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NUMERICAL SIMULATION OF ONE-DIMENSIONAL THREE-PHASE FLUID FLOW IN A STRATIFIED PETROLEUM RESERVOIR WITH NO CROSS-FLOW

BY

ALI MOHAMMED AKBAR, 1941 -

A

THESIS

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ABSTRACT

A numerical model for linear, three-phase fluid flow in a stratified petroleum reservoir has been developed for the purpose of studying water-flooding performance characteristics. Gravity forces, capillary pressure and cross-flow were neglected, but the fluids were considered compressible and evaluations of dissolved gas concentrations have been accounted for.

The model was used in evaluating the performance of a water flood project for a hypothetical reservoir which contained the three phases of oil, gas, and water. The results were then compared with those obtained using the Dykstra-Parsons technique. The model produces results which were found to be similar to those obtained by the Dykstra-Parsons method for the displacement of oil by water with no gas present, but poor agreement resulted when free gas saturations were formed. The results obtained using the numerical model are considered to be more realistic than those obtained by the Dykstra-Parsons method in depleted oil sands with a substantial gas saturation since the Dykstra-Parsons model was built on the assumption that only one phase flows at a point and water displace oil in a piston-like manner. When a free gas saturation is formed at the production well, the oil mobility is reduced substantially resulting in a lower oil recovery at water breakthrough than can be predicted by the Dykstra-Parsons technique.

ACKNOWLEDGEMENT

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I. INTRODUCTION

Fluid injection methods have been used for many years for secondary and tertiary recovery and have contributed substantially to marked improvements in oil recovery. The number of such projects should increase in the future due to the rising costs of discovering and developing primary reserves.

To evaluate any proposed fluid injection program, it is first necessary to know how much oil can be recovered economically. This information must include accurate knowledge of oil rates, water-oil ratios, and gas-oil ratios with time or pressure declines. In order to best evaluate these parameters, it is desirable to simulate (by a complete mathematical description) the multi-phase fluid flow in the porous medium, and the complex interaction of natural forces and physical properties of a petroleum reservoir during depletion with various fluid injection programs.

The approach undertaken in this project, to simulate this multi-phase flow, was to sum the three continuity equations for the three mobil phases (oil, water and gas) in one dimension, and to force the sum of saturation derivatives to vanish, thus forming one partial differential equation in pressure, dependent upon the simultaneous multi-phase flow in the reservoir. The differential equation, with pressure as the dependent variable, was then replaced by a finite difference equation to expedite solution. Writing this equation about each discrete point in the system results in

a tridiagonal matrix with coefficients which are both pressure and saturation dependent. This tridiagonal matrix is solved for pressure and yields pressure variances at the specific time level at the end of each time step. The method requires knowledge of rock and fluid properties, field geometry, well spacing, and known or proposed fluid injection rates.

The Dykstra-Parsons technique[1] was used as a reference to check the numerical model for cases in which the Dykstra-Parsons method is applicable and to test the Dykstra-Parsons technique for its applicability where a third phase, gas, appears and where the assumption of piston-like displacement of oil by water is not accurate.

The Dykstra-Parsons method employs a system of horizontal layers, not connected, except at the wells, in which permeability may vary vertically from layer to layer, with each layer being homogeneous. At any point in a layer only water or oil is flowing, but water is allowed to exist on one side of the front and oil on the other such that the velocity of fluids within a layer is dependent on the position of the front as well as the mobility ratio. No allowance is made for a third phase, gas, to exist in the system and all fluids are considered incompressible. Water-oil ratio is dependent on the fluid velocities in the individual layers and the number of layers which have experienced water breakthrough.

---

[1] See References.

The Dykstra-Parsons technique was chosen as a reference method to indicate the validity of the mathematical model because it is believed that the Dykstra-Parsons method is very accurate within the limitations of the inherent assumptions.

With this introductory preface, one can say that the purpose of this research is two-fold:

1. To simulate a three-phase, one-dimensional petroleum reservoir, with water being injected at one boundary and oil, water and gas being produced at the other boundary. It can be readily noted that the approach is also useful for estimating vertical coverages.
2. To compare predicted performance of a hypothetical reservoir as evaluated by the numerical model with results obtained by the Dykstra-Parsons method.

The mathematical model and the Dykstra-Parsons method were programmed for an IBM 360 computer in Fortran IV language. The flow diagrams and programs are presented in Appendix D.

II. LITERATURE REVIEW

In 1856, the French engineer Henry Darcy[2] empirically formulated that the rate of flow of a single fluid through a porous medium is directly proportional to the potential gradient and the cross-sectional area normal to the direction of flow and inversely proportional to the viscosity of the fluid. This relationship is known today as Darcy's Law and has served as the cornerstone for describing fluid flow in porous media. Later it was shown that Darcy's Law is only valid for the viscous flow region usually defined for flow in porous media as that region where Reynolds number is equal to or less than one. Hubbert[3] showed that Darcy's Law can be deduced from the classical Navier-Stokes hydrodynamics equations. Other investigators found that Darcy's Law can be extended to fluids other than water insofar as Darcy's investigations were confined to water flow. The generalized form of Darcy's Law is written as,

$$V = -\frac{k}{\mu}\nabla\Phi^{*}$$

where

$\Phi = P-\rho gh$, h is taken positive downward.

In 1930, experimental studies were made to investigate the flow of immiscible fluids in porous media.[4] Experiments showed that the presence of a second phase reduces the conductance for both phases. The work of Wyckoff and Botset[5]

---

*Terms defined in Nomenclature.

made it evident that the relative permeability to each phase of an immiscible two-phase system is a function of the phase saturation within the porous medium. Leverett[6] investigated the effect of fluid viscosity, pressure gradient and interfacial tension on the relative permeability and concluded that relative permeability is independent of fluid viscosity but is some function of pore size distribution, displacement pressure, pressure gradient and fluid saturation. In his work he introduced the concept of and defined the J-function, which relates capillary pressure to porosity and permeability. In 1941 Leverett and Lewis[7] reported the results of steady state flow tests on unconsolidated sands with three phases (oil, water and gas) present. From his work, the concept of three-phase relative permeabilities was established.

For a system containing two or more immiscible phases, a mathematical analysis consists of formulating differential equations obtained by combining Darcy's Law (with capillary pressure) and an equation of state for each phase with the Continuity Equation for each phase flowing. This results in a non-linear, second-order partial differential equation for each mobile phase. Due to the complexity of the resulting differential equation, a solution cannot be obtained by classical methods.

After the advent of high speed digital computing equipment, serious attempts at numerical solution were made. West, Garvin and Sheldon[8] studied horizontal, linear and radial systems produced under gas drive excluding capillary

effects. Their numerical treatment employed a finite difference grid system and they used implicit methods for solving the resulting system of linear equations.

Hovanessian and Fayers[9] reported a numerical simulation for one-dimensional, inclined flow of two immiscible, incompressible fluids. They showed that the inclusion of capillary pressure and gravitational forces had a pronounced effect on both phase saturations and pressure distributions. Douglas et al.[10] studied a horizontal, one-dimensional system including capillary pressure and concluded that at high flow rates this model yields results similar to those obtained by the Buckley-Leverett method.[11]

Two dimensional techniques have also been attempted but these early studies indicated that such models were highly unstable. However, in 1958, Douglas[12] published a paper on the "Alternating Direction Implicit Procedure" (ADIP) which provided a stable method for the treatment of these two dimensional problems. In 1959, Douglas, Peaceman and Rachford[13] used this classic finite difference treatment (ADIP) to successfully simulate two-dimensional flow of immiscible fluids. They reported good agreement between predicted performance for displacement of oil by water and observed values obtained from laboratory experiments.

Thus, in the 1950's, digital computing equipment and techniques had been developed sufficiently to allow numerical solution of multi-phase, multi-dimensional flow problems. In 1955, Rapoport[14] combined the three dimensional partial differential equations describing immiscible, incompressible

two-phase flow to obtain a single equation. He investigated the displacement of oil by water in terms of production history before and after water breakthrough.

In 1965, Fagin and Stewart[15] presented a two-dimensional, multi-phase reservoir simulator. This was a general flow model yielding accurate representations of pressure and multi-phase saturation changes with time.

In 1967, Coats, et al[16] presented the concept of vertical equilibrium in a horizontal model and adjusted a two-dimensional, two-phase model to account for saturation variation in the third dimension. Good agreement was reported when compared with three-dimensional calculations for a reservoir having definitive vertical communication.

In 1965, Quon, et al[17] reported the applicability of an "Alternating Direction Explicit Procedure" (ADEP) to a two-dimensional reservoir analysis where large time increments can be used to significantly reduce the time involved in solving such models. Their application of the ADEP technique was to simulate a gas reservoir and its flow characteristics and reported good agreement with similar results from an ADIP solution on the same model.

III. ASSUMPTIONS

The following assumptions are made to permit the desired solution;

1. The model simulates a reservoir consisting of horizontal layers of varying vertical thickness. The permeability and thickness are permitted to vary from layer to layer. Each layer consists of blocks of variable length and width adjacent to each other horizontally with the capability of varying permeability from block to block. It is assumed that there is an impermeable barrier between layers thus implying that there is no vertical cross-flow between layers.

2. Rock heterogeneity with respect to permeability (but not porosity) is assumed.

3. Capillarity, gravity and rock compressibility are considered negligible.

4. Water is injected in one end at constant rate, and oil, water and gas are produced at the other end at rates such that reservoir voidage equals water influx at reservoir conditions.

5. It is assumed that the simulation accurately accounts for three-phase flow, fluid compressibilities, and evolution of dissolved gas.

IV. THEORY

A. PARTIAL DIFFERENTIAL EQUATIONS

The basic concept employed in describing fluid flow in a porous media is based on the law of mass conservation and Darcy's Law.

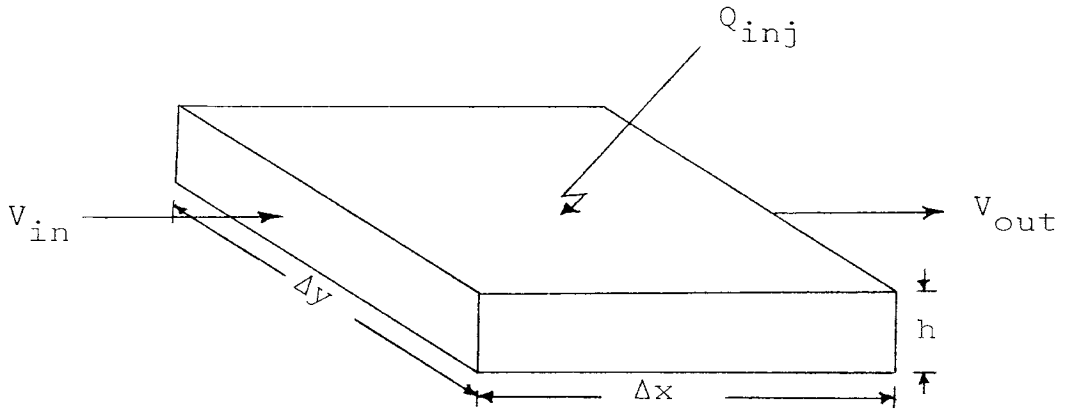


Fig. 1. A Typical Reservoir Elemental Block

If one considers three-phase fluid flow through an elemental block in the x-direction as shown in Figure (1), then one can apply the law of mass conservation on each phase making use of the Darcy's Law in the following manner:

Oil Phase:

a) Mass rate in - mass rate out = rate of mass accumulation (1)

b) Mass rate in = $\vec{V}o_{in} A\rho_{os}/\beta_o + Q_{oinj}\rho_{os}$ (2)

where Q_{oinj} is negative if it is production instead of injection and,

$\vec{V}o_{in}$ is the oil velocity entering the elemental block from the left hand side which is defined by Darcy's Law as:

$$\vec{V}o = - \frac{k_o}{\mu_o} \frac{\partial \Phi_o}{\partial x}$$

where;

Φ_o is the velocity potential defined as:

$$\Phi_o = P_o + \rho_o gh \quad .$$

c) The mass rate out can be expressed as;

Mass rate out = $\vec{V}o_{out} A\rho_{os}/\beta_o$

where $\vec{V}o_{out}$ is the velocity of oil leaving the elemental block through the right hand side and face of the elemental volume and is equal to

$$\vec{V}o_{in} + \frac{\partial \vec{V}_o}{\partial x} \Delta x. \qquad (3)$$

The mass accumulation rate can be defined as;

d) Rate of mass accumulation = $\frac{\partial M_o}{\partial t}$ =

$$h\Delta x\Delta y\phi\rho_{os} \frac{\partial}{\partial t} (\frac{S_o}{\beta_o}) \qquad (4)$$

where h, ϕ and ρ_{os} are constant factors.

Substituting Equations (2), (3) and (4) into Equation (1) and defining V as defined by Darcy's Law yields:

$$h\Delta x\Delta y\rho_{os} \frac{\partial}{\partial x}(\frac{k_o}{\mu_o\beta_o} \frac{\partial \Phi_o}{\partial x}) + \rho_{os}Q_{oinj} = \phi h\Delta x\Delta y\rho_{os} \frac{\partial}{\partial t}(\frac{S_o}{\beta_o}) \quad . \qquad (5)$$

Dividing both sides of Equation (5) by $h\Delta x\Delta y\rho_{os}$ yields:

$$\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o \beta_o}\frac{\partial \Phi_o}{\partial x}\right) + \frac{Q_{oinj}}{h\Delta x\Delta y} = \phi \frac{\partial}{\partial t}\left(\frac{S_o}{\beta_o}\right) . \qquad (6)$$

Expanding the time derivative (the right hand side of Equation (6)), and multiplying both sides by β_o yields:

$$\beta_o \frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o \beta_o}\frac{\partial \Phi_o}{\partial x}\right) + \beta_o \frac{Q_{oinj}}{h\Delta x\Delta y} = \phi\left[\frac{\partial S_o}{\partial t} - \frac{S_o}{\beta_o}\frac{\partial \beta_o}{\partial P_o}\frac{\partial P_o}{\partial t}\right]. \qquad (P\text{-}1)$$

Equation (P-1) is the partial differential equation describing the flow of oil in one dimension (x-direction) in a porous medium.

Water Phase:

Applying the logic similar to that employed in deriving the equation for the oil phase, a partial differential equation for the water phase flowing through an elemental block can be developed as follows:

$$\beta_w \frac{\partial}{\partial x}\left(\frac{k_w}{\mu_w \beta_w}\frac{\partial \Phi_w}{\partial x}\right) + \frac{\beta_w Q_{winj}}{h\Delta x\Delta y} = \phi\left[\frac{\partial S_w}{\partial t} - \frac{S_w}{\beta_w}\frac{\partial \beta_w}{\partial P_w}\frac{\partial P_w}{\partial t}\right] \qquad (P\text{-}2)$$

where $\Phi_w = P_w + \rho_w g h$.

Gas Phase:

The derivation of a partial differential equation for the gas phase differs from that of oil and water phases only in that gas evolved from solution in oil must be accounted for. Therefore, the law of mass conservation may be written in the following manner:

Mass rate in + mass rate of gas evolution - mass rate out = rate of mass accumulation. (7)

Mathematically, Equation (7) is defined as

$$-h\Delta x\Delta y\ \rho_{gs}\ \frac{\partial}{\partial x}\left(\frac{\vec{V}g_{in}}{\beta_g}\right) + \rho_{gs}Q_{ginj} = -\frac{\partial M_{FG}}{\partial t} + \frac{\partial M_g}{\partial t} \qquad (8)$$

where $\frac{\partial M_{FG}}{\partial t}$ is the rate of change in the mass of gas evolved, and its derivation is based on the mass rate of gas evolution and is related to the oil phase present, the solution gas oil ratio (R_s), and their rate of change as follows:

$$\frac{\partial M_{FG}}{\partial t} = -h\Delta x\Delta y\ \rho_{gs}\ \phi\ \frac{\partial}{\partial t}\left(\frac{S_o}{\beta_o}\ R_s\right) . \qquad (9)$$

The minus sign in Equation (9) accounts for the fact that a decrease in the solution gas oil ratio is accompanied by an increase in free gas volumes. Also, in the same equation, R_s remains inside the derivative because it also varies with time being uniquely a function of pressure which is a function of time.

Substituting the Darcy's Law equivalent of the velocity potential and the results of Equation (9) into Equation (8) and then dividing both sides of the resulting equation by $h\Delta x\Delta y\ \rho_{gs}$, and multiplying through by β_g, yields:

$$\beta_g\ \frac{\partial}{\partial x}\left(\frac{k_g}{\mu_g\beta_g}\ \frac{\partial \Phi_g}{\partial x}\right) + \frac{\beta_g Q_{ginj}}{h\Delta x\Delta y} = \phi\beta_g\ \frac{\partial}{\partial t}\left(\frac{S_g}{\beta_g} + \frac{S_o}{\beta_o}\ R_s\right) \qquad (10)$$

where $\Phi_g = P_g + \rho_g gh$.

Expanding the time derivative of the right hand side of Equation (10) yields:

$$\phi\beta_g \left[\frac{\partial}{\partial t}\left(\frac{S_g}{\beta_g} + \frac{S_o}{\beta_o} R_s\right)\right] = \left[\frac{1}{\beta_g}\frac{\partial S_g}{\partial t} - \frac{S_g}{\beta_g^2}\frac{\partial \beta_g}{\partial P_g}\frac{\partial P_g}{\partial t} + \frac{S_o}{\beta_o}\frac{\partial R_s}{\partial P_g}\frac{\partial P_g}{\partial t} + R_s \frac{\partial}{\partial t}\left(\frac{S_o}{\beta_o}\right)\right] \phi\beta_g \quad . \tag{11}$$

Substituting the value of $\phi \frac{\partial}{\partial t}\left(\frac{S_o}{\beta_o}\right)$ as given by Equation (6) into Equation (11) yields:

$$\phi \frac{\partial S_g}{\partial t} + \left(\phi\beta_g \frac{S_o}{\beta_o}\frac{\partial R_s}{\partial P_g} - \phi \frac{S_g}{\beta_g}\frac{\partial \beta_g}{\partial P_g}\right)\frac{\partial P_g}{\partial t} + \beta_g R_s \frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o \beta_o}\frac{\partial \Phi_o}{\partial x}\right) + \beta_g R_s \frac{Q_{oinj}}{h\Delta x \Delta y} = \phi\beta_g \left[\frac{\partial}{\partial t}\left(\frac{S_g}{\beta_g} + \frac{S_o}{\beta_o} R_s\right)\right] \tag{12}$$

Substituting the results shown in Equation (12) for the right hand side of Equation (10) and rearranging yields:

$$\beta_g \frac{\partial}{\partial x}\left(\frac{k_g}{\mu_g \beta_g}\frac{\partial \Phi_g}{\partial x}\right) + \phi \frac{S_g}{\beta_g}\frac{\partial \beta_g}{\partial P_g}\frac{\partial P_g}{\partial t} - \frac{\phi S_o \beta_g}{\beta_o}\frac{\partial R_s}{\partial P_g}\frac{\partial P_g}{\partial t} - \beta_g R_s \frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o \beta_o}\frac{\partial \Phi_o}{\partial x}\right) - \frac{\beta_g R_s Q_{oinj}}{h\Delta x \Delta y} + \frac{\beta_g Q_{ginj}}{h\Delta x \Delta y} = \phi \frac{\partial S_g}{\partial t} \quad . \tag{P-3}$$

There is no need to account for loss of oil due to the evolution of gas since this loss is totally accounted for by changes in the oil formation volume factor.

Since capillary pressure and gravity are considered negligible in this model, the following potential equivalences are plausible;

$$\Phi_o = \Phi_w = \Phi_g = P \ , \tag{13}$$

and individual phase pressures are identical, i.e.,

$$P_o = P_w = P_g = P \ .$$

Recognizing that the potential terms appearing in Equations (P-1), (P-2), and (P-3) are identical as defined by Equation (13), it may be noted that these three partial differential equations are simultaneous equations relating pressure and saturations (four dependent variables) to the independent variables, position and time. This system cannot be solved as there are three equations and four unknowns. This problem is readily overcome, however, by introducing another equation in the dependent variables. This equation can be based on the definition of saturations as follows:

$$S_o + S_w + S_g = 1.0 \ . \tag{14}$$

It may be further noted that the partial differential equations have been judiciously arranged so that they may be added to yield one equation in one dependent variable, notably pressure. Note that the three time derivatives of the saturations are isolated such that their only coefficient is a constant, porosity, and that taking the time derivative of Equation (14) yields:

$$\frac{\partial S_o}{\partial t} + \frac{\partial S_w}{\partial t} + \frac{\partial S_g}{\partial t} = 0 \ . \tag{15}$$

Summing Equations (P-1), (P-2), and (P-3), rearranging terms, and simplifying as discussed immediately above based on Equations (13), (14), and (15) yields:

$$\beta_g \frac{\partial}{\partial x}\left(\frac{k_g}{\mu_g \beta_g}\frac{\partial P}{\partial x}\right) + (\beta_o - \beta_g R_s)\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o \beta_o}\frac{\partial P}{\partial x}\right) + \beta_w \frac{\partial}{\partial x}\left(\frac{k_w}{\mu_w \beta_w}\frac{\partial P}{\partial x}\right)$$

$$= -\left[\frac{\beta_g Q_{ginj}}{h\Delta x\Delta y} + (\beta_o - \beta_g R_s)\frac{Q_{oinj}}{h\Delta x\Delta y} + \frac{\beta_w Q_{winj}}{h\Delta x\Delta y}\right]$$

$$-\phi\left[\frac{S_g}{\beta_g}\frac{\partial \beta_g}{\partial P} + \frac{S_o}{\beta_o}\left(\frac{\partial \beta_o}{\partial P} - \beta_g \frac{\partial R_s}{\partial P}\right) + \frac{S_w}{\beta_w}\frac{\partial \beta_w}{\partial P}\frac{\partial P}{\partial t}\right] \qquad \text{(P-4)}$$

Equation (P-4) is a non-homogeneous, second-order non-linear, partial differential equation and no known technique for solving it by classical means exists. It is non-linear because the coefficients are in themselves pressure and saturation dependent. The equation is solvable, however, by finite difference approximation techniques.

B. FINITE-DIFFERENCE EQUATION

Equation (P-4) is in a form where a difference equation can be written, from which the value of the dependent variable (pressure) at time level t(n+1) can be calculated when the values of all parameters are known at the previous time level t(n). The terms of the left hand side of Equation (P-4) can be represented by a finite difference equation if the time-space plane is divided into discrete cells as shown in Figure (2) (for a single time), then expanded about some point (i) in the spatial grid pattern at some fixed time level.

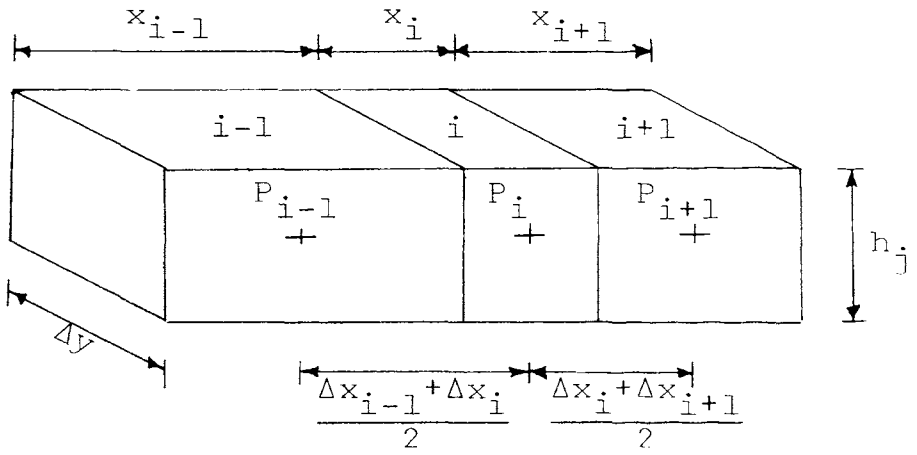


Fig. 2. Central Grid Point Illustration

The point at which all parameters are defined in each block is arbitrarily selected as its mid-point. Thus, the center is the focal point at which the pressure is assumed concentrated and this forms the basis for calculating pressure gradients.

Since there are three phases flowing, it is necessary to use relative permeabilities rather than absolute permeabilities. The former are defined by the following relations:

$$k_g = k\, k_{rg} \ ;$$

$$k_o = k\, k_{ro} \ ;$$

$$k_w = k\, k_{rw} \ .$$

Utilizing this fact the following equations may be written:

$$\beta_g \frac{\partial}{\partial x}\left(\frac{k\, k_{rg}}{\mu_g \beta_g}\frac{\partial P}{\partial x}\right) \simeq \frac{\beta_{gi}^{n+\frac{1}{2}}}{\Delta x_i}\left[\left\{\frac{k\, k_{rg}}{\mu_g \beta_g}\right\}_{i+\frac{1}{2}}^{n+\frac{1}{2}} \frac{(P_{i+1}-P_i)^{n+1}}{\left(\frac{\Delta x_{i+1}+\Delta x_i}{2}\right)}\right.$$

$$\left. - \left\{\frac{k\, k_{rg}}{\mu_g \beta_g}\right\}_{i-\frac{1}{2}}^{n+\frac{1}{2}} \frac{(P_i-P_{i-1})^{n+1}}{\left(\frac{\Delta x_i+\Delta x_{i-1}}{2}\right)}\right], \qquad \text{(1-D)}$$

and,

$$(\beta_o-\beta_g R_s)\frac{\partial}{\partial x}\left(\frac{k\, k_{ro}}{\mu\ \beta}\frac{\partial P}{\partial x}\right) \simeq \left[\frac{\beta_o-\beta_g R_s}{\Delta x_i}\right]^{n+\frac{1}{2}}\left[\left\{\frac{k\, k_{ro}}{\mu_o \beta_o}\right\}_{i+\frac{1}{2}}^{n+\frac{1}{2}} \frac{(P_{i+1}-P_i)^{n+1}}{\left(\frac{\Delta x_{i+1}+\Delta x_i}{2}\right)}\right.$$

$$\left. - \left\{\frac{k\, k_{ro}}{\mu_o \beta_o}\right\}_{i-\frac{1}{2}}^{n+\frac{1}{2}} \frac{(P_i-P_{i-1})^{n+1}}{\left(\frac{\Delta x_i+\Delta x_{i-1}}{2}\right)}\right], \text{ and,} \qquad \text{(2-D)}$$

$$\beta_w \frac{\partial}{\partial x}\left(\frac{k\, k_{rw}}{\mu_w \beta_w}\frac{\partial P}{\partial x}\right) = \frac{\beta_{wi}^{n+\frac{1}{2}}}{\Delta x_i}\left[\left\{\frac{k\, k_{rw}}{\mu_w \beta_w}\right\}_{i+\frac{1}{2}}^{n+\frac{1}{2}} \frac{(P_{i+1}-P_i)^{n+1}}{\left(\frac{\Delta x_{i+1}+\Delta x_i}{2}\right)}\right.$$

$$\left. - \left\{\frac{k\, k_{rw}}{\mu_w \beta_w}\right\}_{i-\frac{1}{2}}^{n+\frac{1}{2}} \frac{(P_i-P_{i-1})^{n+1}}{\left(\frac{\Delta x_i+\Delta x_{i-1}}{2}\right)}\right]. \qquad \text{(3-D)}$$

where the superscript $n+\frac{1}{2}$ represents the midpoint representation of the time level.

It is noted from Equations (1-D), (2-D), and (3-D), that a time average of the pressure and saturation dependent variables has been employed. Therefore, the pressure dependent variables should be evaluated at $\left(\frac{P^{n+1}+P^n}{2}\right)$ and the relative permeabilities should be evaluated at $\left(\frac{S^{n+1}+S^n}{2}\right)$ where the saturations have not been shown with subscript

depending on the phase under evaluation. The parameters describing rock properties and model configuration are not time-dependent and can be isolated, forming a term, HKX, which may be defined as follows:

$$HKX_i = \frac{2k_{i+\frac{1}{2}}}{\Delta x_{i+1}+\Delta x_i} = \frac{2k_i k_{i+1}}{\Delta x_i k_{i+1}+\Delta x_{i+1} k_i} \tag{16}$$

for the ith spatial point, and

$$HKX_{i-1} = \frac{2k_{i-\frac{1}{2}}}{\Delta x_{i-1}+\Delta x_i} = \frac{2k_{i-1} k_i}{\Delta x_{i-1} k_i+\Delta x_i k_{i-1}} \tag{17}$$

for the i-1 spatial point.

$K_{i+\frac{1}{2}}$ and $K_{i-\frac{1}{2}}$ are series averaged values for rock permeability, evaluated at the interfaces bounding the ith block. Equations (16) and (17) are derived in Appendix .

At this point a difference equation for the left hand side of Equation (P-4) has been written. On the right hand side of Equation (P-4), the only term to be written as a difference is $\frac{\partial P}{\partial t}$ which may be expressed as follows;

$$\frac{\partial P}{\partial t} \cong \frac{P_i^{n+1}-P_i^n}{\Delta t} . \tag{4-D}$$

Now substituting Equations (1-D), (2-D), (3-D), (16), (17) and (4-D) in Equation (P-4) yields:

$$\left\{\frac{\beta_{gi}}{\Delta x_i}\right\}^{n+\frac{1}{2}} \left[HKX_i \gamma_{gi+\frac{1}{2}}^{n+\frac{1}{2}} (P_{i+1}-P_i)^{n+1} - HKX_{i-1} \gamma_{gi-\frac{1}{2}}^{n+\frac{1}{2}} (P_i-P_{i-1})^{n+1}\right]$$

$$+ \left\{\frac{\beta_o-\beta_g R_s}{\Delta x_i}\right\}^{n+\frac{1}{2}} \left[HKX_i \gamma_{oi+\frac{1}{2}}^{n+\frac{1}{2}} (P_{i+1}-P_i)^{n+1} - HKX_{i-1} \gamma_{oi+\frac{1}{2}}^{n+\frac{1}{2}} (P_i-P_{i-1})^{n+1}\right]$$

$$+ \left\{\frac{\beta_{wi}}{\Delta x_i}\right\}^{n+\frac{1}{2}} \left[HKX_i \gamma_{wi+\frac{1}{2}}^{n+\frac{1}{2}} (P_{i+1}-P_i)^{n+1} - HKX_{i-1} \gamma_{wi+\frac{1}{2}}^{n+\frac{1}{2}} (P_i-P_{i-1})^{n+1}\right]$$

$$= - \frac{1}{h\Delta x_i \Delta y} \left[(\beta_g Q_{ginj})_i + (\beta_o-\beta_g R_s)_i (Q_{oinj})_i + (\beta_w Q_{winj})_i\right]^{n+\frac{1}{2}}$$

$$- \phi \left[\frac{S_{gi}}{\beta_{gi}} \beta'_{gi} + \frac{S_{oi}}{\beta_{oi}} (\beta'_o-\beta_g R'_s)_i - C_w S_{wi}\right]^{n+\frac{1}{2}} \frac{P_i^{n+1}-P_i^n}{\Delta t} \qquad (5\text{-D})$$

where

$$\gamma_g = \frac{k_{rg}}{\mu_g \beta_g} , \quad \gamma_o = \frac{k_{ro}}{\mu_o \beta_o} , \quad \gamma_w = \frac{k_{rw}}{\mu_w \beta_w} ;$$

$$\beta'_g = \frac{\partial \beta_g}{\partial P} , \quad \beta'_o = \frac{\partial \beta_o}{\partial P} , \quad R'_s = \frac{\partial R_s}{\partial P} ; \quad \text{and}$$

$$C_w = - \frac{1}{\beta_w} \frac{\partial \beta_w}{\partial P} .$$

Equation (5-D) is a complete solution of the partial differential Equation (P-4) in difference form. For simplicity, Equation (5-D) can be rearranged by collecting all terms associated with P_{i-1}^{n+1}, P_i^{n+1}, and P_{i+1}^{n+1}, and rewritten in the following form:

$$A_i P_{i-1}^{n+1} + B_i P_i^{n+1} + C_i P_{i+1}^{n+1} = D_i \qquad (6\text{-}D)$$

where,

$$A_i = AOX1 + AWX1 + AGX1 \qquad (7\text{-}D)$$

$$C_i = AOX2 + AWX2 + AGX2 \qquad (8\text{-}D)$$

$$B_i = - A_i - C_i + TRM_i \qquad (9\text{-}D)$$

$$D_i = QTERM_i + TRM_i \, P_i^n \qquad (10\text{-}D)$$

$$AOX1 = \frac{HKX_{i-1}}{\Delta x_i} \left[(\beta_o - \beta_g R_s)_i \gamma_{oi-\frac{1}{2}} \right]^{n+\frac{1}{2}} \qquad (11\text{-}D)$$

$$AWX1 = \frac{HKX_{i-1}}{\Delta x_i} \left[\beta_{wi} \gamma_{wi-\frac{1}{2}} \right]^{n+\frac{1}{2}} \qquad (12\text{-}D)$$

$$AGX1 = \frac{HKX_{i-1}}{\Delta x_i} \left[\beta_{gi} \gamma_{gi-\frac{1}{2}} \right]^{n+\frac{1}{2}} \qquad (13\text{-}D)$$

$$AOX2 = \frac{HKX_i}{\Delta x_i} \left[(\beta_o - \beta_g R_s)_i \gamma_{oi+\frac{1}{2}} \right]^{n+\frac{1}{2}} \qquad (14\text{-}D)$$

$$AWX2 = \frac{HKX_i}{\Delta x_i} \left[\beta_{wi} \gamma_{wi+\frac{1}{2}} \right]^{n+\frac{1}{2}} \qquad (15\text{-}D)$$

$$AGX2 = \frac{HKX_i}{\Delta x_i} \left[\beta_{gi} \gamma_{gi+\frac{1}{2}} \right]^{n+\frac{1}{2}} \qquad (16\text{-}D)$$

$$TRM_i = \frac{\phi}{\Delta t} \left[\frac{S_{gi}}{\beta_{gi}} \beta'_{gi} + \frac{S_{oi}}{\beta_{oi}} (\beta'_o - \beta_g R'_s)_i - C_w S_{wi} \right]^{n+\frac{1}{2}} \qquad (17\text{-}D)$$

$$QTERM_i = -\frac{1}{h\Delta x_i \Delta y}\left[(\beta_g Q_{ginj})_i + (\beta_o - \beta_g R_s)_i (Q_{oinj})_i + (\beta_w Q_{winj})_i\right]^{n+\frac{1}{2}} . \qquad (18\text{-}D)$$

Equation (6-D), written about each point in the grid system, forms a set of simultaneous linear equations whose coefficient matrix is tridiagonal. The solution for a tridiagonal system is,

$$P_{m-1} = \alpha_{m-1}$$

$$P_i = \alpha_i - \frac{C_i P_{i-1}}{\lambda_i} , \qquad i = m-2, m-3, \ldots, 1$$

where α's and λ's are determined from the recursion formulae,

$$\lambda_1 = B_1, \quad \alpha_1 = \frac{D_1}{\lambda_1}$$

$$\lambda_i = B_i - \frac{A_i C_{i-1}}{\lambda_{i-1}} , \qquad i = 2, 3, 4, \ldots, m-1$$

$$\alpha_i = \frac{D_i - A_i \alpha_{i-1}}{\lambda_i} . \qquad i = 2, 3, 4, \ldots . m-1$$

It has been previously mentioned that the coefficients of Equation (6-D) are dependent on both pressure and saturation which are unknowns. Thus, the method involves a trial and error procedure. An estimate of future pressures

(P^{n+1}) and saturations (S^{n+1}) are made by linear extrapolation. Then the fluid properties are evaluated at $P^{n+\frac{1}{2}} = \frac{P^{n+1}+P^{n}}{2}$ while the rock properties (relative permeabilities) are evaluated at $S^{n+\frac{1}{2}} = \frac{S^{n+1}+S^{n}}{2}$. The unknown pressures are then calculated. Using these calculated pressures, an explicit determination for future saturations are made as described in the Material Balance Section. Both calculated pressures and saturations are then compared with the previously assumed values. If agreement is not within a prespecified tolerance, then an iteration is made. Otherwise, the calculations may proceed to the next time step. After the first time loop through, a linear extrapolation is made to predict pressures and saturations for the next time step. The extrapolation technique is shown in Appendix B.

C. STRATIFICATION

Since it is assumed that no cross-flow occurs between layers, then the performance of each layer is calculated independently, and the performance of the total reservoir is obtained by summing the results of each individual layer's performance. This method of treatment represents the approach used by both Stiles and Dykstra.

Injection and production (through wells) occurs through selected sources or sinks, respectively, in the mathematical simulation. The assignment of water injection into the left edge of each layer and the calculation of production of oil, water and gas from the right edge of each layer is the

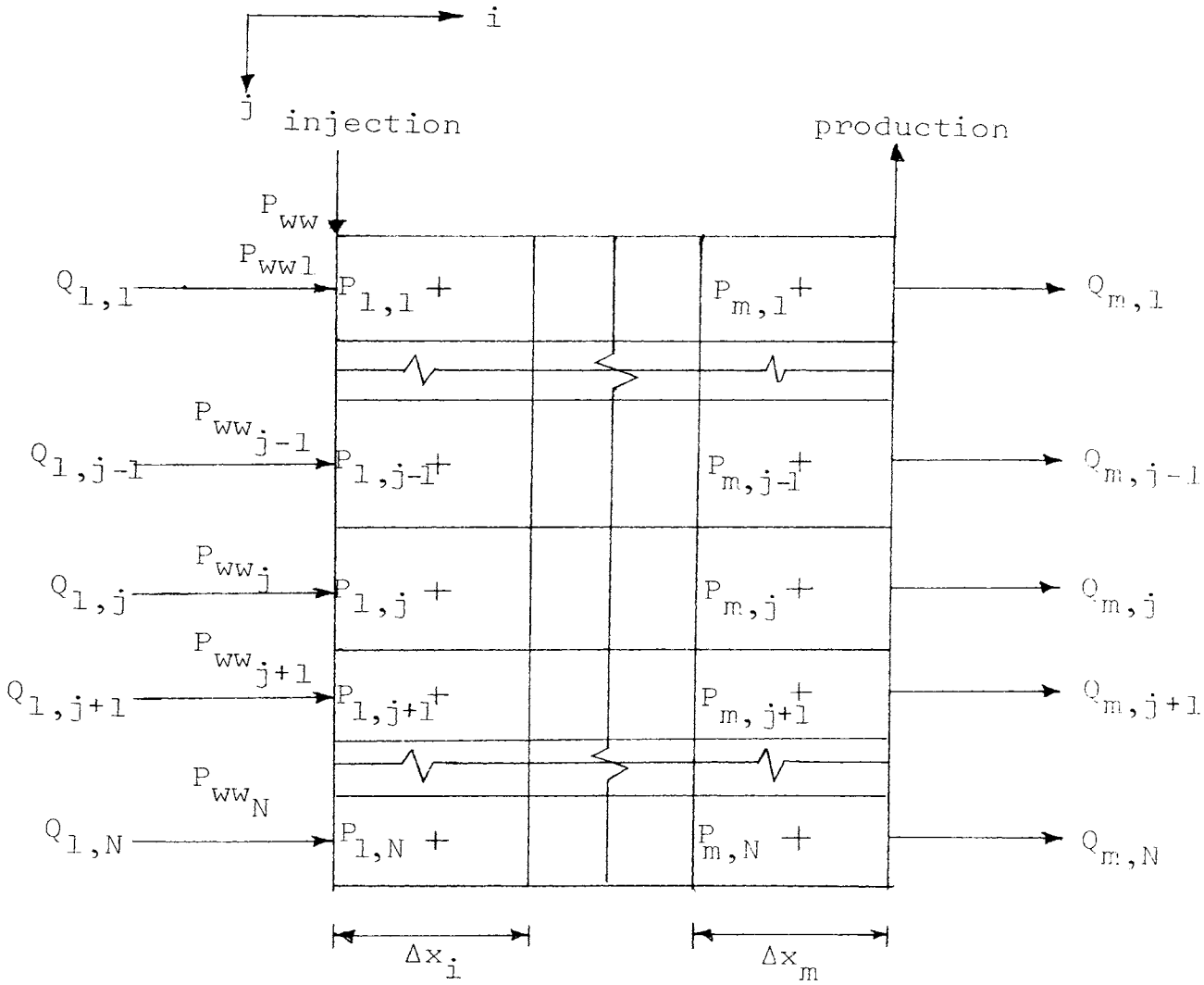


Fig. 3. Injection and Production Wells

convention assumed in this model and is treated as part of the boundary conditions. This is illustrated in Figure (3).

D. BOUNDARY CONDITIONS

The system simulated by this model is assumed to be closed at all boundaries except for those blocks containing wells, which for this model are those assumed to be at either end. No fluid is permitted to flow across these boundaries except that being produced or injected at the wells. The boundary barrier can be effected by either assigning zero permeability to an imaginary block outside the model or by assigning a pressure gradient of zero at these boundaries. The effect is identical. A schematic drawing of this model follows as Figure (4).

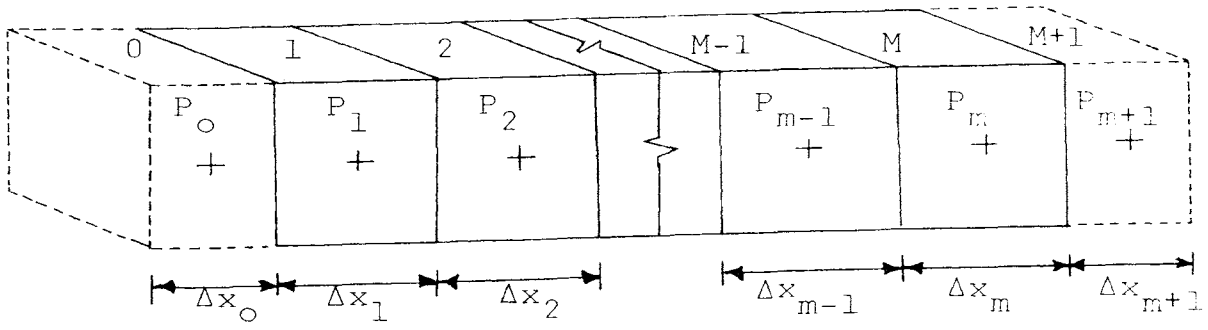


Fig. 4. Boundary Elements

Applying the latter assumption to Equation (6-D) and writing an equation about point i = 1, (see Figure (3)) with no flow across the left-hand boundary gives:

$$A_1 P_o^{n+1} + B_1 P_1^{n+1} + C_1 P_2^{n+1} = D_1 \quad . \tag{18}$$

The left hand boundary is closed by setting the pressure gradient to zero $\left[\left(\frac{\partial P}{\partial x}\right)_{1-\frac{1}{2}} = 0\right]$, which is equivalent to setting $P_o = P_1$.

Substituting this result into Equation (18) for i = 1 yields:

$$(A_1+B_1)\ P_1^{n+1} + C_1 P_2^{n+1} = D_1$$

but, from Equation (9-D),

$$B_1 = - A_1 - C_1 + TRM_1$$

then the coefficient of P_1^{n+1} becomes

$$A_1 + B_1 = - C_1 + TRM_1 \quad .$$

This result is equivalent to setting $A_1 = 0$, its value when the external block has a permeability of zero as shown below.

Equation (18) for the first block may be written as

$$B_1 P_1^{n+1} + C_1 P_2^{n+1} = D_1 \quad . \tag{19}$$

Closing the left hand boundary by assigning a zero permeability at the boundary ($k_o = 0$) gives the following result:

The coefficient A_1 in Equation (18) is defined as:

$$A_1 = \frac{HKX_o}{\Delta x_i}\left[(\beta_o - \beta_g R_s)_1 \gamma_{oi-\frac{1}{2}} + \beta_{wi}\gamma_{wi-\frac{1}{2}} + \beta_{gi}\gamma_{gi-\frac{1}{2}}\right] \tag{20}$$

where,

$$HKX_o = \frac{2k_i k_o}{\Delta x_o k_i + \Delta x_i k_o} = \frac{2(k_i)\ (D)}{\Delta x_o k_i} = 0 \ .$$

Substituting this result into Equation (20) gives a value of

$$A_1 = 0 \ .$$

Equation (18) may, then, be written about the first point as shown by Equation (19), both methods giving identical results.

Second, using the same logic about point m (no flow across the right hand boundary), will result in $C_m = 0$, and an equation about point m, may be written as,

$$A_m P^{n+1}_{m-1} + B_m P^{n+1}_m = D_m \qquad (21)$$

where

$$B_m = - A_m + TRM_m \ .$$

1. Injection End (left boundary).

The layer-wise distribution of water injection rate is obtained by first assuming a constant total injection rate in the "well", the left side of the model. A wellbore pressure at the top of the sand face (P_{ww}) is assigned and the wellbore pressure at the middle of each layer may be calculated as follows:

$$P_{ww_j} = P_{ww_{j-1}} + \rho_w \left(\frac{h_j + h_{j-1}}{2}\right) .$$

The horizontal pressure gradient entering element i,j is

$$\left(\frac{\partial P}{\partial x}\right)_{i,j} = \frac{2\left(P_{ww_{j-1}} + \frac{\rho_w}{2}(h_{j-1} + h_j) - P_{i,j}\right)}{\Delta x_i} . \qquad (22)$$

Replacing the gradient in Darcy's Law by the definition of the gradient as provided by Equation (22) yields:

$$(Q_{wl,j})_{inj} = \frac{2k_{l,j}k_{rwl,j}h_j\Delta y}{\beta_{wl,j}\mu_{wl,j}\Delta x_l}\left(P_{ww_{j-1}} + \frac{\rho_w}{2}(h_{j-1} + h_j) - P_{l,j}\right) \qquad (23)$$

The average pressure for elements l,j (for all layers) may be defined as:

$$\bar{P} = \frac{\sum_{j=1}^{n} P_{l,j}h_j}{h_T} .$$

Then the average pressure gradient is:

$$\left(\frac{\bar{\partial P}}{\partial x}\right) = \frac{2\left(\left(P_{ww} + \frac{\rho_w}{2}h_T\right) - \bar{P}\right)}{\Delta x_l} . \qquad (24)$$

Applying the definition of gradient shown in Equation (24) to Darcy's Law and summing the capacity terms (kA) yields total flow rate to give the following relationship:

$$(Q_{Tw})_{inj} = \frac{2\sum_{j=1}^{n}(k_{l,j}k_{rwl,j}h_j)\Delta y}{\bar{\beta}_w\bar{\mu}_w\Delta x_l}\left(P_{ww} + \frac{\rho_w}{2}h_T - \sum_{j=1}^{n}\frac{P_{l,j}h_j}{h_T}\right) . \qquad (25)$$

The fraction of the total injection rate entering the jth layer is obtained by dividing Equation (23) by Equation (25) as follows:

$$\frac{(Q_{1,j})_{inj}}{(Q_{Tw})_{inj}} = \frac{k_{1,j}k_{rw1,j}h_j(P_{ww_{j-1}}+\frac{\rho_w}{2}(h_j+h_{j-1})-P_{1,j})}{(\sum_{j=1}^{n}(k_{1,j}k_{rw1,j}h_j))(P_{ww}+\frac{\rho_w}{2}h_T-\sum_{j=1}^{n}\frac{P_{1,j}h_j}{h_T})} \qquad (26)$$

and the actual injection rate into the 1,jth block is given as

$$(Q_{1,j})_{inj} = (Q_{Tw})_{inj}\frac{k_{1,j}k_{rw1,j}h_j(P_{ww_{j-1}}+\frac{\rho_w}{2}(h_j+h_{j-1})-P_{1,j})}{(\sum_{j=1}^{n}(k_{1,j}k_{rw1,j}h_j))(P_{ww}+\frac{\rho_w}{2}h_T-\sum_{j=1}^{n}\frac{P_{1,j}h_j}{h_T})} \qquad (27)$$

Equation (27) is used to calculate the water injection rate into first element of each layer at the injection end. It should be noted that for elements between the "injection end" and "production end" with no external flows, the QTERM is zero. The application of Equation (27) requires knowledge of the wellbore pressure at the top of the sand. A major difficulty arises from the fact that wellbore pressure is not constant when constant injection is specified. The most realistic approach to solving the problem is to assume a constant wellbore pressure rather than constant injection rate and calculate water injection rate into first element of each layer by using Equation (23). For simplicity, it is assumed that pressure gradient across the first element of each layer is constant; then, Equation (27) can be written as

$$(Q_{l,j})_{inj} \tilde{=} (Q_{Tw})_{inj} \frac{k_{l,j} k_{rwl,j} h_j}{\sum_{j=1}^{n} k_{l,j} k_{rwl,j} h_j} , \qquad (28)$$

since pressure gradient cancels out under this set of conditions.

2. Production End (right boundary).

The sum of oil, water and gas production rates evaluated at reservoir conditions at the production well from a particular layer is equal to the water injection rate (at reservoir conditions) into that layer at the injection well. The convention adopted in this model is that injection is positive and production is negative. Thus, the production-injection balance stated above may be written as follows:

$$(Q_{w_{inj}} \beta_w)_{l,j} = - (Q_{o_{prd}} \beta_o)_{m,j} - (Q_{w_{prd}} \beta_w)_{m,j} - (Q_{g_{prd}} \beta_g)_{m,j} . \qquad (29)$$

Darcy's Law is used to calculate water-oil ratio at reservoir conditions for each layer at the production well and may be written as follows including simplification:

$$\frac{(Q_{w_{prd}} \beta_w)_{m,j}}{(Q_{o_{prd}} \beta_o)_{m,j}} = \left(\frac{k_{rw}}{k_{ro}} \frac{\mu_o}{\mu_w}\right)_{m,j} ,$$

and

$$(Q_{w_{prd}} \beta_w)_{m,j} = (Q_{o_{prd}} \beta_o)_{m,j} \left(\frac{k_{rw}}{k_{ro}} \frac{\mu_o}{\mu_w}\right)_{m,j} . \qquad (30)$$

Writing an equation for gas-oil ratio in the same manner and solving for the gas production rate at reservoir conditions yields the following relation:

$$(Q_{g_{prd}}\beta_g)_{m,j} = (Q_{o_{prd}}\beta_o)_{m,j} \left(\frac{k_{rg}}{k_{ro}}\frac{\mu_o}{\mu_g}\right)_{m,j} . \qquad (31)$$

Substituting results shown by Equations (30) and (31) into the reservoir voidage balance given by Equation (29), and then solving for oil production rate at reservoir conditions, yields

$$(Q_{o_{prd}}\beta_o)_{m,j} = \frac{-(Q_{w_{inj}}\beta_w)_{1,j}}{1+\left\{\frac{k_{rw}}{k_{ro}}\frac{\mu_o}{\mu_w} + \frac{k_{rg}}{k_{ro}}\frac{\mu_o}{\mu_g}\right\}_{m,j}} . \qquad (32)$$

Having available reservoir oil production rates for each layer now allows values of the water and gas rates at reservoir conditions to be calculated by Equations (30) and (31), respectively, for each layer. Then all three rates are converted to stock tank conditions by dividing the reservoir rates calculated (which include volume factors) by the appropriate formation volume factor for each phase.

The total production rates for the well for each phase are now obtained by adding the respective rates from each layer. The water-oil ratio and gas-oil ratio from the well for the time period under consideration are computed as follows:

$$(Q_{To})_{m,prd} = \sum_{j=1}^{n} (Q_{om,j})_{prd} , \tag{33}$$

$$(Q_{Tw})_{m,prd} = \sum_{j=1}^{n} (Q_{wj,j})_{prd} , \tag{34}$$

$$(Q_{Tg})_{m,prd} = \left(\sum_{j=1}^{n} ((Q_{gm,j})_{prd} + (Q_{om,j})_{prd} \left(\frac{R_{sm,j}^{n+1} + R_{sm,j}^{n}}{2} \right) \right)) . \tag{35}$$

$$WOR = (Q_{Tw})_{m,prd} / (Q_{To})_{m,prd} , \tag{36}$$

and

$$GOR = (Q_{Tg})_{m,prd} / (Q_{To})_{m,prd} . \tag{37}$$

E. MATERIAL BALANCE

Material balance calculations were made on each block in deriving the pressure equation and are also used to update saturations across each time step.

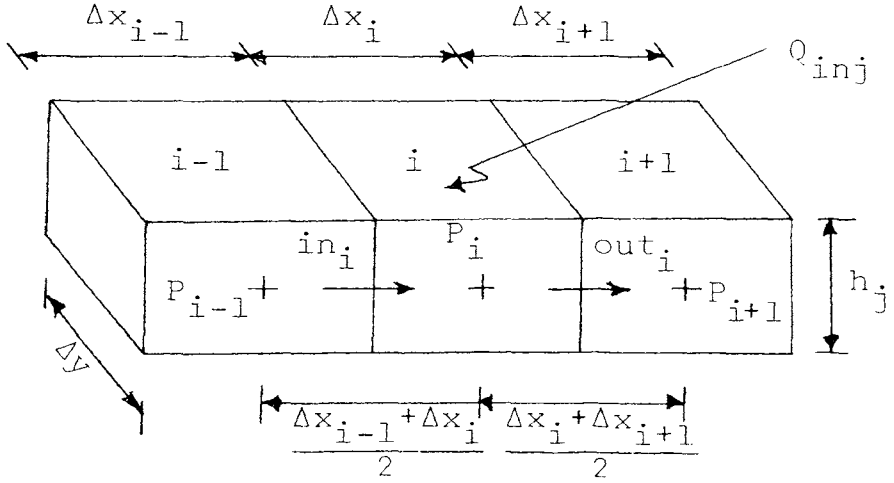


Fig. 5. Grid System for Material Balance

Referring to Figure (5), the saturation in block (i) at a new time level t(n+1) can be stated as the saturation at the beginning of the time step (old time level t(n)) plus the change in saturation during the time increment (Δt) (from n to n+1 time level), may be mathematically defined as,

$$S_i^{n+1} = S_i^n + \Delta S_i \ .$$

ΔS_i for a phase is the change in volume of that phase in block (i) during the time increment, divided by that block's pore volume.

However, in allowing for the compressibility of phases in this model, the accuracy of the calculation will be enhanced by making a mass balance rather than a volumetric balance and then converting the change in mass in the block to an equivalent change in saturation.

Thus, the following relation will be applicable for oil and water phases,

$$\Delta S_i = \frac{(\text{mass in})_i - (\text{mass out})_i + (\text{mass injected})_i}{\rho_{ri}(PV)_i} \ . \qquad (1\text{-}M)$$

Darcy's Law is used at the (i-½) and (i+½) interfaces to evaluate the $(\text{mass in})_i$ and $(\text{mass out})_i$ terms, respectively, in Equation (1-M). The following combination of terms are applicable:

$$\Delta t \frac{\rho_s Q}{\beta} = - \frac{\rho_s k \, k_r A(\frac{\partial P}{\partial x})}{\beta \mu} \Delta t \ . \qquad (38)$$

Recognizing that the mass entering block (i) comes from block (i-1) as indicated in Figure (5), and taking the convention that flow is from left to right, it concluded that the following relations apply for the $(\text{mass in})_i$ term:

$$\beta = \beta_{i-1},\ k = k_{i-\frac{1}{2}},\ \mu = \mu_{i-\frac{1}{2}},$$

$$(\partial P/\partial x)_{i-\frac{1}{2}} \tilde{=} (P_i - P_{i-1})/((\Delta x_i + \Delta x_{i-1})/2). \quad (39)$$

and similarly for the $(\text{mass out})_i$ term:

$$\beta = \beta_i,\ k = k_{i+\frac{1}{2}},\ \mu = \mu_{i+\frac{1}{2}},$$

$$(\partial P/\partial x)_{i+\frac{1}{2}} \tilde{=} (P_{i+1} - P_i)/((\Delta x_{i+1} + \Delta x_i)/2). \quad (40)$$

Oil Phase:

Introducing the results shown immediately above into Equation (37) gives the following relations:

$$(\text{Mass in})_i = - \frac{\rho_{os} k_{i-\frac{1}{2}} \gamma_{oi-\frac{1}{2}} A(P_i - P_{i-1})\Delta t}{\beta_{oi-1} \left\{ \frac{\Delta x_i + \Delta x_{i-1}}{2} \right\}} \quad (2\text{-M})$$

$$(\text{Mass out})_i = - \frac{\rho_{os} k_{i+\frac{1}{2}} \beta_{oi+\frac{1}{2}} A(P_{i+1} - P_i)\Delta t}{\beta_{oi} \left\{ \frac{\Delta x_{i+1} + \Delta x_i}{2} \right\}} \quad (3\text{-M})$$

$$(\text{Mass injected})_i = (Q_{oinj})_i\ \rho_{os} \Delta t \quad (4\text{-M})$$

$$\rho_{or}\ (PV)_i = \Delta x_i A\ \frac{\rho_{os}}{\beta_{oi}}\ \phi\ . \quad (5\text{-M})$$

The equality $\rho_{or} = \frac{\rho_{os}}{\beta_o}$ is assumed applicable and is shown in detail in Appendix B.

Substituting the results shown in Equations (2-M), (3-M), (4-M) and (5-M) into Equation (1-M) and introducing the terms HKX_i and HKX_{i-1} as defined by Equations (16) and (17), yields:

$$\Delta S_o = - \frac{\beta_{oi}\Delta t}{\Delta x_i \phi} \left[\frac{HKX_{i-1}\gamma_{oi-\frac{1}{2}}(P_i - P_{i-1})}{\beta_{oi-1}} - \frac{HKX_i\gamma_{oi+\frac{1}{2}}(P_{i+1} - P_i)}{\beta_{oi}} - \frac{(Q_{oinj})_i}{h\Delta y} \right]^{n+\frac{1}{2}} \qquad (6\text{-M})$$

where $A = h\Delta y$. It is well to note that all rock and fluids properties are evaluated at an average time, namely the $(n+\frac{1}{2})$ time level.

Water Phase:

Following a similar development as that shown for the oil phase, an equation for the change of water saturation during time increment Δt may be written as,

$$\Delta S_w = - \frac{\beta_{wi}\Delta t}{\Delta x_i} \left[\frac{HKX_{i-1}\gamma_{wi-\frac{1}{2}}(P_i - P_{i-1})}{\beta_{wi-1}} - \frac{HKX_i\gamma_{wi+\frac{1}{2}}(P_{i+1} - P_i)}{\beta_{wi}} - \frac{(Q_{winj})_i}{h\Delta y} \right]^{n+\frac{1}{2}} . \qquad (7\text{-M})$$

Gas Phase:

For the gas phase, an additional term should be incorporated into Equation (1-M) to account for the gas evolution as defined by Equation (9). With this additional term, the following equation describes the gas saturation incremental change:

$$\Delta S_{gi} = (\text{mass in})_i + (\text{mass of gas evolved})_i - (\text{mass out})_i + (\text{mass injected})_i / \rho_{gri} (PV)_i \ .$$

An explanation of the term accounting for solution gas was given previously and the term is listed as Equation (9). The present handling of this term is identical except that the ratio $(\frac{S_o}{\beta_o})$ will be factored out of the derivation as a constant at the average time over the increment. This is a realistic assumption since permitting $(\frac{S_o}{\beta_o})$ to vary may give rise to a situation in which all the gas in solution in the ΔS_o increment would be released during the time step. Thus, the "gas evolved term" in difference form will be,

$$\frac{\partial M_{FG}}{\partial t} \Delta t = - \Delta x_i A \phi \rho_{gs} \left(\frac{S_o}{\beta_o}\right)^{n+\frac{1}{2}} (R_s^{n+1} - R_s^n)_i \ . \qquad (9\text{-M})$$

Applying logic like that used to derive ΔS_i for the oil and water phases and incorporating the gas evolution term, an equation may be written for the change in gas saturation in block (i) during time increment (Δt) as follows:

$$\Delta S_{gi} = - \frac{\beta_{gi} \Delta t}{\Delta x_i \phi} \left[\frac{HKX_{i-1} \gamma_{gi-\frac{1}{2}} (P_i - P_{i-1})}{\beta_{gi-1}} - HKX_i \gamma_{gi+\frac{1}{2}} (P_{i+1} - P_i) - \frac{(Q_{ginj})_i}{h \Delta y} \right]^{n+\frac{1}{2}} - \left\{ \frac{S_o}{\beta_o} \right\}_i^{n+\frac{1}{2}} (R_s^{n+1} - R_s^n)_i \beta_{gi}^{n+\frac{1}{2}} \ . \qquad (10\text{-M})$$

For simplicity, Equations (6-M), (7-M) and (10-M) may be written as follows:

$$\Delta S_{oi} = ZZ(AMOX1-AMOX2-QOTERM) \qquad (11\text{-}M)$$

$$\Delta S_{wi} = ZZ(AMWX1-AMWX2-QWTERM) \qquad (12\text{-}M)$$

$$\Delta S_{gi} = ZZ(AMGX1-AMGX2-QGTERM)-RSTERM \qquad (13\text{-}M)$$

where,

$$ZZ = -\Delta t/\phi\Delta x_i$$

$$AMOX1 = HKX_{i-1}\left[\gamma_{oi-\frac{1}{2}}(P_i-P_{i-1})\left(\frac{\beta_{oi}}{\beta_{oi-1}}\right)\right]^{n+\frac{1}{2}}$$

$$AMWX1 = HKX_{i-1}\left[\gamma_{wi-\frac{1}{2}}(P_i-P_{i-1})\left(\frac{\beta_{wi}}{\beta_{wi-1}}\right)\right]^{n+\frac{1}{2}}$$

$$AMGX1 = HKX_{i-1}\left[\gamma_{gi-\frac{1}{2}}(P_i-P_{i-1})\left(\frac{\beta_{gi}}{\beta_{gi-1}}\right)\right]^{n+\frac{1}{2}}$$

$$AMOX2 = HKX_i\left[\gamma_{oi+\frac{1}{2}}(P_{i+1}-P_i)\right]^{n+\frac{1}{2}}$$

$$AMWX2 = HKX_i\left[\gamma_{wi+\frac{1}{2}}(P_{i+1}-P_i)\right]^{n+\frac{1}{2}}$$

$$AMGX2 = HKX_i\left[\gamma_{gi+\frac{1}{2}}(P_{i+1}-P_i)\right]^{n+\frac{1}{2}}$$

$$QOTERM = \frac{\beta_{oi}(Q_{winj})_i}{h\Delta y}$$

$$QWTERM = \frac{\beta_{wi}(Q_{winj})_i}{h\Delta y}$$

$$QGTERM = \frac{\beta_{gi}(Q_{ginj})_i}{h\Delta y}$$

$$RSTERM = \left(\frac{S_o}{\beta_o}\right)^{n+\frac{1}{2}} (R_s^{n+1}-R_s^n)\ \beta_{gi} \ .$$

The equations for updating oil, water and gas saturations are,

$$S_{oi}^{n+1} = S_{oi}^n + \Delta S_{oi} \ ;$$

$$S_{wi}^{n+1} = S_{wi}^n + \Delta S_{wi} \ ;$$

and,

$$S_{gi}^{n+1} = S_{gi}^n + \Delta S_{gi} \ .$$

V. RESULTS

Computational procedures for each of the analyses were developed and programmed in Fortran IV and the calculations were made by an IBM 360-50 computer. Flow diagrams and digital computer programs are presented in Appendix D.

Results of seventeen studies using four basic models are shown graphically by Figures 9, 10, 11, 12 and 13, and a summary of study runs with results are shown in Table I. Reservoir configuration and computational data are presented in Appendix C. Of the four basic models used in the study, two were numerical models, and two were computer-programmed versions of the Dykstra'-Parsons' technique.

Variable permeability in the vertical direction was simulated numerically by combining four linear models such that it simulates four vertical layers of different permeabilities. The first of these models included horizontal variations in permeability and is referred to as the "heterogeneous layered model". The results from a single run for an oil-water viscosity ratio of 1.96 using this model are presented in Figures 9, 10, and 11 as curve Number 4 in each, and as indicated on the graph in Figures 12 and 13.

The second of the numerical models used is layered in four vertical zones just as in the first model; however, each layer is homogeneous and the value of permeability for a layer was computed as a series average of the heterogeneous permeabilities in the first model. This second model is

referred to as the "homogeneous layered model" and results of six runs obtained using this model are presented. The results for an oil-water viscosity ratio of 1.96 are shown for comparison purposes in Figures 9, 10 and 11, and referred to as curve Number 1. The results for oil-water viscosity ratios of 0.5, 1.96, 5.0, 10.0, 20.0 and 40.0 are shown in Figures 12 and 13.

The first Dykstra-Parsons model is layered in four vertical zones, and the value for permeability of a layer is the same as in the second version of the numerical model. The results of five runs are presented. The results for an oil-water viscosity ratio of 1.96 are shown in Figures 9, 10 and 11 and referred to as curve Number 2 and results using oil-water viscosity ratios of .5, 1.96, 5.0, 10.0 and 20.0 are shown in Figures 12 and 13.

The second Dykstra-Parsons model is layered in twenty-five vertical zones with an equal thickness of one foot in each of the layers. The permeability of each layer was read from Figure 8, which was obtained by plotting the series-averaged value for permeability of each of the four layers used previously as a function of cumulative thickness. Results of five runs using this model are presented. The results for an oil-water viscosity ratio of 1.96 are shown in Figures 9, 10 and 11, and referred to as curve Number 3 and results for oil-water viscosity ratios of 0.5, 1.96, 5.0, 10.0 and 20.0 are shown in Figures 12 and 13.

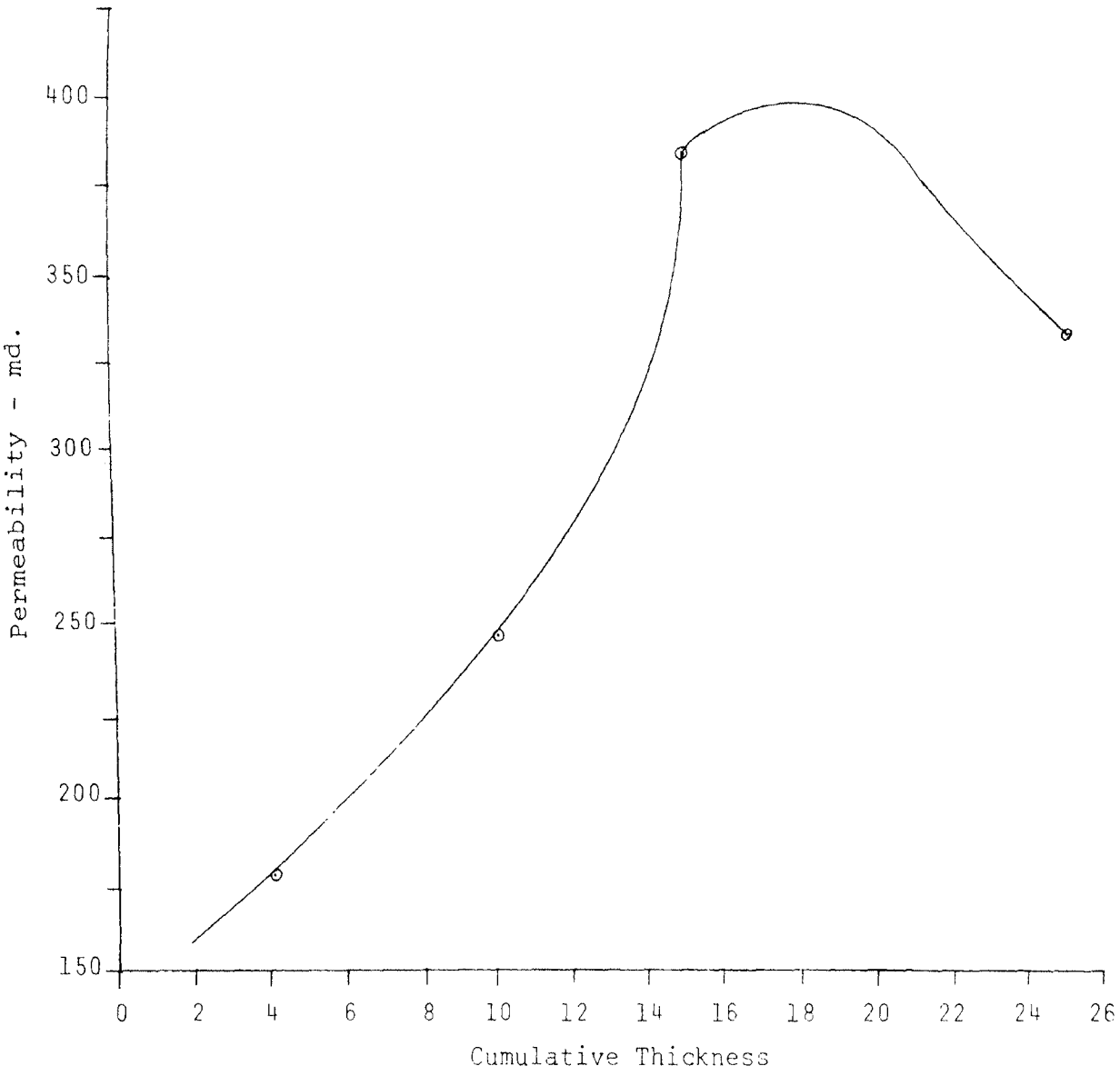


Fig. 8. Permeability Variations with Depth

TABLE I. Summary of Results

Model	% Recovery at WOR = 10 for Oil-Water Viscosity Ratio of					
	.5	1.96	5	10	20	40
Numerical (Heterogeneous Layered)	--	51.4	--	--	--	--
Numerical (Homogeneous Layered)	79.5	79	9.2	7.05	5.3	3.18
Dykstra-Parsons (25 Layers)	74.6	73.6	68.9	62.3	56	--
Dykstra-Parsons (4 Layers)	72.7	71.7	70.9	68.1	61.5	--

VI. DISCUSSION OF RESULTS

Figure (9) illustrates cumulative oil produced as a function of time for an oil-water viscosity ratio of 1.96 based on results from the four basic models described previously. Water breakthrough in all models occurred at approximately the same time (120-130 days). Curve 1 (numerical-homogeneous layers) and Curves 2 and 3 (Dykstra-Parsons, 4 layers and 25 layers, respectively) showed close agreement until water breakthrough occurs and slight deviation thereafter. Curve 2 was in closer agreement with Curve 1 than Curve 3 as expected, since the models of Curves 1 and 2 are in closer agreement, theoretically. However, final recovery was the same for both Dykstra-Parson models. Curve 4 (the numerical-heterogeneous layered model) showed a great reduction in recovery compared to Curves 1, 2 and 3. This was primarily due to the formation of free gas resulting from the heterogeneity in the fourth layer, which in turn caused the reservoir pressure to decline below the bubble point pressure. Although the other layers in this model were also heterogeneous, there was less variation in the permeabilities in these blocks prohibiting notable gas formation.

Figure (10), illustrates cumulative oil produced as a function of water-oil ratio. Curves 1 and 4 show that water-oil ratio changes gradually over most of the region except for regions with periodic rapid increases. This is explained by the fact that when any layer breaks through, the composite water-oil ratio increases significantly.

Thereafter, the water-oil ratio increases were gradual due to continually changing saturations until another layer breaks through resulting in another sudden increase. This face is not as pronounced in Curves 2 and 3 (Dykstra-Parsons), even though the results show some gradual changes in water-oil ratio. Because of the assumption of piston-like displacement in the Dykstra-Parsons method, only water is flowing in any layer after water breakthrough. The gradual change in water-oil ratio is due to changing velocities within the layers due to the front continually advancing in layers of different permeabilities. The changes are not due to saturation changes at the production face. In fact, Curves 2 and 3 are very nearly step-functions. It is also of interest to note that areal non-uniformity in permeability increased the life of the reservoir but resulted in less oil recovery and consistently higher water-oil ratios.

Figure (11) illustrates oil production rate as a function of time. Curve 1 shows fluctuations at the beginning of water injection which is due to the unsteady state nature of the model. A sudden decrease in oil production rate occurs when water breaks through in the most permeable layer. Curve 4 shows a sudden decrease in oil production rate until water breakthrough, and this is due to the formation of free gas.

Figure (12) illustrates the relationship between cumulative oil produced and cumulative water injected for various oil-water viscosity ratios. As expected, the recovery

decreased as oil-water viscosity ratio increased for a particular model. This is also shown in Figure (13) by results from both numerical and Dykstra-Parson models. The deviation and reduction in oil recovery as compared by results from the numerical model from Dykstra-Parsons for oil-water viscosity ratios of 5.0, 20.0, and 40.0, results from the fact that the Dykstra-Parsons approach is not adapted for conditions where free gas is formed or is present as occurs in depleted oil sands. The model assumes only two-phase fluid flow (oil and water), and the method is not suitable for gas flow due to the assumption of piston-like displacement. It is also of interest to note that an increase in oil-water viscosity ratio results in a decline of reservoir pressure and an increase in the life of the reservoir by producing less oil at a higher water-oil ratio.

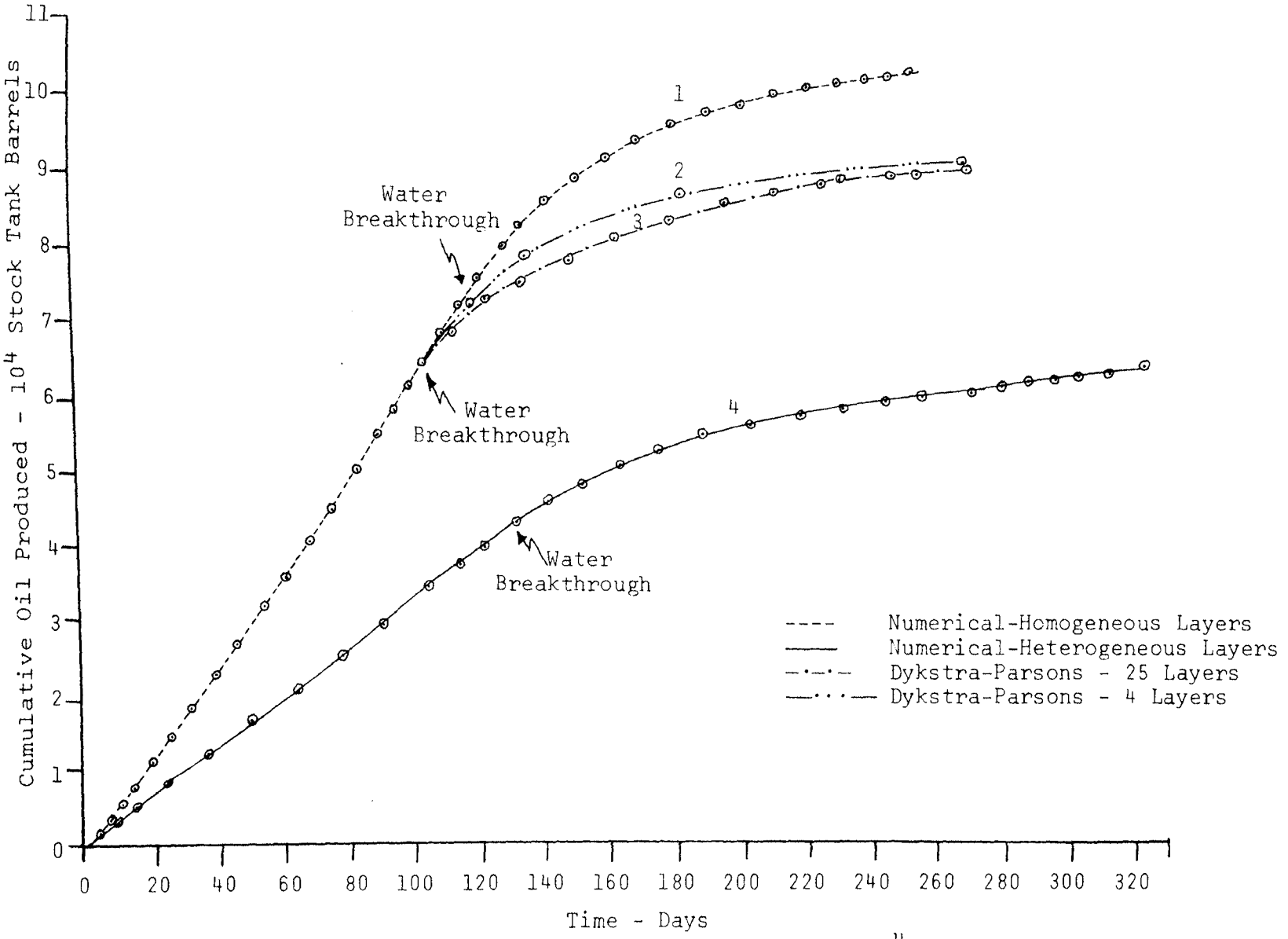


Fig. 9. Cumulative Oil Produced vs. Time for $\frac{\mu_o}{\mu_w} = 1.96$

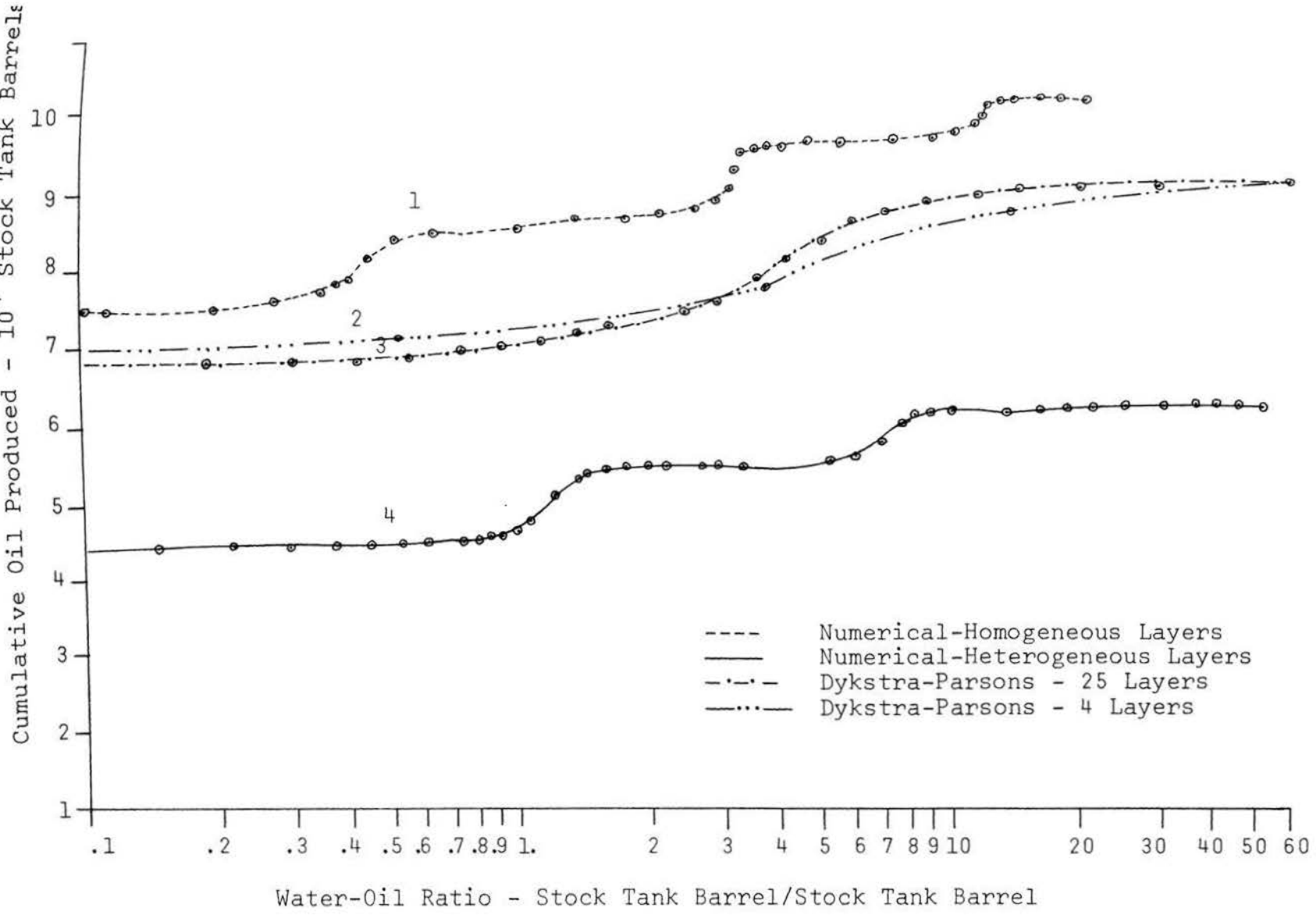


Fig. 10. Cumulative Oil Produced vs. Water-Oil Ratio for $\frac{\mu_o}{\mu_w} = 1.96$

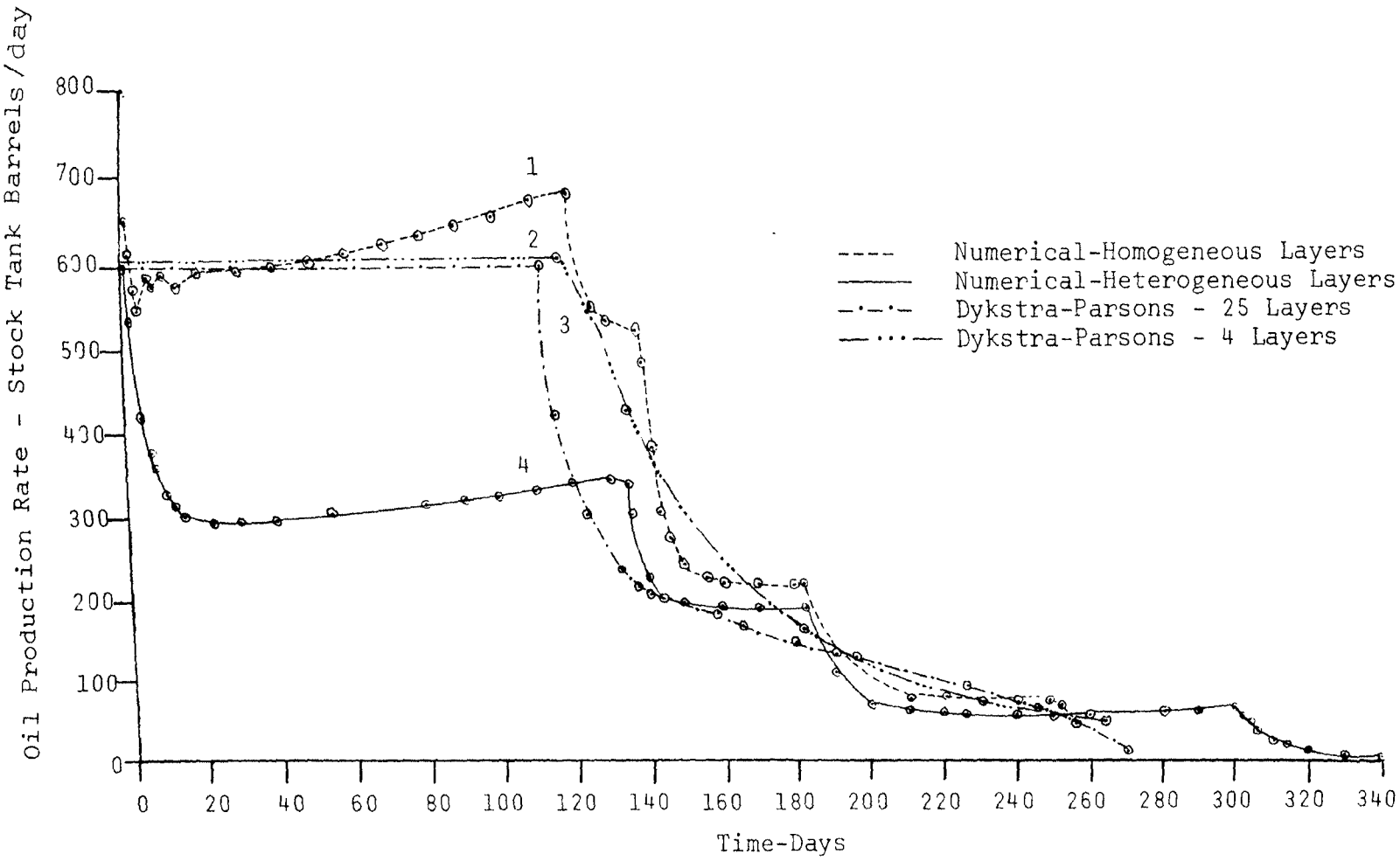


Fig. 11. Oil Production Rate vs. Time for $\frac{\mu_o}{\mu_w} = 1.96$

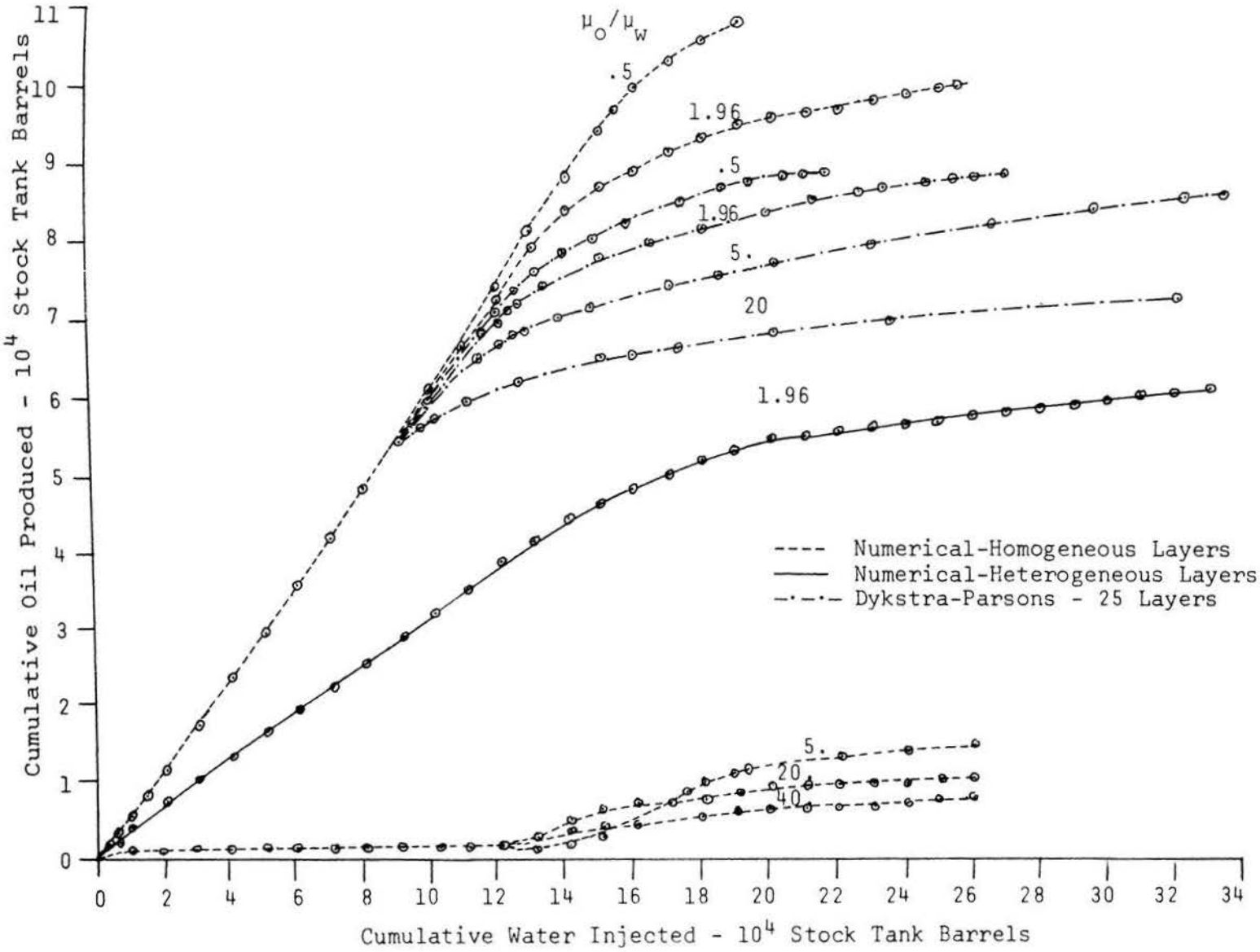


Fig. 12. Cumulative Oil Produced vs. Cumulative Water Injected for Various $\frac{\mu_o}{\mu_w}$ Ratios

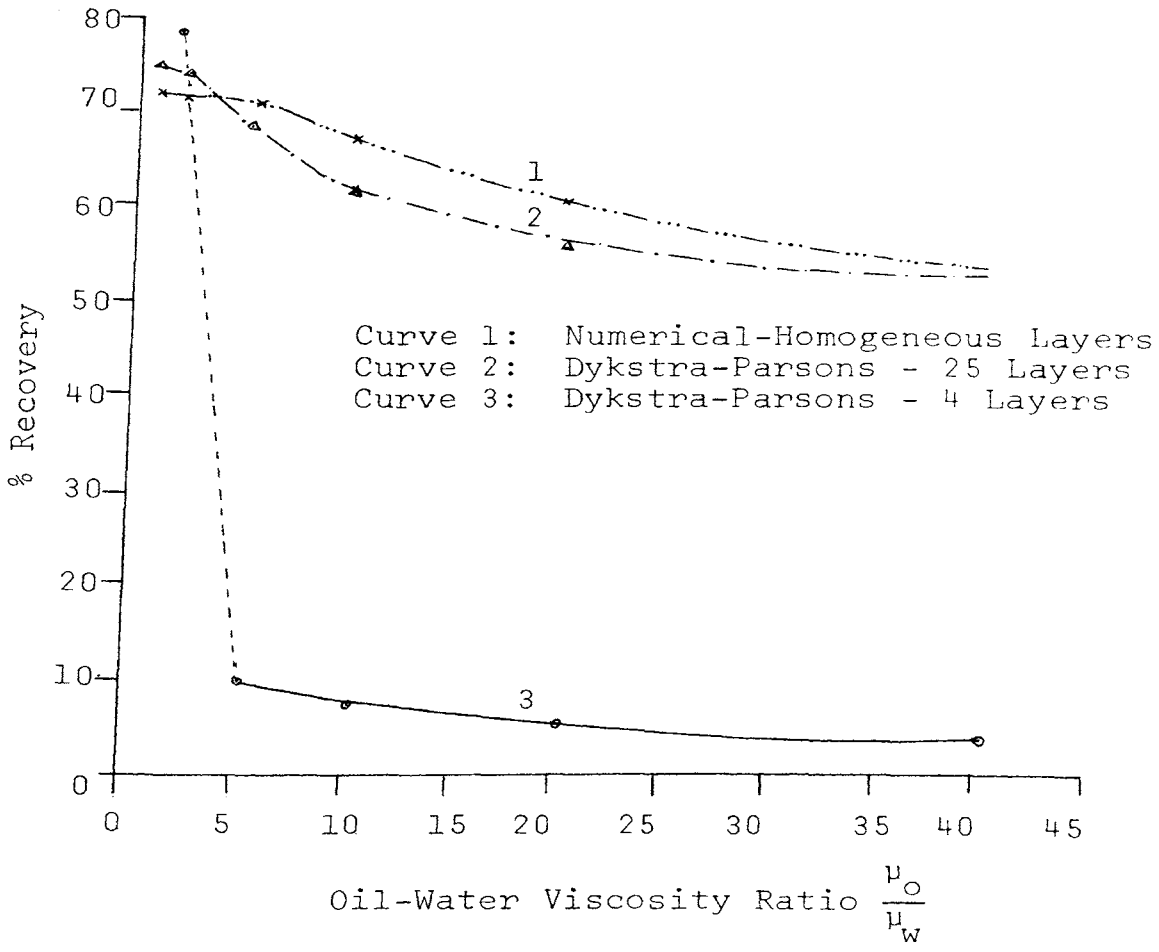


Fig. 13. Percent Recovery of Initial Oil in Place vs. Oil-Water Viscosity Ratio at WOR = 20:1

VII. CONCLUSIONS

1. Prediction of reservoir performance under a water drive by a three-phase numerical simulation model is shown to be feasible due to the favorable comparison of results with the Dykstra-Parsons method for those cases where the Dykstra-Parsons assumptions are satisfied.
2. The numerical method is valid for more general use than the Dykstra-Parsons method since the latter does not account for changes in fluid properties as a function of pressure, reservoir geometry, rock heterogeneity, and three-phase flow.
3. Results show that the prediction of performance of a water flood project by the Dykstra-Parsons method will not be valid when a mobile gas is present. The gas phase reduces oil mobility to such an extent that considerably less oil may be produced at water breakthrough than predicted by the Dykstra-Parsons technique.
4. Areal rock heterogeneity is not adequately accounted for by series-averaging individual permeabilities within a layer. Thus, a numerical simulator is needed for predictions where areal heterogeneity is obvious.
5. The Dykstra-Parsons calculations are faster than those by the numerical model and should be used in preference where conditions fit the assumptions of the Dykstra-Parsons method.

VIII. APPENDICES

APPENDIX A

1. NOMENCLATURE

A = cross sectional area normal to the flow direction, $A = \Delta y h$ - sq-cm

C = compressibility - atm^{-1}

g = acceleration of gravity - cm/sq-sec

h = layer thickness - cm

h_T = total reservoir thickness - cm

k = absolute permeability - Darcy

k_g, k_o, k_w = phase effective permeability

k_r = relative permeability

M = mass of accumulation - gm

m = number of elemental blocks in each layer

M_{FG} = mass of free gas evolved from solution gas for time period t_n to t_{n+1}

n = number of layers

P = pressure - tam

P_{md} = pressure at the interface between blocks - atm

PV = pore volume - cc

P_{ww} = pressure at top of the sand - atm

Q = production or injection rate - cc/sec

R_s = solution gas-oil ratio cc of gas/cc of oil

$R_s' = \frac{\partial R_s}{\partial P}$

S = saturation - fraction

SGC = equilibrium gas saturation - fraction

SOR = residual oil saturation - fraction

SWC = connate water saturation - fraction

t = time - sec

V = Darcy's velocity potential - cm/sec

v = volume - cc

Greek Letters and Derivatives:

β = formation volume factor - res cc/std cc

β' = $\frac{\partial \beta}{\partial P}$

γ = $\frac{k_r}{\mu \beta}$

ϕ = porosity - fraction

μ = viscosity - cp

Φ = pressure potential - atm

ρ = density - gm/cc

Δx = length of elemental block - cm

ΔP = pressure drop

ΔS_i = change in saturation to the ith elemental block from time t_n to t_{n+1} - fraction

Δt = time increment - sec

Δy = width of elemental block - cm

$\frac{\partial M}{\partial t}$ = mass rate of accumulation - gm/sec

$\frac{\partial M_{FG}}{\partial t}$ = mass rate of gas evolved from solution gas from time t_n to t_{n+1}

Subscripts:

b = bubble point

g = gas phase

i = indicating ith elemental block in x-direction

inj = injection

j = indicating jth layer in Z-direction

max = maximum

prd = production

s = standard conditions of pressure and temperature (1 atm and $60^{\circ}F$)

T = total

Superscript:

n = indicating nth time level

2. LIST OF COMPUTER SYMBOLS

FORTRAN SYMBOL	ALBEGRAIC SYMBOLS OR DEFINITION
AX	A
BGPRM	β_g'
BOPRM	β_o'
BTA	λ
BTAG	β_g
BTAO	β_o
BTAW	β_w
BTOBP	β_o at bubble point
BTWBP	β_w at bubble point
BX	B
COV	C = coverage
CUMOP	cumulative oil produced
CUMWIN	cumulative water injected
CX	C
DELT	Δt
DELTP	ΔN_p
DELX	Δx
DELY	Δy
DELZ	$Z_i - Z_{i-1}$
DX	D
GMA	α
GOR	gas-oil ratio

M	number of elemental blocks
N	number of layers
OILPR	cumulative oil production - N_P
P	pressure at n time level
PB	bubble point pressure
PERM	permeability - k
PERMAX	k_{max}
PHI	porosity - ϕ
PIN	initial pressure
PM	M
PN1	trial pressure for n+1 time level
PN2	pressure at n+1 time level
PRTOL	prespecified pressure tolerance check
QOR	oil production rate
RECOV	recovery = $\frac{\text{cumulative oil production}}{\text{initial oil in place}}$ x100
RKG	k_{rg}
RKO	k_{ro}
RKW	k_{rw}
RSPRM	R_s'
SATOL	prespecified saturation tolerance check
SGI	initial gas saturation
SOI	initial oil saturation
STOIP	initial oil in place
SUBROUTINE BETA	formation volume factors calculation
SUBROUTINE BETPRM	β_g', β_o', R_s' calculation

SUBROUTINE COFF	calculation of coefficients A, B, C and D
SUBROUTINE MBAL	material balance calculation
SUBROUTINE RPERM	relative permeabilities calculation
SUBROUTINE SOLGAS	solution gas-oil ratio calculation
SUBROUTINE TRIDAG	tri-diagonal solution
SUBROUTINE VISC	viscosity calculation
SUMH	total thickness
SUMKH	$\sum_{j=1}^{N} (kh)_j$
SUMQG	cumulative gas produced
SUMQO	cumulative oil produced
SUMQW	cumulative water produced
SUMQWI	$\sum_{j=1}^{N} QW_{inj,1,j}$
SW	saturation at n+1 time level
SWI	initial water saturation
SW1	saturation at n time level
SW2	trial saturation for n+1 time level
TEMP1	$(\beta_o - \beta_g R_s)_{i,j}^{n+\frac{1}{2}}$
TEMP2	$\beta_{w_{i,j}}^{n+\frac{1}{2}}$
TEMP3	$\beta_{g_{i,j}}^{n+\frac{1}{2}}$
TEMP4	$\Delta x_{i-1}/\Delta x_i$
TEMP5	$\beta_{o_{i,j}}^{n+\frac{1}{2}}$
TEMP6	$(\beta_o - \beta_g R_s)_{i-1,j}^{n+\frac{1}{2}}$
WI	cumulative water injected
WIR	water injection rate

WOR	water oil ratio
WP	cumulative water produced
XMUG	μ_g
XMUO	μ_o
XMUW	μ_w
XMUOBP	μ_o at bubble point
XMUWBP	μ_w at bubble point

APPENDIX B.

DERIVATION OF MISCELLANEOUS RELATIONSHIPS

1. Derivation of HKX

$K_{i+\frac{1}{2}}$ and $k_{i-\frac{1}{2}}$ are the series averaged values of permeability evaluated downstream and upstream, respectively, for the ith block. Each evaluation is based on the length and permeability of the two blocks bounding each respective interface horizontally.[18] The derivation of this parameter for the downstream interface is given below and is based on the geometric configuration shown in Figure (6).

Consider only that portion of the elemental block (i) and (i+1) between their mid points, an average permeability ($\bar{k}$) can be derived by Darcy's Law evaluated separately between the two blocks.

$$Q_i = \frac{k_i A_i \Delta P_i}{\mu \frac{\Delta x_i}{2}} \qquad (1\text{-A})$$

and

$$Q_{i+1} = \frac{k_{i+1} A_{i+1} \Delta P_{i+1}}{\mu \frac{\Delta x_{i+1}}{2}} \qquad (2\text{-A})$$

Note that pressures are evaluated at the mid point of each elemental block, therefore Δx's are divided by 2.

$$\Delta P_i = P_{md} - P_i$$

$$\Delta P_{i+1} = P_{i+1} - P_{md}$$

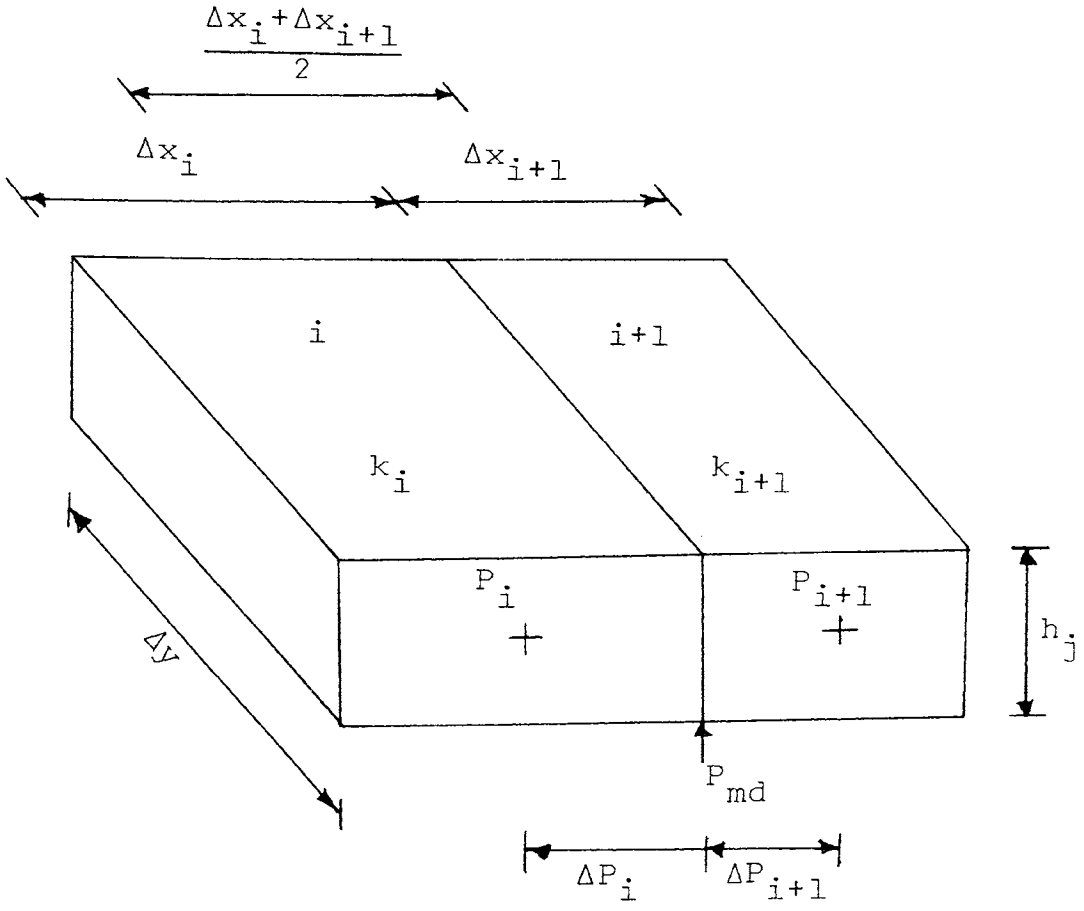


Fig. 6. Illustration of Series-Averaging Technique for Permeabilities

The total flow rate through both blocks in series is:

$$Q_T = \frac{\bar{k}A(P_{i+1}-P_i)}{\mu\frac{\Delta x_{i+1}+\Delta x_i}{2}} \quad , \bar{k} = k_{i+\frac{1}{2}} \tag{3-A}$$

Considering the flow rate constant through a series of elemental blocks yields:

$$Q_T = Q_i = Q_{i+1} \tag{4-A}$$

Further noting that,

$$\Delta P_i+\Delta P_{i+1} = (P_m-P_i) + (P_{i+1}-P_m) = P_{i+1}-P_i \tag{5-A}$$

Equations (1-A), (2-A) and (3-A) are solved for their respective ΔP's and these results are substituted into Equation (5-A) to yield:

$$\frac{Q\mu}{A}\left(\frac{\Delta x_{i+1}+\Delta x_i}{2\bar{k}}\right) = \frac{Q\mu}{A}\left(\frac{\Delta x_i}{2k_i} + \frac{\Delta x_{i+1}}{2k_{i+1}}\right)$$

or

$$\bar{k} = k_{i+\frac{1}{2}} = \frac{\Delta x_{i+1}+\Delta x_i}{\frac{\Delta x_i}{k_i}\quad\frac{\Delta x_{i+1}}{k_{i+1}}} = \frac{k_{i+1}k_i(\Delta x_{i+1}+\Delta x_i)}{\Delta x_i k_{i+1}+\Delta x_{i+1}k_i} \tag{6-A}$$

HKX_i is defined as,

$$HKX_i = \frac{2k_{i+\frac{1}{2}}}{\Delta x_i+\Delta x_{i+1}} \tag{7-A}$$

Substituting Equation (6-A) into Equation (7-A) yields:

$$HKX_i = \frac{2k_i k_{i+1}}{\Delta x_i k_{i+1} + \Delta x_{i+1} k_i} \qquad (8\text{-A})$$

and in the same manner:

$$HKX_{i-1} = \frac{2k_{i-1} k_i}{\Delta x_{i-1} k_i + \Delta x_i k_{i-1}} \qquad (9\text{-A})$$

2. Derivation of the Relationship $\rho_r = \frac{\rho_s}{\beta}$

$$\rho_r = \frac{\text{mass}}{V_r}, \quad \rho_s = \frac{\text{mass}}{V_s}$$

$$\frac{\rho_r}{\rho_s} = \frac{\text{mass}}{V_r} \times \frac{V_r}{\text{mass}} = \frac{V_s}{V_r} = \frac{1}{\beta}$$

Thus,

$$\rho_r = \frac{\rho_s}{\beta} \qquad (10\text{-A})$$

3. Linear Extrapolation

Assume a linear relationship between time and pressure Figure (7), then,

$$\frac{P^n - P^{n+1}}{t^{n+1} - t^n} = \frac{P^{n+1} - P^{n+2}}{t^{n+2} - t^{n+1}} \qquad (11\text{-A})$$

Letting

$$\Delta t_1 = t^{n+1} - t^n$$

and

$$\Delta t = t^{n+2} - t^{n+1}$$

Then solving Equation (11-A) for P^{n+2}, gives,

$$P^{n+2} = P^{n+1} + (P^{n+1} - P^{n}) \frac{\Delta t}{\Delta t_1} \qquad (12\text{-A})$$

In the same manner the extrapolated saturation is defined as:

$$S^{n+2} = S^{n+1} + (S^{n+1} - S^{n}) \frac{\Delta t}{\Delta t_1} \qquad (13\text{-A})$$

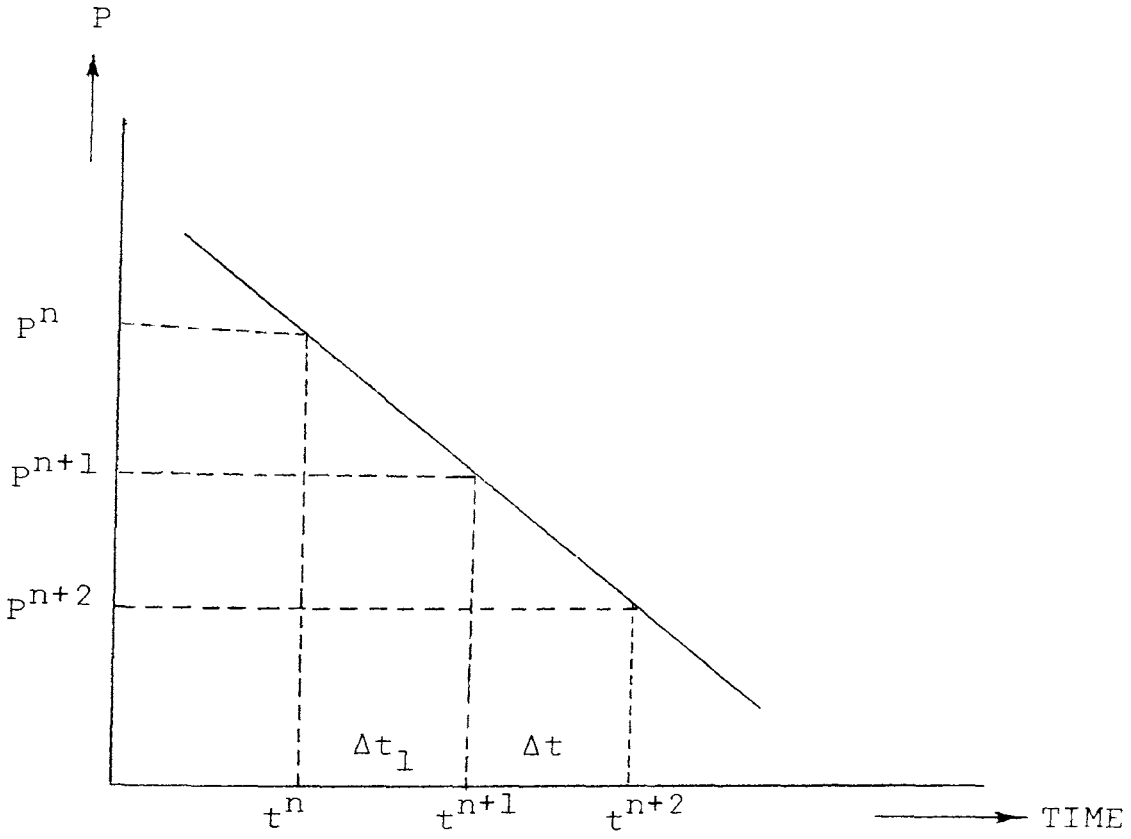


Fig. 7. Linear Extrapolation

APPENDIX C.

RESERVOIR DATA

A reservoir to be water flooded will be considered to have the following geometry, rock and fluid properties,*

Numerical Method (Heterogeneous Layered Case):

j \ i	1	2	3	4	5	6	7	8	
	Δx	Δx	Δx	Δx	Δx	Δx	Δx	Δx	
1	.15	.18	.17	.16	.2	.19	.19	.20	h = 4 ft.
2	.25	.23	.27	.22	.25	.27	.26	.25	h = 6 ft.
3	.35	.38	.37	.38	.40	.42	.39	.38	h = 5 ft.
4	.38	.36	.35	.32	.30	.38	.35	.30	h = 10 ft.
	100	150	200	250	250	200	150	100	ft.

x direction →

The number in each block in the above diagram is an absolute permeability value.

Total reservoir length = 1400 feet

Total reservoir thickness = 25 feet

Reservoir width Δy = 200 feet

Total water injection rate = 1000 barrels/day

*The data should be input in the computer program using field units. Conversion of field units to cgs units is made within the computer program.

Bubble point pressure P_b = 3487 psi

Oil compressibility CO = .003 atm^{-1}

Water compressibility CW = .0000045 atm^{-1}

Porosity ϕ = .20

Connate water saturation SWC = .2

Residual oil saturation SOR = .2

Initial oil saturation SOI = .8

Initial water saturation SWI = .2

Initial gas saturation SGI = 0.0

Equilibrium gas saturation SGC = .05

Oil formation volume factor at bubble point β_{ob} = 1.65

Water formation volume factor at bubble point β_{wb} = .95

Initial pressures in all the blocks of the first layer = 3487 psi

Initial pressures in the blocks of other layers are calculated by:

$$P_{i,j} = P_{i,j-1} + .433\left(\frac{h_j + h_{j-1}}{2}\right)$$

The fluid properties were suitably curve-fitted with the following empirical equations (using the least squares technique).

$$\left.\begin{aligned} \beta_o &= 1.09816 + 1.3487\times10^{-4}\,P + 6.8235\times10^{-9}\,P^2 \\ \frac{\partial \beta_o}{\partial P} &= 1.3487\times10^{-4} + 2(6.82357)\times10^{-9}\,P \end{aligned}\right\} \quad P < P_b$$

$$\left.\begin{aligned} \beta_o &= \beta_{ob}\, e^{-C_o(P-P_b)} \\ \frac{\partial \beta_o}{\partial P} &= -\,C_o\beta_o \end{aligned}\right\} \quad P > P_b$$

$$\beta_w = \beta_{wb}\, e^{-C_w(P-P_b)}$$

$$\beta_g = 1./(-19.937+9.126\times10^{-2}\ P-2.1086\times10^{-6}\ P^2)$$

$$\frac{\partial \beta_g}{\partial P} = -\beta_g{}^2\ (.09126-2(2.1086)P)$$

$$R_s = 44.2725+.266273P+7.7775\times10^{-6}\ P^2 \qquad P \le P_b$$

$$R_s = R_{s_b} \qquad P > P_b$$

$$\frac{\partial R_s}{\partial P} = .266273+2(7.7775\times10^{-6}\ P) \qquad P < P_b$$

$$\frac{\partial R_s}{\partial P} = 0 \qquad P > P_b$$

$$\mu_o = 3.98117-1.992\times10^{-2}P+8.0314\times10^{-7}P^2-1.9869\times10^{-10}P^3 \qquad P \le P_b$$

$$\mu_o = \mu_{o_b} + .0001(P-P_b) \qquad P > P_b$$

$$\mu_w = \mu_{w_b} + .00001(P-P_b)$$

$$\mu_g = .002474+9.244\times10^{-6}P-7.6556\times10^{-10}P^2$$

Relative Permeabilities[19]

$$k_{rg} = \frac{S_g{}^3(2-S_g-2SWC)}{(1-SWC)^4}, \quad S_w > SWC$$

$$k_{rg} = \frac{S_g{}^3(2-S_g-2SW)}{(1-SW)^4}, \quad S_w \le SWC$$

$$k_{rw} = \left[\frac{S_w - SWC}{1-SWC}\right]^4, \quad S_w > SWC$$

$$k_{rw} = 0, \quad S_w \leq SWC$$

$$k_{ro} = \frac{(1-S_g-SW)^3(1-S_g+SW-2SWC)}{(1-SWC)^4}, \quad S_w > SWC$$

$$k_{ro} = \left[\frac{1-S_g-S_w}{1-SWC}\right]^4, \quad S_w \leq SWC$$

Numerical Method (Homogeneous Layered Case):

These data are the same as those for the heterogeneous case except that the layers are homogeneous. A series-averaged value of permeability was calculated for each heterogeneous layer and was assigned to that same layer in this model. These values are:

Layer	Permeability-Darcy
1	.18
2	.25
3	.38
4	.34

Dykstra-Parsons Method:

Initial gas saturation = 0.0

Initial oil saturation = .8

Initial water saturation = .2

Residual oil saturation = .2

Connate water saturation = .2

Reservoir length = 1400 feet

Reservoir thickness = 25 feet

Reservoir width = 200 feet

β_o = 1.65

μ_w = .75

μ_o = 1.47

μ_w = 1.0

k_{rw} at the residual oil saturation = .316

k_{ro} at the connate water saturation = .75

Layer's Thickness feet	Permeability md
4	180
6	250
5	380
10	340

These permeability variations were plotted versus cumulative thickness as in Figure (8) and rearranged for equal intervals of one foot as shown in Table II.

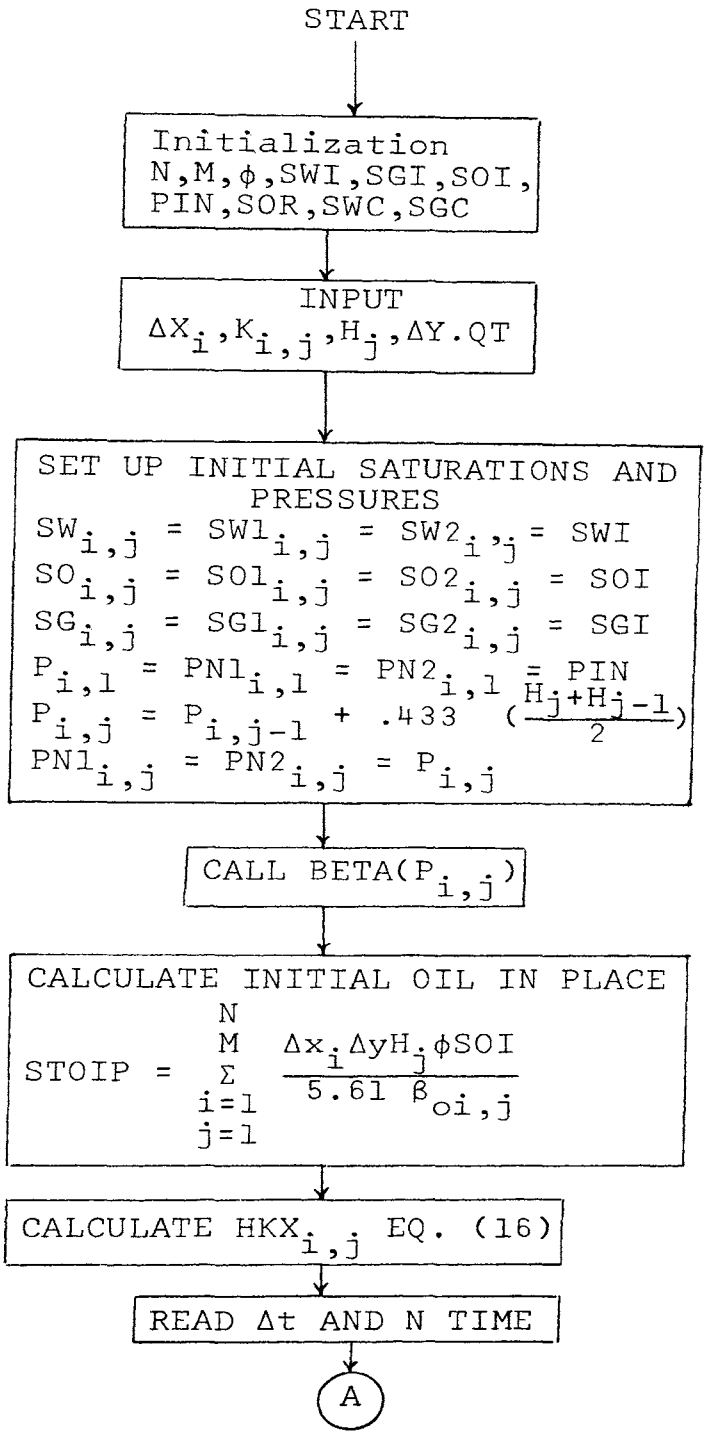
TABLE II. Rearranged Permeabilities

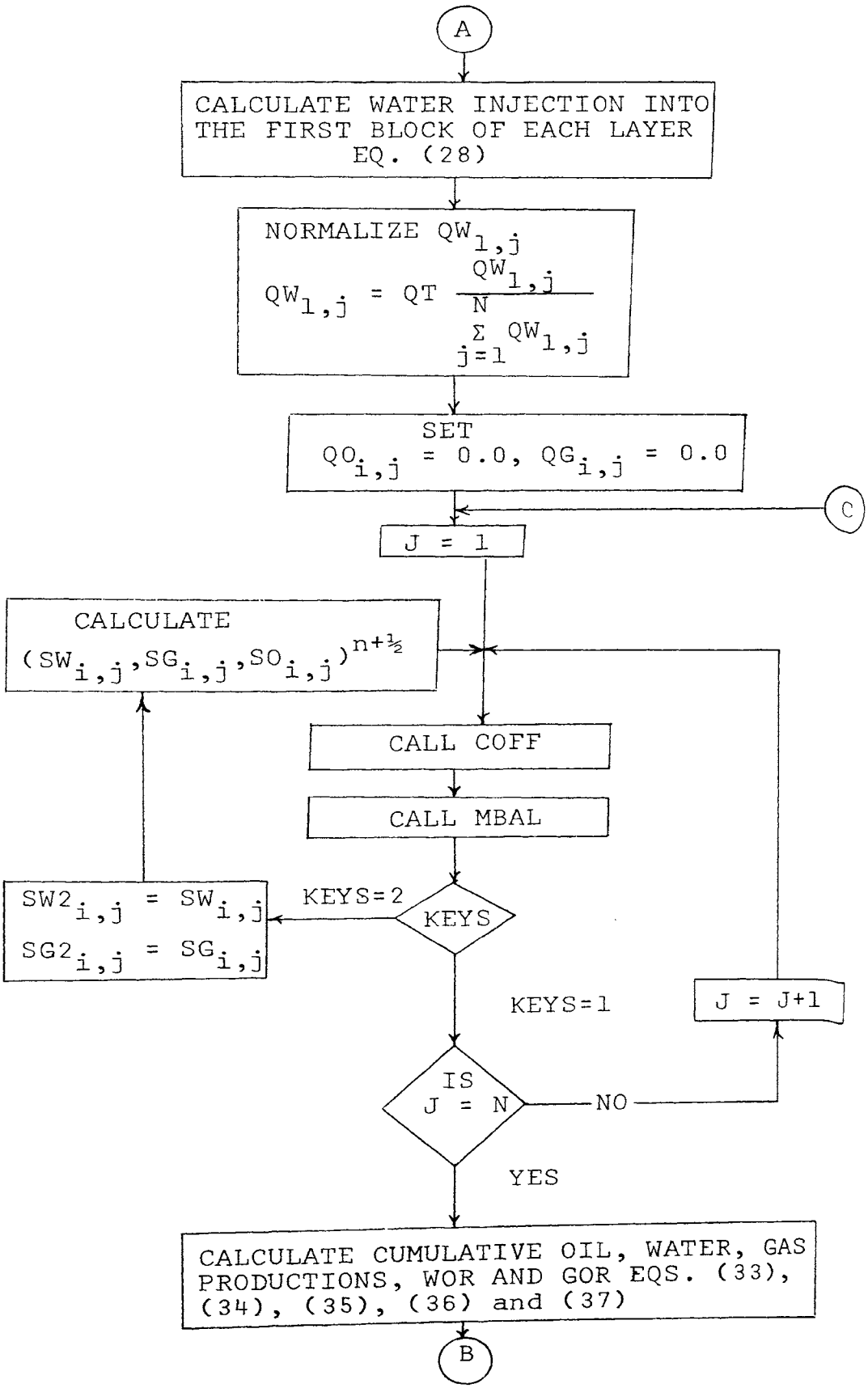
Cumulative Thickness feet	Permeability From Fig. 8 md.	Rearranged Permeability md.
1	170	398
2	172	398
3	175	397
4	180	395
5	183	388
6	189	385
7	200	380
8	212	376
9	230	365
10	250	358
11	272	350
12	298	340
13	327	327
14	358	298
15	380	272
16	395	250
17	398	230
18	398	212
19	397	200
20	388	189
21	385	183
22	376	180
23	365	175
24	350	172
25	340	170

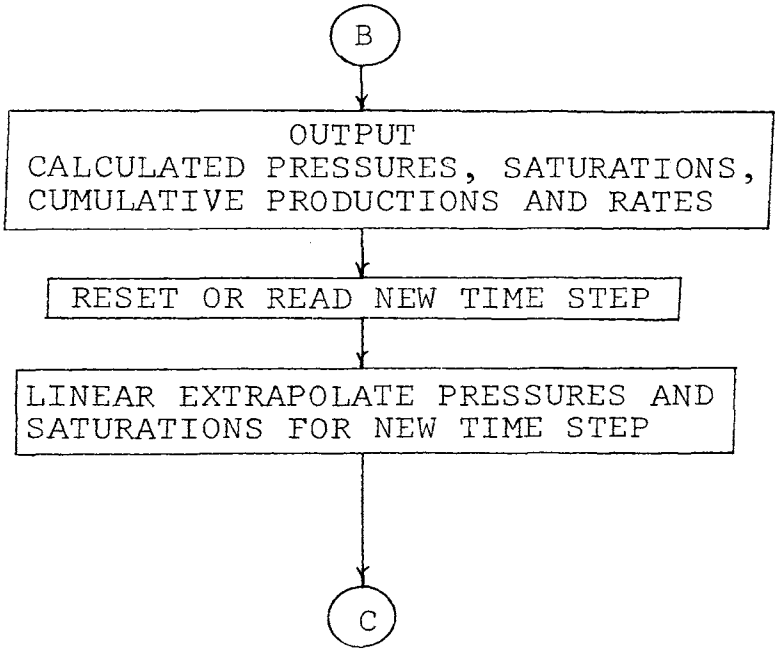
APPENDIX D

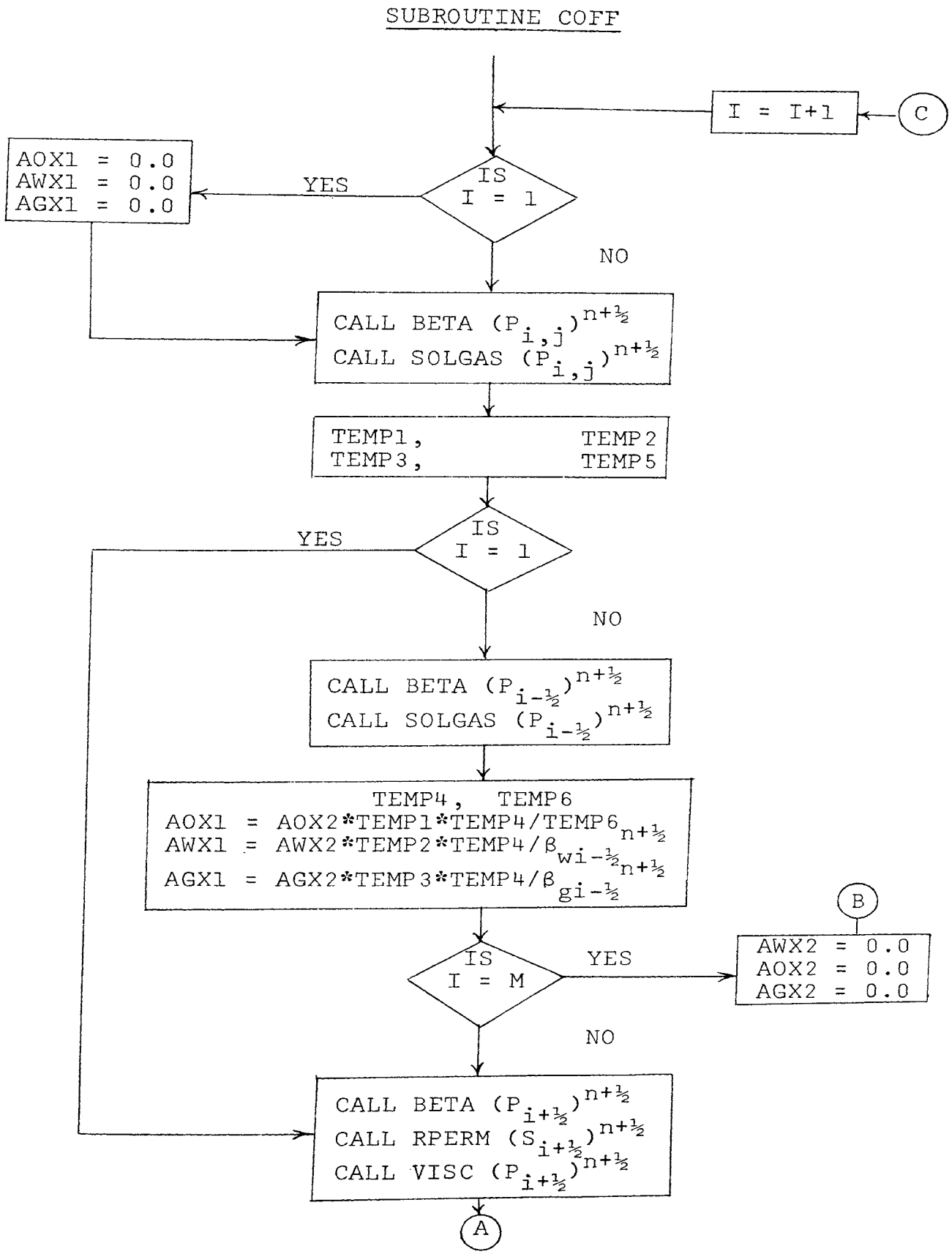
1. COMPUTER FLOW DIAGRAM FOR NUMERICAL MODEL

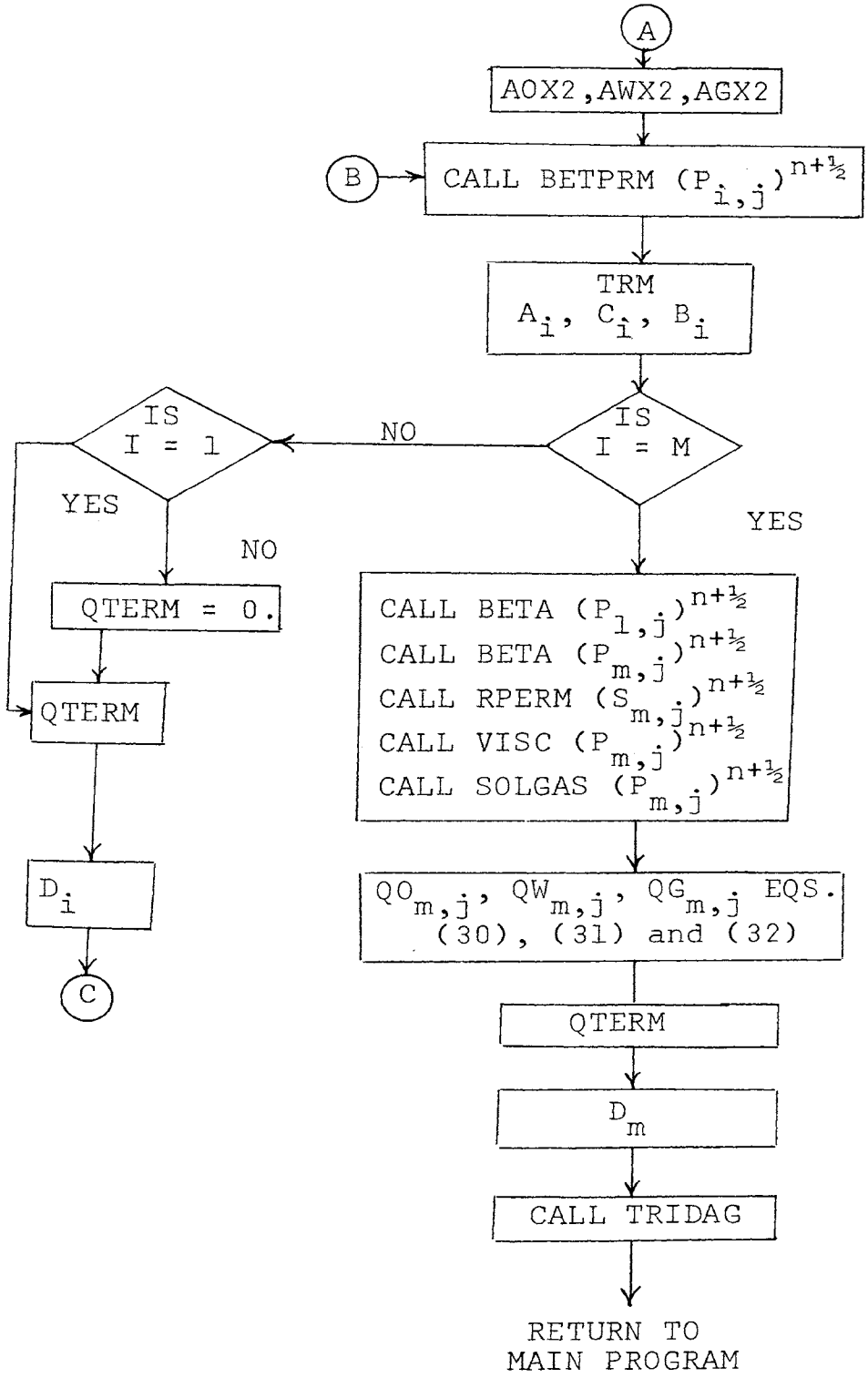
MAIN PROGRAM

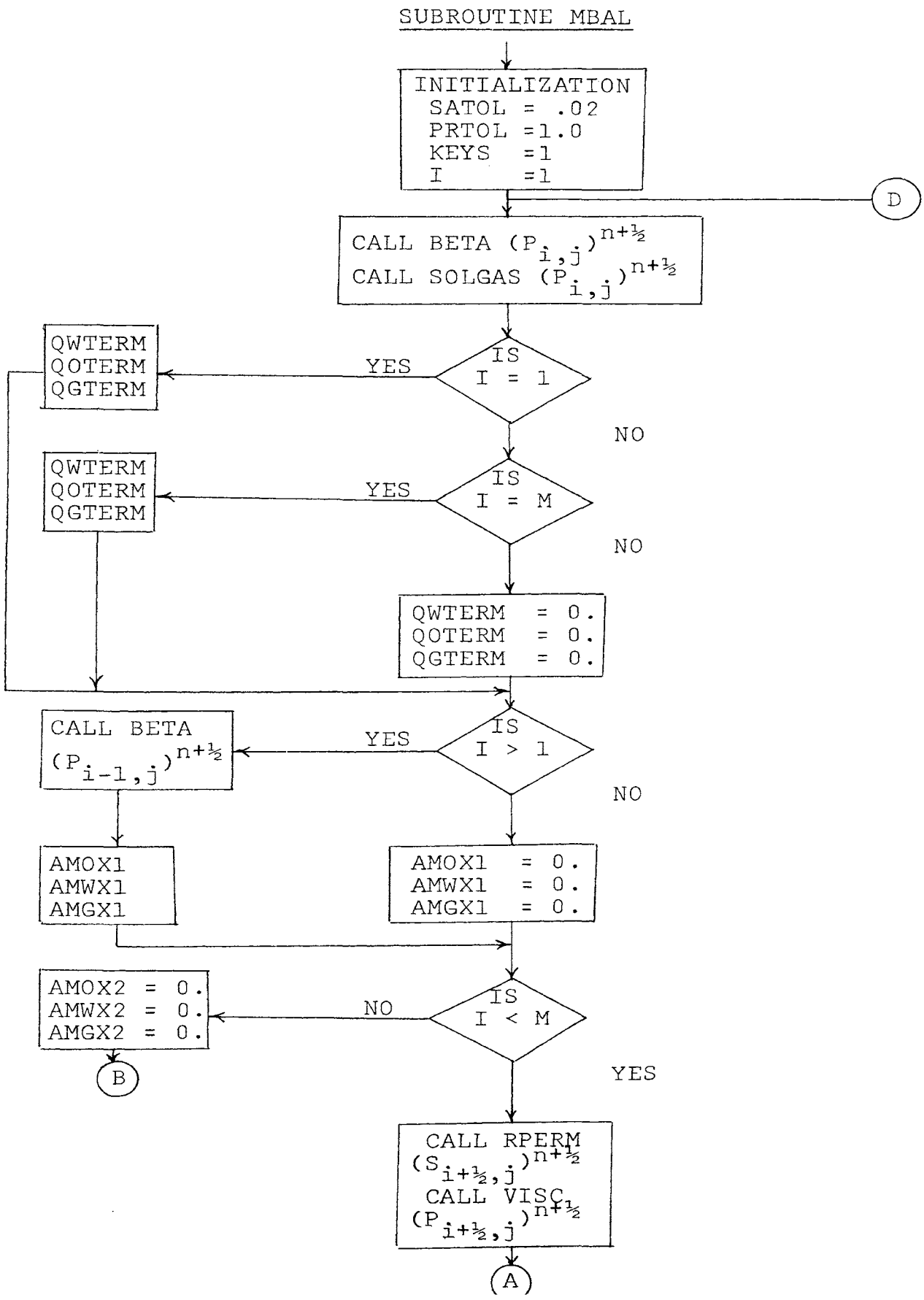


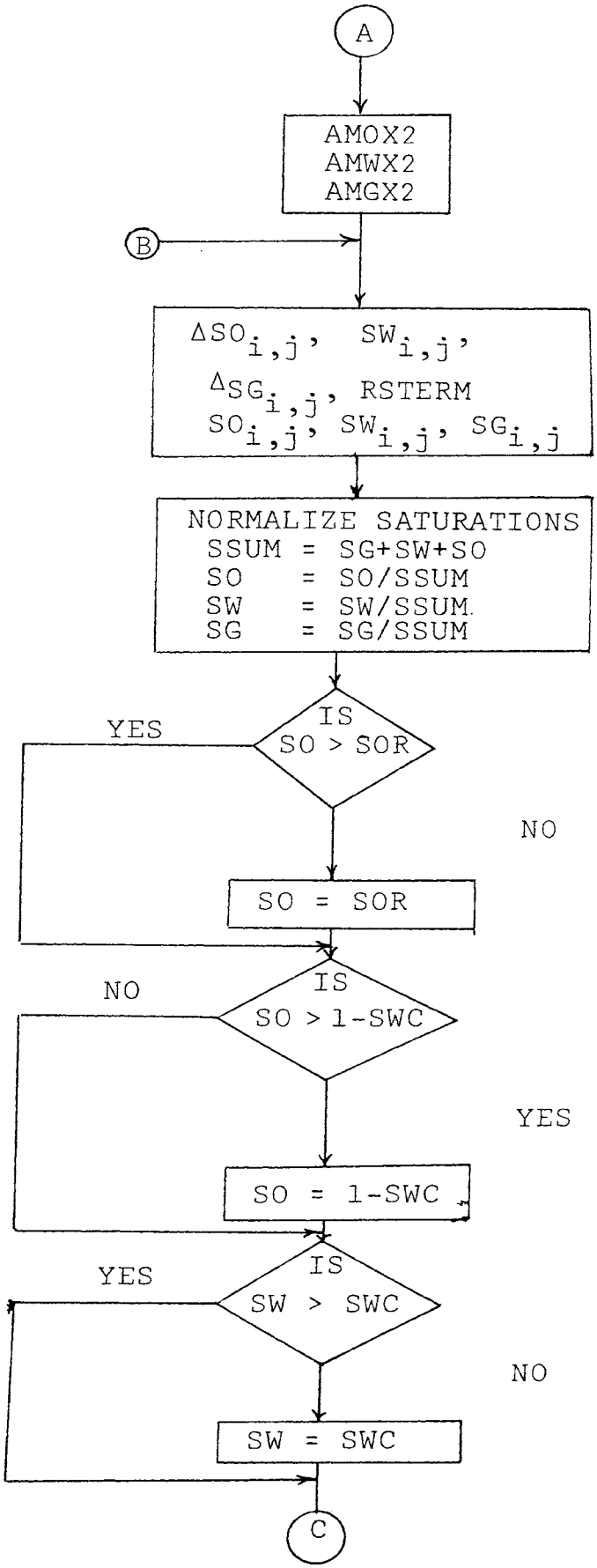

A
CALCULATE WATER INJECTION INTO THE FIRST BLOCK OF EACH LAYER EQ. (28)
NORMALIZE $QW_{1,j}$
$QW_{1,j} = QT \frac{QW_{1,j}}{\sum_{j=1}^{N} QW_{1,j}}$
SET $QO_{i,j} = 0.0$, $QG_{i,j} = 0.0$
C
J = 1
CALCULATE $(SW_{i,j}, SG_{i,j}, SO_{i,j})^{n+1/2}$
CALL COFF
CALL MBAL
KEYS
KEYS=2
$SW2_{i,j} = SW_{i,j}$
$SG2_{i,j} = SG_{i,j}$
KEYS=1
J = J+1
IS J = N
NO
YES
CALCULATE CUMULATIVE OIL, WATER, GAS PRODUCTIONS, WOR AND GOR EQS. (33), (34), (35), (36) and (37)
B

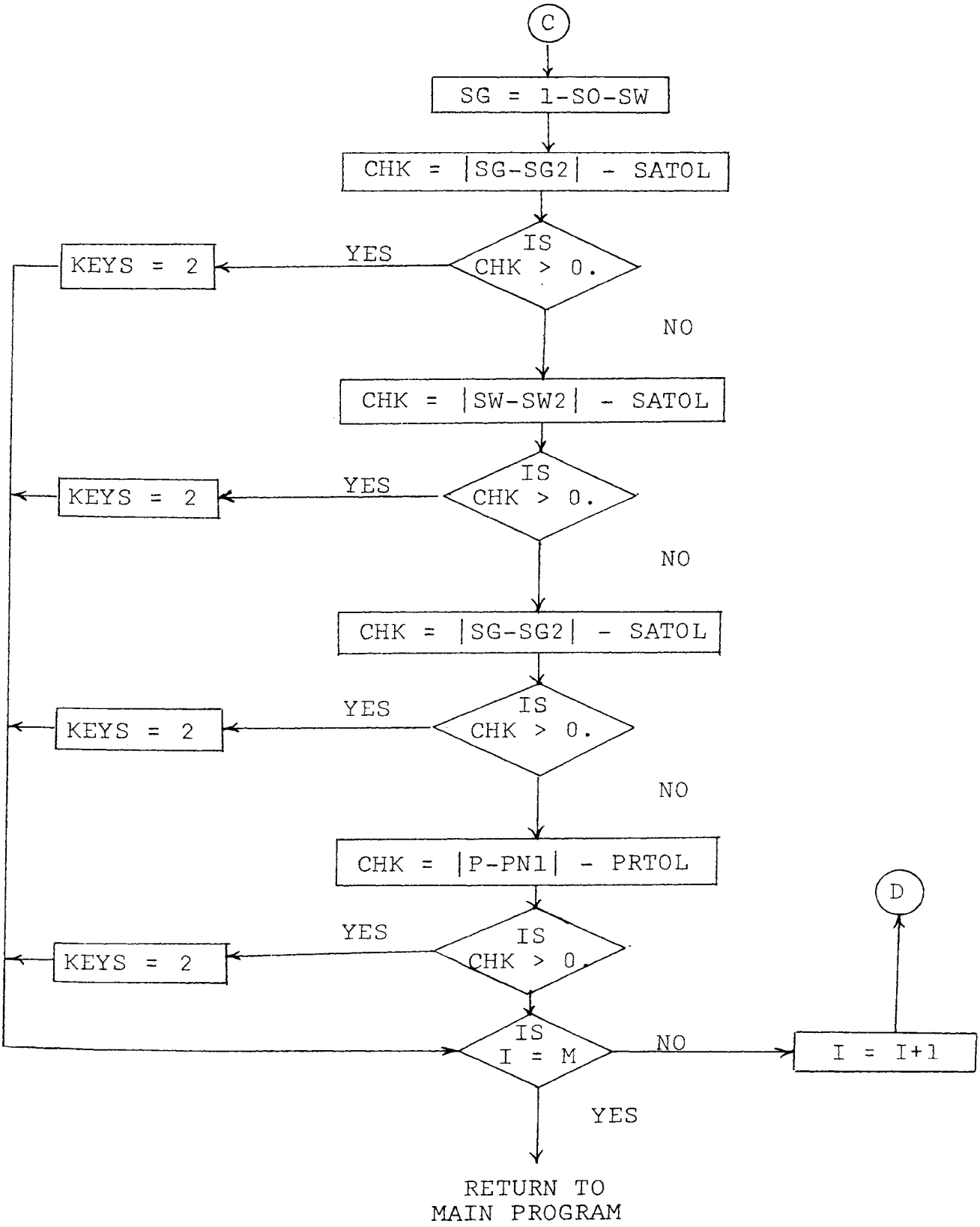

B
OUTPUT
CALCULATED PRESSURES, SATURATIONS,
CUMULATIVE PRODUCTIONS AND RATES
RESET OR READ NEW TIME STEP
LINEAR EXTRAPOLATE PRESSURES AND
SATURATIONS FOR NEW TIME STEP
C

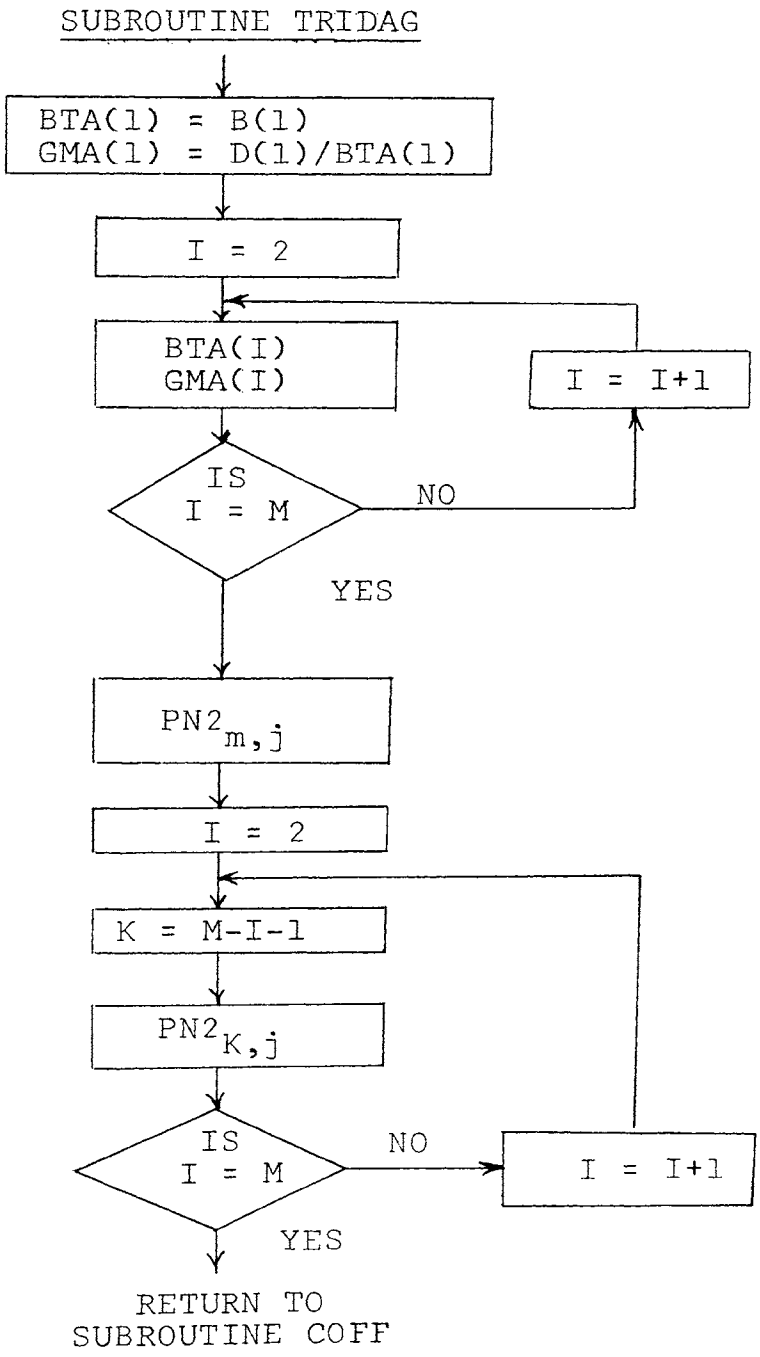

SUBROUTINE COFF
I = I+1
C
AOX1 = 0.0
AWX1 = 0.0
AGX1 = 0.0
YES
IS
I = 1
NO
CALL BETA (P i,j) n+½
CALL SOLGAS (P i,j) n+½
TEMP1, TEMP2
TEMP3, TEMP5
YES
IS
I = 1
NO
CALL BETA (P i-½) n+½
CALL SOLGAS (P i-½) n+½
TEMP4, TEMP6
AOX1 = AOX2*TEMP1*TEMP4/TEMP6 n+½
AWX1 = AWX2*TEMP2*TEMP4/β wi-½ n+½
AGX1 = AGX2*TEMP3*TEMP4/β gi-½
B
AWX2 = 0.0
AOX2 = 0.0
AGX2 = 0.0
IS
I = M
YES
NO
CALL BETA (P i+½) n+½
CALL RPERM (S i+½) n+½
CALL VISC (P i+½) n+½
A


A
AOX2,AWX2,AGX2
B
CALL BETPRM ($P_{i,j}$)$^{n+\frac{1}{2}}$
TRM
A_i, C_i, B_i
IS I = M
NO
IS I = 1
YES
NO
YES
QTERM = 0.
QTERM
D_i
C
CALL BETA ($P_{1,j}$)$^{n+\frac{1}{2}}$
CALL BETA ($P_{m,j}$)$^{n+\frac{1}{2}}$
CALL RPERM ($S_{m,j}$)$^{n+\frac{1}{2}}$
CALL VISC ($P_{m,j}$)$^{n+\frac{1}{2}}$
CALL SOLGAS ($P_{m,j}$)$^{n+\frac{1}{2}}$
$QO_{m,j}$, $QW_{m,j}$, $QG_{m,j}$ EQS. (30), (31) and (32)
QTERM
D_m
CALL TRIDAG
RETURN TO MAIN PROGRAM

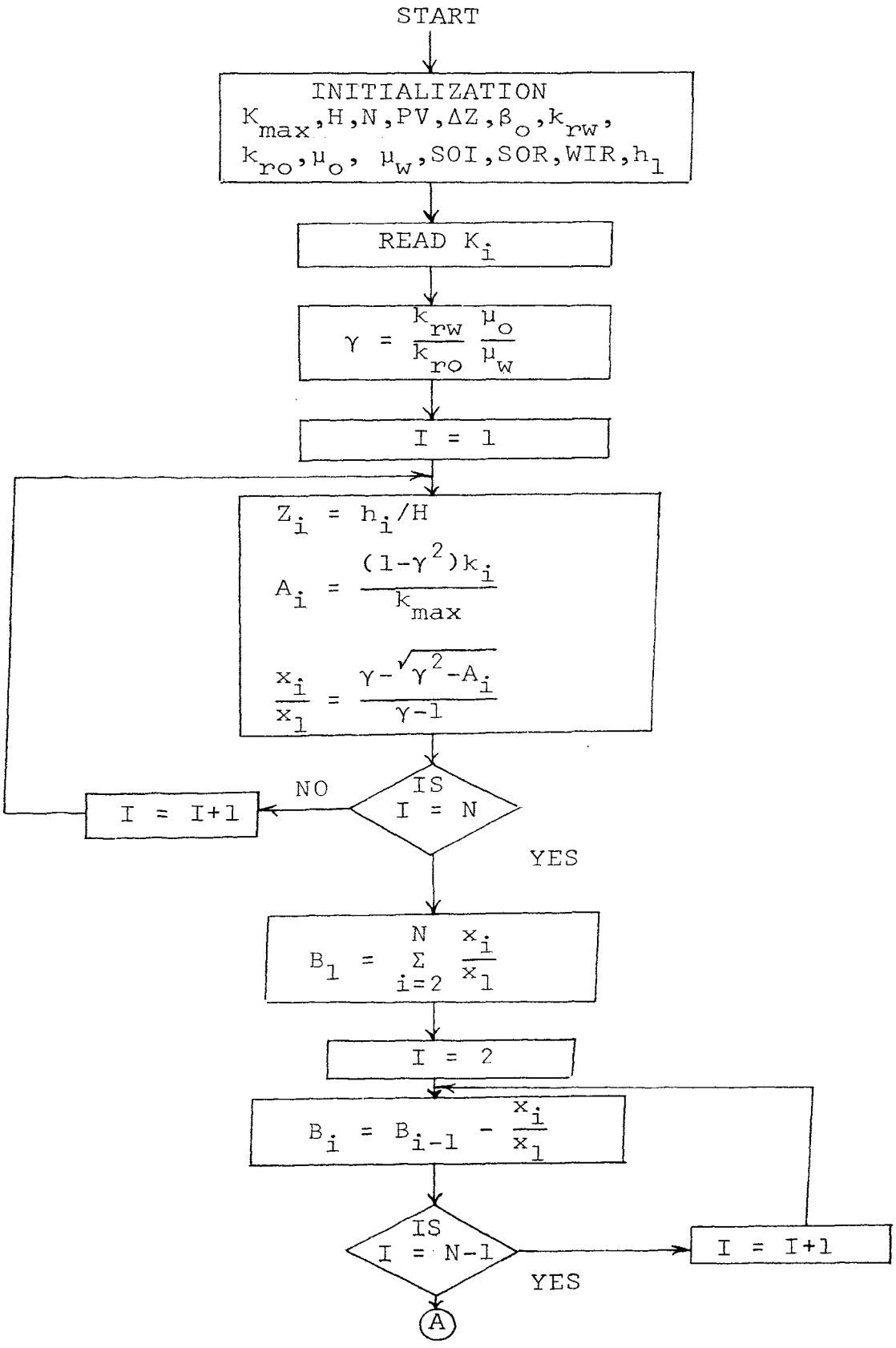

SUBROUTINE MBAL
INITIALIZATION
SATOL = .02
PRTOL = 1.0
KEYS = 1
I = 1
D
CALL BETA $(P_{i,j})^{n+\frac{1}{2}}$
CALL SOLGAS $(P_{i,j})^{n+\frac{1}{2}}$
IS I = 1
YES
QWTERM
QOTERM
QGTERM
NO
IS I = M
YES
QWTERM
QOTERM
QGTERM
NO
QWTERM = 0.
QOTERM = 0.
QGTERM = 0.
IS I > 1
YES
CALL BETA $(P_{i-1,j})^{n+\frac{1}{2}}$
NO
AMOX1
AMWX1
AMGX1
AMOX1 = 0.
AMWX1 = 0.
AMGX1 = 0.
IS I < M
NO
AMOX2 = 0.
AMWX2 = 0.
AMGX2 = 0.
B
YES
CALL RPERM $(S_{i+\frac{1}{2},j})^{n+\frac{1}{2}}$
CALL VISC $(P_{i+\frac{1}{2},j})^{n+\frac{1}{2}}$
A

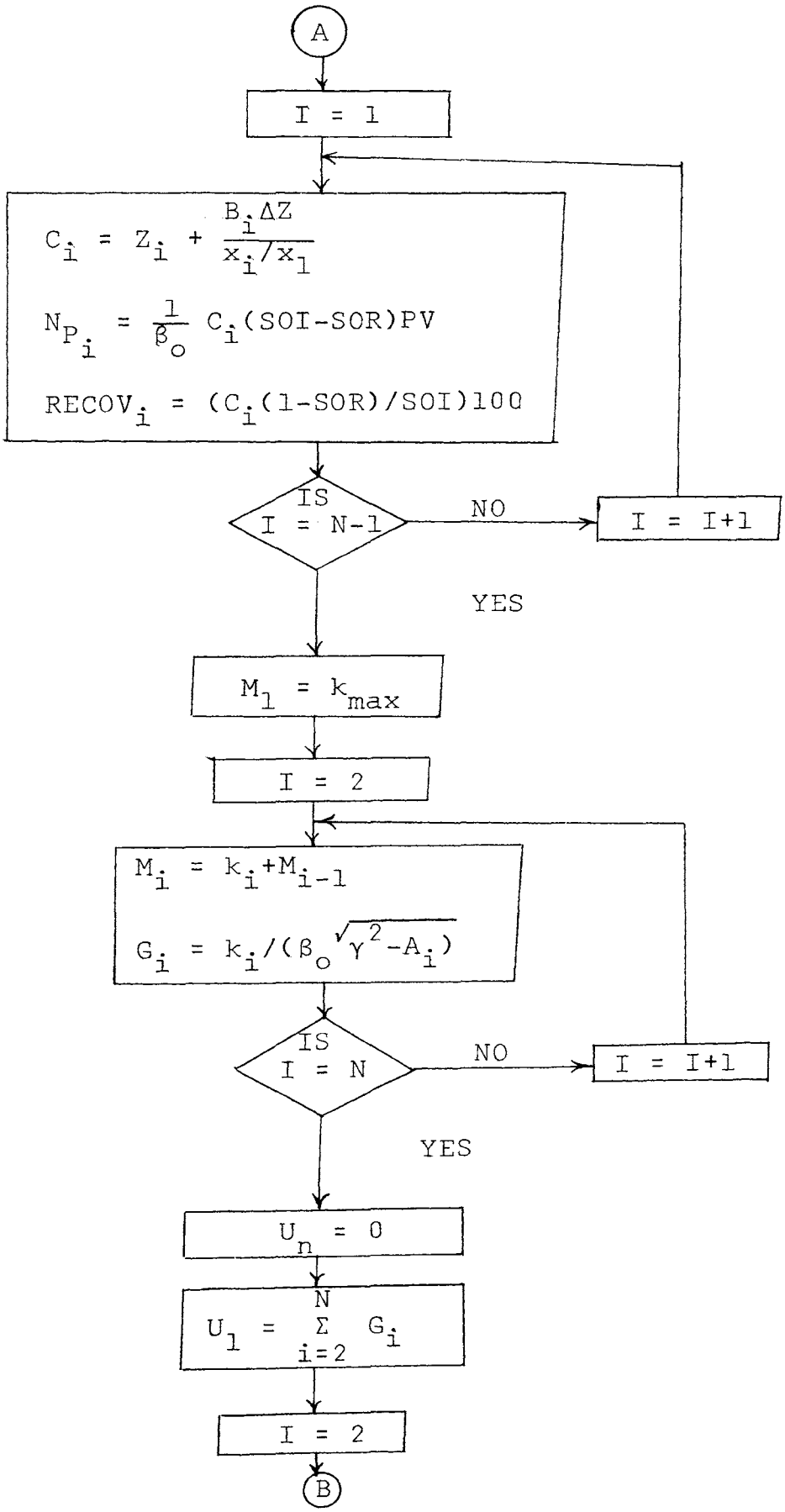

A
AMOX2
AMWX2
AMGX2
B
$\Delta SO_{i,j}$, $SW_{i,j}$,
$\Delta SG_{i,j}$, RSTERM
$SO_{i,j}$, $SW_{i,j}$, $SG_{i,j}$
NORMALIZE SATURATIONS
SSUM = SG+SW+SO
SO = SO/SSUM
SW = SW/SSUM
SG = SG/SSUM
IS SO > SOR
YES
NO
SO = SOR
IS SO > 1-SWC
NO
YES
SO = 1-SWC
IS SW > SWC
YES
NO
SW = SWC
C

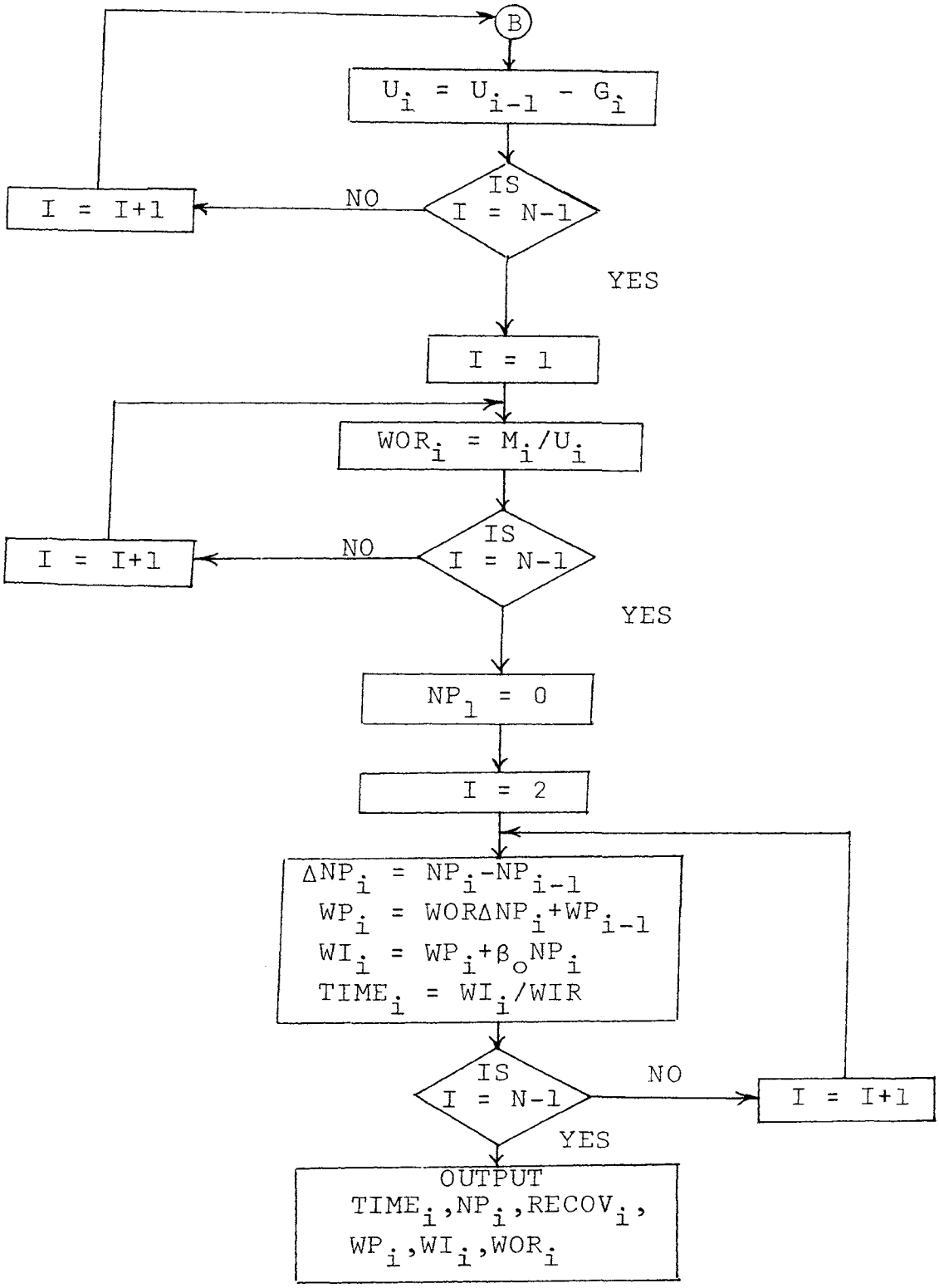

C
SG = 1-SO-SW
CHK = |SG-SG2| - SATOL
IS CHK > 0.
YES
KEYS = 2
NO
CHK = |SW-SW2| - SATOL
IS CHK > 0.
YES
KEYS = 2
NO
CHK = |SG-SG2| - SATOL
IS CHK > 0.
YES
KEYS = 2
NO
CHK = |P-PN1| - PRTOL
IS CHK > 0.
YES
KEYS = 2
IS I = M
NO
I = I+1
D
YES
RETURN TO MAIN PROGRAM


SUBROUTINE TRIDAG
BTA(1) = B(1)
GMA(1) = D(1)/BTA(1)
I = 2
BTA(I)
GMA(I)
I = I+1
IS
I = M
NO
YES
PN2m,j
I = 2
K = M-I-1
PN2K,j
IS
I = M
NO
I = I+1
YES
RETURN TO
SUBROUTINE COFF

2. COMPUTER FLOW DIAGRAM FOR DYKSTRA-PARSONS METHOD




A
I = 1
$C_i = Z_i + \frac{B_i \Delta Z}{x_i/x_1}$
$N_{P_i} = \frac{1}{\beta_o} C_i(SOI-SOR)PV$
$RECOV_i = (C_i(1-SOR)/SOI)100$
IS I = N-1
NO
I = I+1
YES
$M_1 = k_{max}$
I = 2
$M_i = k_i + M_{i-1}$
$G_i = k_i/(\beta_o \sqrt{\gamma^2 - A_i})$
IS I = N
NO
I = I+1
YES
$U_n = 0$
$U_1 = \sum_{i=2}^{N} G_i$
I = 2
B


B
$U_i = U_{i-1} - G_i$
IS
I = N-1
NO
I = I+1
YES
I = 1
$WOR_i = M_i/U_i$
IS
I = N-1
NO
I = I+1
YES
$NP_1 = 0$
I = 2
$\Delta NP_i = NP_i - NP_{i-1}$
$WP_i = WOR\Delta NP_i + WP_{i-1}$
$WI_i = WP_i + \beta_o NP_i$
$TIME_i = WI_i/WIR$
IS
I = N-1
NO
I = I+1
YES
OUTPUT
$TIME_i, NP_i, RECOV_i,$
WP_i, WI_i, WOR_i

3. COMPUTER PROGRAM FOR NUMERICAL MODEL

```
FORTRAN IV G LEVEL 1, MOD 4            MAIN         DATE = 69290          18/06/54
            C     ONE DIMENSIONAL, THREE PHASE, STRATIFID, PETROLEUM RESERVOIR
            C     SIMULATOR    VARIABLE LENGTH(DELT X), VARIABLE LAYERS THICKNES
            C     (H), HETROGENOUS PERMEABILITY IN EACH LAYER, NEGLETING GRAVITY
            C     FORCES, CAPILLARY PRESSERS AND ROCK COPERSIBILITY, NO FLOW BETWEEN
            C     LAYERS,    INJECTING FROM ONE END AND PRODUCING FROM THE OTHER
            C     END
            C
            C                              ***** MAIN PROGRAM *****
            C
0001              DIMENSION DELX(10),PERM(10,10),H(10),SO(10,10),SW(10,10),SG(10,10)
                 1,SO1(10,10),SO2(10,10),SW1(10,10),SW2(10,10),SG1(10,10),SG2(10,10)
                 1,P(10,10),PN1(10,10),PN2(10,10),HKX(10,10),QO(10,10),QW(10,10),
                 1QG(10,10),PN3(10,10)
0002              N=4
0003              M=8
0004              TIME=0.0
0005              ITCNT=0
0006              CUMOP=0.0
0007              CUMWP=0.0
0008              SWC=.2
0009              NN=N-1
0010              PHI=.20
0011              SWI=.2
0012              SGI=0.0
0013              SOI=.80
0014              MM=M-1
0015              KCOUNT=0
            C
0016              READ(1,90)(DELX(I),I=1,M)
0017              READ(1,100)((PERM(I,J),I=1,M),J=1,N)
0018              READ(1,102)(H(I),I=1,N)
0019              DELY=200.
0020              PIN=3487.
            C
```

```
                C     SET UP INITIAL SATURATIONS
                C
0021                  DO 98 I=1,M
0022                  DO 98 J=1,N
0023                  SW(I,J)=SWI
0024                  SG(I,J)=SGI
0025                  SO(I,J)=SOI
0026                  SW1(I,J)=SW(I,J)
0027                  SW2(I,J)=SW(I,J)
0028                  SO1(I,J)=SO(I,J)
0029                  SO2(I,J)=(I,J)
0030                  SG1(I,J)=SG(I,J)
0031               98 SG2(I,J)=SG(I,J)
                C     SW1,SG1,SO1 ARE SATURATIONS AT N TIME LEVEL (OLD TIME LEVEL)
                C     SW2,SG2,SO2 ARE SATURATIONS AT N+1 TIME LEVEL AND K+1 ITERITIVE
                C
                C     SET UP INITIAL PRESSURES
                C
0032                  DO 97 I=1,M
0033                  P(I,1)=PIN
0034                  PN1(I,1)=P(I,1)
0035               97 PN2(I,1)=P(I,1)
0036                  DO 96 J=2,N
0037                  DO 96 I=1,M
0038                  P(I,J)=P(I,J-1)+.433* (H(J)+H(J-1))/2.
0039                  PN1(I,J)=P(I,J)
0040               96 PN2(I,J)=P(I,J)
                C     P IS PRESSURE AT N TIME LEVEL (OLD TIME LEVEL)
                C     PN1 IS PRESSURE AT N+1 TIME LEVEL AND K ITRITIVE
                C     PN2 IS PRESSURE AT N+1 TIME LEVEL AND K+1 ITERITIVE
                C
                C     OUTPUT INITIAL CONDITIONS
0041                  WRITE(3,90)(DELX(I),I=1,M)
0042                  WRITE(3,102)(H(I),I=1,N)
0043                  WRITE(3,100)((PERM(I,J),I=1,M),J=1,N)
0044                  WRITE(3,121) TIME
0045                  WRITE(3,241)
```

```
0046              WRITE(3,240)((PN2(I,J),I=1,M),J=1,N)
0047              WRITE (3,253)
0048              WRITE(3,251)((SO(I,J),I=1,M),J=1,N)
0049              WRITE(3,254)
0050              WRITE(3,251)((SW(I,J),I=1,M),J=1,N)
0051              WRITE(3,255)
0052              WRITE(3,251)((SG(I,J),I=1,M),J=1,N)
0053              DO 92 J=1,N
0054              DO 92 I=1,M
0055              P(I,J)=P(I,J)/14.7
0056              PN1(I,J)=PN1(I,J)/14.7
0057           92 PN2(I,J)=PN2(I,J)/14.7
          C
0058              STOIP=0.0
0059              DO 91 J=1,N
0060              DO 91 I=1,M
0061              CALL BETA (P(I,J),BTAO,BTAW,BTAG)
0062           91 STOIP=STOIP+DELX(I)*DELY*H(J) *PHI*SOI/(5.61*BTAO)
0063              WRITE(3,256)STOIP
0064              DO 93 I=1,M
0065           93 DELX(I)=DELX(I)*30.48
0066              DO 94 J=1,N
0067           94 H(J)=H(J)*30.48
0068              DELY=DELY*30.48
          C
          C       CALCULATE HKX
          C
0069              DO 1 J=1,N
0070              DO 2 I=1,MM
0071            2 HKX(I,J)=(2.*PERM(I,J)*PERM(I+1,J))/(DELX(I)*PERM(I+1,J)+DELX(I+1)
                  1*PERM(I,J))
          C
          C       READ TIME STEP
          C
0072              READ(1,108)DELT,NTIME
0073              TIME=TIME+DELT
0074              DELT=DELT*24.*3600.
```

```
0075            ITCNT=ITCNT+1
0076          897 CONTINUE
              C
0077            QT=1000.*5.615*30.48*30.48*30.48/(24.*3600.)
0078            SUMKH=0.0
0079            SUMQWI=0.0
              C     CALCULATE WATER INJECTION INTO THE FIRST BLOCK OF EACH LAYER
0080            PBIG=0.0
0081            DO 50 J=1,N
0082            IF(PBIG.LT.P(1,J)) PBIG=P(1,J)
0083           50 CONTINUE
0084            PBIG=PBIG+20.
0085            DO 899 J=1,N
0086          899 SUMKH=SUMKH+PERM(1,J)*H(J)*(PBIG-P(1,J))
0087            DO 898 J=1,N
0088            QW(1,J)=QT*PERM(1,J)*H(J)*(PBIG-P(1,J))/SUMKH
0089            SUMQWI=SUMQWI+QW(1,J)
0090            QO(1,J)=0.0
0091          898 QG(1,J)=0.0
0092            DO 901 J=1,N
0093            DO 901 I=2,MM
0094            QW(I,J)=0.0
0095            QO(I,J)=0.0
0096          901 QG(I,J)=0.0
0097            DO 892 J=1,N
0098          892 QW(1,J)=QT*(SW(1,J)/SUMQWI)
              C
0099           53 CONTINUE
0100            J=1
0101            NCOUNT=0
0102           52 CONTINUE
0103            CALL COFF(SW1,SW2,SG1,SG2,HKX,P,PN1,PN2,M,MM,DELT,DELY,DELX,QO,1QW,QG,
                PHI,H,J,SO,SW,SG,PN3)
              C
0104            CALL MBAL(PHI,DELX,DELY,HKX,QO,QW,QG,SO,SW,SG,PN2,P,DELT,H,SO1,1SO2,
                SW1,SW2,SG1,SG2,M,KEYS,J,PN1)
0105            NCOUNT=NCOUNT+1
```

```
0106              GO TO (55,59), KEYS
0107           59 CONTINUE
0108              IF(NCOUNT.GT.2) GO TO 55
0109              DO 60 I=1,M
0110              SW2(I,J)=SW(I,J)
0111              SW(I,J)=.5*(SW1(I,J)+SW2(I,J))
0112              SG2(I,J)=SG(I,J)
0113              SG(I,J)=.5*(SG1(I,J)+SG2(I,J))
0114              SO(I,J)=.5*(SO(I,J)+1.-SW1(I,J)-SG1(I,J))
0115           60 PN1(I,J)=PN2(I,J)
0116              GO TO 52
          C
0117           55 IF(J.EQ.N) GO TO 56
0118              J=J_1
0119              NCOUNT=0
0120              GO TO 52
0121           56 CONTINUE
0122              WRITE(3,121)TIME
          C
          C       CALCULATE OIL,WATER AND GAS PRODUCTIONS,WOR,GOR AND OIL RECOVERY
          C
0123              SUMQO=0.0
0124              SUMQW=0.0
0125              SUMQG=0.0
0126              DO 63 J=1,N
0127              SUMQO=SUMQO+QO(M,J)
0128              RQO=SUMQO*24.*3600./(30.48*30.48*30.48*5.615)
0129              SUMQW=SUMQW+QW(M,J)
0130              RQW=SUMQW*24.*3600./*30.48*30.48*30.48*5.615)
0131              CALL SOLGAS(P(M,J),RS1)
0132              CALL SOLGAS(PN2*M,J),RS2)
0133           63 SUMQG=SUMQG_QG(M,J)+QO(M,J)*(RS1+RS2)/2.
0134              IF(SUMQO.FQ.0.0) GO TO 58
0135              WOR=SUMQW/SUMQO
0136              GOR=5.615*SUMQG/SUMQO
0137              GO TO 54
0138           58 WOR=0.0
```

```
0139                GOR=0.0
0140             54 CONTINUE
0141                CUMOP=CUMOP+SUMQO*DELT/(30.48*30.48*30.48*5.615)
0142                CUMWP=CUMWP+SUMQW*DELT/(30.48*30.48*30.48*5.615)
0143                RECOV=100.*CUMOP/STOIP
0144                CUMWIN=QT*TIME*24.*3600./(30.48*30.48*30.48*5.615)
          C
0145                WRITE(3,241)
0146                WRITE(3,240)((PN3(I,J),I=1,M),J=1,N)
0147                WRITE(3,253)
0148                WRITE(3,251)((SO(I,J),I=1,M),J=1,N)
0149                WRITE(3,254)
0150                WRITE(3,251)((SW(I,J),I=1,M),J=1,N)
0151                WRITE(3,255)
0152                WRITE(3,251)((SG(I,J),I=1,M),J=1,N)
0153                WRITE(3,257) RQO,RQW
0154                WRITE(3,252)CUMOP,CUMWP,RECOV,CUMWIN,WOR,GOR
0155                IE(WOR.GT.20.) GO TO 99
0156                KCOUNT=KCOUNT+1
             C
             C      RESET OR READ TIME STEP
             C
0157                DELTN1=DELT
0158                IF(ITCNT.LT.NTIME) GO TO 57
0159                ITCNT=0
0160                READ(1,108,END=99) DELT,NTIME
0161                DELT=DELT*24.*3600.
0162             57 ITCNT=ITCNT+1
0163                TIME=TIME+DELT/(24.*3600.)
             C
             C      LINEAR INTERPOLATION FOR PRESSURES AND SATURATIONS FOR THE NEXT STEP TO
             C      ACCELERATE CONVERGENCE
             C
0164                DO 30 J=1,N
0165                DO 30 I=1,M
0166                SW2(I,J)=SW(I,J)+(SW(I,J)-SW1(I,J))/DELTN1*DELT
0167                SW1(I,J)=SW(I,J)
```

```
0168            SG2(I,J)=SG(I,J)+(SG(I,J)=SG1(I,J))/DELTN1*DELT
0169            SG1(I,J)=SG(I,J)
0170            SW(I,J)=.5*(SW1(I,J)+SW2(I,J))
0171            SG(I,J)=.5*(SG1(I,J)+SG2(I,J))
0172            SO(I,J)+1.=SW(I,J)-SG(I,J)
0173            PN1(I,J)=PN2(I,J)+(PN2(I,J)-P(I,J))/DELTN1*DELT
0174            P(I,J)=PN2(I,J)
0175         30 PN2(I,J)=PN1(I,J)
0176            IF(KCOUNT.GT.60) GO TO 500
0177            GO TO 53
0178        500 TIME=TIME-DELT/(24.*3600.)
0179            WRITE(2,502)((P(I,J),I=1,M),J=1,N)
0180            WRITE(2,502)((PN1(I,J),I=1,M),J=1,N)
0181            WRITE(2,502)((PN2(I,J),I=1,M),J=1,N)
0182            WRITE(2,503) TIME,DUMOP,CUMWP
0183            WRITE(2,504)((SW(I,J),I=1,M),J=1,N)
0184            WRITE(2,504)((SW1(I,J),I=1,M),J=1,N)
0185            WRITE(2,504)((SW2(I,J),I=1,M),J=1,N)
0186            WRITE(2,504)((SO(I,J),I=1,M),J-1,N)
0187            WRITE(2,504)((SO2(I,J),I=1,M),J=1,N)
0188            WRITE(2,504)((SG(I,J),I-1,M),J=1,N)
0189            WRITE(2,504)((SG1(I,J),I=1,M),J=1,N)
0190            WRITE(2,504)((SG2(I,J),I=1,M),J=1,N)
0191         99 STOP
0192         90 FORMAT(8F8.2)
0193        100 FORMAT(8F8.2)
0194        102 FORMAT(8F8.2)
0195        108 FORMAT(F10.5,14)
0196        121 FORMAT(/,40X,'*****TIME ELAPSED=',F16.7,'DAYS*****',//)
0197        240 FORMAT(20X,8F10.1)
0198        241 FORMAT(60X,'PRESSURE IN PSI',//)
0199        251 FORMAT(20X,8F10.4)
0200        253 FORMAT(//,60X,'OIL SATURATIONS',//)
0201        254 FORMAT(//,60X,'WATER SATURATIONS',//)
0202        255 FORMAT(//,60X,'GAS SATURATIONS',//)
0203        252 FORMAT(/,5X,'CUMULATIVE OIL PRODUCTION = ',F18.5,'STB',10X,'CUMLATIVE
               1WATER PRODUCTION = ',F18.5,'STB',/,15X,'PERCENT RECOVERY = 'F10.4,10X,
```

```
     1'CUMULATIVE WATER INJECTED = ',F18.5,'STB',/,5X,'WOR=',F10.4,10X,
     1'GOR = ',F10.4,//)
0204      256 FORMAT(/,10X,'ORIGINAL OIL IN PLACE',F16.2,'STB',//)
0205      257 FORMAT(/,5X,'OIL PRODUCTION RATE = ',F18.5,'STB/DAY',5X,'WATER
     1PRODUCTION RATE = ',F18.5,'STB/DAY')
0206      502 FORMAT(8F8.2)
0207      503 FORMAT(3F20.6)
0208      504 FORMAT(8F8.5)
0209          END
```

```
      C                    *****CALCULATION OF COEFFICIENT*****
0001        SUBROUTINE COFF(SW1,SW2,SG1,SG2,HKX,P,PN1,PN2,M,MM,DELT,DELY,DELX,QO,
           1SQ,QG,PHI,H,J,SO,SW,SG,PN3)
0002        DIMENSION SW1(10,10),SW2(10,10),SG1(10,10),SG2(10,10),HKX(10,10),
           1P(10,10),PN1(10,10),PN2(10,10),DELX(10),QO(10,10),SQ(10,10),QG(10,120),
           1AX(10),BX(10),CX(10),DX(10),SO(10,10),SW(10,10),SG(10,10),H(10),PN3
           1(10,10)
0003        CW=.0000045
0004        SOR=.20
0005        SWC-.20
      C
      C     AT THIS POINT SW,SG,SO ARE TIME AVERAGED SATURATIONS BETWEEN N AND N+1
      C     TIME LEVEL
      C
0006        DO 50 I=1,M
0007        IF(I.EQ.1) GOTO 10
0008       2 PX=.5*(P(I,J)+PN2(I,J))
0009        CALL BETA (PX,BTAO,BTAW,BTAG)
0010        CALL SOLGAS(PX,RS)
0011        TEMP1=BTAO-BTAG*RS
0012        TEMP2=BTAW
0013        TEMP3=BTAG
0014        TEMP5=BTAO
0015        IF(I.EQ.1) GO TO 8
0016       3 PX=.5*(P(I-1,J)+PN2(I,J))
0017        CALL BETA(PX,BTAO,BTAW,BTAG)
0018        CALL SOLGAS(PX,RS)
0019        TEMP6=BTAO-BTAG*RS
0020        TEMP4=DELX(I-1)/DELX(I)
0021        AOX1=AOX2*TEMP1*TEMP4/TEMP6
0022        AWX1=AWX2*TEMP2*TEMP4/BTAW
0023        AGX1=AGX2*TEMP3*TEMP4/BTAG
      C
0024        IF(I.EQ.M) GO TO 30
      C
0025       8 PX=.25*(P(I,J)+P(I+1,J)+PN2(I,J)_PN2(I+1,J))
0026        SWA=.5*(SW(I,J)+SW(I+1,J))
```

```
0027              SGA=.5*(SG(I,J)+SG(I+1,J))
0028              SOA=.5*(SO(I,J)+SO(I+1,J))
0029              IF(SO(I,J).LE.SOR) SOA=0.0
0030              CALL BETA(PX,BTAO,BTAW,BTAG)
0031              CALL RPERM(SOA,SWA,SGA,RKO,RKW,RKG)
0032              CALL VISC (PX,XMUO,XMUW,XMUG)
0033              AOX2=HKX(I,J)/DELX(I)*TEMP1/BTAO*RKO/XMUO
0034              AWX2=HKX(I,J)/DELX(I)*TEMP2/BTAW*RKW/XMUW
0035              AGX2=HKX(I,J)/DELX(I)*TEMP3/BTAG*RKG/XMUG
0036              GO TO 11
                C
0037           10 AOX1=0.0
0038              AWX1=0.0
0039              AGX1=0.0
0040              GO TO 2
                C
0041           30 AOX2=0.0
0042              AWX2-0.0
0043              AGX2=0.0
                C
0044           11 CONTINUE
0045              PX=(P(I,J)+PN1(I,J))*.5
0046              CALL BETPRM (PX,BOPRM,BGPRM,RSPRM)
0047              TRM=-SW(I,J)*CW+SG(I,J)*BGPRM/TEMP3+SO(I,J)*(BOPRM-RSPRM*TEMP3)/TEMP5
0048              TRM=PHI*TRM/DELT
0049              AX(I)=AOX1+AWX1+AGX1
0050              CX(I)=AOX2+AWX2+AGX2
0051              BX(I)=-AX(I)-CX(I)+TRM
                C
0052              IF(I.EQ.M) GO TO 40
0053              IF(I.EQ.1) GO TO 12
                C
0054              QTERM=0.0
0055              GO TO 13
0056           12 QTERM=QG(I,J)*TEMP3+QO(I,J)*TEMP1+QW(I,J)*TEMP2
0057              QTERM=-QTERM/(DELX(I)*DELY*H(J))
                C
```

```
0058          13 DX(I)=QTERM+TRM*P(I,J)
0059             GO TO 50
              C
0060          40 PX=.5*(P(1,J)+PN2(1,J))
0061             CALL BETA(PX,BTAO,BTAW,BTAG)
0062             TEMP2=BTAW
0063             PX=.5*(P(M,J)+PN2(M,J))
0064             SOA=SO(M,J)
0065             SWA=SW(M,J)
0066             SGA-SG(M,J)
0067             CALL BETA(PX,BTAO,BTAW,BTAG)
0068             CALL RPERM(SOA,SWA,SGA,RKO,RKW,RKG)
0069             CALL VISC(PX,XMUO,XMUW,XMUG)
0070             CALL SOLGAS(PX,RS)
              C
0071             IF(RKO.LE.0.0) GO TO 41
0072             TEMP1=BTAO-BTAG*RS
0073             QO(M,J)=TEMP2*QW(1,j)/(1.+RKW/RKO*XMUO/XMUW+RKG/RKO*XMUO/XMUG)
0074             QO(M,J)=QO(M,J)/BTAO
0075             QW(M,J)=QO(M,J)*RKW*BTAO*XMUO/(RKO*BTAW*XMUW)
0076             QG(M,J)=QO(M,J)*RKG*BTAO*XMUO/(RKO*BTAG*XMUG)
0077             QTERM-QG(M,J)*BTAG+QO(M,J)*TEMP1+QW(M,J)*BTAW
0078             QTERM=QTERM/(DELX(M)*DELY*H(J))
              C
0079             DX(M)=QTERM+TRM*P(M,J)
              C
0080             GO TO 50
0081          41 QO(M,J)=0.0
0082             IF(RKG.LE.0.0) GO TO 42
0083             QG(M,J)=TEMP2*QW(1,J)/(1.+RKW/RKG*XMUG/XMUW)
0084             QG(M,J)=QG(M,J)/BTAG
0085             GO TO 43
0086          42 QG(M,J)=0.0
0087          43 QW(M,J)=(TEMP2*QW(1,J)-BTAG*QG(M,J))/BTAW
0088             QTERM=QG(M,J)*BTAG+QW(M,J)*BTAW
0089             QTERM-QTERM/(DELX(M)*DELY*H(J))
0090             DX(M)=QTERM+TRM*P(M,J)
```

```
      C
0091      50 CONTINUE
0092         CALL TRIDAG(AX,BX,CX,DX,PN2,M,J,PN3)
0093         RETURN
0094         END
```

```
      C
      C                    *****MATERIAL BALANCE CALCULATION*****
0001            SUBROUTINE MBAL(PHI,DELX,DELY,HKX,QO,QW,QG,SO,SW,SG,PN2,P,DELT,H,SO1,
               1SO2,SW1,SW2,SG1,SG2,M,KEYS,J,PN1)
0002            DIMENSION SW1(10,10),SW2(10,10),SW(10,10),SG1(10,10),SG2(10,10),
               1SG(10,10),HKX(10,10),P(10,10),PN2(10,10),PN1(10,10),DELX(10),QO(10,10),
               1QW(10,10),QG(10,10),H(10),SO(10,10),SO1(10,10),SO2(10,10)
      C
0003            SWC=.2
0004            SGC=.05
0005            SOR-.20
0006            SATOL=.02
0007            PRTOL=1.0
0008            KEYS=1
      C
0009            DO 100 I=1,M
0010            ZZ=-DELT/(DELX(I)*PHI)
0011            PX=(P(I,J)+PN2(I,J))*.5
0012            CALL BETA (PX,BTOI,BTWI,BTGI)
0013            CALL SOLGAS(P(I,J),RSN)
0014            CALL SOLGAS(PN2(I,J),RSNP1)
      C
      C         CALCULATE QWTERM,QOTERM,QGTERM
      C
0015            IF(I.EQ.1) GO TO 904
0016            IF(I.EQ.M) GO TO 905
      C
0017            QWTERM=0.0
0018            QOTERM=0.0
0019            QGTERM=0.0
0020            GO TO 900
0021          904 QWTERM=QW(I,J)*BTWI/(H(J)*DELY)
0022            QOTERM=0.0
0023            QGTERM=0.0
0024            GO TO 900
0025          905 AWTERM=-QW(M,J)*BTWI/(H(J)*DELY)
0026            QOTERM=-QO(M,J)*BTOI/(H(J)*DELY)
```

```
0027            QGTERM=-QG(M,J)*BTGI/(H(J)*DELY)
0028          900 CONTINUE
      C
0029            IF(I.GT.1) GO TO 5
      C
0030            AOX1=0.0
0031            AWX1=0.0
0032            AGX1=0.0
      C
0033            GO TO 7
      C
0034            5 PX=(P(I-1,J)+PN2(I-1,J))*.5
0035              CALL BETA(PX,BTAO,BTAW,BTAG)
0036              AOX1=(BTOI/BTAO)*AOX2
0037              AWX1=(BTWI/BTAW)*AWX2
0038              AGX1=(BTGI/BTAG)*AGX2
      C
0039            7 IF(I.LT.M) GO TO 8
      C
0040              AOX2-0.0
0041              AWX2=0.0
0042              AGX2=0.0
0043              GO TO 17
      C
0044            8 SWA=.5*(SW(I,J)+SW(I+1,J))
0045              SOA=.5*(SO(I,J)+SO(I+1,J))
0046              SGA=.5*(SG(I,J)+SG(I+1,J))
0047              IF(SO(I,J).LE.SOR) SOA=0.0
0048              CALL RPERM(SOA,SWA,SGA,RKO,RKW,RKG)
0049              PAV=.25*(P(I,J)+P(I+1,J)+PN2(I,J)+PN2(I+1,J))
0050              CALL VISC(PAV,XMJO,XMUW,XMUG)
0051              AOX2=HKX(I,J)*(RKO/XMUO)*(.5*(P(I+1,J)+PN2(I+1,J)-P(I,J)-PN2(I,J)))
0052              AGX2=HKX(I,J)*(RKG/XMUG)*(.5*(P(I+1,J)+PN2(I+1,J)-P(I,J)-PN2(I,J)))
0053              AWX2=HKX(I,J)*(RKW/XMUW)*(.5 (P(I+1,J)+PN2(I+1,J)-P(I,J)-PN2(I,J)))
       C
0054           17 DELSW=ZZ*(AWX1-AWX2-AWTERM)
0055              DELSO=ZZ*(AOX1-AOX2-QOTERM)
```

```
0056              DELSG=ZZ*(AGX1-AGX2-QGTERM)
                C
0057              RSTERM=SO(I,J)*(RSNP1-RSN)*(BTGI/BTOI)
0058              SG(I,J)=SG1(I,J)+DELSG-RSTERM
0059              SW(I,J)=SW1(I,J)+DELSW
0060              SO(I,J)=1.0-SW1(I,J)-SG1(I,J)+DELSO
                C
                C       SATURATION TOLERANCE CHECK
                C
0061              SSUM=SG(I,J)+SW(I,J)+SO(I,J)
0062              SG(I,J)=SG(I,J)/SSUM
0063              SW(I,J)=SW(I,J)/SSUM
0064              SO(I,J)=SO(I,J)/SSUM
0065              IF(SO(I,J).GT.(1.-SWC)) SO(I,J)=1.-SWC
0066              IF(SO(I,J).GE.SOR) GO TO 20
0067              SO(I,J)=SOR
0068           20 IF(I.EQ.1) GO TO 21
0069              IF(SW(I,J).GT.SW(I-1,J)) SW(I,J)=SW(I-1,J)
0070           21 IF(SW(I,J).GE.SWC) GO TO 22
0071              SW(I,J)=SWC
0072           22 SG(I,J)=1.0=SO(I,J)-SW(I,J)
0073              CHK=ABS(SG(I,J)-SG2(I,J))-SATOL
0074              IF(CHK.GT.0.) KEYS=2
0075              CHK=ABS(SW(I,J)-SW2(I,J))-SATOL
0076              IF(CHK.GT.0.) KEYS=2
0077              CHK=ABS(SO(I,J)+SG2(I,J)+SW2(I,J)-1.)-SATOL
0078              IF(CHK.GT.0.) KEYS=2
0079              CHK=ABS(PN2(I,J)-PN1(I,J))-PRTOL
0080              IF(CHK.GT.0.) KEYS=2
0081              IF(SG(I,J).GT.0.0) GO TO 100
0082              SG(I,J)=0.0
0083              SW(I,J)=1.-SO(I,J)
                C
0084          100 CONTINUE
0085              RETURN
0086              END
```

```
      C                    *****TRI-DIAGONAL SOLUTION*****
0001            SUBROUTINE TRIDAG(AX,BX,CX,DX,PN2,M,J,PN3)
0002            DIMENSION AX(10),BX(10),CX(10),DX(10),PN2(10,10),BTA(10),GMA(10),PN3
               1(10,10)
0003            BTA(1)=BX(1)
0004            GMA(1)=DX(1)/BTA(1)
0005            DO 10 I=2,M
0006            BTA(I)=BX(I)-AX(I)*CX(I-1)/BTA(I-1)
0007          10 GMA(I)=(DX(I)-AX(I)*GMA(I-1))/BTA(I)
0008            PN2(M,J)=GMA(M)
0009            PN3(M,J)=GMA(M)*14.7
0010            DO 20 I=2,M
0011            K=M0I+1
0012            PN2(K,J)=GMA(K)-(CX(K)*PN2(K+1,J)/BTA(K))
0013          20 PN3(K,J)=PN2(K,J)*14.7
0014            RETURN
0015            END
```

```
0001              SUBROUTINE BETA(P,BTAO,BTAW,BTAG)
0002              AO=.1098168E01
0003              A1=.13487E-03
0004              A2=.682357E-08
0005              A3=-19.937
0006              A4=.9126E-1
0007              A5=-.21086E-5
0008              CW=.0000045
0009              CO=.0003
0010              PB=3487./14.7
0011              BTWBP=.95
0012              BTOBP=1.650
0013              IF(P.LT.PB) GO TO 4
0014              BTAO=BTOBP*(EXP(-CO*(P-PB)))
0015              GO TO 5
0016            4 BTAO=AO+A1*(14.7*P)+A2*(14.7*P)**2
0017            5 BTAG=A3+A4*14.7*P+A5*(14.7*P)**2
0018              BTAG=1./BTAG
0019              BTAW=BTWBP*(EXP(-CW*(P-PB)))
0020              RETURN
0021              END
```

```
0001            SUBROUTINE VISC(P,XMUO,XMUW,XMUG)
0002            AO=.397117E01
0003            A1=-.199219E-02
0004            A2=.80314E-06
0005            A3=-.1986899E-09
0006            A4=.21027E-13
0007            A5=.0024738
0008            A6=.9244E-5
0009            A7=-.76556E-9
0010            PB=3487./14.7
0011            CVISO=.0001
0012            CVISW=.00001
0013            XMUOBP=1.47
0014            XMUWBP=.75
0015            IF(P.LT.PB) GO TO 6
0016            XMUO=XMUOBP+CVISO*(P-PB)
0017            GO TO 7
0018          6 XMUO=AO+A1*14.7*P+A2*(14.7*P)**2+A3*(14.7*P)**3+A4*(14.7*P)**4
0019          7 XMUW=XMUWBP+CVISW*(P-PB)
0020            XMUG=A5+A6*14.7*P+A7*(14.7*P)**2
0021            RETURN
0022            END
```

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```
0001            SUBROUTINE SOLGAS(PA,RS)
0002            PX=PA
0003            PB=3487./14.7
0004            IF(PX.GT.PB) PX=PB
0005            RS=44.27252+.266273*14.7*PX+.000007775651*(14.7*PX)**2
0006            RS=RS/5.615
0007            RETURN
0008            END
```

```
0001            SUBROUTINE RPERM(SOA,SWA,SGA,RKO,RKW,RKG)
0002            SGC=.05
0003            SWC=.20
0004            SOR=.20
0005            IF(SWA.GT.SWC) GO TO 10
0006            RKG=(SGA***3)*(2.-SGA-2.*SWA)/((1.-SWA)**4)
0007            RKW=0.0
0008            RKO=((1.-SGA-SWA)/(1.-SWC))**4
0009            GO TO 20
0010         10 CONTINUE
0011            RKG=(SGA**3)*(2.-SGA-2.*SWC)/((1.-SWC)**4
0012            RKW=((SWA-SWC)/(1.-SWC))**4
0013            RKO=((1.-SGA-SWA)**3)*(1.-SGA+SWA-2.*SWC)/((1.-SWC)**4)
              C
              C     ADJUST RKO FOR SOR AND RKG FOR SGC
0014         20 RKO=RKO*(SOA-SOR)/(1.-SOR)
0015            RKG=RKG*(SGA-SGC)/(1.-SGC)
              C
0016            IF(RKO.LT.0.) RKO=0.0
0017            IF(SOA.LE.SOR) RKO=0.0
0018            IF(RKG.LT.0.) RKG=0.0
0019            IF(SGA.LE.SGC) RKG=0.0
0020            IF(RKW.LT.0.) RKW=0.0
0021            RKSUM=RKO+RKW+RKG
0022            IF(RKSUM.LE.1.) GO TO 6
0023            RKO=RKO/RKSUM
0024            RKW=RKW/RKSUM
0025            RKG=RKG/RKSUM
0026          6 CONTINUE
0027            RETURN
0028            END
```

```
0001              SUBROUTINE BETPRM(PX,BOP,BGP,RSP)
0002              A1=.13487E-03
0003              A2=.682357E-08
0004              A3=-19.937
0005              A4=.09126
0006              A5=-.21086E-5
0007              A6=.266273
0008              A7=.7775651E-5
0009              BTOBP=1.65
0010              CO=.0003
0011              PB=3487./14.7
0012              IF(PX.LT.PB) GO TO 2
0013              BTAO=BTOBP*(EXP(-CO*(PX-PB)))
0014              BOP=-CO*BTAO
0015              RSP=0.0
0016              GO TO 3
0017            2 BOP=A1+2.*A2*PX*14.7
0018              RSP=A6+2.*A7*PX*14.7
0019              RSP=RSP/5.615
0020            3 BTAG=A3+A4*14.7*PX+A5*(14.7*PX)**2
0021              BTAG=1./BTAG
0022              BGP=-BTAG**2*(A4+2.*A5*PX*14.7)
0023              RETURN
0024              END
```

4. COMPUTER PROGRAM FOR DYKSTRA-PARSONS METHOD

```
      /WAT 4     PT140710,TIME=01,PAGES=010   AKBAR ALI M          DEFAULT W JOB 75
      /          CLASS=W,PRIORITY=13,READER=2
      C              MODIFIED DYKSTRA-PARSONS METHOD
 1                DIMENSION PERM(50),H(50),Z(50),A(50),XRATIO(50),B(50),COV(50),OILPR(50),
                 1PM(50),G(50),U(50),WOR(50),DELTP(50),WP(50),WI(50),TIME(50),RECOV(50) ,
                 1QOR(50)
 2                TIME(1)=0.0
 3                PERMAX=399.
 4                SUMH=25.
 5                N=25
 6                NN=N-1
 7                PV=200.*25.*1400.*.2/5.615
 8                DELZ=.04
 9                BO=1.65
10                RKW=.316
11                RKO=.75
12                XMUW=.75
13                SOI=.8
14                SOR=.2
15                WIR=1000.
16                H(1)=1.
17                DO 4 I=2,N
18              4 H(I)=1.+H(I-1)
19                READ(1,100)(PERM(I),I=1,N)
20              2 READ(1,400,END=50) XMUO
21                VRATIO=XMUO/XMUW
22                WRITE(3,410) VRATIO
23                GAMA=RKW/RKO*XMUO/XMUW
24                DO 10 I=1,N
25                Z(I)=H(I)/SUMH
26                A(I)=(1.-GAMA**2)*PERM(I)/PERMAX
27             10 XRATIO(I)=(GAMA-(SQRT(GAMA**2+A(I))))/(GAMA-1.)
28                BETA=0.0
29                B(N)=0.0
30                DO 11 I=2,N
```

```
31              11 BETA=BETA+XRATIO(I)
32                 B(1)=BETA
33                 DO 12 I=2,NN
34              12 B(I)=B(I-1)-XRATIO(I)
35                 DO 13 I=1,NN
36                 COV(I)=Z(I)+B(I)*DELZ/XRATIO(I)
37                 RECOV(I)=COV(I)*(1.-SOR/SOI)*100.
38              13 OILPR(I)=COV(I)*(SOI-SOR)*PV/BO
39                 PM(1)=PERMAX
40                 DO 14 I=2,N
41                 PM(I)=PERM(I)+PM(I-1)
42              14 G(I)=PERM(I)/(BO*(SQRT(GAMA**2+A(I))))
43                 SUMU=0.0
44                 U(N)=SUMU
45                 DO 15 I=2,N
46              15 SUMU=SUMU+G(I)
47                 U(1)=SUMU
48                 DO 16 I=2,NN
49              16 U(I)=U(I-1)-G(I)
50                 DO 17 I=1,NN
51              17 WOR(I)=PM(I)/U(I)
52                 WRITE(3,200)
53                 WP(1)=0.0
54                 DO 18 I=2,NN
55                 DELTP(I)=OILPR(I)-OILPR(I-1)
56                 WP(I)=WOR(I)*DELTP(I)+WP(I-1)
57                 WI(I)=WP(I)+BO*OILPR(I)
58                 TIME(I)=WI(I)/WIR
59                 QOR(I)=DELTP(I)/(TIME(I)-TIME(I-1))
60                 WRITE(3,400)QOR(I)
61                 WRITE(3,300) TIME(I),OILPR(I),RECOV(I),WP(I),WI(I),WOR(I)
62              18 CONTINUE
63                 GO TO 2
64              100 FORMAT(7F10.3)
65              200 FORMAT(//,8X,'TIME',8X,'CUMULATIVE OIL PROD',8X,'RECOVERY',7X,
                   1'CUMULATIVE WATER PROD',5X,CUMULATIVE WATER INJ',15X,'WOR',//,
                   18X,'DAYS',16X,'STB',19X,'%',21X,'STB',21X,'STB',21X,'BBL/STB',//)
```

```
66            300 FORMAT(5X,F10.2,1X,F20.2,9X,F10.5,5X,F20.2,5X,F20.2,5X,F20.5,/)
67            400 FORMAT(F10.5)
68            410 FORMAT(/,5X,'OIL VISCO RATIO=',F10.3)
69             50 STOP
70                END
```

REFERENCES

1. Dykstra, H. and Parsons, R. L.: The Prediction of Oil Recovery by Water Flood, Secondary Recovery of Oil in the United States, API, New York, 2nd ed. (1950) p. 160.

2. Muskat, M.: Physical Principles of Oil Production, McGraw-Hill Book Co., New York, N. Y. (1949).

3. Hubbert, M. K.: Darcy's Law and the Fluid Equations of Flow of Underground Fluid, Trans. AIME (1956) 207, p. 222.

4. Cloud, W. F.: Variation of Pressure Gradient with Distance of Rectilinear Flow of Gas Saturated Oil and Unsaturated Oil through Unconsolidated Sands, Trans. AIME (1930) 86, p. 337.

5. Wyckoff, R. D. and Botset, H. G.: Flow of Gas Liquid Mixtures through Sands, Physics (1936) p. 325.

6. Leverett, M. C.: Capillary Behavior in Porous Solids, Trans. AIME (1941) 142, p. 151.

7. Leverett, M. C. and Lewis, W. B.: Steady Flow of Gas-Oil-Water Mixtures through Unconsolidated Sands, Trans. AIME (1941) 142, p. 107.

8. West, W. J., Garvin, W. W. and Sheldon, J. W.: Solution of the Equation of Unsteady State Two-phase Flow in Oil Reservoirs, Trans. AIME (1954) 201.

9. Hovanessian, S. A. and Fayers, F. J.: Pressure Studies in Bounded Reservoirs, Soc. Pet. Eng. Jour. (1961) 32.

10. Douglas, J., Jr., Blair, P. M. and Wagner, R. J.: Calculation of Linear Water Flood Behavior Including the Effects of Capillary Pressure, Trans. AIME (1958) 213, p. 96.

11. Buckley, S. E. and Leverett, M. C.: Mechanism of Fluid Displacement in Sands, Trans. AIME (1942) 146, p. 107.

12. Douglas, J., Jr.: On the Relation Between Stability and Convergence in the Numerical Solution of Parabolic and Hyperbolic Differential Equations, Jour. Soc. Ind. Appl. Math. (1958) 4, p. 20.

13. Douglas, J., Jr., Peaceman, D. W. and Rachford, H. H., Jr.: A Method for Calculating Multi-Dimensional Immiscible Displacement, Trans. AIME (1959) 216, pp. 297-308.

14. Rapoport, L. A.: Scaling Laws for Use in Design and Operation of Water-Oil Flow Models, Trans. AIME (1955) 204, p. 143.

15. Fagin, R. G. and Stewart, C. H., Jr.: A New Approach to the Two-Dimensional, Multi-phase Reservoir Simulator, Soc. Pet. Eng., (June, 1966), 175-182.

16. Coats, K. H., Nielsen, R. H., Terhune, M. H. and Weber, A. G.: Simulation of Three-Dimensional, Two-phase Flow in Oil and Gas Reservoirs, Soc. Pet. Eng., (December, 1967), 377-388.

17. Quon, D., Dranchuk, P. M., Allada, S. R. and Leung, P. K.: Application of the Alternating Direction Explicit Procedure to Two-Dimensional Natural Gas Reservoirs, Soc. Pet. Eng., (June, 1966).

18. Amyx, J. W., Bass, D. M., Jr. and Whiting, R. L.: Petroleum Reservoir Engineering, McGraw-Hill Book Co., New York, 1960.

19. Gottfried, B. S.: A Mathematical Model of Thermal Oil Recovery in Linear Systems, SPE Jour., (September, 1965), 196-210.

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