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Computer Simulation Of Three-dimensional Mechanical Assemblies: Part I - General Formulation

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COMPUTER SIMULATION OF THREE-DIMENSIONAL MECHANICAL ASSEMBLIES: PART I — GENERAL FORMULATION

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ABSTRACT

In Part I of this paper, a dynamic modeling system for the simulation of three dimensional mechanical assemblies is presented. With this simulation tool, a designer can interactively create an assembly of mechanical components ready for dynamic analysis. The modeling system presented in this paper includes the derivation of the equations of motion of spatial multi-body systems, and the formulation of the equations to model the associated collision detection and collision responses. Part II of this paper is to introduce the geometry modeling and computer simulation of 3D systems.

INTRODUCTION

In engineering applications, there is often a need to examine an assembly of components rather than the individual component itself. Conventionally, if a designer wants to inspect the dynamic or kinematic performance of an assembly, he or she has to generate a model of assembly for the analysis from the assembly drawing. The mating condition for the kinematic and dynamic analysis will have to be specified as idealized joints, such as revolute, prismatic, spherical, gear joint, etc. Although there are good reasons for using these standard joints, the following problems may occur when joints are assumed to be perfect:

1. Accuracy analysis is difficult to perform, due to the existing clearances between two mating parts;

2. Dynamic analysis of a mechanical assembly can be unrealistic because the parts may incur some geometric defects;

3. If the mating condition is far beyond the traditional joint category, a designer will have a difficult time in performing kinematic and dynamic analysis. This will most likely force the designer to follow the traditional design pattern and limit his or her design creativity;

4. One may be able to use a solid modeling system to model the individual components, and manually assemble them. However, essential questions such as "are all the components tightly fitted to each other?", or "is it easy to assemble these components?", are hard to answer.

Presented in this paper is an advanced modeling approach for general mechanical assemblies. A designer can interactively create an assembly of components in a simulated physical environment, and each component will respond according to physical laws. Therefore, without specifying the mating conditions between the individual components, this approach can automatically animate the kinematic and dynamic response of the system.

PREVIOUS RESEARCH

The literature related to this research is summaried as follows:

Animation of Mechanical Assemblies

To animate the dynamic and kinematic analysis of mechanical assemblies, most of the contemporary solid geometric modeling systems would allow a user to define the geometry of each component using solids, however, the user would also have to manually simulate the motion of a mechanical assembly. Tilove [1983] presented a general method to allow coordinate systems to be rigidly attached to solids so that each component can properly be "hooked together" as its free parameters are varied. Revolute and prismatic joints were discussed in the paper. Lieberman and Wesley [1977] described a geometric modeling system, AUTOPASS, that generated a database in which components and

assemblies are represented by nodes in a graph structure. Eastman [1981] created a graph called the "location graph" which stores the relative location between each component in a hierarchical manner. Kim and Lee [1989] developed an assembly modeling system so that the designer can create an assembly by providing only the joint mating conditions as a spatial relation between components, and information for dynamic or kinematic analysis is derived automatically. Although the foregoing investigations have been very interesting and can solve certain problems, their usage has also been limited by employing the perfect joints.

Configuration Changing Modeling

Mechanical systems can be treated as subsystems which are coupled together through constraints. When subsystems change, the configuration of the entire system changes. Cipra and Uicker [1981] applied this approach to provide an accurate and efficient simulation and analysis of complex nonlinear systems. This dynamic analysis simulation strategies can be classified as application specific and general approaches. The application-specific approaches typically deal with systems which have small relative distances between the bodies[Dubowsky and Freudenstein, 1971; Wang and Lee, 1983]. However, the applications of these approaches are somewhat limited to joint modeling. General approaches intend to handle systems with large relative displacement require the user to indicate when and where the contacts between bodies occur and the type of resulting kinematic constraint [Winfrey et al. 1973; Wehage and Haug 1982]. Recently, a strategy to simulate the dynamics of planar mechanical systems was presented by Gilmore and Cipra [1991]. This approach is able to handle systems with unpredictable changes in the kinematic constraints, large relative displacement, and unpredictable configurations.

Collision Detecting and Response in Computer Graphics

If general mating conditions between components are to be modeled, then the fundamental force analysis of rigid bodies will need to be included during computer simulation. Initial work in computer animation for controlling object motion has been done by several researchers [Steketee and Badler, 1985; Sturman, 1987; Reynolds, 1982; Armstrong and Green, 1985; Isaacs and Cohen, 1987; Lundin, 1984; Terzopoulous, et al., 1987; Wilhelms, 1987; Moore and Wilhelms, 1988]. For the current development in the area of collision detecting and response of objects by computer simulation, Moore and Wilhelms [1988] described two collision detection algorithms and two collision response modeling methods. In the collision response modeling methods, linear momentum and angular momentum, as well as surface friction are modeled. One method is based on temporary springs introduced at collision points, and the other method is an analytical linear system solution. Although the collision detection model has been very expensive, among different approaches, simple, robust, and inexpensive are the goals for the animators.

FORCE CLOSURE FORMULATION

This modeling of three-dimensional muti-body systems is developed based on force closed kinematic joint[Reuleaux, 1876] which maintains the kinematic constraint between two bodies by the use of closing force. This is in contrast to a form closed kinematic joint which is constructed such that the kinematic constraint is physically provided by the bodies. The closing force in the force closed joint acts in the same direction as the reaction force would in an equivalent form closed joint. Figure 1 shows the spatial kinematic joints of various degrees of freedom in their form closed and force closed versions. The approach presented here is able to automatically determine if a joint breaks due to an unforeseen disturbance and does not require the user to indicate the possible changes in topology.

From a kinematic viewpoint, if a point on a rigid body is constrained to remain in contact with a surface of another rigid body, the point is restricted to move in the direction tangent to the surface. Hence, a degree of freedom is lost. In addition, a reaction force exists at each point contact. By considering the kinematic constraints in this manner, it becomes possible to consider a kinematic joint, which may change its degrees of freedom at any time depending on its contact force condition.

Equivalent Points Contacts for Three-Dimensional Bodies

The location of a constrained point on a surface is significant in allowing the proper motion and in modeling the system for a changing topology. Considering the spatial motion, each body has several boundary surfaces (faces), each face has several boundary lines (edges), and each edge has two endpoints (vertices). The vertices of one body may come into contact with a face of another body thus establishing a point to surface contact. When an edge comes into contact with a face, the bodies are in line contact. Line contact is kinematically equivalent to two point to surface contact. In a similar way, a surface to surface contact can be substituted by three point to surface contact.

The point to surface contact and force closure concepts lend a great deal of flexibility to the method. They allow a modelled system to change their topology by changing the number of point to surface contacts. The point to surface kinematic constraint method simply adds or deletes point to surface kinematic constraint equations, eliminating the need to determine the type of joint between the bodies and to maintain a joint constraint library.

Point to Surface Contraint Equation

A general equation for point to surface kinematic constraint as shown in Figure 2 is derived in this section. The reaction force F between two bodies results in a force closed kinematic constraint at point i_a. The (x, y, z) coordinate system is the global coordinate system while the local coordinate system (ξ, η, ζ) is fixed to each contact body. Point i_a on body j is instantaneously in contact with one surface of the body i. While contact is

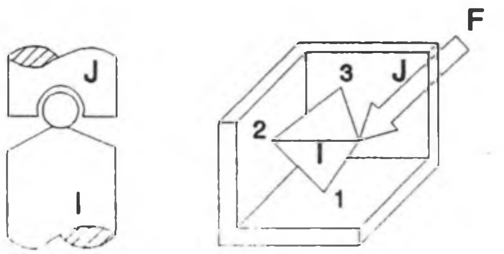

FIGURE 1.A EQUIVALENT POINT CONSTRAINT OF SPHERE JOINT

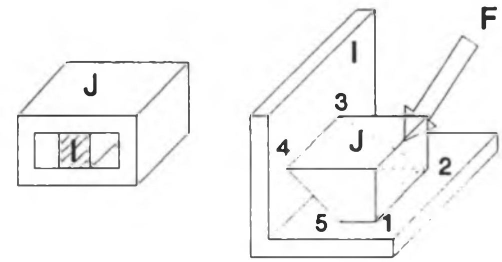

FIGURE 1.C EQUIVALENT POINT CONSTRAINT OF SLIDER JOINT

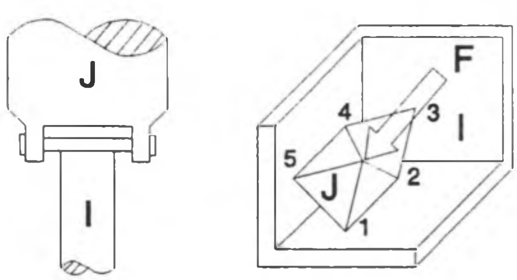

FIGURE 1.B EQUIVALENT POINT CONSTRAINT OF REVOLUTE JOINT

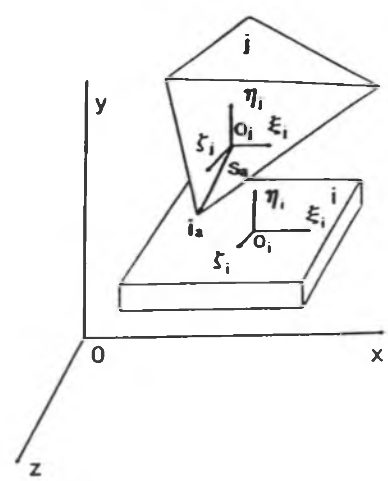

FIGURE 2 POINT TO SURFACE CONSTRAINT

maintained, point i_a is constrained in the surface, such that it does not separate from body i but is able to slide along the surface or rotate with respect to i_a. Thus the two-body system has five relative degrees of freedom. This point to surface constraint can be expressed as

$$\phi(q,t)=Ax_a+By_a+Cz_a+D=0 \tag{1}$$

where A, B, C, and D are constants. (x_a, y_a, z_a) is the coordinate of the point i_a with respect to the global coordinate system, and q is the generalized coordinate:

$$q_i=(x_i,y_i,z_i,e_{0i},e_{1i},e_{2i},e_{3i})^t \tag{2}$$

where (x_i, y_i, z_i) is the coordinate of o_i in the xyz coordinate system and $(e_{0i}, e_{1i}, e_{2i}, e_{3i})$ are Euler parameters [Wittenburg, 1977]. The four Euler parameters are required to satisfy the following equation

$$e_{0i}^2+e_{1i}^2+e_{2i}^2+e_{3i}^2=1 \tag{3}$$

As shown in Figure 3, a local coordinate system (ξ_i, η_i, ζ_i) fixed to a rigid body is rotating about a unit vector u_i. The parameters e_{1i}, e_{2i}, and e_{3i} are the projections of a vector e_i (along the unit vector u_i) on the three axes of the fixed coordinate system, and

vector e_i is defined by

$$\bar{e}_i=\bar{u}_i \sin\frac{\theta_i}{2} \tag{4}$$

where θ_i is the angle of rotation. The fourth parameter e_{0i} is given by

$$e_{0i}=\cos\frac{\theta_i}{2} \tag{5}$$

The advantage of using Euler parameters, in contrast to Euler angles and Bryant angles [Wittenburg, 1977], is due to the fact that there is no singular case in calculating the key parameters to define a spatial rotation.

Thus from Figure 2, one can obtain

$$(x_a, y_a, z_a)^t = (x_j, y_j, z_j)^t + A^j \bar{S}^j_a \tag{6}$$

where (x_j, y_j, z_j) is the global coordinate of the mass center of the rigid body j, and

$$\bar{S}^j_a = (\xi^j_a + \eta^j_a + \zeta^j_a)^t \tag{7}$$

is the coordinate of the point i_a with respect to body fixed coordinate system, and A^j is the Euler transformation matrix of body j

$$A^j=2\begin{pmatrix} e_0^2+e_1^2-\frac{1}{2} & e_1e_2-e_0e_3 & e_1e_3+e_0e_2 \\ e_1e_2+e_0e_3 & e_0^2+e_2^2-\frac{1}{2} & e_2e_3-e_0e_1 \\ e_1e_3-e_0e_2 & e_2e_3+e_0e_1 & e_0^2+e_3^2-\frac{1}{2} \end{pmatrix} \tag{8}$$

Equations of Motion

In either spatial or planer system dynamics, a set of generalized coordinates is defined as

$$q=(q_1,q_2,\dots,q_n)^t \tag{9}$$

where the vector q_i has been defined in equation (2).

The kinetic energy of the multi-body system can be written in the form

$$T=\frac{1}{2}\dot{q}^t M(q)\dot{q} \tag{10}$$

where

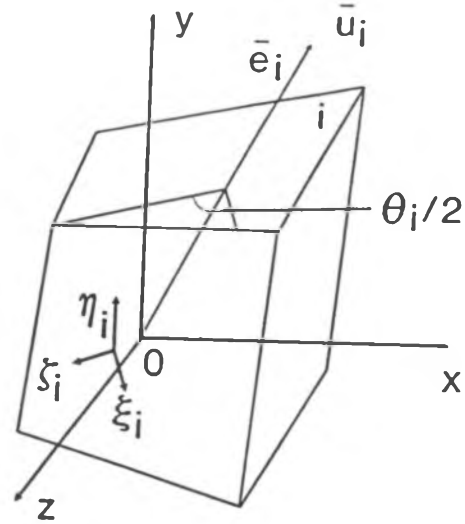

FIGURE 3 ORIENTATION OF THE BODY USING EULER PAREMETERS

$$\dot{q}=\frac{dq}{dt} \tag{11}$$

and dq/dt is the time derivative of the generalized coordinate vector q.

The effect of forces and torques that act on the system from external source is defined by calculating the virtual work done by these forces and torques as the system of bodies undergoes an infinitesimal virtual displacement δq i.e.,

$$\delta w^e=(Q^e)^t\delta q \tag{12}$$

where Q^e is the vector of generalized forces.

The generalized coordinate vector q defines the position and orientation of each body in the system respect to the global reference frame. In the simulation system of a multi-body system, the bodies are connected by physical joints or force closed kinematic constraints. Such kinematic constraints are defined by a set of algebraic constraint equations of the form

$$\Phi(q,t)=(\Phi_1(q,t),\Phi_2(q,t),\dots,\Phi_n(q,t))^t=0 \tag{13}$$

It is assumed that each component $\phi_i(q,t)$ in the vector constraint equation has two continuous derivatives with respect to

its arguments.

Equation (13) is differentiated with respect to time to yield the velocity constraint

$$\Phi_q \dot{q} = -\Phi_t \qquad (14)$$

where subscript q denotes partial differentiation: i.e.,

$$\Phi_q = \frac{\partial \Phi_i}{\partial q_j} \qquad j=1,\ldots,n \qquad (15)$$

The matrix Φ_q is called the constraint Jacobian and plays a critical role in analysis of the system. One may similarly differentiate equation (14) to obtain the acceleration constraint equation

$$\Phi_q \ddot{q} = -[(\Phi_q \dot{q})_q \dot{q} + 2\,\Phi_{tq} + \Phi_{tt}] \qquad (16)$$

For convenience, the following equation has been defined

$$r(q) = -[(\Phi_q \dot{q})_q \dot{q} + 2\,\Phi_{tq} + \Phi_{tt}] \qquad (17)$$

Equations (13)-(17) comprise the set of kinematic constraint equations for the simulation of a 3D multi-body system.

The equations of motion of the system can be written using Lagrangian formulation, with Lagrangian multipliers λ to account for constraint force in the form

$$M\ddot{q} + \Phi_q^t \lambda = Q^* + Q^e \qquad (18)$$

where

$$\ddot{q} = \frac{dq^2}{dt^2} \qquad (19)$$

and Q^* is a vector of quadratic terms in velocity which is a function of the generalized coordinates (in planar system, Q^* is zero). Equations (13) and (18) comprise a mixed system of differential and algebraic equations that govern the dynamics of the system. Combining equations (16) and (18), a set of matrix equations are obtained. It determines acceleration and Lagrane multipliers.

$$\begin{pmatrix} M & \Phi_q^t \\ \Phi_q & 0 \end{pmatrix} \begin{pmatrix} \ddot{q} \\ \lambda \end{pmatrix} = \begin{pmatrix} Q^* + Q^e \\ r \end{pmatrix} \qquad (20)$$

In the above analysis the effects of friction, damper, and spring force have not yet been considered. In order to obtain more general equations for the dynamic analysis, more terms need to be considered.

The Lagrangian multipliers in the equation (20) represent the reaction forces of the system's force closed point-to-surface kinematic constraint. Specifically, the normal force F_n of constraint between bodies that slide relative to one another is given by the equation

$$F_n = \Phi_q^t \lambda \qquad (21)$$

If sliding occurs, the friction force determined by Coulomb's law is

$$F_f = f_c(q,dq/dt)\, \Phi_q^t \lambda \qquad (22)$$

where $f_c(q,dq/dt)$ is the coefficient of friction that determines the magnitude and the direction of the friction force. In many applications, the coefficient of friction is constant and the algebraic sign of the relative velocity of the slide between bodies in the contact point determines the direction of the friction force.

The generalized force of sliding friction is calculated for each constraint, using the principle of virtual work and equation (22). Thus the following equation can be obtained:

$$Q^f = f_c(q,dq/dt)\, B(q)\, \Phi_q^t \lambda \qquad (23)$$

where B(q) depends on the coordinate system used[Haug *et al.*, 1986]. This generalized friction force can be included in the external force term of the equations of motion. If one considers the damping force and spring force at the same time, an expression of the form

$$\begin{pmatrix} M & \Phi_q^t \\ \Phi_q & 0 \end{pmatrix} \begin{pmatrix} \ddot{q} \\ \lambda \end{pmatrix} + \begin{pmatrix} c & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{q} \\ \lambda \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} q \\ \lambda \end{pmatrix} = \begin{pmatrix} Q^* + Q^e + fB\,\Phi_q^t \lambda \\ r \end{pmatrix} \qquad (24)$$

can be obtained which is the general equation governing the whole system during the motion.

Note that the Lagrangian multipliers λ appears in equation (24) on both the left and right hand sides, that indicates that the friction force influences the reaction force in other joints of the system.

CHANGING TOPOLOGY

The topology of the system may change during motion at any time. To model such phenominon, one must first detect the change and then add or delete the corresponding constraint equations from the system's equations of motion. The mathematical modeling of changing topology is summarized below:

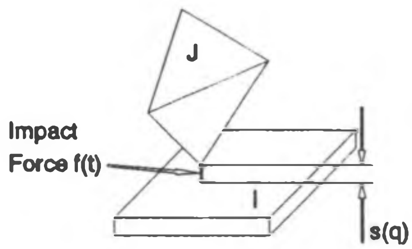

FIGURE 4 COLLISION RESPONSE

Topology Changing Detection

When two or more objects attempt to interpenetrate each other, it is called a collide/contact. If two bodies collide and remain in contact, then one or more point-to-surface constraints are formed. However, an impact in the system may cause bodies to separate due to the resulting impulsive motion. Two other reasons exist for a constraint breaking: when the contact point between two bodies ceases to be in compression, or when a point in contact slides off the surface.

For the constraint breaking due to insufficient closing force, the reaction force of each joint needs to be monitored during the motion. When the Lagrangian multipliers λ_i in equation (24) become zero or change signs, the corresponding constraint is broken.

For the formation of the new constraints and constraints breaking due to sliding off with respect to each other, the collision detection techniques and impact analysis techniques are required.

Collision Response

Consider the case in which an impulsive force occurs between a pair of bodies. The impulsive force acts over a very short period of time. Since the contact point is known, the distance s(q(t)) can be analytically determined as shown in Figure 4. The virtual work of the impact force f(t) is defined as

$$\delta w^I = f\delta s = f\frac{\partial s}{\partial q}\delta q \tag{25}$$

The generalized impact force Q^I is

$$Q^I = f\frac{\partial s}{\partial q} \tag{26}$$

Consider the case in which the impulsive force Q^I is applied to the system. This will yield an equation of motion as

$$M\ddot{q} + \Phi_q^t\lambda = Q^* + Q^e + Q^I \tag{27}$$

Since the impact time period is very short, the friction and other small forces can be neglected. Integrating the equation (27) from t_1 to t_2, the following equation can be obtained:

$$\int_{t_1}^{t_2} M\ddot{q}dt + \int_{t_1}^{t_2}\Phi_q^t\lambda dt = \int_{t_1}^{t_2}(Q^*+Q^e)dt + \int_{t_1}^{t_2}Q^I dt \tag{28}$$

Taking the limit (t_2-t_1) to zero, one can obtain

$$\lim_{t_1-t_2\to 0}\int_{t_1}^{t_2} M\ddot{q}dt = M(t_1)[\dot{q}(t_1+)-\dot{q}(t_1-)] = M\Delta\dot{q}(t_1) \tag{29}$$

where $\Delta q(t_1)$ is the jump discontinuity in velocity across the impact instance, and

$$\lim_{t_2-t_1\to 0}\int_{t_1}^{t_2}\Phi_q^t\lambda\, dt = \Phi_q^t(t_1)P^\lambda \tag{30}$$

where

$$P^\lambda = \lim_{t_2-t_1\to 0}\int_{t_1}^{t_2}\lambda\, dt \tag{31}$$

Since Q^* and Q^e depend only on generalized coordinates and velocities, they are bounded functions of time. Thus, as the time interval approaches zero, the following equation can be obtained

$$\lim_{t_2-t_1\to 0}\int_{t_1}^{t_2}(Q^*+Q^e)dt = 0 \tag{32}$$

and

$$P^I = \int_{t_1}^{t_2} Q^I dt \tag{33}$$

Substituting the results of equations (28) to (33) into equation (27), one obtains

$$M\Delta\dot{q} + \Phi_q^t P^\lambda = P^I \tag{34}$$

In order to solve the equation, the velocity constraint equation (14) is required. One can evaluate equation (14) at t_2 and t_1 to obtain

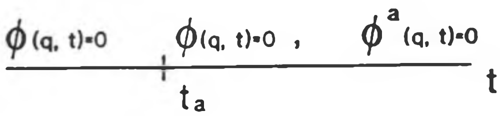

FIGURE 5 CONSTRAINT ADDITION

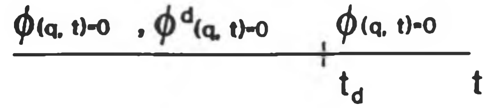

FIGURE 6 CONSTRAINT DELETION

$$\lim_{t_2-t_1\to 0}(\Phi_q(t_2)\dot{q}(t_2)-\Phi_q(t_1)\dot{q}(t_1)) = -\lim_{t_2-t_1\to 0}(\Phi_t(t_2)-\Phi_t(t_1)) \tag{35}$$

Since ϕ_q and ϕ_t are continuous, from equation (35) one obtains

$$\Phi_q \Delta\dot{q} = 0 \tag{36}$$

Combining equations (26) and (33), one obtains

$$P^I = \int_{t_1}^{t_2} Q^I dt = \left(\frac{\partial s}{\partial q}\right)^t P \tag{37}$$

where

$$P = \lim_{t_2-t_1\to 0}\int_{t_1}^{t_2} f(t)\, dt \tag{38}$$

The coefficient of restitution e is defined by

$$\dot{s}(t_1+) = -e\,\dot{s}(t_1-) \tag{39}$$

where

$$\dot{s} = \frac{\partial s}{\partial q}\dot{q} \tag{40}$$

which represents the relative velocity of two impacting bodies. Substituting equation (40) into equation (39), one obtains

$$\frac{\partial s}{\partial q}\dot{q}(t_1+) = -e\frac{\partial s}{\partial q}\dot{q}(t_1-) \tag{41}$$

and

$$\frac{\partial s}{\partial q}\Delta\dot{q} = -(1+e)\frac{\partial s}{\partial q}\dot{q}(t_1-) \tag{42}$$

Combining equations (34), (36), (37), and (42), we obtain the matrix equation as

$$\begin{pmatrix} M & \Phi_q^t & (\frac{\partial s}{\partial q})^t \\ \Phi_q & 0 & 0 \\ \frac{\partial s}{\partial q} & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta\dot{q} \\ P^\lambda \\ -P \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -(1+e)\frac{\partial s}{\partial q}\dot{q}(t_1-) \end{pmatrix} \tag{43}$$

Solving this equation for Δq and using

$$\dot{q}(t_1+) = \dot{q}(t_1-) + \Delta\dot{q} \tag{44}$$

the post impact velocity can be obtained. If this post impact velocity violates velocity constraint equation (14), the corresponding kinematic constraint equation should be removed from the system equations of motion. The violation of the velocity constraint also indicates that the point of a point-to-surface constraint has sufficient velocity to move away from the contacted surface so that the newly formed kinematic constraint will be destroyed. however, there is a potential problem on how to decide whether a point has sufficient velocity to leave the surface. If the relative velocity is approaching zero, the kinematic constraint should be imposed; otherwise the constraint will be destroyed. An easier way to deal with this problem is to estimate the time Δt for that specific point to move away from the boundary surface a specified distance Δs. If the estimating time is below a specified value, then the point has sufficient velocity to move away and is not physically constrained by the surface, thus the corresponding constraint equation will not be added to the equations of motion.

Constraint addition and deletion

Consider the motion of the system that is subjected to constraint in equation (1) prior to time t_a, at which an additional constraint

$$\Phi^a(q,t)=0 \tag{45}$$

is added. The time sequence diagram in Figure 5 shows constraint equations that are active before and after the event

time t_a.

The constraint in equation (45) is active throughout a time interval $t > t_a$. Thus the time derivative of the constraint equation must also be zero, leading to a pair of constraint equations

$$\phi^a(q,t)=0 \qquad t > t_a \quad (46)$$

$$\frac{\partial\phi^a(q,t)}{\partial t} = \phi^a_q \dot{q} + \phi^a_t = 0 \qquad t > t_a (47)$$

Equation (46) is a position constraint and equation (47) is a velocity constraint. If only equation (46) is satisfied, an impact occurs. Therefore, the criterion for determining the time t_a at which the constraint is added is equations (46) and (47).

Consider the next situation in which a constraint is deleted due to joint breakage or due to sliding off at time t_d. The time sequence diagram shown in Figure 6 illustrates a situation in which the constraint

$$\phi^d(q,t)=0 \qquad (48)$$

is deleted at time t_d, leaving the basic kinematic constraint equation (1). To implement these processes in computer, a phase parameter can be set to every potential point and face of each body. When the point comes into contact with a surface of another body, the corresponding phase parameter is set to 1 and the constraint equation will be added to the system's equations of motion. When these parameters become zero, the corresponding constraint equation will be deleted from the system's equation. The method is very flexible to handle the change of the system's topology since the user is not required to predict the constraint changes and the resulting topology.

CONCLUSION

Using the force closed constraint concept and the formulations described above, a dynamic mechanical system with unpredicted or unforeseen kinematic constraint changes can be modelled. Dynamic mechanical systems whose bodies establish contact and separate are described by discontinuous equations of motion. Through the addition and deletion of the constraint equations, the modelling system can automatically reformulate the equations of motion. Thus, the constraint changes or resulting system topology can be taken care of automatically.

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