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Infrared cooling near atmospheric temperature inversions and absorber concentration variations

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INFRARED COOLING NEAR ATMOSPHERIC TEMPERATURE INVERSIONS AND ABSORBER CONCENTRATION VARIATIONS

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ABSTRACT

Cooling due to infrared radiation near temperature inversions is investigated. Temperature inversions tend to hold pollution below the inversion level. The pollution itself may contribute to the stability of the inversion by selectively cooling certain portions of the atmosphere. A method is developed for evaluating infrared cooling rates at discreet points in the atmosphere. Several sample calculations are given to demonstrate the effects of variations of absorber concentration and lapse rate.

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LIST OF SYMBOLS

γ	Lapse rate
T	Temperature (°K or °C as needed)
z	Height above ground
ρ_a	Density of air
g	Acceleration or gravity
p	pressure
Φ	Geopotential
V	Specific volume
R_m	Molecular gas constant
F_v	Force per unit volume
a	Acceleration
Q	Heat energy
U	Internal energy
C_v	Specific heat at constant volume
C_p	Specific heat at constant pressure
Γ	Adiabatic lapse rate
S_z	Atmosphere stability
θ	Potential temperature
h	Planck's constant
c	Velocity of light
λ	Wavelength
k	Boltzman's constant
I	Intensity of black body radiation
b	Stephan-Boltzman constant

θ	Zenith angle
ω	Solid angle
B	Flux from black body
σ	Black body flux constant
a_λ	Monochromatic absorptivity
τ_λ	Monochromatic transmissivity
r_λ	Monochromatic reflectivity
e_λ	Monochromatic emissivity
J_λ	Monochromatic intensity
k_λ	Monochromatic mass absorption coefficient
m	Mass
dl	Increment of path length
ρ_w	Density of absorbing material
w	Optical depth
$\bar{e}$	Gray body emissivity
B_g	Flux from gray body
F_n	Net flux
U	Upward flux
G	Downward (or ground) flux
I_λ	Monochromatic Plankian intensity
σ_λ	Monochromatic scattering coefficient
r	Direction vector
γ_λ	Monochromatic scattering function
G_λ	Monochromatic downward intensity or flux
U_λ	Monochromatic upward intensity or flux
B_λ	Monochromatic Planckian flux
J_j	Intensity of spectral region j

k_j	Mean absorption coefficient for region j
p_j	Fractional part function in region j
A_f	Mean absorption function
B_j	Proportional part of black body flux in region j
ε_o	Thermal roughness
P_f	Mean transmission function
β	Constant for absolute humidity model

I. INTRODUCTION

It has been pointed out by several researchers, e.g. MÖLLER (1941) and FEIGLSON (1965), that cooling and heating of the atmosphere by infrared radiation may considerably affect the formation and stability of certain atmospheric conditions. Frequently, the atmosphere over a city is characterized by a temperature inversion aloft. The temperature lapse rate below an elevated inversion can easily be superadiabatic which allows bubbles of warm air (containing gaseous and particulate pollution) to rise through the superadiabatic zone. Upon reaching the subadiabatic lapse rate of the inversion the bubble may overshoot the level slightly and then mix horizontally thus causing a stratified layer of haze. The air displaced from the top of the layer is absorbed into lower altitude layers and the haze can remain near the inversion. Along with the abrupt change in particle concentration at the inversion, there may also be a sharp change in the water vapor content. Both of these conditions may lead to radiational cooling which would tend to perpetuate the inversion.

To evaluate the effects of radiational cooling at and near abrupt changes in absorber concentrations, it is necessary to develop some sort of scheme to characterize the transfer of radiation through the atmosphere. Some of the commonly used methods for evaluating radiational heating and cooling rates involve the calculation of a radiation balance at several levels and then, using a good deal of intuition, arrive at the rate of change of temperature with time. It would be an asset to have a method of quickly determining approximate heating and cooling rates for a given set of atmospheric conditions. Given such a method one could evaluate the relative importance of radiational heating and cooling with respect to the stability of certain pollution conditions.

II. DEFINITIONS OF TERMS AND BASIS OF TECHNIQUES

A. THERMAL STRUCTURE OF THE TROPOSPHERE

The lower 10 kilometers of the atmosphere, or troposphere, is usually characterized by a decrease in temperature with increase in altitude up to about 10 km where the stratosphere begins. This vertical temperature gradient, or lapse rate (γ) where

$$\gamma = -\left(\frac{\partial T}{\partial z}\right) \qquad \text{II-1}$$

determines whether the troposphere is statically stable or unstable.

If a parcel of air displaced a very small distance from its initial location tends to return to that position, then the atmosphere is called statically stable. If after such a displacement, the parcel is again in equilibrium then the atmosphere is called adiabatic. A parcel of air that tends to remain in motion after a displacement indicates a statically unstable atmosphere.

1. THE HYDROSTATIC EQUATION

If ρ_a represents the mass per unit volume of a packet of air in the atmosphere, then the force due to gravity acting on the packet is $\rho_a g$, where g is the gravitational acceleration. The hydrostatic pressure (p) at any given height (z) will then be

$$p = \int_z^\infty \rho_a g \, d\mu \qquad \text{II-2}$$

where (μ) is a dummy variable of integration. If we take the partial derivative of p with respect to z, while the lateral coordinates and time are held constant, the result is called the hydrostatic equation,

$$-(\partial p/\partial z) = \rho_a g \qquad \text{II-3}$$

where the minus sign appears as a result of z being the lower limit of the integral in equation II-2. Equation II-3 is usually written as

$$-(\partial p/\partial \Phi) = \rho_a \qquad \text{II-4}$$

where Φ is the geopotential ($\Phi = \int_0^z g \, d\mu$), to take into account the changing gravitational acceleration. However, such corrections may be neglected for our purposes.

2. ATMOSPHERIC STABILITY

The equation of state for an ideal gas is

$$pv = R_m T \quad \text{or} \quad v = R_m T/p \qquad \text{II-5}$$

where (v) is the specific volume ($\equiv \frac{1}{\rho_a}$), (R_m) is the molecular gas constant, and (T) is the absolute temperature. If we use the gas law, and consider that the net upward force per unit volume of air (F_v) on a parcel of air of density (ρ_a') imbedded in air of density (ρ_a) is given by

$$F_v = (\rho_a - \rho_a')g \qquad \text{II-6}$$

then we can equate the force to an upward acceleration (a)

$$a = \frac{F_v}{\rho_a'} = \frac{(\rho_a - \rho_a')}{\rho_a'} g \qquad \text{II-7}$$

Substituting the ideal gas law, noting that ($v \equiv 1/\rho_a$) or ($\rho_a = p/R_m T$), results in

$$a = g \frac{(p/T - p_a'/T')}{p_a'/T'} \qquad \text{II-8}$$

In the atmosphere, each parcel of gas is in pressure equilibrium with the surrounding gas and $p \approx p'$, which after multiplying equation II-8 top and bottom by TT' leads to

$$a = g\left(\frac{T'-T}{T}\right) \qquad \text{II-9}$$

For infinitesimal, quasi-static processes the first law of thermodynamics can be written as

$$dQ = dU + p\ dv \qquad \text{II-10}$$

where (dQ) is an inexact differential representing the heat added to the system, and (dU) is the change in internal energy. The differential of internal energy is related to the specific heat of an ideal gas at constant volume (c_v) and temperature by

$$dU = c_v\ dT \qquad \text{II-11}$$

Also, it can be shown that

$$R_m = c_p - c_v \qquad \text{II-12}$$

where (c_p) is the specific heat at constant pressure. In the case under consideration, the system is approximately adiabatic implying that dQ=0, and substituting this condition into equation II-10 and letting $dU = c_v\ dT$,

$$c_v\ dT + p\ dv = 0 \qquad \text{II-13}$$

For an ideal gas $v=R_m T/p$ and

$$dv = R_m\left(\frac{dT}{p} - \frac{Tdp}{p^2}\right) \qquad \text{II-14}$$

Substituting II-14 into II-13 and dividing by T

$$\frac{c_v dT}{T} + \frac{pR_m}{T}\left(\frac{dT}{p} - \frac{Tdp}{p^2}\right) = 0$$

or

$$\frac{c_v dT}{T} + \frac{R_m dT}{T} - \frac{R_m dp}{p} = 0 \qquad \text{II-15}$$

using the relation II-12 and dividing through by R_m results in

$$\frac{c_p}{R_m}\ \frac{dT}{T} - \frac{dp}{p} = 0$$

or

$$\frac{dT}{T} - \frac{R_m}{c_p}\ \frac{dp}{p} = 0 \qquad \text{II-16}$$

which can be integrated to give

$$Tp^{-\frac{R_m}{c_p}} = \text{constant} \qquad \text{II-17}$$

Substituting equation II-3 into II-16, multiplying by T, and dividing by dz gives

$$\left(\frac{dT}{dz}\right)_{\text{Adiabatic}} = -\frac{TR_m}{p}\frac{\rho_a g}{c_p} \qquad \text{II-18}$$

However, $TR_m/p = v$ and $v\rho_a = 1$ resulting in

$$\left(\frac{dT}{dz}\right)_{\text{Adiabatic}} = -\frac{g}{c_p} \qquad \text{II-19}$$

which is defined as the adiabatic lapse rate (Γ)

$$\Gamma \equiv -\left(\frac{dT}{dz}\right)_{\text{Adiabatic}} = \frac{g}{c_p} \qquad \text{II-20}$$

For a displaced parcel of air, the final temperature of the parcel (T') would be given by

$$T' = T_o - \Gamma\Delta z \qquad \text{II-21}$$

where (T_o) is the initial temperature and (Δz) is the vertical displacement. The temperature of the area that the parcel moved into would be given by

$$T = T_o - \gamma\Delta z \qquad \text{II-22}$$

where (γ) is the actual lapse rate as in equation II-1. Substituting equations II-21 and II-22 into II-9 we obtain the upward acceleration (a),

$$a = -\frac{g\Delta z}{T}(\Gamma-\gamma) \qquad \text{II-23}$$

Atmospheric static stability (s_z) is defined as

$$s_z \equiv -\frac{a}{g\Delta z} = \frac{1}{T}(\Gamma-\gamma) \qquad \text{II-24}$$

the downward acceleration of the displaced parcel of air per unit geopotential. This shows that the atmosphere is statically stable if s_z is positive, i.e. $\Gamma>\gamma$ and unstable if $\Gamma<\gamma$.

The potential temperature (θ) is defined as that temperature which a parcel of air, of initial temperature (T) would attain were it brought isentropically to 1000 millibars pressure, or from II-17

$$Tp^{-\frac{R_m}{c_p}} = \theta(1000\ \text{mb})^{-\frac{R_m}{c_p}} = \text{constant}$$

or

$$\theta = T(1000\ \text{mb}/p)^{R_m/c_p} \qquad \text{II-25}$$

Differentiating with respect to (z) we obtain

$$\frac{\partial \theta}{\partial z} = \frac{\partial T}{\partial z}\left(\frac{1000}{p}\right)^{R_m/c_p} + T\frac{R_m}{c_p}\left(\frac{1000}{p}\right)^{(R_m/c_p)-1} 1000\,\frac{\partial}{\partial z}(1/p)$$

Dividing both sides by $T\left(\frac{1000}{p}\right)^{\frac{R_m}{c_p}}$

$$\frac{1}{T\left(\frac{1000}{p}\right)^{R_m/c_p}}\frac{\partial \theta}{\partial z} = \frac{1}{T}\frac{\partial T}{\partial z} - \frac{R_m}{c_p}\,p\,\frac{1}{p^2}\frac{\partial p}{\partial z}$$

or

$$\frac{1}{\theta}\frac{\partial \theta}{\partial z} = \frac{1}{T}\frac{\partial T}{\partial z} - \frac{R_m}{c_p}\frac{1}{p}\frac{\partial p}{\partial z}$$

Since $-\left(\frac{\partial p}{\partial z}\right) = \rho_a g$ and $\rho = p/R_m T$

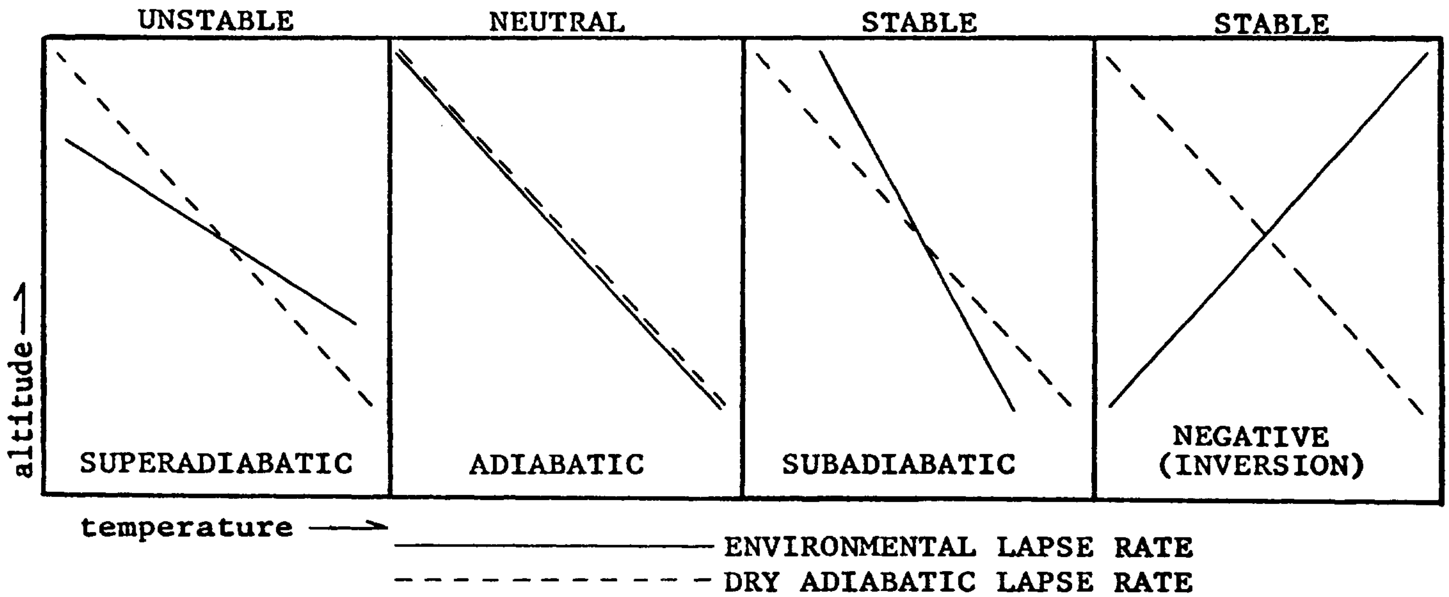
$$\frac{1}{\theta}\frac{\partial \theta}{\partial z} = \frac{1}{T}\left(\frac{\partial T}{\partial z} + \frac{g}{c_p}\right)$$

and

$$\frac{1}{\theta}\frac{\partial \theta}{\partial z} = \frac{1}{T}(\Gamma - \gamma) = s_z = \frac{\partial \ln\theta}{\partial z} \qquad \text{II-26}$$

We see that $s_z > 0$ if the potential temperature increases with height (requirement for a statically stable atmosphere), and an unstable atmosphere is indicated if θ decreases with altitude.

The adiabatic lapse rate (Γ), which we have been discussing, is the dry-adiabatic lapse rate (i.e. excluding the condensation of water vapor) and is numerically equal to 9.86°C/km.

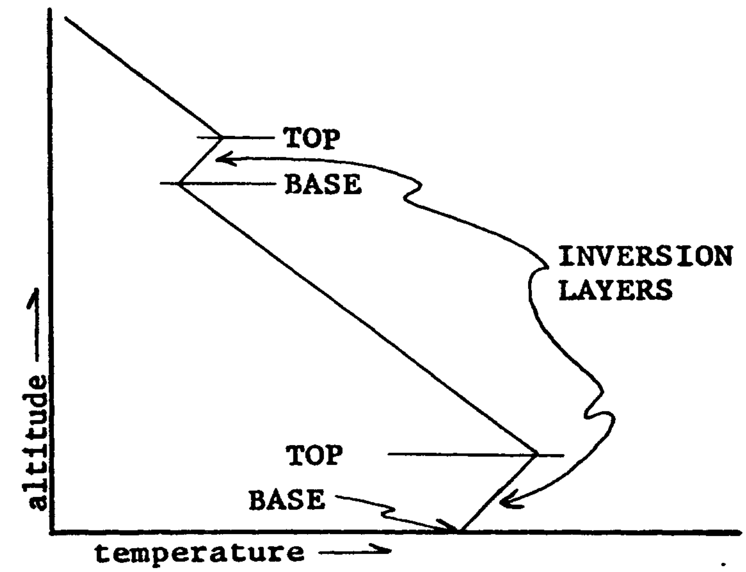


TYPES OF LAPSE RATES
FIGURE II-1

Temperature gradients are frequently grouped into two classes, lapse and inversion. Lapse is defined to include superadiabatic and adiabatic lapse rates and some or all subadiabatic lapse rates. It should be noted that lapse and inversion are not strictly synonomus with instability and stability.

The saturation adiabatic lapse rate, for air saturated with water vapor, is smaller than the dry-adiabatic rate because of the release of the latent heat of condensation as the air parcel ascends and cools and varies with temperature and height. The diagrams in Figure II-1 illustrate various lapse conditions and indicates their names and effects.

An inversion is characterized by the heights of its base and top, and the value of the lapse rate. As shown in Figure II-2, the inversion base may be either at the ground or aloft and more than one inversion can exist at the same time in the vertical structure.



INVERSION CHARACTERISTICS
FIGURE II-2

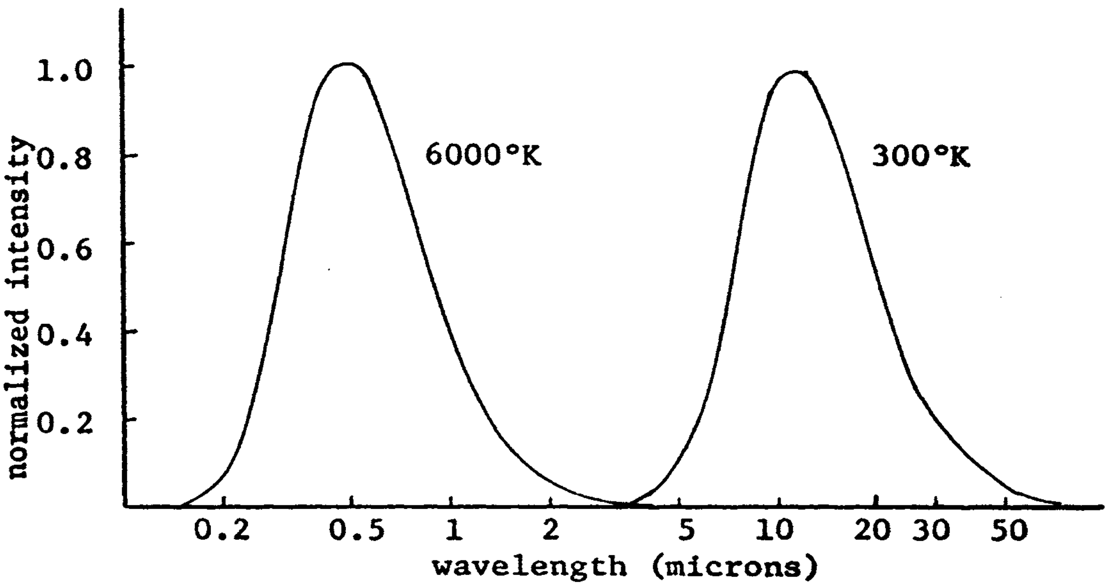
The temperature lapse rate often varies significantly with height. Above a surface inversion, the lapse rate may be zero or positive. A surface inversion, commonly observed at night with clear skies and weak winds, is frequently due to cooling of the ground by radiation and exchange of heat between the ground and lower air layers. Above a surface adiabatic or superadaibatic layer, the lapse rate may be zero or negative. This condition is frequently observed for an hour or two after sunrise, when solar heating of the ground and subsequent convective mixing of the layer nearest the surface have converted a deep surface inversion into a shallow layer of adiabatic or super adiabatic lapse rate with an isothermal layer or inversion above. In the coastal regions of southern California it has been observed that an inversion with base height, on the average, some hundreds of meters above the surface, is present as a semipermanent feature of the large scale circulation (AIR POLLUTION VOLUME I).

B. RADIATION IN THE ATMOSPHERE

1. TYPES OF RADIATION

Energy transfer by radiation is of extreme importance in the earth-atmosphere system. Radiant energy from the sun supplies the earth with 1.79×10^{24} ergs/second. The energy leaving the earth-atmosphere system must be largely radiational in nature, and must be equal to the total input to prevent continued heating or cooling. This is pointed out by the fact that the system as a whole is in long term equilibrium. Local and short term atmospheric anomalies, as well as long term and widespread effects, may arise from the interaction of radiation with the atmosphere.

The types of radiation of interest are solar radiation, which is very nearly that of a black body at 6000°K, and terrestial radiation which approximates the radiation from a 300°K black body. It is convenient that the radiational energy distribution of a 6000°K black body and a 300°K black body are, for practical purposes, separated in wavelength as shown in Figure II-3.



NORMALIZED BLACK-BODY INTENSITY PER UNIT WAVELENGTH
FIGURE II-3

This separation allows the effects of solar and terrestial radiation to be studied separately. The curves in Figure II-3 were taken from Planck's Law of Black Body Radiation with the maximum intensity being normalized to one. Planck's Law states that the monochromatic intensity (I_λ) is given by

$$I_\lambda = \frac{2hc^2 \lambda^{-5}}{(e^{hc/k\lambda T} - 1)} \qquad \text{II-27}$$

where (c) is the velocity of light, (λ) is the wavelength, (k) is Boltzmann's constant, (T) is absolute temperature, and (h) is Planck's constant. By integrating Planck's Law over all wavelengths the Stephan-Boltzmann Law for Black Body Intensity (I) is derived,

$$I = b\, T^4 \qquad \text{II-28}$$

where (b) is a constant. Since radiation from a black body is isotropic, integration over a hemispherical (2π) solid angle (ω), with azimuthal angle (θ) in spherical coordinates, gives the radiational flux density (B, the energy crossing a unit area in a unit time) from a black body

$$B = \int_{o}^{2\pi} I \cos\theta \; d\omega$$

$$= \int_{o}^{\pi/2} 2\pi \; I \sin\theta \cos\theta \; d\theta$$

$$= \pi I = \pi b T^4$$

and

$$B \equiv \sigma T^4 \qquad \text{II-29}$$

where $\sigma = 0.817 \times 10^{-10}$ cal.cm^{-2}min^{-1} $^{o}K^{-4}$.

To work with radiation fields and their effects on various materials, it is necessary to consider the monochromatic absorptivity, transmissivity, and reflectivity (a_λ, τ_λ, and r_λ respectively) of the materials. These quantities are the ratios of the absorbed, transmitted, and reflected monochromatic flux densities to the incident monochromatic flux densities. Also of importance is the monochromatic emissivity (e_λ) which is the ratio of the emitted monochromatic flux density to the flux density expected from a black body with the same temperature. Kirchoff's Law states that

$$e_\lambda = a_\lambda \qquad \text{II-30}$$

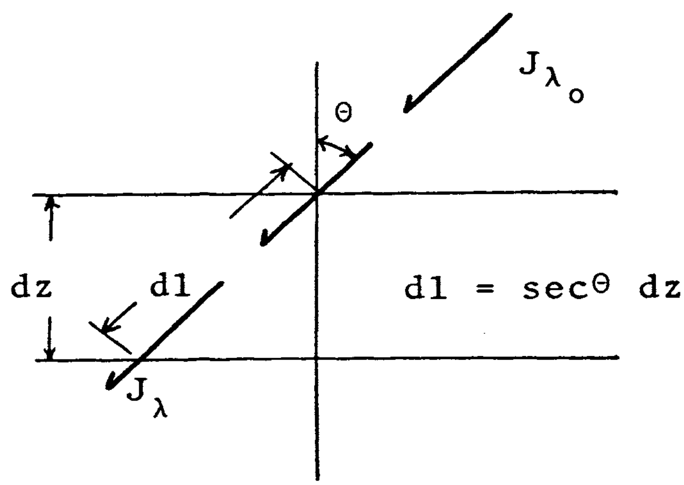
and it is also true that

$$a_\lambda + \tau_\lambda + r_\lambda = 1 \qquad \text{II-31}$$

The emitted flux density of a black body radiator varies only with the fourth power of the temperature. For a non-black body it is sometimes possible to determine an average emissivity ($\bar{e}$) and write the flux from the "gray body" (B_g) as

$$B_g = \bar{e}\ \sigma T^4 \qquad \text{II-32}$$

Beer's Law of Absorption describes the effect of a layer of absorbing material of geometric thickness dz and density ρ_w, on a parallel beam of monochromatic radiation of initial intensity J_{λ_o} from a direction θ as shown in Figure II-4.



SCHEMATIC OF BEER'S LAW
FIGURE II-4

The mass absorption coefficient (k_λ) is defined as the fractional change of monochromatic intensity per unit mass of absorbing material

$$k_\lambda \equiv - \frac{1}{J_\lambda} \frac{dJ_\lambda}{dm} \qquad \text{II-33}$$

where ($dm = \rho_w \sec\ \theta\ dz$). Equation II-33 can be rewritten

$$\frac{dJ_\lambda}{J_\lambda} = -k_\lambda \rho_w \sec \theta \, dz$$

and integrated

$$\int_{J_{\lambda_o}}^{J_\lambda} \frac{dJ}{J} = -k_\lambda \sec \theta \int_o^z \rho_w \, d\mu$$

to give

$$\ln \frac{J_\lambda}{J_{\lambda_o}} = -k_\lambda \sec \theta \int_o^z \rho_w \, d\mu \qquad \text{II-34}$$

Defining an increment of optical depth (dw)

$$dw \equiv \rho_w \, dl$$

it can be seen that

$$w = \int_o^l \rho_w \, d\mu \qquad \text{II-35}$$

Putting II-35 into II-34 results in

$$J_\lambda = J_{\lambda_o} e^{-k_\lambda \sec \theta \, w}$$

and if the optical depth (w) is measured along the direction of the radiation, θ is zero and sec $\theta = 1$ giving

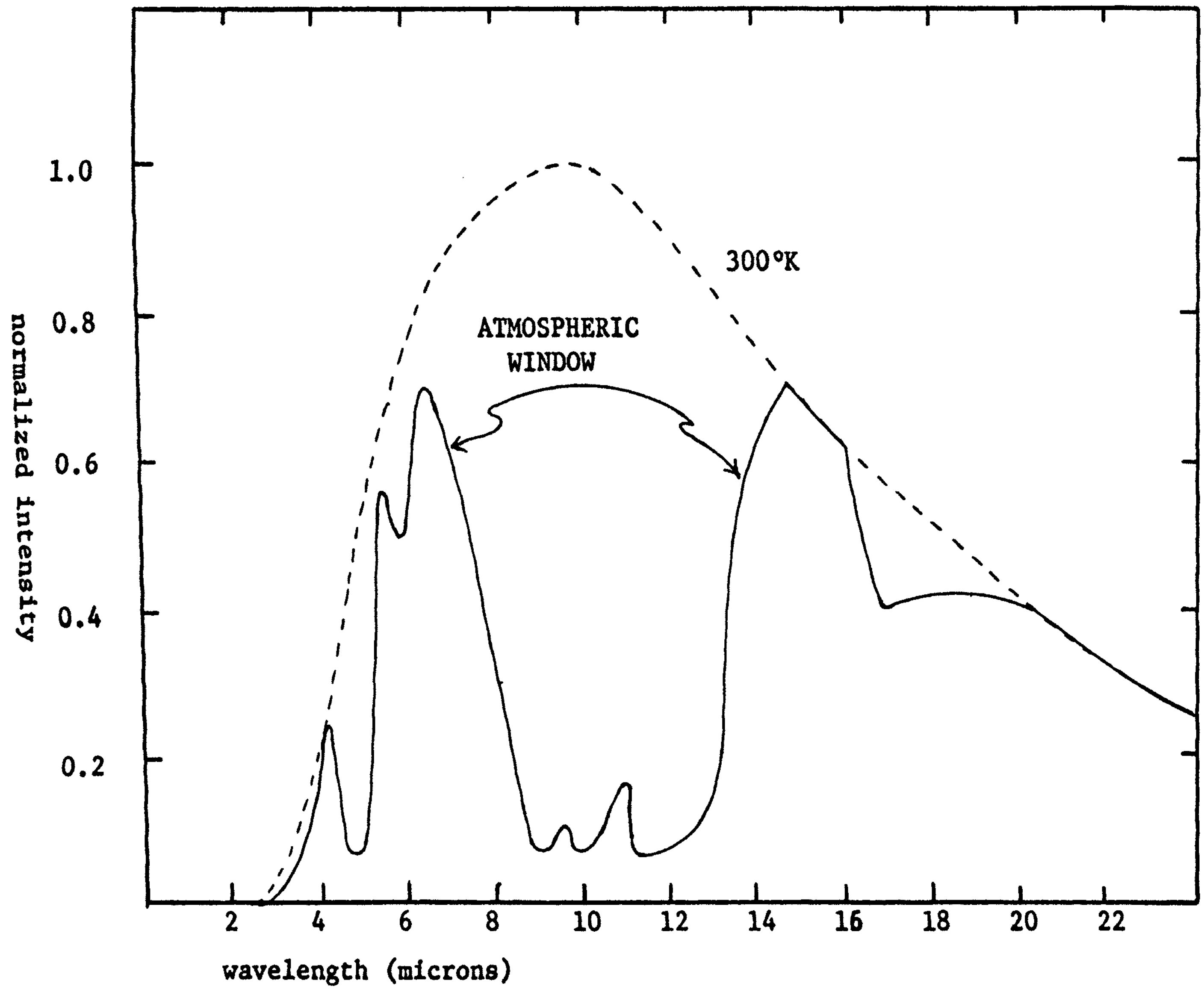
$$J_\lambda = J_{\lambda_o} e^{-k_\lambda w} \qquad \text{II-36}$$

which is the usual form of Beer's Law.

2. TRANSFER OF RADIATION IN THE ATMOSPHERE

It is generally true that a relatively small amount (about 17%) of impingent solar short wave radiation is absorbed directly by the atmosphere. The rest of it is absorbed by the earth's surface or reflected back into space. On the other hand, about 95% of the infrared radiation emitted by the earth is absorbed in the atmosphere and then reradiated back toward the earth or into outer space. The global mean value of solar radiation flux at the top of the atmosphere (0.5 Cal cm^{-2} min^{-1}) is energetically approximately the same as the flux emitted by the earth's surface (0.572 Cal cm^{-2} min^{-1} on the average). Thus it can be seen that the mechanisms of infrared absorption and emission are usually of considerably more importance in atmospheric phenomena than is the absorption of solar radiation. For this reason, and others to be mentioned later, we will confine our investigation of transfer of radiation in the atmosphere to those wavelengths associated with terrestial radiation.

The most important infrared absorber and emitter in clear air is water vapor. Other absorbers of interest include CO_2, ozone, liquid water (clouds and fog), and pollution both gaseous and particulate. A schematic representation of the absorption spectrum, for terrestial radiation, for a thin layer of moist air at sea level (no clouds or pollution) is given in Figure II-5. The dotted line in Figure II-5 is the normalized energy distribution for radiation from a 300°K black body. The ordinate value of the absorption curve for a particular wavelength, relative to the corresponding value of the black body curve, is the percentage absorption at that wavelength. The region between 7 and 14 microns of wavelength is known as the atmospheric window, and a certain percentage of the energy radiated at these wavelengths escapes directly to outer space.



CLEAR AIR ABSORPTION SPECTRUM
FIGURE II-5

To determine the I.R. flux density at a particular height in the atmosphere it is necessary to consider flux density as a function of the vertical temperature distribution and the absorbing-emitting characteristics of the atmosphere. A general approach to the problem is to determine the monochromatic intensity arriving at the level of interest (reference level) from a horizontal slab, or stratified layer, of optical thickness dw in a direction (θ), i.e. ($dJ_{\theta\lambda w}$). Expressing the quantity in terms of flux density and integrating out θ, results in the monochromatic flux $dJ_{\lambda w}$ from the entire slab. Next, one would integrate over all wavelengths to get the total flux contribution of the slab dJ_w. This is the difficult step as it requires a detailed knowledge of the absorption coefficients for all wavelengths. Finally, integration over all the horizontal slabs would result in the total flux density at the reference level (J).

A knowledge of the flux density at all levels of the atmosphere is necessary to determine accurately the details of the radiation balance of the earth-atmosphere system, and it is toward this end that a great deal of the work in this field has been applied. It is also possible to consider the effects of IR absorption and emission leading to heating and cooling of various layers of the atmosphere.

3. RADIATIONAL HEATING

To discuss the mechanisms of radiational heating it is convenient to define the net flux at a given level (F_n) as the difference between the upward flux from the layers of atmosphere below (U) and the downward flux from layers above (G), i.e.

$$F_n \equiv U - G \qquad \text{II-37}$$

If the law of conservation of energy is applied to a stratified slab of air, and the net flux (F_n) is different at one edge of the layer as compared to the other edge of the same layer, then the energy lost or gained by the layer must manifest itself by a cooling or heating of the layer in the absence of other non-adiabatic processes.

It has been shown by KONDRAT'YEV (1965) and others that the rate of change of temperature with time (t) for a given layer of air with density (ρ_a), due to the effects of radiation, is given by

$$\frac{dT}{dt} = \frac{1}{c_p \rho_a} \nabla F_n \qquad \text{II-38}$$

where ∇F_n is the vector divergence of the net flux (F_n). For practical purposes the net flux changes only in a vertical direction and

$$\frac{\partial T}{\partial t} = \frac{1}{c_p \rho_a} \frac{\partial F_n}{\partial z} = \frac{1}{c_p \rho_a} \left(\frac{\partial U}{\partial z} - \frac{\partial G}{\partial z}\right) \qquad \text{II-39}$$

To use this result it is necessary to develope a method of calculating the transfer of radiation flux in the atmosphere.

4. APPROXIMATE FLUX TRANSFER EQUATION

Kondrat'yev derived equation II-40 to describe the transfer of radiant energy in a steady field of radiation for the case of local thermodynamic equilibrium.

$$\frac{\cos\theta}{\rho_w} \frac{\partial J_\lambda}{\partial z} = k_\lambda I_\lambda + \frac{\sigma_\lambda}{4\pi} \int J_\lambda(z,r') \, \gamma_\lambda(z;r',r) \, d\omega' - (k_\lambda + \sigma_\lambda) J_\lambda \qquad \text{II-40}$$

Where (J_λ) is the intensity of monochromatic radiation of wavelength (λ), (θ) is the angle made by the beam with the vertical, (ρ_w) is the density of the substance absorbing the radiation, (z) is vertical height, (k_λ) and (σ_λ) are mass absorption and scattering coefficients respectively, (I_λ) is the Planckian intensity, and (γ_λ) is a function characterizing the law of scattering. The quantity $1/4\pi\ \gamma_\lambda\ (z;r',r)$ is equal to the fraction of radiant energy incident upon the scattering volume from the direction r' which is scattered in the direction r and the integral extends over all possible directions of r' in the solid angle (ω). The following development, beginning with equation II-40 and extending through equation II-53 and the associated radiation chart, is outlined by Kondrat'yev. Comments and intermediate steps have been inserted to clarify the development and provide continuity. Equations denoted by a cross (†) are given explicitly by Kondrat'yev.

For the conditions of negligible absorption of solar radiation, and negligible scattering of thermal radiation, equation II-40 reduces to

$$\frac{\cos\theta}{\rho_w}\ \frac{\partial J_\lambda}{\partial z} = k_\lambda\ (I_\lambda - J_\lambda) \qquad \text{II-41}\ ^\dagger$$

where the variables are the same as in II-40, and (I_λ) represents the Plankian intensity emitted by a black body at the temperature of the absorbing material with absorption coefficient (k_λ), and (J_λ) is the flux that is passing through the absorbing material.

It is convenient to break up the radiation into downward and upward parts,

$$G_\lambda(z,\theta) = J_\lambda(z,\pi - \theta) \qquad \text{for} \qquad 0 \leq \theta \leq \frac{\pi}{2}$$

where (G_λ) is the intensity of monochromatic radiation in a downward direction (from the upper hemisphere of air layers) and (U_λ) is the intensity in an upward direction (from the layers below) giving

$$U_\lambda(z,\theta) = J_\lambda(z,\theta) \qquad \text{for} \qquad 0 \leq \theta \leq \frac{\pi}{2}$$

Due to the fact that $\cos(\pi-\theta)=-\cos\theta$, equation II-41 transforms into the following two equations

$$\frac{\cos\theta}{\rho_w} \frac{\partial G_\lambda(z,\theta)}{\partial z} = k_\lambda[G_\lambda(z,\theta) - I_\lambda]$$

$$\frac{\cos\theta}{\rho_w} \frac{\partial U_\lambda(z,\theta)}{\partial z} = k_\lambda[I_\lambda - U_\lambda(z,\theta)] \qquad \text{II-42}^{\dagger}$$

It should be noted that equations II-42 apply only to parallel beam monochromatic radiation.

The intensity of radiation emitted by a layer of absorbing material may be converted to an emitted flux by integrating over a hemisperical solid angle of 2π and considering the emitted intensity as dependent on sec θ. It turns out that for extremely small optical thicknesses the emitted flux is twice the emitted intensity since absorption by the intervening material is very small. For large optical thicknesses, radiation emitted by the slab becomes black body radiation for all directions and emitted flux equals emitted intensity. It has been shown by a number of authors, e.g. FLEAGLE and BUSINGER (1963), that for values of optical thickness in the range of interest in the atmosphere the emitted flux is approximately equal to 1.6 times the parallel beam emission. Using this approximation, equations II-42 may be used to describe diffuse monochromatic radiation providing that the zenith angle (θ) be considered as 0, and I_λ is replaced by B_λ, the monochromatic Planckian flux, and (G_λ) and (U_λ) are considered as fluxes rather than intensities, in the downward and upward directions. Also, the absorption (or emission) function or the absorbing (emitting) mass should be multiplied by 1.6 when the equations are applied.

From the above assumptions, approximate radiative transfer equations for monochromatic radiation may be written as total derivatives with respect to z since the θ dependance is removed, thus

$$\frac{dG_\lambda(z)}{dz} = k_\lambda \rho_w \,[G_\lambda(z) - B_\lambda(z)]$$

$$\frac{dU_\lambda(z)}{dz} = k_\lambda \rho_w \,[B_\lambda(z) - U_\lambda(z)] \qquad \text{II-43}$$

To transform these equations further, it is necessary to develope a scheme for determining the absorption coefficient (k_λ). Two rules for the behavior of atmospheric absorption spectra have been proposed. The "square root law" is used by ELSASSER (1942) to describe the absorption of nonoverlapping line spectra. In actuality most absorption lines overlap, and the "exponential law" of absorption is widely used for describing the absorption of long wave radiation in the atmosphere. In this scheme the entire infrared absorption spectrum is broken up into a large number (25-30) of sufficiently narrow spectral regions in each of which the absorption coefficient is independent of the wavelength. For each region (j) the absorption is given by Beer's Law

$$J_j = J_{j_o}\, e^{-k_j w}$$

where (w) is the optical depth and (k_j) is determined by

$$k_j = \frac{1}{w} \ln \frac{J_{j_o}}{J_j}$$

If the number of spectral regions is (n) and the j th region receives a p_j th fractional part of the radiant energy falling upon the layer of the absorbing material, the mean absorption function (A_f) can be written as

$$A_f = 1 - \sum_{j=1}^{n} p_j\, e^{-k_j w} \qquad \text{II-44}^{\dagger}$$

where (k_j) is the mean mass absorption coefficient over the interval (j).

With a knowledge of a usable absorption function it is possible to integrate the transfer equations II-43 over all wavelengths. Using summation instead of integration, and summing over all wavelengths from 0 to ∞ (actually, the only wavelengths of current interest are those in the atmospheric infrared spectrum, since we are excluding solar radiation), the following results

$$\frac{d}{dz} \sum_{\lambda=0}^{\infty} G_\lambda = \sum_{\lambda=0}^{\infty} k_\lambda \rho_w G_\lambda - \sum_{\lambda=0}^{\infty} k_\lambda \rho_w B_\lambda$$

$$\frac{d}{dz} \sum_{\lambda=0}^{\infty} U_\lambda = \sum_{\lambda=0}^{\infty} k_\lambda \rho_w B_\lambda - \sum_{\lambda=0}^{\infty} k_\lambda \rho_w U_\lambda \qquad \text{II-45}$$

These sums are sums of very narrow but finite spectral intervals where Beer's Law of Absorption applies.

If the intervals in the summations are arranged in groups with similar absorption coefficients, rather than by ascending frequency (that is, the j th interval is composed of sub-intervals which may be drawn from widely separated spectral regions, but they cover only a narrow range of absorption coefficients) the mean absorption coefficient is over a smoother set of absorption coefficients in each interval. As a result, the mean absorption function is more well defined since the mean absorption coefficients are smoother and better defined. Using such a grouping, putting in k_j, and breaking up the black body flux ($B=\sigma T^4$) into contributions due to each interval ($B_j = p_j B$), equations II-45 representing each interval (j) become

$$\frac{dG_j}{dz} = k_j \rho_w (G_j - p_j B)$$

$$\frac{dU_j}{dz} = k_j \rho_w (p_j B - U_j) \qquad \text{II-46}$$

To integrate equations II-46 it is necessary to assume boundary conditions. Such conditions might be

$$G_j = 0 \qquad \text{at} \qquad z = z_\infty$$

$$U_j = \bar{e}\, p_j B + (1-\bar{e}) G_j \qquad z = \varepsilon_o$$

$$j = 1, 2, 3, \ldots n$$

where ($\bar{e}$) is the gray body emissivity of the earth's surface, the reflectivity from Kirchoff's Law and equation II-31 is ($1-\bar{e}$), z_∞ is the tropopause altitude, and ε_o

is the thermal roughness or the height of the effective radiation surface of the earth assuming that the earth-atmosphere interface is in local thermodynamic equilibrium (ε_o may vary from 10^{-5} cm to 10 cm for surfaces such as smooth glass or tall grass respectively).

By introducing the optical depth

$$w = \int_0^z \rho_w d\mu \qquad \text{or} \qquad \frac{dw}{dz} = \rho_w$$

and dividing through by ρ_w, equations II-46 and their boundary conditions become

$$\frac{dG_j}{dw} = k_j(G_j - p_j B)$$

$$\frac{dU_j}{dw} = k_j(p_j B - U_j)$$

$$G_j = 0 \qquad \text{at} \qquad w = w_\infty \qquad\qquad \text{II-47}$$

$$U_j = \bar{e} p_j B + (1-\bar{e}) G_j \qquad \text{at} \qquad w = w_o$$

$$(j = 1, 2, 3, \ldots n)$$

Considering the first of equations II-47 as a normal first order nonhomogeneous differential equation and multiplying through by (dw) and an appropriate integrating factor ($e^{-k_j w}$) we get

$$e^{-k_j w}\,[dG_j(w) - k_j G_j(w)dw] = -e^{-k_j w}\, k_j p_j B(w)dw$$

or

$$d[G_j(w)\, e^{-k_j w}] = -e^{-k_j w}\, k_j p_j B(w)dw$$

Integrating both sides from w to w_∞, the left side results in

$$G_j(w_\infty)\, e^{-k_j w_\infty} - G_j(w)\, e^{-k_j w}$$

or, considering the boundary condition $[G_j(w_\infty)=0]$ just

$$-G_j(w)\, e^{-k_j w}$$

The right side of the equation requires a change of the variable (w) to a dummy variable (μ) (to avoid confusion) and integration by parts

$$udv = uv - \int vdu$$

with

$$u = p_j B(\mu) \qquad du = p_j dB(\mu) = p_j \frac{dB}{d\mu}\, d\mu$$

$$dv = d(e^{-k_j \mu}) \qquad v = e^{-k_j \mu}$$

and results in

$$p_j B(\mu)\, e^{-k_j \mu} \Big|_{w}^{w_\infty} - \int_{w}^{w_\infty} p_j \frac{dB}{d\mu}\, e^{-k_j \mu}\, d\mu$$

Thus

$$-G_j(w)\, e^{-k_j w} = p_j B(w_\infty)\, e^{-k_j w_\infty} - p_j B(w)\, e^{-k_j w}$$

$$- p_j \int_{w}^{w_\infty} \frac{dB}{d\mu}\, e^{-k_j \mu}\, d\mu$$

and multiplying through by $(-e^{k_j w})$ gives

$$G_j(w) = p_j B(w) - p_j B(w_\infty)\, e^{-k_j (w_\infty - w)}$$

$$+ p_j \int_{w}^{w_\infty} \frac{dB}{d\mu} \cdot e^{-k_j (\mu - w)}\, d\mu \qquad \text{II-48}$$

$$(j = 1, 2, 3, \ldots n)$$

Considering the second of equations II-47 and the associated boundary condition

$$dU_j(w) + k_j U_j(w)\, dw = k_j p_j\, B dw$$

and

$$U_j = \bar{e}\, p_j\, B + (1-\bar{e})G_j \qquad \text{at} \qquad w = w_o$$

steps analogous to those used on the first equation, with integrating factor $(e^{k_j w})$ and integration from (w_o) to (w), result in

$$U_j(w)\, e^{k_j w} - U_j(w_o)\, e^{k_j w} = p_j B(w)\, e^{k_j w} - p_j B(w_o)\, e^{k_j w_o}$$

$$- p_j \int_{w_o}^{w} \frac{dB}{d\mu} e^{k_j \mu}\, d\mu$$

By putting in the boundary condition (with $G_j(w_o)$ being evaluated from equation II-48), multiplying through by $(e^{-k_j w})$, and performing some algebraic simplification we obtain

$$U_j(w) = p_j B(w) - (1-\bar{e})(e^{-k_j(w-w_o)})(p_j\, B(w_\infty)\, e^{-k_j w_\infty}$$

$$- p_j \int_{w_o}^{w_\infty} \frac{dB}{d\mu} e^{-k_j \mu}\, d\mu) + p_j\, B(w_o)\, [e^{-k_j w} - e^{-k_j(w-w_o)}]$$

$$- p_j \int_{w_o}^{w} \frac{dB}{d\mu} e^{-k_j(w-\mu)}\, d\mu$$

But for levels well above ε_o , $w-w_o=w$ which further simplifies the above equation to

$$U_j(w) = p_j\ B(w) - (1-\bar{e})[p_j B(w_\infty)\ e^{-k_j w_\infty} - p_j \int_w^{w_\infty} \frac{dB}{d\mu}\ e^{-k\mu}\ d\mu]e^{-k_j w}$$

$$- p_j \int_{w_o}^{w} \frac{dB}{d\mu}\ e^{-k_j(w-\mu)}\ d\mu \qquad \text{II-49}$$

$$(j = 1, 2, 3, \ldots n)$$

Now let us recall equation II-39 describing the heating rate and substitute into it the transfer equations II-47, resulting in

$$\rho_a\ c_p\ \frac{\partial T}{\partial t} = -\rho_w \sum_{j=1}^{n} [k_j(U_j - p_j B) + k_j(G_j - p_j B)]$$

Then the use of the integrated equations for (U_j) and (G_j) (II-48 and II-49) in w coordinates allows the heating rate to be written

$$\rho_a\ c_p\ \frac{\partial T}{\partial t} = -\rho_w\ \{B(w_\infty) \sum_{j=1}^{n} p_j k_j\ e^{-k_j(w_\infty - w)} - \int_w^{w_\infty} \frac{dB}{d\mu} \sum_{j=1}^{n} p_j k_j\ e^{-k_j(\mu-w)} d\mu$$

$$+ \int_o^{w} \frac{dB}{d\mu} \sum_{j=1}^{n} p_j k_j\ e^{-k_j(w-\mu)}\ d\mu\} \qquad \text{II-50}^{\dagger}$$

Where the approximations $\bar{e}=1$ and $w_o=0$ have been made in equation II-49. These approximations are justifiable since the earth is very nearly a black body and the ground temperature may be adjusted to offset any errors.

If the reflectivity of a substance is negligible, then the transmissivity from Kirchoff's Law is equal to one minus the absorptivity, and the transmission function (P_f) is equal to one minus the mean absorption function

$$P_f = 1 - A_f$$

For the absorption function equation II-44

$$A_f = 1 - \sum_{j=1}^{n} P_j \, e^{-k_j w}$$

and

$$P_f = \sum_{j=1}^{n} P_j \, e^{-k_j w}$$

Now it is interesting to note that some of the derivatives of the (P_f) function are

$$\frac{dP_f(w)}{dw} = - \sum_{j=1}^{n} p_j k_j \, e^{-k_j w}$$

$$\frac{dP_f(w_\infty - w)}{dw} = \sum_{j=1}^{n} p_j k_j \, e^{-k_j (w_\infty - w)}$$

$$\frac{dP_f(\mu - w)}{dw} = - \sum_{j=1}^{n} p_j k_j \, e^{-k_j (\mu - w)}$$

$$\frac{dP_f(w - \mu)}{dw} = \sum_{j=1}^{n} p_j k_j \, e^{-k_j (w - \mu)}$$

and that the quantities on the right side of these equations are found in the heating rate equation II-50. Substituting in these derivatives in place of the summations in the heating rate equation results in

$$\rho_a c_p \frac{\partial T}{\partial t} = -\rho_w \{ B(w_\infty) \frac{dP_f(w_\infty - w)}{dw} + \int_w^{w_\infty} \frac{dP_f(\mu - w)}{dw} \frac{dB}{d\mu} d\mu$$

$$+ \int_o^w \frac{dP_f(w-\mu)}{dw} \frac{dB}{d\mu} d\mu \} \qquad \text{II-51}^\dagger$$

or changing to integrals over B(w)

$$\rho_a c_p \frac{\partial T}{\partial t} = -\rho_w \{ \int_o^{B(w_\infty)} \frac{dP_f(w_\infty - w)}{dw} dB + \int_{B(w)}^{B(w_\infty)} \frac{dP_f(\mu - w)}{dw} \cdot dB$$

$$+ \int_{B(o)}^{B(w)} \frac{dP_f(w-\mu)}{dw} \cdot dB \} \qquad \text{II-52}^\dagger$$

From the definitions of the derivatives it can be seen that, in every case

$$\frac{dP_f(w_\infty - w)}{dw} \geq 0, \qquad \frac{dP_f(\mu - w)}{dw} \leq 0, \qquad \text{and} \qquad \frac{dP_f(w-\mu)}{dw} \geq 0$$

Thus the heating rate equation II-52 can be written as

$$\frac{\partial T}{\partial t} = \frac{\rho_w}{c_p \rho_a} \{ - \int_0^{B(w_\infty)} \left| \frac{dP_f(w_\infty - w)}{dw} \right| dB$$

$$- \int_{B(w_\infty)}^{B(w)} \left| \frac{dP_f(\mu - w)}{dw} \right| dB + \int_{B(w)}^{B(o)} \left| \frac{dP_f(w-\mu)}{dw} \right| dB \} \qquad \text{II-53}^\dagger$$

This selection of signs permits a numerical or graphical solution of the integrals in the B(w), dP_f/dw coordinate system. The temperature change from equation II-53 has three physically meaningful components. The first term on the right represents the cooling due to radiation into outer space. The second term gives the cooling due to the radiation balance at the reference level with respect to the atmosphere above, and the third term is the heating due to the radiation balance with respect to the atmosphere below.

Some time should be spent in explaining how to evaluate the integrals in equation II-53, both graphically and numerically. The integrals can be interpreted as areas on a chart with a $B(w)$, dP_f/dw coordinate system (See Figure II-7). The areas representing the first two integrals are negative, and the last positive, contributions to the total area. The total area obtained is multiplied by $\rho_w/c_p\rho_a$ to give the heating rate. The ordinate of such a chart is simply the numerical value of dP_f/dw which has units of cm^2/gram. These units arise from the facts that the optical depth (w) has units of gm/cm^2, the absorption function (A_f) is unitless, and

$$\frac{dP_f}{dw} = - \frac{dA_f}{dw}$$

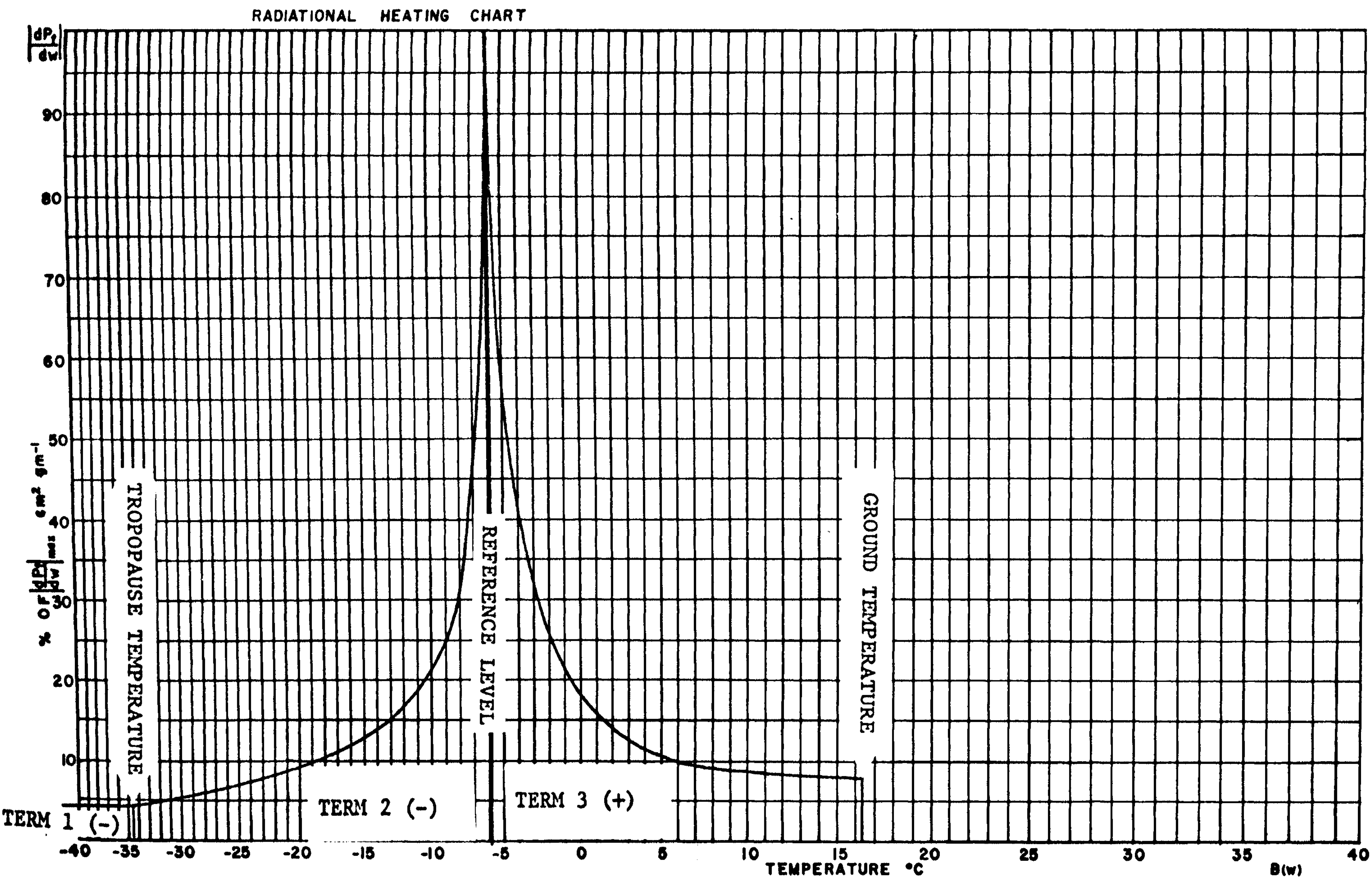
It is necessary to know dP_f/dw as a function of w and it is convenient to label the ordinate of the chart in terms of w, thus providing a monographic conversion of w (which is a number that can be obtained from sounding data) to units of dP_f/dw. Of course, labeling the ordinate in terms of w requires knowledge of the relation between w and dP_f/dw, and the chart in a given form is usable only so long as this relationship is valid. The abscissa is in units of cal/cm^2 min which are the units of $B(w)$. Since

$$B(w) = \sigma T^4$$

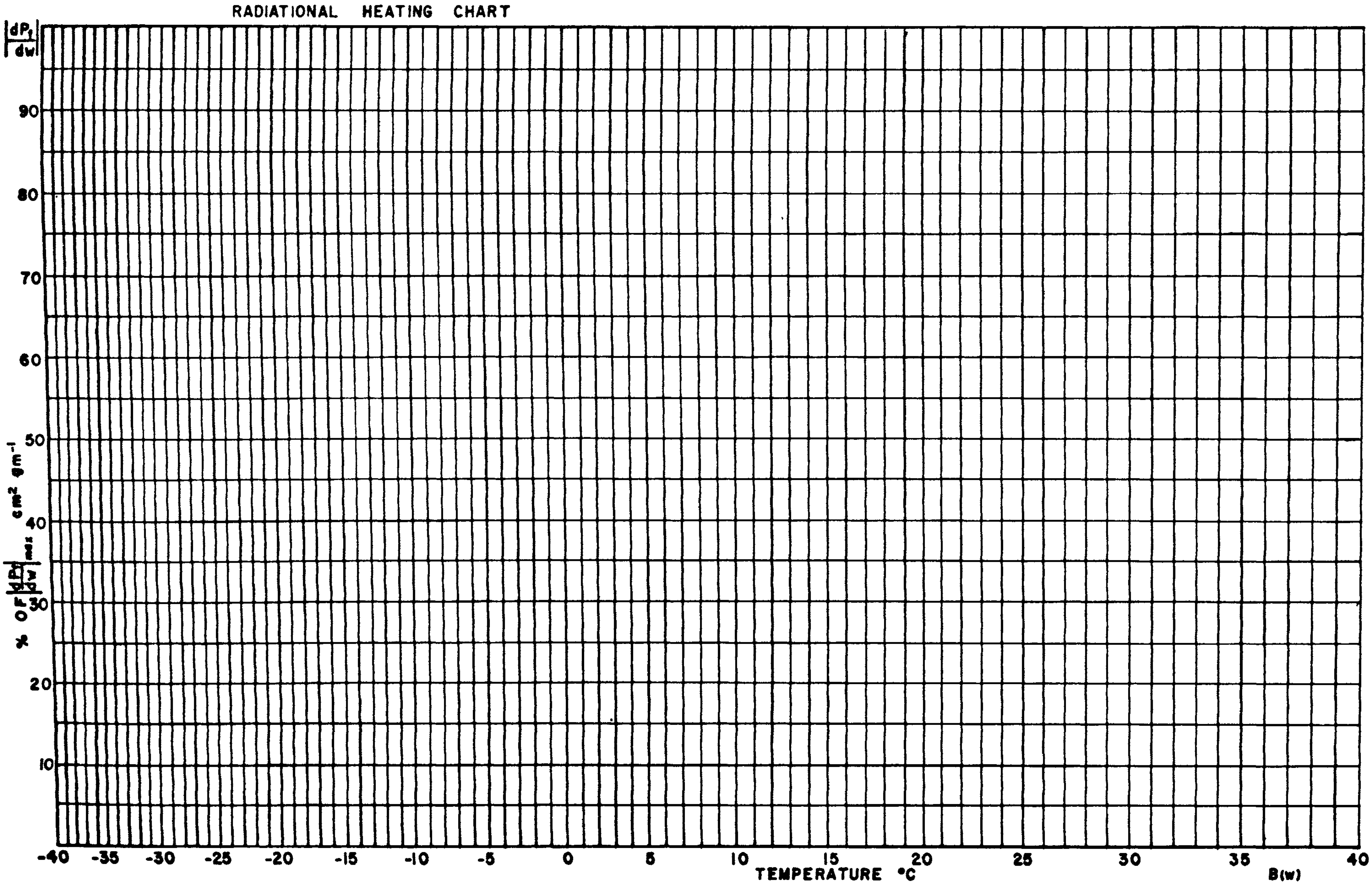
it is convenient to label the abscissa in terms of temperature so that the physical distance between T_1 and T_2 represents the radiant power difference $[\sigma(T_2^4 - T_1^4)]$.

To use the chart it is necessary to have data giving the temperature and absorber concentration as a function of altitude. A reference level (the altitude at which the heating rate is desired) is picked and this altitude, along with the entire range of altitudes in the troposphere, is converted to the temperature of each respective level. To plot the points on the chart, one must determine, from sounding data or a model, the optical depth (and thus dP_f/dw) between the temperature point being plotted and the reference level temperature. A schematic plot of an atmosphere in which temperature and absolute humidity decreases with height is shown in Figure II-6. Each area shows the sign which should be used when adding it to the total area. If the temperature is not monotonically decreasing with height the curve will be multivalued at certain temperatures and one must take care to assign the proper algebraic sign before adding the contribution due to an area with increasing temperature. The total area obtained is multiplied by $(\rho_w/c_p\rho_a)$ for the reference level to give the heating rate, a positive result indicates heating and a negative result cooling.

A blank chart with a linear ordinate labeled in percentage values is given in Figure II-7. This chart is easily adapted to data for dP_f/dw as a function of w by considering each value of dP_f/dw as a given percentage of the maximum value and plotting accordingly. A calibration area for the chart may be arrived at by considering the



SCHEMATIC OF THE HEATING RATE EQUATION
FIGURE II-6



RADIATIONAL HEATING CHART
FIGURE II-7

physical dimensions of the chart and the power differences at the extremes of the chart, then adjusting the location of a square to fit the maximum value of dP_f/dw. A calibration area of 0.1 cal/gm min is useful in a part of the atmosphere where a very large temperature gradient exists (such as the surface layer) and the resulting heating rate is large (tenths of a degree per minute). For applications in the free atmosphere where heating rates are usually in terms of degrees per day, a calibration area equal to 0.001 cal/gm min would be more appropriate. To use such an area requires drastic stretching of the ordinate axis to allow the area to span a temperature difference of one degree. This is more easily done with a numerical calculation than on a chart and for these purposes the chart serves mainly as an aid to visualization by providing a schematic of a particular situation.

A description of the numerical methods used to evaluate equations II-53 is given in Section III.

III. NUMERICAL SOLUTION OF HEATING RATE EQUATIONS

A. GENERAL DESCRIPTION OF PROGRAM

A FORTRAN program has been written to evaluate equation II-53, a listing of which is given in Appendix 1. The program utilizes the technique of integration by summing trapezoidal areas. This technique is easily visualized on a schematic chart (Figure II-7) as described in Section II. To insure the accuracy of this method of integration, the width of each trapezoid is automatically adjusted so that the change of the ordinate of the function being integrated is less than one percent of the maximum value of the function. This change in width is accomplished by adjusting the size of the increment of altitude between calculation points. The normal altitude increment is 100 meters and this value is reduced as necessary to a minimum of one millimeter. An altitude change of one millimeter corresponds to a very small change in temperature in the atmosphere, and it follows that the variation in black body radiation between two such temperatures is also very small. The trapezoidal areas being summed are formed by the difference in radiation between two altitudes and the average value of the function dP_f/dw between these altitudes. From the above considerations, it can be seen that errors resulting from integration by summing areas are kept to a minimum. It is very important to minimize integration errors since the final value is the small difference of two large values. The program assigns the proper algebraic sign to each incremental area depending on whether the working level is above or below the reference level and whether the temperature is increasing or decreasing with altitude. The program assumes that the stratosphere is isothermal, any outgoing radiation at the tropopause is lost into space and that there is no incoming radiation.

The total area under the curve is multiplied by $\rho_w/c_c\rho_a$ (calculated at the reference level) to give the heating rate. The calculations of the densities of the air (ρ_a) and the absorber (ρ_w) are handled by subprograms with some additional calculations done in the main program. The program prints out the value of several variables at each reference level. Among these are the reference level altitude, temperature, mixing ratio of absorbing material, and the heating rate. The last three variables are also put into the form of graphs relating the altitude to each of the other variables. These graphs are printed out on the high speed line printer. Computer time required is about ten seconds per reference level on an IBM/360 model 50.

1. <u>INPUT DATA REQUIREMENTS</u>

Data must be provided for some of the subprograms and this is read in before any parameters for the main program are read. The main program first reads the number of sets of data (i.e., the number of heating rate graphs to be produced) to be processed, and then the number of lapse rates to be used with the current set of data (minimum of two). This is followed by the lapse rates in units of °C/meter and altitudes, in meters, representing the base level of each respective lapse rate. The last lapse rate should be equal to zero and its base altitude is the tropopause altitude. Next, the ground temperature in °C and the constants used to determine the absorber concentrations plus the number of reference levels to be used with the current set of data are read. Lastly, the various reference altitudes in meters are read. The next data to be read is the number of lapse rates used in the following data set, and so on for the specified number of data sets.

B. DESCRIPTION OF SUBPROGRAMS

The temperature subprogram uses the data given for the lapse rates and their respective base altitudes. The subprogram can handle up to 50 different lapse rates, but it assumes that all altitudes at and above the last given altitude are isothermal. It would be a simple matter to substitute a subprogram that would interpolate between values from a "table look up" of observed temperature points of actual data. The temperature is handled as a double precision variable (i.e., 16 significant digits rather than the normal 8) since the temperature differences between two closely spaced altitudes may be very small.

A subprogram is provided to look up atmospheric pressure for given altitudes. This subprogram requires a table of altitudes in feet with appropriate pressures in inches of mercury which is among the data read in for the subprograms. The table of pressures and altitudes is from a standard model given by TREWARTHA (1954). Actual data could be substituted but it is not critical. The pressure at each reference altitude is needed in order to calculate the density of air. The density is calculated from the ideal gas law, using the temperature given by the temperature program, and is corrected for water vapor content.

A subprogram is used to calculate the optical depth of absorbing material between the working and reference level. This program would normally have to accept sounding data, or similar actual data, and perform an integration to find the optical depth. The listing in Appendix 1 shows an integration technique satisfactory for calculating the optical depth of water vapor when the density of this absorber is given by an exponential model. The model used is written

$$\rho_w(z) = \rho_{w_o} e^{-\beta z} \qquad \text{III-1}$$

where (ρ_w) is the density of water vapor, (ρ_{w_o}) is the density at altitude z=0, and (β) is a constant. To give a constant relative humidity in a standard atmosphere β would be 0.00045.

With a given set of data for dP_f/dw versus optical depth, a "table look up program" with interpolation is probably the easiest method of calculating values for dP_f/dw. For purposes of comparison with previous claculations concerning water vapor absorption, the data of ELSASSER (1960), for the rate of change of emissivity with optical thickness, was approximately fitted with a power function and the values of dP_f/dw calculated directly (see Section IV B).

IV. RESULTS AND CONCLUSIONS

A. CALCULATION OF HEATING RATES IN PARTICULATE POLLUTION

To use the methods we have developed for the study of an actual pollution situation would require a knowledge of the particular atmospheric temperature structure, the concentration of absorbers in the atmosphere and, most importantly, the variation of dP_f/dw as a function of optical depth for the absorbers under consideration. Although this information for polluted atmospheres near the ground is not available, it would be possible to make measurements to determine it. To make these measurements, one might use a device in which various gas and/or particulate mixtures could be introduced into a chamber where the infrared absorption and its rate of change could be measured with respect to the optical mass of intervening material. To get the long pathlengths needed, one might use some sort of mirror folding arrangement. Also, it might be possible to use a measuring scheme involving an actual atmosphere and a parallel-beam radiometer as described by BROOKS (1952).

Some data (arrived at by the above method) relating the integrated absorption to optical depth have been published, but the rate of change of absorption is rarely mentioned. An effort was made to differentiate graphically these data with respect to optical depth. However, heating is due largely to conditions near the area of interest (within a few hundred meters). In the free atmosphere (with normal temperature gradients), working altitudes very near the reference level are extremely close to the same temperature. The large values of dP_f/dw obtained for small optical depths are largely offset by the fact that these small optical depths are nearly isothermal. The optical depths corresponding to altitude changes of a few meters to a few hundred meters result in relatively small values of dP_f/dw and it is impractical to measure these small values from graphs (the form of most published data for absorption versus optical depth).

F. Möller performed calculations of heating rates near discontinuities in water vapor and haze concentration and an English translation of these results are reported in a paper by SHEPPARD (1958). Möller's calculations concerning the effects of haze on the heating rates were based on the assumption that the particulate material was in the form of gray body spheres one micron in diameter, with a mean absorption coefficient of 0.7×10^{-4}/cm. He assumed that these gray spheres were twice as numerous at the ground as at the top of the haze layer (1.67 km), that there were no particles above 1.67 km, and that the mean concentration was 1000/cm^3. With this model Möller predicted a cooling rate of 13°C/day at 1.67 km, or about 5.5°C/day more than with the water vapor discontinuity alone. The parameters used by Möller do not lend themselves to use in the heating rate program described in this thesis, since it would require that for the average particulate concentration

$$dP_f/dw = 0.7 \times 10^{-4}/\text{cm} \quad \text{for } w \leq 1.4 \times 10^4 \text{ cm}$$

$$dP_f/dw = 0 \quad \text{for } w > 1.4 \times 10^4 \text{ cm}$$

This would imply that a layer of haze only 140 meters thick would absorb all of the radiation incident upon it. The applicability to actual pollution conditions of such a model as Möller's is very doubtful. Current data indicate that the size of the particles is too large for the concentrations assumed (or conversly the concentrations too great), CLARK (1967) and that the absorption coefficient was too high, ROACH (1958).

Roach points out that it is unlikely that particulate absorption is as important as had been assumed earlier. However, he concludes that there is an important cooling effect due to particulate pollution and/or water vapor that serves to sustain a "pollution dome" over London. In any case, if data for dP_f/dw in polluted conditions can be supplied to the program, the calculated cooling rates can be compared to other factors (such as convective mixing, etc.) affecting the stability of the inversion and thus determine the relative importance of radiational cooling.

B. CALCULATION OF HEATING RATES FOR MOIST AIR

Water vapor is the most important single infrared absorber in the atmosphere. It is common to find an inversion where the water vapor content is larger below the inversion than above. This condition is similar to the case of particulate matter trapped under an inversion. A change in water vapor content will lead to increased infrared cooling or heating near the change. The heating effects of water vapor are an excellent indication of what might be expected from particulate matter. ELSASSER (1960) has calculated the value of

dP_f/dw (which he calls rate of change of emissivity) as a function of optical depth for water vapor and carbon dioxide. The Elsasser data on water vapor have been used to test the ability of the heating rate program to give results comparable to previous calculations made by Möller. Möller's calculations of heating rates near discontinuities of relative humidity were based on a surface temperature of 10°C, a lapse rate of 6°C/km, and a relative humidity of 100% below 1.67 km, and 20% above 1.67 km. When these parameters were put into the program, the results at all altitudes agreed very closely (within 10%) with those of Möller and a plot of both results appears in Appendix 2, Data Set 1.

Appendix 2 contains sets of output data from the program for a variety of input conditions. It should be kept in mind that the scales of the graphs are adjusted so that the maximum and minimum value of the variable being displayed correspond to the extremes of the graph. Data Set 2 shows the effects of a constant lapse rate of 6°C/km and a discontinuity of relative humidity from 80% to 20% at 500 meters altitude. A cooling rate ranging from 3.3°C/day at 50 meters to a maximum of 6.25°C/day at 500 meters is indicated and above 500 meters the cooling rate quickly returns to about 1°C/day. This cooling would tend to create a radiation inversion at 500 meters.

Data Set 3 illustrates the effects of a constant 80% relative humidity with an inversion characterized by a lapse rate of -6°C/km from 500 to 700 meters and 6°C/km elsewhere. The higher temperature peak shows a cooling rate of 2.85°C/day and the lower temperature peak at 500 meters, as would be expected, shows a relatively smaller cooling rate of 0.24°C/day. The cooling rate near the ground is about 1.7°C/day and well above the inversion about 1.25°C/day cooling is indicated. Results from a constant relative humidity of 20% are similar, indicating a maximum cooling rate of 2.3°C/day and a minimum of 0.2°C/day. These conditions will act to "smooth" the temperature inversion and cause it to dissipate.

It is very interesting to combine the humidity discontinuity of Data Set 2 with the temperature inversion of Data Set 3. The results of such a combination are given as Data Set 4, which shows that the cooling effect of the absorber discontinuity completely overshadows the relative warming effect of the temperature minimum at 500 meters. The cooling rate is about 3.3°C/day at 50 meters (the same as in Data Set 2) and increases to a maximum of about 4.1°C/day between 450 and 475 meters. The cooling rate at 500 meters is about the same as in data set 3 (2.8°C/day). The minimum cooling of about 1°C/day is shown to be between 500 and 525 meters. A relative cooling peak of 2.4°C/day is indicated at 700 meters and above this the cooling returns to normal. The maximum cooling occurs about 35 meters below the inversion and absorber discontinuity while the relative minimum occurs about 10 meters above. These conditions will tend to sustain the inversion and prevent the penetration of moisture and pollution laden air from below and thus form a "pollution dome".

Data Set 5 shows the result of a change in relative humidity from 80% to 20% at 500 meters with an isothermal atmosphere below 500 meters and a lapse rate of 6°C/km above. A cooling rate of

7.5°C/day is indicated at 500 meters, which will tend to create an inversion at 500 meters similar to the one in Data Set 4. Data Set 6 is the same as Set 5 except that the lapse rate below 500 meters is -6°C/km. The cooling rate indicated is 8.9°C/day; the increase over the result in Data Set 5 is due to the temperature peak and the increased temperature at 500 meters.

C. DISCUSSION OF ERRORS

Data Set 7 shows the results obtained from the program for a constant lapse rate of 6°C/km and a constant relative humidity of 50% over the entire troposphere. The cooling of 2.2°C/day indicated near the tropopause is incorrect and can be attributed to the fact that the program considers the stratosphere as a black body at 0°k. This effect is negligible below 7 km and does not affect the calculations made near the ground. To eliminate the error near the tropopause would require that the integration be extended through the stratosphere. Considering the approximations made in equation II-49 there is doubt that the program functions properly in the boundary layer extremely close to the ground. Again this fault is not serious in the region of interest. The heating rate equation used by the program contains no method of correcting for pressure broadening (or lack of it), but in the regions of interest this does not present a problem. Also, CO_2 has been neglected, but this should cause no problems since its effect is very small compared to that of water vapor.

Since the entire development of the program was intentionally approximate, the values calculated for heating rates should be considered as good indicators and not highly accurate. The most outstanding feature of this program is the ability to give excellent detail concerning the relative heating rates very near temperature and absorber anomalies.

D. SUGGESTED FUTURE INVESTIGATIONS

An effort should be made to provide some good data for dP_f/dw in polluted air, a study which will require a great deal of work to provide trustworthy results. However, the applications of such work should prove very interesting. A feasible experiment would be to make measurements of moisture content, temperature, and the radiation balance at each level in an actual pollution dome. With radiation balance data it would be possible to predict the actual heating rates. The program could be used to predict the contribution due to water vapor and perhaps infer the effects of the pollution.

BIBLIOGRAPHY

1. Air Pollution Volume I (1968) "Air Pollution and its Effects", edited by Arthur C. Stern, second edition, New York, Academic Press, Inc., 694 pp.

2. Brooks, F.A. (1952) "Atmospheric Radiation and its Reflection from the Ground", Journal of Meteorology, Vol. 9, No. 1, pp. 41-52.

3. Clark, William E. (1967) "Concentration and Size Distribution Measurements of Atmospheric Aerosols and a Test of the Theory of Self-Preserving Size Distributions", Journal of Atmospheric Sciences, Vol. 24, No. 6, Nov. 1967, pp. 677-687.

4. Elsasser, W.M. (1942) "Heat Transfer by Infrared Radiation in the Atmosphere", Harvard Meterological Monograph, Cambridge, Massachusetts, Harvard Printing Office, 69 pp.

5. Elsasser, W.M. (1960) Meteorological Monographs, Vol. 4, No. 23, 43 pp.

6. Feigel'son, E.M. (1965) "The Time Variation of Temperature in Stratus Clouds", Studies of Clouds, Precipitation and Thunderstorm Electricity, edited by N.I. Vul'fson and L.M. Levine, Translated from Russian by the American Meterological Society.

7. Fleagle, R.G. and Businger, J.A. (1963) "An Introduction to Atmospheric Physics", first edition, New York, Academic Press, Inc., 346 pp.

8. Kondrat'yev, K. Ya. (1965) "Radiative Heat Exchange in the Atmosphere", Translated and edited by O. Tedder and C.D. Walshaw, first edition, London, Pergamon Press Ltd., 411 p.

9. Möller, F. (1941) Beitr. Geophys., Vol. 58, p. 11.

10. Roach, W.T. and Goody, R.M. (1958) "Absorption and Emission in the Atmospheric Window from 770 to 1,250 cm-1", Quarterly Journal of Royal Meteorological Society, Vol. 84, pp. 319-333.

11. Sheppard, P.A. (1958) "The Effect of Pollution on Radiation in the Atmosphere", International Journal of Air Pollution, Vol. 1, Pergamon Press, pp. 31-43.

12. Trewartha, G.T. (1954) "An Introduction to Climate", third edition, New York, McGraw-Hill Book Co., 407 pp.

APPENDIX I
Program Listings

This appendix contains a listing of the programs described in the text. Also included is a typical set of input data satisfactory for the program as it appears in the listing. A physical description of each input variable appears as a comment in the program just prior to its usage.

```
      100 DOUBLE PRECISION T,TT,DT,TEMP,Z,DZ,RLEV
        1 DIMENSION RL(100),ATEM(100),RMIX(100),HRTD(100)
        2 COMMON WL(50),DP(50),G(50),H(50),TO,ALT(40),PR(40),RW1,RW2,HCHG,B,
         1NLR
C
C         READ IN DATA FOR SUBPROGRAMS
C
        3 READ (1,104) (ALT(J),PR(J),J=1,19)
      104 FORMAT (2E18.8)
C
C         READ IN THE NUMBER OF DATA SETS (NDS)
C
      204 READ (1,304) NDS
      304 FORMAT (I6)
      404 DO 369 IDATA = 1,NDS
C
C         READ IN THE NUMBER OF LAPSE RATES (NLR)  AND
C         READ IN LAPSE RATES (G(J) AND BASE ALTITUDES FOR SAME (H(J))
C
      504 READ (1,304) NLR
        5 READ (1,104) (G(J),H(J),J=1,NLR)
      106 FORMAT (5F14.6)
C
C         READ IN NUMBER OF REFERENCE LEVELS (K)  GROUND TEMPERATURE (TG)
C         CONSTANT FOR EXPONENTIAL MODEL OF VAPOR DENSITY (B)  INITIAL
C         VAPOR DENSITY (RW1)  SECOND VAPOR DENSITY (RW2)    AND THE
C         ALTITUDE WHERE THE DENSITY CHANGES (HCHG)
C
        7 READ (1,108) K,TG,B,RW1,RW2,HCHG
      108 FORMAT (I4,F12.0,4F12.6)
C
C         READ IN THE  K  REFERENCE LEVELS (RL(J))
C
        9 READ (1,106) (RL(J),J=1,K)
      109 WRITE (3,209)
      209 FORMAT (1H1)
C
C         DEFINE MOLECULAR GAS CONSTANT (RM)  SPECIFIC HEAT OF AIR (CP) AND
C         BOLTZMANS CONSTANT (SIGMA)
C
       10 RM = 0.287
       11 CP = 0.239
       12 SIGMA = 0.817E-10
       13 TO = TG + 273.16
       14 DO 57 I=1,K
C
C         LOOP CONTROL FOR CALCULATION OF HEATING RATE AT EACH REFERENCE
C         LEVEL
```

```
C
   15 AREA = 0.0
   16 DZ = 0.0
   17 TT = TO
   18 RLEV = RL(I)
   19 CONTINUE
   20 Z = DZ
  124 W1 = OPTD(Z,RLEV)
  224 IF (W1) 324,524,524
  324 W1S = -1.0
  424 GO TO 25
  524 W1S = 1.0
   25 T1 = DPDW(ABS(W1))
  125 INC = 0
C
C       CONTROLS FOR ADJUSTING THE RATE OF INCREMENT (ADZ)
C
  225 IF (DABS(RLEV-Z)-100.0) 325,325,825
  325 IF (DABS(RLEV-Z)-10.00) 425,425,625
  425 ADZ = 1.0
  525 GO TO 925
  625 ADZ = 10.0
  725 GO TO 925
  825 ADZ = 100.0
  925 CONTINUE
   26 DZ = Z + ADZ
   31 T2 = DPDW(ABS(OPTD(DZ,RLEV)))
   32 IF (0.01 - ABS((T1-T2)/313.0)) 33,33,35
   33 ADZ = ADZ/10.0
  133 INC = INC + 1
   34 GO TO (26,26,26,35),INC
   35 DPAVE = (T1 + T2)/2.0
   36 T = TT
   37 TT = TEMP(DZ)
   38 DT = T - TT
   39 DE = SIGMA*(4.*T*T*T*DT+6.*T*T*DT*DT+4.*T*DT*DT*DT+DT*DT*DT*DT)
C
C       ACCUMULATION OF AREA BY INTEGRATION   WITH SIGMA*T**4 CALCULATION
C
   40 AREA = AREA + DE*DPAVE*W1S
   41 IF (DZ-H(NLR)) 19,42,42
   42 AREA = AREA - DPAVE*SIGMA*TT*TT*TT*TT
   43 IF (RL(I)-HCHG) 44,44,46
   44 RWC = RW1
   45 GO TO 47
   46 RWC = RW2
```

```
C
C        CALCULATION OF VAPOR DENSITY AT REFERENCE LEVEL (RWL)  AIR
C        TEMPERATURE AT REFERENCE LEVEL (ATEM(I))  AIR DENSITY AT
C        REFERENCE LEVEL (RAIRL)  VAPOR DENSITY CORRECTION FOR AIR
C
C        DENSITY (RAM)    MIXING RATIO IN GRAMS/KILOGRAM (RMIX(I))
C        HEATING RATE IN DEGREES/MINUTE  (HRTM)  HEATING RATE IN DEGREES
C        PER DAY (HRTD(I))
C
   47 RWL = RWO*EXP(-B*RL(I))
   48 ATEM(I) = TEMP(RLEV) - 273.16
   49 RAIRL = PRES(RLEV)/(RM*(ATEM(I)+273.16))
   50 RAM = RAIRL*RAIRL/(RAIRL+0.61*RWL)
   51 RMIX(I) = 1000.0*RWL/RAM
   52 HRTM = (AREA*RWL)/(CP*RAM)
   53 HRTD(I) = HRTM*1440.0
C
C        WRITE OUT RESULTS AT REFERENCE LEVELS ALONG WITH CONSTANTS USED
C        IN THE CALCULATIONS    ALSO PRODUCE PLOTS OF THE RESULTS
C
   54 WRITE (3,255) RL(I),AREA,ATEM(I),RMIX(I),HRTD(I)
  255 FORMAT (////10X,'RL = ',F8.1,5X,'AREA = ',1PE16.8,5X,'TEMP = '0PF1
     10.4,5X,'MIX RATIO = ',F6.2,5X,'HEAT RATE = ',F10.3)
  355 WRITE (3,455) RWL,RAIRL,RAM,W1,T1
  455 FORMAT (/,10X,'RWL = ',F6.2,'   RAIR(RL) = ',F8.2,'   RAM = ',F8.2
     1,'   W AT TR-PAUSE = ',F10.6,'   DPDW = ',F12.6)
   57 CONTINUE
   58 CALL PPLT (HRTD,RL,K)
   59 WRITE (3,260)
  260 FORMAT (//16X,'HEATING RATE (DEG C/DAY) VS ALTITUDE (METERS)')
   61 CALL PPLT (ATEM,RL,K)
   62 WRITE (3,263)
  263 FORMAT (//16X,'TEMPERATURE (DEG C) VS ALTITUDE (METERS)')
   64 CALL PPLT (RMIX,RL,K)
   65 WRITE (3,266)
  266 FORMAT (//16X,'MIXING RATIO OF ABSORBER (GRAMS/KG) VS ALTITUDE (ME
  2671TERS)',////)
   68 WRITE (3,269) TG,B,RW1,RW2,HCHG,(G(J),H(J),J=1,NLR)
  269 FORMAT (/5X,'GRND TEMP = ',F6.2,'   BETA = ',1PE12.3,'   RW1 = ',0
     1PF6.2,'   RW2 = ',F6.2,'   ALT OF HCHG = ',1PE12.5,////,T22,'LAPSE
     2 RATE',T42,'ALTITUDE',/,(/,20X,1PE12.4,T41,0PF9.1))
 1269 WRITE (3,2269)
 2269 FORMAT (1H1,T22,'ALTITUDE',T32,'TEMPERATURE',T45,'MIX RATIO',T55,'
     1HEAT RATE',//)
 3269 WRITE(3,4269) (RL(J),ATEM(J),RMIX(J),HRTD(J),J=1,K)
 4269 FORMAT (T23,F9.1,T33,F8.4,T46,F6.2,T56,F8.3)
  369 CONTINUE
   70 CALL EXIT
   71 END
```

```
      1 FUNCTION DPDW(W)
C   ELSASSER FUNCTION
C
C      FUNCTION SUPROGRAM FOR CALCULATION OF RATE OF CHANGE OF
C      ABSORPTION WITH OPTICAL DEPTH
C
      2 IF (2.0E-05 - W) 3,5,5
      3 DPDW = 0.105/(W**0.845562)
      4 RETURN
      5 DPDW = 987.3
      6 RETURN
      7 END
```

```
      1 DOUBLE PRECISION FUNCTION TEMP(ZDUM)
C
C      FUNCTION SUBPROGRAM FOR CALCULATION OF TEMPERATURE AS A
C      FUNCTION OF ALTITUDE  WITH GIVEN LAPSE RATES
C
        DOUBLE PRECISION T
      2 COMMON WL(50),DP(50),G(50),H(50),TO,ALT(40),PR(40),RW1,RW2,HCHG,B,
     1NLR
    101 Z = ZDUM
    102 IF (Z-H(NLR)) 302,202,202
    202 Z = H(NLR) - 1.0
    302 CONTINUE
      3 I = 1
      4 T = TO
      5 IF (Z-H(I+1)) 9,9,6
      6 T = T + G(I)*(H(I+1)-H(I))
      7 I = I + 1
      8 GO TO 5
      9 TEMP = T + G(I)*(Z-H(I))
     10 RETURN
     11 END
```

```
      1 FUNCTION OPTD(Z,RLEV)
C
C     FUNCTION SUBPROGRAM FOR CALCULATION OF OPTICAL DEPTH
C
      2 COMMON WL(50),DP(50),G(50),H(50),TO,ALT(40),PR(40),RW1,RW2,HCHG,B,
       1NLP
      3 IF (RLEV-HCHG) 4,4,9
      4 IF (Z-HCHG) 5,5,7
      5 OPTD = RW1*(EXP(-B*Z)-EXP(-B*RLEV))/(B*10000.0)
      6 RETURN
      7 OPTD = ((RW1-RW2)*EXP(-B*HCHG) + RW2*EXP(-B*Z) - RW1*EXP(-B*RLEV))
       1/(B*10000.0)
      8 RETURN
      9 IF (Z-HCHG) 12,10,10
     10 OPTD = RW2*(EXP(-B*Z)-EXP(-B*RLEV))/(B*10000.0)
     11 RETURN
     12 OPTD = ((RW2-RW1)*EXP(-B*HCHG) + RW1*EXP(-B*Z) - RW2*EXP(-B*RLEV))
       1/(B*10000.0)
     13 RETURN
     14 END
```

```
      1 FUNCTION PRES(Z)
C
C     SUBPROGRAM FOR CALCULATION OF ATMOSPHERIC PRESSURE AS A
C     FUNCTION OF ALTITUDE  FROM A GIVEN SET OF PARAMETERS
C
      2 COMMON WL(50),DP(50),G(50),H(50),TO,ALT(40),PR(40),RW1,RW2,HCHG,B,
       1NLR
      3 AL = 3.28084*Z
      4 IF (AL-18000.0) 5,5,11
      5 I = 1
      6 I = I + 1
      7 IF (AL - ALT(I)) 8,8,6
      8 P = PR(I) + (PR(I-1)-PR(I))*(ALT(I)-AL)/(ALT(I)-ALT(I-1))
      9 PRES = P*3386.4
     10 RETURN
     11 PRES  = 50529.8*EXP(-4.38106E-05*(AL - 18000.0))
     12 RETURN
     13 END
```

STATEMENT NUMBER	Cont.	FORTRAN STATEMENT

```
         0.0            29.92
      1000.0            28.86
      2000.0            27.82
      3000.0            26.81
      4000.0            25.84
      5000.0            24.89
      6000.0            23.89
      7000.0            23.09
      8000.0            22.22
      9000.0            21.38
     10000.0            20.85
     11000.0            19.97
     12000.0            19.03
     13000.0            18.29
     14000.0            17.57
     15000.0            16.88
     16000.0            16.21
     17000.0            15.56
     18000.0            14.94
```

INPUT DATA FOR THE

PRESSURE SUBPROGRAM

(19 CARDS)

STATEMENT NUMBER	Cont.	FORTRAN STATEMENT

```
     1   (NUMBER OF DATA SETS TO BE PROCESSED)

     4   (NUMBER OF LAPSE RATE CARDS TO FOLLOW IN THIS DATA SET)
            -0.0060                 0.0        (LAPSE RATE CARD NO. 1)
            -0.0030               500.0        (LAPSE RATE CARD NO. 2)
            -0.0060               700.0        (LAPSE RATE CARD NO. 3)
                0.0             10000.0        (LAPSE RATE CARD NO. 4)
         (LAPSE RATES)       (BASE ALTITUDES)


  20 (K)     20.0 (TG)  0.00045 (B)    13.8 (RW1)    3.46 (RW2)   500.0 (HCHG)


        50.000         150.00        250.00        350.00        400.00
        450.00         475.00        500.00        525.00        550.00
        600.00         650.00        675.00        700.00        725.00
        750.00         1000.0        1250.0        1500.0        2000.0
                              (REFERENCE LEVELS)
```

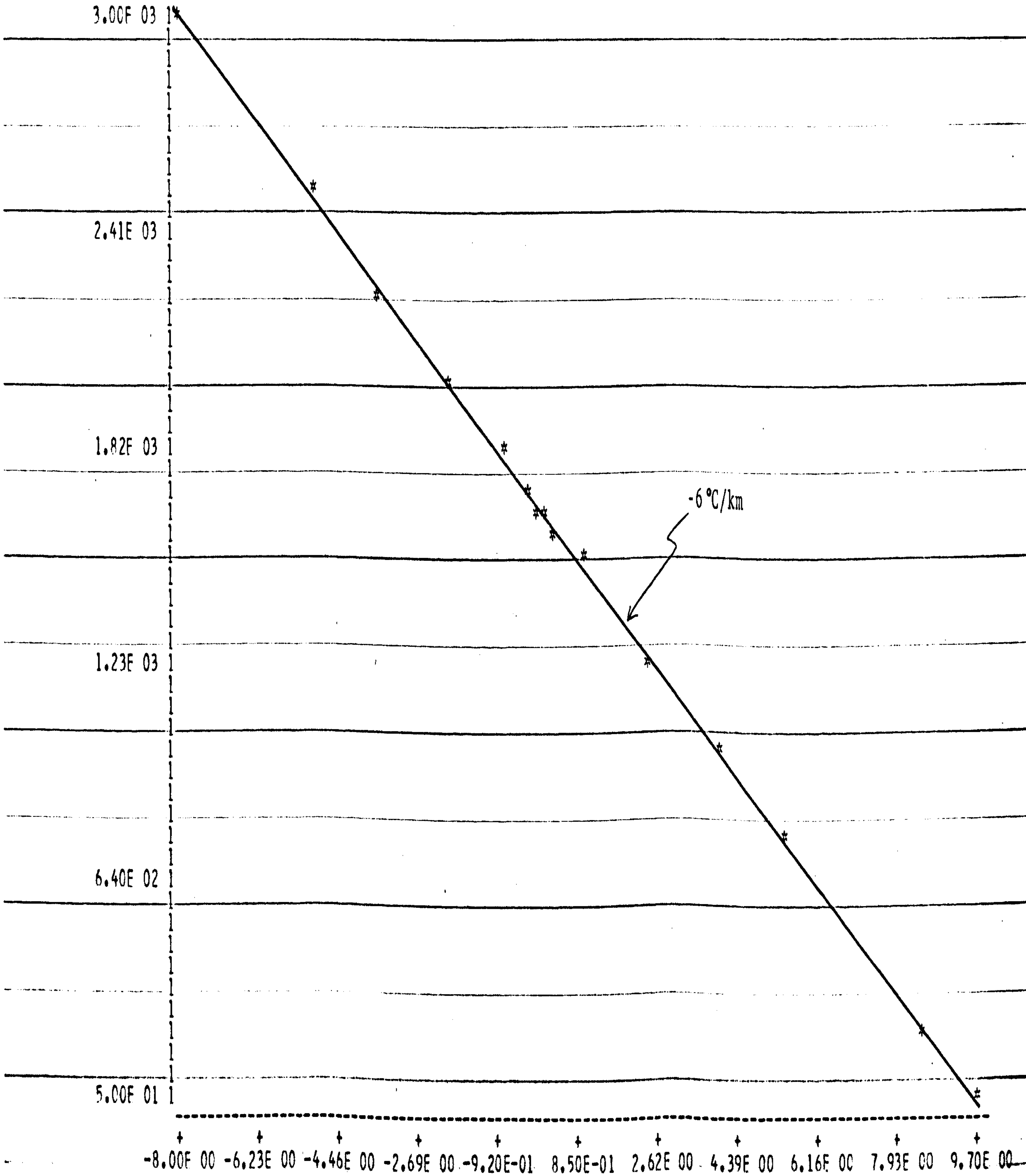
TYPICAL SET OF INPUT DATA FOR THE MAIN PROGRAM

APPENDIX II

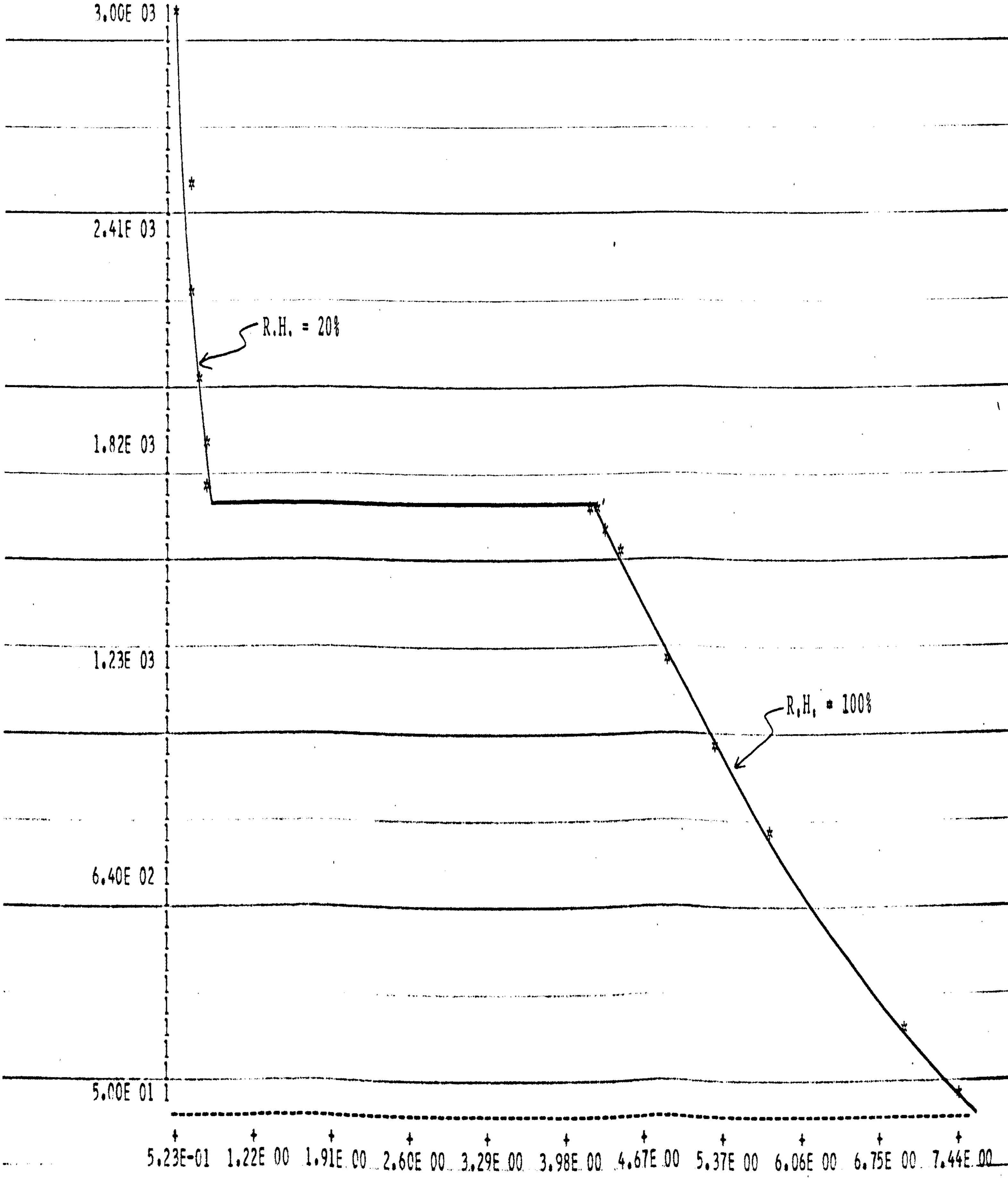
Program Output for a Variety of Input Conditions

This appendix contains output from the program as described in the text. The output is arranged into sets and is largely self-explanatory.

Pages 57 to 72 appear in a pocket at the end of this paper.

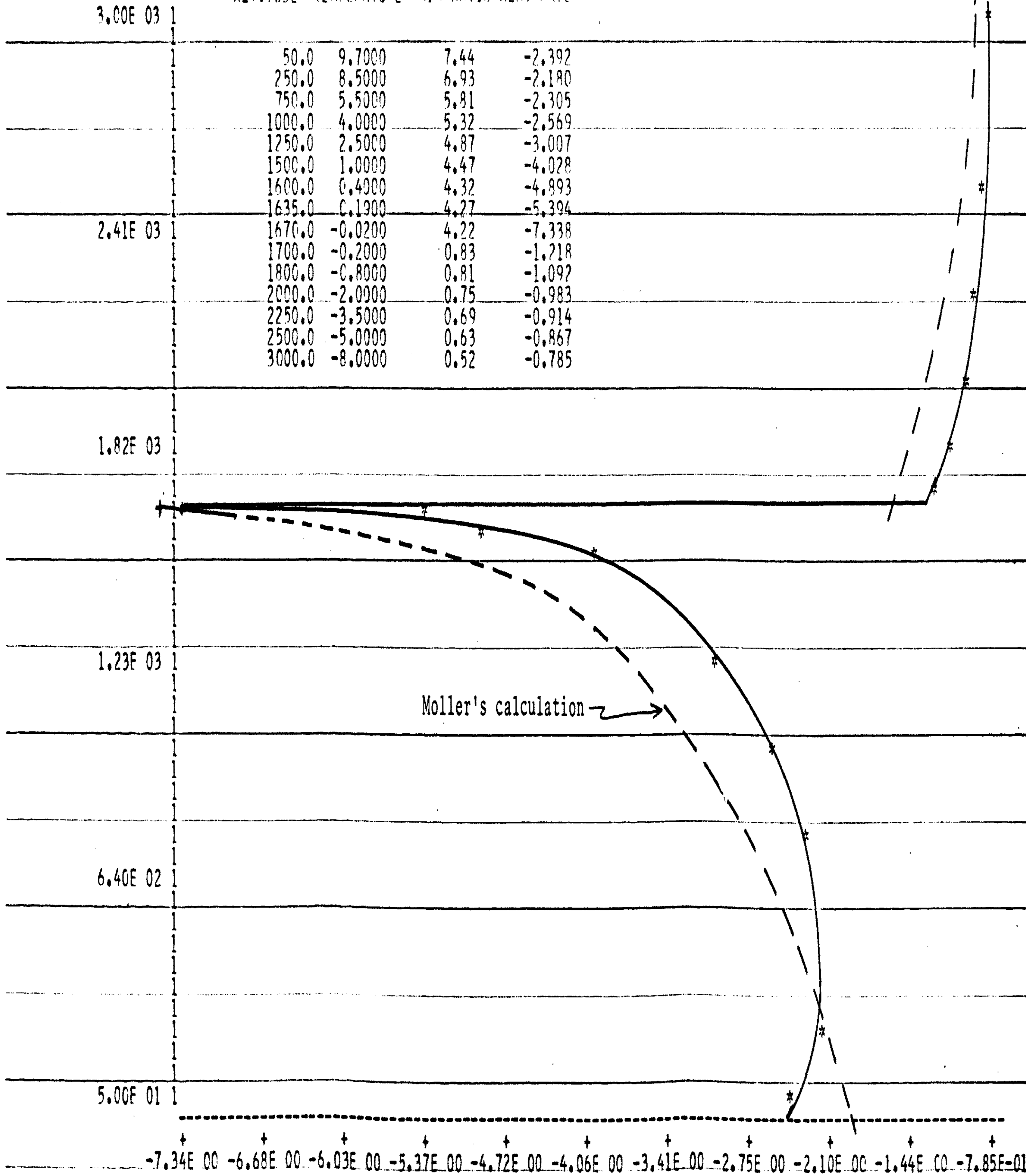


TEMPERATURE (DEG C) VS ALTITUDE (METERS) for DATA SETS 1 and 2

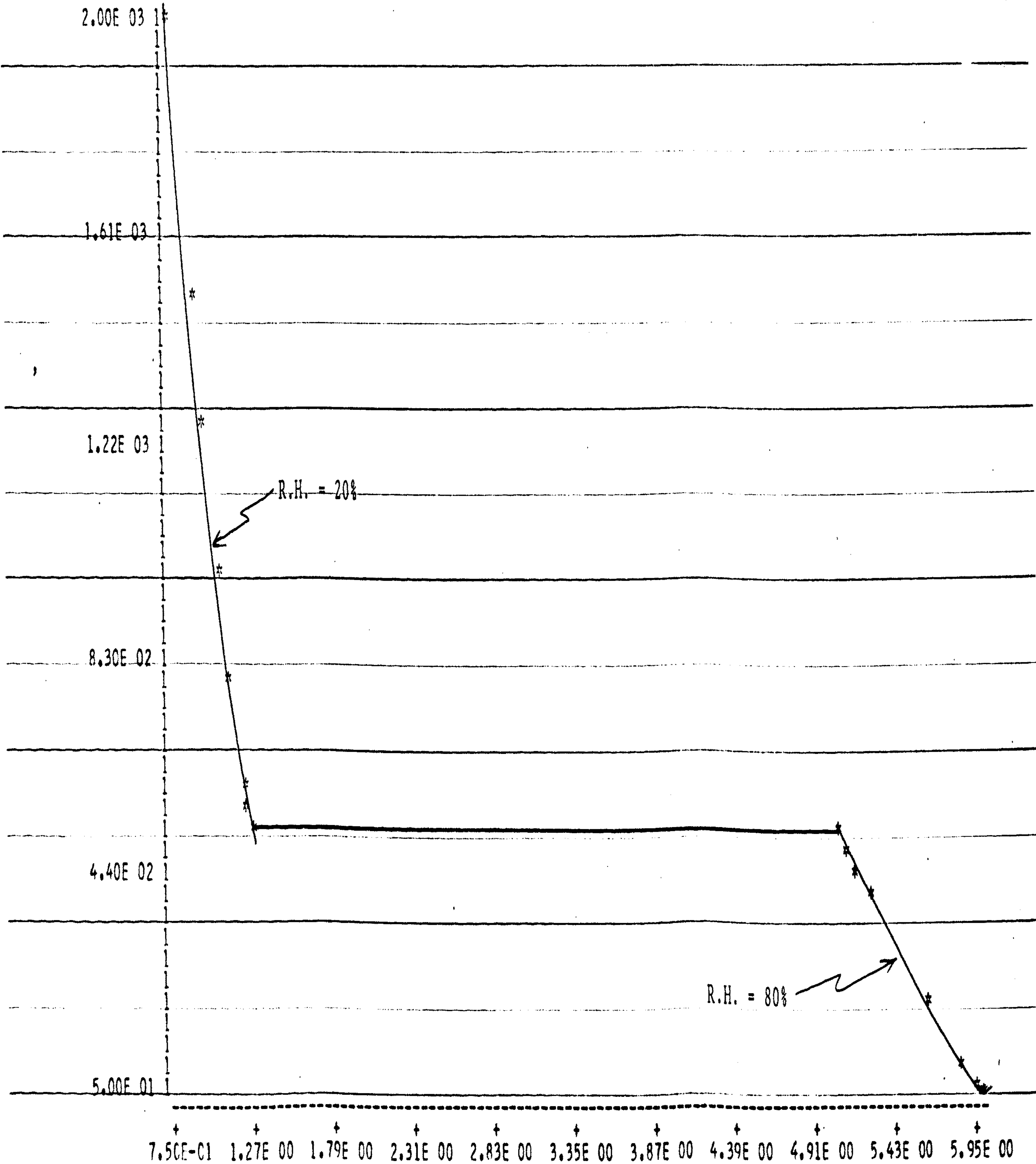


MIXING RATIO OF ABSORBER (GRAMS/KG) VS ALTITUDE (METERS) for DATA SET 1

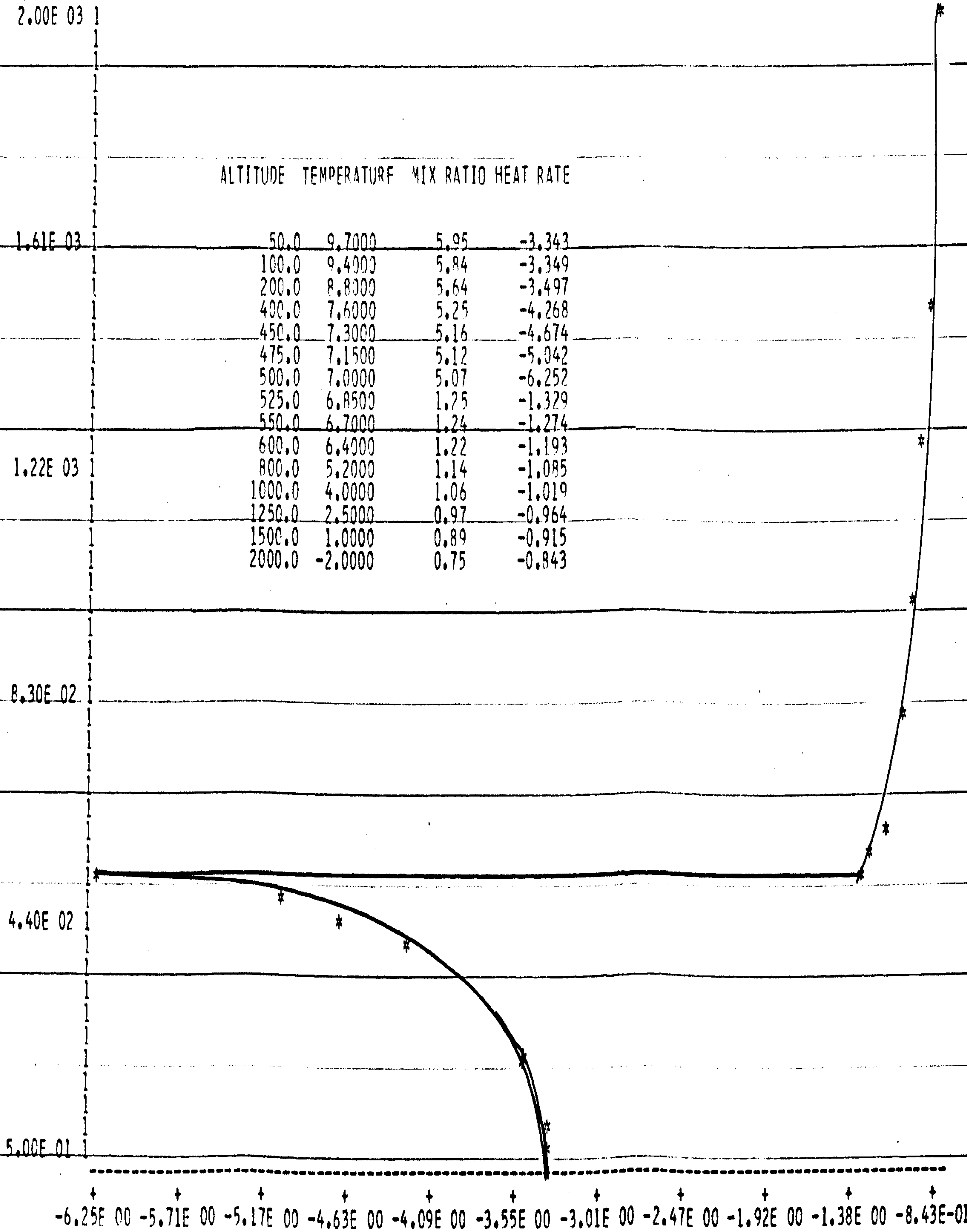
ALTITUDE	TEMPERATURE	MIX RATIO	HEAT RATE
50.0	9.7000	7.44	-2.392
250.0	8.5000	6.93	-2.180
750.0	5.5000	5.81	-2.305
1000.0	4.0000	5.32	-2.569
1250.0	2.5000	4.87	-3.007
1500.0	1.0000	4.47	-4.028
1600.0	0.4000	4.32	-4.893
1635.0	0.1900	4.27	-5.394
1670.0	-0.0200	4.22	-7.338
1700.0	-0.2000	0.83	-1.218
1800.0	-0.8000	0.81	-1.092
2000.0	-2.0000	0.75	-0.983
2250.0	-3.5000	0.69	-0.914
2500.0	-5.0000	0.63	-0.867
3000.0	-8.0000	0.52	-0.785



HEATING RATE (DEG C/DAY) VS ALTITUDE (METERS) DATA SET 1

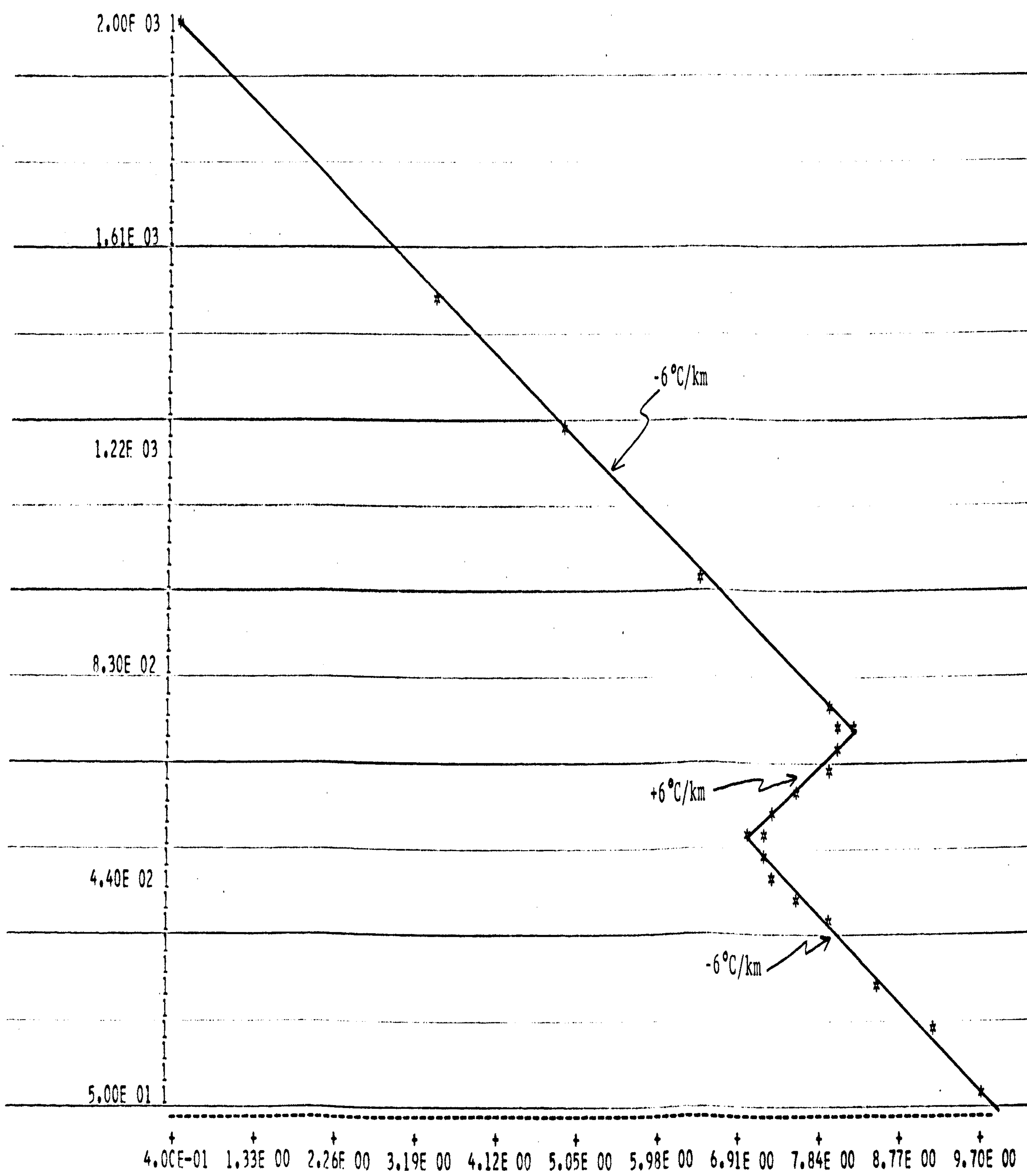


MIXING RATIO OF ABSORBER (GRAMS/KG) VS ALTITUDE (METERS) for DATA SETS 2, 4, 5, and 6

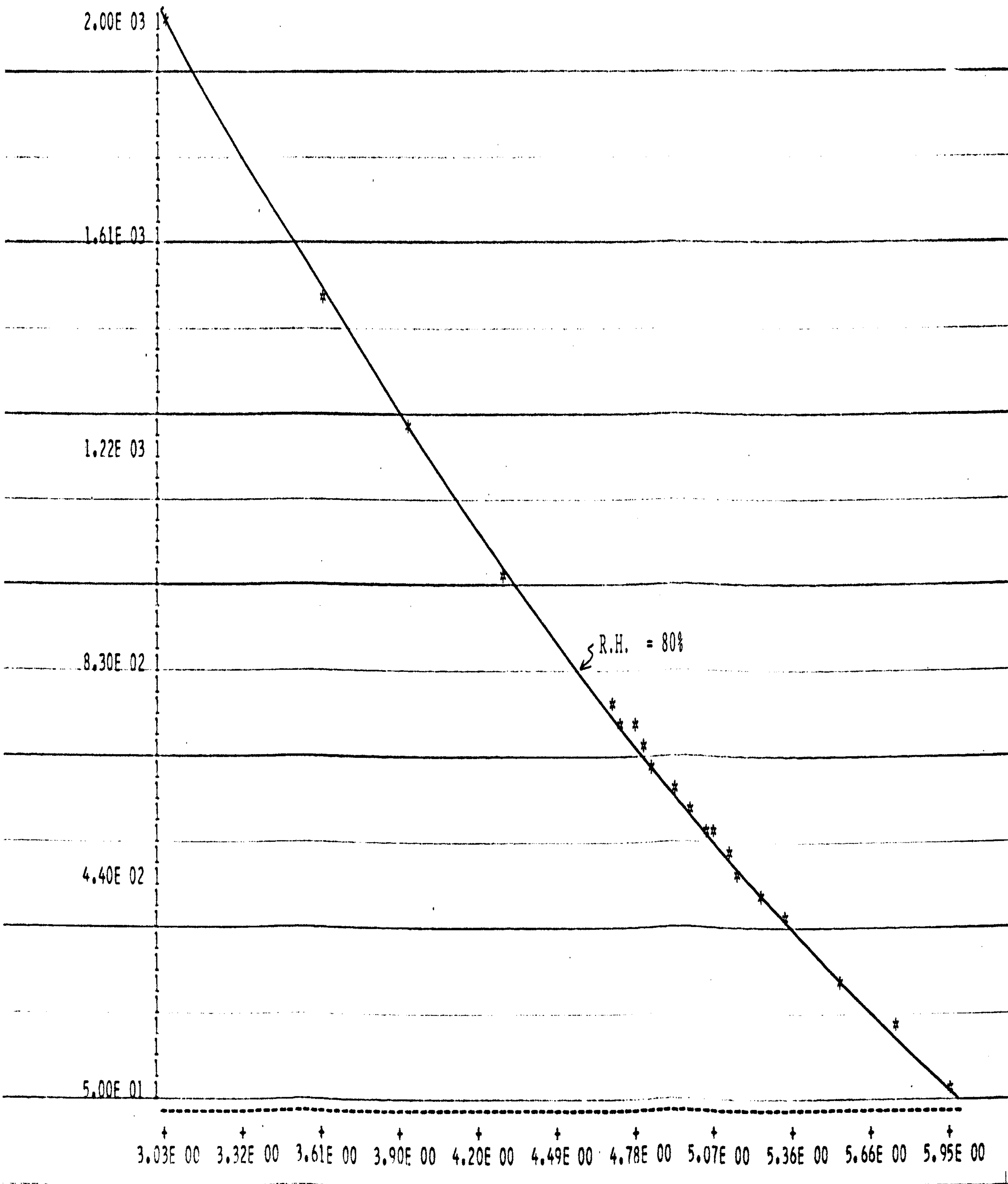


ALTITUDE	TEMPERATURE	MIX RATIO	HEAT RATE
50.0	9.7000	5.95	-3.343
100.0	9.4000	5.84	-3.349
200.0	8.8000	5.64	-3.497
400.0	7.6000	5.25	-4.268
450.0	7.3000	5.16	-4.674
475.0	7.1500	5.12	-5.042
500.0	7.0000	5.07	-6.252
525.0	6.8500	1.25	-1.329
550.0	6.7000	1.24	-1.274
600.0	6.4000	1.22	-1.193
800.0	5.2000	1.14	-1.085
1000.0	4.0000	1.06	-1.019
1250.0	2.5000	0.97	-0.964
1500.0	1.0000	0.89	-0.915
2000.0	-2.0000	0.75	-0.843

HEATING RATE (DEG C/DAY) VS ALTITUDE (METERS) DATA SET 2

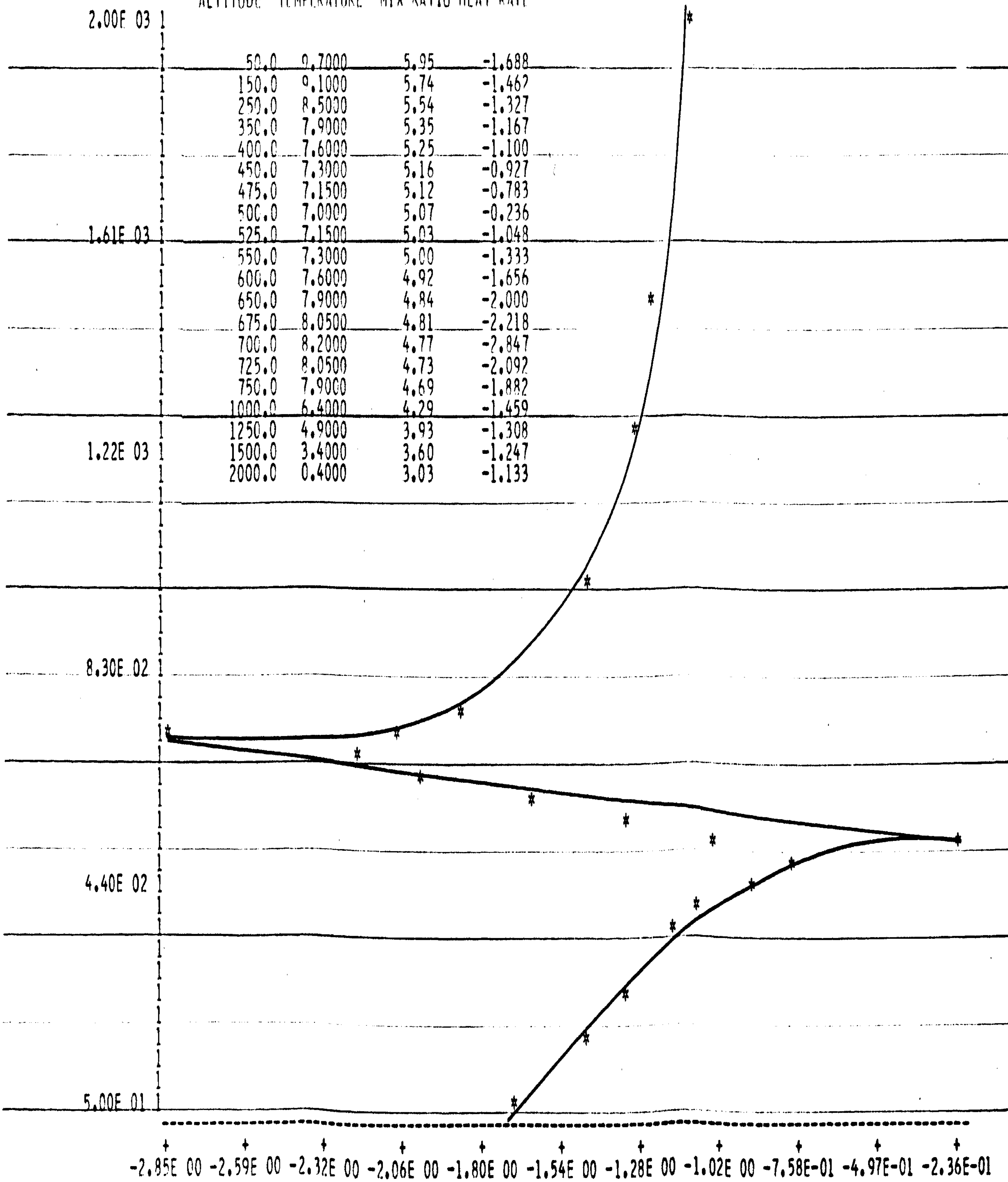


TEMPERATURE (DEG C) VS ALTITUDE (METERS) for DATA SETS 3 and 4

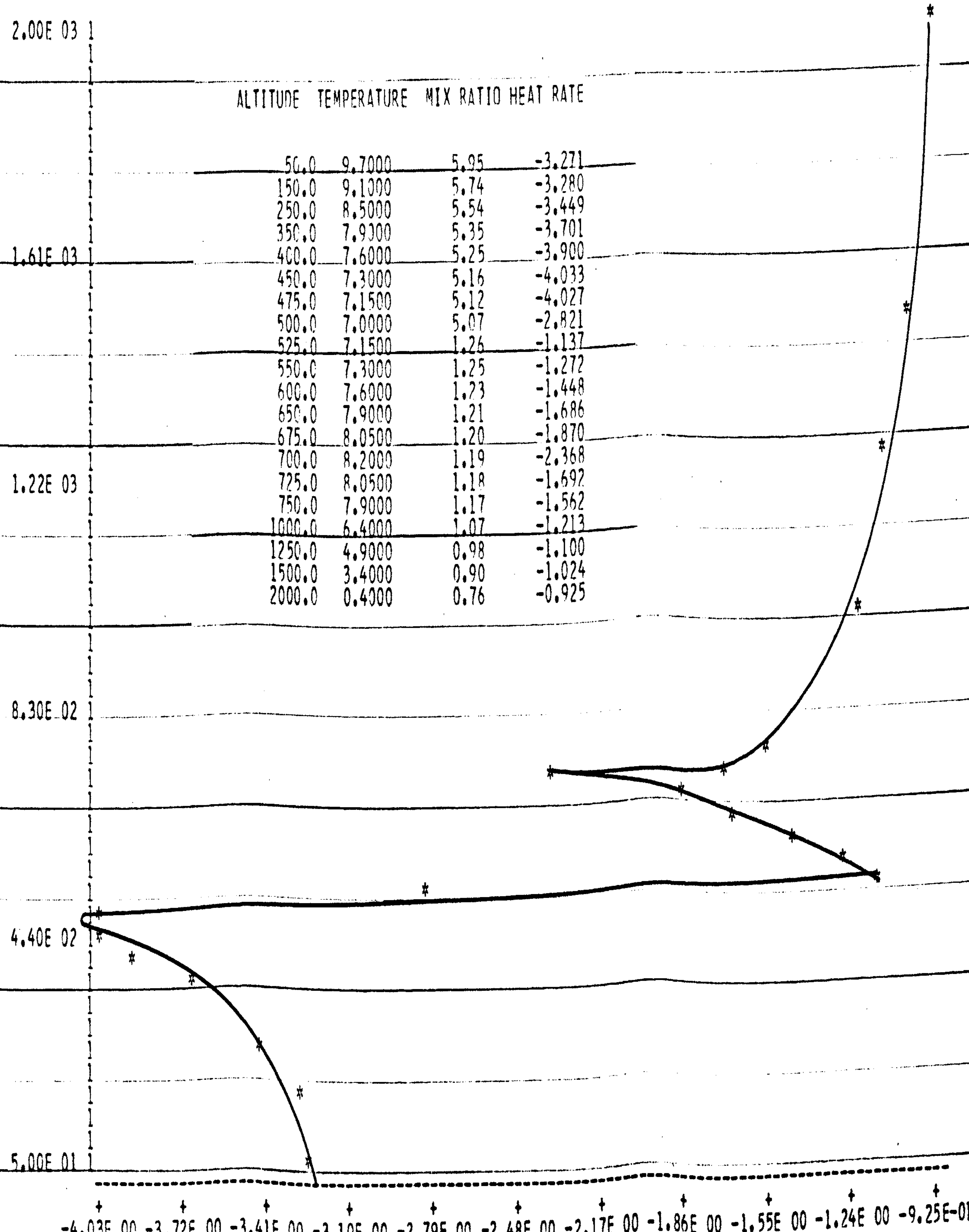


MIXING RATIO OF ABSORBER (GRAMS/KG) VS ALTITUDE (METERS) DATA SET 3

ALTITUDE	TEMPERATURE	MIX RATIO	HEAT RATE
50.0	9.7000	5.95	-1.688
150.0	9.1000	5.74	-1.462
250.0	8.5000	5.54	-1.327
350.0	7.9000	5.35	-1.167
400.0	7.6000	5.25	-1.100
450.0	7.3000	5.16	-0.927
475.0	7.1500	5.12	-0.783
500.0	7.0000	5.07	-0.236
525.0	7.1500	5.03	-1.048
550.0	7.3000	5.00	-1.333
600.0	7.6000	4.92	-1.656
650.0	7.9000	4.84	-2.000
675.0	8.0500	4.81	-2.218
700.0	8.2000	4.77	-2.847
725.0	8.0500	4.73	-2.092
750.0	7.9000	4.69	-1.882
1000.0	6.4000	4.29	-1.459
1250.0	4.9000	3.93	-1.308
1500.0	3.4000	3.60	-1.247
2000.0	0.4000	3.03	-1.133

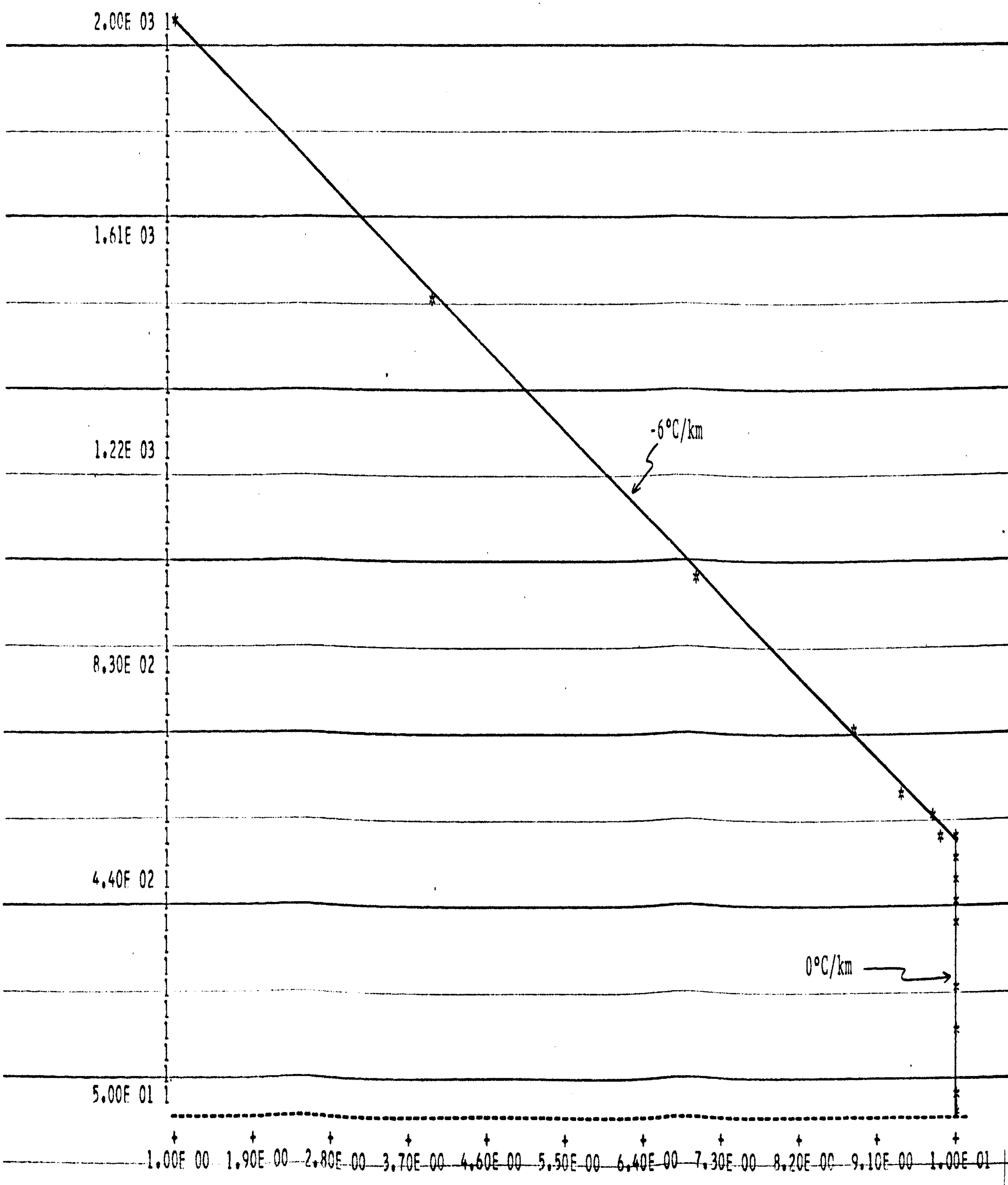


HEATING RATE (DEG C/DAY) VS ALTITUDE (METERS) DATA SET 3

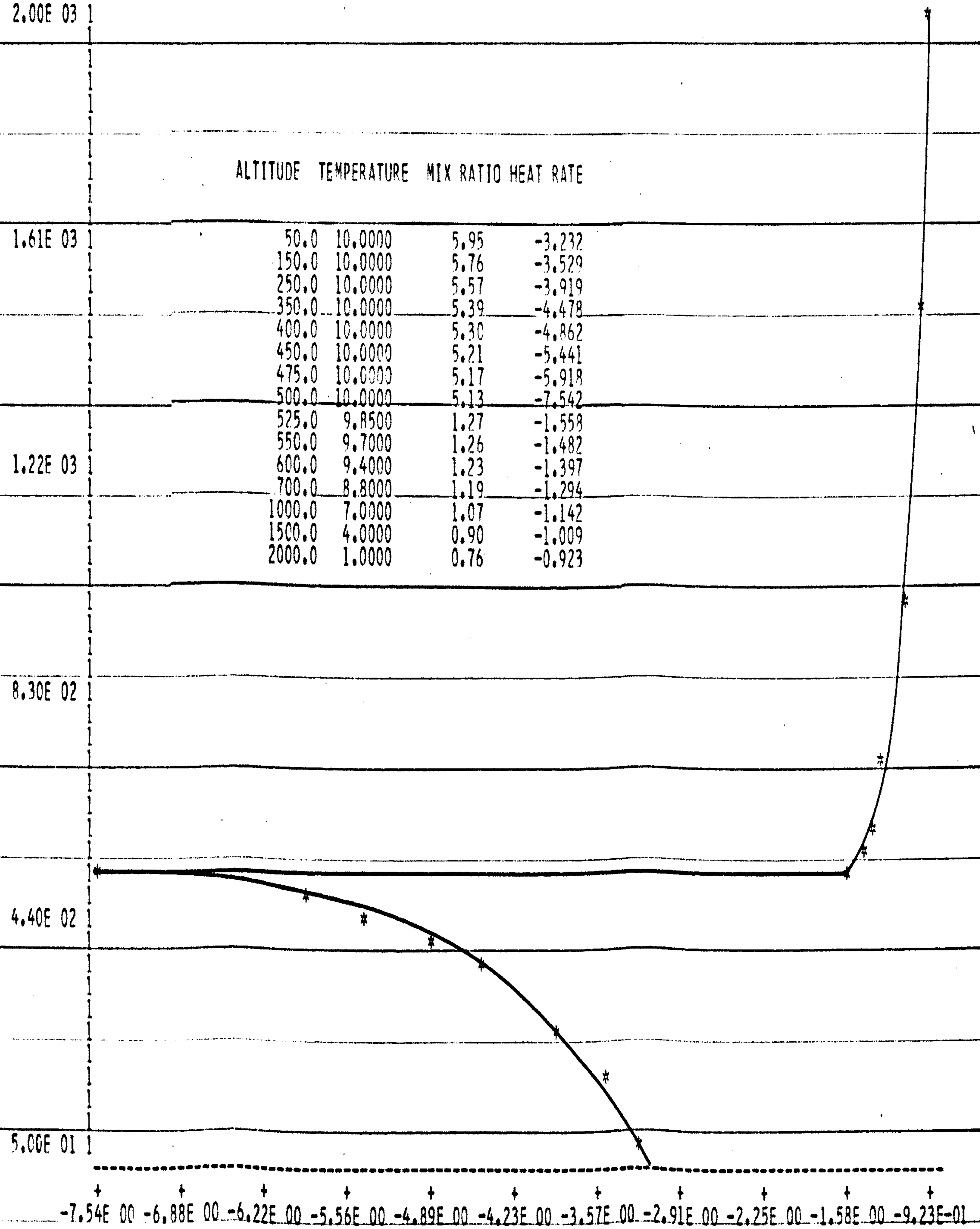


ALTITUDE	TEMPERATURE	MIX RATIO	HEAT RATE
50.0	9.7000	5.95	-3.271
150.0	9.1000	5.74	-3.280
250.0	8.5000	5.54	-3.449
350.0	7.9000	5.35	-3.701
400.0	7.6000	5.25	-3.900
450.0	7.3000	5.16	-4.033
475.0	7.1500	5.12	-4.027
500.0	7.0000	5.07	-2.821
525.0	7.1500	1.26	-1.137
550.0	7.3000	1.25	-1.272
600.0	7.6000	1.23	-1.448
650.0	7.9000	1.21	-1.686
675.0	8.0500	1.20	-1.870
700.0	8.2000	1.19	-2.368
725.0	8.0500	1.18	-1.692
750.0	7.9000	1.17	-1.562
1000.0	6.4000	1.07	-1.213
1250.0	4.9000	0.98	-1.100
1500.0	3.4000	0.90	-1.024
2000.0	0.4000	0.76	-0.925

HEATING RATE (DEG C/DAY) VS ALTITUDE (METERS) DATA SET 4

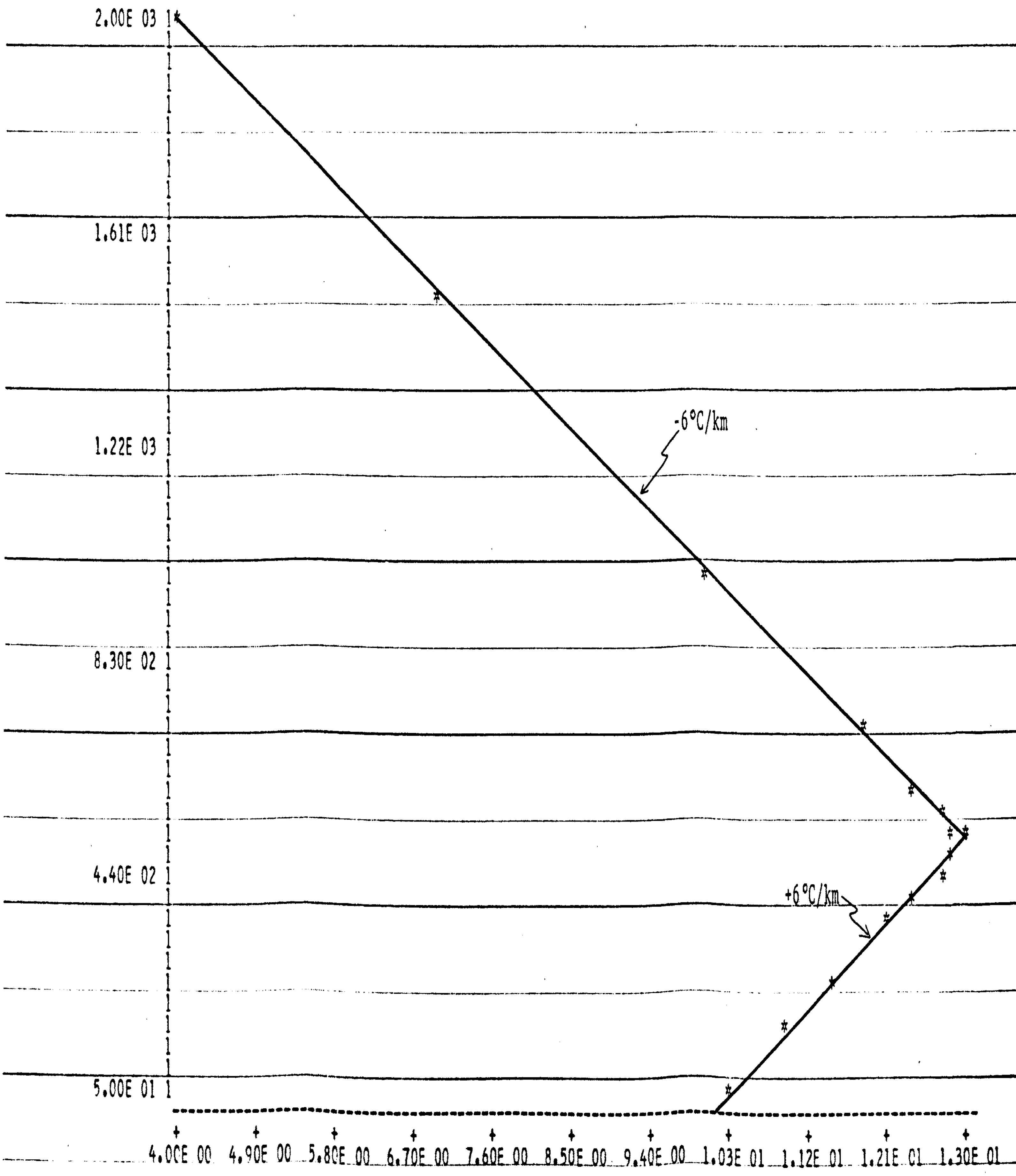


TEMPERATURE (DEG C) VS ALTITUDE (METERS) DATA SET 5

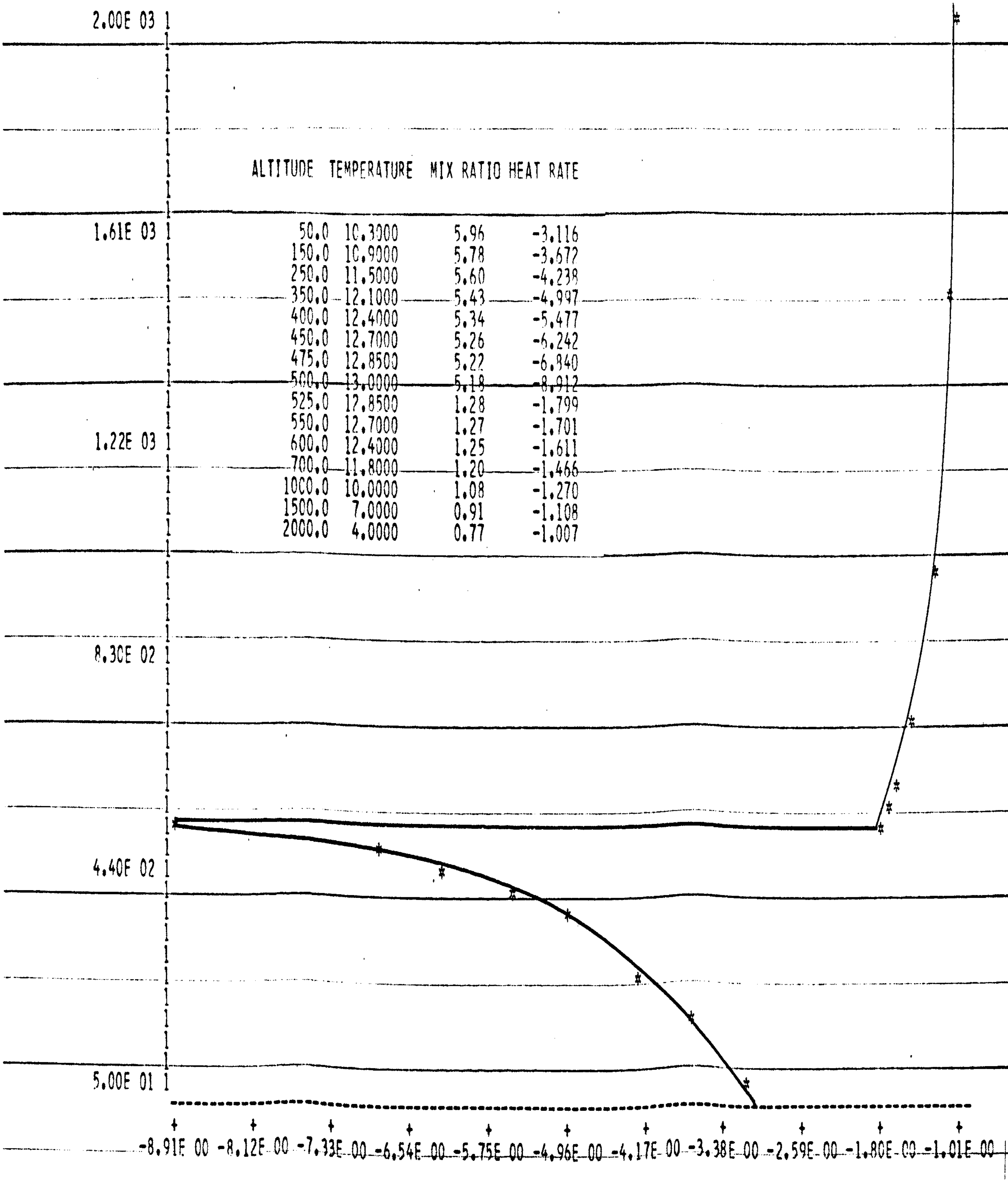


ALTITUDE	TEMPERATURE	MIX RATIO	HEAT RATE
50.0	10.0000	5.95	-3.232
150.0	10.0000	5.76	-3.529
250.0	10.0000	5.57	-3.919
350.0	10.0000	5.39	-4.478
400.0	10.0000	5.30	-4.862
450.0	10.0000	5.21	-5.441
475.0	10.0000	5.17	-5.918
500.0	10.0000	5.13	-7.542
525.0	9.8500	1.27	-1.558
550.0	9.7000	1.26	-1.482
600.0	9.4000	1.23	-1.397
700.0	8.8000	1.19	-1.294
1000.0	7.0000	1.07	-1.142
1500.0	4.0000	0.90	-1.009
2000.0	1.0000	0.76	-0.923

HEATING RATE (DEG C/DAY) VS ALTITUDE (METERS) DATA SET 5

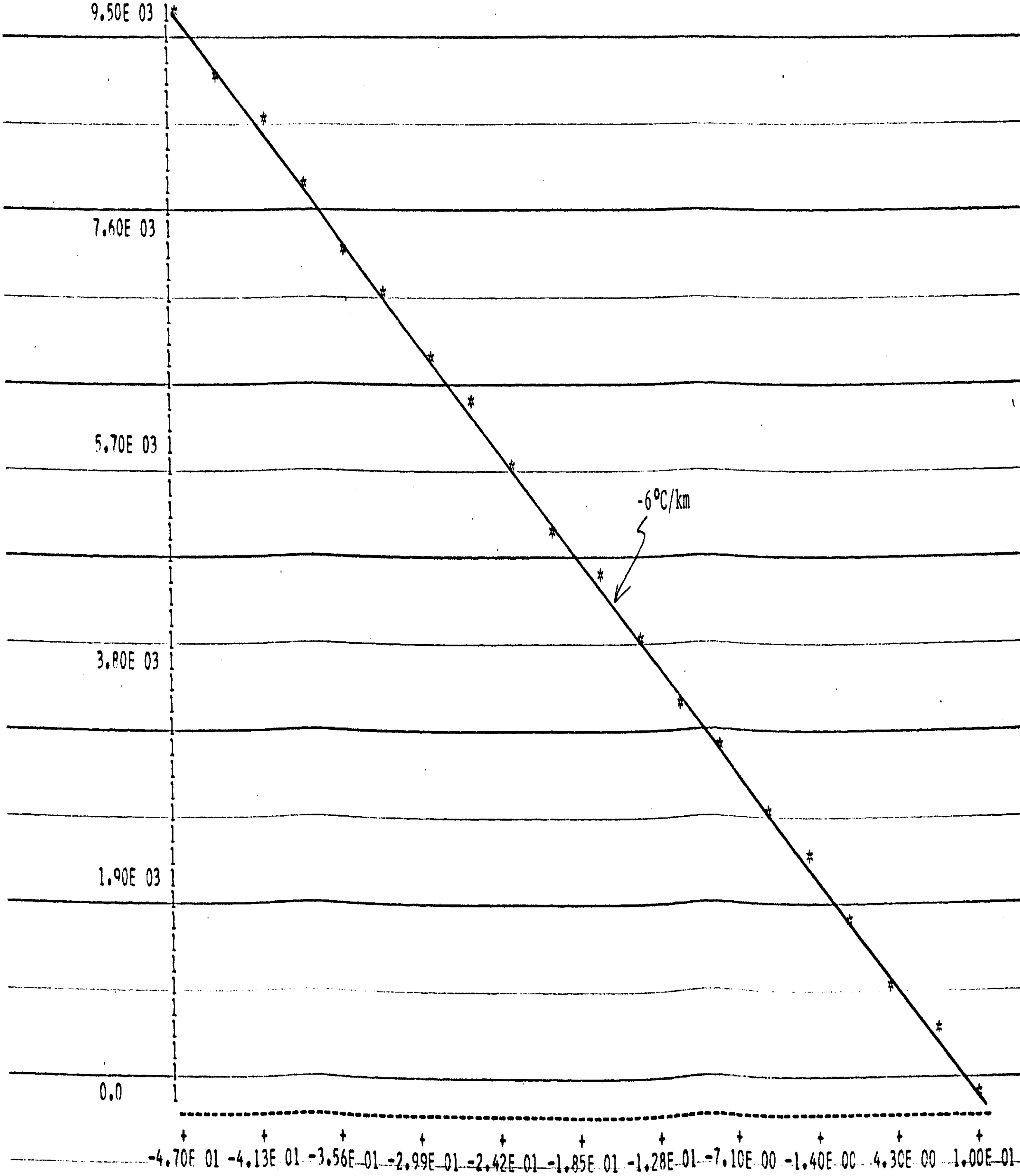


TEMPERATURE (DEG C) VS ALTITUDE (METERS) DATA SET 6

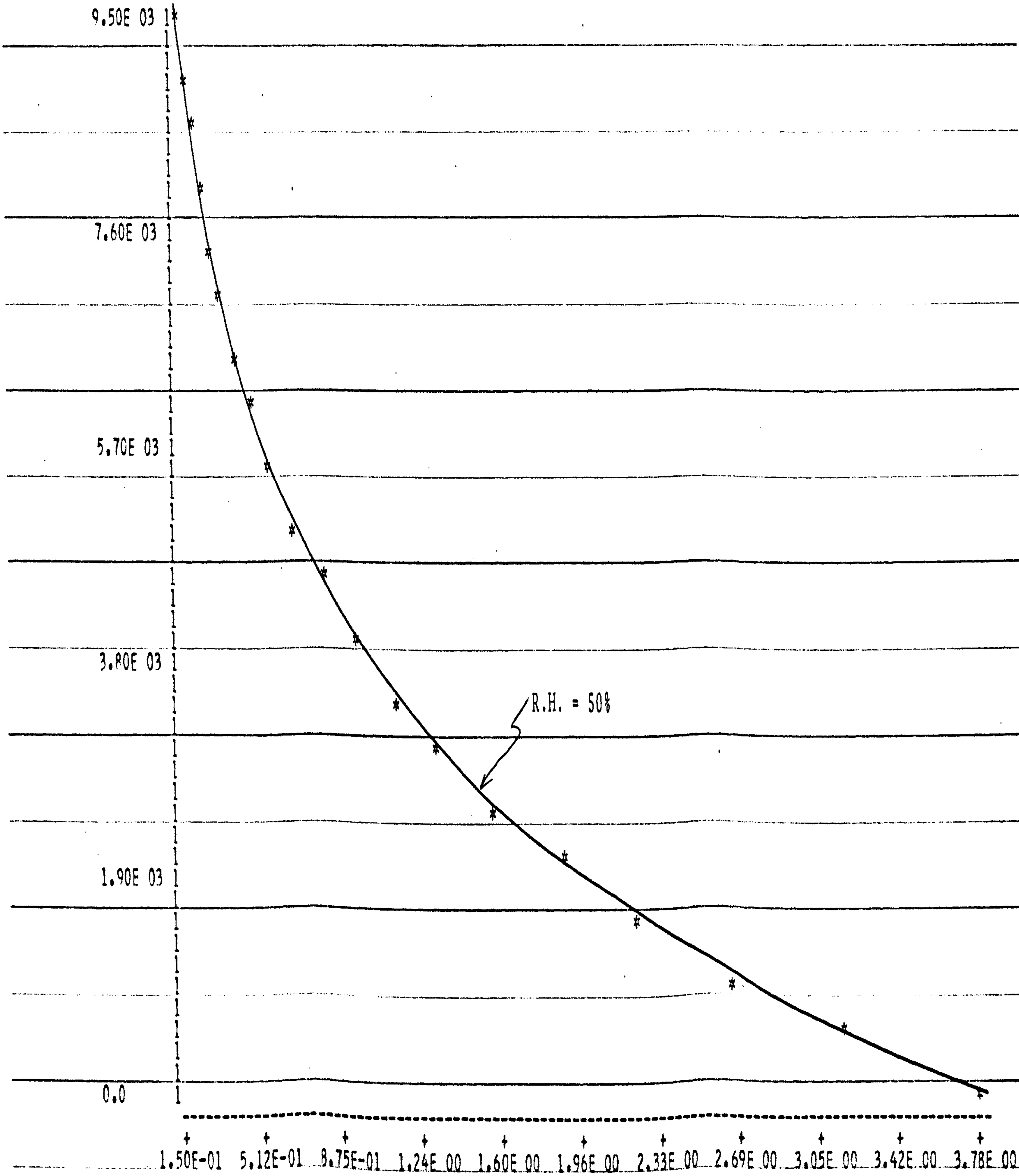


ALTITUDE	TEMPERATURE	MIX RATIO	HEAT RATE
50.0	10.3000	5.96	-3.116
150.0	10.9000	5.78	-3.672
250.0	11.5000	5.60	-4.238
350.0	12.1000	5.43	-4.997
400.0	12.4000	5.34	-5.477
450.0	12.7000	5.26	-6.242
475.0	12.8500	5.22	-6.840
500.0	13.0000	5.18	-8.912
525.0	12.8500	1.28	-1.799
550.0	12.7000	1.27	-1.701
600.0	12.4000	1.25	-1.611
700.0	11.8000	1.20	-1.466
1000.0	10.0000	1.08	-1.270
1500.0	7.0000	0.91	-1.108
2000.0	4.0000	0.77	-1.007

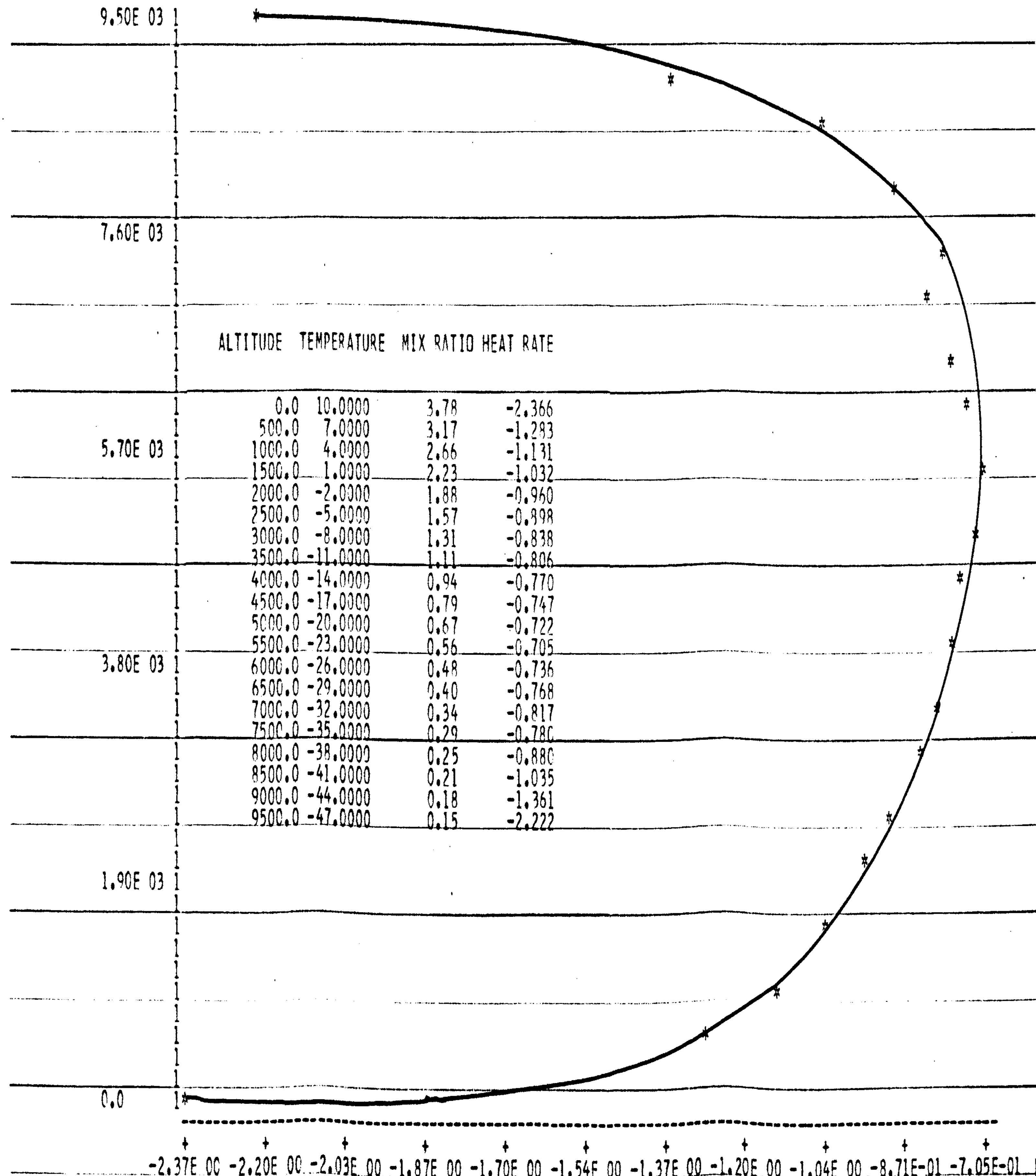
HEATING RATE (DEG C/DAY) VS ALTITUDE (METERS) DATA SET 6



TEMPERATURE (DEG C) VS ALTITUDE (METERS) DATA SET 7



MIXING RATIO OF ABSORBER (GRAMS/KG) VS ALTITUDE (METERS) DATA SET 7



ALTITUDE	TEMPERATURE	MIX RATIO	HEAT RATE
0.0	10.0000	3.78	-2.366
500.0	7.0000	3.17	-1.283
1000.0	4.0000	2.66	-1.131
1500.0	1.0000	2.23	-1.032
2000.0	-2.0000	1.88	-0.960
2500.0	-5.0000	1.57	-0.898
3000.0	-8.0000	1.31	-0.838
3500.0	-11.0000	1.11	-0.806
4000.0	-14.0000	0.94	-0.770
4500.0	-17.0000	0.79	-0.747
5000.0	-20.0000	0.67	-0.722
5500.0	-23.0000	0.56	-0.705
6000.0	-26.0000	0.48	-0.736
6500.0	-29.0000	0.40	-0.768
7000.0	-32.0000	0.34	-0.817
7500.0	-35.0000	0.29	-0.780
8000.0	-38.0000	0.25	-0.880
8500.0	-41.0000	0.21	-1.035
9000.0	-44.0000	0.18	-1.361
9500.0	-47.0000	0.15	-2.222

HEATING RATE (DEG C/DAY) VS ALTITUDE (METERS) DATA SET 7

VITA

Joey Keith Tuttle was born on April 29, 1942, in Corvallis, Oregon. He received his primary and secondary education in Houston, Missouri. He received a Bachelor of Science Degree in Physics from the Missouri School of Mines and Metallurgy in June, 1964. The author worked for the International Business Machines Corporation from June, 1964, until September, 1967.

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