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Analog simulation of a synchronous machine

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ANALOG SIMULATION OF A SYNCHRONOUS MACHINE

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A

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ABSTRACT

The problem presented here is a study by an analog computer simulation of a synchronous machine under balanced three phase, short-circuit conditions. The purpose is to obtain a solution that is more accurate than one obtained by approximate methods using transient and subtransient reactances as is often done. This simulation is accomplished by using an idealized synchronous machine. The equations relating voltage and currents in the machine, as well as speed and torque, are presented for simulation. The equations for the machine include the saliency of one member, as well as the effect of speed and the damper windings.

A transformation of axes, as suggested by Park, is used to simplify the set of equations to be simulated. The set of tranformed equations are simulated on an EAI, TR-48 analog computer. The solution of the equations is obtained in terms of the direct and quadrature axis currents and then transformed into phase currents. The phase currents obtained from the simulation are then compared with actual test data for a synchronous machine.

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LIST OF SYMBOLS

$i_{a,b,c}$	phase current, phase a, b, or c
i_f	field current
i_d	direct axis current
i_q	quadrature axis current
i_o	zero sequence current
$i_{d_{1,2\cdots n}}$	current in direct axis damper winding $1,2,\cdots n$
$i_{q_{1,2\cdots n}}$	current in quadrature axis damper winding $1,2,\cdots n$
$e_{a,b,c}$	phase voltage, phase a, b, or c
e_f	field voltage
e_d	direct axis voltage
e_q	quadrature axis voltage
e_o	zero sequence voltage
v_d	direct axis load voltage
v_q	quadrature axis load voltage
x_l	leakage reactance of coil a
x_n	mutual reactance between two armature coils
$x_a=x_l+x_m$	effective leakage reactance of either of the axis coils
$x_o=x_l-3x_m$	zero sequence reactance associated with zero sequence current.
x_f	reactance of field coil
$x_{d_{1,2\cdots n}}$	reactance of damper coils $1,2,\cdots n$
x_{md}	per-unit mutual reactance between coils on the direct axis

$x_{d_{1,2\cdots n}}$ mutual reactance between coils D and $D_{1,2,\cdots n}$

$x_{d_{1,2}}$ mutual reactance between coils D_1 and D_2

x_d self reactance of coil D

$x_{q_{1,2\cdots n}}$ mutual reactance between coils Q and $Q_{1,2,\cdots n}$

$x_{q_{1,2}}$ mutual reactance between coils Q_1 and Q_2

x_q self reactance of coil Q

x_{mq} per-unit mutual reactance between coils on the quadrature axis

x_a per-unit leakage reactance of armature

x_f per-unit leakage reactance of field

x_{kd} per-unit leakage reactance of damper winding on the direct axis

x_{kq} per-unit leakage reactance of damper winding on the quadrature axis

r_a per-unit resistance of the armature

r_f per-unit resistance of the field

$r_{d_{1,2\cdots n}}$ resistance of damper windings $1,2,\cdots n$ on the direct axis

$r_{q_{1,2\cdots n}}$ resistance of damper winding $1,2,\cdots n$ on the quadrature axis

$r_{d_{1,2}}$ resistance between coils D_1 and D_2

$r_{q_{1,2}}$ resistance between coils Q_1 and Q_2

r_{kd} per-unit direct axis damper winding resistance

r_{kq} per-unit quadrature axis damper winding resistance

ψ_d	direct axis flux linkage
ψ_q	quadrature axis flux linkage
ψ_{md}	direct axis mutual flux linkage
ψ_{mq}	quadrature axis mutual flux linkage
$v = \frac{d\theta}{dt}$	actual speed in radians/second
θ	actual position in radians
ω	nominal speed of machine
δ	power angle
p	d/dt and $p^2 = d^2/dt^2$
G	quadrature axis field winding
F	direct axis field winding
D	direct axis armature winding
Q	quadrature axis armature winding
$D_{1,2 \cdots n}$	direct axis damper windings
$Q_{1,2 \cdots n}$	quadrature axis damper windings
H	per-unit inertia constant
P	total electric power
J	moment of inertia
f_e	instantaneous electrical torque defined as positive when mechanical power is applied at positive speed
f_t	total instantaneous torque defined as positive as above

The symbols used in Table I and II under functions are listed as follows:

S.C.	sign changer
G	gain
I	integrator
M	multiplier
S	summer

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CHAPTER I

INTRODUCTION

Synchronous machines must be protected, as do all electrical machines, against high currents that sometimes occur during accidental short-circuits. In order to design or select the proper protective equipment, it is necessary to study the short-circuit behavior of synchronous machines. In this thesis the case of a three phase short-circuit occuring at the terminals of an isolated synchronous machine running at no-load will be considered. The equations developed are simulated on the analog computer and compared with actual test data for two machines.

In general, a sudden short-circuit occuring in an electric network disturbs the power balance that exists during steady-state operation. That is to say, in the case of the power system, the mechanical input no longer equals the electrical output of the machine. This is taking into account, of course, the efficiency of conversion. When the short-circuit takes place, the machine will decelerate or accelerate and will continue to do so until the power balance is restored. This observation indicates that a short-circuit initiates mechanical as well as electrical transients.

Concordia (1)* et. al. have shown mathematically and from experience that mechanical transients are quite slow compared to electrical transients. This statement is found to be true because of the very high inertia of the rotating masses of the generators and turbines. This high inertia indicates that, in effect, the speed can be assumed to be a constant during the analysis of the electrical transient. In this thesis, both the case of constant speed and changing speed during the transient situation will be investigated.

It is assumed that the reader has a basic understanding of the construction and operation of a synchronous machine. However, for those who might wish to refresh their memories, the use of references (3) or (9) is recommended.

The voltage produced in a synchronous machine is due to the relative motion of the two structures, the field and armature, and the flux linkages produced. It is possible, then, to write down the voltage equations in terms of the self and mutual inductances of all the windings and the speed of the machine. Figure 1 shows the arrangement of the rotor and stator for a three-phase salient-pole machine. In this thesis, the investigation will be limited to a machine with only one salient member. Figure 1 has direct-

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* Numbers in parentheses are references to the Bibliography.

axis and quadrature-axis damper windings, as well as the main field winding shown (2).

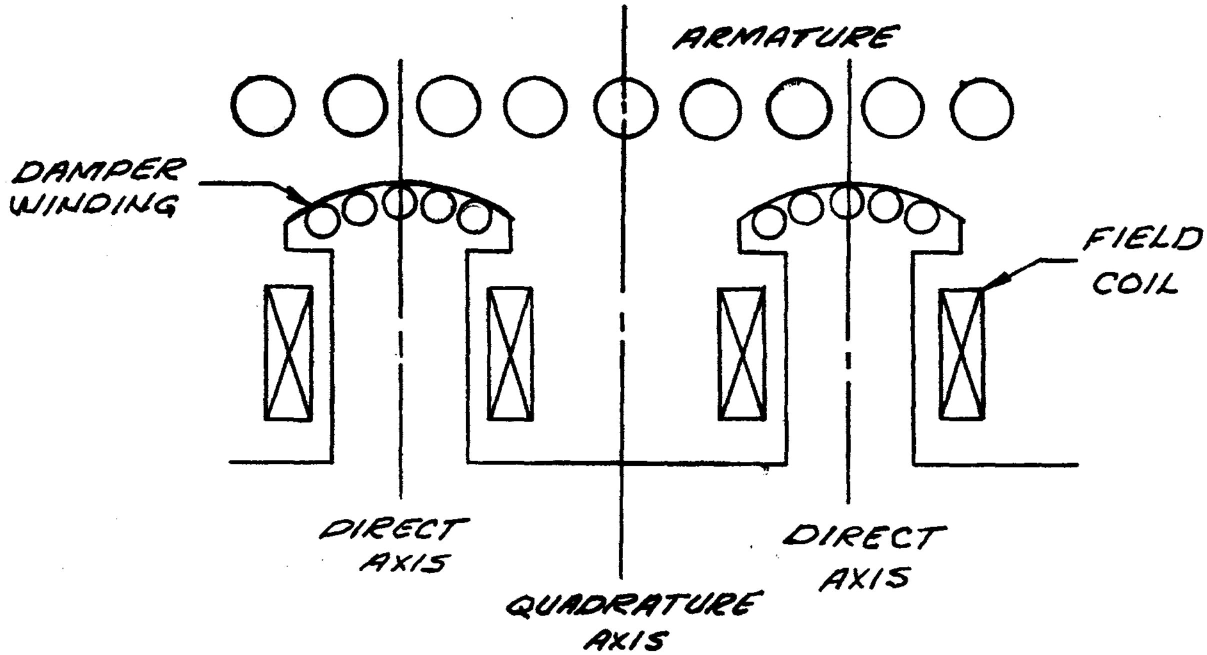

Figure 1. Salient-Pole Synchronous Machine With Damper Windings

All mutual inductances between stator and rotor circuits are periodic functions of the rotor angular position. In addition, the mutual inductances between any two stator phases are also functions of the rotor angular position (3). Therefore, upon writing a set of equations for the voltages produced, the result is a set of differential equations, and most of their coefficients are functions of the rotor angle. So, even in the case of constant speed, which makes the set of equations linear, and with saturation neglected, the equations are still difficult to solve.

It has been found that if a few reasonable assumptions are made, a relatively simple transformation of variables will eliminate all of the troublesome functions of the angle from the equations. In Chapter III and the Appendices, the equations, assumptions, and transformations described above are presented.

CHAPTER II

REVIEW OF THE LITERATURE

At the 1929 winter convention of the American Institute of Electrical Engineers in New York City, R. H. Park presented a paper entitled "Two-Reaction Theory of Synchronous Machines, Generalized Method of Analysis" (4). In this paper, Mr. Park generalized and extended the work of Blondel, Dreyfus, Doherty, and Nickle (5), (6), (7), (8). Park presented a general method of calculating power, current, and torque under both transient and steady-state conditions. Although he presented his paper in 1929, there is very little in the literature concerning his generalized method until the late forties and early fifties. It seems that although the direct and quadrature axis reactances are used, most of the solutions are performed by graphical or simplified math methods using the synchronous reactances (9). As late as 1954, Van Ness (10) suggests a transient analyzer analog which consists of a transient voltage in series with the transient reactance.

In 1942, Kron developed a method of applying tensors to the analysis of electrical machines, and he indicates that his method can be applied to any type of electrical machine (11). In his method, Kron applied a transformation tensor to, as he calls it, a primitive polyphase machine. In his work he used Park's idea of the transformation of axes. This treatise seems to have interested a number of people, because several authors (1), (2), (12) began writing concerning the generalized machine with reference to the works of Kron and Park.

In 1956, Breedon and Ferguson (18) used Park's set of transformation equations to derive a set of equations for simulation on the analog computer. The equations, however, are a set of linear equations which indicates that the speed was assumed to be constant. The equations are also much simplified and involve a set of relations using the direct and quadrature reactances.

In 1951, Boast and Rector (17) presented a paper concerning an analog method for determining power system swing curves. In 1954, Shen and Lisser (21) used an analog computer method for automatically determining the swing curves for the power system. In 1957 and 1958, Aldred and Shackshaft (13), (15) made use of a much simplified set of equations for the synchronous machine to study the effect of clearing time and the voltage regulator on the transient stability of the machine. In 1957, Aldred and Doyle (14) made a study, using the same set of equations as Aldred and Shackshaft, to investigate the transient stability of a synchronous machine.

Aldred and Coreless team up (20), (16) to introduce a power system simulator. In these works, the authors present systems involving one or two generators tied to an infinite bus as the power system. The simulation of the synchronous machine is kept as simple as possible for their work. In 1962, Aldred (19) extended the work of references (20) and (16) to an analog simulation of a multi-machine power system network. The equations for the machine are the same as those used by Aldred previously, but in this work he presents equations for the simulation of the transmission system and load.

In the works that have been cited here, the authors have simplified as much as possible the simulation of the synchronous machine. In order to simplify the equations of the machines, more assumptions were made than will be made in this thesis. None of the works cited investigated the phase currents of the machine under short-circuit conditions, and most of the authors were interested only in the stability of the machine or system or the swing curves that indicate the system stability.

CHAPTER III

GENERALIZED MACHINE THEORY

The theory used in this thesis is developed for an idealized machine. This idealized, two-pole machine is then converted, by use of transformations, to a generalized machine. The theory is then developed for the a.c. machine by deriving a set of equations relating voltages and currents in the generalized machine.

A. DISCUSSION OF THE PER-UNIT SYSTEM

Although the equations that will be developed to represent the generalized machine are valid for either actual or per-unit values, it is preferable to use the per-unit system. It is often difficult to understand the per-unit system that is being used and, therefore, it would be well to present here the per-unit system used in this thesis. The quantities, such as voltage, current, power, resistance, impedance, etc., can be translated to per-unit quantities as follows:

$$\text{Quantity in per-unit} = \frac{\text{actual quantity}}{\text{base quantity}}$$

Once the reference quantities are selected, the rest of the per-unit system relating various values must be consistent (3).

For any given machine, the reference quantities and the units of voltage and current are the rated values of the machine.

The unit of flux is defined such that a unit rate of change of flux induces a unit of voltage.

The unit of reactance is defined so that $\psi_{ab} = x_{ab}(i_a+i_b)$, for a mutually coupled circuit. Also, $\psi_a = x_a i_a$ and $\psi_b = x_b i_b$, where x_{ab} is called the mutual reactance, and x_a and x_b are called the effective leakage reactances.

Unit of impedance is defined so that a unit of current produces a unit voltage drop.

The unit of power is the power developed when unit voltage and current exist in all the main circuits. The unit of power can then be written as the sum of the powers supplied to the individual circuits times k_p, $P = k_p \Sigma ei$, where k_p is the reciprocal of the number of main circuits.

The unit of torque is defined as the torque which produces unit power at the nominal speed (12). Nominal speed in an a.c. machine is the synchronous speed of the two-pole machine in radians per-second, or 2π times the frequency of the source.

The moment of inertia is called the inertia constant in the per-unit system and is defined as follows (22):

$$H = \frac{0.231 \times 10^{-6}\ (Wk^2)n^2}{\text{rating in kva}}$$

where n = synchronous speed in rpm

W = weight in pounds

k = radius of gyration in feet

The unit of time is defined as the time for the rotor to pass through one electrical radian at nominal speed, and the unit of speed is the nominal speed as defined above.

B. GENERALIZED MACHINE

In the idealized machine, each winding or each part of a winding forming a complete circuit will be represented by a single coil. In this analysis, it is not important which element is rotating, since the operation depends only on the relative motion between the coils.

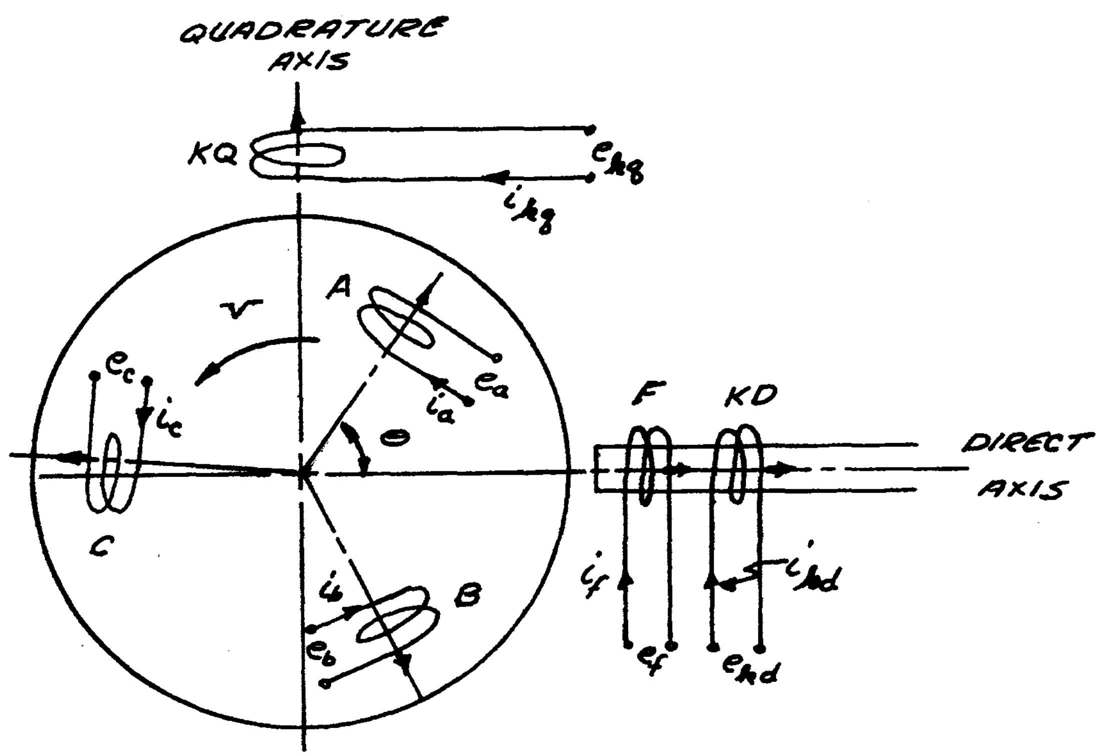

Figure 2. Diagram of an Idealized Synchronous Machine

In an actual machine with salient poles, either the stator or the rotor may have the salient poles. It is convenient, however, to represent a machine with the salient poles stationary and indicating them as the outer members, as shown in Figure 2. It is to be understood that the theory applies to either situation, and as is pointed out earlier, only one member is considered to be salient.

The diagram in Figure 2 shows only two coils on the direct axis and one on the quadrature axis. However, the number of coils is determined only by the accuracy desired in representing the machine.

The next step in the development of the machine theory is to transform the phase values shown in Figure 2 into direct and quadrature axis components. The general theory is then developed for a generalized machine having a number of coils with their axes along the fixed direct and quadrature axes. Figure 3 shows a diagram of such a generalized machine.

The machine shown in Figure 3 has F and G on the stationary member, and D and Q on the rotating member of the machine. According to this theory, any machine may be represented by the generalized machine if an appropriate number of coils are placed on each axis.

If the coils of the actual machine are on the axes, they are exactly the same as the ones represented in the

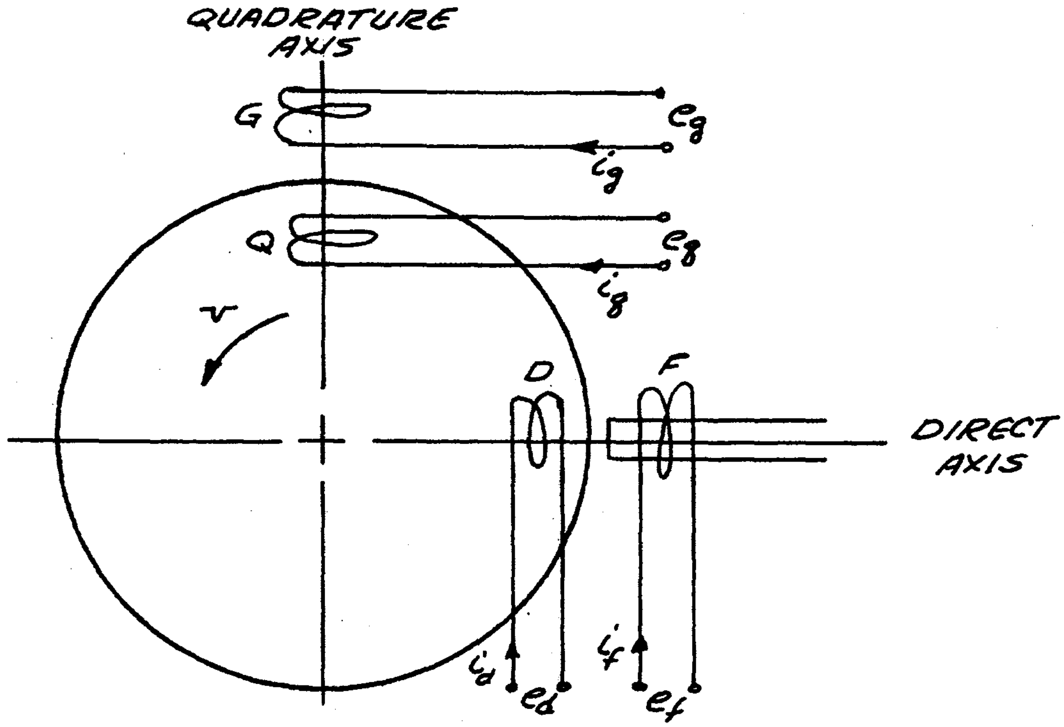

Figure 3. Diagram of a Generalized Machine (Four Coils)

generalized machine. If the coils are not permanently located on the axes, then it is necessary to make a conversion. The process of converting the actual coils of the machine to equivalent coils on the direct and quadrature axes, or vice versa, is known as a transformation of axes.

The idea of using transformations was first suggested by Park (4). The equations of the a.c. machine are of a much simpler form and are easier to solve if the actual phase voltages and currents e_a, e_b, e_c, i_a, i_b, and i_c are replaced by ficticious quantities e_d, e_q, e_o, i_d, i_q, and i_o. This method of analysis of machines by use of the transformed equations was termed by Park as the "two-reaction theory". The transformation equations will be found in Appendix A.

The basic ideas of the generalized machine theory have been presented. It is possible to apply the general theory to any type of rotating machine in the following manner. Set up an idealized two-pole machine using the smallest number of coils required to obtain the desired accuracy. This idealized two-pole machine is then converted, by appropriate transformations, to the generalized machine. The theory is then developed for the machine by deriving a set of equations relating the voltages and currents in the generalized machine. In addition to the voltage and current relationships, an equation relating torque to the currents and torque to speed must be derived.

The generalized machine has been called ideal, and the following assumptions have been made in considering the machine ideal. It has been assumed that the MMF wave, due to a sinusoidal current in the armature coils, is also sinusoidal. Because of the above assumption, the harmonic MMF waves set up are negligible. This assumption has been explained by Park in stating that the winding is sinusoidally distributed and, therefore, the armature flux wave is sinusoidally distributed in space. It has been assumed that saturation is negligible. The effect of eddy-current and hysteresis loss has also been assumed to be negligible (4). The effect of hysteresis and eddy-current loss in the member which contains the damper windings could be included by the addition of a damper circuit in the equations. However, hysteresis and eddy-current loss will have to be neglected in the member that does not have damper windings.

The method for transformation of voltages and currents from phase values to two-axis values has been presented. The two-axis values of flux will now be considered. In the per-unit system, a unit of flux linkage is such that a unit rate of change of flux linkage induces a unit voltage in any coil. To apply this idea to a machine, the axis of the flux wave must be the same as the axis of the coil in which the voltage will be induced. The phase flux may be resolved into two components expressed as the direct and quadrature axis flux linkages. As Park has shown, this resolution provides a simpler method of investigating instantaneous values of flux. Continuing with Park's idea, it is understood that each of the two components of flux linkages is the flux linkage with a coil on the corresponding axis. With this stipulation, ψ_d and ψ_q represent the flux linkages with any of the various coils, regardless of the number of turns. This is true because of the way in which the unit of voltage has been previously defined.

C. GENERAL EQUATIONS FOR A.C. MACHINES

Figure 4 is a diagram of a generalized a.c. synchronous machine that will be used in this thesis.

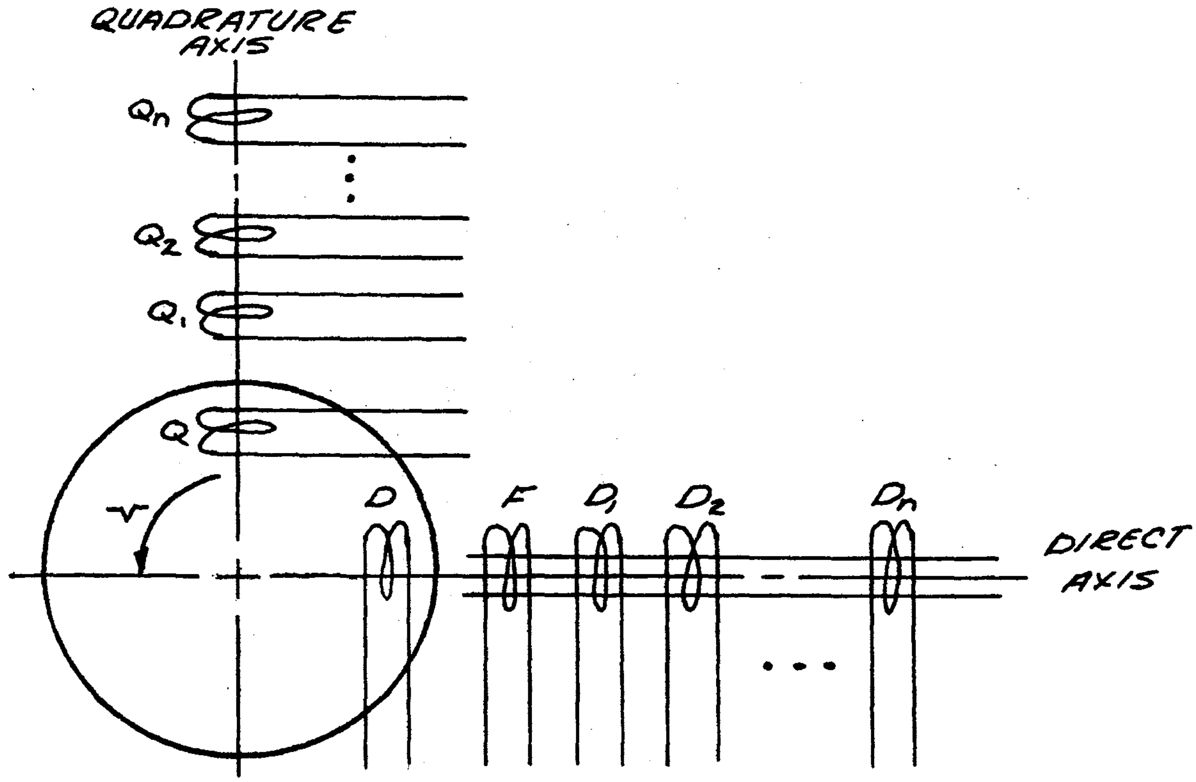

Figure 4. Diagram of Generalized Synchronous Machine

In Figure 4, the field is represented as a single coil on the direct axis, and this coil is normally supplied a voltage, e_f, from an exciter or other d.c. source. The other windings, D_1 through D_n and Q_1 through Q_n, represent the damper circuits of the machine. D and Q are again the direct and quadrature axis coils of the machine. The eddy-current paths in the iron could be considered as being an infinite number of damper bars and represented by coils. The equations for the field and direct and quadrature-axis damper coils are:

$$
\begin{aligned}
e_f &= (r_f+x_f p)i_f+x_{md}\, p i_{d_1}+x_{md} p i_{d_2}+ \cdots +x_{md} p i_f \\
0 &= x_{md} p i_f+(r_{d_1} +x_{d_1}\, p)i_{d_1}+(r_{d_2}+x_{d_2}p)i_{d_2}+ \cdots +x_{md} p i_d \\
0 &= x_{md} p i_f+(r_{d_{12}}+x_{d_{12}}p)i_{d_1}+(r_{d_2}+x_{d_2}p)i_{d_2}+ \cdots +x_{d_2} p i_d \\
&\;\vdots
\end{aligned}
\qquad (1)
$$

$$0 = x_{fd_n} p i_f + (r_{d_{1n}} + x_{d_{1n}} p) i_{d_1} + (r_{d_{2n}} + x_{d_{2n}} p) i_{d_2} + \cdots + x_{d_n} p i_d$$

$$\psi_d = x_{md} i_f + x_{d_1} i_{d_1} + x_{d_2} i_{d_2} + \cdots + x_d i_d \qquad (2)$$

$$\psi_q = x_{q_1} i_{q_1} + x_{q_2} i_{q_2} + \cdots + x_q i_q$$

$$0 = (r_{q_1} + x_{q_1} p) i_{q_1} + (r_{q_{12}} + x_{q_{12}} p) i_{q_2} + \cdots + x_{q_1} p i_q$$

$$0 = (r_{q_{12}} + x_{q_{12}} p) i_{q_1} + (r_{q_2} + x_{q_2} p) i_{q_2} + \cdots + x_{q_2} p i_q \qquad (3)$$

$$\vdots$$

$$0 = (r_{q_{1n}} + x_{q_{1n}} p) i_{q_1} + (r_{q_{2n}} + x_{q_{2n}}) i_{q_2} + \cdots + x_{q_n} i_q$$

This set of equations, (1), (2), and (3), along with those derived in Appendix B, equations (5B), (6B), and (7B), and from Appendix D, equations (11D) and (12D), represent the synchronous machine in general. The equations from the appendices mentioned above are stated here.

$$e_d = p\psi_d + v\psi_q + r_a i_d \qquad (4)$$

$$e_q = p\psi_q - v\psi_d + r_a i_q \qquad (5)$$

$$e_o = (r_a + x_o p) i_o \qquad (6)$$

$$\delta = \omega t - \theta \quad \text{and} \quad v = d\theta/dt \qquad (7)$$

$$f_t = \omega/2(\psi_d i_q - \psi_q i_d) + \frac{2H}{\omega} p^2 \delta \qquad (8)$$

CHAPTER IV

SIMULATION OF THE EQUATIONS FOR A SYNCHRONOUS MACHINE WITH ONE DAMPER WINDING

A synchronous machine with one damper winding has been chosen for the simplified machine to be studied in this thesis. This generalized machine is shown in Figure 5.

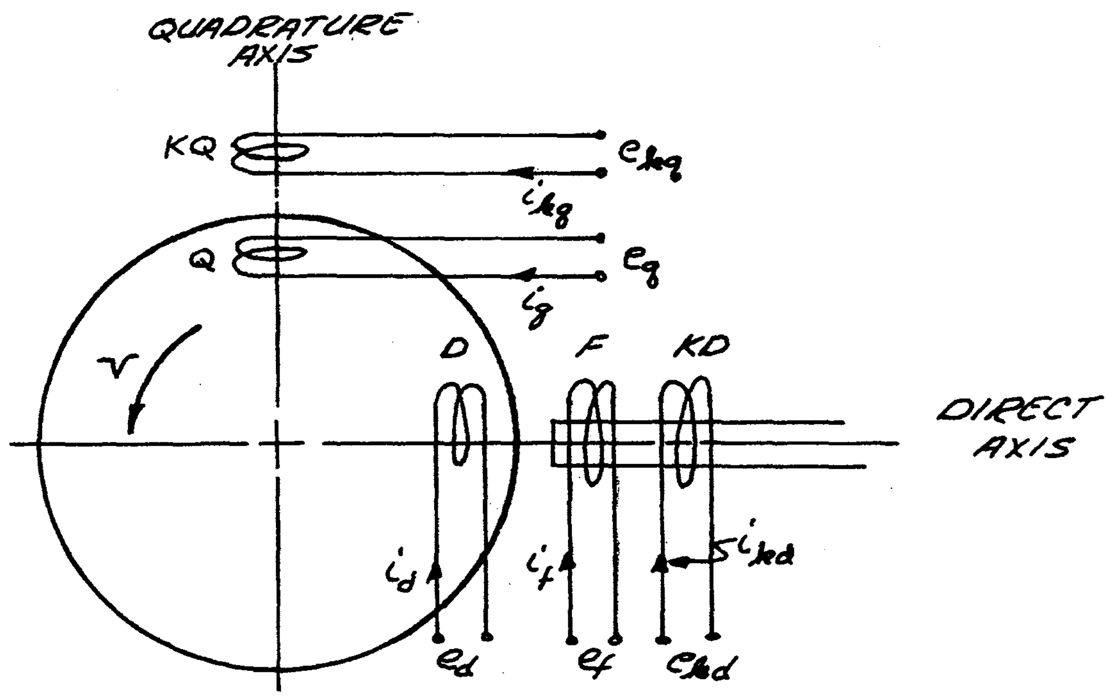

Figure 5. Synchronous Machine With One Damper Winding

The equations describing the synchronous machine with one damper winding have been derived in the various appendices. The set of equations representing the machine shown in Figure 5 are restated here in the following order: (9C), (11D), and (12D).

$$e_f = r_f i_f + (x_{md} + x_f) p i_f + x_{md} p i_{kd} + x_{md} p i_d \qquad (9)$$

$$0 = x_{md}pi_f+r_{kd}i_{kd}+(x_{md}+x_{kd})pi_{kd}+x_{md}pi_d \quad (10)$$

$$0 = r_{kq}i_{kq}+(x_{mq}+x_{kq})pi_{kq}+x_{mq}pi_q \quad (11)$$

$$e_d = x_{md}pi_f+x_{md}pi_{kd}+x_{mq}vi_{kq}+r_ai_d+(x_{md}+x_a)pi_d+(x_{mq}+x_a)vi_q \quad (12)$$

$$e_q = -x_{md}vi_f-x_{md}vi_{kd}+x_{mq}pi_{kq}-(x_{md}+x_a)vi_d+r_ai_q+(x_{mq}+x_a)pi_q \quad (13)$$

$$\delta = \omega t-\theta \quad \text{and} \quad v = d\theta/dt \quad (14)$$

$$f_t = \frac{\omega}{2}\left\{[x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d]i_q-[x_{mq}i_{kq}+(x_{mq}+x_a)i_q]i_d\right\} + \frac{2H}{\omega}p^2\delta \quad (15)$$

The set of equations (9) through (15) is the set of equations representing the machine to be simulated. The first step in the simulation of the equations is to solve each equation for the highest ordered derivative of one of the variables in each equation. This has been done as follows:

$$pi_f = \frac{1}{(x_{md}+x_a)}(e_f-r_fi_f-x_{md}pi_{kd}-x_{md}pi_d) \quad (16)$$

$$pi_{kd} = \frac{1}{(x_{md}+x_{kd})}(-r_{kd}i_{kd}-x_{md}pi_f-x_{md}pi_d) \quad (17)$$

$$pi_{kq} = \frac{1}{(x_{mq}+x_{kq})}(-r_{kq}i_{kq}-x_{mq}pi_q) \quad (18)$$

$$pi_d = \frac{1}{(x_{md}+x_a)}\left[-r_ai_d+e_d-x_{md}pi_f-x_{md}pi_{kd}+ -x_{mq}i_{kq} -(x_{mq}+x_a)i_q\, v\right] \quad (19)$$

$$pi_q = \frac{1}{(x_{mq}+x_a)}\left[e_q-r_ai_q-x_{mq}pi_{kq}+ x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d\right] \quad (20)$$

$$\theta = \omega t-\delta \quad \text{and} \quad v = d\theta/dt \quad (21)$$

$$p^2\delta = \frac{\omega}{2H}f_t-\frac{\omega^2}{4H}\left\{[x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d]i_q - [x_{mq}i_{kq}+(x_{mq}+x_a)i_q]i_d\right\} \quad (22)$$

Equations (16) through (22) are in the proper form for simulation on the electronic analog computer.

The problem considered here is the balanced three phase short-circuit at the terminals of a machine that is operating at no load and rated voltage. Under these conditions, the initial current flowing in phases a, b, and c is zero. The field current flowing is the necessary amount to produce rated voltage at the terminals of the machine and the field voltage applied is that required to cause the proper field current to flow under steady-state conditions. The speed is synchronous speed. Upon applying the three phase short-circuit, the terminal voltages v_a, v_b, and v_c are reduced to zero. Since v_a, v_b, and v_c are reduced to zero, it follows that v_d and v_q are zero as shown by:

$$v_d = 2/3\left[v_a \cos\theta + v_b \cos(\theta - 120^\circ) + v_c \cos(\theta + 120^\circ)\right]$$

$$v_q = 2/3\left[v_a \sin\theta + v_b \sin(\theta - 120^\circ) + v_c \sin(\theta + 120^\circ)\right]$$

As previously stated, i_a, i_b, and i_c are zero, therefore, i_d and i_q are zero initially as shown by:

$$i_d = 2/3\left[i_a \cos\theta + i_b \cos(\theta - 120^\circ) + i_c \cos(\theta + 120^\circ)\right]$$

$$i_q = 2/3\left[i_a \sin\theta + i_b \sin(\theta - 120^\circ) + i_c \sin(\theta + 120^\circ)\right]$$

The value of field current is found as being the current necessary to produce unit flux such that unit voltage or rated voltage will be generated. For steady-state operation preceeding the short-circuit, the field excitation will be the value of per-unit field current times the per-unit resistance of the field winding.

Incorporating the above initial conditions and conditions existing for a short-circuit at the terminals into equations (10) to (15), the simulation diagram for the machine is found to be Figure 6. The pot settings and amplifier outputs are listed in Tables I and II in terms of the variables involved so that different machines can be simulated.

The simulation diagram shown in Figure 6 is not as complete as is desired. Due to the lack of equipment on the TR-48 computer, the phase current output could not be obtained in this simulation. Therefore, the values of i_d and i_q were obtained from the analog, and selected points from the analog data were put into an IBM 1620 digital computer to perform the conversion from d, q values to a, b, and c values of current. Figure 7 is a simulation diagram showing the process by which i_a, i_b, and i_c would have been found had there been enough multipliers on the TR-48 analog computer.

Since the author had no idea of the form of the solution that should be obtained, the determinant representing the machine with constant speed was set up and solved for i_d and i_q by Laplace transforms. The inverse transform was taken to find the time domain representation. This solution was very helpful in realizing the proper solution from the analog simulation. The check solution is shown in Figure 8, and the check solution transformed into phase currents is shown in Figure 9.

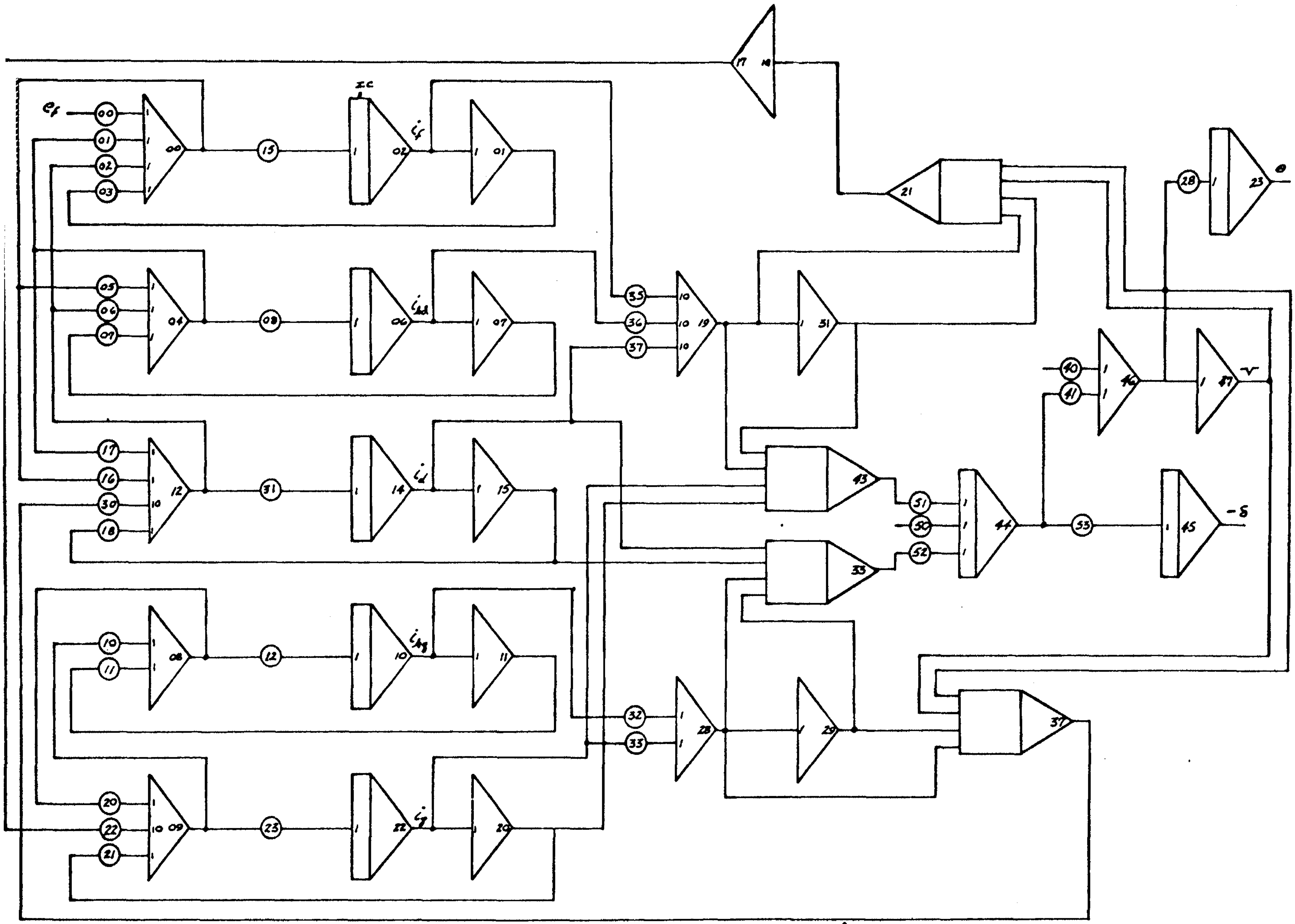

FIGURE 6. SIMULATION DIAGRAM FOR SYNCHRONOUS MACHINE

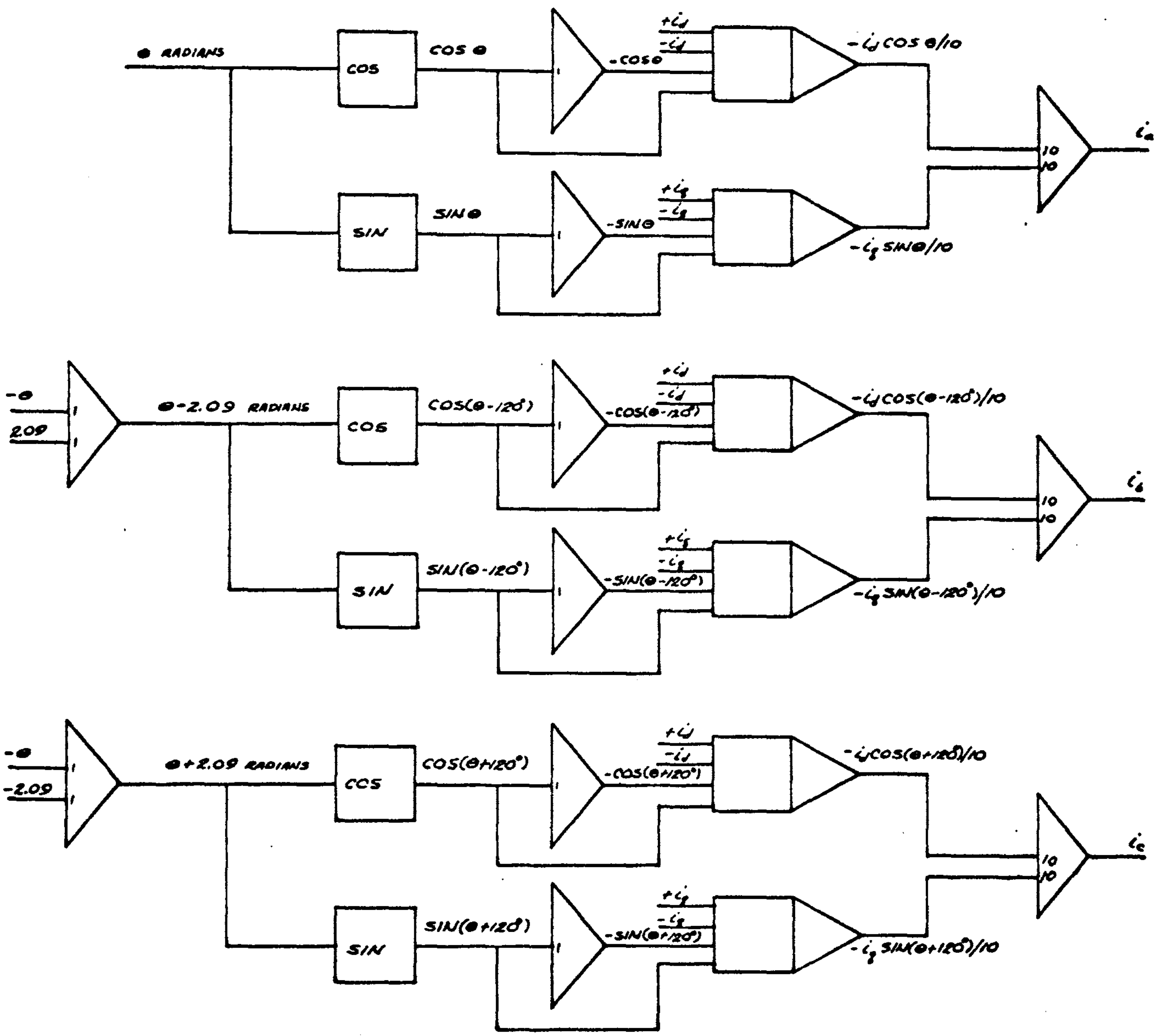

FIGURE 7. SIMULATION DIAGRAM FOR TRANSFORMATION FROM AXIS TO PHASE CURRENTS

TABLE I

POTENTIOMETER ASSIGNMENT LIST

Potentiometer Number	Parameter Description
00	$1/(x_{md}+x_f)$
01	$x_{md}/x_{md}+x_f)$
02	$x_{md}/(x_{md}+x_f)$
03	$r_f/(x_{md}+x_f)$
05	$x_{md}/(x_{md}+x_{kd})$
06	$x_{md}/(x_{md}+x_{kd})$
07	$r_{kd}/(x_{md}+x_{kd})$
08	Time Scaling
10	$x_{mq}/(x_{mq}+x_{kq}$
11	$r_{kq}/(x_{mq}+x_{kq})$
12	Time Scaling
15	Time Scaling
16	$x_{md}/(x_{md}+x_a)$
17	$x_{md}/(x_{md}+x_a)$
18	$r_a/(x_{md}+x_a)$
20	$x_{mq}/(x_{mq}+x_a)$
21	$r_a/(x_{mq}+x_a)$
22	$1/(x_{mq}+x_a)$
23	Time Scaling
28	Time Scaling

TABLE I (continued)

Potentiometer Number	Parameter Description
30	$1/(x_{md}+x_a)$
31	Time Scaling
32	x_{mq}
33	$x_{mq}+x_a$
35	x_{md}
36	x_{md}
37	$x_{md}+x_a$
40	1 p.u. speed
50	$\omega/2H$
51	$\omega^2/4H$
52	$\omega^2/4H$
53	Time Scaling

TABLE II

AMPLIFIER ASSIGNMENT LIST

Amplifier Number	Function	Output Variable
00	S	$-si_f$
01	S.C.	$-i_f$
02	I	i_f
04	S	$-si_{kd}$
06	I	i_{kd}
07	S.C.	$-i_{kd}$
08	S	$-si_{kq}$
09	S	$-si_q$
10	I	i_{kq}
11	S.C.	$-i_{kq}$
12	S	$-si_d$
14	I	i_d
15	S.C.	$-i_d$
17	S.C.&G.	$+[x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d]v$
19	S	$-[x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d]$
20	S.C.	$-i_q$
21	M	$-[x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d]v/10$
22	I	i_q
23	I	θ
28	S	$-[x_{mq}i_{kq}+(x_{mq}+x_a)i_q]$

TABLE II (continued)

Amplifier Number	Function	Output Variable
29	S.C.	$+\left[x_{mq}i_{kq}+(x_{mq}+x_a)i_q\right]$
31	S.C.	$+\left[x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d\right]$
33	M	$-\left[x_{mq}i_{kq}+(x_{mq}+x_a)i_q\right]i_d/10$
37	M	$+\left[x_{mq}i_{kq}+(x_{mq}+x_a)i_q\right]v/10$
43	M	$+\left[x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d\right]i_q/10$
44	I	$+s\delta$
45	I	$-\delta$
46	S	$-v$
47	S.C.	$+v$

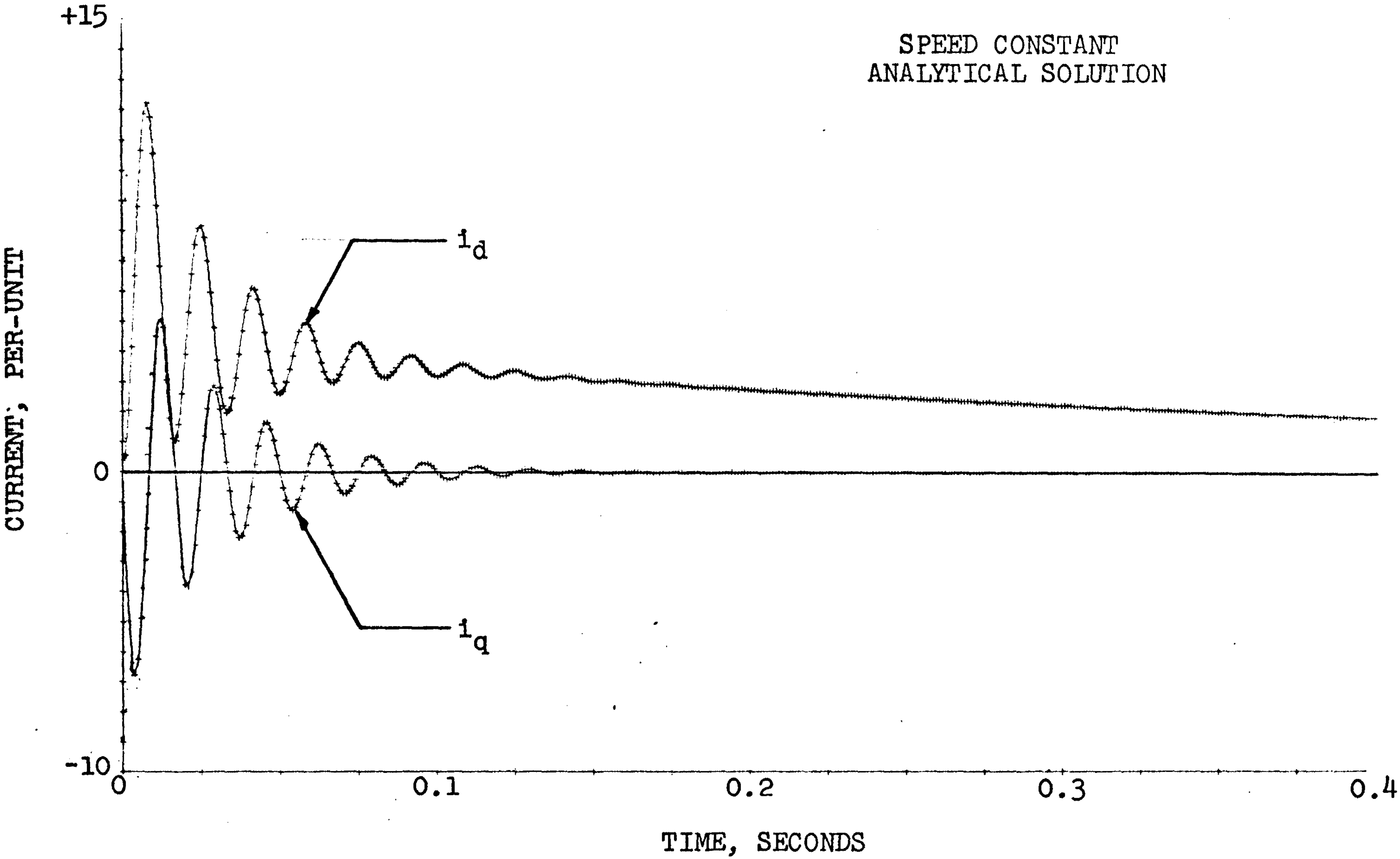

FIGURE 8. DIRECT AND QUADRATURE AXIS CURRENTS

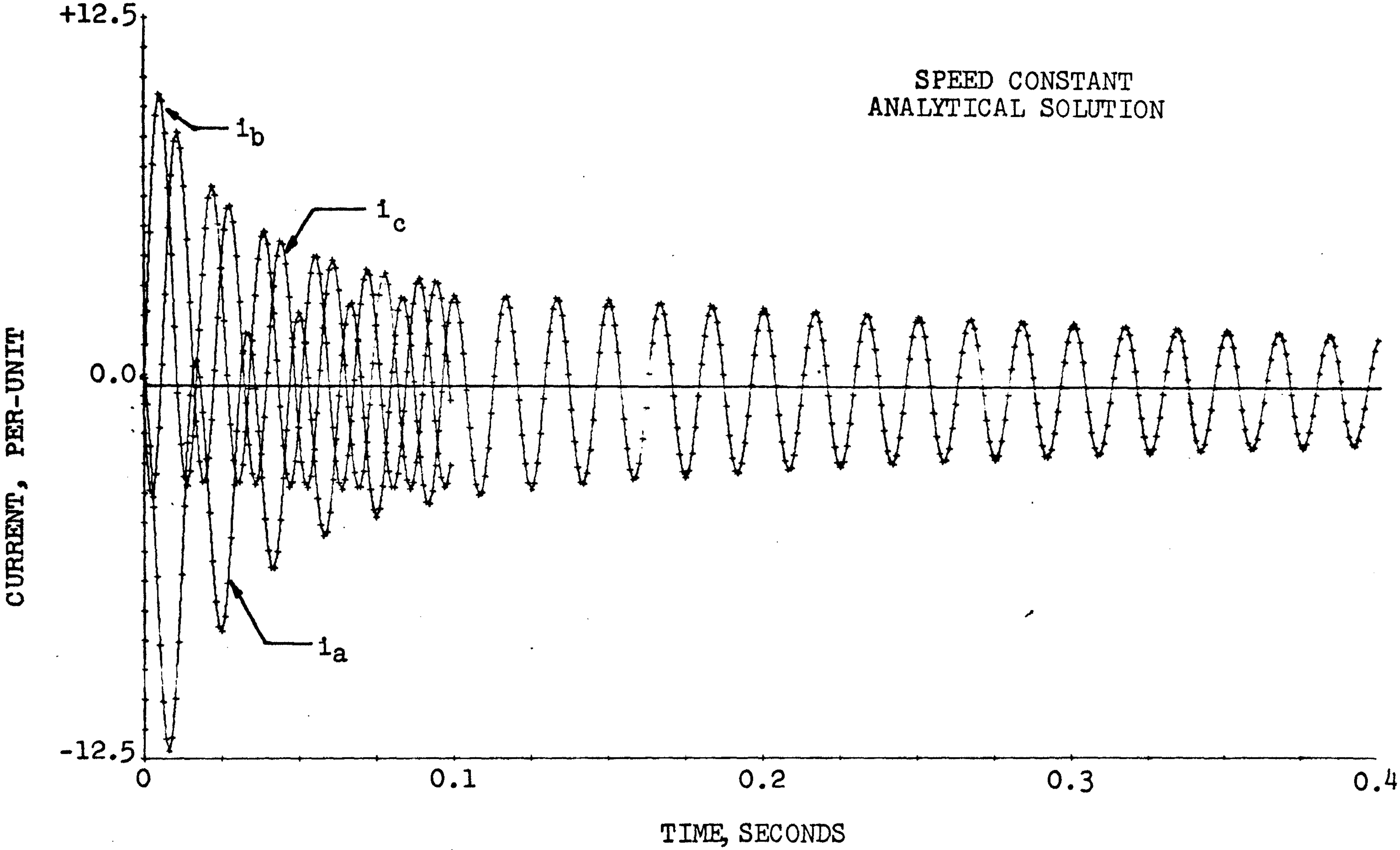

FIGURE 9. PHASE CURRENTS

CHAPTER V

RESULTS AND CONCLUSIONS

The results of this thesis are presented in Figures 10 through 21. Although a picture is worth a thousand words, it is important that the work done be summarized at this time. As has been stated earlier in this work, the results of the simulation are to be compared with test data for an actual machine to determine how well the results of the simulation compare with actual short-circuit data.

In Appendix F, the test data for two different machines is presented in Figures F-1 through F-7. Figures F-1 and F-2 are for the machine which will be called number one, and F-3 through F-7 are taken from a machine hereafter referred to as machine number two.

Figure F-1 is for no-load, and Figure F-2 is for full-load short-circuit conditions of machine one. Also included in the test data for machine one is the effect of a voltage regulator, the effects of which were not considered in this thesis. Figure F-2 is of short-circuit conditions, including regulator, when machine one is operating at full-load, which was not considered, but is offered in the next chapter as a possibility for extended study of the machine.

It is difficult to compare the results of the simulation to the test results on a peak-for-peak and cycle-for-cycle basis. This is due, in part, to the difference in the angle of the generated voltage at the time of the application of the fault. While the angle of the simulation was chosen to be zero, the angle at which the faults were applied in the test situation is not known. Therefore, the results of the test and simulation will be compared on the following basis: (1) the decrement of the transient currents, (2) the sustained values of current, and (3) the approximate time constant as read from the graphs.

It will not be possible to make a comparison between the test data and simulation data for machine one. The comparison cannot be made because the test data for machine one includes the effects of a voltage regulator. The field current under test conditions continues to increase as the regulator attempts to restore rated voltage; therefore, the test data would not be the same as the simulation which did not include a regulator. The simulation field current and speed are shown in Figures 10 and 11, and the phase currents in Figures 12 and 13 for machine one.

For a check on the simulation, the data for machine two is relied upon. Figures F-3 through F-7 are the test data for machine two, Figure F-3 being for phase a and the field current, while Figures F-4 to F-7 are of all three phases and the field current.

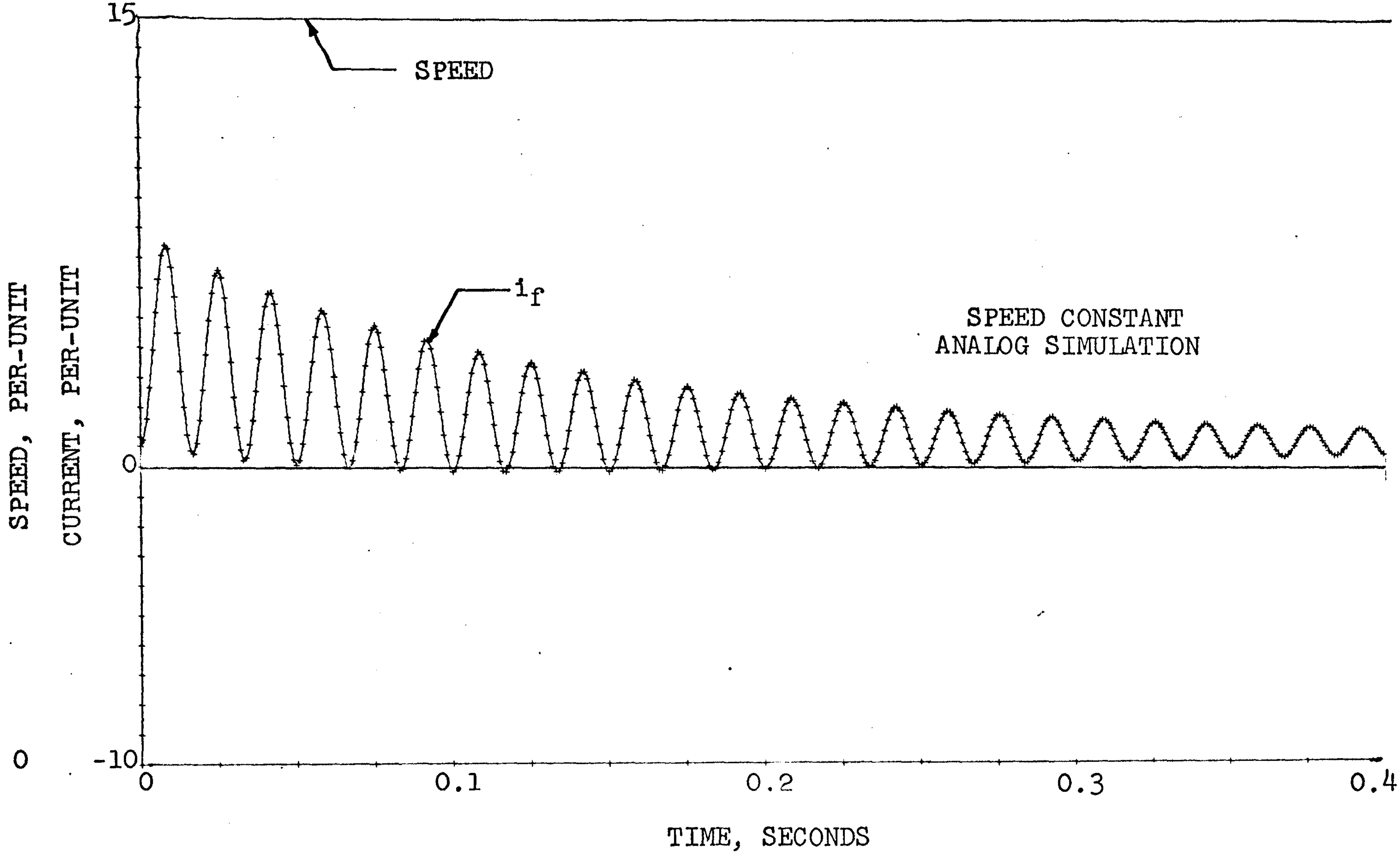

FIGURE 10. FIELD CURRENT AND SPEED, MACHINE ONE

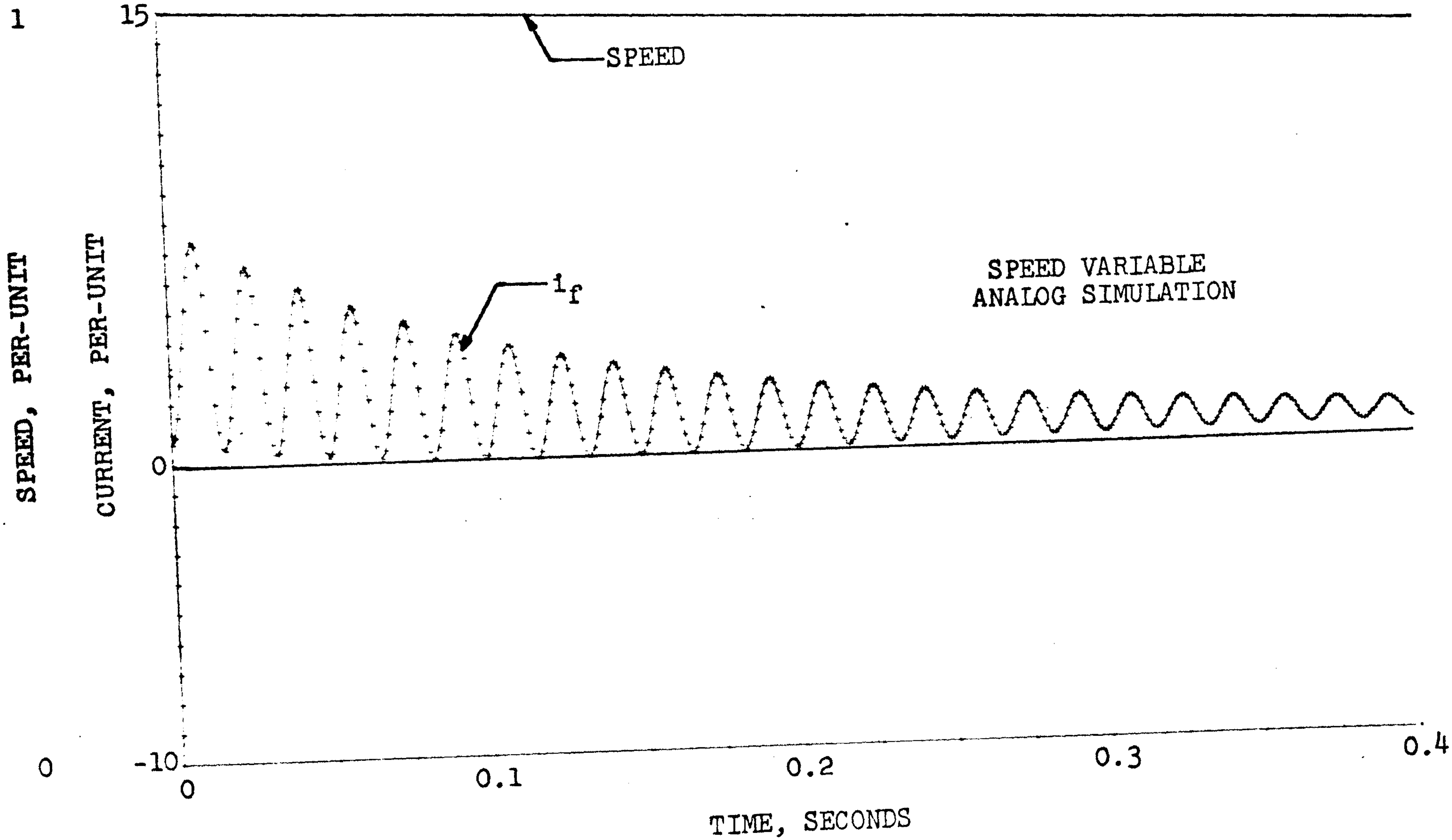

FIGURE 11. FIELD CURRENT AND SPEED, MACHINE ONE

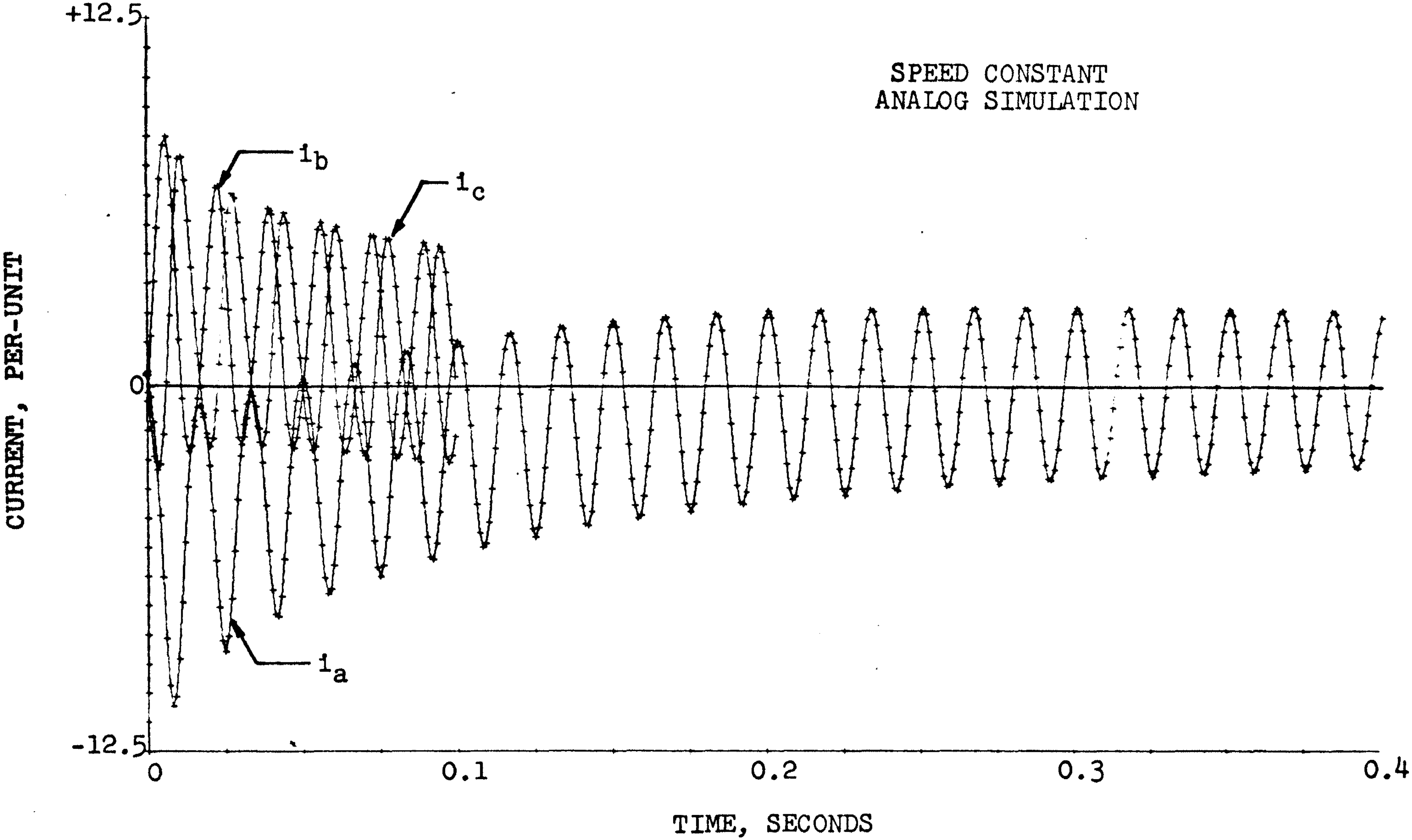

FIGURE 12. PHASE CURRENTS, MACHINE ONE

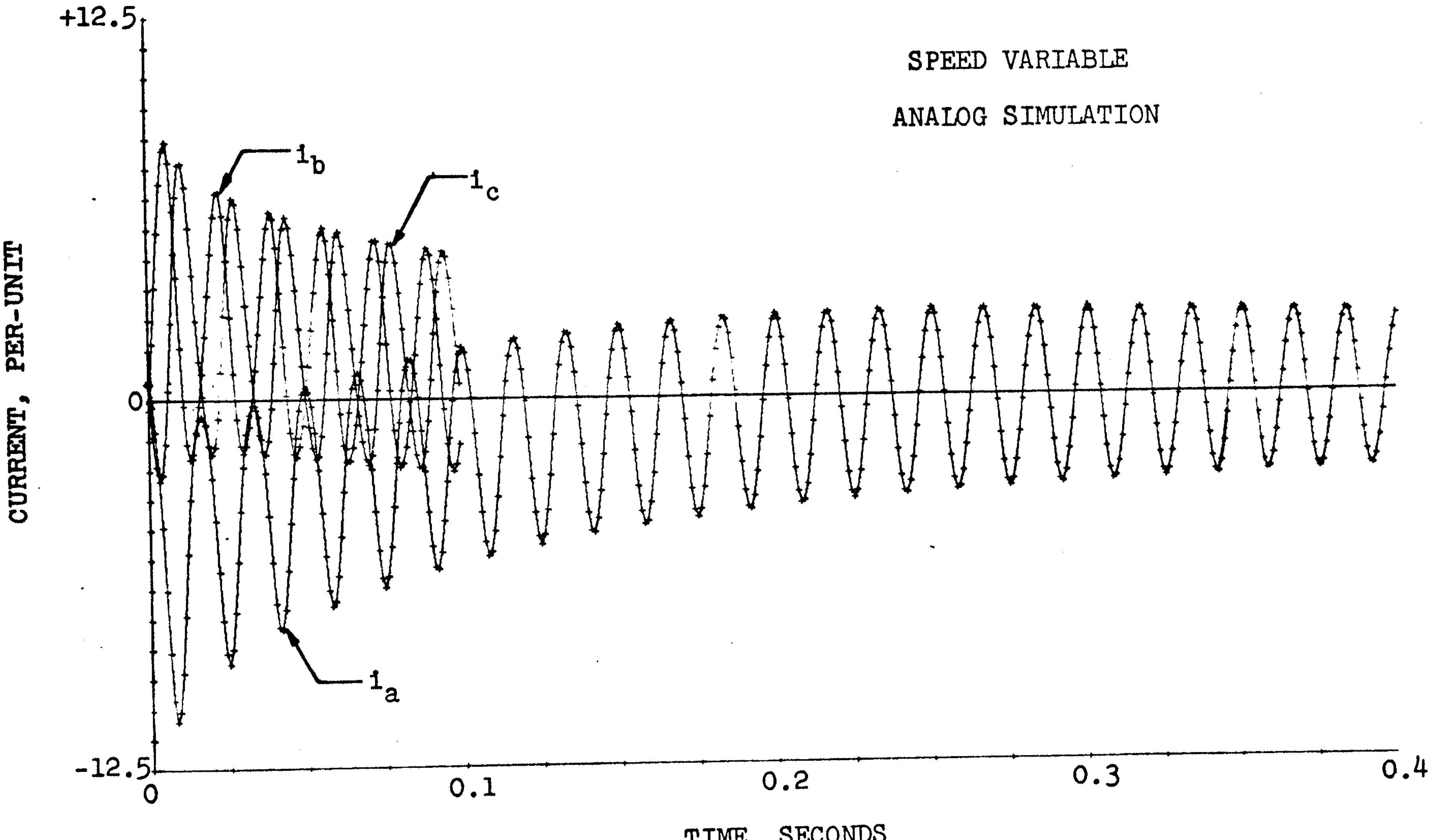

FIGURE 13. PHASE CURRENTS, MACHINE ONE

Figures 14 and 15 are of field current and speed for the simulation of machine two. Figures 16 and 17 are of the phase currents for machine two. In these figures, the results of two different simulations are given. One result is for the constant speed case, and the other for variable speed. There is very little difference in the results of the two cases, for the speed does not change appreciably during the transient conditions. However, it is noticed that the transient current damps out just slightly faster for the variable speed case. Also, after about forty-five cycles, the speed variation begins to show up in the frequency of the sustained current.

The conditions above for machine two were also found to exist for the simulation of machine one. A comparison between the two machines can be drawn. The speed of machine two drops more rapidly than machine one, as would be expected because of the difference in the moments of inertia.

Figures 18 and 19 show the D, Q values of current for machine one, and Figures 20 and 21 show the D, Q values for machine two. Figures 18 and 20 are for constant speed, and 19 and 21 are for variable speed conditions. The only comparison that can be made of these values is to compare them with the shape of Figure 8. D and Q values of current are ficticious quantities, and it has been shown that the D, Q values can be obtained by an analog simulation and trans-

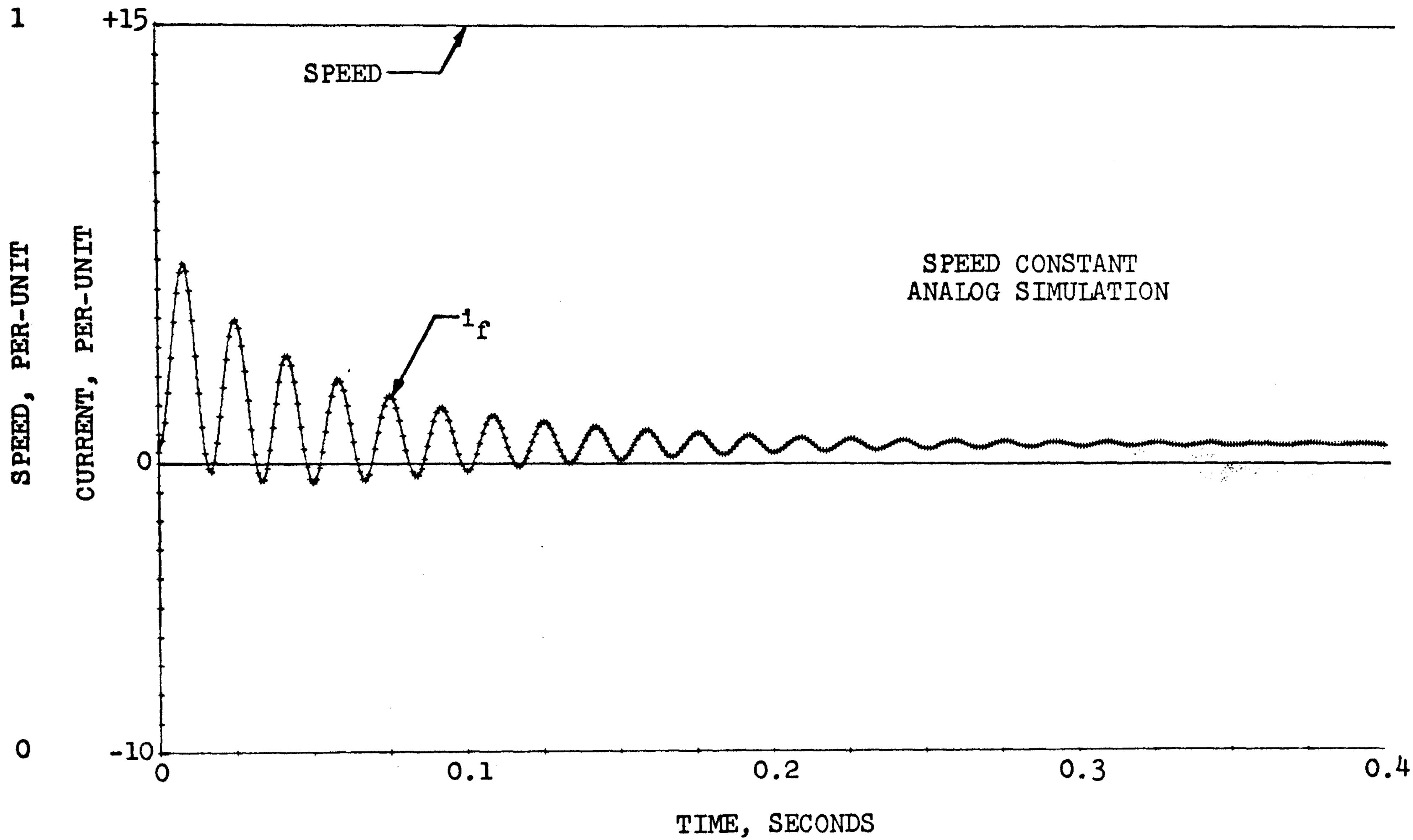

FIGURE 14. FIELD CURRENT AND SPEED, MACHINE TWO

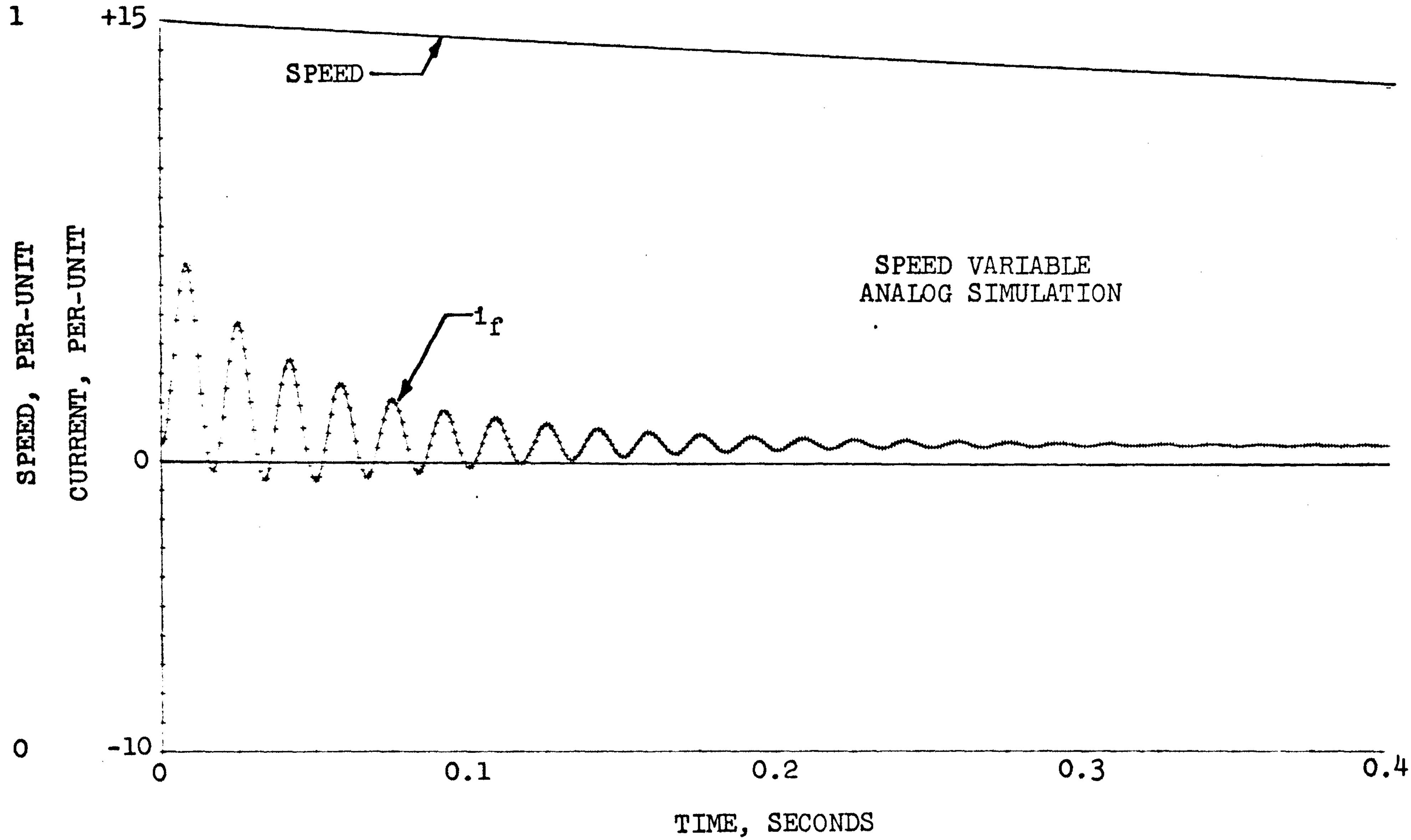

FIGURE 15. FIELD CURRENT AND SPEED, MACHINE TWO

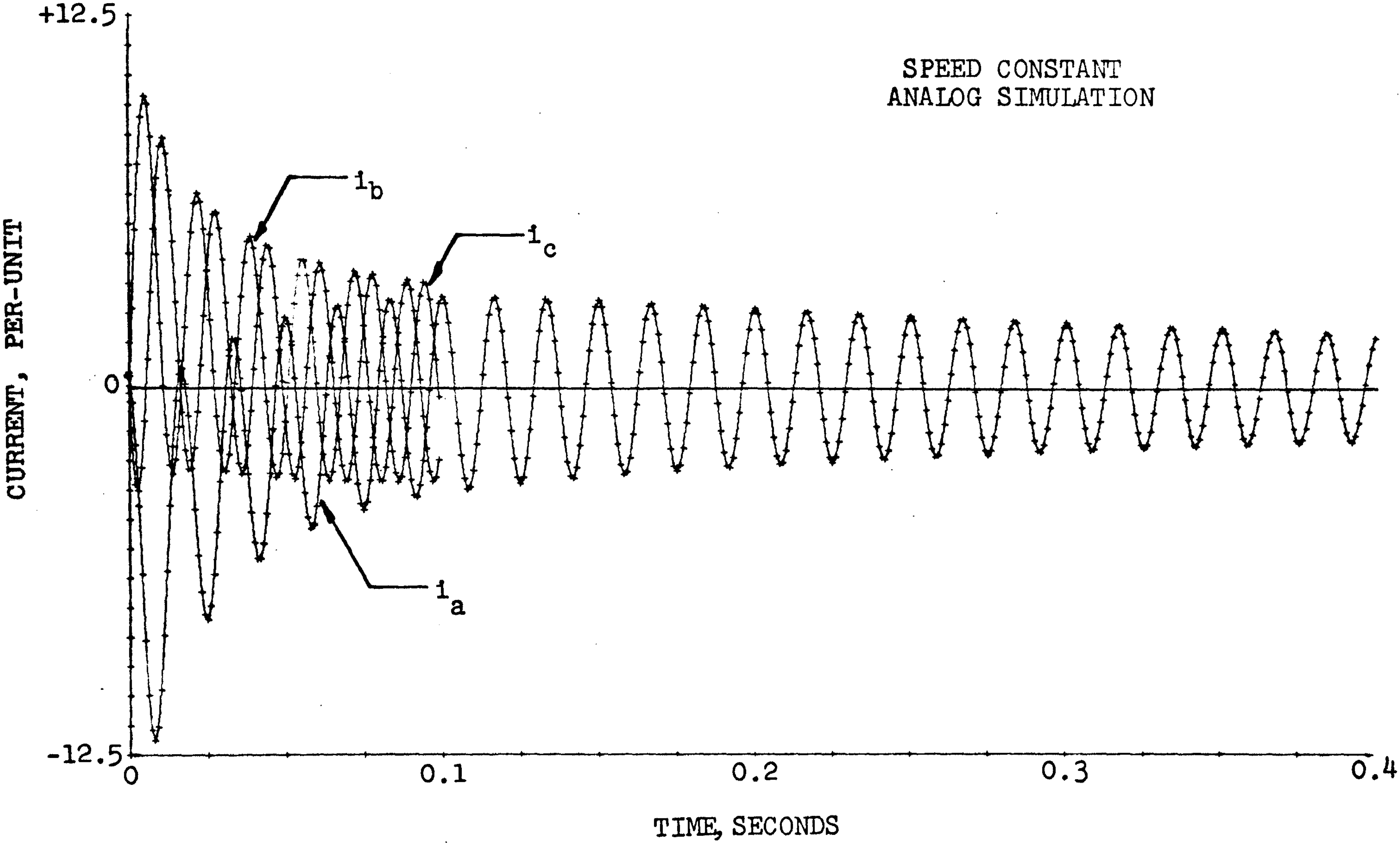

FIGURE 16. PHASE CURRENTS, MACHINE TWO

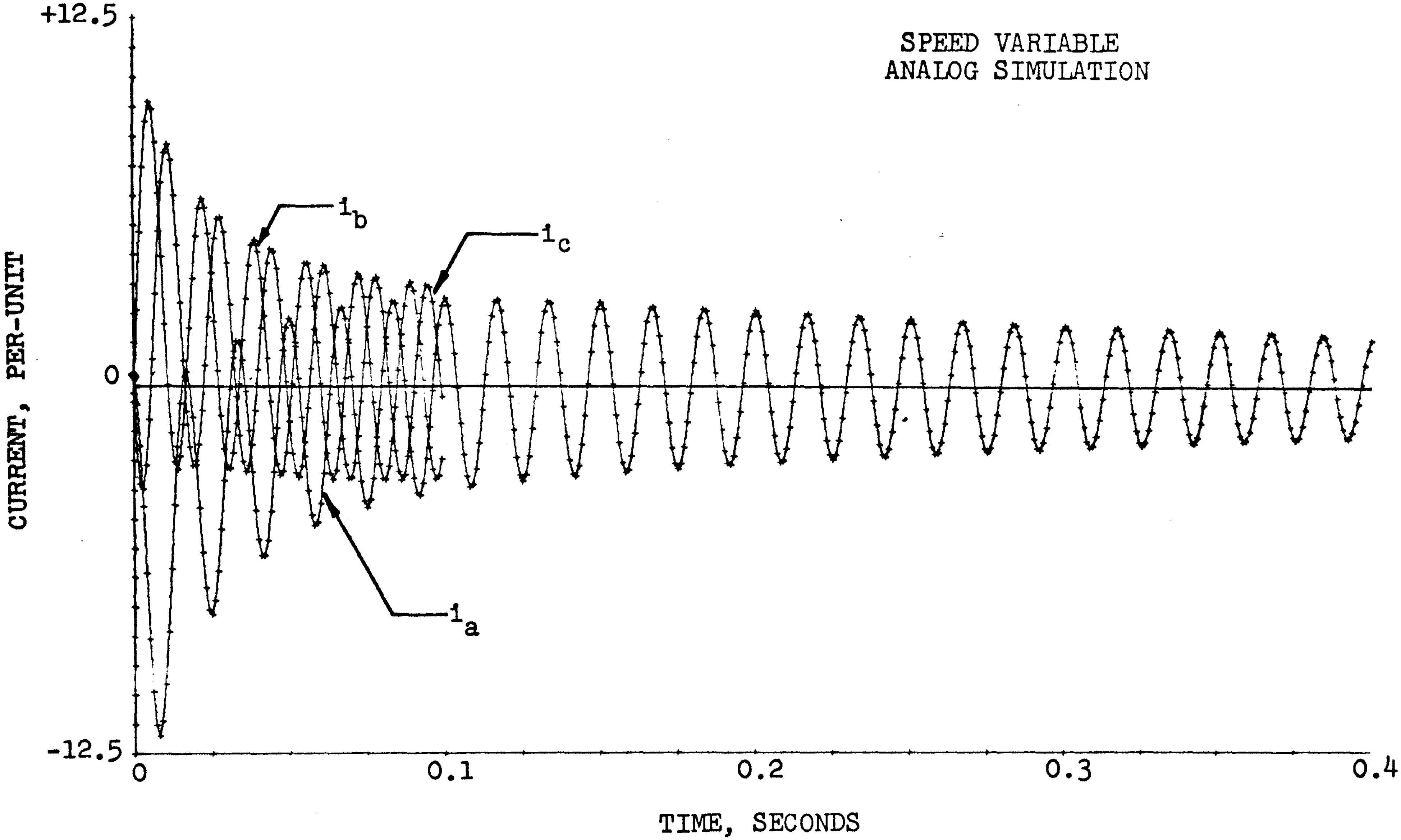

FIGURE 17. PHASE CURRENTS, MACHINE TWO

formed into phase quantities by the set of transformations in Appendix A.

In comparing the simulation data with the test data for machine two, it was found that the plot of the transient currents gave a transient current of 3.96 per-unit for the test data, and 3.91 per-unit for the simulation. A plot of sub-transient currents gave a sub-transient current of 22 per-unit for the test data, and 21 per-unit for the simulation. The sustained values of current are 0.806 per-unit for the test data, and 0.79 for the analog simulation. The time for the current to reduce to its sustained value was found to be 46 cycles for the test situation, and approximately 44 cycles for the analog simulation. A plot of the d.c. component was made and the time constant determined from this plot. The time constants for the test data and simulation were found to be in close agreement at about 4.6 cycles or 0.078 seconds.

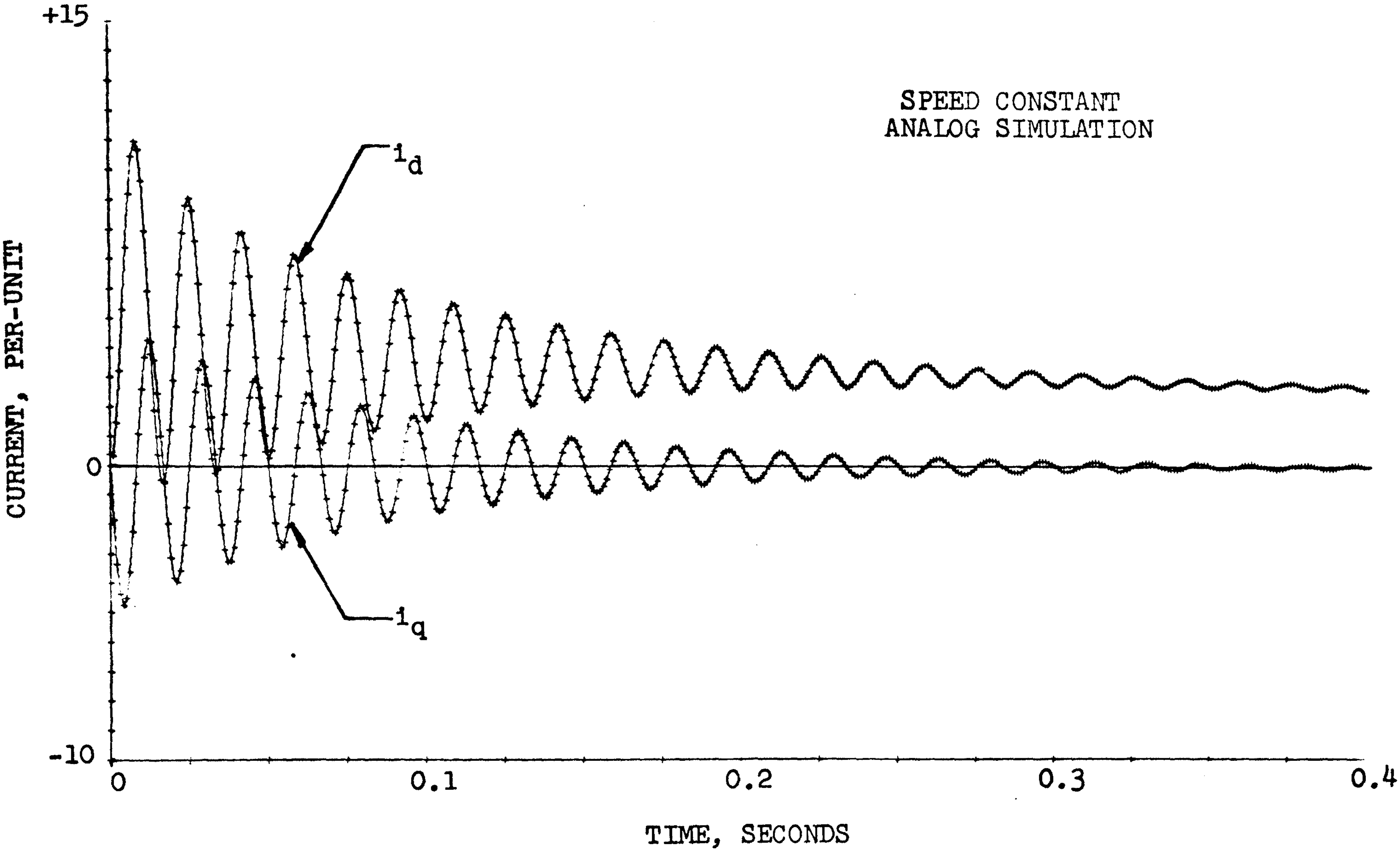

FIGURE 18. DIRECT AND QUADRATURE AXIS CURRENTS, MACHINE ONE

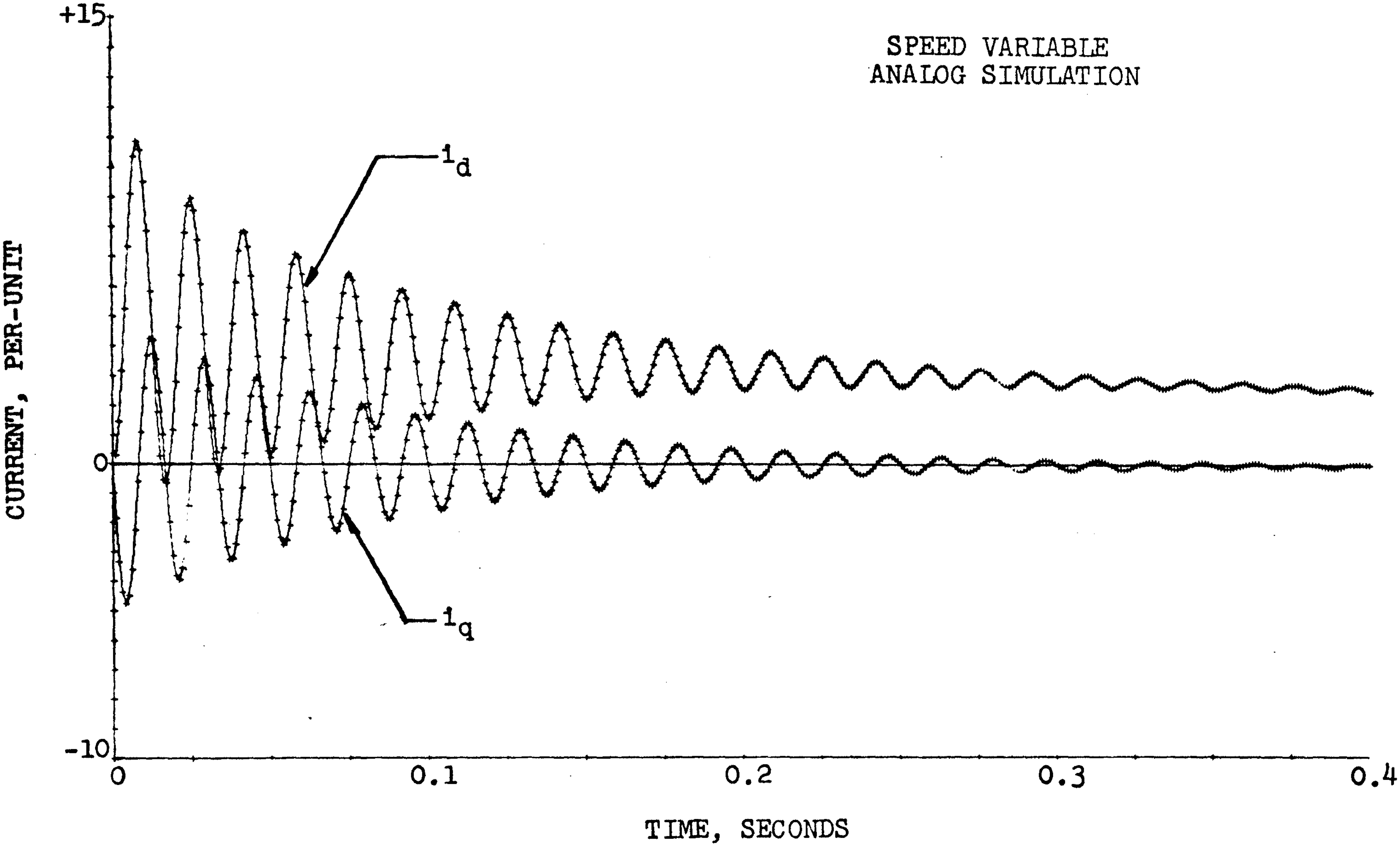

FIGURE 19. DIRECT AND QUADRATURE AXIS CURRENTS, MACHINE ONE

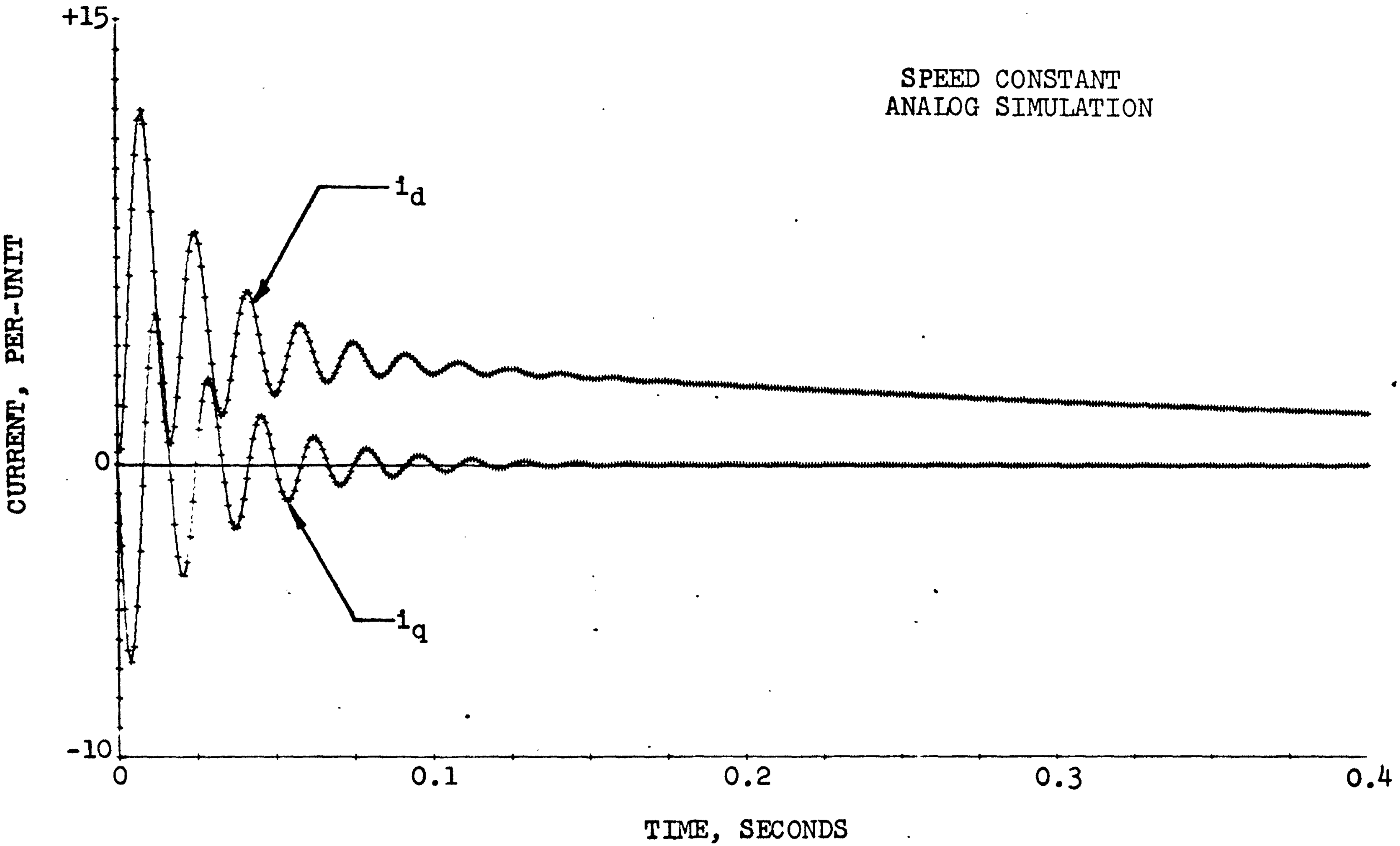

FIGURE 20. DIRECT AND QUADRATURE AXIS CURRENTS, MACHINE TWO

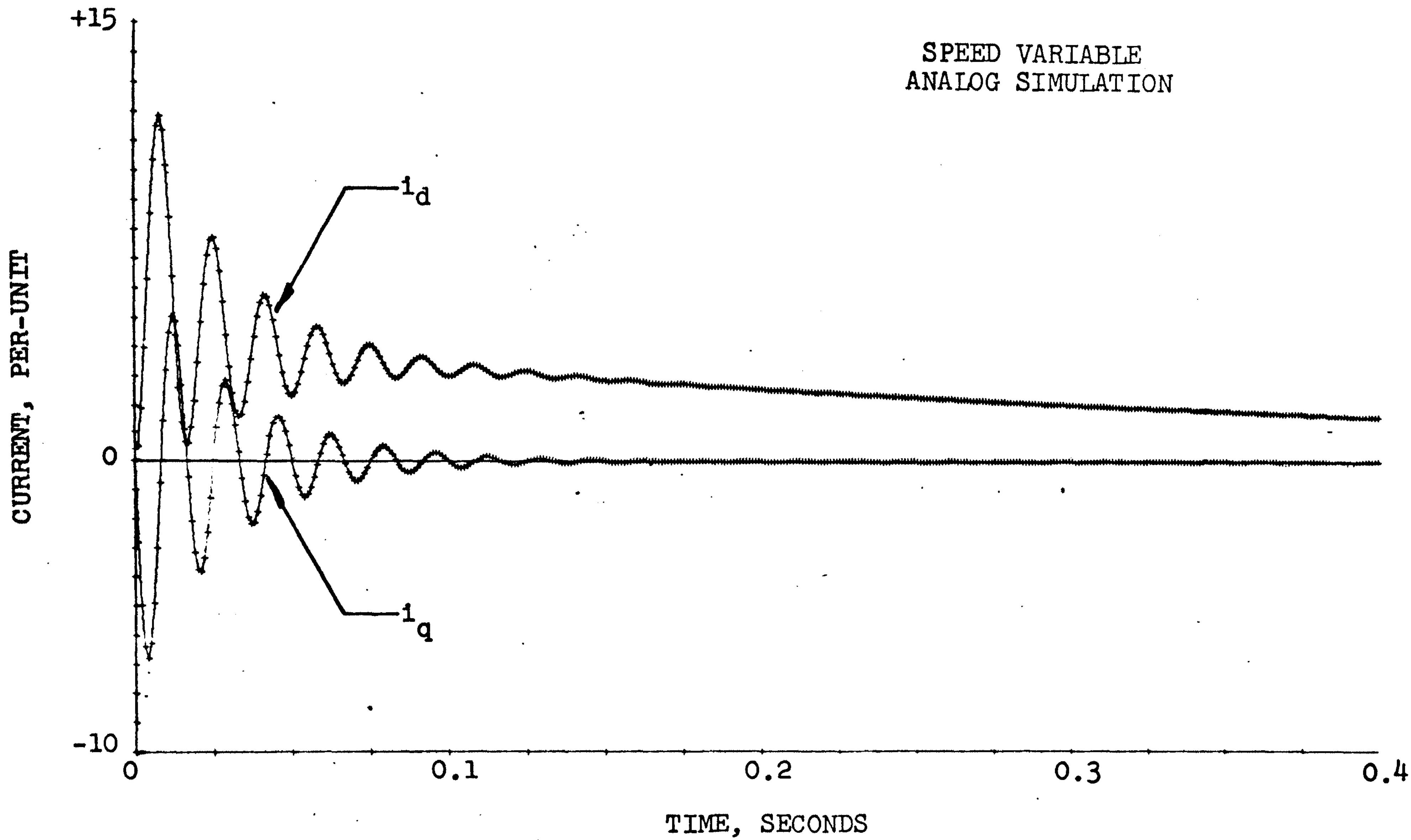

FIGURE 21. DIRECT AND QUADRATURE AXIS CURRENTS, MACHINE TWO

CHAPTER VI

RECOMMENDATIONS FOR FURTHER STUDY

The study presented in this thesis has been of the three-phase short-circuit conditions with the machine operating unloaded. The investigation opened the way to further studies and some of the possibilities presented here. As is true in most investigations, more problems are generated than time allows for solution.

A. THREE-PHASE SHORT-CIRCUIT WITH MACHINE LOADED

The equations for the load are presented in Appendix E. The voltage at the terminals of the machine can be set equal to the equations for the terminal voltage of the load if a lossless transmission system is assumed. The equations relating the machine voltage to the load voltage would then be solved for i_d and i_q. The machine simulation would then represent a generator supplying various amounts and types of loads. Since the load voltage equations were set equal to the machine voltage equations, the effect of a short-circuit at the terminals could be obtained by suddenly reducing the load terms to zero.

B. THREE-PHASE SHORT-CIRCUIT CONDITIONS FOR A MACHINE WITH A VOLTAGE REGULATOR

Aldred and Shankshaft studied the effects of a voltage regulator on the stability of the faulted generator (15). In their paper, there is a good presentation of the voltage regulator transfer function. It would be interesting to see what effects the inclusion of the regulator in the simulation would have on the phase currents. This type of study could embrace the loaded and unloaded generator with regulator.

C. THREE-PHASE SHORT-CIRCUIT CONDITIONS FOR A MACHINE WITH A GOVERNOR

The influence of the governor in the transient situation is another factor that would be interesting to investigate. J. L. Dineley and M. W. Kennedy, in reference (23), present the transfer function for three types of governors: (1) the velocity governor, (2) the acceleration governor, and (3) the combined velocity and acceleration governor. These two men investigated the effect of the governor on the stability problem and not on the effect it had on the phase currents. J. L. Dineley and E. T. Powner, in reference (24), present a study of power system governors and present a transfer function involving (1) the actuating mechanism, (2) the oil servo-system, (3) the throttle valve, and (4) the turbine into a combined expression. This investigation could, as in the previous situation, involve both the loaded and unloaded machine.

D. THREE-PHASE SHORT-CIRCUIT CONDITIONS FOR A MACHINE WITH A GOVERNOR AND VOLTAGE REGULATOR

After the presentation of points II and III, the next step would be to include the effects of both the governor and the voltage regulator in the simulation. It would, of course, require a great deal of analog equipment to accomplish such a study.

E. UNBALANCED SHORT-CIRCUIT CONDITIONS

There are three basic types of unbalanced short-circuits that might occur which will be considered.

1. Line to neutral short-circuit

In this case, the following conditions exist: $e_a = o$ and $i_b = i_c = o$ for an unloaded generator. If these conditions are put into the transformation equations in Appendix A, it can be shown that:

$$i_d^2 + i_q^2 = 4i_o^2 \qquad (23)$$

$$e_d^2 + e_q^2 = 4e_o^2 \qquad (24)$$

$$-i_d\cos\theta - i_q\sin\theta = o \qquad (25)$$

$$e_d\cos\theta + e_q\sin\theta = o \qquad (26)$$

The set of equations (23) through (26), plus the set of equations (9) through (15) and the known initial conditions, could be simulated to find the line to neutral short-circuit conditions.

2. Line to line short-circuit

The phase conditions existing in the line to line short-circuit case are:

$i_a = o$; $i_b + i_c = o$; $i_o = o$

$e_b = e_c$; $e_o = o$

If these conditions are substituted for phase values in the transformation equations given in Appendix A, the following equations result:

$$i_d \cos\theta + i_q \sin\theta = o \tag{27}$$

$$e_d \sin\theta - e_q \cos\theta = o \tag{28}$$

Equations (27) and (28), along with equations (9) through (15) and the known initial conditions, are sufficient to simulate the conditions that occur during a line to line short-circuit.

3. Double line to neutral short-circuit

When a double line to neutral fault occurs, the following phase conditions will exist:

$e_b = e_c = o$

$i_a = o$

Again substituting these phase conditions into the transformation equations of Appendix A, the following equations result:

$$e_d^2 + e_q^2 = 4e_o^2 \tag{29}$$

$$i_d^2 + i_q^2 = 4i_o^2 \tag{30}$$

$$-e_d\cos\theta - e_q\sin\theta = o \tag{31}$$

$$i_d\cos\theta + i_q\sin\theta = o \tag{32}$$

This set of equations, (29) through (32), in addition to equations (9) through (15) and the known initial conditions, can be simulated to yield the conditions for a double line to neutral fault.

Sets of equations using direct and quadrature axis values for the simulation of unbalanced faults have been developed. It is noticed, however, that the equations are quite cumbersome, and solution by an analog simulation would require a great deal of equipment. It would appear, then, that the development of a different transformation could be made to solve for the three short-circuit cases developed above. The d, q transformation is extremely useful in the symmetrical case, but the principle advantage in transforming to d, q quantities is lost in the unsymmetrical cases. It is suggested, then that the two-phase α, β, o components be investigated for use in simulating unsymmetrical faults.

APPENDIX A

TRANSFORMATION EQUATIONS

Current Transformations (4)

$$i_d = 2/3\left[\cos\theta i_a+\cos(\theta-2\pi/3)i_b+\cos(\theta-4\pi/3)i_c\right] \tag{1A}$$

$$i_q = 2/3\left[\sin\theta i_a+\sin(\theta-2\pi/3)i_b+\sin(\theta-4\pi/3)i_c\right] \tag{2A}$$

$$i_o = 1/3(i_a+i_b+i_c) \tag{3A}$$

$$i_a = \cos\theta\ i_d+\sin\theta\ i_q+i_o \tag{4A}$$

$$i_b = \cos(\theta-2\pi/3)i_d+\sin(\theta-2\pi/3)i_q+i_o \tag{5A}$$

$$i_c = \cos(\theta-4\pi/3)i_d+\sin(\theta-4\pi/3)i_q+i_o \tag{6A}$$

Voltage Transformations (4)

$$e_d = 2/3\left[\cos\theta e_a+\cos(\theta-2\pi/3)e_b+\cos(\theta-4\pi/3)e_c\right] \tag{7A}$$

$$e_q = 2/3\left[\sin\theta e_a+\sin(\theta-2\pi/3)e_b+\sin(\theta-4\pi/3)e_c\right] \tag{8A}$$

$$e_o = 1/3(e_a+e_b+e_c) \tag{9A}$$

$$e_a = \cos\theta\ e_d+\sin\theta\ e_q+e_o \tag{10A}$$

$$e_b = \cos(\theta-2\pi/3)e_d+\sin(\theta-2\pi/3)e_q+e_o \tag{11A}$$

$$e_c = \cos(\theta-4\pi/3)e_d+\sin(\theta-4\pi/3)e_q+e_o \tag{12A}$$

APPENDIX B

INDUCED VOLTAGES IN THE ARMATURE

The voltage induced by ψ_{md} is $-p(\psi_{md}\cos\theta)$ and that by ψ_{mq} is $-p(\psi_{mq}\sin\theta)$. Therefore, the total voltage induced by the main air-gap flux is:

$$p(\psi_{md}\cos\theta + \psi_{mq}\sin\theta) \tag{1B}$$

The armature currents also cause fluxes which are not a part of the main air-gap flux, but link with phase A. For example, i_a produces a leakage reactance drop $px_l i_a$. Currents i_b and i_c produce drops in coil A, $-px_m i_b$ and $-px_m i_c$.

The total impressed voltage e_a is equal to the voltage induced by the main flux, the leakage and mutual inductance voltages, and the armature resistance drop.

$$e_a = p(\psi_{md}\cos\theta + \psi_{mq}\sin\theta)+(r_a+sx_l)i_a-px_m i_b-px_m i_c \tag{2B}$$

Using equations (3A) and (4A) of Appendix A:

$$i_o = 1/3(i_a+i_b+i_c) \tag{3A}$$

$$i_a = \cos\theta\ i_d+\sin\theta\ i_q + i_o \tag{4A}$$

$$\begin{aligned} e_a &= p(\psi_{md}\cos\theta+\psi_{mq}\sin\theta)+(x_l+x_m)p(i_d\cos\theta+i_q\sin\theta+i_o \\ &\quad -3x_m p i_o+r_a i_a \\ &= p\ (\psi_{md}+x_a i_d)\cos\theta+(\psi_{mq}+x_a i_q)\sin\theta\ +x_o p i_o+r_a i_a \\ &= p\ (\psi_d\cos\theta+\psi_q\sin\theta)+x_o p i_o+r_a i_a \end{aligned} \tag{3B}$$

Using equations (4A) and (10A)

$$i_a = \cos\theta\ i_d + \sin\theta\ i_q + i_o$$

$$e_a = \cos\theta\ e_d + \sin\theta\ e_q + e_o$$

In (3B)

$$e_a = p(\psi_d\cos\theta + \psi_q\sin\theta) + x_o p i_o + r_a(\cos\theta i_d + \sin\theta i_q + i_o)$$

$$(e_d - r_a i_d)\cos\theta + (e_q - r_a i_q)\sin\theta + e_o - r_a i_o - x_o p i_o$$

$$= \cos\theta(p\psi_d + v\psi_q) + \sin\theta(p\psi_q - v\psi_d) \qquad (4B)$$

This equation must hold for all values of θ, and coefficients can be equated as follows:

$$(e_d - r_a i_d)\cos\theta = \cos\theta(p\psi_d + v\psi_q)$$

$$e_d = p\psi_d + v\psi_q + r_a i_d \qquad (5B)$$

$$(e_q - r_a i_q)\sin\theta = \sin\theta(p\psi_q - v\psi_d)$$

$$e_q = p\psi_q - v\psi_d + r_a i_q \qquad (6B)$$

$$e_o = (r_a + x_o p)i_o \qquad (7B)$$

If equations were written for e_b or e_c, the same transformations made, and coefficients equated, the same results would be obtained as given in equations (5B), (6B), and (7B).

APPENDIX C

EQUATIONS FOR SYNCHRONOUS MACHINES WITH ONE DAMPER WINDING

The equations derived in Appendix B and equations (1), (2), and (3) hold, whether the quantities are on a per-unit basis or not. For this simpler machine, equations (1), (2), and (3) reduce to the following set of equations:

$$\begin{aligned} e_f &= (r_f+x_f p)i_f+x_{fkd}pi_{kd}+x_f pi_d \\ o &= x_{fkd}pi_f+(r_{kd}+x_{kd}p)i_{kd}+x_{kd}pi_d \end{aligned} \qquad (1C)$$

$$\begin{aligned} \psi_d &= x_f i_f+x_{kd}i_{kd}+x_d i_d \\ \psi_q &= x_{kq}i_{kq}+x_q i_q \end{aligned} \qquad (2C)$$

$$o = (r_{kq}+x_{kq}p)i_{kq}+x_{kq}pi_q \qquad (3C)$$

These equations become easier to handle if per-unit quantities are used. Another simplification can also be made, and that is the assumption that the three per-unit mutual reactances on the direct axis are all equal. With this assumption:

$$x_{kd} = x_f = x_{fkd} = x_{md}$$

$$x_{kq} = x_{mq}$$

The complete self reactance of each coil is the sum of the mutual reactance and the leakage reactance.

$$x_d = x_{md} + x_a$$

$$x_f = x_{md} + x_f$$

$$x_{kd} = x_{md} + x_{kd}$$

$$x_q = x_{mq} + x_a$$

$$x_{kq} = x_{mq} + x_{kq}$$

Equations (1C), (2C), and (3C) become

$$\begin{aligned} e_f &= [r_f+(x_{md}+x_f)p]\,i_f+x_{md}pi_{kd}+x_{md}pi_d \\ o &= x_{md}pi_f+[r_{kd}+(x_{md}+x_{kd})p]\,i_{kd}+x_{md}pi_d \end{aligned} \tag{4C}$$

$$\begin{aligned} \psi_d &= x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d \\ \psi_q &= x_{mq}i_{kq}+(x_{mq}+x_a)i_q \end{aligned} \tag{5C}$$

$$o = [r_{kq}+(x_{mq}+x_{kq})p]\,i_{kq}+x_{mq}pi_q \tag{6C}$$

The equations for e_d and e_q from Appendix B are:

$$e_d = p\,\psi_d-v\,\psi_q+r_ai_d \tag{5B}$$

$$e_q = p\psi_q+v\psi_d+r_ai_q \tag{6B}$$

Substituting equation (5C) in these two equations:

$$\begin{aligned} e_d = {} & x_{md}pi_f+x_{md}pi_{kd}+x_{mq}vi_{kq}+[r_a+(x_{md}+x_a)p]\,i_d \\ & +(x_{mq}+x_a)vi_q \end{aligned} \tag{7C}$$

$$\begin{aligned} e_q = {} & -x_{md}vi_f-x_{md}vi_{kd}+x_{mq}pi_{kq}-(x_{md}+x_a)vi_d \\ & +[r_a+(x_{mq}+x_a)p]\,i_q \end{aligned} \tag{8C}$$

Taking equations (4C), (6C), (7C), and (8C) and arranging them in matrix form:

$$
\begin{bmatrix} e_f \\ o \\ o \\ e_d \\ e_q \end{bmatrix}
=
\begin{bmatrix}
r_f + (x_{md}+x_f)p & x_{md}p & o & x_{md}p & o \\
x_{md}p & r_{kd} + (x_{md}+x_{kd})p & o & x_{md}p & o \\
o & o & r_{kq} + (x_{mq}+x_{kq})p & o & x_{mq}p \\
x_{md}p & x_{md}p & x_{mq}\nu & r_a + (x_{md}+x_a)p & (x_{mq}+x_a)\nu \\
-x_{md}\nu & -x_{md}\nu & x_{mq}p & -(x_{md}+x_a)\nu & r_a + (x_{mq}+x_a)p
\end{bmatrix}
\begin{bmatrix} i_f \\ i_{kd} \\ i_{kq} \\ i_d \\ i_q \end{bmatrix}
\qquad (9C)
$$

APPENDIX D

GENERAL TORQUE EQUATIONS

The equation for the actual torque applied to the machine, where positive torque and speed indicate generating action, is:

$$f_t = f_e + J\frac{dv}{dt} \tag{1D}$$

As in previous situations, it is desirable to express equation (1D) in the per-unit system. In the per-unit system, a unit of torque is defined as the torque which produces unit power at synchronous or nominal speed. The unit of power in the per-unit system is defined as the power in all the main circuits for a unit of voltage and current.

The inertia constant is the moment of inertia of the machine for the per-unit system. H is the symbol used, and the inertia constant is defined as:

$$H = \frac{\text{stored energy at synchronous speed in KW sec.}}{\text{rated KVA}} \tag{2D}$$

Since in the per-unit system a unit of energy is a unit of power acting for one second, H is numerically equal to the per-unit of energy. The stored energy at synchronous speed is equal to $\frac{1}{2}J_o\ ^2$. The rated KVA can be shown to equal $f_o\omega$. If f_o is one per-unit torque, and ω is per-unit speed, then

$$H = \frac{\frac{1}{2}J_o\omega^2}{f_o\omega} = \frac{1}{2}J\omega$$

where J_o is the actual moment of inertia and J is given in per-unit.

$$J = \frac{2H}{\omega} \tag{3D}$$

and equation (1D) becomes in per-unit

$$f_t = f_e + \frac{2H}{\omega} p\nu \tag{4D}$$

The calculation of the electrical torque can be made in terms of the various currents and voltages in the machine. The total electrical power supplied to a machine is the sum of the powers supplied to the individual circuits.

$$P = k_p \sum ei \tag{5D}$$

The factor k_p depends on the number of main circuits in the machine. In the per-unit system, it is desired that the total power be unity when the voltage and currents in the main windings are unit values. For example, in a three-phase machine, unit power is supplied when unit RMS voltages and currents exist at unity power factor. Therefore, for a three-phase machine, k_p must be equal to 1/3. In general, then, k_p is the reciprocal of the number of main circuits in the machine.

Using equation (5D) for a three-phase machine, the power input is given by:

$$P = 1/3\ (e_a\, i_a + e_b\, i_b + e_c\, i_c)$$

Substituting equations from Appendix A for the phase voltages and currents into the above expression, the following is obtained:

$$P = \tfrac{1}{2}(e_d\, i_d + e_q\, i_q) + e_o\, i_o \tag{6D}$$

In the case of balanced polyphase conditions, it is recognized that e_o and i_o are zero. Substituting the rotation voltage terms of equations (5B), (6B), and (7B) into equation (6D), the following equation results:

$$P = \frac{v}{2}(\psi_q i_d - \psi_d i_q) + r_a i_o^2 + x_o i_o p i_o \qquad (7D)$$

The last two terms of equation (7D) are either absorbed as ohmic loss or stored magnetic energy and, therfore, contribute nothing to the torque.

The torque developed by the interaction between the flux and the currents is called electrical torque. Using this definition and the definition of unit torque:

$$P_e = \frac{-v}{\omega} f_e$$

The negative sign appears because P_e is derived for the electrical power passing into the terminals, and f_e is defined as a torque applied to the shaft. The following expression is then obtained for the electrical torque of the machine shown in Figure 5:

$$f_e = \omega k_p(\psi_q i_d - \psi_d i_q) \qquad (8D)$$

where $k_p = \frac{1}{2}$ for the machine used.

From the above equation, it can be seen that no torque results by interaction of flux and current on the same axis. The substitution of (8D) into (4D) yields an expression for the total torque in terms of electrical quantities.

$$f(t) = \frac{\omega}{2}(\psi_d i_q - \psi_q i_d) + \frac{2H}{\omega} pv \qquad (9D)$$

Substitution of equation (5C) for ψ_d and ψ_q in (9D) yields an equation for the total torque in terms of previously derived quantities.

$$f(t) = \frac{\omega}{2}\left\{\left[x_{md}i_f+x_{md}i_{kd}+(x_{md}+x_a)i_d\right]i_q - \left[x_{mq}i_{kq}+(x_{mq}+x_q)i_q\right]i_d\right\} + \frac{2H}{\omega}pv \qquad (10D)$$

In the general problem, the speed of the machine is not constant, and with varying speed, the machine position is

$$\theta = \omega t - \delta \qquad (11D)$$

where δ for a generator is negative. The speed is found by differentiating (11D).

$$v = p\theta = \omega - p\delta$$

The acceleration is

$$pv = p^2\delta$$

The torque equation becomes

$$f_t = f_e + \frac{2H}{\omega}p^2\delta$$
$$f_t = \frac{\omega}{2}\left[\psi_d i_q - \psi_q i_d\right] + \frac{2H}{\omega}p^2\delta \qquad (12D)$$

APPENDIX E

LOAD EQUATIONS

The per-unit equations for a wye-connected, grounded neutral, balanced load are as follows:

$$
\begin{aligned}
v_a &= R_L\, i_a + X_L\, pi_a \\
v_b &= R_L\, i_b + X_L\, pi_b \qquad \text{(1E)} \\
v_c &= R_L\, i_c + X_L\, pi_c
\end{aligned}
$$

Transforming the above equations into direct and quadrature components by means of the transformation equations in Appendix A, the following set of equations is obtained:

$$
\begin{aligned}
&\cos\theta\, v_d + \sin\theta\, v_q + v_o \\
&\quad = R_L(\cos\theta\, i_d + \sin\theta\, i_q + i_o) + X_L p(\cos\theta\, i_d + \sin\theta\, i_q + i_o) \\
&\quad = R_L i_d \cos\theta + R_L i_q \sin\theta + R_L i_o + X_L \cos\theta\, pi_d \qquad \text{(2E)} \\
&\qquad - X_L i_d \sin\theta\, p\theta + X_L \sin\theta\, pi_q + X_L i_q \cos\theta\, p\theta + X_L pi_o
\end{aligned}
$$

Equating coefficients:

$$v_d\cos\theta = R_L i_d \cos\theta + X_L pi_d \cos\theta + X_L i_q \cos\theta\, \nu$$

$$v_d = R_L i_D + X_L pi_d + X_L i_q \nu \qquad \text{(3E)}$$

$$v_q \sin\theta = R_L i_q \sin\theta - X_L i_d \sin\theta\, \nu + X_L \sin\theta\, pi_q$$

$$v_q = R_L i_q + X_L pi_q - X_L i_d \nu \qquad \text{(4E)}$$

$$v_o = R_L i_o + X_L pi_o \qquad \text{(5E)}$$

APPENDIX F*

SYNCHRONOUS MACHINE DATA

Rating:	Machine #1	Machine #2
Kw	2500	750
Kva	3125	937.5
Rpm	1200	1200
Poles	6	6
Line voltage	450	450
Phases	3	3
Frequency	60	60
Power factor	0.8	0.8
Armature connected	6 circuit Delta	6 circuit Wye
Rotor moment of inertia, lb.-ft.^{2}	8050	1186
Field turns per pole	163.5	106.5
Field excitation (amps) for:		
Full load, rated voltage	108	102.5
No-load, rated voltage	45.6	43.8
Short circuit, rated current	57.2	55.2
Per unit resistances:		
Field	.000911	.00172
Armature	.00515	.0131
Direct axis damper	.0483	.0226
Quadrature axis damper	.0410	.0270
Per unit reactances:		
Direct axis magnetizing	1.4045	1.67
Quadrature axis magnetizing	.619	.888
Armature leakage	.082	.0708
Field leakage	.1872	.177
Direct axis damper leakage	.143	.091
Quadrature axis damper leakage	.156	.112

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* Data provided by Mr. J. C. White, Manager of Production Engineering, General Electric Company, Schenectady, New York.

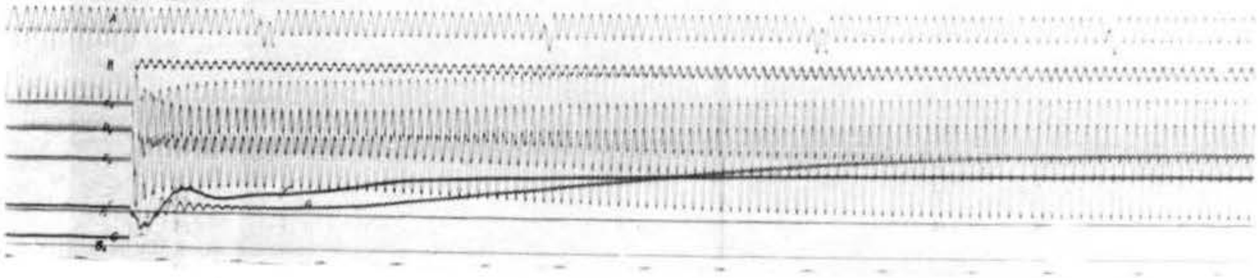

Figure F-1. Machine One, Three Phase Short-Circuit Test With Automatic Regulator, No Load

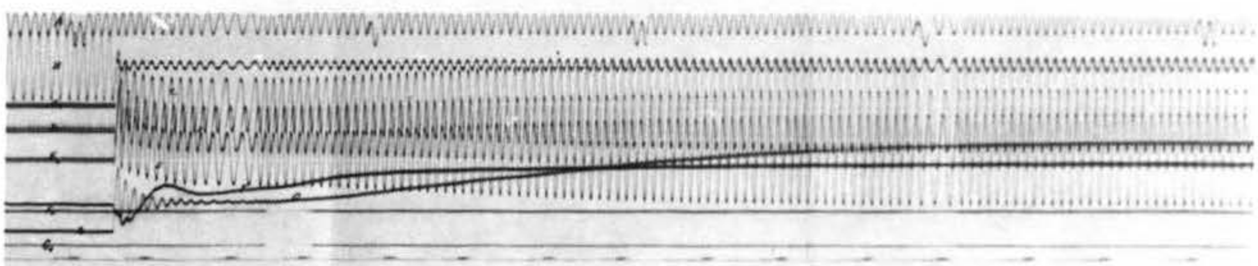

Figure F-2. Machine One, Three Phase Short-Circuit Test With Automatic Regulator, Full Load

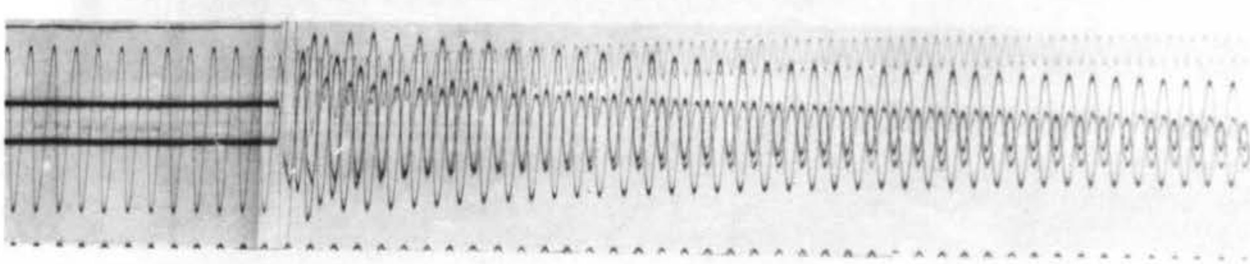

Figure F-3. Machine Two, Three Phase Short-Circuit Test, No Load, Run #1

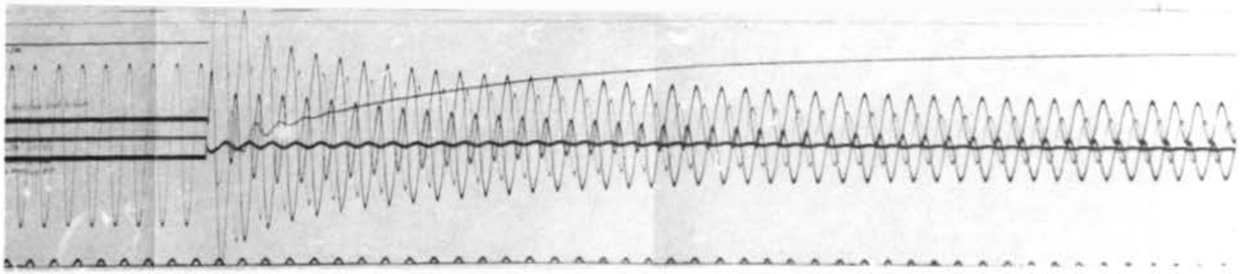

Figure F-4. Machine Two, Three Phase Short-Circuit Test, No Load, Run #2

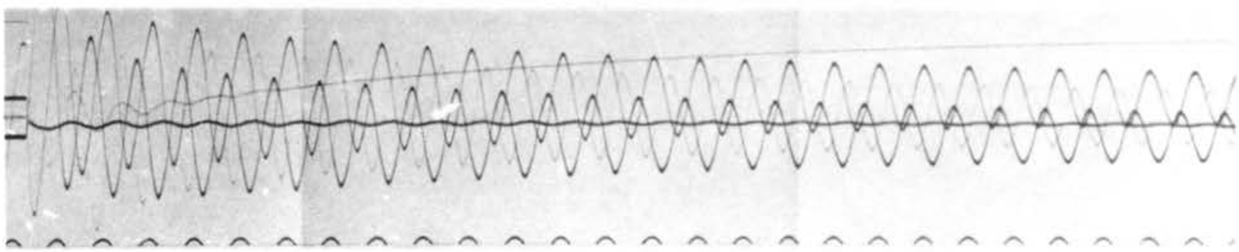

Figure F-5. Machine Two, Three Phase Short-Circuit Test, No Load, Run #3

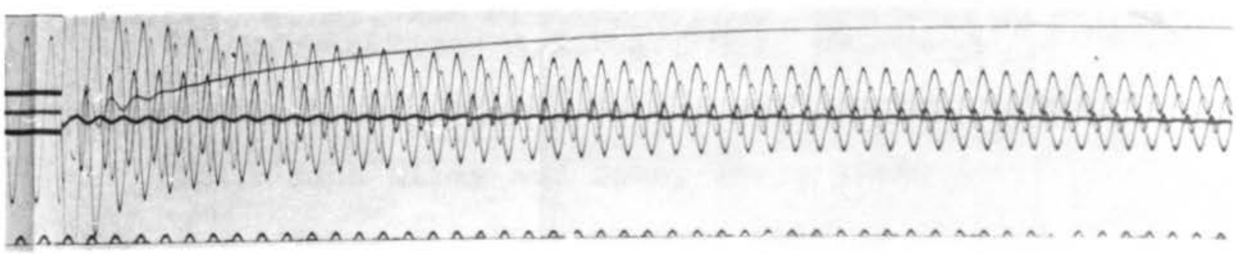

Figure F-6. Machine Two, Three Phase Short-Circuit Test, No Load, Run #4

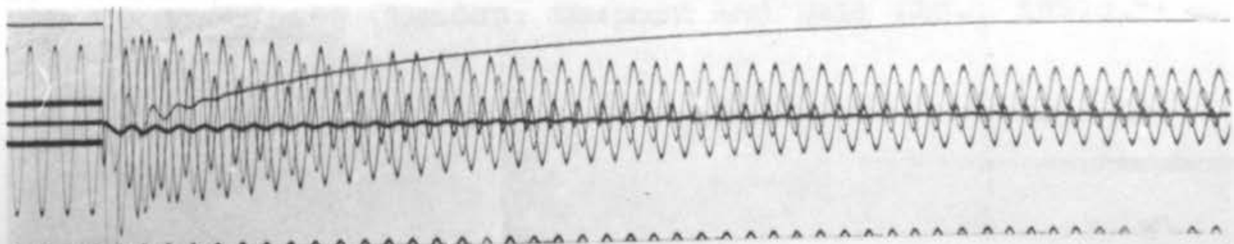

Figure F-7. Machine Two, Three Phase Short-Circuit Test, No Load, Run #5

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VITA

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