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A comment on 'Failure modes of hybrid composites . . .'

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Abstract. This paper comments on an earlier paper on the failure modes of hybrid composites by Pitkethly and Bader, which appeared in the Carbon Fibre Conference issue.

In [1], a Weibull two-parameter distribution

$$P_f(L) = [1 - \exp(-L(\sigma/\sigma_0)^{\omega})] \qquad (1)$$

is used to model the probability of failure of a fibre of length L under an applied stress σ where ω and σ_0 are the Weibull shape and scale parameters, respectively. The authors considered that these parameters depend on the specific material and the length of the fibre being tested. This interpretation leads to the determination of a set of constants σ_0 and ω for each fibre length with the same material. In this paper, it will be shown that a unique set of parameters allows (1) to model the probability of failure of one type of fibre.

The Weibull distribution (equation (1)) follows from the weak-link model in which the fibre of length L is regarded as a series of n elements of length l. The probability of failure of the fiber $P_f(L)$ is related to that of a segment $P_f(l)$ as

$$P_f(L) = [1 - (1 - P_f(l))^n]. \qquad (2)$$

Equation (1) is derived from the weak-link model (equation (2)) and includes the effect of fibre length along with the effect of applied stress [2]. A unique choice of the parameters σ_0 and ω should allow a complete representation of the experimental data for a given type of fibre.

A unique set of parameters σ_0 and ω should be selected to properly fit all of the tests results on a given type of fibre as in figure 1 of [1], for example. Table 1 of [1] gives values of the parameters corresponding to a best fit for each of the gauge lengths. For four different gauge lengths four sets of parameters are proposed. The parameter ω, represents the slope of the lines in figure 1(b) of [1] and is almost constant with an average of 5.8 [1]. Table 1 also gives experimental σ_0 values for each of the gauge lengths. In fact, these values are the applied stress levels σ^* corresponding to a probability of failure of $(1 - 1/e) = 0.632$ where e is the base of natural logarithms. That is when the relation $\ln\ln[1/(1 - P_f)] = 0$ is satisfied as pointed out in [1]. Starting with (1) it can be shown that

$$\sigma_0 = \sigma^* L^{1/\omega}. \qquad (3)$$

This equation indicates that the parameter σ_0 does not have the dimension of a stress. It can be interpreted as the stress that will produce a probability of failure of $(1 - (1/e))$ for a fibre of unit length. Therefore, it is a measure of the strength of the material and ω accounts for the variability in test results.

Taking the 'experimental σ_0' values from table 1 of [1] as σ^* and $\omega = 5.8$, a value for σ_0 for each of the gauge lengths can be computed.

Table 1. Values of the σ_0 parameter determined from the data presented in table 1 of [1].

Carbon fibre gauge length (mm)	σ^* (GPa)	σ_0 (GPa)
1	4.53	4.53
10	3.25	4.84
20	2.63	4.41
50	2.41	4.73

Table 2. Values of σ_0 determined from the data in table 2 of [1].

Carbon fibre bundle type	Gauge length (mm)	σ^* from [1] (GPa)	σ_0	Weibull exponent ω [1]	Average value of ω
IB	50	2.88	3.41	21	
IB	100	2.56	3.12	30	23.33
IB	200	2.45	3.07	19	
H	20	3.90	4.30	30	
H	50	3.83	4.35	30	31
H	100	3.77	4.37	30	
H	200	3.71	4.40	34	

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Those values for σ_0 (table 1) are very close with an average of 4.63 which indicates that, given the original test data, a unique set of parameters can be determined so that equation (1) will fit all of the experimental data in figure 1 of [1].

Considering the data presented in table 2 of [1], the same calculations can be performed. A constant value for the parameter ω was taken as the average of the values given in [1] for each type of fibre as shown in table 2. The parameter σ_0 averages 3.15 GPa for the impregnated bundle (IB) data and 4.36 GPa for the hybrid composed (H) data.

The same calculations are performed for the data in table 3 of [1] and results are shown in table 3.

The results presented in tables 1–3 show that for a given material, a unique set of constants σ_0 and ω is necessary to represent the experimental data. In some instances (table 3, bundle spacings of 0.5 and 0 mm) large variations in the Weibull exponent ω are reported [1]. This might indicate that for bundles of fibres and composites, the probability of failure does not follow a Weibull distribution. The weak-link model was developed for single fibres and some existing models [2] show that for fibre bundles the probability of failure follows a different distribution. A complete proof would require analysing the original data which is not readily available in [1]. A method for determining the parameters σ_0 and ω due to Freudenthal and Gumbel is discussed in [2].

Table 3. Values of σ_0 determined from the data in table 3 of [1].

Carbon fibre bundle spacing (mm)	Gauge length (mm)	σ^* from [1] (GPa)	σ_0	Weibull exponent ω [1]	Average value of ω
1.55	20	3.94	4.30	31	
	50	3.90	4.38	33	34
	100	3.85	4.41	38	
1.0	20	3.67	3.97	41	
	50	3.55	3.93	38	38.33
	100	3.52	3.97	36	
0.5	20	3.75	3.99	54	
	50	3.68	4.01	45	46
	100	3.62	4.00	39	
0	20	3.77	4.06	50	
	50	3.66	4.03	34	40.67
	100	3.58	4.01	38	

References

[1] Pitkethly M H and Bader M S 1987 *J. Phys. D: Appl. Phys.* **20** 315–22

[2] Garg S K, Svalbonas V and Gurtman G A 1973 *Analysis of Structural Composite Materials* (New York: Dekker)