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A comparison of Liapunov functions used to determine power system stability for multimachine power systems

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A COMPARISON OF LIAPUNOV FUNCTIONS USED TO DETERMINE POWER SYSTEM STABILITY FOR MULTIMACHINE POWER SYSTEMS

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ABSTRACT

Four Liapunov functions used in stability studies are compared for accuracy on an eight machine power system. This comparison is made to determine which function is best in multimachine stability applications. Morgan's Estimating Technique is used to estimate when the Liapunov function would exceed its maximum value and thus to predict the critical switching time for the system.

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I. INTRODUCTION

The significance of power system stability was realized only after 1920. Since then it has become an important problem, particularly, in connection with transmission over long distance and with the development of large and interconnected power systems. Tests have been made and methods of analysis developed for the improvement of reliability and stability of the system. The most important of these methods of analysis are the circle diagrams, the method of symmetrical components, the point-by-point method, the equal-area criterion, the phase-plane method and the energy-integral criterion.

A set of nonlinear differential equations describes the power system behavior during transient disturbances. For complex systems calculations for a stability study become cumbersome and very costly, since the conventional point-by-point method involves assumption of different values of clearing times and computation of the corresponding swing curves. The equal-area criterion and the phase-plane method are applicable in practice to one or two machine systems only. Therefore in some recent papers, the application of the direct method of Liapunov to the problem of transient power system stability is described and is developed to permit the practical study of n-machine systems. The method consists of using a suitable Liapunov function

$V(\delta,\omega)$ to determine the region and extent of asymptotic stability and to predict when $V(\delta,\omega)$ would exceed its maximum value of $V(\delta,\omega)$ max. for stability as the system swings, thereby determining the critical switching time.

Fault, critical switching time and stability are defined in Appendix B. The symbol δ is used to represent the angular displacement in electrical radians measured from the synchronously rotating reference axis to the rotor of the machine in question. Many times it is customary to choose, in the system, the machine with largest moment of inertia as the reference and measure angle δ from the rotor of the reference machine to the rotor of the machine in question. In this study of eight machine power system, synchronously rotating axis is chosen as reference except for the plotting of swing curves, where machine number eight is chosen as reference machine (Figures 6 and 7). The symbol ω is used to denote the time derivative of δ. $V(\delta,\omega)$ is a function of δ_i and ω_i, where $i = 1,2,\ldots, 8$.

In this paper, four different Liapunov functions have been used for the purpose of studying stability and predicting the critical switching time of an eight machine power system. A comparative study of these is then made for accuracy and to determine which function is best in the multimachine stability application.

II. REVIEW OF LITERATURE

The direct method of Liapunov, which has recently received a great deal of attention in control system literature, has been used by various authors in studying the power system transient stability problem concerning the single-machine and multimachine power system. The difficulty in the application of this method is that no standard procedure has yet been found to determine whether a suitable Liapunov function $V(\delta,\omega)$ exists for the given system or if the chosen $V(\delta,\omega)$ is the best one available.

If, for a given system, there exists a real scalar function $V(\bar{x})$, a continuous function with continuous first partial derivatives, such that

1. $V(\bar{x}) = 0$ for $\bar{x} = 0$
2. $V(\bar{x}) > 0$ for $\bar{x} \neq 0$
3. $dV(\bar{x})/dt < 0$ for $\bar{x} \neq 0$

then the given system is asymptotically stable in the neighborhood of the origin (stable equilibrium point).

In the absence of an unique method, trials based on the physical nature of the system are helpful for finding the $V(\bar{x})$ function. Thus by trial and error and with the help of physical considerations Gless[1], as well as, El-Abiad and Nagappan[2], have used an energy integral as a Liapunov function for a multimachine system. Yu and Vongsuriya[10] have applied Zubov's method to construct a

Liapunov function for a single-machine system model with nonlinear damping. Pai, AnandMohan and J. Rao[8] have applied methods developed by V.M. Popov and R.E. Kalman to a single-machine system model, with a velocity governer represented by linear dynamics, for the construction of a Liapunov function.

The first Liapunov function, under consideration in this paper, developed by El-Abiad and Nagappan[2] and the second and third Liapunov functions developed by M. Ribbens-Pavella[7] were based on energy considerations. J.L. Willems and J.C. Willems have applied the Popov Criterion for nonlinear feedback systems to the dynamic equations of power systems[9]. The Liapunov function developed by them is the fourth Liapunov function under consideration. A detailed description of the development of the four Liapunov functions used is given in Appendix A .

III. EQUILIBRIUM STATES AND STEADY-STATE STABILITY

A. Equations of Motion

In most transient stability investigations the following assumptions are usually made.

1. The input power to all the machines in the system remain constant during the entire transient period.

2. Angular momentum of the synchronous machine is constant.

3. Each machine may be represented in the network by a constant voltage behind its transient reactance.

4. The mechanical angle of each machine rotor coincides with the electrical phase of the voltage behind transient reactance.

Considering the assumptions made above, the set of differential equations that describe the power system is as follows:

$$M_j \frac{d^2\delta_j}{dT^2} + D_j \frac{d\delta_j}{dT} + F_j(\delta_1, \delta_2, \ldots, \delta_n) = P_{mj} \quad \text{(III-I)}$$

$$j = 1, 2, \ldots, n$$

M_j = inertia constant per unit power rad^2/rad

D_j = damping constant per unit power rad/rad

P_{mj} = mechanical power input minus losses at the jth machine

T = $2\pi ft$ time in radians

f = frequency in cycles per second

t = time in seconds

$F_j(\delta_1, \delta_2, \ldots, \delta_n)$ = electrical power output

$$= \sum_{k=1}^{n} E_j E_k Y_{jk} \cos[\theta_{jk} - (\delta_j - \delta_k)] \qquad \text{(III-2)}$$

$E_j \underline{/\delta_j}$ = internal voltage of the jth machine expressed in polar form

$Y_{jk} \underline{/\theta_{jk}}$ = transfer admittance between jth and kth machines expressed in polar form

$Y_{kk} \underline{/\theta_{kk}}$ = driving-point admittance of kth machine expressed in polar form

The form of the set of differential equations as the system goes through three stages; the prefault system, the system during fault and the postfault system, is the same as equation (III-1), except that the parameters of the system are different for different stages.

It is of primary importance to find the system conditions after clearing the fault and to determine the steady-state stability of the equilibrium states.

B. Determination of the Stable Equilibrium State

The nonlinear algebraic equation describing the equilibrium state is given by

$$F_j(\delta_1, \delta_2, \ldots, \delta_n) = P_{mj}; \quad j = 1,2,\ldots,n \qquad \text{(III-3)}$$

The solution to this equation is obtained by a gradient method in the following manner:

Form a function $\emptyset[F_j(\delta_1,\delta_2,...,\delta_n),P_{mj}]$ as

$$\emptyset[F_j(\delta_1,\delta_2,...,\delta_n),P_{mj}] = \sum_{j=1}^{n} [F_j(\delta_1,\delta_2,...,\delta_n)-P_{mj}]^2 \quad \text{(III-4)}$$

The solution of equation (III-3) exists where

$$F_j(\delta_1,\delta_2,...,\delta_n)-P_{mj} = 0.$$

Now, if $[F_j(\delta_1,\delta_2,...,\delta_n)-P_{mj}]$ is squared, then $\emptyset[F_j(\delta_1,\delta_2,...,\delta_n),P_{mj}] \geq 0$ and the solution to the set of simultaneous equations exists where $\emptyset[F_j(\delta_1,\delta_2,...,\delta_n),P_{mj}]$ is a minimum or zero. The gradient method used, minimizes $\emptyset[F_j(\delta_1,\delta_2,...,\delta_n),P_{mj}]$ and obtains the solution to the set of equation of (III-3). The particular gradient method used is one which locates a local minimum of a function of several variables by the method of conjugate gradients.

As the function $F_j(\delta_1,\delta_2,...,\delta_n)$ is nonlinear, the solution depends considerably on the right selection of the starting values of δ_i; $i = 1,2,...,n$. Normally, in practice, the prefault values are chosen to be the starting values as they are very close to the values of postfault equilibrium.

C. Determination of the Unstable Equilibrium States

The unstable equilibrium state is found in the same manner as the stable equilibrium state. However, the problem is to select the initial estimate of δ_i; i=1,2, ...,n.

For steady-state conditions, if damping is neglected the zero angular acceleration condition between two machines can be determined from a knowledge of their inertias and power angles[3]. Selecting the machine with the largest inertia as the reference and applying the above criterion to each of the remaining machines in the system, we have

$$M_R \frac{d^2\delta_R}{dT^2} = P_{aR}; \quad M_j \frac{d^2\delta_j}{dT^2} = P_{aj} \text{ for all } j \neq R \qquad \text{(III-5)}$$

$$\frac{d^2(\delta_R - \delta_j)}{dT^2} = \frac{P_{aR}}{M_R} - \frac{P_{aj}}{M_j} = 0; \text{ for all } j \neq R \qquad \text{(III-6)}$$

i.e.

$$\frac{P_{aR}}{M_R} = \frac{P_{aj}}{M_j}; \text{ for all } j \neq R \qquad \text{(III-7)}$$

i.e.

$$[P_{mR} - E_R^2 G_{RR} - E_R E_j G_{Rj} \cos(\delta_R - \delta_j) - E_R E_j B_{Rj} \sin(\delta_R - \delta_j)]/M_R = [P_{mj} - E_j^2 G_{jj} - E_j E_R G_{jR} \cos(\delta_j - \delta_R) - E_j E_R B_{jR} \sin(\delta_j - \delta_R)]/M_j, \text{ for all } j \neq R \qquad \text{(III-8)}$$

As G_{jR} is relatively small for all $j \neq R$, equation (III-8) becomes

$$[P_{mR} - E_R^2 G_{RR} - E_R E_j B_{Rj} \sin(\delta_R - \delta_j)]/M_R$$

$$= [P_{mj} - E_j^2 G_{jj} - E_j E_R B_{jR} \sin(\delta_j - \delta_R)]/M_j; \quad \text{(III-9)}$$

for all $j \neq R$

$B_{Rj} = B_{jR}$, so rearranging

$$(M_j + M_R) E_j E_R B_{jR} \sin(\delta_j - \delta_R) \approx M_R P_{mj} - M_j P_{mR}$$

$$- M_R E_j^2 G_{jj} + M_j E_R^2 G_{jj}; \text{ for all } j \neq R \quad \text{(III-10)}$$

from equation (III-10)

$$(\delta_j - \delta_R) = \delta_o = \sin^{-1} \left(\frac{M_R P_{mj} - M_j P_{mR} - M_R E_j^2 G_{jj} + M_j E_R^2 G_{jj}}{(M_j + M_R) E_j E_R B_{jR}} \right) \quad \text{(III-11)}$$

or

$$(\delta_j - \delta_R) = \pi - \delta_o = \pi - \sin^{-1} \left(\frac{M_R P_{mj} - M_j P_{mR} - M_R E_j^2 G_{jj} + M_j E_R^2 G_{jj}}{(M_j + M_R) E_j E_R B_{jR}} \right) \quad \text{(III-12)}$$

The values for equations (III-11) and (III-12) are shown in figure 1 for the example of a two machine system.

For the minimization of $\emptyset[F_j(\delta_1, \delta_2, \ldots, \delta_n), P_{mj}]$; $j=1,2,\ldots,n$, the initial value of the angle for the most unstable machine is chosen as given by equation (III-12) and the initial value of the angles for the rest of the machines as given by equation (III-11).

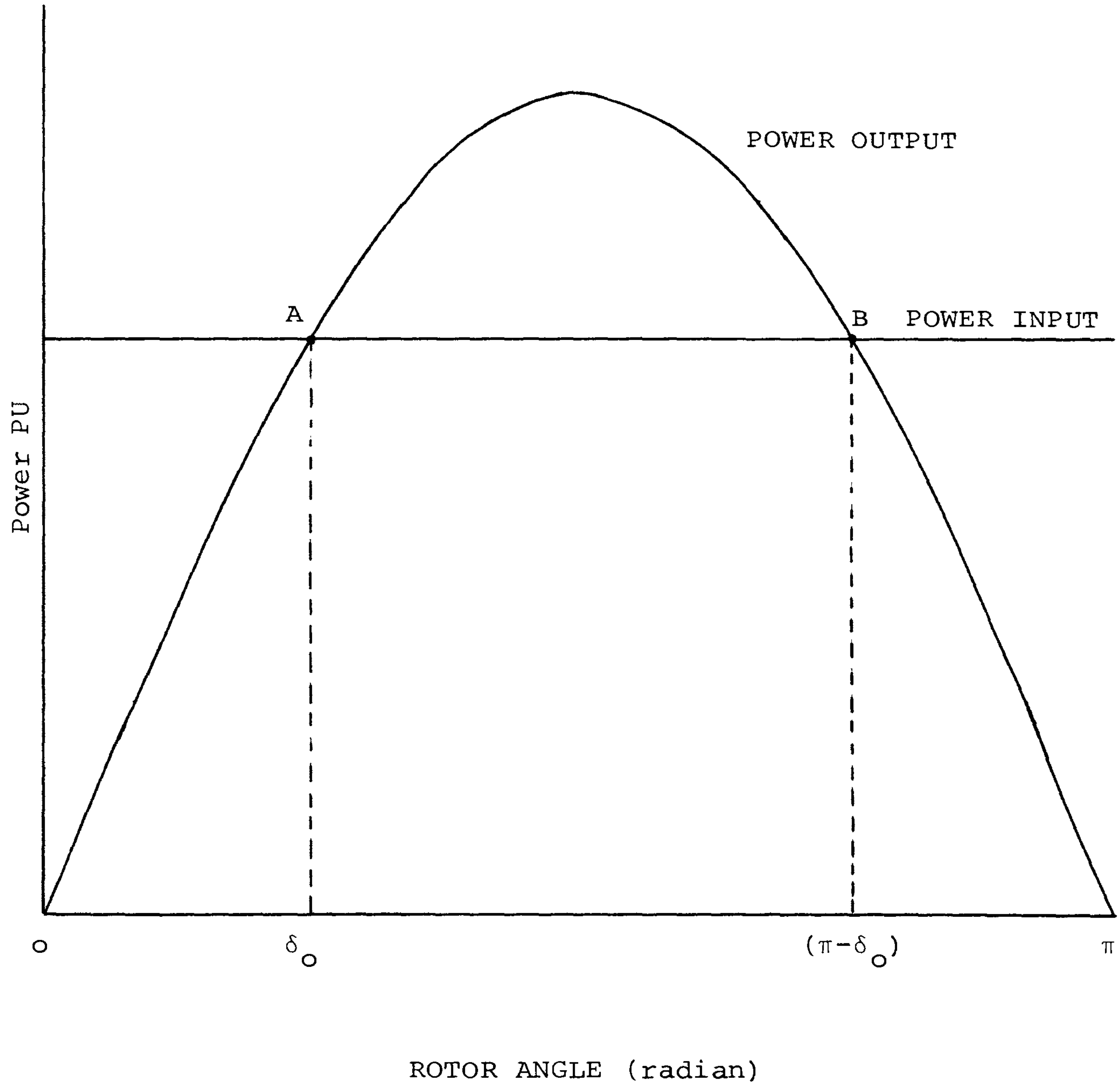

Figure 1. Power vs Rotor Angle

To determine the most unstable machine and therefore the minimum function value, each machine in turn, except the reference machine, will be selected as the one that will swing out the farthest and then the minimization of equation (III-4) will be carried out as described. The solution will converge to the nearest unstable equilibrium for the most unstable machine.

D. Determination of Steady-State Stability for Equilibrium States

The solution of equation (III-3) is tested at the calculated states to determine their steady-state stability. If $\partial[F_j(\delta_i)]/\partial\delta_i \geq 0$ for i = 1,2,...,n, the system is steady-state stable; if $\partial[F_j(\delta_i)]/\partial\delta_i \ngeq 0$ for any i, i = 1,2,...,n, the equilibrium state is unstable.

E. Description of the Method for Predicting Critical Switching Time

After determining the stable and unstable equilibrium states the procedure outlined below is used to predict the critical switching time for a multimachine system.

Step 1 Determination of the prefault initial conditions and postfault transfer and driving point admittances for the power system.

Step 2 Determination of the stable and unstable equilibrium states following the procedure outlined above.

Step 3 Determination of the maximum value of $V(\delta,\omega)$. This is done by substituting in the expression for $V(\delta,\omega)$, $\omega=0$ and values of the stable and unstable equilibrium states for δ. This will be the maximum value that $V(\delta,\omega)$ can attain for the system and still have the system remain stable.

Step 4 Determination of $V(\delta,\omega)$ as the system swings, at increments of time, by substituting the swing angle δ and the rate of change of the angles ω in the expression for $V(\delta,\omega)$.

Step 5 Prediction of critical switching time using the information obtained in previous steps and applying Morgan's estimating technique[6] as shown in figure 2, which can be described as follows:

After calculating three values of $V(\delta,\omega)$ as described in step 4, a curve fit is passed through these three points and an estimate is made of the time at which $V(\delta,\omega)$ will exceed $V(\delta,\omega)_{maximum}$ as calculated in step 3. As each additional increment of time elapses, the same procedure is followed until the estimate of critical switching time does not vary more than a certain amount, Δt, from the previously estimated switching time. When

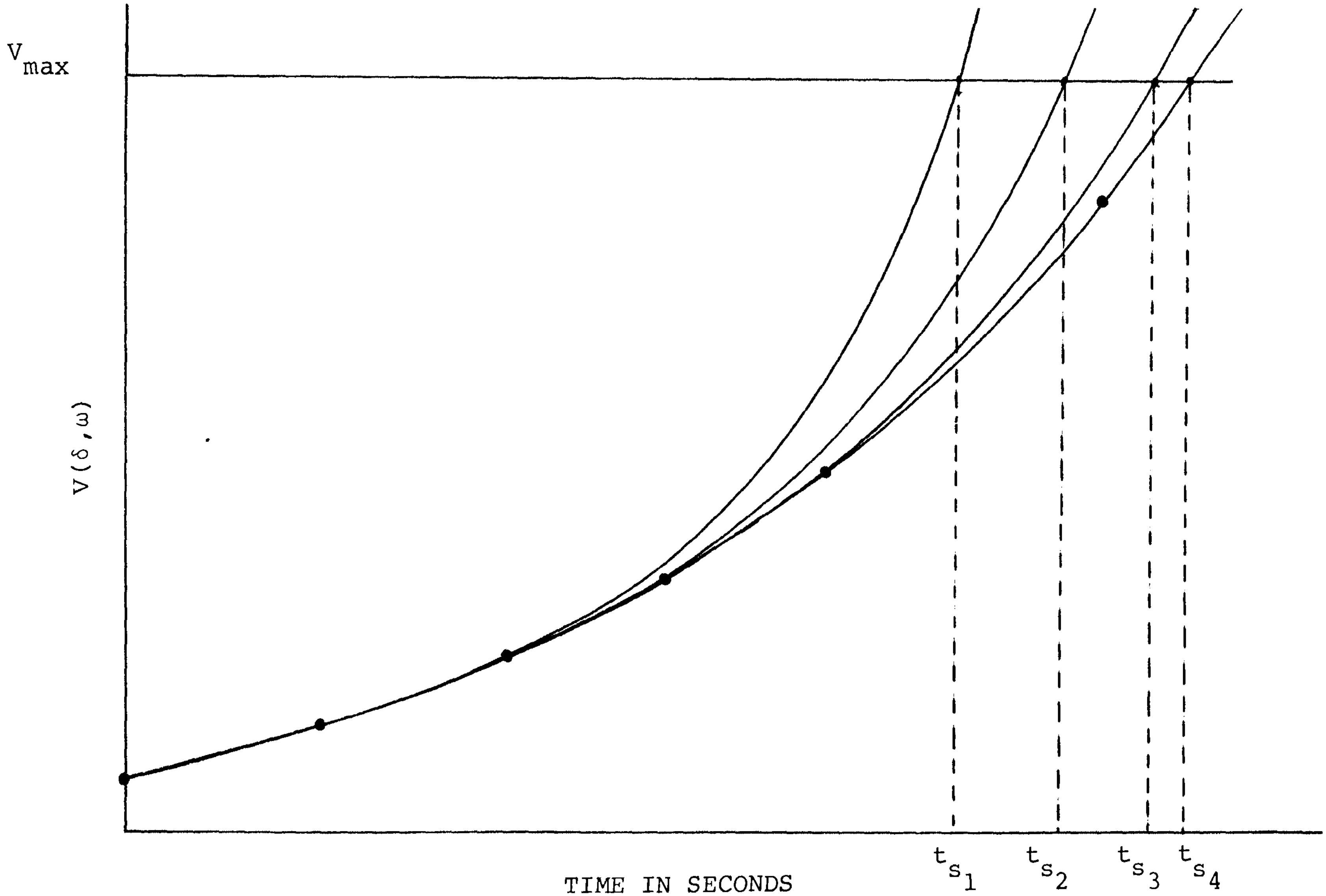

Figure 2. Morgan's Estimating Technique for Predicting Critical Switching Time

this occurs, the last time estimate is adopted as the predicted critical switching time for the system.

IV. NUMERICAL EXAMPLE

A. Description of the System

The system used in the numerical example is an eight machine system. The one-line diagram of the above system is given in figure 3. The line data and the generator data are given in tables I and II, respectively. The internal bus voltages and the load flow data obtained for prefault condition are presented in table III and IV. The driving point and transfer admittances, between the internal busses of the machines, are calculated for the postfault system and are given in table V. Tables VI and VII give the stable and unstable equilibrium states respectively.

A three phase fault is considered on the line between busses thirteen and eighteen on the line side of bus thirteen. The critical switching time is evaluated, for each of the four Liapunov functions, using the method described in section III.

B. Results Obtained

Figures 4 and 5 are plots of $V_1(\delta,\omega)$, $V_4(\delta,\omega)$ and $V_2(\delta,\omega)$, $V_3(\delta,\omega)$ respectively, showing the times at which each of them will exceed their respective maximum value $V(\delta,\omega)\max$.

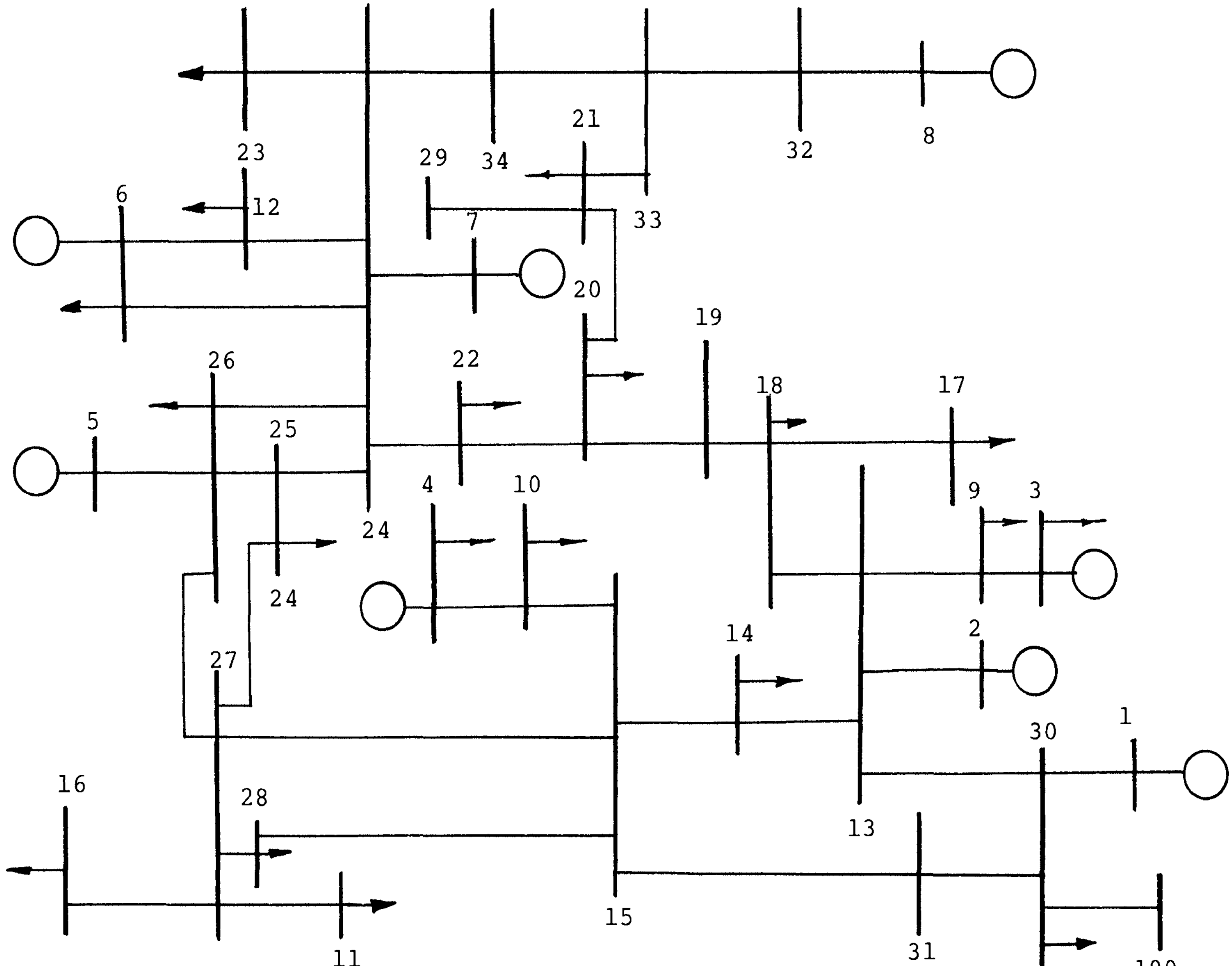

Figure 3. One Line Diagram of 8 Machine Test System

TABLE I

Line Data

From Bus	To Bus	R(P.U.)	X(P.U.)
1	30	0.0	0.0257
3	9	0.026	0.153
4	10	0.0	0.1610
6	12	0.0	0.4313
6	24	0.04	0.314
9	13	0.0	0.1831
10	15	0.0	0.100
11	27	0.0	0.063
12	24	0.0	0.0385
13	14	0.0608	0.2357
13	2	0.0	0.082
13	18	0.1606	0.6221
13	17	0.074	0.4912
14	15	0.0619	0.2401
15	27	0.0378	0.1453
15	31	0.0	0.0151
15	28	0.009	0.0359
16	27	0.1715	0.4814
17	18	0.0512	0.1997
18	19	0.0668	0.2588
19	20	0.0701	0.2716
20	21	0.1459	0.3313
20	22	0.0647	0.1469
22	24	0.0537	0.1219
23	24	0.0861	0.3474
24	25	0.0512	0.1486
24	26	0.0611	0.2371
24	7	0.0	0.0780
25	26	0.0153	0.0598

TABLE I (Continued)

From Bus	To Bus	R(P.U.)	X(P.U.)
25	27	0.0921	0.3569
26	27	0.0906	0.3529
26	5	0.0	0.035
27	28	0.0285	0.1094
29	21	0.0	0.086
30	31	0.0105	0.06365
30	13	0.0	0.0302
32	33	0.00725	0.05955
32	8	0.0	0.0445
33	34	0.0086	0.07
33	21	0.0	0.016
34	24	0.0	0.0131

TABLE II

Synchronous Machine Constants

Generator	Bus	MVA Capacity	Xd'(100MVA Base)	M(100MVA Base)
1	1	334	0.0729	6458.6
2	2	155	0.1550	6448.2
3	3	76	0.2317	2985.8
4	4	129	0.1459	4071.5
5	5	187	0.1431	5234.1
6	6	131	0.1113	5285.4
7	7	175	0.1049	6441.24
8	8	314	0.0938	10306.9

TABLE III

Internal Bus Voltages for Prefault Conditions

Generator Number	Internal Bus	E P.U.	δ Radians
1	35	0.995	0.588
2	36	1.100	0.576
3	37	0.998	0.563
4	38	1.042	0.139
5	39	1.018	0.602
6	40	1.018	0.449
7	41	1.075	0.454
8	42	1.061	0.916

TABLE IV

Bus Data and Initial Conditions

Bus	Volt Mag	Volt Angle	Load MW	Load MVAR	Gen MW	Gen MVAR	Var Lim MIN	Var Lim MAX	Shunt MVAR
1	1.0	21.84	0	0	280	-36.17	-112	144	
2	1.0	23.26	0	0	120	54.64	- 30	81	
3	1.0	23.59	3.0	1.0	65	- 5.66	- 15	40	
4	1.0	2.36	86.0	36.0	70	25.51	- 30	60	
5	1.0	21.06	0.0	0.0	165	- 6.96	- 90	90	
6	1.0	19.44	58.0	11.0	100	11.11	- 22	61	
7	1.0	20.62	0.0	0.0	96.25	67.30	- 50	98	
8	1.0	38.13	0.0	0.0	280	29.87	-138	138	
9	.999	18.04	55	17					
10	1.017	3.81	103	10					15
11	1.02	8.92	35	6					
12	.999	13.6	135	19					
13	1.008	17.38							
14	.982	11.28	42	11					
15	.999	10.74							
16	.910	1.2	30	7					
17	.895	359.88	61	7					
18	.889	.77	54	12					
19	.894	4.29							
20	.886	8.29	66	21					
21	.968	20.64	88	15					
22	.934	10.95	41	0					
23	.883	3.89	56	13					
24	.998	16.09							
25	.994	15.67	39	7					
26	1.00	17.76	58	2					
27	.989	10.21							

TABLE IV (Continued)

Bus Data and Initial Conditions

Bus	Volt Mag	Volt Angle	Load MW	Load MVAR	Gen MW	Gen MVAR	Var Lim MIN	Var Lim MAX	Shunt MVAR
28	.995	10.13	30						
29	.994	20.64							-30
30	1.012	17.76	99.96	-33.77					
31	.995	12.1							
32	1.044	30.93							
33	1.024	22.05							
34	1.024	17.05							

TABLE V

Matrix for Postfault System (Y_{eq})

G Matrix

	1	2	3	4	5	6	7	8
1	1.172	.346	.166	.394	.144	.052	.105	.085
2	.346	.115	.057	.106	.043	.019	.037	.032
3	.166	.057	.220	.047	.020	.008	.017	.014
4	.394	.106	.047	.794	.103	.026	.052	.035
5	.144	.043	.020	.103	.6279	.105	.211	.129
6	.052	.019	.008	.026	.105	.632	.306	.186
7	.105	.037	.017	.052	.211	.306	.6140	.373
8	.085	.032	.014	.035	.129	.186	.373	.690

B Matrix

	1	2	3	4	5	6	7	8
1	-5.987	2.357	.996	1.180	.665	.176	.353	.251
2	2.357	-3.821	.448	.305	.177	.052	.104	.079
3	.996	.448	-1.763	.128	.075	.022	.044	.033
4	1.180	.305	.128	-2.546	.270	.064	.129	.083
5	.665	.177	.075	.270	-3.714	.558	1.117	.692
6	.176	.052	.022	.064	.558	-3.150	1.246	.770
7	.353	.104	.044	.129	1.117	1.246	-5.225	1.542
8	.251	.079	.033	.083	.692	.770	1.542	-3.625

TABLE VI

Stable Equilibrium State of the Postfault System

Internal Bus	δ (rad)	$\partial F_i / \partial \delta_i$	
35	0.595	Positive	Stable
36	0.656	Positive	Stable
37	0.635	Positive	Stable
38	0.193	Positive	Stable
39	0.583	Positive	Stable
40	0.433	Positive	Stable
41	0.416	Positive	Stable
42	0.775	Positive	Stable

TABLE VII

Unstable Equilibrium State Closest to the Stable Equilibrium STate of Table VI

Internal Bus	δ (rad)	$\partial F_i / \partial \delta_i$	
35	2.164	Negative	Unstable
36	-1.013	Negative	Unstable
37	-0.871	Negative	Unstable
38	-0.832	Negative	Unstable
39	0.633	Positive	Stable
40	0.362	Positive	Stable
41	0.379	Positive	Stable
42	0.730	Positive	Stable

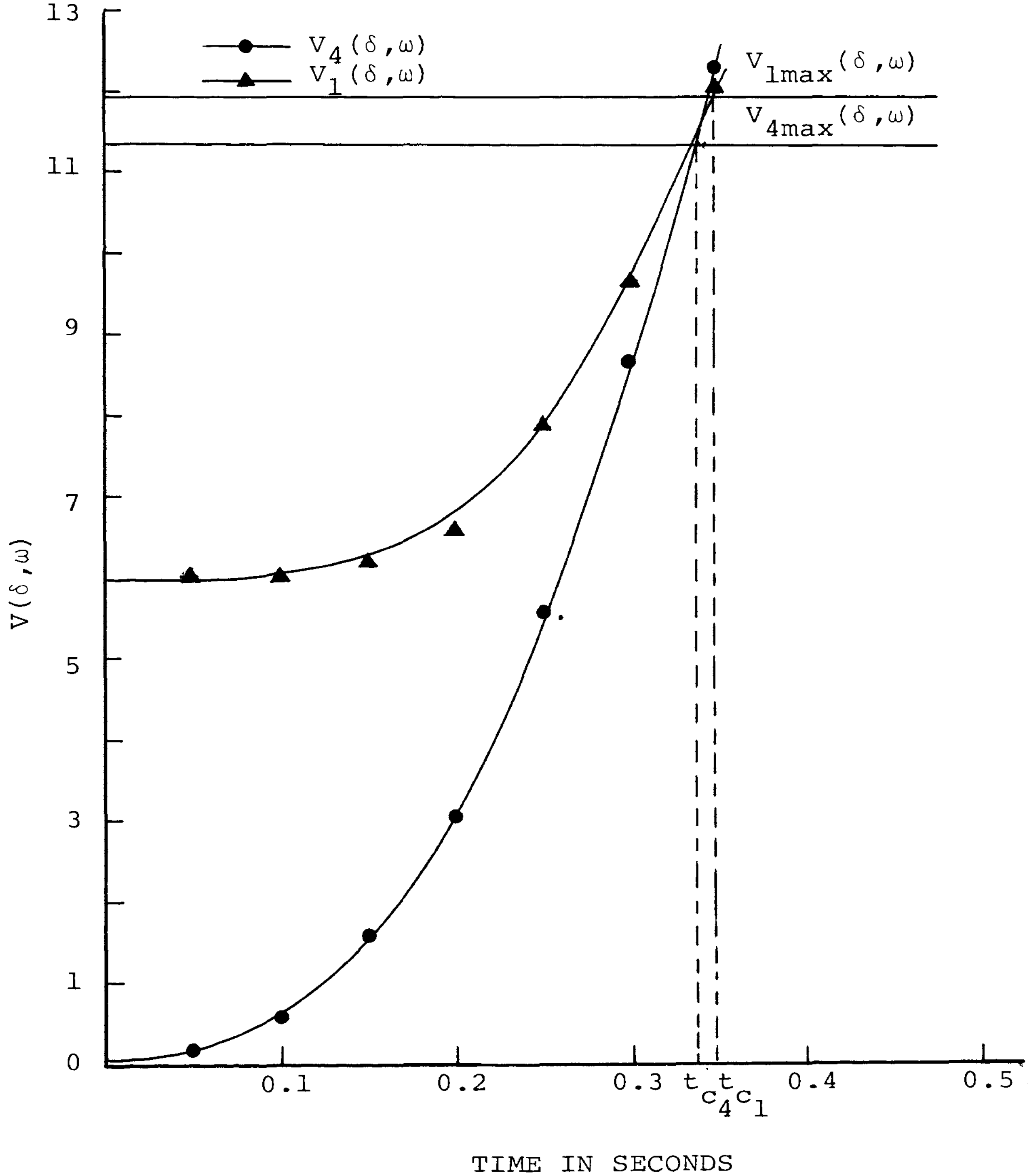

Figure 4. $V_1(\delta,\omega)$ and $V_4(\delta,\omega)$ Vs Time

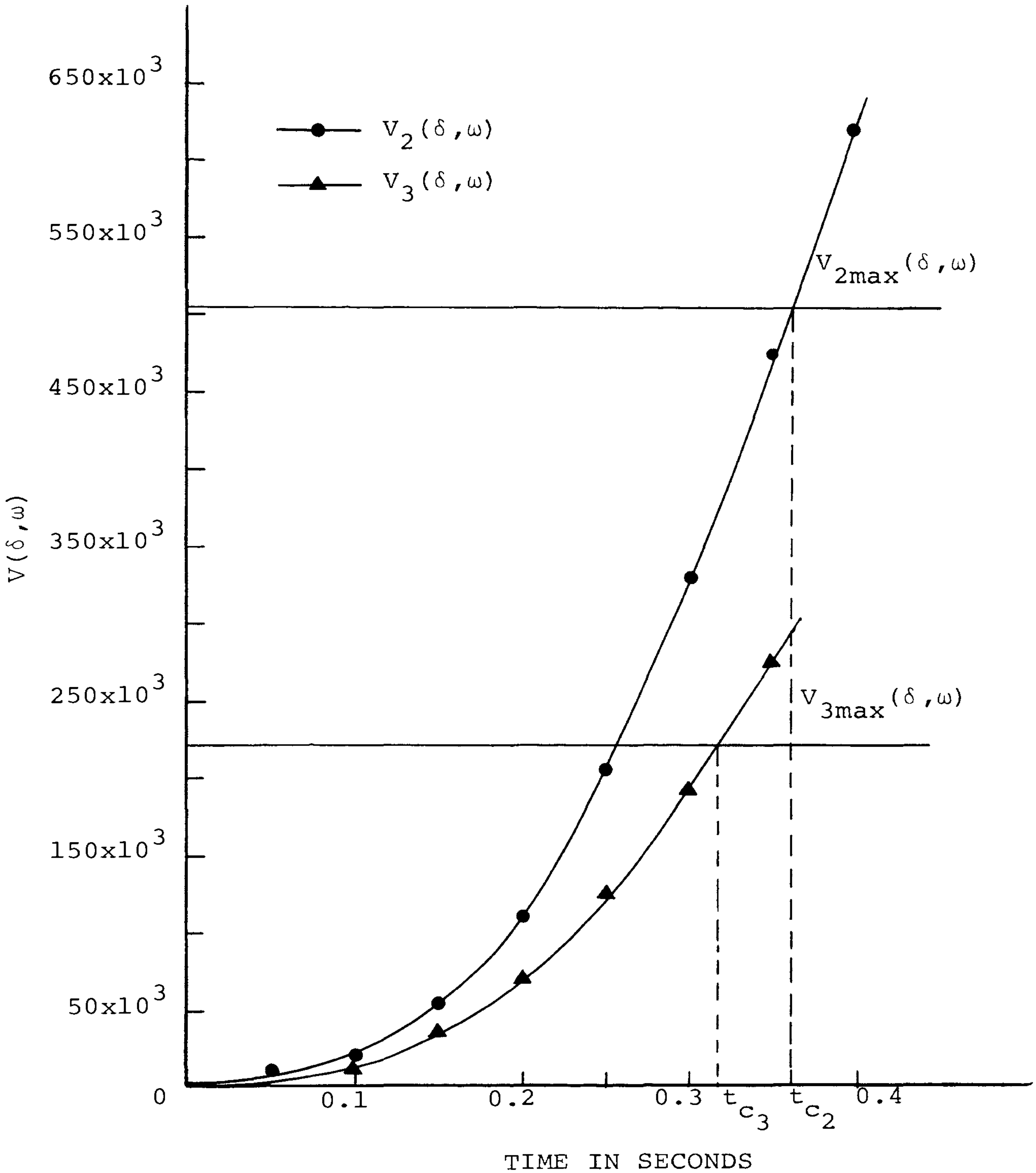

Figure 5. $V_2(\delta,\omega)$ and $V_3(\delta,\omega)$ Vs Time

The maximum value and the critical switching time obtained for each of the four Liapunov functions are tabulated in table VIII.

Figure 6 shows the results of the swing curves for a three-phase fault on bus thirteen cleared at 0.3 seconds. The indication is distinctly one of stability for this clearing time. Figure 7 shows the swing curve that results when the fault is cleared at 0.31 seconds. The results indicate that the system is unstable for clearing the fault at this time.

TABLE VIII

Maximum Value of Liapunov Functions and Predicted Critical Switching Time

Liapunov Function	Maximum Value	Predicted Critical Switching Time in Sec.	Predicted Critical Switching Time in Cycles
V_1 (δ, ω)	11.941	0.347	20.82
V_2 (δ, ω)	506628.3	0.366	21.96
V_3 (δ, ω)	219820.9	0.321	19.26
V_4 (δ, ω)	11.349	0.340	20.40

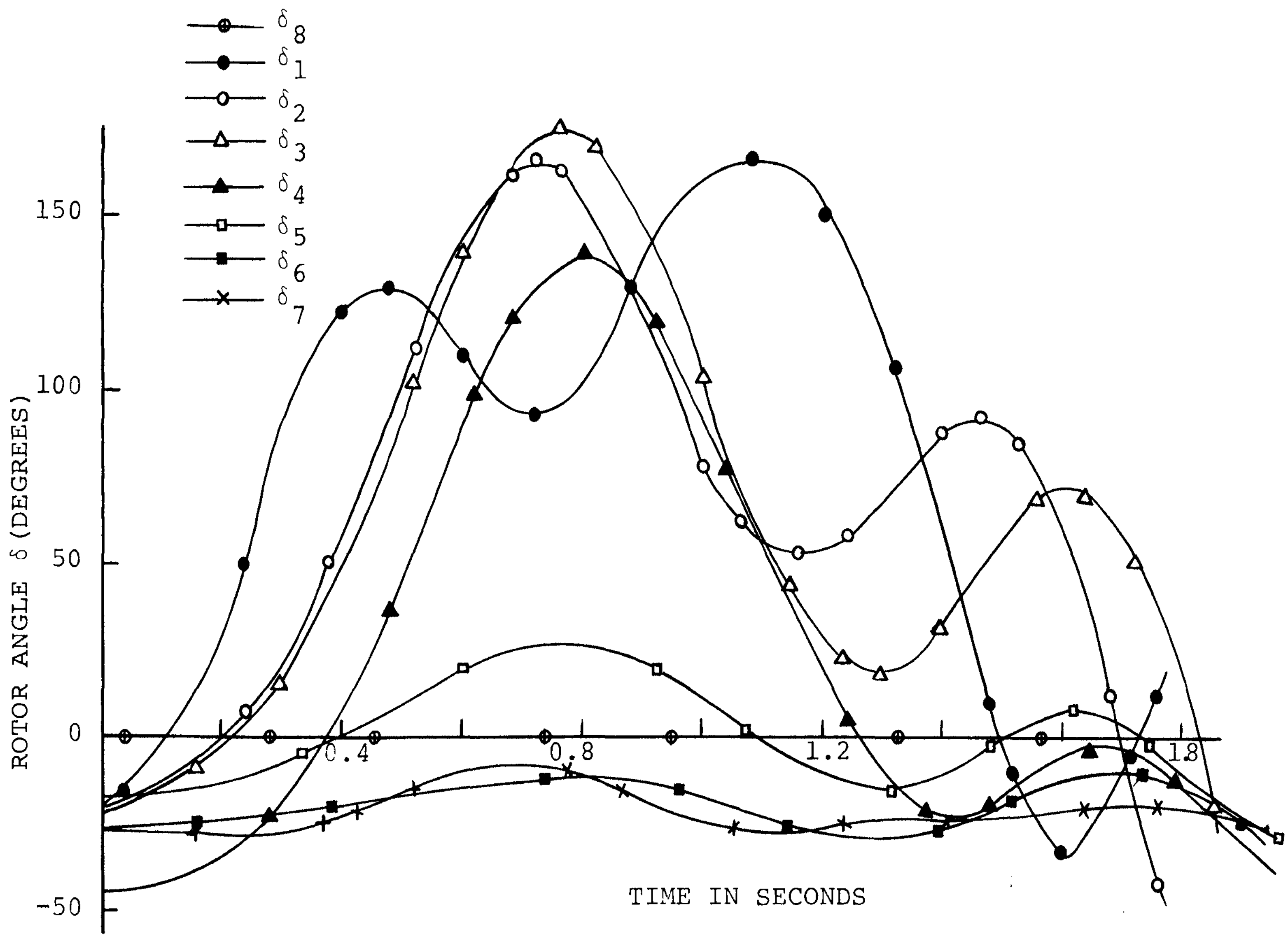

Figure 6. Swing Curves for Eight-Machine System Fault Cleared at 0.3 Seconds (18.00 Cycles)

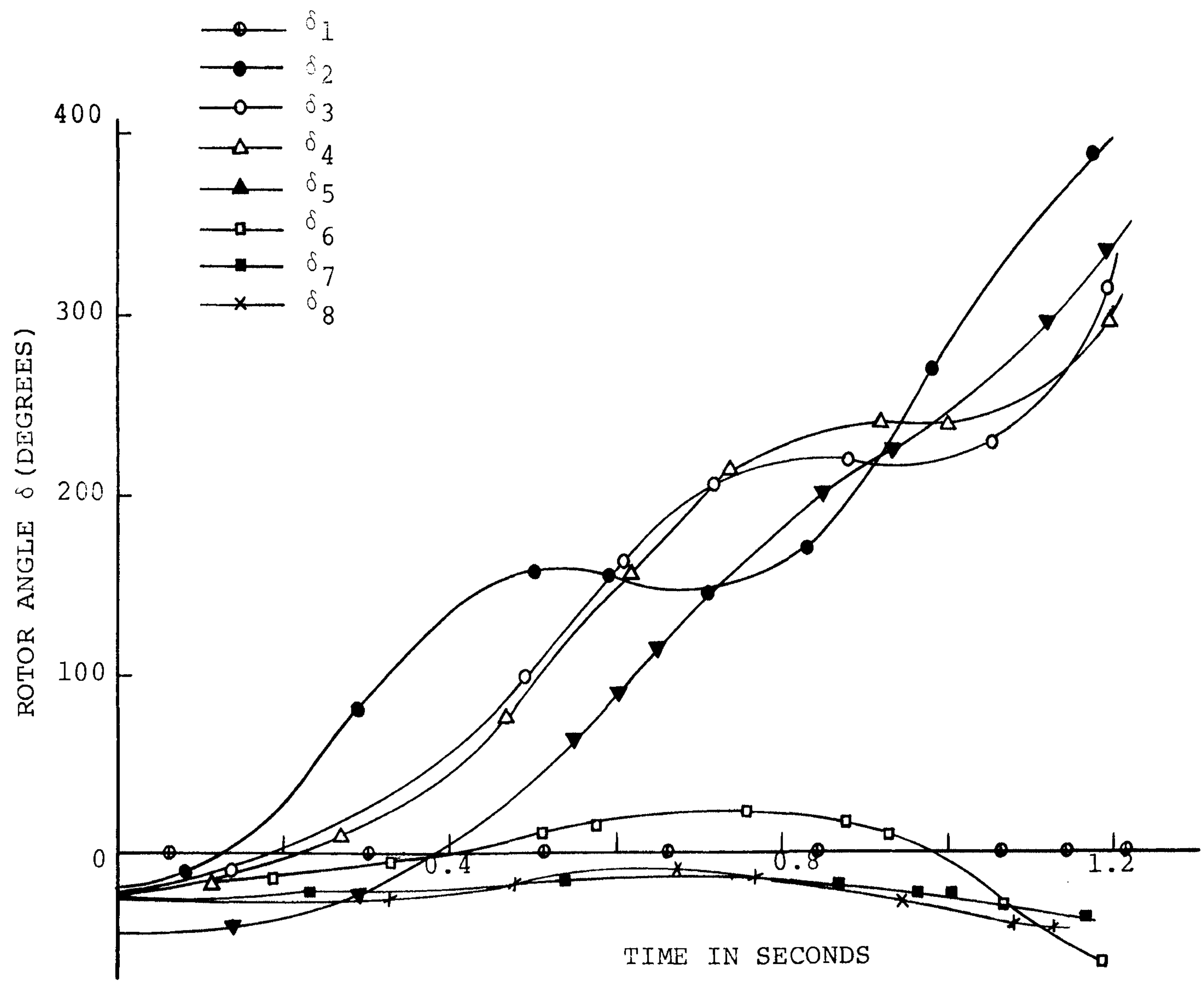

Figure 7. Swing Curves for Eight-Machine System Fault
Cleared at 0.31 Seconds (18.60 Cycles)

V. CONCLUSION

The results obtained for the prediction of critical switching time, using four different Liapunov functions, differ by about 3 cycles. As all the four Liapunov functions were developed by using different criterion, this difference in the result was expected.

The critical switching time was obtained from the swing curves. Comparing this with the results obtained by using Liapunov functions for predicted critical switching times, it was found that the difference was between 1.2 and 4 cycles. This difference does not limit the usefulness of the results obtained, if the following facts are considered:

(i) the assumption made in the stability study, listed on page 5, gave conservative results

(ii) the fact that the Liapunov function always assures a conservative result

(iii) the change in step size for the increment of Δt, in the numerical method used to solve the swing equations on a digital computer, can make a change in the results

(iv) small changes in the result can also be justified by the use of modeling techniques in which two generators at the same bus are combined into one and parallel lines between the same busses are also combined.

In this study, the predicted critical switching time obtained, using the third Liapunov function $V_3(\delta,\omega)$ developed by M Ribbens-Pavella, is found to be close (1.2 cycle) to the critical switching time obtained from the swing curves.

This study has revealed that there is still work that needs to be done

(i) to show that the best function for multi-machine power systems is independent of system or fault locations. For this, similar studies can be made taking different fault locations and also considering different power systems,

and (ii) to include the effects of voltage regulator, governer action and saliency.

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VITA

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APPENDIX A

DESCRIPTION OF THE FOUR LIAPUNOV FUNCTIONS USED

A. Development of First Liapunov Function $V_1(\delta,\omega)$

As stated earlier in the introduction to this paper, there is no systematic method available for constructing a Liapunov function. A suitable function can therefore be determined by trials for a particular system or equations to be tested. For the system equations, similar to equation III-1, El-Abiad and Nagappan have developed a Liapunov function[2] as follows:

$$V(\delta,\omega) = \sum_{k=1}^{n} 1/2\ M_k \omega_k^{\ 2} + \sum_{k=1}^{n} (E_k^2 G_{kk} - P_{mk})(\delta_k - \delta_k^s) + \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} E_k E_j \{B_{kj}[\cos(\delta_k^s - \delta_j^2) - \cos(\delta_k - \delta_j)] + G_{kj}[\sin(\delta_k^s - \delta_j^s) - \sin(\delta_k - \delta_j)]\} \qquad (A1-1)$$

The construction of this function was on the basis that the Liapunov's theory of stability is a generalized extension of the energy concept, which states that, near an equilibrium state of a physical system, if the transient energy of the system is always decreasing then the equilibrium state is stable.

The first summation of equation (Al-1) represents the kinetic energy of the system. The next two summations represent the potential energies of the system with respect to the equilibrium state. The potential energy at the equilibrium state is zero, and so equation (Al-1) is the sum of kinetic and potential energies thus giving the total entrgy of the system.

The function represented by equation (Al-1) above is the first Liapunov function $V_1(\delta,\omega)$ considered.

B. Development of Second and Third Liapunov Functions $V_2(\delta,\omega)$ and $V_3(\delta,\omega)$

The next two Liapunov functions $V_2(\delta,\omega)$ and $V_3(\delta,\omega)$ considered were developed by M. Ribbens-Pavella[7].

1. Equation of Motion

M. Ribbens-Pavella considered an n-machine power system without damping effects. The equation describing the motion of the ith machine for this system is given by

$$M_i \ddot{\delta}_i = P_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} A_{ij} \cos(\delta_{ij} - \theta_{ij}); \; i=1,2,\dots,n. \tag{Al-2}$$

where

$$P_i = P_{mi} - E_i^2 Y_{ii} \cos\theta_{ii}$$

$$A_{ij} = E_i E_j Y_{ij}$$

Taking one of the machine angles as a comparison angle, and realizing that now the stability of the system is given by (n-1) equations instead of n equations of form (Al-2), (n-1) equations can be derived from the pre-ceeding n equations using the variable $(\delta_i - \delta_n)$, i=1,2,..., n-1 as follows

$$\delta_{in} = D_{in} + A_{in}[M_n^{-1} \cos(\delta_{in} + \theta_{in}) - M_i^{-1} \cos(\delta_{in} - \theta_{in})] + \sum_{\substack{j=1 \\ j\neq 1}}^{n-1} [M_n^{-1} A_{nj} \cos(\delta_{nj} - \theta_{nj}) - M_i^{-1} A_{ij} \cos(\delta_{ij} - \theta_{ij})] \qquad \text{(Al-3)}$$

where $D_{in} = \frac{P_i}{M_i} - \frac{P_n}{M_n}$.

Thus the motion of the n-machine system is described by (n-1) differential equations of the second order of form (Al-3) or by 2(n-1) differential equations of the form

$$\begin{aligned} \delta_{in} &= \omega_{in} \\ \dot{\omega}_{in} &= F_{in} \end{aligned} \qquad \text{(Al-4)}$$

The equilibrium points of the above system represented in a 2(n-1) dimensional phase space are given by

$$\left.\begin{aligned} \dot{\omega}_{in} &= 0 \\ F_{in} &= 0 \end{aligned}\right\} ; \; i = 1,2,\ldots,n-1 \qquad \text{(Al-5)}$$

2. Construction of Liapunov Functions

For the construction of Liapunov functions M. Ribbens-Pavella considered n(n-1) variables given as

$$\begin{aligned} &\delta_{ik} \quad ; \; i = 1,2,\ldots,n-1 \\ &\omega_{ik} \quad\;\; k = i+1,\ldots,n \end{aligned} \tag{A1-6}$$

instead of 2(n-1) variables as in equation (A1-4). This is because the construction of a Liapunov function, using a first integral of the (n-1) set of equation (A1-3), is not possible as the system lacks 'symmetry'.

Incorporating a few modifications the equations of motion in this new n(n-1) dimensional phase space become

$$\dot{\delta}_{ik} = \omega_{ik}$$

$$\dot{\omega}_{ik} = D_{ik} + A_{ik}[M_k^{-1} \cos(\delta_{ik}+\theta_{ik}) - M_i^{-1} \cos(\delta_{ik} - \theta_{ik})]$$

$$+ \sum_{\substack{j=1 \\ j\neq i,k}}^{n} [M_k^{-1} A_{kj} \cos(\delta_{kj}-\theta_{kj}) - M_i^{-1} A_{ij} \cos(\delta_{ij}-\theta_{ij})]$$

$$i=1,2,\ldots,n-1 \qquad k=i+1,\ldots,n \tag{A1-7}$$

The calculation of a first integral becomes possible in two cases described below which give rise to two distinct Liapunov functions.

(i) substituting the value $\pi/2$ for all θ_{ij} $(i\neq j)$ represented in the right hand side of equation (A1-7)

(ii) substituting the value $\pi/2$ for all θ_{ij} $(i \neq j)$ represented only in the third term of the right hand side of equation (A1-7)

To give the value $\pi/2$ to the arguments of all or of some of the non-diagonal terms of the admittance matrix means that the conductances of the transfer admittances are to be neglected. This is an assumption which comes very close to the reality in most cases.

Calculating a first integral of the n(n-1) equations system derived from equation (A1-7) where all θ_{ij} $(i \neq j)$ are taken as equal to $\pi/2$ we obtained

$$A_1(\omega) = \phi_1(\delta) + K \qquad \text{(A1-8)}$$

where K is integration constant appropriately chosen.

Then the following relation is proposed as a Liapunov function

$$V_A(\delta, \omega) = A_1(\omega) - \phi_1(\delta) + K \qquad \text{(A1-9)}$$

where

$$A_1(\omega) = \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} 1/2\ M_i M_k \omega_{ik}^2 \qquad \text{(A1-10)}$$

$$\phi_1(\delta) = \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} [B_{ik}\delta_{ik} + A_{ik}(M_i + M_k)\cos\delta_{ik}] + $$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n-1} \sum_{\substack{k=j+1 \\ k \neq i}}^{n} M_i A_{jk} \cos\delta_{jk} \qquad \text{(A1-11)}$$

$$B_{ik} = P_i M_k - P_k M_i \tag{A1-12}$$

To assure asymptotic stability of the system one of the conditions which the function $V_A(\delta,\omega)$ given by equation (A1-9) should satisfy is

$$V_A(\delta^s,0) = 0$$

which follows

$$V_A(\delta^s,0) = A_1(0) - \emptyset_1(\delta^s) + K = 0$$

$$K = \emptyset_1(\delta^s)$$

Thus, equation (A1-9) becomes

$$V_A(\delta,\omega) = A_1(\omega) - \emptyset_1(\delta) + \emptyset_1(\delta^s) \tag{A1-13}$$

The function represented by equation (A1-13) is the second Liapunov function $V_2(\delta,\omega)$ considered.

Substituting θ_{ij} $(i \neq j)$, appearing in the third term of the right hand side of equation (A1-7), equal to $\pi/2$, another Liapunov function was obtained follows

$$V_B(\delta,\omega) = A_2(\omega) - \emptyset_2(\delta) + \emptyset_2(\delta^s) \tag{A1-14}$$

where

$$A_2(\omega) = \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} 1/2 \; M_i M_k \omega_{ik}^2$$

$$\phi_2(\delta) = \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} B_{ik}\delta_{ik} + A_{ik}[M_k \sin(\delta_{ik}-\theta_{ik})$$

$$- M_i \sin(\delta_{ik}+\theta_{ik})] + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n-1} \sum_{\substack{k=j+1 \\ k\neq i}}^{n} M_i A_{jk} \cos\delta_{jk}$$

The function thus represented by equation (A1-14) is the third Liapunov function $V_3(\delta,\omega)$ considered.

C. Development of Fourth Liapunov Function $V_4(\delta,\omega)$

The Popov Criterion, which gives a sufficient condition for the stability of a feedback system, has recently received a great deal of attention in the automatic control literature. J.L. Willems and J.C. Willems have computed a Liapunov function to prove a generalized theorem of the Popov stability theorem for systems with multiple nonlinearities.

For an n-machine power system a Liapunov function developed by them is given as follows[9]:

$$V(X) = \sum_{i=1}^{n} \frac{M_i\omega_i^2}{2} + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_iE_jB_{ij}[\cos(\delta_i^o - \delta_j^o) -$$

$$\cos(\delta_i-\delta_j) - (\delta_i-\delta_j-\delta_i^o+\delta_j^o)\sin(\delta_i^o-\delta_j^o)] \qquad \text{(A1-15)}$$

where $(\delta_1^o,\delta_2^o,\ldots,\delta_n^o)$ are defined as the equilibrium load angles.

The function given by equation (A1-15) is the fourth Liapunov function considered.

APPENDIX B

DEFINITIONS

The term "fault" is understood to mean reducing all three line voltages instantaneously and simultaneously to zero at a known point along one of the transmission lines, which interconnect the system loads and generators[11]. Removal or clearing of the fault is accomplished by opening the faulted line at both ends. The fault is defined to occur at $t = 0$, and is cleared at $t = T$. The problem is to find the maximum value of T such that the system will remain stable. This value is referred to as the critical clearing time, and is denoted as T_c.

The term stability is defined in "American Standard Definitions of Electrical Terms" as follows:

"Stability, when used with reference to a power system, is that attribute of the system, or part of the system, which enables it to develop restoring forces between the elements thereof, equal to or greater than the disturbing forces so as to restore a state of equilibrium between the elements."

Roughly interpreted this means that although the system generator rotors are not moving at the same speed during the interval $0 < t \leq T$, and for some finite interval $t > T$, the system will eventually settle to stable equilibrium condition such that all the machines are again synchronized (i.e., the rotors all have the same speed).