time Needed	0 min	7724 min		

After running the program with different inputs it was noticed that the model rarely suggests 100% inspection. This is especially the case with large lot sizes and it is to be expected given that the probability of accepting the lot with defects exponentially decreases and, therefore, further inspection is redundant.

The advantage of this model is that it provides an optimal solution for the problem quickly. However, it operates under the assumption that the time available for inspection is flexible. This means that the management can hire more workers to perform the inspection, which is particularly applicable for companies that utilize outside inspection services. This may not be the case for some companies. Also, one of the disadvantages could be that the management is not able to acquire all the necessary data needed in order for model to work. However, if the assumption are satisfied this would be a useful tool to use to find the most economical inspection plan.

5. CONCLUSIONS AND FUTURE WORK

This research was focused on determining the most economical solution for quality control by determining optimal sampling levels for the incoming lots. In order to achieve this goal the model had to find an optimal tradeoff between the cost of inspection and the expected cost of defect reaching assembly. The proposed model is able to solve and provide an optimal solution to the problem. Several assumptions were made for the model development. The first assumption is that the probability of having a specific number of defects in the lot follows the binomial distribution. The second assumption is that the probability of accepting the lot with a certain number of defect after inspecting a sample size of items for part i follows the hypergeometric distribution. Another assumption is that if one item in the sample size was found to be defective the entire lot is rejected. The final assumption is that the time needed for inspecting sample sizes provided by the model is available. If these assumptions are satisfied and management is able to provide the necessary information, then the model gives an optimal solution to the problem. Examples were provided in the paper as well as the results that show the optimal solution for minimizing the total cost of quality control.

Future work would include developing a model that would not just look into the number of items that need to be inspected but also the specific characteristic of the item that is proven to have a possible issue. This would increase the efficiency of inspection, which means that operators could inspect more items if they know which particular characteristic needs more attention.

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SECTION

2. CONCLUSIONS

This research was able to provide solution to the problem that the modern industry facing. It looked into two aspects of the problem facing the management. First it looked into minimizing the total cost of nonconformance with in the limited time available for inspection. Many companies face this problem where the time and labor available for inspection is limited and cannot be easily expanded or expanded at all. Therefore, the management has to make due with the recourses it has provided. The model was able to solve the problem and provide a reasonable solution within a reasonable time. It should be noted that the evolutionary algorithm used to solve the problem does not give an optimal solution but rather a better one.

The second model was developed under the assumption that the time available for inspection is expandable. Here the management is able to hire more people to do the inspection. Model was then set to find the minimal total cost of inspection where the optimal tradeoff between cost of inspection and the expected cost associated with a nonconformance reaching assembly is minimized. If all the assumptions are satisfied then the model is able to provide an optimal solution to the problem.

VITA

Zlatan Hamzic was born on 19th January 1987 in Pancevo, Serbia. He received his Bachelor of Science degree in Applied Mathematics and Bachelor of Arts degree in Economics from Missouri University of Science and Technology in Rolla, Missouri. He graduated with Masters of Science degree in Engineering Management in December 2013 from Missouri University of Science and Technology.