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# Analysis of shear bands in simple shearing deformations of nonpolar and dipolar viscoplastic materials

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During the past few years we have studied numerically the initiation and growth of shear bands in nonpolar and dipolar viscoplastic materials being deformed in simple shear, and in nonpolar materials undergoing plane strain deformations. We summarize here our work for the former problem.

## 1. INTRODUCTION

Narrow regions of intense plastic deformation have been observed during high strain-rate plastic deformation of many metals. These have been called adiabatic shear bands since there is not enough time available for the heat to be conducted away from these regions, and the primary mode of deformation is that of shearing. Tresca (1878) and Massey (1921) observed such bands in the form of a cross during the hot forging of a metal and called them hot lines. The research activity in this field has increased significantly since the time Zener and Hollomon (1944) reported 32  $\mu\text{m}$  wide shear bands during the punching of a hole in a steel plate. Backman and Finnegan (1965) have reported that shear bands initiate at flaws, pits, scratches, and inhomogeneities in the material, and these zones of inhomogeneous deformation propagate from their initiation site like a running crack. The study of shear bands is important because once a shear band initiates, the subsequent deformations of the body are concentrated in this narrow region with the rest of the body undergoing very little, if any, deformations. Also, shear bands usually precede shear fractures. We refer the reader to the papers by Shawki and Clifton (1989) and Batra and Zhu (1991) for additional references.

The foregoing remarks suggest that the modeling of a material inhomogeneity and the material behavior at high temperatures and strain-rates should play significant roles in the study of shear bands. We address these and other issues below.

## 2. FORMULATION OF THE PROBLEM

In terms of non-dimensional variables, the equations governing the dynamic thermomechanical deformations of a viscoplastic block, shown in Fig 1, and undergoing overall simple shearing deformations, are

$$\alpha w \dot{v} = (ws - l(w\sigma)_{,y})_{,y}, \quad -1 < y < 1, \quad (1)$$

$$w\dot{\theta} = \beta(w\theta_{,y})_{,y} + w(s\dot{\gamma}_p + l\sigma\dot{d}_p), \quad -1 < y < 1, \quad (2)$$

$$\dot{s} = \mu(v_{,y} - \dot{\gamma}_p), \quad (3)$$

$$\dot{\sigma} = \mu l(v_{,yy} - \dot{d}_p), \quad (4)$$

$$\dot{\gamma}_p = f(s, \sigma, \psi, \theta, \gamma_p, d_p, l), \quad (5)$$

$$\dot{d}_p = l g(s, \sigma, \psi, \theta, \gamma_p, d_p, l), \quad (6)$$

$$\dot{\psi} = (s\dot{\gamma}_p + l\sigma\dot{d}_p)/(1 + \psi/\psi_0)^n, \quad (7)$$

where the non-dimensional numbers

$$a = \rho \dot{\gamma}_0^2 / s_0 H^2 \quad \text{and} \quad \beta = k / \rho c \dot{\gamma}_0 H^2 \quad (8)$$

give, respectively, the effect of inertia forces relative to the flow stress of the material and that of heat conduction. Here  $\rho$  is the mass density,  $v$  the velocity of a material particle in the direction of shearing,  $w$  the thickness of the block,  $s$  the shearing stress,  $\sigma$  the dipolar shearing stress,  $l$  is a material characteristic length,  $\theta$  the temperature rise,  $\dot{\gamma}_0$  the average applied strain rate,  $\dot{\gamma}_p$  the plastic strain-rate,  $\dot{d}_p$  the dipolar plastic strain-rate,  $2H$  the height of the block,  $c$  the specific heat,  $s_0$  the yield stress in a quasistatic isothermal simple shearing test conducted at the room temperature, and  $k$  the thermal conductivity. The length, time, temperature, and stresses are scaled respectively by  $H$ ,  $\dot{\gamma}_0$ ,  $\theta_r$ , and  $s_0$  to obtain their nondimensional counterparts where

$$\theta_r = s_0 / \rho c. \quad (9)$$

Throughout this paper, a superimposed dot indicates the material time derivative and a comma followed by  $y$  signifies partial differentiation with respect to  $y$ . Equations (1) and (2) express, respectively, the balance of linear momentum and the balance of internal energy. Equations (3) through (7) are constitutive assumptions, wherein  $\mu$  is the shear modulus,  $\psi$  is an internal variable used to describe the effect of the history of deformation upon the response of the material, and the functional forms of  $f$  and  $g$  characterize the material of the body. In Eq (7)  $\psi_0$  and  $n$  are material parameters.

For the initial and boundary conditions, we take

$$v(y,0) = 0, \quad s(y,0) = 0, \quad \sigma(y,0) = 0, \quad \theta(y,0) = 0, \quad (10)$$

$$\theta_{,y}(\pm 1,t) = 0, \quad \sigma(\pm 1,t) = 0, \quad v(\pm 1,t) = \pm t/t_r, \quad 0 \leq t \leq t_r, \quad (11)$$

$$= \pm 1, \quad t \geq t_r.$$

That is, the block is initially at rest, is stress free, and is at a uniform temperature. The lower and upper surfaces of the block are thermally insulated, and equal and opposite shearing speeds are prescribed on them. The boundary conditions for  $\sigma$  are justified since a narrow band forms around  $y = 0$  and the velocity gradients are constants near  $y = \pm 1$ .

Our account of dipolar effects differs from that of Coleman and Hodgdon (1985) and Zbib and Aifantis (1988). Whereas we include dipolar stresses corresponding to second-order gradients of velocity in the balance of linear momentum and the balance of internal energy, the two papers cited above include gradients of strain only in the constitutive function  $f$ . Note that our formulation of the problem reduces to that for simple materials by taking  $l = 0.0$ .

Sometimes we used initial and/or boundary conditions different from those given in (10) and (11), and also modeled a material inhomogeneity by perturbing the temperature field. Such variations to the formulation of the problem are delineated whenever applicable. Also whenever possible we exploited the symmetry or antisymmetry of deformations about  $y = 0$  and solved the problem on the domain  $[0,1]$ .

We note that the coupled partial differential equations (1) through (7) are highly nonlinear. Their approximate solution under the side conditions (10) and (11) has been obtained by the finite element method. This is accomplished by first reducing the governing equations to a set of coupled ordinary differential equations by using the Galerkin approximation, which are then integrated with respect to time  $t$  by using the Gear (1971) method included in the subroutine LSODE developed by Hindmarsh (1983). The details of the aforesaid solution technique have been given by Batra and Kim (1990a). We note that Batra (1987) integrated the ordinary differential equations by using the Crank-Nicolson method. His results are in qualitative agreement with those obtained subsequently by using the Gear method.

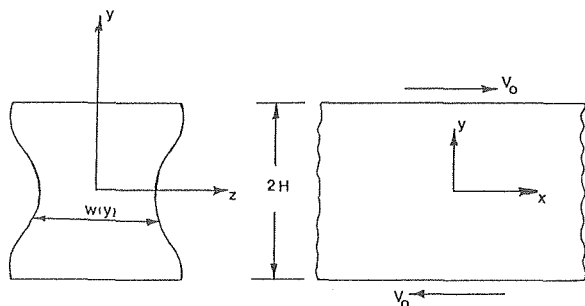


FIG 1. A schematic sketch of the problem studied.

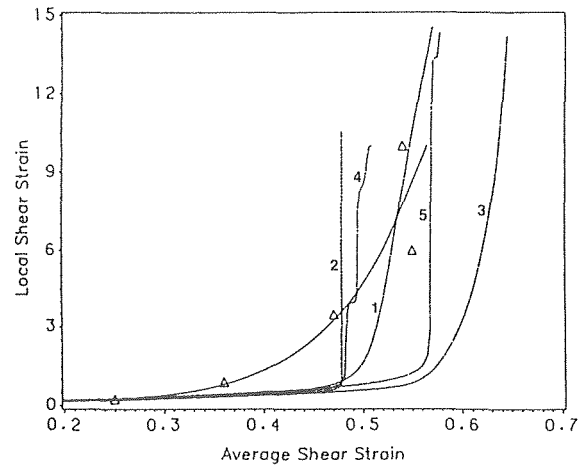


FIG 2. Growth of the local shear strain within the band as the specimen deforms. Curve 1, Bodner-Partom; Curve 2, Litonski (nonpolar); Curve 3, Wright-Batra dipolar; Curve 4, power law; Curve 5, Johnson-Cook;  $\Delta$  experimental data from Marchand and Duffy's paper.

### 3. RESULTS

#### 3.1 Effect of constitutive relations

Batra and Kim (1990b) assumed the block to be of uniform thickness and modeled the material inhomogeneity by assuming that the initial temperature of the material near  $y = 0$  was slightly higher than that of the rest of the material. They found values of material parameters in the Bodner-Partom (1975) constitutive relation, Litonski law (1977), Johnson-Cook law (1983), Power law (e.g. see Costin et al 1980), and the Wright-Batra (1987) dipolar theory by ensuring that the computed shear stress-shear strain curve for a block without any defect matched well with that reported by Marchand and Duffy (1988) for a HY-100 steel specimen subjected to torsional loading at room temperature and deformed at a nominal strain-rate of  $3300 \text{ sec}^{-1}$ . The magnitude of the temperature perturbation was determined by numerical experiments so that the sharp drop in stress

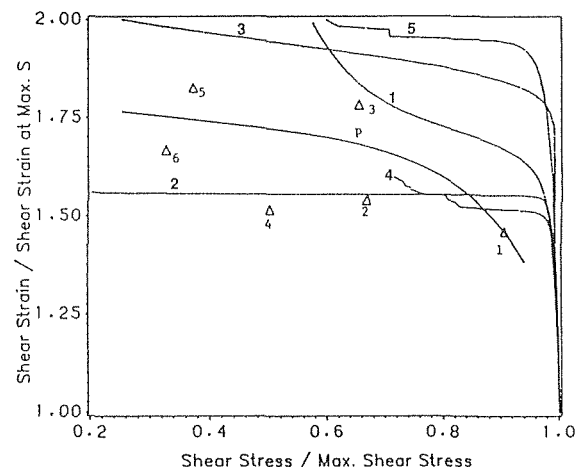


FIG 3. Plot of the normalized shear strain vs the normalized shear stress during the time shear stress is dropping with increasing strain. See FIG 2 for legends of curves.

occurred at about the test value of the nominal strain. Subsequently, various solution variables for tests done at nominal strain rates of  $1600 \text{ sec}^{-1}$  and  $1400 \text{ sec}^{-1}$  were compared with the experimental findings.

Figure 2 depicts the growth of the shear strain within the band, taken here to be the strain at  $y = 0$ , as the specimen deforms. Whereas the Litonski law, the Power law, and the Johnson-Cook law give a rapid increase in the local strain once a shear band initiates, the Bodner-Partom law and the Wright-Batra dipolar theory give general trends in agreement with the experimental data. A similar conclusion can be drawn from the results plotted in Fig 3, which exhibits normalized shear strain versus the normalized shear stress during the time the shear stress is dropping. Note that the abscissa represents a nonuniformly stretched out time scale. The band width, defined as the width of the region over which the plastic shear strain varies by no more than 5% of its value at the center, was found to be  $2 \mu\text{m}$ ,  $14 \mu\text{m}$ ,  $14 \mu\text{m}$ ,  $6 \mu\text{m}$ , and  $51 \mu\text{m}$ , respectively, for the Litonski law, the Power law, the Bodner-Partom law, the Johnson-Cook law,

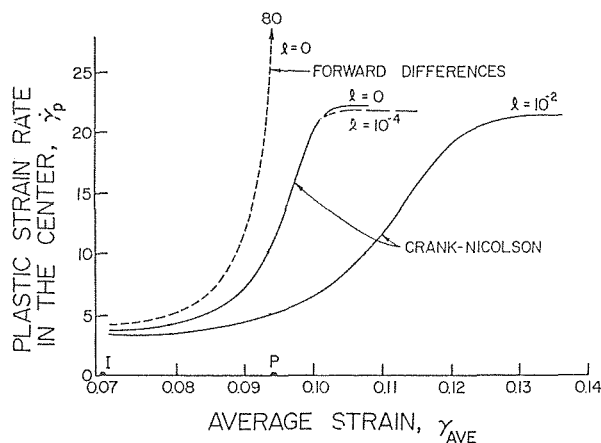


FIG 4. Evolution of the plastic strain-rate at the band center for nonpolar and dipolar materials.

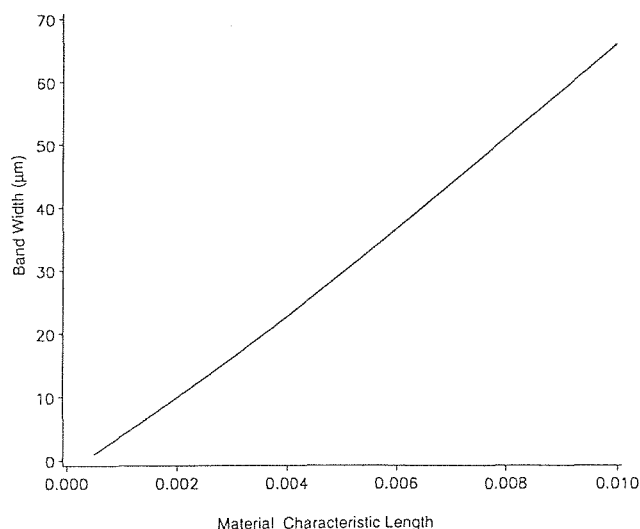


FIG 5. Dependence of the band width upon the material characteristic length  $l$ .

and the Wright-Batra dipolar theory. When the shear stress at the specimen center had dropped to 66% of its maximum value, Marchand and Duffy (1988) found the band width to be between  $20 \mu\text{m}$  and  $55 \mu\text{m}$  depending upon the point of observation around the circumference of the steel tube. Marchand and Duffy defined the band width to be the width of the region, surrounding the band center, over which the plastic strain remains uniform.

### 3.2 Effect of material characteristic length $l$ .

Wright and Batra (1987) perturbed the homogeneous solution for the simple shearing problem by introducing a temperature perturbation just before the shear stress attained its peak value and assumed simple expressions for the constitutive functions  $f$  and  $g$ . From the evolution of the plastic strain-rate at the specimen center plotted in Fig 4, it is clear that the consideration of dipolar effects delays the initiation of the shear band. The shear band is assumed to initiate when the plastic strain-rate at the specimen center grows rapidly. Also the strain-rate increases gradually for  $l = 10^{-2}$  but quite rapidly for  $l = 0.0$ . The dependence of the band-width upon  $l$ , as computed by Batra and Kim (1988) and depicted in Fig 5, shows that the band-width increases with an increase in the value of  $l$ .

Zhang and Batra (1992) analyzed the stability of the homogeneous solution of equations (1) through (7) and assumed that the flux of linear momentum is essentially uniform in the spatial variable and  $|s, y| \ll \Gamma |\theta_y|$ , where  $\Gamma$  is defined in terms of the material variables. They showed that an increase in the thermal conductivity and the material characteristic length has a stabilizing effect, and an increase in the block height has a destabilizing effect on small perturbations superimposed on the homogeneous solution. The specific heat did not appear in the stability criterion.

Batra (1987) studied the interaction between two shear bands and found that the bands that would grow independently in simple materials ( $l = 0.0$ ) coalesced in dipolar materials even when the material characteristic length equalled  $1/20$  of the distance between them at the time of their initiation. Kwon and Batra (1988) introduced multiple defects by perturbing the uniform temperature within the block when the material just starts deforming plastically to that given by a cosine function which assumes relative maximum values at several points in the block. They found that for simple materials, the deformation localized at points where the perturbed temperature had relative minima for  $\dot{\gamma}_0 = 500 \text{ sec}^{-1}$  and at the locations of the relative maxima at  $\dot{\gamma}_0 \geq 1000 \text{ sec}^{-1}$ . For dipolar materials with  $l = 0.01$ , the deformation localized near the block boundaries at  $\dot{\gamma}_0 = 500 \text{ sec}^{-1}$  and at the locations of the relative maxima when  $\dot{\gamma}_0 = 50,000 \text{ sec}^{-1}$ . For both simple and dipolar materials the initiation of the localization was considerably delayed as compared to that when the temperature perturbation had only one maxima at the specimen center.

### 3.3 Unloading wave from the shear band

Wright and Walter (1987) found that for thermally softening and strain-rate hardening materials the shear stress within the

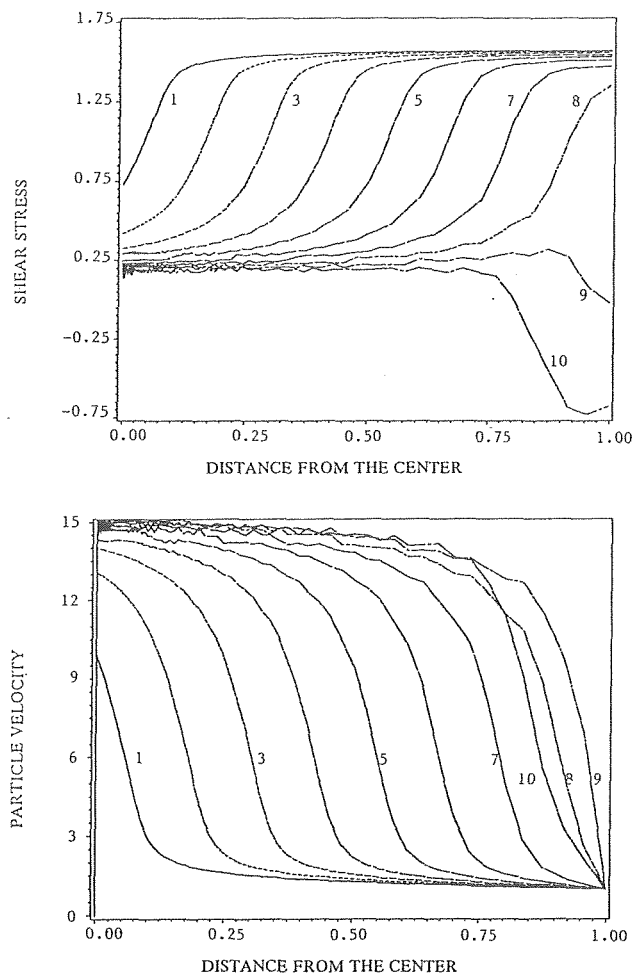


FIG 6. Distribution of the shear stress and the particle velocity within the specimen at different times during the localization of the deformation for nonpolar materials. These curves are plotted at intervals of  $0.1 \mu\text{s}$  with Curve 1 at  $t = 64.0 \mu\text{s}$ , Curve 2 at  $t = 64.1 \mu\text{s}$ , etc; the time being reckoned from the instant of introducing the perturbation.

band dropped precipitously. Batra and Kim (1990) also considered the effect of material elasticity and work-hardening and modeled the material by the Litonski flow rule. Their computed distribution of the shear stress and the particle velocity within the specimen at intervals of one-tenth of a microsecond starting from the instant when the deformation begins to localize is shown in Fig 6. It is clear that an unloading elastic shear wave emanates outwards from the region of severe deformation. The computed speed, 3,178 m/sec, of this wave essentially equals the analytical value of 3,190 m/sec. It takes  $0.807 \mu\text{s}$  for the shear wave to reach the outer boundary where it is reflected back with a negative value of the shear stress.

Computations for dipolar materials, and for simple materials but using the other four flow rules did not result in the emanation of an unloading elastic wave from the severely deformed region. Of course, no such wave would be found if the inertia forces were neglected.

### 3.4 Effect of thermal conductivity

Batra and Kim (1991) used the Litonski law, the Bodner-Partom law, and the Johnson-Cook law to model the viscoplastic response of the material and computed results for five different values, namely, 0, 5, 50, 500, and  $5000 \text{ W/m}^2\text{C}$  of the thermal conductivity  $k$ . The thickness of the block was assumed to vary smoothly, with the thickness at the specimen center being 5% smaller than that at the boundary. For each of these three constitutive relations, the rates of evolution of the temperature and the shear strain at the specimen center were steepest for  $k = 0$ , and decreased with an increase in the value of  $k$ . The computed band-width depends upon how far the localization has progressed, or how much the shear stress at the specimen center has dropped. This is evidenced by the results plotted in Fig 7. For  $k = 50$  and  $500 \text{ W/m}^2\text{C}$  the band-width seems to reach a stable value as the shear stress within the band drops. For  $k = 5,000 \text{ W/m}^2\text{C}$  an interesting situation developed in that the band-width decreased at first with the drop in the shear stress at the specimen center, reached a plateau at  $s_a/s_{\max} \approx 0.85$ , and then started to increase. Here  $s_a$  equals the average shear stress in the block. The rate of change of the band-width with respect to  $s_a/s_{\max}$  does depend upon the constitutive relation employed. A plausible explanation for this computed decrease and increase of the band-width is that as the shear stress at the specimen center drops and the plastic strain-rate increases sharply, the heat generated due to plastic working raises the temperature there more than at other points in the block. Initially, the rate of heat loss to outer parts of the block is less than the rate of heat generation at the specimen center, and the temperature there rises, making the material there softer and thus easier to deform. As the temperature gradients build up, the rate of heat loss increases, more of the material surrounding the specimen center is heated up enough so as to deform severely and the band-width increases.

The plots of the band-width  $w$  computed when  $s_a/s_{\max}$  equalled 0.95, 0.90, 0.85, 0.80, 0.75, and 0.70 versus  $(k)^{1/2}$  revealed that  $w$  decreased with a decrease in the value of  $k$ , for each of the three constitutive relations used, and that the relationship between  $w$  and  $(k)^{1/2}$  was not linear as asserted by Dodd and Bai (1985). The Litonski and the Johnson-Cook flow rules gave zero band-width for  $k = 0$ , but the Bodner-Partom law gave a finite value of the band-width for  $k = 0$ .

### 3.5 Effect of initial temperature

Wang et al (1988) tested titanium alloy TB2 (Ti-8Cr-5 Mo-5V-3 Al) specimens in compression at different environmental temperatures and concluded that the susceptibility to adiabatic shearing increased at lower environmental temperatures. In order to assess the effect of the initial specimen temperature on the initiation of shear bands, Kim and Batra (1992) assumed the dependence, shown in Fig 8, of the specific heat, thermal conductivity and the shear modulus of a plain carbon steel upon the temperature. They modeled the viscoplastic response of the material by the Bodner-Partom flow rule and computed results for six different values, namely,  $\theta_0 = 83^\circ\text{K}$ ,  $200^\circ\text{K}$ ,  $300^\circ\text{K}$ ,  $343^\circ\text{K}$ ,  $407^\circ\text{K}$ , and  $423^\circ\text{K}$

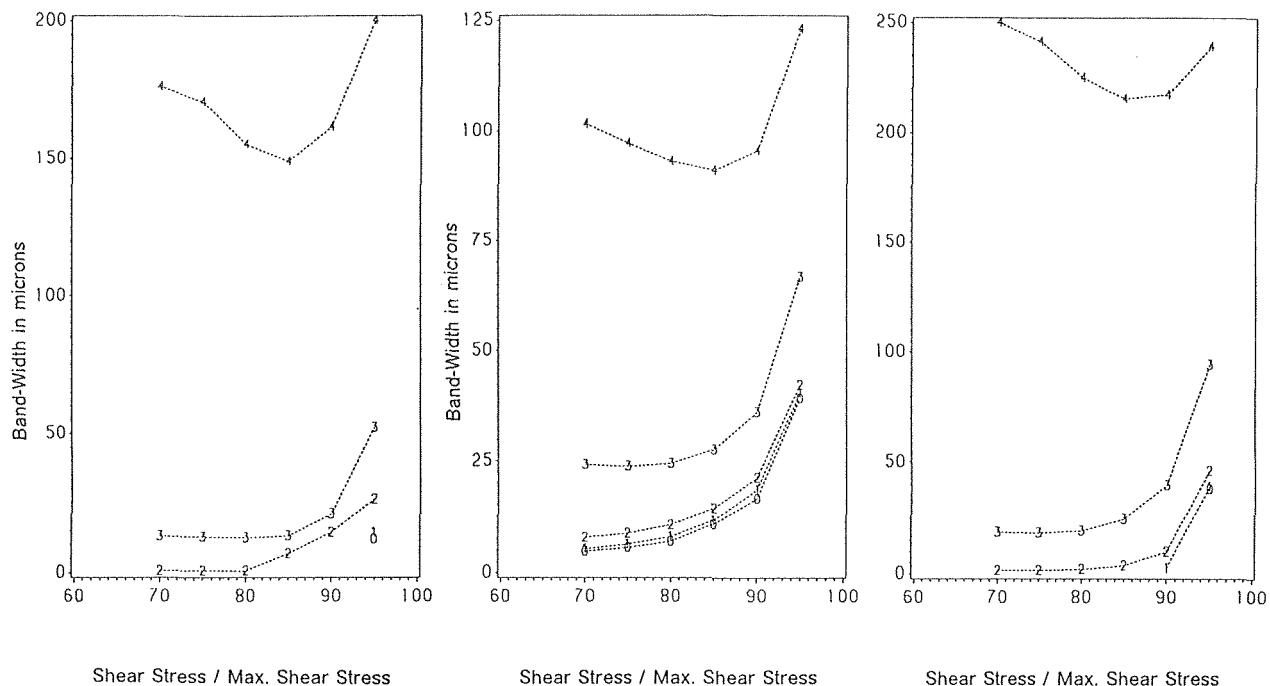


FIG 7. Dependence of band-width upon  $s_a/s_{max}$ . Curve 0 for  $k = 0$ ; Curve 1,  $k = 5$  W/m°C; Curve 2,  $k = 50$  W/m°C; Curve 3,  $k = 500$  W/m°C; curve 4,  $k = 5,000$  W/m°C.

of the initial temperature. The specimen thickness at its center was taken to be 5% smaller than that at its edges. One way to study the localization of the deformation is to observe at different instants the deformed position of an initially straight line. We recall that Marchand and Duffy (1988) used this technique to find the plastic strain at a point. In Fig. 9, we have plotted the deformed positions of an initially straight line at  $s_a/s_{max} = 1.0, 0.95, 0.85$ , and  $0.75$ . For each value of  $\theta_0$  considered, the deformation has become nonhomogeneous by the time the average shear stress  $s_a$  attains its peak value  $s_{max}$ . As  $\theta_0$  increases, the width of the central severely deformed region increases. From the plots given in Fig 10 of the shear strain  $\gamma_{loc}$  at the band center versus the average strain, it is clear that higher initial temperatures of the specimen delay the initiation of the localization of the deformation.

### 3.6 Effect of defect size

Batra (1987) studied the effect of different temperature perturbations on the initiation and growth of shear bands in a material obeying the Litonski law. He found that a shear band formed sooner with an increase in the height of the temperature perturbation. Larger temperature perturbations caused the shear band to initiate even before the peak in the shear stress-shear strain curve was reached. Also, a wider perturbation resulted in the shear strain localization at a lower rate as compared to the narrow perturbation of the same amplitude.

Batra and Kim (1992) recently studied the effect of the thickness variation on the development of shear bands in twelve different materials. The viscoplastic response of each

material was modeled by the Johnson-Cook law. Figure 11 depicts the localization ratio versus  $\log \delta$  when  $s_a/s_{max} = 1.0$  and  $0.85$ , where  $\delta$  is the percentage decrease in the thickness of the specimen at the center relative to that at its edges, and the localization ratio equals the maximum shear strain within the severely deforming region divided by the nominal strain in the specimen. The localization ratio depends strongly upon  $\log \delta$ ; larger defects result in more severe localization of the deformation for the same value of  $s_a/s_{max}$ . The defect

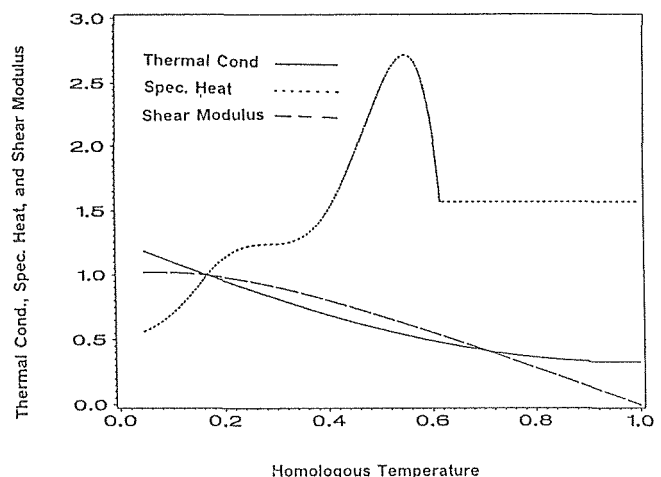


FIG 8. Dependence of the non-dimensional specific heat, thermal conductivity, and shear modulus upon the homologous temperature. The values of material variables are scaled so as to equal 1.0 at the room temperature.

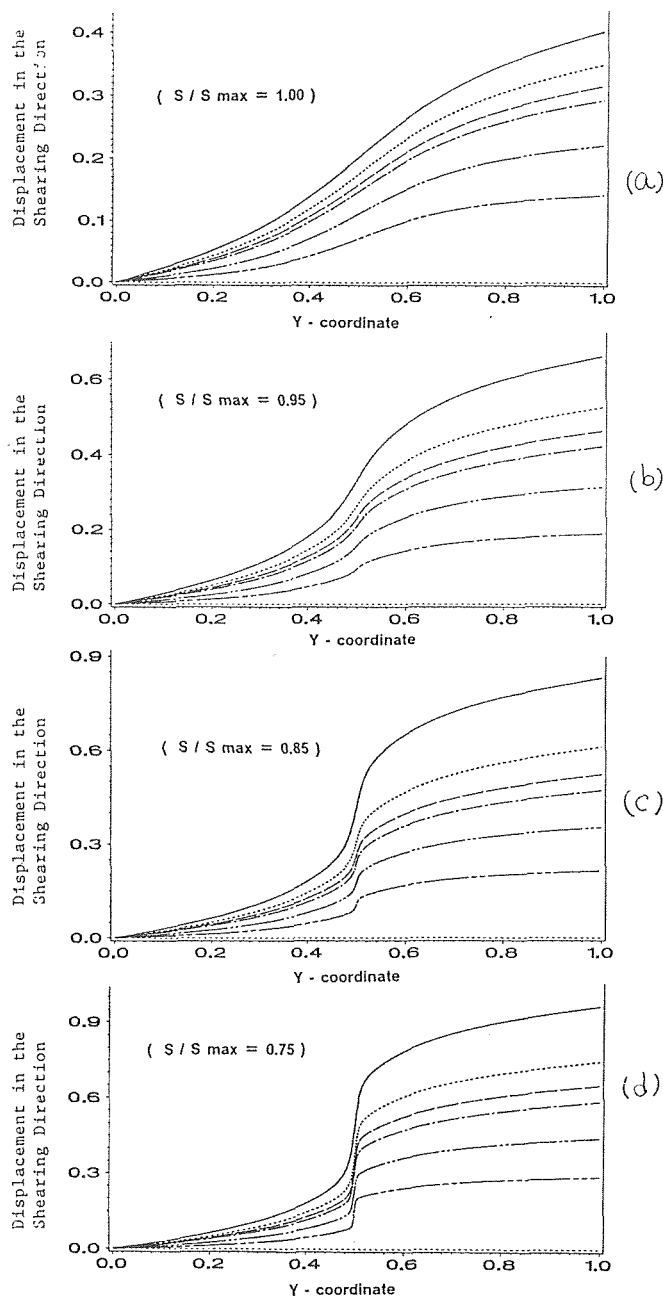


FIG 9. Deformed positions of an initially straight line at  $s_a/s_{\max}$  = 1.0, 0.95, 0.85, and 0.75. —  $\theta_0 = 83^\circ\text{K}$ , .....  $\theta_0 = 200^\circ\text{K}$ , ----  $\theta_0 = 300^\circ\text{K}$ , - · - ·  $\theta_0 = 343^\circ\text{K}$ , - - - -  $\theta_0 = 407^\circ\text{K}$ , and - · - ·  $\theta_0 = 523^\circ\text{K}$ .

size affects not only the nominal strain when a shear band initiates, but also the growth of the shear band. The homologous temperature, defined as the ratio of the absolute temperature of a material point to the melting temperature of the material, versus  $\log \delta$  plots in Figs 12a and 12b reveal that the homologous temperature is essentially independent of the defect size at  $s_a/s_{\max} = 1.0$  and 0.85. Note that the vertical scales in these figures are such that small differences in the values of the homologous temperatures are greatly exaggerated.

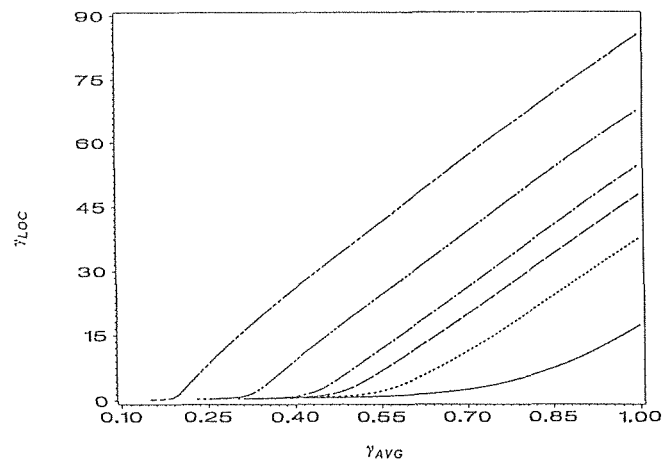


FIG 10. Evolution of plastic strain within the band for different values of the initial temperature. See Fig 9 for legends to curves.

### 3.7 Effect of inertia forces

Figure 13 exhibits the evolution of the plastic strain-rate at the specimen center with and without the consideration of inertia forces. In order to disregard the effect of inertia forces, the value of  $\alpha$  in Eq (1) was reduced by a factor of  $10^5$ . It is clear that the consideration of inertia forces delays the initiation of the shear band when the material is modeled by the Bodner-Partom flow rule. Similar results were obtained for the other four flow rules stated in Sec 3.1. Batra (1988) studied the shear banding problem for  $\dot{\gamma}_0 = 50 \text{ s}^{-1}$ ,  $500 \text{ s}^{-1}$ ,  $5,000 \text{ s}^{-1}$ , and  $50,000 \text{ s}^{-1}$ , and concluded that inertia forces start playing a significant role at  $\dot{\gamma}_0 = 5,000 \text{ s}^{-1}$ . Since only discrete values of  $\dot{\gamma}_0$  were considered, this conclusion is tenuous. For a quasistatic problem, no unloading wave will emanate outwards from the severely deformed region.

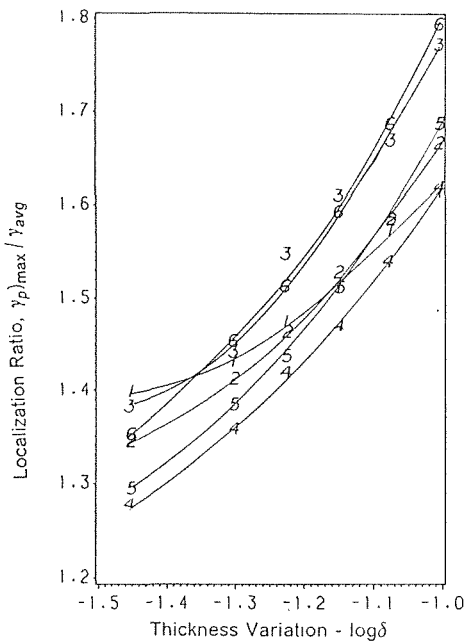
### 3.8 Effect of mass matrix

In the aforesaid results, we used a consistent mass matrix in the discrete formulation of the problem. Figure 14 shows the effect of different mass matrices on the evolution of the plastic strain-rate at the specimen center. For beam bending problems, the higher-order (average) mass matrix accelerates the rate of convergence of the solution. This can not be verified for the present problem since the analytical solution is not known.

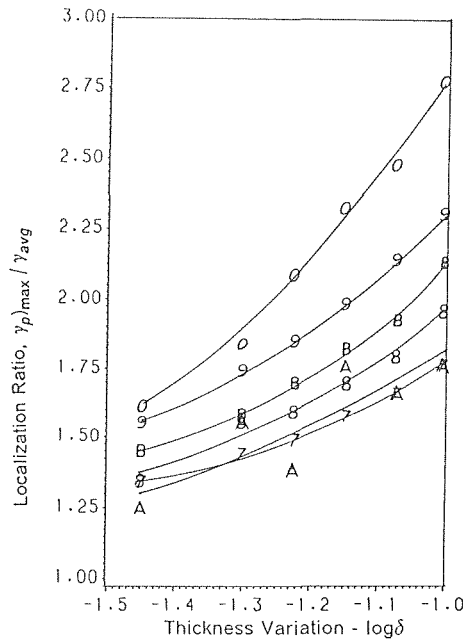
### 3.9 Shear bands in a bimetallic body

Batra and Kwon (1989) studied the phenomenon of shear banding in a bimetallic body. The domains  $[-1,0)$  and  $(0,1]$  were assumed to be made of different materials and a temperature perturbation symmetric about  $y = 0$  was considered. For a fixed set of material properties, the effect of the applied average strain-rate; and for a given prescribed strain-rate, the effect of varying the shear modulus, thermal conductivity, and the coefficient of thermal softening of one material relative to the other were examined. It was found that a shear band formed in the material that softened more readily.

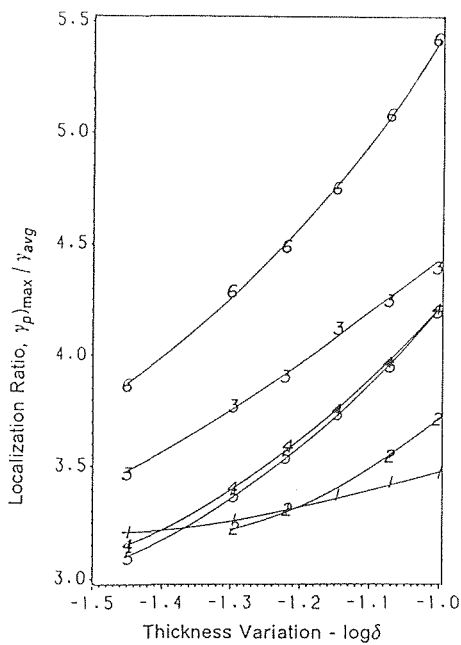




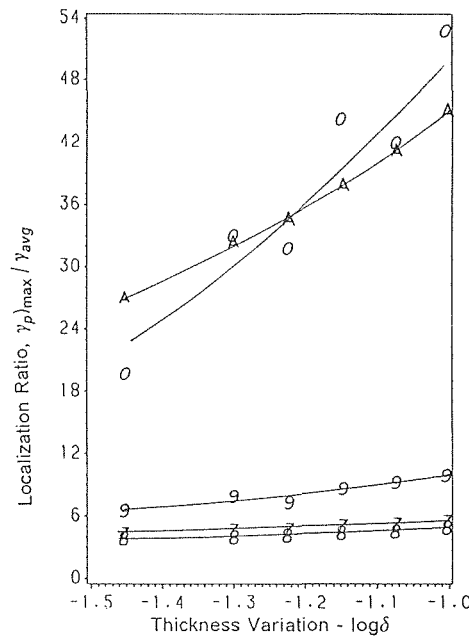
When Shear Stress reaches Maximum.



When Shear Stress reaches Maximum.



When Shear Stress drops to 85 % of Max. S.



When Shear Stress drops to 85 % of Max. S.

(a)

(b)

FIG 11. Dependence of the localization ratio upon the defect size for (a)  $s_a/s_{max} = 1.0$  and (b)  $s_a/s_{max} = 0.85$ . (1 - Copper, 2 - Cartridge brass, 3 - Nickel 200, 4 - Armco IF iron, 5 - Carpenter electric iron, 6 - 1006 steel, 7 - 2024 aluminum, 8 - 7039 aluminum, 9 - low alloy steel, O - S-7 tool steel, A - Tungsten alloy, B - Depleted Uranium).

#### 4. CONCLUSIONS

We have summarized some of our results on the initiation and growth of shear bands in a thermally softening elastic-viscoplastic body undergoing overall simple shearing deformations. It is unrealistic to expect a good agreement between the computed results and the experimental findings since we have studied a one-dimensional problem and most test configurations require analysis of either two- or three-dimensional problems. One of the unresolved issues is finding a suitable constitutive model valid for high temperatures and strain-rates found in a shear band.

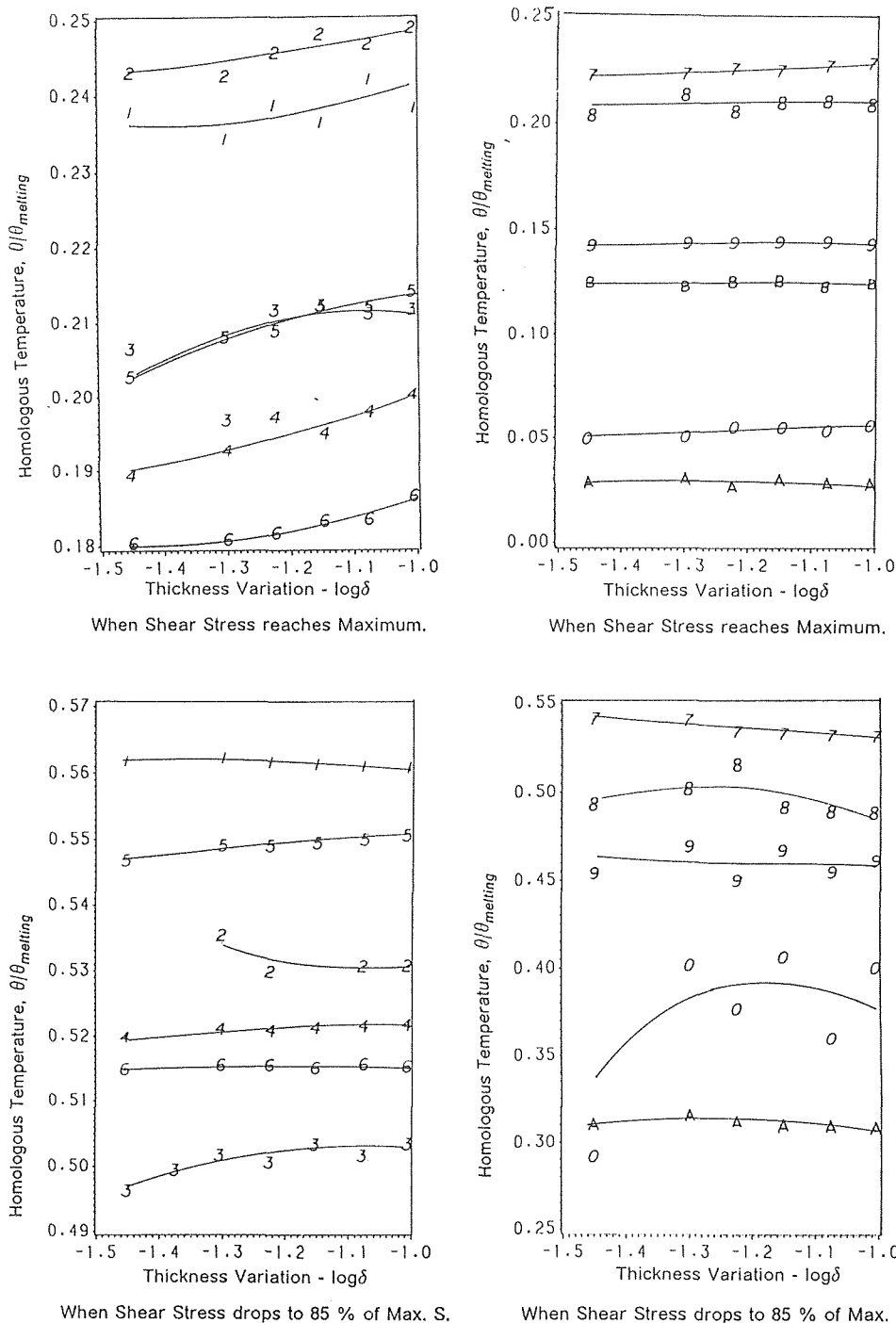
#### ACKNOWLEDGEMENTS

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(a)

(b)

FIG 12. Dependence of the homologous temperature at the band center upon the defect size for (a)  $s_a/s_{max} = 1.0$  and (b)  $s_a/s_{max} = 0.85$ . (See Fig 11 caption for explanations.)

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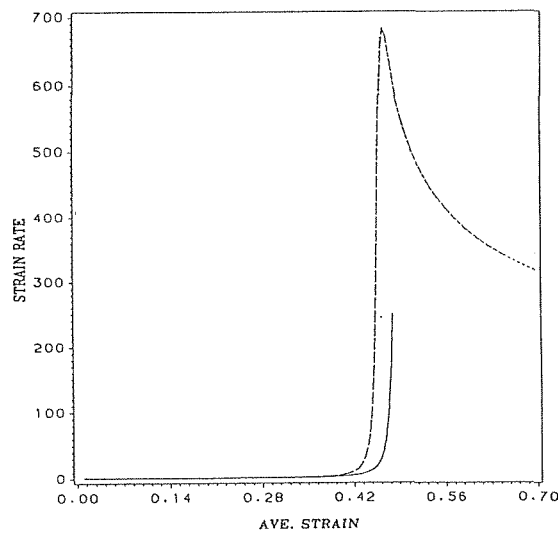


FIG 13. Evolution of the plastic strain-rate at the specimen center for the Bodner-Partom flow rule; — with inertia forces, ---- without inertia forces.

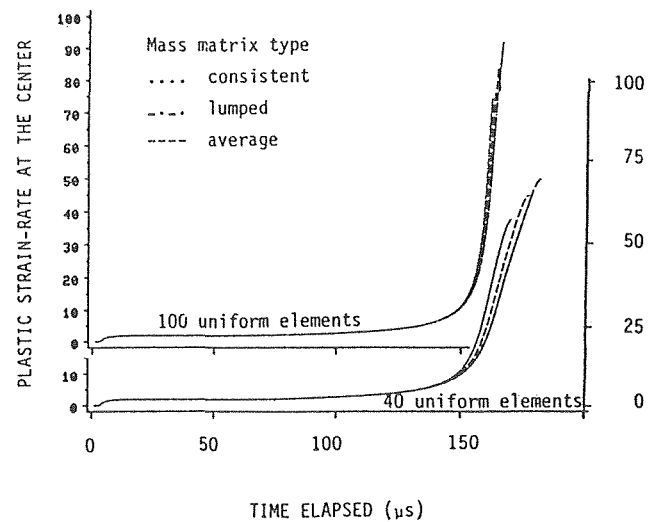


FIG 14. Effect of different mass matrices on the evolution of the plastic strain-rate at the specimen center ( $\dot{\gamma}_0 = 5,000 \text{ sec}^{-1}$ ).

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