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Risk-sensitive preventive maintenance policies using semivariance

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RISK-SENSITIVE PREVENTIVE MAINTENANCE POLICIES USING SEMIVARIANCE

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ABSTRACT

Preventive maintenance is an important tool that increases the reliability of the production system by reducing downtime due to failures. In the literature, maintenance and replacement policies for productions systems have been widely studied and modeled. Traditionally preventive maintenance has focused only on minimizing expected costs without considering variability in costs. Cost variability is also commonly known as risk. In a 2003 paper, Chen and Jin used variance of costs as a measure of risk for formulating preventive maintenance policies. The variance criterion that they used ignored the probability of costs exceeding monthly or yearly budgets provided to managers. The goal of the present work is to develop a performance metric for preventive maintenance that will not only consider long-run average cost but also minimize the chance that costs will exceed pre-specified budgets. Therefore the model introduced here uses a relatively less known risk metric called *semivariance.* The semivariance model developed here relies on an objective function that combines average cost with risk via the framework developed by Markowitz in 1952. It uses renewal theory and semi-Markov decision processes to develop mathematical expressions for the average cost and risk. These mathematical models are implemented within MATLAB, but they can also be implemented in spreadsheet software such as Microsoft Excel. We show via numerical experiments that the semivariance-penalized model outperforms cost-based and variance penalized models.

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Symbol	Description
θ :	Risk-aversion factor
T :	Time for maintenance
τ :	Target cost.

1. INTRODUCTION

Total productive maintenance (TPM) is a management program that has been widely practiced in industry. It is a well-defined and time-tested concept that maximizes overall equipment effectiveness and reduces machinery downtime (Wang, 2002). Its overall goal is to avoid waste in the production environment and produce quality goods which increase the customer satisfaction rate. In the past, management has been known to overlook preventive maintenance (PM) that can lead to frequent machine breakdowns which increase lead time and variability in the system. TPM uses a proactive system that monitors and corrects root causes and emphasizes the importance of maintenance as a necessary activity in managing a production system. The advantage of productive maintenance is that it optimizes the life cycle cost of a production system by minimizing unexpected machine breakdowns that result in production losses, delays in meeting customer demands, and high manufacturing costs.

An important tool of a TPM program is the underling statistical model, which helps determine the optimal schedule of machine maintenance (Askin and Goldberg, 2002). The objective of any PM program is to maximize the value of machines and other equipment to ensure the optimal functioning of a production system at the minimum cost to management. PM can reduce the need for unpredicted repairs when the failure rate is increasing (Das and Sarkar, 1999).

Traditional PM policies, such as age replacement and periodic replacement, have a critical drawback in that they only consider expected costs and overlook management risk due to variability in costs. In other words, with traditional PM policies the costs can become occasionally large. Such policies are called risk-neutral policies. Usually, the maintenance manager is provided with a budget. The risk associated with these policies can result in costs significantly exceeding the target maintenance budget. Additionally, risk-neutral PM policies lead to undesirable solutions with high variability in costs.

Maintenance cost variability is often significant due to the unexpected nature of failures. Effective and efficient supervision of maintenance costs can significantly reduce variability and expected costs. These important considerations have compelled managers to employ risk-penalized PM policies that consider both expected costs and variance

(Chen and Jin, 2003; Gosavi, 2006; Shewade, 2006). Managers actually require a more sophisticated approach that quantifies risk and determines the optimal maintenance time. This work introduces a new approach that defines the long-run semivariance of costs to represent the management risk. An objective function that combines costs with risks (semivariance) to achieve an optimal cost-and-semivariance maintenance policy is used.

A well-known approach to deal with cost-variability risk emerged from the Nobel Prize winning work of Markowitz (1952) on portfolio analysis. His work considered both expected value and the variance of the cost to solve problems of portfolio management. Many PM problems can be formulated based on the stochastic models that underlie TPM programs; these models include renewal reward theory (Kao, 1997) and Markov decision processes (MDPs) (Bertsekas, 1995). Detailed surveys and analyses of cost-variability in MDPs can be found in White (1988), Filar et al., (1989), and Sobel (1985, 1994). Renewal theory identifies a cyclical phenomenon in a stochastic system and determines the expected total cost and expected total time incurred in a single cycle. According to renewal theory principles, the ratio of the expected total costs to the expected total time is equal to the average cost per unit time in the system. The theory of MDPs uses Markov chains to model the behavior of stochastic systems. A reward structure is then provided to the Markov chains in order to generate a performance metric, e.g., average cost. The semi-Markov decision process (SMDP) is a more general version of the MDP. In the SMDP model, the time spent in any transition of the Markov chain is not necessarily unity; and the time spent for each transition could be either a deterministic or a random variable. This research uses the renewal process and the SMDP to model the PM problem of interest here. These tools, i.e., renewal processes and SMDPs, provide the necessary mathematical framework to adequately capture the complex dynamics of the machine maintenance system of interest to us.

Computer programs to generate solutions are written in MATLAB software. MATLAB is a high-performance language developed by the Math Works for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include math and computation, algorithm development, modeling, simulation, prototyping, data analysis, exploration, and visualization.

The concept of semivariance is discussed here as a measure of risk and emphasize its applicability to risk-sensitive PM. As stated above, much of the research in risk-sensitive PM focuses on using variance of costs as a measure of management risk. However, the variance of cost is not always the best measure of risk. Its most significant limitation is that it fails to distinguish between upside and downside risk. Risk-sensitive PM policies that consider the variance as a measure of risk penalize instances of cost below the mean. However, realizations of cost below the mean are favorable to the decision maker and therefore must not be considered as risk. To overcome these drawbacks, semivariance can be used as a more suitable measure of risk.

The objective here is to determine the optimal time to perform PM on a production system using the semivariance criterion. A new risk-sensitive framework was developed to measure risk via the semivariance of cost per unit time. The semivariance of the cost earned in the n^{th} instance of renewal process is defined as,

$$S_{var}\,[R_n] = [(R_n - \tau \times L_n)]_+^2$$

where R_n denotes the expected cost of a given policy in the n^{th} instance, τ denotes the target PM cost established by a manager, and L_n denotes the length of the n^{th} instance. Clearly, semivariance considers deviations above the target τ, to calculate variability, and therefore is a much more accurate measure of risk. The PM optimization problem calculates the optimal time for PM. Let T denote the age of the unit or system when PM is performed. The age T is the time elapsed since the last repair or preventive maintenance. The assumptions which we considered for our model are: First, the unit or equipment is as good as new when it is repaired or PM is performed. Second, when the machine is out of order, the unit is considered not to age.

Then our objective is to

Minimize $$g(T) = \mu_C + \theta {\sigma_{Svar}}^2 \quad with\ \theta > 0,$$

where μ_C and ${\sigma_{Svar}}^2$ denote the long-run mean cost per unit time and the long-run semivariance of the cost per unit time respectively, and θ is the risk-aversion factor. Usually, θ assumes small values, e.g., 0.1. The value of θ depends on how risk-averse the manager is.

The advantages of using the above in optimization are as follows. First, the above model captures cost and risk in one metric. Secondly, it leads one to solutions in which the cost is sacrificed to a certain degree in order to accommodate a lower value for the risk.

This research incorporates the semivariance metric into PM models that employ both renewal theory and Markov chains. Renewal theory is used for PM of smaller units, e.g., pumps, generators, fork-lift trucks, whereas the Markov-chain model is used to formulate PM policies for larger units, such as production lines. As applied here, the Markov-chain model uses an SMDP and determines a solution by implementing a mathematical program using software like CPLEX or MATLAB.

The remainder of the thesis is organized as follows. Section 2 provides a brief overview of the literature on various preventive maintenance approaches. Section 3 explains the renewal theory model and its underlying framework. Section 4 discusses in detail the SMDP model. Section 5 presents the numerical results of a preventive maintenance problem based on the renewal theory and semi-Markov models. Section 6 offers some conclusions and future research.

2. LITERATURE REVIEW

This research focused on implementing renewal processes and Markov chains in a semivariance-penalized PM problem. Ross (1997) provides an introduction to the application of renewal reward theory and Markov chains in stochastic processes. Most of the work published on this topic evaluates PM policies based on the expected cost criteria. Barlow and Proschan (1965) have done some of the seminal work in this area. In the 1960s and 1970s, traditional PM policies similar to the Barlow and Proschan model were developed by many researchers like Fox (1966), Glasser (1967), Nakagawa and Osaki (1974). These traditional PM policies for a production system, such as age replacement and periodic replacement, focused only on minimizing the expected cost without considering management risk.

The renewal process and the Markov decision processes are frequently used as the underlying stochastic models in a TPM program. The ultimate goal of implementing TPM in production systems is to reduce unexpected machine breakdowns and optimize productivity. TPM establishes a PM system for the entire life span of the equipment. Pomorski (2004) provides a comprehensive review of TPM concepts. Although TPM is historically equipment-focused, its effective implementation needs a continuous improvement methodology for increasing overall manufacturing productivity. PM is regularly performed at specific intervals on devices that are operated continuously to reduce or eliminate deterioration (Endrenyi and Anders, 2004). PM is worthwhile when the cost incurred by an equipment failure is greater than the cost of maintenance.

Generally, there are two types of PM schemes, condition-based and time-based (Billinton and Allan, 1996). In condition-based PM, the action taken after each inspection is dependent on the state of the system. In time-based PM, however, PM is carried out at predetermined intervals (Chen and Trivedi, 2004). Numerous maintenance and replacement models have been developed in the past several decades. However, each model falls into some category of maintenance policies, such as age replacement, repair cost limit, failure limit, reference time, and so on. Wang (2002) presents a survey of these maintenance policies for both single-unit and multi-unit systems. Each kind of policy has distinct characteristics, advantages, and disadvantages.

The growing importance of maintenance has generated an increasing interest in the development and implementation of optimal maintenance strategies for improving system reliability. Wang (2002) developed a classification scheme for maintenance models so that a decision maker can recognize the model that best fits a specific problem. If PM policies are not designed properly, frequent machine breakdowns occur, causing losses that can exceed millions of dollars annually. These unexpected breakdowns make it difficult to transition from make-to-stock to make-to-order (Suri, 1998), thereby making the system inflexible. They also increase inventory-holding costs by requiring the storage of safety stocks (Askin and Goldberg, 2002).

Das and Sarkar (1999) address the problem of optimal PM in a production-inventory system. This work models the PM problem as a semi-regenerative process and uses the expected cost benefit as the system performance measure. They developed a PM model for a production-inventory system using information on system conditions and continuous probability distributions for the machine failure process. They considered a model similar to that of Srinivasan and Lee (1996) in which the decision to perform PM depends on the inventory level and the number of items produced since the last repair or maintenance. Both Srinivasan and Lee (1996) and Das and Sarkar (1999) considered a single operating state production facility. In such a facility, the production rate does not change with equipment use, and repair costs are independent of the age of a facility. The machine replacement problem has been widely studied, and it is an important topic in operations research (Gertsbakh, 2000).

Markowitz’s (1952) portfolio analysis is the best known approach for dealing with cost-variability risk. His framework uses both expected cost and variance of cost to characterize the system rewards and variability. Some recent research results on risk-penalized MDPs are presented by Filar et al., (1984) and Sobel (1985, 1994), in which some of Markowitz’s principles are applied in the MDP context. A review of the literature indicates that MDPs are now used for variability-sensitive or risk-sensitive decision making (White, 1988).

In the real world, risk occurs due to cost variability. PM policies that neglect risk result in inappropriate maintenance budget allocations and financial crises. Risk-sensitive managers have used variance per unit time as a measure of risk in strategic decision making (Ruefli, et al., 1999). Chen and Jin (2003) for the first time proposed a method for formulating PM policies that considered the effects of both expected cost and cost variability. They modeled maintenance management risk using a long-run variance of the cost in the renewal process. They developed optimal cost-variability-sensitive maintenance policies by altering the objective function. The Chen and Jin model significantly reduced the maintenance management risk with only a small increase in the expected cost. Gosavi (2006) proposed an alternate approach to measure the long-run variance. His approach is based on MDPs and for renewal processes in which the cycle time is not necessarily one. Shewade (2006) develops an MDP model for semivariance, but his renewal reward model for semivariance does not take time into consideration.

Quirk and Saposnik (1962) offer a theoretical analysis of semivariance. Bawa (1975) defines a family of risk measures called *lower partial moments*, which further modify target semivariance. Porter (1974) provided an early analysis similar to that of Bawa. Nawrocki (1999) presents a brief history of downside risk measures and semivariance concepts and applications. To date, semivariance has not been used as a measure of risk in PM policies.

MDPs and SMDPs can be solved via classical dynamic programming (DP) methods (Tijms, 2003). Two well-known approaches in DP are policy iteration and value iteration (Bertsekas, 1995). The policy iteration technique uses an iterative approach to solve a linear system of equations developed by Bellman (1954), whereas the value iteration approach uses Bellman transformation to compute an optimal value function iteratively. DP methods require exact computation and storage of transition probability matrices. This research uses the linear programming technique to solve SMDPs.

3. A RENEWAL THEORY MODEL

3.1. INTRODUCTION

Renewal theory is an important paradigm that has been used successfully in reliability applications and very frequently in TPM programs. The model known as age-replacement is also based on renewal theory and has strong mathematical roots, providing robust heuristics. The present work uses renewal theory to formulate a problem to determine the optimal replacement time for a machine or unit. The renewal process assumes that the life cycle is complete when the unit either fails or requires maintenance. The renewal reward theorem (RRT) is a classical result that provides an expression for the expected reward or cost per unit time in a renewal process. Renewal-reward processes are very useful for computation of important performance measures, such as long-run costs and reward rates. Chen and Jin (2003) extended the concept to determine the variance in the rewards of the renewal process. This section models the PM problem as a renewal reward process, and the results can be used to calculate the long-run semivariance of the cost per unit time.

This approach addresses the need of managers in local industry for a means to measure risk, and it is particularly helpful to senior management. The unit of risk it uses is *dollar*^{2}*/hour* or *Euro*^{2}*/hour*. In the real world, many strategic managers use variance to measure risk (Ruefli et al., 1999); this research shows that semivariance is also a useful approach to determine risk. This thesis demonstrates how renewal theory can be applied in modeling the risk-penalized machine replacement problem. The model assumes that the unit or machine is as good as new after repair or maintenance; that is, its age is zero after each repair or maintenance. The machine ages only when it is in operation. The following section discusses the notation and basic definitions involved in the renewal process. A recursive computational procedure is presented which can be used to obtain the optimal replacement policy and the expected average cost.

3.2. NOTATION

This section uses the following variables:

X : Random variable denoting the time between system failures

T : The age of the unit at which PM is performed

$F(\cdot)$: The cumulative distribution function of X

$f(\cdot)$: The probability density function of X

C_r : The expected cost of one repair

C_m: The expected cost of one PM activity

t_r : The expected time of one repair

t_m : The expected time of one PM activity

3.3. DEFINITIONS

3.3.1. Renewal Reward Process. Consider a counting process, $\{\boldsymbol{N(t), t \geq 0}\}$, and let $\boldsymbol{L_n}$ denote the time between the $\boldsymbol{(n-1)^{th}}$ and the *n*th event in this process with $\boldsymbol{n \geq 1}$. A counting process is a stochastic process if $\boldsymbol{N(t)}$ represents the total number of events that have occurred in $[\boldsymbol{0, t}]$. This process must satisfy two conditions i.e. $\{\boldsymbol{N(t) \geq 0}\}$ and $\{\boldsymbol{N(t)}\}$ is an integer for all *t*. If $\{\boldsymbol{L_1, L_2, \ldots\ldots}\}$ denotes a sequence of non-negative random variables that are independent and identically distributed, then the counting process is called a *renewal process*. When an event is triggered, a renewal is said to have taken place. In the context of the PM problem, every failure or maintenance of the system constitutes a renewal. The time between the successive failures or PM activities represents a sequence of non-negative random variables, which for a given system can be assumed to be independent and identically distributed. When a reward is associated with each renewal event, the above counting process is known as a *renewal reward process*. The reward could be the net revenues associated with the event.

3.3.2. Expected Reward. Let $\boldsymbol{R_n}$ be the reward earned at the time of the *n*th renewal process. The total reward $\boldsymbol{R(t)}$ earned by time t can then be expressed as:

$$R(t) = \sum_{n=1}^{N(t)} R_n \ . \qquad (3.1)$$

This above expression calculates the sum of the individual rewards earned by time *t*. The total reward squared earned by time t can be represented as:

$$R^2(t) = \sum_{n=1}^{N(t)} (R_n)^2 \ . \qquad (3.2)$$

This expression denotes the sum of the square of the individual rewards earned by time *t*. Further, let us consider $E[R] = E[R_n]$, $E[R^2] = E[(R_n)^2]$ and $E[L] = E[L_n]$ where *E* denotes the expectation operator. The elementary renewal theorem as stated and proved by Ross (1997) and which defines the rate of the renewal process is given as:

Theorem 3.3.2.1. The elementary reward theorem *with probability* 1,

$$\lim_{t\to\infty} \frac{N(t)}{t} = \frac{1}{E[L]} \ . \qquad (3.3)$$

The renewal reward theorem (see e.g., Ross, 1997, Proposition 7.3, p. 417) for the expected reward per unit time is shown below.

Theorem 3.3.2.2. *If* $E[R] < \infty$, $E[L] < \infty$, *then with probability* 1,

$$\mu_R \equiv \lim_{t\to\infty} \frac{R(t)}{t} = \frac{E[R]}{E[L]} \ . \qquad (3.4)$$

The above expression states that the long-run expected reward per unit time is equal to the expected reward earned in a renewal cycle divided by the expected length of the renewal cycle. Hence if *C* denotes the cost in one cycle, then

$$\mu_C = \frac{E[C]}{E[L]} \ . \qquad (3.5)$$

3.3.3 Long-run Variance. The following definition for the long-run variance, which measures a time average of the total variance in infinite renewals, was first presented by Gosavi, (2006).

Theorem 3.3.3.1. Variance 1 model (Gosavi, 2006)

If $E[R] < \infty$, $E[L] < \infty$, $E[(R)^2] < \infty$, *then with probability 1,*

$$\sigma^2 = \lim_{t\to\infty} \frac{V(t)}{t} = \frac{E[(R)^2] - (E[R])^2}{E[L]} \ , \qquad (3.6)$$

where $V(t) = \sum_{n=1}^{N(t)} (R_n - E[R])^2$ and σ^2 represents the sum of the squared deviations of the renewal cycle from their means divided by the total duration of the cycle over the infinite number of renewals.

Theorem 3.3.3.2. Variance 2 model (Gosavi, 2008)

If $E[R] < \infty$, $E[L] < \infty$, $E[(R)^2] < \infty$, *then with probability 1,*

$$\sigma^2 = \frac{E[R]}{E[L]} - 2 \times \rho \times E[L] + \rho^2 \times \frac{E[L^2]}{E[L]} \ , \qquad (3.7)$$

where

$$\rho = \frac{E[R]}{E[L]} \ . \qquad (3.8)$$

3.3.4 Long-run Semivariance. This section now proposes a result for the long-run semivariance and explains the proof in detail. The target rate $\boldsymbol{\tau}$ is used to measure the average semivariance over an infinite number of renewal cycles.

Theorem 3.3.4.1. *If* $E[L] < \infty$ *and* τ *is the target reward(cost) per renewal cycle, then with probability 1, the mean long-run semivariance of the rewards of the renewal cycle will be:*

$$\rho_T = \frac{S_{var}\,[R,L]}{E[L]} = \frac{E[S_{var}\,(R,L)]}{E[L]} \ , \qquad (3.9)$$

where, $E[S_{var}\,(R,L)]$ *is the expected semivariance in one renewal cycle.*

Proof:

The semivariance of the reward earned in the *n*th renewal cycle will be defined as

$$S_{var}\,[R_n] = [(R_n - \tau \times L_n)]_+^2 \ . \tag{3.10}$$

The total semivariance of the reward measured over an infinite number of renewals is given as:

$$\rho_T = \lim_{l\to\infty} \frac{S_{var}\,[R,L]}{E[L]} \quad , \tag{3.11}$$

where, $S_{var}\,[R,L]$ is the semivariance of the reward earned by time t, which is represented as,

$$S_{var}\,[R_n] = \sum_{n=1}^{N(t)} [(R_n - \tau \times L_n)]_+^2 \ . \tag{3.12}$$

Therefore,

$$\rho_T = \lim_{t\to\infty} \frac{S_{var}\,(t)}{t} = \lim_{t\to\infty} \left(\frac{S_{var}\,(t)}{N(t)}\right)\left(\frac{N(t)}{t}\right)$$

$$= \lim_{t\to\infty} \left(\frac{\sum_{n=1}^{N(t)} [(R_n - \tau \times L_n)]_+^2}{N(t)}\right)\left(\frac{N(t)}{t}\right)$$

$$= \frac{E[S_{var}\,(R,L)]}{E[L]} \ .$$

This equality follows from the strong law of large numbers.■

Theorem 3.3.4.1 proves that the long-run total semivariance per unit time is equal to the expected semivariance of the reward in one renewal cycle divided by the expected length of cycle. In the next section, an expression for calculating the expected semivariance of the cost, of a given PM policy, in one renewal cycle is presented.

3.4. OBJECTIVE FUNCTION

A general objective function or cost function using the Markowitz criterion with T denoted as the age of PM is represented as shown below:

Minimize $$g(T) = \mu_C + \theta\sigma^2 \quad with\ \theta > 0, \tag{3.13}$$

where μ_C and σ^2denotes the long-run mean and the long-run variance of the net cost per unit time incurred from PM, respectively.

An alternate way of representing the objective function in terms of rewards and in which the function is maximized is expressed as

Maximize $$g(T) = \mu_R - \theta\sigma^2 \quad with\ \theta > 0, \tag{3.14}$$

where μ_C and σ^2denotes the long-run mean and the long-run variance of the net reward per unit time incurred from PM, respectively.

Risk-neutral (RN) statistical models for PM have the objective function which is equal to the expected cost because $\theta = 0$. Generally the value of θ, i.e., risk-aversion factor, is selected by the manager through experimentation. Value of θ plays an important role for managers. When the value of θ is very low the model does not include risks and when the value of θ is very high, it results in low variability but high cost.

Using the definitions and the theorems presented in Section 3.1 above, the objective function for evaluating the performance of a known PM policy can be developed as follows. Each renewal event results in a failure or PM of the system, and the associated cost is the expected cost of the failure or PM. As a result, if T is the time required to perform PM, the expected cost in a renewal cycle is given as

$$E[C] = C_r P(X < T) + \ C_m P(X \geq T) = C_r F(T) + \ C_m [1 - F(T)]. \tag{3.15}$$

Similarly, the expected semivariance of the cost in a renewal cycle is given as

$$E[S_{var}\,(R, L)] = \int_0^T [C_r - \tau(x + t_r)]_+^2 f(x)dx + [1 - F(t)][C_m - \tau(T + t_m)]_{+.}^2 \tag{3.16}$$

Also, the expected length of the renewal cycle is given as

$$E[L] = \int_0^T (x + t_r) f(x) dx + (T + t_m)\ [1 - F(T)]. \tag{3.17}$$

and expected length square is given as

$$E[L^2] = \int_0^T (x + t_r)^2 f(x) dx + (T + t_m)^2\ [1 - F(T)]. \tag{3.18}$$

Finally, using the Markowitz criterion as stated in Equation (3.9) and the results of Equations (3.11), (3.12), and (3.13) the following expression for the objective function can be used as the objective function.

$$g(T) = \frac{E[C]}{E[L]} + \theta \frac{E[S_{var}(R, L)]}{E[L]}. \tag{3.19}$$

This expression can be optimized with respect to T to determine the optimal time, T^* for performing PM. The overall objective function of renewal theory model can then be shown to be:

$$g(Svar) = \frac{C_r F(T) + \ C_m [1 - F(T)]}{\int_0^T (x + t_r) f(x) dx + (T + t_m)\ [1 - F(T)]} +$$

$$\theta \frac{\int_0^T [C_r - \tau(x + t_r)]_+^2 F(x) dx + [1 - F(t)][C_m - \tau(T + t_m)]_+^2}{\int_0^T (x + t_r) f(x) dx + (T + t_m)\ [1 - F(T)]}\ . \tag{3.20}$$

4. SEMI-MARKOV DECISION PROCESS

4.1 MARKOV PROCESSES

The Markov process is widely used to study real-life systems in engineering. In a Markov process, the transition among states is a probabilistic phenomenon, that is, a random affair. To optimize real life systems using Markov processes, we often define a performance metric for the system. This research uses the expected cost per unit time plus θ times risk as the performance metric.

The Markov process is distinguished by its memoryless property. This property states that when a system in state *i*, whether it goes to state *j* after one transition does not depend on states visited by system before coming to *i*. The Markov process is governed by the following law:

$$Prob\{X(t+1) = j | X(t) = i\} = p(i,j) \,, \tag{4.1}$$

where $p(i,j)$ is the probability that the next state is *j* given that current state is *i* and $p(i,j)$ is a constant for a set of given *i* and *j* values.
A non-Markovian process of three states can be represented by the following law:

$$Prob\{X(t+1) = j | X(t) = i, X(t-1) = k\} = p(i,k,j) \,. \tag{4.2}$$

This equation assumes that the system is at *i* when time is *t*, and at time *t-1* the system is at state *k*. The equation implies that the probability that it goes to *j* from *i* also depends on where the system was at time *t-1*. This is an example of non-Markovian process. For a Markov process, the transition probability depends on the present state and the next state. However, in the non-Markov process, the transition probability depends on the next state, the current state and one or more of the previous states. In each state of a Markov chain, an action can be selected by the decision maker. The Markov decision process (MDP) is a problem of control optimization in which one seeks the optimal action in each state.

The following sections address a variant of the MDP, introducing commonly used notation and definitions. It also describes two methods used to derive an optimal policy under various criteria: exhaustive enumeration and linear programming.

4.2 SEMI-MARKOV PROCESSES

The semi-Markov decision process (SMDP) is a tool for analyzing a sequential decision process with random decision epochs in which the transition time is a random variable. This research develops an SMDP to optimize the maintenance policy for a production system.

The SMDP is a more general version of the MDP. An important assumption with the MDP is that the transition of a system to a new state or to the current state happens after unit time. In an SMDP, the time spent in each transition of the Markov process can be a deterministic quantity or a random variable. The only difference between the SMDP and the MDP lies in the time taken in transitions between states of a system. SMDPs generalize MDPs by allowing the decision maker to choose actions whenever the system state is changed. In the SMDP, the time spent in any transition of the Markov process varies, and is in fact a random variable. That the time spent in the MDP is unity implies that the expected cost per unit time for an MDP is the same as the expected cost per transition, where as in SMDP this is not usually true.

SMDPs model the system evolution in continuous time and allow the time spent in a particular state to follow an arbitrary probability distribution. If the time spent in any transition in an SMDP model is a deterministic quantity, then it is referred to as deterministic time Markov decision problem. If the transition times of SMDP model are exponentially distributed, then it is referred as continuous time Markov decision problem. In SMDPs, there are two stochastic processes associated with the Markov process: the natural process (NP) and the decision making process (DMP).

The natural process keeps a record of every state change in the system. During NP, the system does not return to itself in a single transition. To explain in detail, every natural process remains in a state for a particular time and then jumps to different state. However, the decision-making process has a different approach. DMP is similar to NP but it records those states in which action needs to be selected by a decision-maker. In DMP the system can return to itself in a single transition.

In this section, the problem of finding the optimal policy for performing PM of a production machine is modeled as an SMDP. The SMDP model, developed in this research, uses the transition probability matrix, transition cost matrix (TCM), and transition time matrix (TTM) and is solved using the linear programming approach or the exhaustive enumeration technique. The next section presents the notations and an explanation of elements used to model the objective function. Basic concepts such as the state space, the action space, the transition probability matrix, the transition reward or cost matrix, the transition time matrix and the long run average cost are also reviewed.

4.3 NOTATION

S : Set of states of SMDP

$A(i)$ **:** The set of actions permitted in state *i*

$p(i, a, j)$ **:** Transition probability matrix of going from state i, to state *j* under the condition that action a

$c(i, a, j)$ **:** The cost incurred due to transition from state i, to state *j* under the influence of action a

$t(i, a, j)$ **:** Time spent in going from state i, to state *j* under the condition that action a

$v(i, a, j)$ **:** Long-run semivariance

$\bar{c}(i, a)$ **:** Expected cost incurred in state i where action a belongs to $A(i)$ is chosen

$\bar{t}(i, a)$ **:** Average time spent in a transition from state i, under the influence of action a

$\bar{v}(i, a)$ **:** Expected long-run semivariance

$\pi(j)$ **:** Limiting probability of state *j*.

4.4 DEFINITIONS

This subsection defines the state space, the policy space, the action spaces and other critical elements of the SMDP framework.

4.4.1. State Space. Throughout this thesis, state space is defined as the number of days elapsed since the last repair or PM of a machine. The transition time is one day, and the state is recorded at the beginning of each day. A finite state space is considered, since typically, for large production machines, PM is eventually performed after a finite time period. As mentioned before, the state of the machine is assumed to be zero after a repair or PM.

4.4.2. PM Policy and Action Space. At the beginning of each day, the decision maker has to select an action which defines the PM policy. At each decision making epoch, the following two actions are available: continue production (p) and perform maintenance (m).

4.4.3. Transition Probability Matrix. As stated above, the distribution of the time between failures is used to calculate the transition probabilities. Let *i* be the state of the machine at time *t*. Then, the one step transition probability matrix for a PM policy which recommends T as the time for maintenance is calculated as follows. For $\boldsymbol{0 < i < T}$, the action taken is state i will be $\boldsymbol{a} \in \boldsymbol{A(i)}$, therefore at time t+1, the machine will transition to state i+1 with probability $\boldsymbol{p(i, a, i+1)}$ and state 0 with probability $\boldsymbol{1 - p(i, a, i+1)}$. The probability $\boldsymbol{p(i, a, i+1)}$ can be calculated as $\boldsymbol{P(X > (i+1)d \,|X > id)}$, where d is a constant denoting the number of time units that machine is functional per day. However, for i=T, the action taken will be a∈ $\boldsymbol{A(i)}$, and the machine will transition to state 0 with probability 1. The resulting matrix is irreducible and the limiting probabilities exist for any given policy.

4.4.4. Transition Cost Matrix. For $\boldsymbol{0 < i < T}$, the each transition of the machine from *i* to i+1 will result in a profit equivalent to a day's worth of production. In the transition cost matrix, this profit will be expressed as a negative cost associated with the successful functioning of the machine. If the machine fails, the cost associated with the transition from *i* will be the average cost of failure, And for i=T, the cost associated will be the average cost of maintenance.

4.4.5 Transition Time Matrix. This matrix is similar to the TPM and the TCM and for action a, its element in the i^{th} row and j^{th} column is represented by $\boldsymbol{t(i,a,j)}$. When the transition times are random variables in a system, the values of $\boldsymbol{t(i,a,j)}$ contain the expected values.

4.5 OBJECTIVE FUNCTION

To identify the optimal policy in a system, we need a performance metric or objective function. In our production problem, we consider expected cost per unit time plus a constant (θ) times the risk per unit time. The objective function or cost function for a PM policy that considers *T* as the time for PM can be expressed as:

$$Minimize \qquad g(T) = \mu_C + \theta {\sigma_{Svar}}^2 \quad with\ \theta > 0,$$

where μ_C and ${\sigma_{Svar}}^2$ denotes the long-run mean and the long-run variance of the net cost per unit time incurred from PM, respectively.

4.5.1. Expected Immediate Cost and Immediate Time. The expected immediate cost, which is the average cost in one transition, can be represented as below:

$$\bar{c}(i,a) = \sum_{j\in S} p(i,a,j)c(i,a,j)\ . \tag{4.3}$$

The expected immediate cost $\bar{c}(i,a)$ associated with respective state i and with action a is shown in equation 4.3. Similar to immediate expected cost we can calculate expected immediate time in transition for an SMDP model as shown below:

$$\bar{t}(i,a) = \sum_{j\in S} p(i,a,j)t(i,a,j) \quad . \tag{4.4}$$

4.5.2. Semivariance of the Immediate Cost. For a given target cost of the PM, which is represented as(τ), the semivariance of the immediate cost in the state i when action a is selected can be expressed as:

$$v(i,a,j) = [c(i,a,j) - \tau \times t(i,a,j)]_+^2 \quad . \tag{4.5}$$

The expected semivariance of the immediate cost is then given as,

$$\bar{v}(i,a) = \sum_{j \in S} p(i,a,j) v(i,a,j) \quad . \tag{4.6}$$

4.6. EXHAUSTIVE ENUMERATION APPROACH

Exhaustive enumeration is conceptually one of the simplest discrete optimization techniques. It is easy to understand and implemented on small problems in the real world. In this method, one enumerates every policy selected and evaluates the performance metric associated with each policy. The policy which produces the best value for performance metric is the optimal policy. By using exhaustive enumeration we can determine the long-run expected cost per transition and the long-run semivariance of the cost. Clearly exhaustive enumeration may not work for very large systems.

The long run expected cost per transition(ϕ), for a given policy $\vec{\mu}$, is defined as follows:

$$\phi_{\vec{\mu}} = \sum_{i=1}^{|S|} \pi_{\vec{\mu}}(i) \bar{c}(i,a) \quad , \tag{4.7}$$

where $\pi_{\vec{\mu}}(i)$ is the limiting probability (steady-state probability) of state i if policy $\vec{\mu}$ is pursued. The above is a well-known expression in MDP theory.

Long-run semivariance of the cost of the given PM policy can be calculated as:

$$\sigma^2_{Svar} = \sum_{i=1}^{|S|} \pi_{\vec{\mu}}(i) \bar{v}(i,a) \quad . \tag{4.8}$$

The objective function for the SMDP model can be formulated using the above equations as:

$$g(\vec{\mu}) = \phi_{\vec{\mu}} + \theta \, \sigma^2_{Svar} \quad . \tag{4.9}$$

Substituting the equations of long-run expected cost and semivariance in the above objective function

$$g(\vec{\mu}) = \sum_{i=1}^{|S|} \pi_{\vec{\mu}}(i)\bar{c}(i,a) + \theta \sum_{i=1}^{|S|} \pi_{\vec{\mu}}(i)\bar{v}(i,a)\ . \tag{4.10}$$

An exhaustive enumeration algorithm can then be used to evaluate the above objective function over a set of allowable PM policies. The algorithm helps to find the optimum policy $\overrightarrow{\mu^*}$, which minimizes the cost function.

4.7. LINEAR PROGRAMMING

Linear programming (LP) is the most commonly applied form of constrained optimization when the objective function is a linear function. The constraints are also linear. LP is a useful technique that is computationally superior to exhaustive enumeration for classical risk-neutral MDPs (Bertsekas, 1995). In this thesis, LP has hence been used as the main optimization tool. It turns out that the semivariance penalized problem can be setup as an LP. The main elements in an LP are the decision variables, the objective function and the constraints. The decision variables are the unknowns during the start of the problem, which needs to be determined. The goal is to find the values of these variables that provide the best value of the objective function. In our problem, the objective function is to minimize the semivariance penalized expected cost per unit time.

The objective function for the problem under consideration for a production system using the LP is (Tijms, 2003):

$$Minimize \quad \sum_{i\in S}\sum_{a\in A(i)} \bar{c}(i,a)x(i,a) + \theta \sum_{i\in S}\sum_{a\in A(i)} \bar{v}(i,a)x(i,a) \qquad with\ \theta \geq 0\ , \tag{4.11}$$

subject to

$$\sum_{a\in A(j)} x(j,a) - \sum_{i\in S}\sum_{a\in A(i)} p(i,a,j)x(i,a) = 0 \qquad for\ all\ j \in S, \tag{4.12}$$

$$\sum_{i\in S}\sum_{a\in A(i)} x(i,a)\bar{t}(i,a) = 1 \qquad and\ x(i,a) \geq 0\ \ for\ all\ i \in S\ \ and\ a \in A(i). \tag{4.13}$$

5. NUMERICAL RESULTS AND DISCUSSION

This section presents the numerical results for PM models developed in this research. The first subsection focuses on the renewal reward theory (RRT) model and the second section focuses on the SMDP model. Both models show encouraging numerical performance. The goal of this exercise is to test whether the models developed in previous section produce results that can be implemented and generated within a reasonable period of computer time.

The models developed in this thesis can be used by real world managers for scheduling preventive maintenance activities. In order for a real world manager to use these models, he/she must have (or collect) data related to the following:

- The mean repair time and maintenance time.
- The cost of one repair and that of one maintenance activity.

For the RRT model, the manager also needs the distribution of the failure times, and for the SMDP model transition probabilities are required. The cost of repair can be typically found by multiplying the average labor cost of the repair/maintenance personnel by the mean time of repair plus the costs of spares/defective items. The cost of maintaining is similarly computed. Generally, the maintenance cost is much lower than the repair cost. Unless this is true, it does not make sense to preventively maintain a machine. Once the inputs are gathered, the models presented in previous sections can be used to determine the optimal time for maintenance.

5.1 RENEWAL REWARD THEORY MODEL

This section describes the experiments and results of the RRT model. In the RRT model developed for this research, we used the gamma distribution for modeling the time to failure. The gamma distribution is a suitable choice since it has an increasing failure rate (Lewis, 1994). The results also apply data collected from a manufacturing plant by Shewade (2006). The input parameters and output results are described in Table 5.1 and Table 5.2, respectively. Several experiments were performed using the RRT model to determine the optimal times (τ) for maintenance under risk-sensitive (RS) and risk-neutral (RN) conditions. The objective function in Equation (3.20) was evaluated over a suitable time range and optimization is performed using exhaustive enumeration. The computer programs were written in MATLAB for the exhaustive enumeration algorithm technique.

For the results to be meaningful, the following condition must hold true:

$$\frac{C_m}{t_m} < \tau < \frac{C_r}{t_r} \; . \tag{5.1}$$

Without this condition, the target semivariance term will not appear in any of our calculations. In the PM problem, we determine: (i) the optimal time for performing PM under semivariance (${T^*}_{Svar}$), risk-sensitive variance (${T^*}_{Var}$), and risk-neutral (${T^*}_{RN}$) criteria. (ii) The optimal objective function or cost function value associated with each criterion. Table 5.3 shows objective functions when using the semivariance, risk-neutral and the variance 1 and variance 2 criteria for the renewal theory model. Table 5.3 also shows the improvement factor (in percent), obtained from pursuing the *Svar* criterion. The plot for the objective function of risk-sensitive semivariance (*Svar*) criterion in cases 1 through 4 is presented in Figure 5.1. Also, the plot presented in Figure 5.2 shows the objective function for case 1, under the *RN*, variance and semivariance criteria. The improvement obtained from using *Svar* criterion in comparison to using *RN* is defined as follows:

$$\frac{g_{Svar}\,({T^*}_{RN}) \; - \; g_{Svar}\,({T^*}_{Svar})}{g_{Svar}\,({T^*}_{RN})} \times 100 \;\; , \tag{5.2}$$

Similarly, the percentage improvement obtained from using *Svar* criterion compared to variance is defined as:

$$\frac{g_{Svar}\,({T^*}_{Var}) \; - \; g_{Svar}\,({T^*}_{Svar})}{g_{Svar}\,({T^*}_{Var})} \times 100 \;\; , \tag{5.3}$$

where g_{Svar} is the objective function of semivariance criterion defined in Equation (3.20). In this objective function, when $\theta = 0$ we get the expected cost for RN criterion.

Figure 5.3 provides a graphical representation of the improvement in the objective function for case 1. From the results in Table 5.3, we observe that the percentage improvement depends on the value of target cost τ selected. Table 5.4 denotes the optimal time and objective function of variance 1 and variance 2 respectively. Through our research we found that for both the variance models, the cost function values are almost the same. This is an interesting finding. The variance 2 model measures the asymptotic variance, where as variance 1 measures the cyclical variance. Variance 2 is much more numerically intensive since it needs the second moment of cycle time where as variance 1 does not need it. It appears that variance 1 does not need it. It appears that variance 1 may be a sufficiently accurate surrogate for variance 2, which is the mathematically more appropriate measure of variability in the renewal reward process. Table 5.5 shows the optimal time and objective function of the risk-neutral model.

Table 5.1 Input parameters for experiments with the risk-sensitive RRT model.

Case	**Gamma(n, λ)**	**C_r($)**	**C_m($)**	**τ ($/hr)**	**t_r(hr)**	**t_m(hr)**	**θ**
1	(6,12.5)	33	2	0.3	25	7.5	0.2
2	(8,12.5)	83	2	0.45	50	15	0.2
3	(4,12.5)	83	5	1.8	25	7.5	0.3
4	(12,8.3333)	83	5	0.7	50	15	0.3
5	(6,12.5)	33	2	0.5	25	7.5	0.3
6	(9,10)	33	2	0.16	50	15	0.3
7	(10,11.1111)	83	5	0.7	25	7.5	0.2
8	(11,6.66667)	83	5	0.65	50	15	0.2
9	(10,10)	33	2	0.3	25	7.5	0.3

Table 5.2 Optimal times (in hours) of maintenance for various criterions in RRT model.

Case	T*(Svar)	T*(RN)	T*(var1)	T*(var2)
1	17	24	15	15
2	21	30	18	18
3	8	12	4	4
4	37	43	28	28
5	20	24	14	14
6	25	34	23	23
7	35	46	30	30
8	24	30	20	20
9	34	41	28	28

Table 5.3 Objective functions when using the semivariance, risk-neutral and the variance 1 and 2 criteria for the renewal theory model.

Case	g(T*(Svar))	g(T*(RN))	g(T*(var1))	g(T*(var2))	IMP% RN	IMP% VAR
1	0.0953	0.1103	0.0972	0.0972	13.59	1.95
2	0.0618	0.0858	0.0631	0.0631	27.97	2.06
3	0.4009	0.4375	0.4461	0.4461	8.36	10.13
4	0.107	0.1116	0.1179	0.1179	4.12	9.24
5	0.0892	0.0919	0.0977	0.0977	2.93	8.70
6	0.0548	0.0667	0.0555	0.0555	17.84	1.26
7	0.1348	0.1616	0.1402	0.1402	16.58	3.85
8	0.1394	0.155	0.146	0.146	10.06	4.52
9	0.0555	0.0602	0.0587	0.0587	7.80	5.45

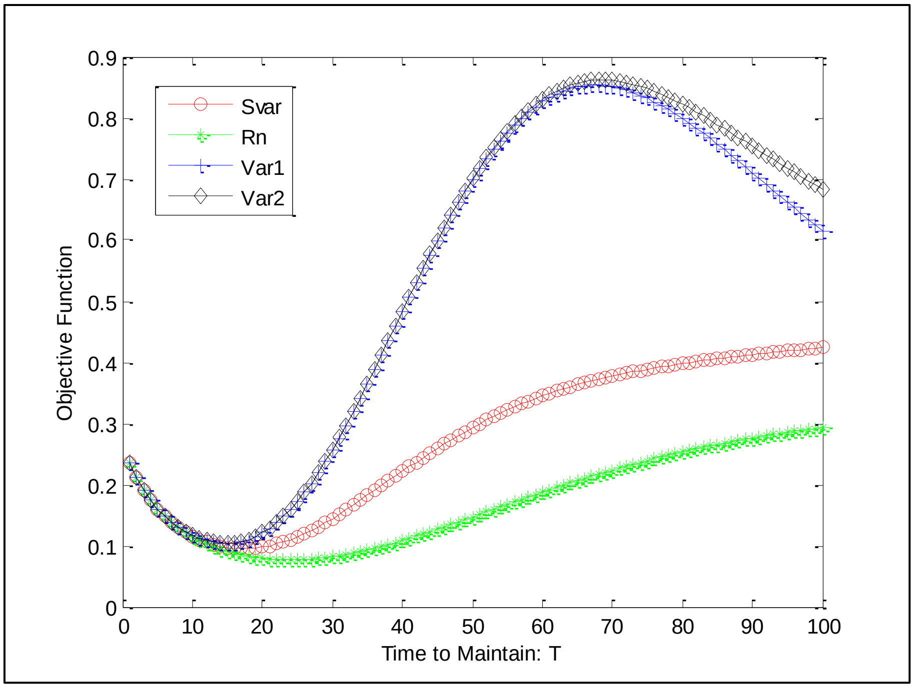

Figure 5.1 Comparison of risk-sensitive (*Svar and Var1*, *Var2*) and the risk-neutral objective functions for case 1, for RRT model.

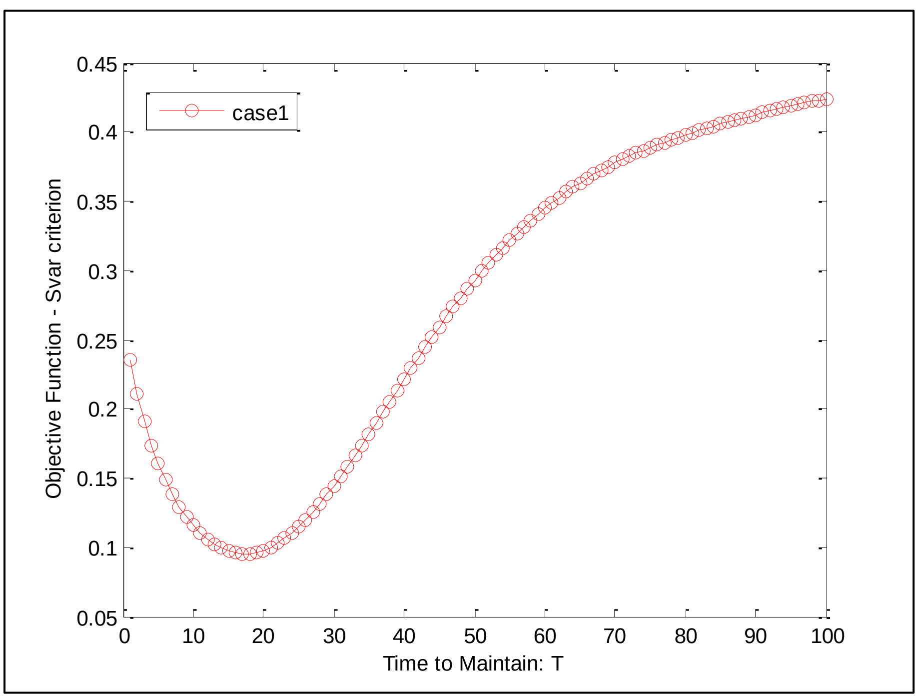

Figure 5.2 Plot of RS semivariance objective function of case 1, for RRT model.

Table 5.4 Optimal times (in hours) and objective functions of variance 1 and variance 2 models.

Case	T*(var1)	T*(var2)	gVar1(Tvar1)	gVar2(Tvar2)
1	15	15	0.1036	0.1037
2	18	18	0.066	0.066
3	4	4	0.4906	0.491
4	28	28	0.1252	0.1252
5	14	14	0.1087	0.1087
6	23	23	0.058	0.058
7	30	30	0.1506	0.1506
8	20	20	0.1536	0.1537
9	28	28	0.0623	0.0623

Table 5.5 Optimal times and objective function of Risk-Neutral model.

Case	T*(RN)	g(T*(RN))
1	24	0.0767
2	30	0.0503
3	12	0.3189
4	43	0.0952
5	24	0.0767
6	34	0.0458
7	46	0.108
8	30	0.1221
9	41	0.0472

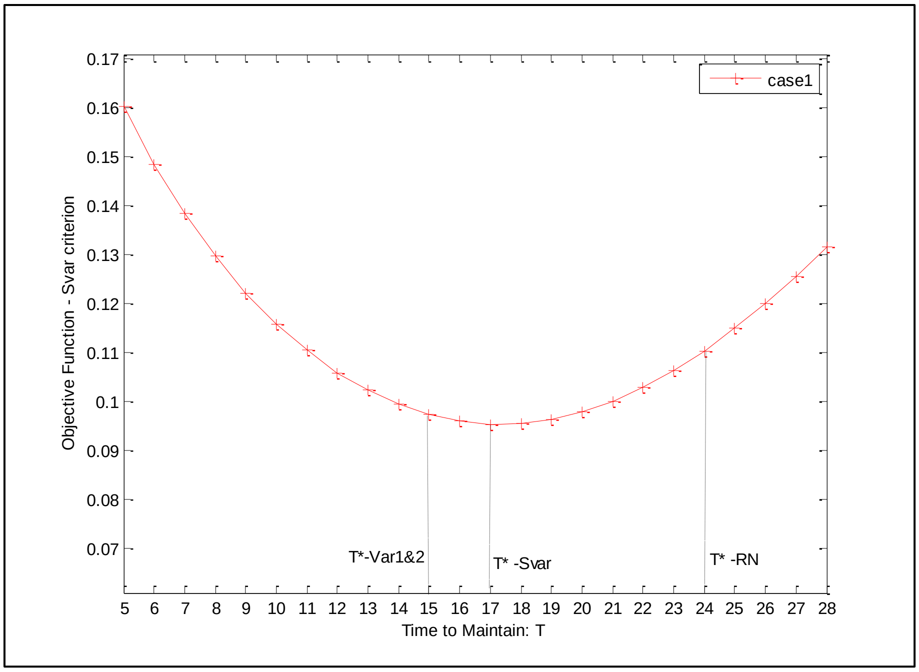

Figure 5.3 RRT model displaying the improvement of the objective function, with semivariance criterion for case1.

5.2 SEMI-MARKOV DECISION PROCESS MODEL

This section presents the numerical results of the SMDP model. Performance metrics of semivariance, and risk-neutral criteria were determined using linear programming. For variance, we use exhaustive enumeration, but we could also use quadratic programming. This research used the linear programming approach to find the objective function and optimal policy for the semivariance and risk-neutral criteria. The exhaustive enumeration approach was used to determine the objective function and optimal policy for the variance 1 criterion. Testing of the production system problem considered various TPMs, cost parameters, and θ values. The set of transition probabilities in our experiments were generated using the law mentioned below as: $p(i, produce, j) = \psi^d$ where i is the current state and j is the next state for the d values are 0, 1, 2,…., |S|-2 and if $p(|S-1|, produce, 0) = 1$. Here the unit of d is days. In our experiments we have used different values of ψ. The time for each production cycle is assumed to be $t_{p.}$ The repair time is assumed to be M1* t_p and the maintenance is M2* $t_{p.}$ M1 and M2 are repair and maintenance factors which are constants. The LP used in our model is more generalized one, even though the TPMs were generated in the above mentioned style for experimentation. The computer programs for the SMDP model is also written in MATLAB and tested to determine the optimal policy. The codes are presented in the appendix section of this thesis. The condition developed in equation 5.1 should be valid for the SMDP model as well. Table 5.6 shows the input parameters for the experiments done with the SMDP linear program and exhaustive enumeration. Table 5.7 displays the optimized policy for semivariance, variance and risk-neutral criteria of the SMDP model using linear programming technique for semivariance, variance and risk-neutral and exhaustive enumeration for variance. The policy prescribes the number of days which PM should be performed.. Table 5.8 denotes the optimized objective function values of semivariance, variance and risk-neutral criteria. Figure 5.4 shows a plot which describes the objective function of case 1 input data for various criterions mentioned in our research of SMDP model. Figure 5.5 shows the graph for improvement of the objective function of *Svar* criterion of case 1, for the semivariance model. Figure 5.6 shows a bar graph of all optimized policies of all input cases for the semivariance, variance and risk-neutral criteria.

Table 5.6 Input parameters for experiments with the SMDP model t_p=15/24 days.

Case	Psi	C_r($)	C_m($)	Theta
1	0.94	5	2	0.2
2	0.92	6	4	0.2
3	0.91	7	5	0.1
4	0.88	8	5	0.3
5	0.93	6	2	0.2
6	0.92	7	5	0.2
7	0.89	6	4	0.3
8	0.96	6	2	0.2
9	0.9	5	2	0.2
10	0.95	10	7	0.1

Table 5.7 Optimized policies for Svar, Var and RN criteria for the SMDP model using linear programming for Svar and RN and exhaustive enumeration for variance.

Case	Policy_Var(days)	Policy_Svar(days)	Policy_Rn(days)
1	3	4	5
2	5	6	9
3	7	8	10
4	1	3	6
5	2	3	4
6	6	7	10
7	4	5	7
8	3	4	5
9	2	3	4
10	8	9	12

Table 5.8 Optimized objective function of Svar, Var and RN criteria for SMDP.

Case	g($\mu Svar$)	g(μVar)	g(μRN)
1	0.0559	0.0775	0.0568
2	0.097	0.1077	0.0976
3	0.1123	0.1141	0.1124
4	0.2109	0.7472	0.2199
5	0.0694	0.1394	0.0725
6	0.1277	0.1363	0.128
7	0.1211	0.141	0.1221
8	0.0564	0.103	0.0579
9	0.0676	0.1018	0.0691
10	0.1529	0.1613	0.1538

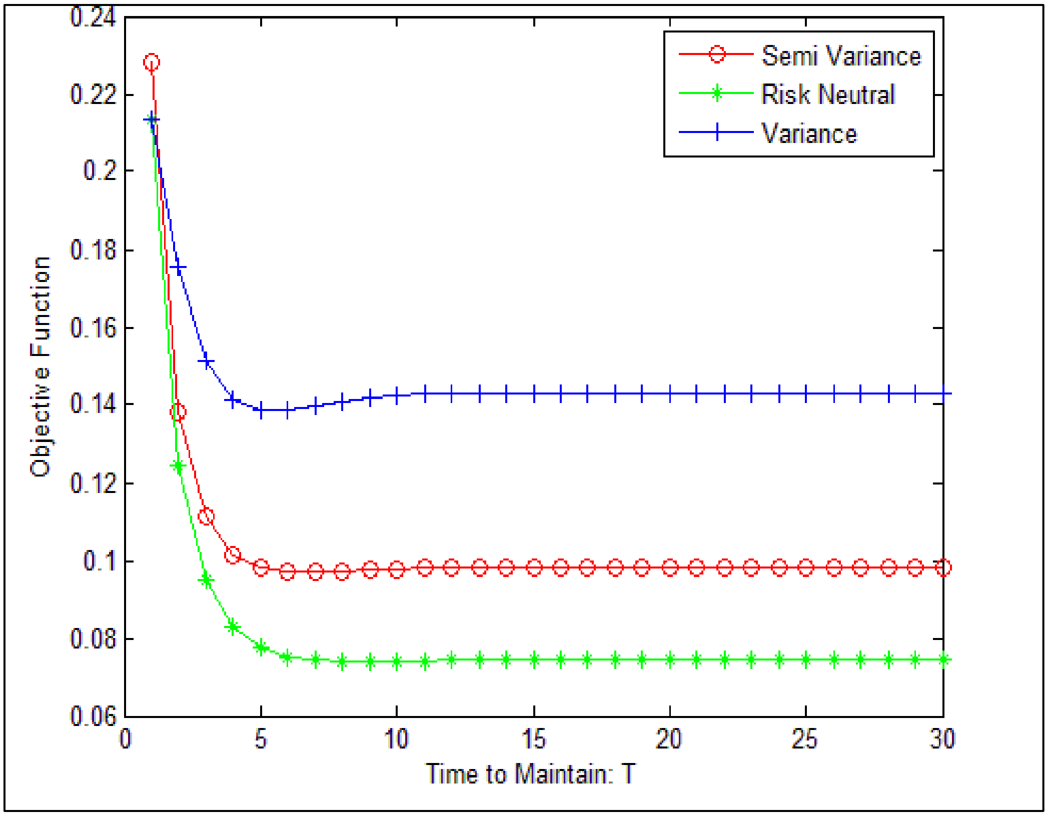

Figure 5.4. A plot of comparing the objective functions of *Svar*, *Var* and *RN* criteria for case1 for the SMDP model.

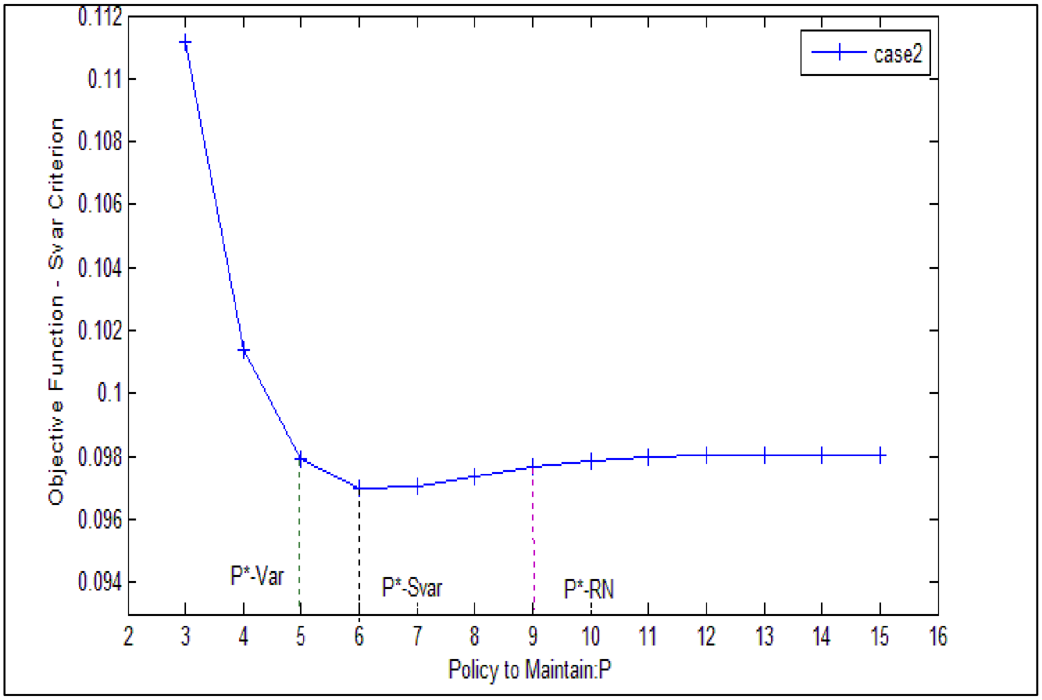

Figure 5.5 Plot of improvement in the objective function of the Svar criterion of case 1 for the SMDP model.

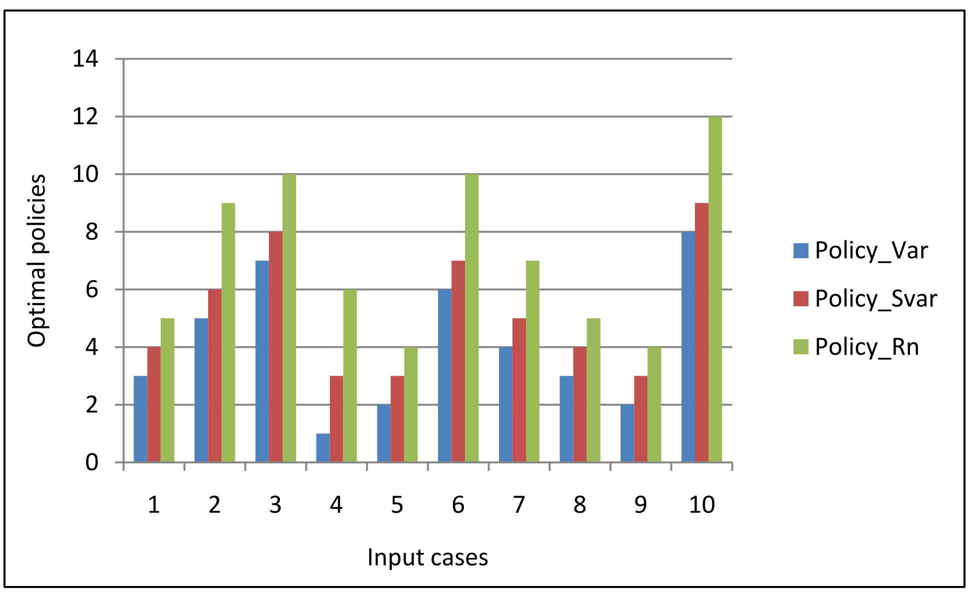

Figure 5.6 Bar graph showing all optimized policies of all input data for SMDP model under Svar, Var and RN criteria.

6. CONCLUSIONS AND FUTURE RESEARCH

This work developed two mathematical models for risk-sensitive PM based on renewal reward theory and SMDPs. In this research, we found the optimal times for performing PM on a production system under the semivariance criterion. The results show the practical superiority of using semivariance over existing risk-neutral and variance-based models. Most published work addresses semivariance in the field of portfolio selection. This work introduced the use of semivariance to model the risk-sensitive objective function of a PM problem.

Shewade (2006) was the first to use semivariance in PM; however his work used an MDP model (as opposed to our SMDP model) and an RRT theory model in which time was assumed to be unity. The RRT model is used generally for PM of smaller and simpler systems e.g., pumps, AGVs, and conveyers. The SMDP model is useful in formulating PM policies for larger systems e.g., production lines. Our RRT model required evaluation of integrals numerically while the SMDP model hinged on linear programming.

Both RRT and the theory of SMDP are frequently applied in other real-world problems. In the RRT model one uses the elementary reward theorem to determine the expected cost. The distinguishing feature of the objective function in the RRT model presented here is that it uses the Markowitz framework with semivariance as a measure of risk. In SMDP model provided here, linear programming is employed for optimization. The computational results clearly show the effectiveness of using semivariance-sensitive PM policies over traditional approaches. In all the experiments we performed, the semivariance model produced optimal policies that recommended maintenance earlier than the risk-neutral model, but later than the variance model. Variance 1 and variance 2 models in the RRT framework have numerically the same objective function value (and optimal times), which is an interesting finding.

Future Research:

A useful direction for future research would be to compare semivariance to other measures of downside risk in the RRT model. This aspect of the study could also be considered for the SMDP model. Another area of future research is to use dynamic programming and develop solutions for value iteration and policy iteration under the semivariance criterion.

APPENDIX

RENEWAL REWARD THEORY MATLAB CODES

Main.m

```
*************************************************************************
% Author: Venkata Manojramam Tirumalasetty
% Model : Renewal Reward Theory
% To find the optimal time for Preventive Maintenance (PM) for a given % problem - Renewal theory model
% Inputs:
% Maximum time (max_T)
% Target cost (tau)
% Cost of Repair (Cr)
% Cost of maintenance (Cm)
% Time to repair (tr)
% Time to maintain (tm)
% Value of theta - small value between 0 and 1(P)
% Value of shape parameter (n)
% Value of scale parameter (lambda); mean n*lambda;
% lambda of the paper is 1/lambda of this code.
************************************************************************

global Cr Cm tr tm theta n lambda tau

max_T=100;
Cr=33;
Cm=2;
tr=25;
tm=7.5;
theta=0.2;
n=6;
lambda=12.5;
tau=0.3;

% Exaustive enumeration through for loop
for i=1:max_T
   S(i)=evaluator(i);
   RN(i)=risk_neutral_eval(i);
   VAR1(i)=variance_eval(i);
   VAR2(i)=variance_new_eval(i);
   T(i)=i;
end;
```

```
%Plot the objective function
plot(T,S,'ro-', T,RN,'g*-', T,VAR1,'b+-', T,VAR2, 'kd-')
xlabel('Time to Maintain: T');
ylabel('Objective Function');
legend('Semi Variance','Risk Neutral','Variance1','Variance2');

% Find the optimal time for maintenance
Opt_maint_cost_Svar=min(S);
Opt_maint_cost_Rn=min(RN);
Opt_maint_cost_Var1=min(VAR1);
Opt_maint_cost_Var2=min(VAR2);
for j = 1:max_T
   if S(j)==Opt_maint_cost_Svar
      Opt_maint_time_Svar=j
   end
   if RN(j)==Opt_maint_cost_Rn
      Opt_maint_time_Rn=j
   end
   if VAR1(j)==Opt_maint_cost_Var1
      Opt_maint_time_Var1=j
   end
   if VAR2(j)==Opt_maint_cost_Var2
      Opt_maint_time_Var2=j
   end
end

Opt_maint_time_Svar
Opt_maint_time_Rn
Opt_maint_time_Var1
Opt_maint_time_Var2

% Here the Objective function g is for Semi variance model
gTSVAR=evaluator(Opt_maint_time_Svar);
gTRN=evaluator(Opt_maint_time_Rn);
gTVAR1=evaluator(Opt_maint_time_Var1);
gTVAR2=evaluator(Opt_maint_time_Var2);

figure
for i=1:max_T
   S(i)=evaluator(i);
    T(i)=i;
end;
x1=T(5:30);
y1=S(5:30);
plot(x1,y1,'b+-')
xlabel('Time to Maintain:T')
```

```
ylabel('Objective Function - Svar Criterion')
figure
plot(T,S,'ro-')
xlabel('Time to Maintain: T');
ylabel('Objective Function - Svar criterion');

%Here the Objective function g is for Variance 1 and 2 model

gVAR1=variance_eval(Opt_maint_time_Var1);
gVAR2=variance_new_eval(Opt_maint_time_Var2);

%Here the Objective function g is for RN Model
gRN=risk_neutral_eval(Opt_maint_time_Rn);

gTSVAR
gTRN
gTVAR1
gTVAR2
gVAR1
gVAR2
gRN
```

Evaluator.m

```
function obj_fun = evaluator(T)

global Cr Cm tr tm theta n lambda tau
% Evaluate the objective function, for Svar criterion for a given value of T
% First, calculate expected cost and the expected renewal time

% Expected cost
R = Cr*gamcdf(T,n,lambda) + Cm*(1-gamcdf(T,n,lambda));
% Expected time
L = quadl(@fun,0,T)+ (T+tm)*(1-gamcdf(T,n,lambda));
% Semivariance
SV = quadl(@fun_sv,0,T)+ (1-gamcdf(T,n,lambda))*max(0,(Cm-tau*(T+tm)))^2;

% Calculate the objective function
obj_fun = (R/L) + theta*(SV/L);
*************************************************************************
```

Risk_neutral_eval.m

```
function [obj_fun_rn] = risk_neutral_eval(T)

% Evaluate objective function, of RN criterion for a given value of T
```

```
global Cr Cm tr tm theta n lambda tau

% Calculate expected cost and the expected renewal time
R = Cr*gamcdf(T,n,lambda) + Cm*(1-gamcdf(T,n,lambda));
L = quadl(@fun,0,T)+ (T+tm)*(1-gamcdf(T,n,lambda));

% Calculate the objective function
obj_fun_rn = R/L;
```

****

Variance_eval.m

```
function obj_fun = variance_eval(T)

global Cr Cm tr tm theta n lambda tau
% Calculate expected cost, expected cyce l length, expected squared
% cycle length and expected squared cost

R = Cr*gamcdf(T,n,lambda) + Cm*(1-gamcdf(T,n,lambda));
L = quadl(@fun,0,T)+ (T+tm)*(1-gamcdf(T,n,lambda));
R2 = (Cr^2)*gamcdf(T,n,lambda)+(Cm^2)*(1-gamcdf(T,n,lambda));
W=(R2-R^2)/L;
obj_fun = (R/L) + theta*(W);
```

****

Variance_new_eval.m

```
function obj_fun = variance_new_eval(T)

global Cr Cm tr tm theta n lambda tau

% Calculate expected cost, expected cycel length, expected squared
% cycle length and expected squared cost

R = Cr*gamcdf(T,n,lambda) + Cm*(1-gamcdf(T,n,lambda));
L = quadl(@fun,0,T)+ (T+tm)*(1-gamcdf(T,n,lambda));
R2 = (Cr^2)*gamcdf(T,n,lambda)+(Cm^2)*(1-gamcdf(T,n,lambda));
L2=quadl(@fun_var,0,T)+(T+tm)^2*(1-gamcdf(T,n,lambda));
rho=R/L;

%Calculate the objective function

W=R2/L-(2*rho^2*L)+(rho^2*L2/L);
obj_fun = (R/L) + theta*(W);
```

****

fun.m

```
function y = fun(x)

global Cr Cm tr tm theta n lambda

y = (x+tr).*gampdf(x ,n,lambda);
```

fun_var.m

```
function y = fun_var(x)

global Cr Cm tr tm theta n lambda

y = (x+tr).^2.*gampdf(x ,n,lambda);
```

fun_sv.m

```
function y = fun_sv(x)

global Cr Cm tr tm theta n lambda tau

y=max(0,Cr-(tau*(x+tr))).^2.*gampdf(x ,n,lambda);
```

****

SEMI-MARKOV DECISION PROCESS

LINEAR PROGRAMMING (LP) APPROACH MATLAB CODES

Main.m

```
% we will assume in our model that when a machine fails it is repaired
% after approximately M1*T time units since the start of the production
% cycle; also when the machine is maintained, the maintenance is
% complete
% after approximately M2*T time units since the decision to maintain is
% made. T is the fixed time for one production. Also, M1 is much
% greater
% than M2.

global NS T PSI CR CM  M1 M2 TAU THETA na

% Declaration of variables
NS=30; % number of states
T=15; % time of one production
PSI =0.94; % TPM variable
```

```
CR=5; % cost of one repair
CM=2; % cost of one maintenance
M1=2; % Repair time factor
M2=1.25; % Maint time factor
TAU=0.15; %Target value
THETA=0.2; %For Risk Neutral put the theta value to 0
na=2; % number of actions in our model: one for production and the other for
maintenance

[tpm,twm,ttm]=matrix_generator(na);
X=lin_prog(tpm,twm,ttm);
```

****

Matrix_generator.m

```
function [tpm,twm,ttm]=matrix_generator(na)

global NS T PSI CR CM CP M1 M2 TAU THETA na

% Generate the TPM
tpm = zeros(NS,NS,na) ;
for state=1:NS
   tpm(state,1,2)=1;
end
for state=1:NS-1
   tpm(state,state+1,1)=PSI^state  ;
   tpm(state,1,1)=1-tpm(state,state+1,1);
end
tpm(NS,1,1)=1;

% generate the TCM
tcm=zeros(NS,na);
 for state=1:NS
   tcm(state,1,1)=CR;
   if state <= (NS-1)
       tcm(state,state+1,1)=0;
   end
   tcm(state,1,2)=CM;
end

% generate the TTM
ttm=zeros(NS,na);
for state=1:NS
   ttm(state,1)= (tpm(state,1,1)*M1*T)+(1-tpm(state,1,1))*T;
   ttm(state,2)=M2*T;
end
```

```
%To calculate v(i,a,j)
v=zeros(NS,NS,na);
for a=1:2
   for i=1:NS
   for j=1:NS
      L=max(0,(tcm(i,j,a)-(TAU)*(ttm(i,a))));
      v(i,j,a)=L^2;
   end
   end
end

% To calculate w(i,j,a)
twm=zeros(NS,NS,na);
  for a=1:2
      for i=1:NS
         for j=1:NS
       twm(i,j,a)=tcm(i,j,a)+(THETA)*v(i,j,a);

         end
      end
  end
```

****

lin_prog.m

```
function [X]=lin_prog(tpm,twm,ttm)

global NS T PSI CR CM CP M1 M2 TAU THETA na

 %Calculate expected wbar
  wbar=zeros(NS,na);
  for a=1:2
   for i=1:NS
   sum=0;
     for j=1:NS
         sum=sum+tpm(i,j,a)*twm(i,j,a);
     end
     wbar(i,a)=sum;
   end
end

% To convert Wbar into 1 D vector
obj_func = zeros(1,NS*na);
l=0;
for i=1:NS
   for j=1:na
```

```
        l=l+1;
        obj_func(1,l)=wbar(i,j);
    end
end

%To calculate ttm_mod - 1D vector of ttm

ttm_mod = zeros(1,NS*na);
l=0;
for i=1:NS
    for j=1:na
        l=l+1;
        ttm_mod(1,l)=ttm(i,j);
    end
end

% To calculate first half of A Constarint

I_matrix=zeros(NS,NS*na);
for i=1:NS
  I_matrix(i,(i-1)* na+1 : i * na)= ones(1,na);
    end
I_matrix

% To calculate second half of A

for j=1:NS
    k=1;
    for i=1:NS
        for a=1:2
            ACP(j,k)=tpm(i,j,a);
            k=k+1;
        end
    end
end
Aeq=I_matrix-ACP
Aeq= [Aeq; ttm_mod]

%To calculate Beq right side of the constraint

Beq=zeros(NS+1,1);
Beq(NS+1)=1
f=obj_func
for i = 1:NS*na
    LB(i)=0;
end
```

```
X=linprog(f,[] ,[] ,Aeq,Beq,LB)

X_mod=X.';
X_mod

val=f*X;
val


%Objective function

Y=zeros(NS,na);
l=0;
for i=1:NS
   for j=1:na
   l=l+1;
   Y(i,j)= X_mod(1,l);
end
end
% To find the optimized policy

q=zeros(NS,na);
for i=1:NS
for a=1:na
   sum=0;
   for j=1:na
      sum=sum+Y(i,j);
   end
   q(i,a) = Y(i,a)/sum;
end
end
*************************************************************************
```

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VITA

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