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A Mathematical Model Used In Conjunction With A Finite Element Analysis To Aid In The Design And Analysis Of Bourdon Tubes

R. C. Weber

Robert L. Davis
Missouri University of Science and Technology, rdavis@mst.edu

R. C. Weber
Asst. Professor,
Department of Mechanical Engineering,
Marquette University,
Milwaukee, Wisc. 53233

R. L. Davis
Dean,
College of Engineering,
University of Missouri-Rolla,
Rolla, Missouri

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A mathematical model that describes a variety of Bourdon tube geometries has been used for the three-dimensional analysis of Bourdon tubes. The variation of the stresses and tip deflections with respect to the variables used in the mathematical model are then presented as an aid in the design and analysis of the Bourdon tube. The finite element program represents a user-orientated package for calculation of tip deflections and stresses. Stresses and deflections are compared to both strain gage data and instrumented tip deflections as well as published data to establish the credibility of both the model and computer program.

Introduction

The Bourdon tube is one of the most widely used primary detectors for pressure. It is rather surprising that it is still being designed by cut-and-dry methods after more than 100 years of development [1]. Until recently, the majority of the efforts in Bourdon Tube Theory were for thin-walled tubes only. Tueda [2], Dressler [3], Clark and Reissner [4], Wolf [5], and Flachbarth [6] all attempted closed form analysis. Due to the complexity of the governing equations and boundary conditions, each eventually made assumptions or simplifications which resulted in an analysis of thin-walled Bourdon tubes. Weydert [7] and Wuest [8] presented thick-walled analysis, each obtained for a specific cross-section. Davis and Webster [9] presented a finite element analysis which compared very favorably with the experimental results obtained by Motherway [10]. However, there was not a sensitivity analysis of parameters presented to aid in the design process.

The mathematical model presented in this paper has no physical restrictions on shape, size, material, or most importantly, wall thickness. It is developed using five variables, which when varied can describe a wide variety of tube shapes and cross-sections. The variation of tip deflection and stresses with respect to each of the variables presents the designer with a tool which will aid his selection and design of thick-walled tubes for specified uses.

Mathematical Model

The cross-sectional area of the Bourdon tube was mathematically modeled using two pairs of circular arcs which are symmetrical with respect to the plane of symmetry (see Fig. 1). Each arc is developed by rotating a radius, R_i, (i denoting the meridian section or arc) through an angle $d\theta$. The focal point of each arc is located by a radius of rotation, a_i, originating at the axis of rotation. To guarantee both continuity at the intersections of the meridian sections and symmetry, the following equations are needed.

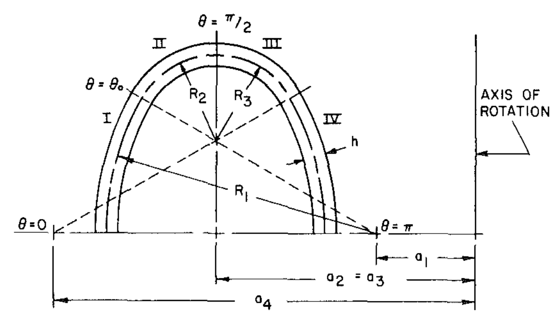

Fig. 1

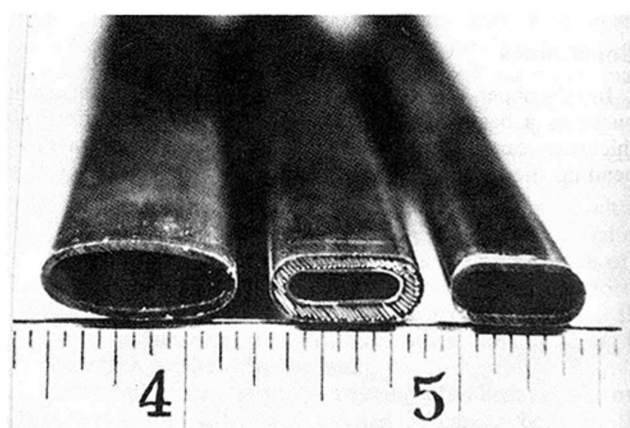

Fig. 2 Cross-sectional areas of various Bourdon tubes

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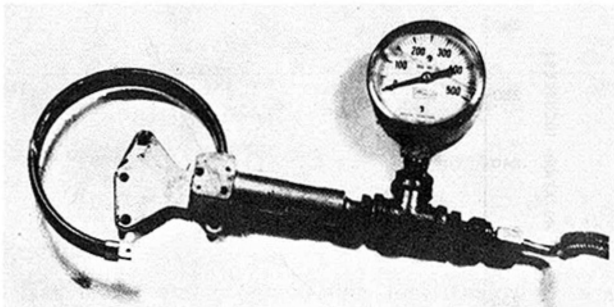

Fig. 3 Experimental apparatus with Bourdon tube attached

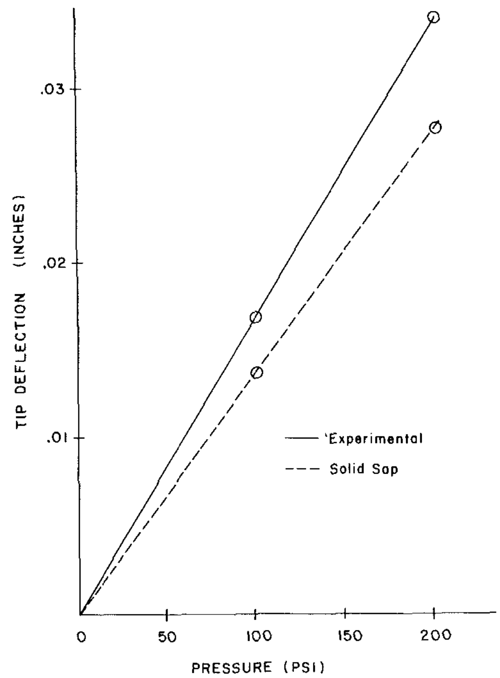

Fig. 4

$$a_1 = a_2 - (R_1 - R_2)\cos\theta_0 \tag{1}$$

$$a_3 = a_2 \tag{2}$$

$$a_4 = a_2 + (R_1 - R_2)\cos\theta_0 \tag{3}$$

$$R_3 = R_2 \quad (5)$$

$$R_4 = R_1 \quad (5)$$

The wall thickness, h, completes the description of the cross-sectional area. This area is now rotated through an angle Ψ_0, completing the description of the tube.

Two displacement boundary conditions were necessary.

$$u = v = w = 0 \qquad @\Psi = 0 \tag{6}$$

and

$$u = 0 \qquad @\Theta = 0, \pi \tag{7}$$

The first condition implies that the deflection at the point of attachment of the tube is zero. The second condition allows for only one-half of the tube to be modeled by creating the plane of symmetry.

Experimental Work

The experimental work was conducted using Bourdon tubes supplied by Precision Tube Company. These tubes were initially straight and of various cross-sections (see Fig. 2). The

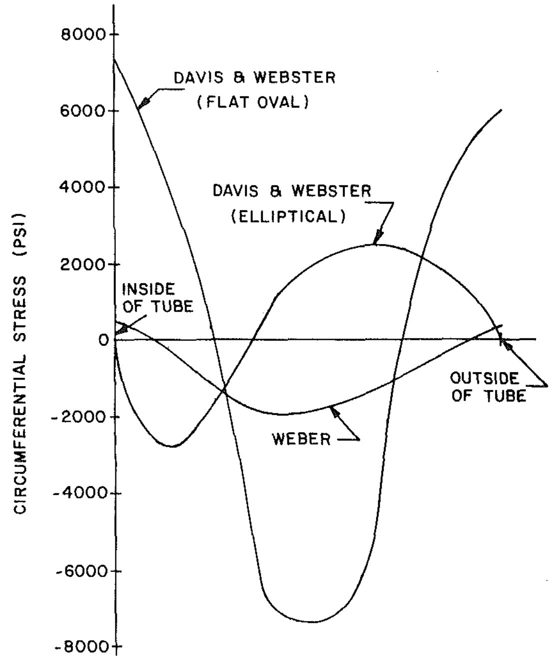

Fig. 5

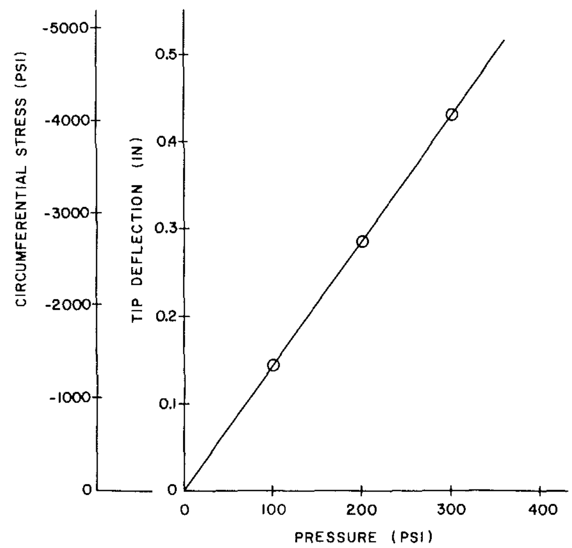

Fig. 6

variation in the cross-sectional areas should be noted here, especially the thick-walled or center tube, since the area generated by the program is perfectly symmetrical with uniform thickness. The tubes were bent to the desired shape and then stress relieved at 400°F for 2 hr and cooled at room temperature. The tubes were then attached to a mounting accessory (Fig. 3) and deflections measured using two mutually perpendicular dial gages.

Finite Element Program

The program used in the analysis of the Bourdon tubes was developed by Edward Wilson [11]. The program uses a 16-node thick-shell element which is isoparametric and has 48 degrees of freedom. The program is written in Fortran IV for use on an IBM 370/168 or similar computer.

The numerical data were obtained by using one tube as the base or control tube and then changing the appropriate variables in succeeding runs. Referring to Fig. 1, the independent variables that were altered were a_2 and h_1 along with Ψ_0 and P. For any given run, only one variable was

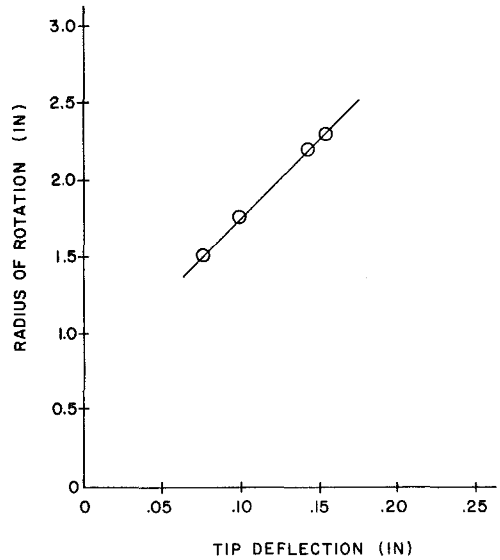

Fig. 7

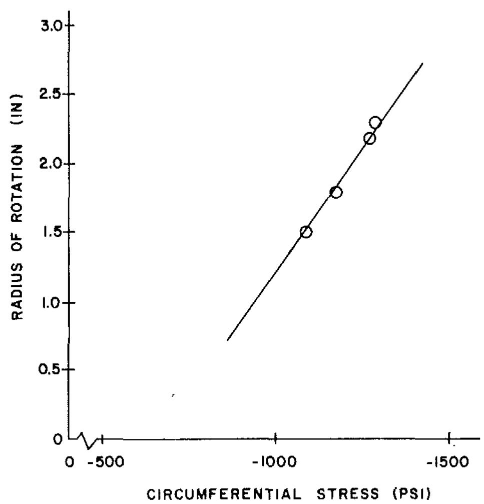

Fig. 8

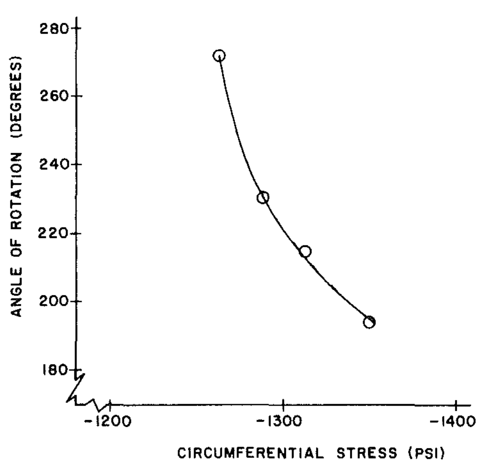

Fig. 9

altered from the control or base tube. By varying the magnitude that the variable altered, the effect the specified variable had on circumferential stress and tip deflection was *thus determined. The data for the base or control run were*

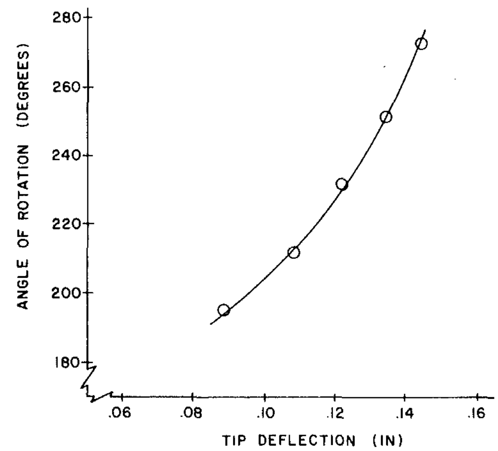

Fig. 10

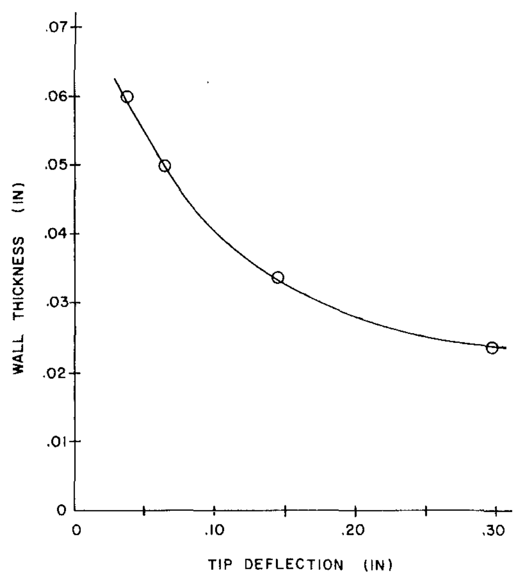

Fig. 11

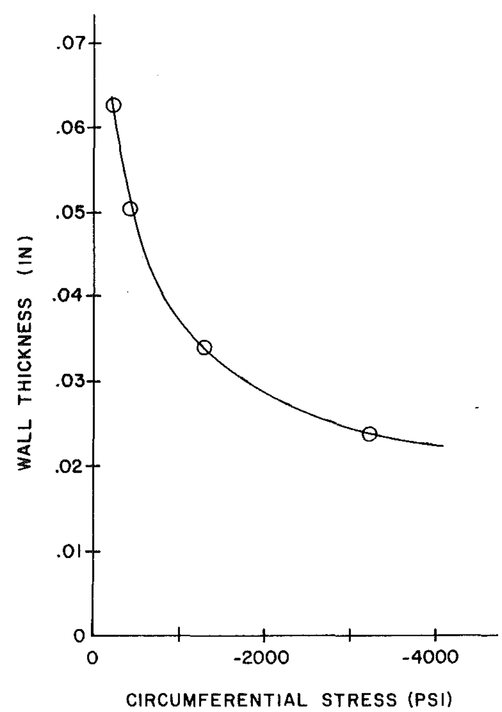

Fig. 12

Variables

$$P = 100 \text{ psi} \qquad h = 0.0340 \text{ in.}$$

$$a_2 = 1.1875 \text{ in.} \qquad \Psi_0 = 271 \text{ deg}$$

Run Constants

$$R_1 = 25.0 \text{ in.} \qquad R_2 = 0.1055 \text{ in.}$$

Results

The model and program were first checked with experimental deflections to verify their accuracy. Figure 4 shows the relationship between tip deflection and pressure. The comparison is considered acceptable when the variation in the actual tubes is taken into account (see Fig. 2). Figure 5 shows the variation of the circumferential stress across the cross-section for various tube shapes. The stress is measured at a cross-section sufficiently removed from the ends so that end conditions are eliminated. The Davis and Webster [9] flat oval accurately matched the data obtained by Motherway [10] for a similar tube. The tubes used in this experiment were similar to the Davis and Webster flat oval with the exception that the flat edges were approximated by circular arcs of relatively large radii. This allows for a smoother transition between arcs, resulting in a similar but reduced stress distribution. As the arcs are developed into an elliptical shape the stress distribution becomes drastically altered. There is a virtually unlimited number of ellipses that could be generated by the varying values of the major and minor diameters. However, the math model indicates the stress distribution should remain similar in shape although the magnitude of the stresses would vary considerably. Figure 5 shows the need for a simplistic method that can predict the behavior of the Bourdon tube. Since the cross-sectional area has a drastic effect on both stress distribution and tip deflection, the model, with its three independent model variables and internal pressure, thus represents a relatively simple, yet quite accurate, method of predicting Bourdon tube behavior.

Figures 6 through 12 represent a series of finite element runs for a specific tube. Because of the complexity of the Bourdon tube, it would be virtually impossible to present a series of figures that would encompass all possible cross-sections of Bourdon tubes. The importance of the following figures lies in the variations or shapes of the curves. Figure 6 shows that the circumferential stress and tip deflection both vary linearly with respect to internal pressure. Since the curve must also pass through the origin, a valuable tool has been presented to the designer. By obtaining one data point in conjunction with the origin, the tip deflection and circumferential stress can now be predicted for any working pressure.

Figures 7 and 8 show the variation of circumferential stress and tip deflection with respect to the radius of rotation (a_2). Both variations are linear, but since the origin has no significance, nothing can be said concerning the relative slopes. Figure 7 implies that the tip deflection will increase as the radius of rotation increases, all other variables remaining constant. Figure 8 shows the circumferential stress also increases proportionally to the increase of radius of rotation. This is in agreement with Fig. 7 in that the larger stresses occur when the deflections are the greatest.

Figures 9 and 10 are concerned with varying angle of rotations (Ψ_0). To increase the angle of rotation the length of the tube is simply increased. Thus a slight decrease in the circumferential stress should not be too surprising. Since the deflection is not measured a constant fixed distance from the end, the tip deflection is actually a combination of tube length and movement. Therefore, the tip deflection increases as the angle of rotation is increased, and in a nonlinear manner.

The final variable to be investigated is wall thickness. The tip deflection decreases as the wall thickness is increased (Fig. 11). This is as expected as is the nonlinear variation, since the shell equations become nonlinear when the wall thickness reaches a given value. The stress varies in a very similar manner (Fig. 12). Since thick-shell analysis has proven to be very cumbersome, if not virtually impossible when applied to the Bourdon tube, this is a very important concept to the designer.

Conclusions

This paper presents the results of a finite element program that was adopted for use in the analysis of Bourdon tubes. The sensitivity of the various parameters presents the designer with a valuable and very powerful tool. Since the stress and deflection are linearly proportional to the pressure, the maximum operating pressure can easily be determined. If an increase in deflection and stress is desired, the radius of rotation can be increased or the wall thickness decreased. However, an increased deflection with decreased stress can be obtained by increasing the angle of rotation.

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