01 Jan 1973

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Recommended Citation

C. Wang and T. L. Noack, "Decision-Directed Detector For Overlapping PCM/NRZ Signals," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES thru 9, no. 3, pp. 442 - 447, Institute of Electrical and Electronics Engineers, Jan 1973.
The definitive version is available at https://doi.org/10.1109/TAES.1973.309730

Decision-Directed Detector for Overlapping PCM/NRZ Signals

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Abstract

A decision-directed (DD) technique for the detection of overlapping PCM/NRZ signals in the presence of white Gaussian noise is investigated. The performance of the DD detector is represented by probability of error P_E versus input signal-to-noise ratio (SNR). To examine how much improvement in performance can be achieved with this technique, P_E's with and without DD feedback are evaluated in parallel. Further, analytical results are compared with those found by Monte Carlo simulations. The results are shown in good agreement.

Manuscript received August 3, 1972.

This work was supported by NASA Grant NGR-26-003-044.

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I. Introduction

A binary communication system can be described as transmitting and receiving a sequence of binary symbols of predetermined duration and form from one point of space to a second. In the transmission process, the duration and form of the symbols may be altered, some symbols may overlap with the others, and the symbol sequence may be perturbed by noise of various kinds. The receiver is designed to detect or recover the original binary data upon receiving the distorted, noise-perturbed signals. In this paper, we investigate the effect of using bit decision to direct the detection process, in order to obtain a less noisy measurement which, in turn, acts on the decision. The result of the above technique is the decision-directed (DD) detector. It is shown [1] that the DD detector is an essential part of solving the self-bit-synchronization problem for overlapping signals. Proakis et al. [2] studied the effect of the DD measurement by Monte Carlo simulation techniques for both orthogonal and anticorrelated signals. The results show the DD approach, in general, yields a lower probability of error than that of the non-DD method, at all SNR levels.

It is well known [3] - [5] that the optimum receiver in studying the adaptive equalizer is nonlinear and complicated in structure when there is a large amount of intersymbol interference. This leads to seeking suboptimum and practical receivers which give reasonable performance. Austin [6] studied a decision feedback equalizer having a filter matched to the isolated received pulse, followed by the baud-rate tapped delay line. An adaptive version of this decision feedback equalizer is investigated by George et al. [7], for the detection of the pulse-amplitude modulated signal through a noisy dispersive linear channel. By means of analysis, computer simulation, and hardware simulation, the performance of the adaptive feedback equalizer is found to be better than that obtained with a similar linear equalizer.

This paper is concerned with the DD technique for the detection of overlapping PCM/NRZ signals in the presence of white Gaussian noise. The performance of the DD detector is represented by the probability of bit error versus various signal-to-noise ratios.

II. The Model

The basic symbol has a duration of one time unit T. Here, for convenience, we assume T to be equal to one second. Further, we assume that m seconds are observed at the input of the detector. The waveforms of the binary NRZ symbols and the binary overlapping NRZ symbols are shown in Fig. 1(A) and (B), respectively. The analytical expression for the overlapping NRZ symbol is

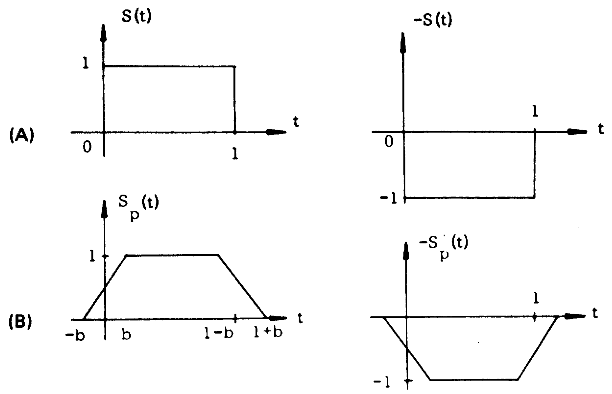


Fig. 1. (A) Binary NRZ symbols. (B) Binary overlapping NRZ symbols.

$$S_p(t) = \begin{cases} \frac{1}{2} + \frac{t}{2b}, & |t| \leqslant b \\ 1, & |t| < 1 - b \\ \frac{1}{2} + \frac{(1-t)}{2b}, & |1-t| \leqslant b \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where b is defined as the overlapping parameter and is in the range from $-1/2$ to $1/2$. The received signal waveforms for the noiseless case are shown in Fig. 2. The noise $n(t)$ is assumed to be a sample function from a Gaussian random process with zero mean and known variance. The input signal to the detector is of the following form:

$$y(t) = \sum_{k=1}^{m} a_k S_p(t-k) + n(t) \quad (2)$$

where $a_k = +1$ or -1 with equal probability.

III. The DD Detector Structure

The block diagram of the DD detector is shown in Fig. 3. Basically, two steps are involved in this approach. First, the symbol most likely received is determined by the post-bit detector as a primary decision. Then this decision is used to direct the detection process to obtain a better estimate of the symbol which will yield a smaller probability of error. The post-bit detector consists of an ordinary matched filter and a sampler, and the output is the primary detected value $\tilde{a}_n$. The function of the square marked "SHAPE" is to maintain a constant level until the next sampling instant. The output of the present-bit detector is the final decision $\hat{a}_n$. The constants K_2 and K_3 represent the areas of integration when the symbol overlaps with the preceding symbol and the following symbol, respectively. Since both detectors in Fig. 3 perform the same function, they can be placed in the front as one detector. The modified structure for the DD detector is shown in Fig. 4.

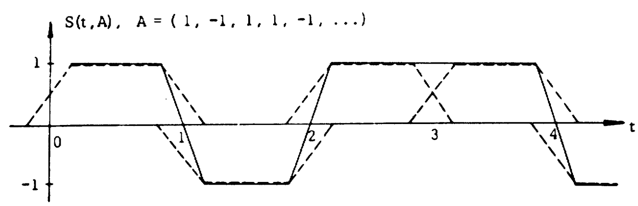


Fig. 2. Received signal without noise.

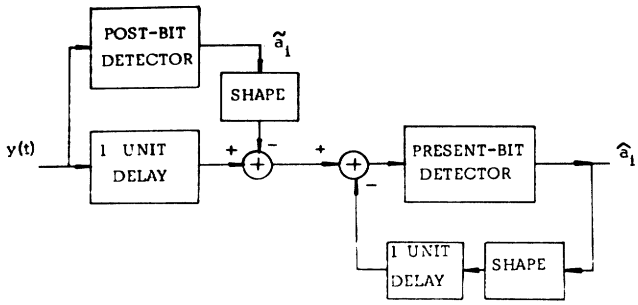
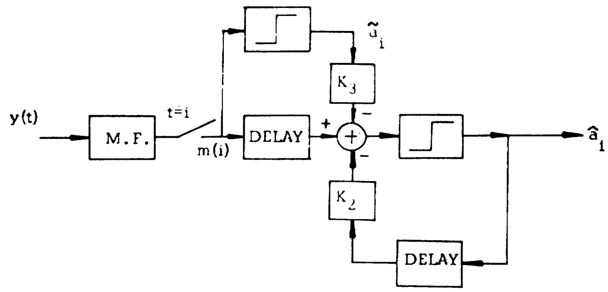


Fig. 3. Decision-directed (DD) detector for overlapping signals.

Fig. 4. Modified block diagram of the DD detector.



IV. Analytical Result

Let the matched filter of Fig. 4 have an impulse response

$$h(t) = S_p(1-t).$$

Then the output of the matched filter can be written as

$$m(t) = \sum_{k=1}^{m} a_k S_p(t-k) * S_p(1-t) + n(t) * S_p(1-t). \quad (3)$$

After integration and sampling, we have, at the sampling time $t = i$,

$$m(t) = \left(K_2 a_{i-1} + K_1 a_i + K_3 a_{i+1}\right) + K_4 n(t) \quad (4)$$

TABLE I

Eight Possible Sequences and Mean Values for Finding the Primary P_E.

	a_{i-1}	a_i	a_{i+1}	$E(m_i) = \mu_i$	$\mathrm{var}(m_i) = \sigma$
A_1	1	1	1	1	
A_2	-1	1	1	1 - (2/3)b	
A_3	1	1	-1	1 - (2/3)b	$\sigma = \sqrt{N_0(1-(2/3)b)}$
A_4	-1	1	-1	1 - (4/3)b	
A_5	1	-1	1	-1 + (4/3)b	
A_6	-1	-1	1	-1 + (2/3)b	
A_7	1	-1	-1	-1 + (2/3)b	
A_8	-1	-1	-1	-1	

where

$$K_1 = \int_{-b}^{1+b} S_p{}^2(t)\,dt = 1 - 2b/3\,,$$

$$K_2 = \int_{-b}^{1+b} S_p(t)S_p(t-1)\,dt = b/3\,, \tag{5}$$

$K_3 = K_2$ due to the symmetry of the overlapping symbol, and

$$K_4 = \sqrt{E\left[\int_{-b}^{1+b} S_p(t)n(t)\,dt\right]^2}$$

$$= \sqrt{\left(\frac{1-2b}{3}\right)N_0} \tag{6}$$

where the variance of the noise is $\sigma^2 = N_0/2$.

Because of the overlapping situation, a sequence of three symbols is involved in determining the probability of bit error of the primary detection for a single bit. Let $A = (a_{i-1}, a_i, a_{i+1})$ be this sequence; the eight possible combinations of the sequence and associated mean values are tabulated in Table I. The P_E of the primary detection is

$$\tilde{P} = P\left\{m_i < 0 \middle| a_i = 1\right\}P\left\{a_i = 1\right\}$$

$$+ P\left\{m_i < 0 \middle| a_i = -1\right\}P\left\{a_i = -1\right\}\,. \tag{7}$$

When we consider the case $a_i = 1$, four possible sequences (namely, A_1, A_2, A_3, A_4) are involved in finding the first term of (6). That is,

$$P\left\{m_i < 0 \middle| a_i = 1\right\} = 1/4 \sum_{j=1}^{4} \int_{-\infty}^{0} N(\mu_j, \sigma^2)\,dt \tag{8}$$

where

$$N(\mu_j, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\frac{(x-\mu_j)^2}{2\sigma^2}\right\}\,dx \tag{9}$$

and

$$\mu_j \triangleq E\left\{m_j\right\}.$$

Treating the case for $a_i = -1$ in the same manner, we have

$$P = \frac{1}{2}\left\{\frac{1}{4}\sum_{j=1}^{4}\int_{-\infty}^{0} N(\mu_j, \sigma^2)\,dx + \frac{1}{4}\sum_{j=5}^{8}\int_{0}^{\infty} N(\mu_j, \sigma^2)\,dx\right\}$$

$$= \frac{1}{8}\left\{\sum_{j=1}^{4}\phi\left(\frac{-\mu_j}{\sigma}\right) + \sum_{j=5}^{8}\phi\left(\frac{\mu_j}{\sigma}\right)\right\} \tag{10}$$

where

$$\phi(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)\,dx\,. \tag{11}$$

TABLE II

Possible Sequences and Mean Values for Finding the Final P_E.

a_i		$E\left[\hat{a}_i\right] \triangleq \eta_i$
1	$a_{i-1} = \hat{a}_{i-1},\ a_{i+1} = \tilde{a}_{i+1}$	$\eta_1 = K_1$
1	$a_{i-1} \neq \hat{a}_{i-1},\ a_{i+1} = \tilde{a}_{i+1}$	$\eta_2 = 2K_2 a_{i-1} + K_1$
1	$a_{i-1} = \hat{a}_{i-1},\ a_{i+1} \neq \tilde{a}_{i+1}$	$\eta_3 = K_1 + 2K_3 a_{i+1}$
1	$a_{i-1} \neq \hat{a}_{i-1},\ a_{i+1} \neq \tilde{a}_{i+1}$	$\eta_4 = 2K_2 a_{i-1} + K_1 + 2K_3 a_{i+1}$
-1	$a_{i-1} = \hat{a}_{i-1},\ a_{i+1} = \tilde{a}_{i+1}$	$\eta_5 = -K_1$
-1	$a_{i-1} \neq \hat{a}_{i-1},\ a_{i+1} = \tilde{a}_{i+1}$	$\eta_6 = 2K_2 a_{i-1} - K_1$
-1	$a_{i-1} = \hat{a}_{i-1},\ a_{i+1} \neq \tilde{a}_{i+1}$	$\eta_7 = -K_1 + 2K_3 a_{i+1}$
-1	$a_{i-1} \neq \hat{a}_{i-1},\ a_{i+1} \neq \tilde{a}_{i+1}$	$\eta_8 = 2K_2 a_{i-1} - K_1 + 2K_3 a_{i+1}$

Substituting the mean value, the probability error of primary detection can be simplified as follows:

$$\tilde{P} = \frac{1}{4}\left\{\phi\left(-\frac{1}{\sigma}\right) + 2\phi\left(-\frac{1-2b/3}{\sigma}\right) + \phi\left(-\frac{1-3b/4}{\sigma}\right)\right\}. \tag{12}$$

Equation (12) is the expression for P_E when the DD techniques are not used. The evaluation of the final P_E is complicated by the fact that the final decision a_i depends on the previous final decision a_{i-1}, as well as on the primary decision on the $(i+1)$th bit, a_{i+1}. That is,

$$\hat{a}_i = m(i) - K_2\tilde{a}_{i-1} - K_3\hat{a}_{i+1}. \tag{13}$$

Substituting $m(i)$ from (4),

$$\hat{a}_i = K_2(a_{i-1} - \hat{a}_{i-1}) + K_1 a_i + K_3(a_{i+1} - \tilde{a}_{i+1}) + K_4 n(t). \tag{14}$$

If the primary decision on this sequence is correct, then the final output of the detector will be just a constant K_1 times a_i (since $a_{i-1} = a_{i-1}$ and $a_{i+1} = a_{i+1}$) with a noise term. The probability of error for the final detection P_i is found in a recursive manner. At the stage i, we have

$$P_i = P\left\{a_i < 0 \,\middle|\, a_i = 1, \tilde{a}_{i+1} = a_{i+1}, \hat{a}_{i-1} = a_{i-1}\right\}\left(\frac{1}{2}\right)(1-P)(1-P_{i-1}) + P\left\{a_i < 0 \,\middle|\, a_i = 1, \tilde{a}_{i+1} = a_{i+1}, \hat{a}_{i-1} \neq a_{i-1}\right\}\left(\frac{1}{2}\right)(1-P)P_{i-1}$$

$$+ P\left\{a_i < 0 \,\middle|\, a_i = 1, \tilde{a}_{i+1} \neq a_{i+1}, \hat{a}_{i-1} = a_{i-1}\right\}\left(\frac{1}{2}\right)P(1-P_{i-1}) + P\left\{a_i < 0 \,\middle|\, a_i = 1, \tilde{a}_{i+1} \neq a_{i+1}, \hat{a}_{i-1} \neq a_{i-1}\right\}\left(\frac{1}{2}\right)PP_{i-1} + \text{four terms for } a_i = -1. \tag{15}$$

Setting $1 - P_{i-1} = Q$, $P_{i-1} = P$, and $1 - \tilde{P} = \tilde{Q}$, we have

$$P_i = \left(\frac{Q\tilde{Q}}{2}\right)\int_{-\infty}^{0} N(\eta_1, \sigma)\,dx + \left(\frac{P\tilde{Q}}{2}\right)\int_{-\infty}^{0} N(\eta_2, \sigma)\,dx + \left(\frac{\tilde{P}Q}{2}\right)\int_{-\infty}^{0} N(\eta_3, \sigma)\,dx + \left(\frac{P\tilde{P}}{2}\right)\int_{-\infty}^{0} N(\eta_4, \sigma)\,dx$$

$$+ \left(\frac{Q\tilde{Q}}{2}\right)\int_{0}^{\infty} N(\eta_5, \sigma)\,dx + \left(\frac{P\tilde{Q}}{2}\right)\int_{0}^{\infty} N(\eta_6, \sigma)\,dx + \left(\frac{Q\tilde{P}}{2}\right)\int_{0}^{\infty} N(\eta_7, \sigma)\,dx + \left(\frac{P\tilde{Q}}{2}\right)\int_{0}^{\infty} N(\eta_8, \sigma)\,dx \tag{16}$$

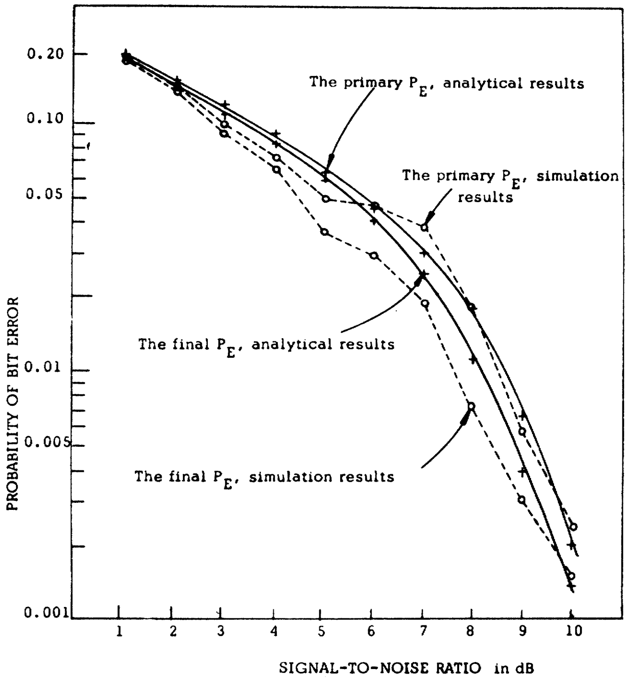


Fig. 5. Comparison between analytical and simulation results of the performance of the DD detector for b = 0.3.

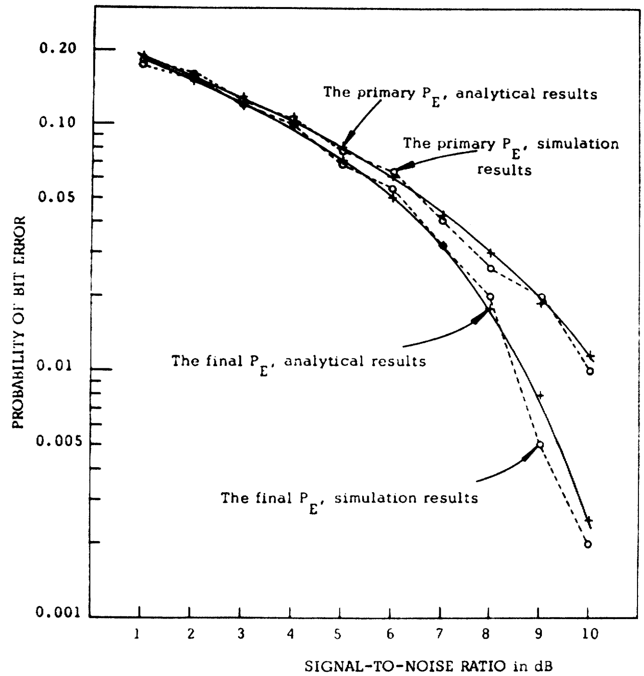


Fig. 6. Comparison between analytical and simulation results of the performance of the DD detector for b = 0.4.

where $N(\eta_i, \sigma)$ is defined in (8). Now we substitute the mean values η_i from Table II, and again write P_i as a function of the ϕ function defined by (11). Note that $\phi(-x) = 1 - \phi(x)$. Therefore, the probability of error of the final detection is

$$\begin{aligned} P_i = Q\tilde{Q}\phi\left(\frac{-K_1}{\sigma}\right) &+ \left(\frac{P\tilde{Q}}{2}\right)\phi\left(\frac{-(K_1 - 2K_2)}{\sigma}\right) \\ &+ \phi\left(\frac{-(K_1 + 2K_2)}{\sigma}\right) + \left(\frac{Q\tilde{P}}{2}\right)\phi\left(\frac{-(K_1 - K_3)}{\sigma}\right) \\ &+ \phi\left(\frac{-(K_1 + K_3)}{\sigma}\right) + \left(\frac{P\tilde{P}}{2}\right)\phi\left(\frac{-(K_1 - 2K_2 - 2K_3)}{\sigma}\right) \\ &+ \phi\left(\frac{-(K_1 + 2K_2 + 2K_3)}{\sigma}\right) + \phi\left(\frac{-(K_1 - 2K_2 + 2K_3)}{\sigma}\right) \\ &+ \phi\left(\frac{-(K_1 + 2K_2 - 2K_3)}{\sigma}\right). \end{aligned} \tag{17}$$

Equations (12) and (17) can be evaluated in parallel by a computer program. The resulting plots of the P_E as a function of SNR are shown in Figs. 5 and 6 for the case $b = 0.3$ and $b = 0.4$, respectively.

V. Simulation Result

The Monte Carlo simulation technique is used to find the P_E of the primary detection and the final detection. The input bit stream for the simulation program is generated by a uniform random number generator subroutine. The SNR in the program is the signal-to-noise power ratio measured in decibels. The P_E in this program is approximated by the ratio of the total number of erroneous bits to the total number of input bits. The resulting plots, compared with those found analytically in the last section, are also shown in Figs. 5 and 6 for b = 0.3 and b = 0.4, respectively.

VI. Conclusions

The performance of the decision-directed (DD) detector for overlapping NRZ signals has been studied, both by an analytical approach and by Monte Carlo simulation techniques. The results show that the probability of error P_E is improved by using the DD technique at all SNR levels. Further, when the overlapping parameter b is large, or the overlapping situation becomes worse, the system tends to correct more errors. For the case when the overlapping parameter is less than or equal to 0.2, however, the curves for the two P_E are very close to each other. It is seen that the DD technique does not significantly improve the performance of this detector.

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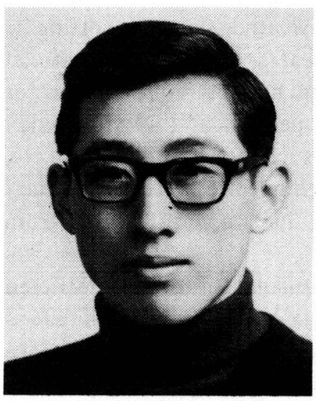

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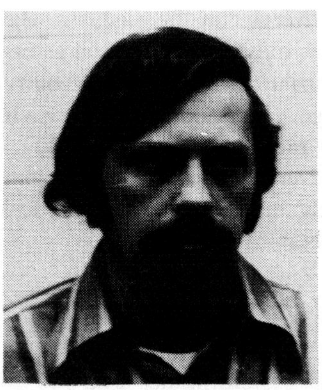

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