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Design of non-uniform linear antenna arrays with digitized spacing and amplitude levels

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DESIGN OF NON-UNIFORM LINEAR ANTENNA ARRAYS WITH DIGITIZED SPACING AND AMPLITUDE LEVELS

BY

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ABSTRACT

Recent developments in space exploration have shown a need for high resolution, high gain antennas. The introduction of non-uniform linear array theory has provided a means by which arrays may be designed to produce narrow main beams with fewer elements than required by uniform arrays. A general theory has not been developed, however, and the design engineer has been left with only trial-and-error methods with which to work.

In this study a technique has been developed by which non-uniform antenna arrays may be synthesized to meet a given set of specifications with reasonable accuracy. The elements of the array are required to occupy any one of a number of preselected positions, and there may be coincidence of elements as the array is built up. Coincidence corresponds to multiplying the current of a single element by a factor equal to the number of elements found in the position in the completed design. It is seen that this method results in quantized, or digitized, amplitudes and spacing of the elements in the linear array.

This method is ideal for solution on a digital computer, where the field pattern may be optimized with respect to any one of several parameters. Several design examples are given, and in general it is found that arrays can be designed, by this method, to have higher directivity, lower sidelobes and fewer elements than uniform linear arrays of the same aperture or with the same number of elements.

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LIST OF SYMBOLS

Symbol	Definition or Remarks
$\bar{E}$	Electric field intensity vector
$\bar{E}(\phi)$	Electric field intensity, as a function of ϕ, with θ and r held constant.
E_θ	Amplitude of θ component of $\bar{E}$
E_ϕ	Amplitude of ϕ component of $\bar{E}$
r,θ,ϕ	Spherical coordinate system variables
d	General distance parameter, or general spacing.
d_i	Distance of i-th element of antenna array from the reference element
d_m	Distance of the farthest element in the array from the reference element; the aperture.

$m = (n - 1)$ = number of elements in the antenna array

$\beta = \frac{2\pi}{\lambda}$ = wave propagation constant

λ = wavelength of electromagnetic radiation

$\psi(\phi) = \beta d \cos\phi$ = derived function used to describe the radiation from a point source as a function of ϕ

$K(\phi) = \frac{\psi(\phi)}{2\pi}$ = derived function used in the array design method developed in this thesis

$K_j = K(\phi = \phi_j) = K(\phi_j)$

ϕ_j	Space angle at which field pattern nulls exist or can be forced to exist
P	Integer multiplier of quantized element postions
$(1/Q)\lambda$	Minimum spacing allowed between elements in the digitized array, fraction of a wavelength
i	Graphic symbol used to represent the i-th element of an antenna array

CHAPTER I

INTRODUCTION

Recent advances in space exploration have shown a great need for antennas with high resolution, high gain, and low sidelobe level. Steerable reflector antennas as large as 200 to 300 feet in diameter have been built. In the last few years antennas larger than these have been considered prohibitive in cost, however, and a number of researchers in the field have turned their attention to large arrays of smaller antennas.

In the analysis of antenna arrays, the conventional approach has been to consider the elements of the array to be equally spaced along a straight line. The line is usually taken as one axis of a coordinate system, and the radiation pattern of the array is then developed in terms of the variables of the system. Schelkunoff (1)* has shown that linear antenna arrays can be represented mathematically by polynomials, and that the characteristics of the radiation pattern of such an array can be analyzed in terms of the properties of its associated polynomial. The closed-form polynomials obtained for arrays of antennas with uniform current amplitudes or for arrays with amplitudes proportional to the coefficients in the binomial expansion, as well as for several other mathematical schemes, have been

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*Numbers in parentheses are references to the Bibliography.

analyzed extensively since Schelkunoff's publication. A theoretical optimum broadside array with equally spaced elements has been obtained by Dolph (2) by making use of the properties of the Tchebycheff polynomials, and other methods of optimization, using various criteria, have been proposed.

Because of the amount of work done in this area in the past, well-developed methods are now available for designing linear arrays with equally spaced elements that will produce a desired radiation pattern with reasonable accuracy. For conventionally designed arrays, however, where all elements are equally spaced, there exists an upper limit to the element spacing if grating lobes (maxima with amplitudes equal the main lobe) are not to appear in the field pattern. This means that for a broadside array, for example, the spacing must be less than one wavelength between elements if there are to be no grating lobes. Unless the excitations of the elements of the array are strongly tapered, the beamwidth of the main lobe is primarily dependent upon the length of the array, and depends only slightly upon the number of elements in the array. As a result, the required number of elements for a uniform linear array becomes astronomically large when very small beamwidths are desired.

The use of arbitrary element positions for pattern synthesis was first suggested by Unz (3) in a short paper in 1960. A paper by King, et al, (4) proposed the use of nonuniform element separation to reduce grating lobes. King also noted that some of the experimental patterns included in the paper had sidelobe levels below those for uniform arrays, and concluded that non-uniform spacing might be used to reduce sidelobe levels.

Because of the complexity of the mathematics, a general theory for non-uniform arrays has not been developed. Some array synthesis has been done with digital and analog computers, and some sophisticated mathematics has been employed in the analysis of a few special configurations of antennas. Recent papers on the subject have employed both perturbation techniques and an examination of the probabilistic properties of large arrays with randomly spaced elements, but have not contributed significantly to a simple and straightforward design method such as is available for the uniform linear array. The problem that has developed in the area of non-uniform antenna array theory is that, because there seems to be no way to treat unequally-spaced arrays by the polynomial method, and the mathematics that has been used in an attempt to develop a general theory has grown more and more elegant with each succeeding paper, the design engineer has been left with only trial-and-error methods with which to work.

The purpose of this study was to investigate the properties of unequally spaced linear antenna arrays and attempt to establish a practical array design procedure. Taking advantage of the fact that any two antennas can be positioned so that their far fields will be 180° out of phase at some given space angle ϕ, it was found that nulls could be forced at any desired position in the far-field pattern of a linear array by suitably positioning additional elements for each desired null. When the elements were then forced to occupy positions which were some integer multiple of any chosen fraction of a wavelength (i.e., antennas spaced $(\frac{1}{2}\lambda)P$ from other antennas in the array, where P is any integer), under certain conditions antennas added as the array was built up fell in the same position as antennas previously placed. This coincidence of elements was seen to correspond to adding the current levels of all antennas which fell in that position, and a technique of array design was developed which results in both digitized spacing and digitized amplitude levels.

The total number of antennas in any array developed by this method for a given set of far-field specifications can not exceed the number for a uniform array of the same aperture, and in general the number will be considerably fewer because of the coincidence of elements and because all of the digitized spaces are not necessarily filled in the design. Using a high speed digital computer, the radiation pattern of a non-uniform array may be optimized with respect to any of several different criteria. Power gain, directivity, and sidelobe level comparisons with a uniform linear array of the same aperture may be made with little effort since the uniform array is always identifiable as a special case in the design values for any given problem. Within the restrictions of the method it has been found that a very good approximation to any desired field pattern may be realized.

CHAPTER 2

REVIEW OF THE LITERATURE

The techniques that have been described in the literature for the design of antenna arrays with unequal spacing between elements may be divided into four general categories: (a) empirical, (b) matrix formulation, (c) space taper, and (d) trial-and-error computer optimization.

The empirical approach is to select a set of element spacings according to some specified law that seems to offer promise of a reasonable radiation pattern. In July, 1960, King, Packard, and Thomas (4) reported on patterns computed for various trial sets of spacings including logarithmic, spacing proportional to prime numbers, and spacing proportional to arithmetic progression, but no unified theory was developed. Lacking better methods, the empirical trial-and-error approach offers a start to the design of unequally spaced arrays, and it has produced quite satisfactory results in some cases.

In March, 1960, Unz (3) was first to report on arbitrary distribution of elements in an array. His short paper proposed a matrix relationship between the elements of an array and its far-field pattern, but was written in extremely general terms. Also in 1960, Sandler (5) suggested an equivalence between equally and unequally spaced arrays. His method of synthesis consists of choosing a spacing scheme and then expanding each term of the unequally spaced array in a Fourier cosine series. The Fourier series is then made to approximate the expression for an equally spaced array. Skolnik, Sherman, and Ogg (6) reported in 1964 on a method of determination of element position and current level in density-tapered arrays. This theoretical approach involved expressing the radiation pattern in a series expansion, truncating the expansion, and inverting a matrix to obtain the desired spacings. These methods, involving matrix relationships, do not lead to a unique solution and are very difficult to apply.

In November, 1962, Ishimaru (7) presented a new approach based on the use of Poisson's sum formula and a new function termed the "source position function". This formulation is in essence the transformation of the unequally spaced array into an equivalent continuous source distribution. By this method it is possible to design unequally spaced arrays which produce a desired pattern, but in general this method is also very difficult to apply.

In a category by themselves are those trial-and-error techniques that take advantage of large-scale computers. Harrington (8) and Andreason (9) have each used an approach which starts with a reasonable set of element spacings selected for some particular feature, and then perturbs the spacings of each element about its initial value. The effect on the pattern is observed and a new set of spacings selected which gives an improved result. The success of this method depends on the correctness of the original set of spacings and on the program for perturbing the spacings.

Skolnik, Nemhauser, and Sherman (10) published a paper in January, 1964, which describes the application of an optimization technique known as "dynamic programming" to the design of unequally-spaced arrays. Using this method a set of digitized, or quantized, positions are chosen about a reference antenna, and a pattern computed for the reference antenna and a single additional antenna when the additional antenna is placed in each quantized position in turn. The best pattern, in some particular sense, is chosen from all of the computed patterns and placed in the computer memory. A third antenna is then tried in each position and the pattern of the third antenna with the "best" arrangement of the first two antennas is computed for each position. This process is continued until a chosen number of elements has been distributed in a chosen aperture.

This method has several shortcomings, the chief of which is that the main beam width is not known until the last element of the array has been positioned. The optimization program must be written to include a large enough angle so that the main lobe will not be affected in the buildup of the array, and it is possible that it may be chosen large enough that a high sidelobe adjacent to the main lobe will be left untouched. Also, since the design of a complete array is built up from successive designs of partial arrays, this method cannot yield a truly optimum design, for the positions of all of the elements are interdependent.

The approach used in this thesis is similar to that of Skolnik, et al, (10) as given above, in that digitized spacings are chosen and the array built up from successive designs of partial arrays. The main differences are (a) the space angle ϕ, measured from the axis of the array, at which the main lobe null occurs is an important parameter in the design, (b) the elements are positioned in groups, and not singly, to effect a sidelobe reduction at a chosen angle in the field pattern, and (c) the number of elements and the aperture of the array are not known until the design is completed, but are determined by the amount of sidelobe reduction desired by the designer as the array is built up.

The method of this thesis and other methods involving trial-and-error computer techniques are similar, also, in that they do not produce unique solutions to a given problem. Even with the additional degree of design freedom gained by removing the requirement for equal spacing, no one array can be designed which is optimum in every sense. Usually it is necessary to determine a single criterion by which any array design will be considered better than others which might be obtained. A computer is ideally suited for work of this kind, and the results obtained by computer designs compare very favorably with those obtained by other design procedures.

CHAPTER III

MATHEMATICS OF ANTENNA ARRAYS

When a transmitting antenna in free space is represented by a point-source radiator located at the origin of spherical coordinates, the radiated energy is said to stream from the source in radial lines (11). The time rate of energy flow per unit area is the Poynting vector, or power density, and has no components in either the θ or ϕ directions. A graph of the magnitude of the time averaged Poynting vector as a function of θ or ϕ is usually called the power pattern of a source. The graph is a relative power pattern when it is normalized with respect to the maximum value of the radiated power density.

The power flow from a point source has only a radial component, and can be considered as a scalar quantity. To describe the vector nature of the field of a point source more completely the electric field intensity, or $\bar{E}$ vector, of the field may be considered. The Poynting vector and the electric field at a point of the far field are related in the same manner as they are in a plane wave, for if r is large, a small section of the spherical wave front may be considered as a plane. Since the Poynting vector around a point source is everywhere radial, it follows that the electric field is entirely transverse, having only E_θ and E_ϕ components. The relationship between the average Poynting vector and the electric field at a point of the far field is given by

$$P_r = \frac{E^2}{2Z_o} \quad \text{watts/meter}^2, \tag{3.1}$$

where P_r = the radial component of the Poynting vector,

Z_o = the intrinsic impedance of free space,

$E = \sqrt{E_\theta^2 + E_\phi^2}$ = the magnitude of the total electric field intensity,

E_θ = the amplitude of the θ component,

and E_ϕ = the amplitude of the ϕ component of the field.

In presenting information concerning the far field of an antenna, the component fields E_θ and E_ϕ are usually given. The total electric field magnitude E can be obtained from the components, but the components cannot be obtained from a knowledge of E alone. If the field pattern is normalized with respect to its maximum value in some direction, it is a relative field pattern, and the relative total field pattern is the square root of the relative total power pattern.

In actual practice the field variation near an antenna, or "near field", is usually ignored and the source of radiation is described only in terms of the "far field" it produces. When observations are made at sufficient distance, any antenna can be represented by a single point source. In theoretical analyses the isotropic point source, a source which radiates energy equally in all directions, has been found convenient even though it is not physically realizable. It has been shown (11) that the radiation pattern of any antenna can be considered to be due to a suitably located array of point sources.

Even the simplest antennas have some directional properties, and may be termed anisotropic sources. When several anisotropic, but similar, antennas are used in an array, the radiation pattern of the array may be determined by using the principle of pattern multiplication (11). That is, the pattern may be considered to be the product of the pattern due to an array of isotropic sources by the pattern of one of the similar isotropic sources. The pattern of an array of isotropic sources located in the same relationship as the physical antennas the sources are to represent is referred to as a universal pattern, and is the basis of most of the theory developed for antenna arrays.

In general, the amplitude, phase, and position of each element except the reference element may be adjusted to obtain a desired field pattern. For an array of n elements there are 3(n-1) parameters which may be varied. Closed-form polynomials, or other concise mathematical expressions, are not obtainable in the general case, however, and pattern characteristics have conventionally been examined by imposing various restrictions on the array parameters. For the work to follow, the arrays will consist of a linear arrangement of isotropic elements, and the universal patterns computed will be identified as field patterns or power patterns as the case may be. Only the total electric field will be found, and only broadside arrays, or arrays with the current fed in-phase to all elements of the array, with resultant main beam perpendicular to the axis of the array, will be considered. Also, throughout the study to follow, it will be assumed that the standard for comparison between arrays will be the maximum sidelobe level for the array. The following figure defines the coordinate system that will be used throughout the study:

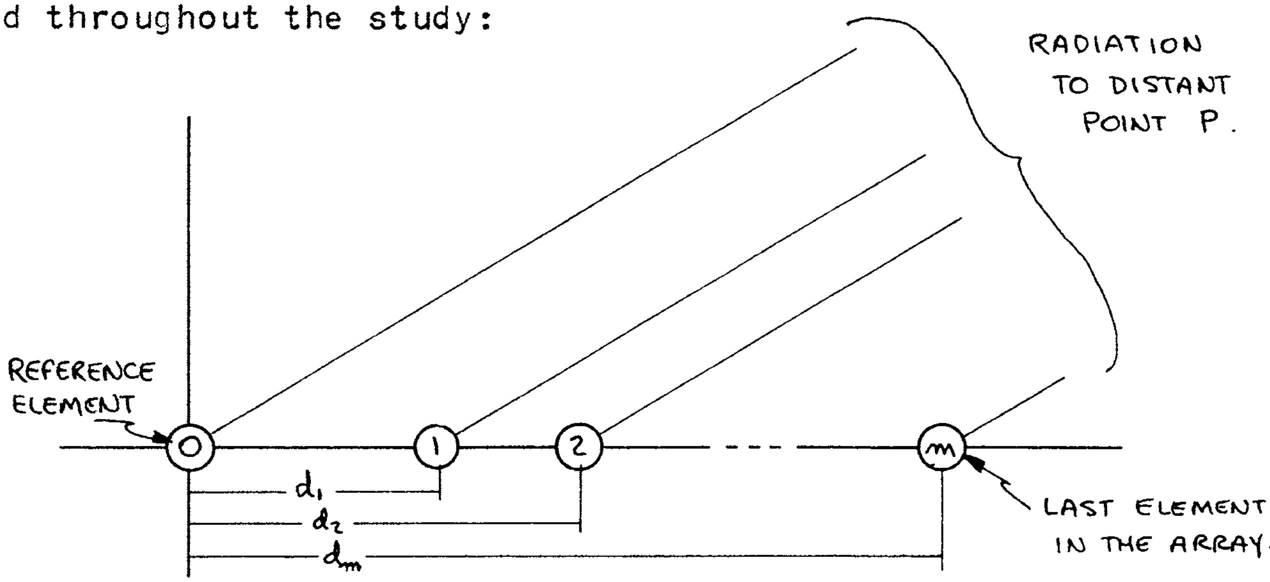


Figure 3.1. Coordinate system for linear array.

For this configuration of elements, with uniform in-phase excitation, the radiated electric field as a function of ϕ, at a constant radius r, may be written as

$$\bar{E}(\phi) = 1 + e^{j\beta d_1 \cos\phi} + e^{j\beta d_2 \cos\phi} + \cdots + e^{j\beta d_m \cos\phi}, \qquad (3.2)$$

where $\beta = \frac{2\pi}{\lambda}$ = the wave propagation constant,

d_i = distance of i-th element from the reference,

d_m = the total length, or aperture, of the array,

and $m = (n-1)$, where n = number of elements in the linear array.

When the spacing between elements is uniform, such that

$d_1 = d$, $d_2 = 2d$, $d_3 = 3d$, etc., then a function $\psi(\phi)$ equal to $\beta d \cos\phi$ may be defined and the field equation written in the form:

$$\bar{E}(\phi) = 1 + e^{j\psi(\phi)} + e^{j2\psi(\phi)} + \cdots + e^{jm\psi(\phi)} . \qquad (3.3)$$

It has been shown by Kraus (11) that the normalized magnitude of equation (3.3) for an n-element uniform linear array may be written

$$\bar{E}(\phi) = \frac{1}{n}\left[\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right] . \qquad (3.4)$$

The field as given by equation (3.4) is referred to as the array factor or universal pattern factor of the array of n isotropic point sources.

Universal field pattern charts for various numbers n of isotropic sources with equal amplitude and spacing have been calculated (11). These show that the maximum sidelobe level for such an array is almost constant for arrays with more than 5 elements. Andreason (9) has shown that the minimum value of this maximum sidelobe is -13.5 db compared to the main beam. In addition, the pattern charts show that the maximum sidelobe is always immediately adjacent the main beam. Thus, when high-resolution is a design requirement, any method which reduces sidelobes adjacent the main beam will constitute an improvement over the uniform linear array.

The null directions for an array occur when $E(\phi) = 0$. Following the procedure given by Schelkunoff (1,11) for an array of n isotropic point sources of equal amplitude and spacing, the nulls occur when equation (3.4) is zero, provided the denominator is not also zero. This requires that

$$n\psi = \pm 2k\pi$$

$$\psi = \beta d \cos\phi = \pm \frac{2k\pi}{n}, \quad \text{OR}$$

$$\phi = \cos^{-1}\left(\frac{\pm 2k\pi}{\beta d n}\right) = \cos^{-1}\left(\pm \frac{k\lambda}{nd}\right). \tag{3.5}$$

Values of k which are integer multiples of n must be excluded from equation (3.5) in order that the denominator of equation (3.4) have non-zero values.

If ϕ is replaced by its complimentary angle, defined here as $\theta = (90^\circ - \phi)$, equation (3.5) may be written

$$\theta = \sin^{-1}\left(\frac{\pm k\lambda}{nd}\right). \tag{3.6}$$

When the angle θ is very small, corresponding to a narrow main lobe, then

$$\theta \cong \pm \frac{k\lambda}{nd}, \qquad nd >> k\lambda .$$

The first nulls either side of the main lobe occur for $k = 1$, and the total beam width of the main lobe between first nulls is then

$$2\theta_1 \cong \frac{2\lambda}{nd}. \tag{3.7}$$

It has been shown by Ishimaru (7) and Andreason (9) that the 3db beam width for a narrow beam may be considered to be $\frac{\lambda}{nd} = \frac{\lambda}{L}$, where $L = nd =$ the total aperture of the array measured in wavelengths.

The main beam maximum for the broadside array occurs at a value of $\psi = 0^\circ$, or $\phi = 90^\circ$, as indicated by equation (3.4). The maxima of the minor lobes are situated between the first and higher-ordered nulls, and it has been pointed out by Schelkunoff (1) that these maxima occur only approximately when the numerator of equation (3.4) is a maximum. Since the numerator of (3.4) varies as a function of Ψ more rapidly than the denominator, the approximation becomes better when n is large. By the same method used to find equation (3.5), it is found that minor lobe maxima for the broadside array occur at angles

$$\phi \cong \cos^{-1}\left[\frac{(2k+1)\lambda}{2md}\right], \quad k = 1, 2, \cdots \qquad (3.8)$$

A special case which occurs is that of the indeterminate form of equation (3.4). Both numerator and denominator will go to zero at particular values of Ψ when the spacing d exceeds one wavelength. A special case of this condition is the separation of elements by an integer number of wavelengths. In this case there will be the same integer number of grating lobes in each quadrant of the angle ϕ, and these grating lobes occur at angles corresponding to the integer values of k restricted from equation (3.5).

This summary of array theory is intended to define the terms, demonstrate the method of analysis, and point out some of the restrictions on linear arrays. The work to follow is an extension of the above material, with the goal of developing a design method by which arrays may be made to produce a pattern having narrow beam width and low sidelobe levels with fewer elements than are required of a uniform linear array of the same aperture, As stated before, the criterion for selecting one array design over another throughout this development is that the maximum sidelobe level is a minimum over the region of interest for the chosen array.

CHAPTER IV

THE DEVELOPMENT OF A DESIGN PROCEDURE

A. Derivation of Design Parameters

The total electric field for a linear array of arbitrarily spaced, uniformly excited, isotropic point-source antennas, as developed in Chapter III, is given by

$$\bar{E}(\phi) = 1 + e^{j\beta d_1 \cos\phi} + e^{j\beta d_2 \cos\phi} + \cdots + e^{j\beta d_m \cos\phi}, \qquad (4.1)$$

where $\beta = \frac{2\pi}{\lambda}$ = propagation constant,

d_i = distance in wavelengths from reference,

$d_m = d_{n-1}$ = distance of n^{th} antenna,

= the aperture of the array.

The electric field intensity, or more simply, the field at any given angle ϕ_o is represented by the vector sum of all of the component fields due to each of the antennas. For the field at an angle ϕ_o to be equal to zero it is necessary that all of the terms, each of which represents a unit radial line in the complex plane, add to zero. In general there is no concise mathematical expression by which it can be established whether or not cancellation of fields will occur. That is, it cannot be said that the arguments of the terms must sum to any particular value, and neither can any other general test or criterion be established beyond straightforward summing of all of the terms, to find whether or not the field is equal zero at a particular angle ϕ_o.

If, however, a zero is required at a specified space angle in the radiation pattern of a single antenna, it is seen that the required zero may be obtained with an additional element so positioned that the arguments of the two antennas are 180^0 out of phase at the specified angle. This means that if an antenna located a distance d_1 from some phase reference has a field at constant r given by

$$\bar{E}(\phi) = e^{j\beta d_1 \cos\phi} ,$$

a zero may be forced in the pattern at an angle ϕ_0 by adding an additional antenna such that

$$\bar{E}(\phi_0) = e^{j\beta d_1 \cos\phi_0} + e^{j\beta d_2 \cos\phi_0} = 0 .$$

For this to be true the arguments must be 180^0 out of phase at the angle ϕ_0, or

$$\beta d_2 \cos\phi_0 = \beta d_1 \cos\phi_0 + \pi , \tag{4.2}$$

$$d_2 = d_1 + \frac{\pi}{\beta \cos\phi_0} . \tag{4.3}$$

If the additional antenna is spaced a distance d_2 from the phase reference, as given in equation (4.3), a zero will appear in the radiation pattern of the two antennas at an angle ϕ_0. Thus it is seen that, while little can be determined about the zeros of an array from its field equation without calculating the expression at every angle ϕ of interest, it is possible to force a zero in the pattern of an array at any angle ϕ_0 desired by adding a new antenna spaced a distance

$$d_2 - d_1 = \frac{\pi}{\beta \cos\phi_0} \quad \text{wavelengths} \tag{4.4}$$

from each existing antenna. This procedure does not seem to be conservative of antennas, but it does provide a fresh approach to array design procedure, and leads to some useful results.

Another fact which comes from uniform linear array theory is that, while main beam width is in general inversely proportional to the aperture for a long linear array (11), a narrow main lobe may also be achieved by spacing two antennas several wavelengths apart. In the latter case grating lobes appear in numbers proportional to antenna separation in wavelengths.

By combining the ideas contained in the paragraphs above, the essence of the design method is obtained: a narrow broadside main beam can be established by positioning two antennas as far apart as is required, and the grating lobes which appear can then be suppressed by the addition of a suitable number of new elements so positioned as to force field pattern nulls at the location of each of the grating lobes. Additional elements may be added to reduce the resultant side lobes as much as required by the designer. A practical limit will of course be obtained, beyond which it is not feasible to continue. Also, when the number of antennas is increased, the spacing between individual elements may become intolerably small. In the work which follows, the range of values of the various design parameters is obtained, and the procedure then extended to include the constraint of minimum spacing. Placing the constraint on the array parameters results in digitized spacing and amplitudes, and a practical design method.

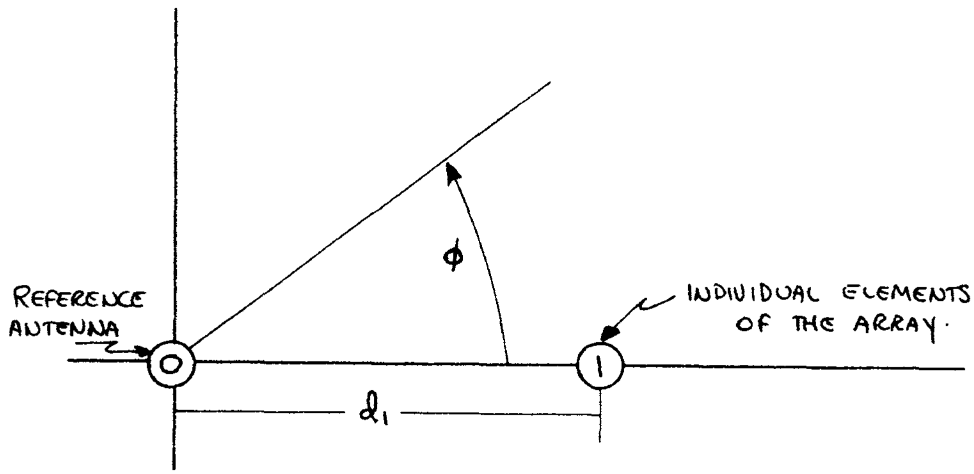


Figure 4.1. Basic array of two elements.

The field due to the two antennas of Figure 4.1, above, is given by $\bar{E}(\phi) = 1 + e^{j\beta d_1 \cos\phi}$, where the spacing d_1 is such that the angle $\phi = \phi_1$ and its supplement are the first nulls either side of the main lobe. To force a new null at an angle $\phi = \phi_2$, which may be taken at the center of a grating lobe if $d_1 > \lambda$, additional antennas are required as shown in Figure 4.2.

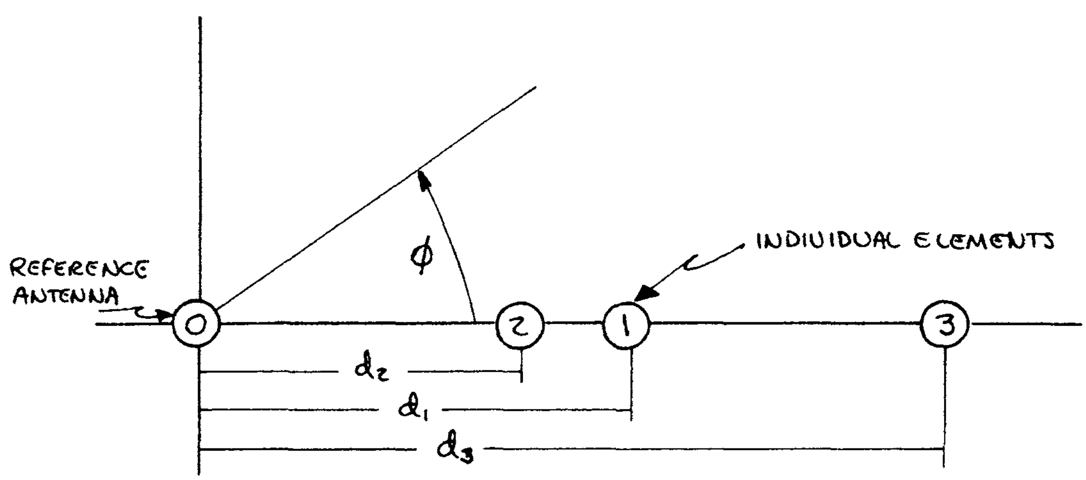


Figure 4.2. Array with zero-producing elements.

For this configuration of antennas the total field is

$$\bar{E}(\phi) = 1 + e^{j\beta d_1 \cos\phi} + e^{j\beta d_2 \cos\phi} + e^{j\beta d_3 \cos\phi} , \qquad (4.5)$$

where d_2 is chosen such that the field from antenna #2 will add to zero with the field from the reference antenna #0 at the space angle $\phi = \phi_2$, and the distance d_3 is chosen such that antennas #1 and #3 will null at the same angle. Then the spacings between pairs are equal, or $d_2 = (d_3 - d_1)$. It is seen that whenever an antenna is paired with the reference antenna to force a zero at a specified angle, an element spaced the same distance must be paired with each existing element of the array so that the total field will go to zero at the specified angle. It is not necessary to calculate the distance of each element from the reference, using this scheme.

To obtain more information about the system, the field equation (4.5) is rewritten as

$$\bar{E}(\phi) = 1 + e^{j\frac{d_1}{d}\beta d \cos\phi} + e^{j\frac{d_2}{d}\beta d \cos\phi} + e^{j\frac{d_3}{d}\beta d \cos\phi} \qquad (4.6)$$

$$= e^{jk_0 \beta d \cos\phi} + e^{jk_1 \beta d \cos\phi} + e^{jk_2 \beta d \cos\phi} + e^{jk_3 \beta d \cos\phi} \qquad (4.7)$$

$$= e^{j2\pi k_0 K(\phi)} + e^{j2\pi k_1 K(\phi)} + e^{j2\pi k_2 K(\phi)} + e^{j2\pi k_3 K(\phi)} \qquad (4.8)$$

$$= \sum_{i=0}^{3} e^{jk_i K(\phi)} , \quad \text{where} \qquad (4.9)$$

$k_i = \dfrac{d_i}{d}$ = an element spacing parameter,

with $k_0 \triangleq \dfrac{d_0}{d} \triangleq 0$ = distance from reference to origin,

$$K(\phi) = \frac{\beta d \cos\phi}{2\pi} = \frac{d\cos\phi}{\lambda}, \text{ and} \tag{4.10}$$

in addition, $\quad K_j = K(\phi_j) = \dfrac{d\cos\phi_j}{\lambda}$, $\qquad$ (4.11)

where ϕ_j = angles at which nulls are forced.

In terms of the variables introduced above, the equation for the original pair of antennas is now written as

$$\bar{E}(\phi) = e^{j2\pi k_0 K(\phi)} + e^{j2\pi k_1 K(\phi)} .$$

The angle ϕ_1 of the main beam null is a design choice by which the distance d_1 is established. To determine this value, the zero of the field is used:

$$\bar{E}(\phi_1) = e^{j2\pi k_0 K_1} + e^{j2\pi k_1 K_1} = 0 \tag{4.12}$$

$$2\pi k_1 K_1 = 2\pi k_0 K_1 + \pi = \pi \tag{4.13}$$

$$k_1 = \frac{1}{2K_1}$$

$$d_1 = \frac{d}{2K_1} = \frac{\lambda}{2\cos\phi_1} . \tag{4.14}$$

When the field is forced to zero at $\phi = \phi_2$ the field equation is (from Figure 4.2)

$$\bar{E}(\phi_2) = e^{jk_0 2\pi K_2} + e^{j2\pi k_1 K_2} + e^{j2\pi k_2 K_2} + e^{j2\pi k_3 K_2} = 0 \tag{4.15}$$

The distances d_2 and d_3 are found from the equations

$$2\pi k_2 K_2 = 2\pi k_0 K_2 + \pi = \pi \tag{4.16}$$

and $\qquad 2\pi k_3 K_2 = 2\pi k_1 K_2 + \pi \qquad$ (4.17)

From (4.16), $\qquad k_2 = \dfrac{1}{2K_2}$

$$d_2 = \frac{d}{2K_2} = \frac{\lambda}{2\cos\phi_2} . \tag{4.18}$$

From (4.17), $\qquad k_3 = \dfrac{1}{2K_2} + k_1$

$$d_3 = \frac{d}{2K_2} + d_1$$

$$d_3 = \frac{\lambda}{2\cos\phi_2} + d_1 \;. \qquad (4.19)$$

Equations (4.18) and (4.19) show that when the spacing of elements for a null at a specified angle has been determined with respect to the reference, additional elements are placed the same distance from each existing antenna.

The values of K_1 and K_2, from equation (4.11), are

$$K_1 = \frac{d\cos\phi_1}{\lambda} \quad , \qquad K_2 = \frac{d\cos\phi_2}{\lambda} \quad ,$$

where d is a constant to be determined. The angle ϕ, measured from the axis of the array, is 90° at the center of the main lobe for a broadside array. The angle of the main beam null, $\phi = \phi_1$, is an angle less than 90°, but greater than the angle of any sidelobe maximum. K_1 will always be the smallest of the K_j's for any given problem, since ϕ_1 is the largest angle to be used as a design parameter. The constant d may be determined if the K_j's are "normalized" by setting $K_1 = 1$. In this case, from (4.11),

$$K_1 = 1 = \frac{d\cos\phi_1}{\lambda}$$

$$d = \frac{\lambda}{\cos\phi_1} \quad \text{wavelengths,} \qquad (4.20)$$

and from (4.10),

$$K(\phi) = \frac{d\cos\phi}{\lambda} = \frac{\cos\phi}{\cos\phi_1} \;. \qquad (4.21)$$

When the first two antennas are spaced a distance d_1 which exceeds one wavelength, one or more grating lobes will appear. If, for example, d_1 is taken as 5λ, there will be five maxima other than the main beam, and five nulls, in the first quadrant of the angle ϕ. To determine how this is reflected in the parameter $K(\phi)$, equation (4.13) must be used. The distance d_1 was determined from (4.13) by assuming the radiation from the two sources is 180^o out of phase at $\phi = \phi_1$, $K(\phi) = K_1$. That is,

$$2\pi k_1 K_1 = 2\pi k_0 K_1 + \pi . \tag{4.13}$$

When d_1 is greater than λ there are additional values of $K(\phi)$ at which zeros occur. For these values, the arguments must be odd multiples of 180^o out of phase, or

$$2\pi k_1 K(\phi) = 2\pi k_0 K_1 + (2k-1)\pi , \tag{4.22}$$

where $k = 1, 2, 3, \ldots.$

When (4.13) is divided by K_1 and (4.22) divided by $K(\phi)$, the resulting equations may be used to obtain

$$2\pi k_0 + \frac{\pi}{K_1} = \frac{2\pi k_0 K_1}{K(\phi)} + \frac{(2k-1)\pi}{K(\phi)} . \tag{4.23}$$

There is no loss in generality in the fact that $k_o = 0$, and (4.23) may be reduced to

$$K(\phi) = (2k-1)K_j \tag{4.24}$$

This important result indicates that zeros will occur in the field pattern at all angles

$$\phi = \cos^{-1}\left[(2k-1)(\cos\phi_1)\, K_j\right] \tag{4.25}$$

$$(2k-1)K_j \leq \frac{1}{\cos\phi_1} = \frac{\cos 0^\circ}{\cos\phi_1} = K_{MAX} . \tag{4.26}$$

This means that, for any specified design value K_j, a zero, which will be termed a primary zero, will occur in the field pattern at the angle $\phi = \phi_j$, but secondary zeros will also occur in the pattern at angles corresponding to all odd multiples of K_j up to the value K_{max} as given by equation

(4.26). Zeros are then forced in the field pattern, and sidelobe levels reduced accordingly, without the expenditure of additional elements. Equation (4.26) also indicates that sidelobes near the main beam will be easily suppressed, since odd multiples of a small quantity are still small, and the angles at which many of the secondary zeros occur will still be near 90^o.

The effect of the additional antennas on the original pattern may be examined in a qualitative manner if the added elements are considered to be a separate, self-contained array. The new array is a broadside array, but it is translated from the phase center of the original array by the distance between each element and the element with which it is paired. By the principle of pattern multiplication (11) it is seen that the resultant pattern will be the product of either array pattern, since they are similar, by the pattern of two isotropic point sources separated a distance equal to the translation of the added array. The two point sources separated by the distance specified will produce the required primary zero at some ϕ_j, and the resultant secondary zeros. The product of the two patterns will then contain all the original zeros and all the new zeros. The fields will add in phase in the broadside direction, but there can not be an increase in the relative field pattern in any direction. In general, there will be a decrease in the relative field pattern in all directions except broadside, and all zeros will be retained from step to step in the design.

B. An Application Of The Design Method

To build up an array using the parameters determined in the previous section it is only necessary to establish a required beam width and the maximum allowable sidelobe level. If, for example, a main lobe width between 3 db points of approximately 5^o is desired, the design might proceed in the following manner:

An angle between main beam nulls of approximately 10^o will produce a 3 db beam angle near 5^o for a large antenna array. For convenience in calculations, choose $\phi_1 = 84.26^o$ so that $\cos(\phi_1) = 0.10$. From equation (4.14), the distance between the first two antennas is then $d_1 = \frac{\lambda}{2\cos\phi_1} = 5\lambda$.

Using the methods of Chapter III, the zeros and maxima of the pattern for the two antennas can be determined. A polar plot of the field pattern, given in the figure below, clearly

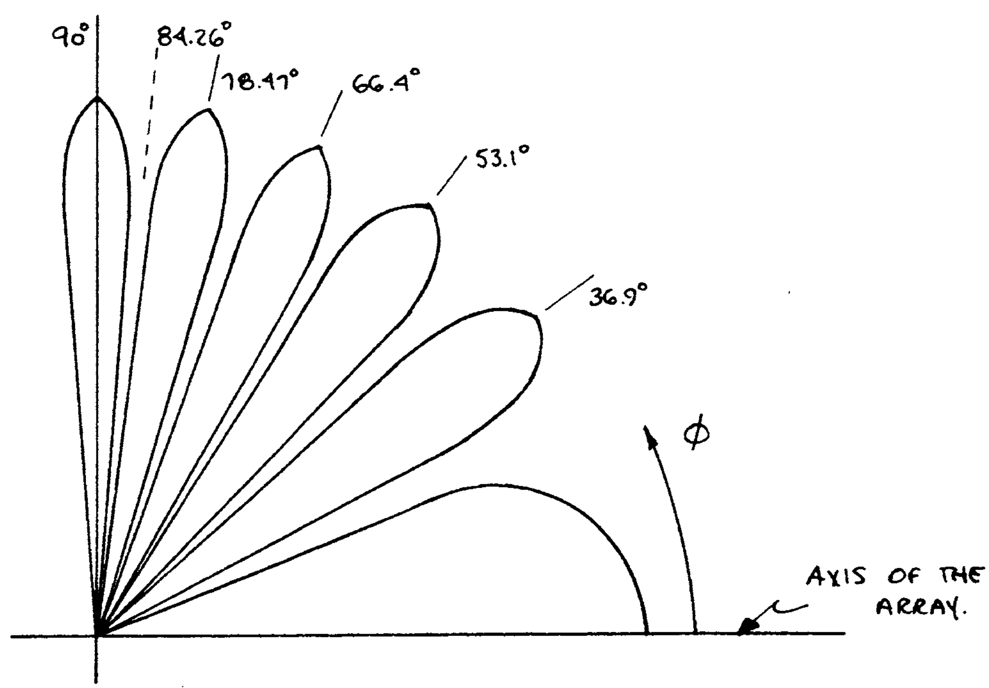


Figure 4.3. Field pattern of two antennas spaced 5λ apart.

shows the zeros and grating lobes which result when elements are placed more than one wavelength apart.

To force a null in the pattern at $\phi_2 = 78.47^{\circ}$, which is the location of the first grating lobe, it is necessary to place antennas

$$d_2 = \frac{\lambda}{2 \cos \phi_2} = 2.5 \lambda$$

from each of the other, or previously existing, antennas in the array. This will yield, in addition, the secondary zeros given by a modification of equation (4.25):

$$\phi = \cos^{-1}\left[(2k+1)(\cos\phi_1)\, K_2\right]$$

$$= \cos^{-1}\left[(2k+1)(\cos\phi_2)\right]$$

$$= 53.1^{\circ},\ 0^{\circ},\quad \text{FOR } k = 1, 2.$$

These values correspond to grating lobes also, in this particular design. Thus, two additional grating lobes have been suppressed without additional elements.

Another application of equation (4.14), to suppress the maximum at $\phi = 66.4^{\circ}$, yields

$$d_4 = \frac{\lambda}{2 \cos (66.4^{\circ})} = 1.25 \lambda .$$

The array now consists of 8 elements: the original two elements spaced 5λ apart, the two added elements spaced 2.5λ from each of the original, and four new elements, each one spaced 1.25λ from one of the four previously positioned antennas. The array at this stage has eight equally spaced elements, with a separation of 1.25λ between them. One of the grating lobes still remains, located at $\phi = 36.9^{\circ}$.

The last grating lobe can be suppressed with the addition of elements spaced

$$d_8 = \frac{\lambda}{2\cos(36.9^\circ)} = .625\lambda$$

from each existing element. This again results in a uniform linear array. This is not the general case, however, but results from the values of K in the example ($K_j = 1,2,4,8$).

The important result of this example is that, although a uniform linear array resulted from suppression of only the grating lobes, if additional nulls are now forced at any chosen location in the pattern, the spacing required must result in an intolerably small separation between elements. For example, the spacing required to force an additional null at $\emptyset = 15^\circ$, which is near the center of the broad endfire lobe, is

$$d_{16} = \frac{\lambda}{2\cos 15^\circ} = .517\lambda .$$

These elements cannot be positioned $.517\lambda$ from each existing element in a space $.625\lambda$ wide and still maintain adequate separation from the next antenna.

The problem that has developed is that, while this procedure in its present form allows the addition of elements to force a zero at any desired angle in the field pattern, the resultant spacing quickly reduces to a value below the practical minimum of $\frac{\lambda}{2}$ required to avoid strong mutual coupling effects. If it is assumed that $\frac{\lambda}{2}$ is to be the minimum distance allowed between any two adjacent elements, the procedure above must be constrained so as to produce such a result. The development of such a constraint and its correlated equation is the subject of the next section.

C. The Constraint Equation: Digitized Spacing

From equation (4.14), the distance of the first element from the reference element is

$$d_1 = \frac{\lambda}{2\cos\phi_1} = \frac{d}{2K_1}, \qquad (4.14)$$

and from equation (4.18) and (4.19), the distance of each added element from each existing element is

$$d_2 = \frac{\lambda}{2\cos\phi_2} = \frac{d}{2K_2} \qquad (4.18)$$

$$d_3 - d_1 = \frac{\lambda}{2\cos\phi_2} = d_2\,. \qquad (4.19)$$

If now the restriction is imposed that the spacing between elements must be an integer multiple P of some chosen fraction $\frac{1}{Q}$ of a wavelength, the equations become

$$d_1 = \frac{d}{2K_1} = \frac{d}{2} = \frac{P_1}{Q}\lambda$$

$$d_2 = \frac{d}{2K_2} = \frac{P_2}{Q}\lambda$$

$$d_3 = d_1 + \frac{d}{2K_2} = d_1 + \frac{P_2}{Q}\lambda\,, \text{ or in general,}$$

the spacing $S_j = \frac{d}{2K_j} = \frac{P}{Q}\lambda$, where (4.27)

$$K_j = \frac{dQ}{2P\lambda} = \frac{Q}{2P\cos\phi_1}\,. \qquad (4.28)$$

Equation (4.28) gives the available, or allowed, values of K_j in terms of integer multiples of some preselected fraction of a wavelength. Thus, if Q is allowed only the values $0 < Q \leq 2$, the antennas of an array can never be closer than $\frac{\lambda}{2}$ to any adjacent antenna, or element, of the array.

Q and $\cos \phi_1$ are constants in equation (4.28), and it is seen that the minimum value of K_j corresponds to the maximum value of the integer P.

$$K_1 = 1 = \frac{Q}{(2\cos\phi_1)P_1} \tag{4.29}$$

$$P_1 = P_{MAX} = \frac{Q}{2\cos\phi_1} . \tag{4.30}$$

Then combining equations (4.28) and (4.30),

$$K_j = \frac{P_{MAX}}{P} , \quad P = 1, 2, 3, \cdots P_{MAX} . \tag{4.31}$$

This important result indicates that when the desired minimum spacing $\frac{1}{Q}$ between elements, and the desired main beam null angle ϕ_1, have been established, all of the available primary zero values are fixed by equations (4.31) and (4.11), where P can take on any or all integer values from $P_1 = P_{max}$ down to a minimum value of unity. The secondary zeros, given by $(2k + 1)K_j \leq K_{max}$, are also available, and greatly extend the number and location of field pattern nulls available for the purpose of reducing minor lobe maxima.

The only remaining restriction to come from the constraint equation is that which is indicated by equation (4.30): The main beam null angle ϕ_1 and the spacing $\frac{1}{Q}$ must be chosen so that

$$P_{MAX} = \frac{Q}{2\cos\phi_1} = \text{an integer.} \tag{4.32}$$

This is easier to realize if ϕ_1 is an angle whose cosine is a ratio of integers, but the choice of an angle with an irrational cosine does not prevent the use of the method. If q and cos ϕ_1 can be chosen so that P_{max} can be approximated by an integer with very small error the effect on the design will be negligible.

D. A Sample Array Design

In this section a complete array design is developed which shows the use and limits of the constrained procedure. Patterns for the non-uniform array and a uniform array with the same number of elements are presented which show the advantage gained with an array designed by this method.

Assume that an array is required to have a broadside main beam with a 3db beam width of approximately 5°. The minimum spacing between elements is arbitrarily chosen to be $\frac{5}{8}\lambda$ for this example, but it is understood that this choice would be subject to an optimization procedure in a computer design. For the values chosen, the equations of the previous section yield

$$Q = \frac{8}{5}$$

$$\cos \phi_1 = 0.10$$

$$d_1 = \frac{\lambda}{2\cos\phi_1} = 5\lambda$$

$$P_{MAX} = \frac{Q}{2\cos\phi_1} = 8$$

$$K_j = \frac{P_{MAX}}{P} = 1,\ \frac{8}{7},\ \frac{4}{3},\ \frac{8}{5},\ 2,\ \frac{8}{3},\ 4,\ 8.$$

The pattern for the system of two antennas spaced 5λ apart will contain 5 grating lobes. The values of K_j determined above are values for which we can force zeros in the field pattern according to equation (4.25):

$$\phi = \cos^{-1}\left[(2k-1)(\cos\phi_1) K_j\right] , \quad k = 1, 2, \cdot \cdot \cdot .$$

A plot of the normalized field pattern for the two antennas is given in Figure 4.4 on the following page. Superimposed on this graph are X's indicating location of available zeros, plotted vs. the integer values of P. With reference to the chart, it is seen that by choosing the spacing multiplier P = 7 we can force zeros at approximately the same angles as given by the values P = 1 and P = 2. In general this will be true: When a choice exists between values of P to obtain a specified zero, the higher value will usually be the better choice. More secondary zeros are available from the larger value of P, and therefore lower sidelobes will be obtained without the use of additional elements.

By the type of analysis indicated above, the design values P = 7, 6, and 4 are chosen, in addition to $P_{max} = 8$ which is required for the first null and grating lobe zeros. The array will require $2^N = 2^4 = 16$ elements, where N is the number of values of P chosen for the design. The array may be built up by solving for each of the necessary spacings, or the following approach can be used: Since each of the P values represents the integer multiple of the quantized spaces that each added antenna must be located away from the element it is being paired with, the positions may be numbered from 0 to some maximum value. If the P values are ordered such that $P_1 = P_{max}$, P_2 = the next largest value of P selected, etc., then the aperture (in terms of the number

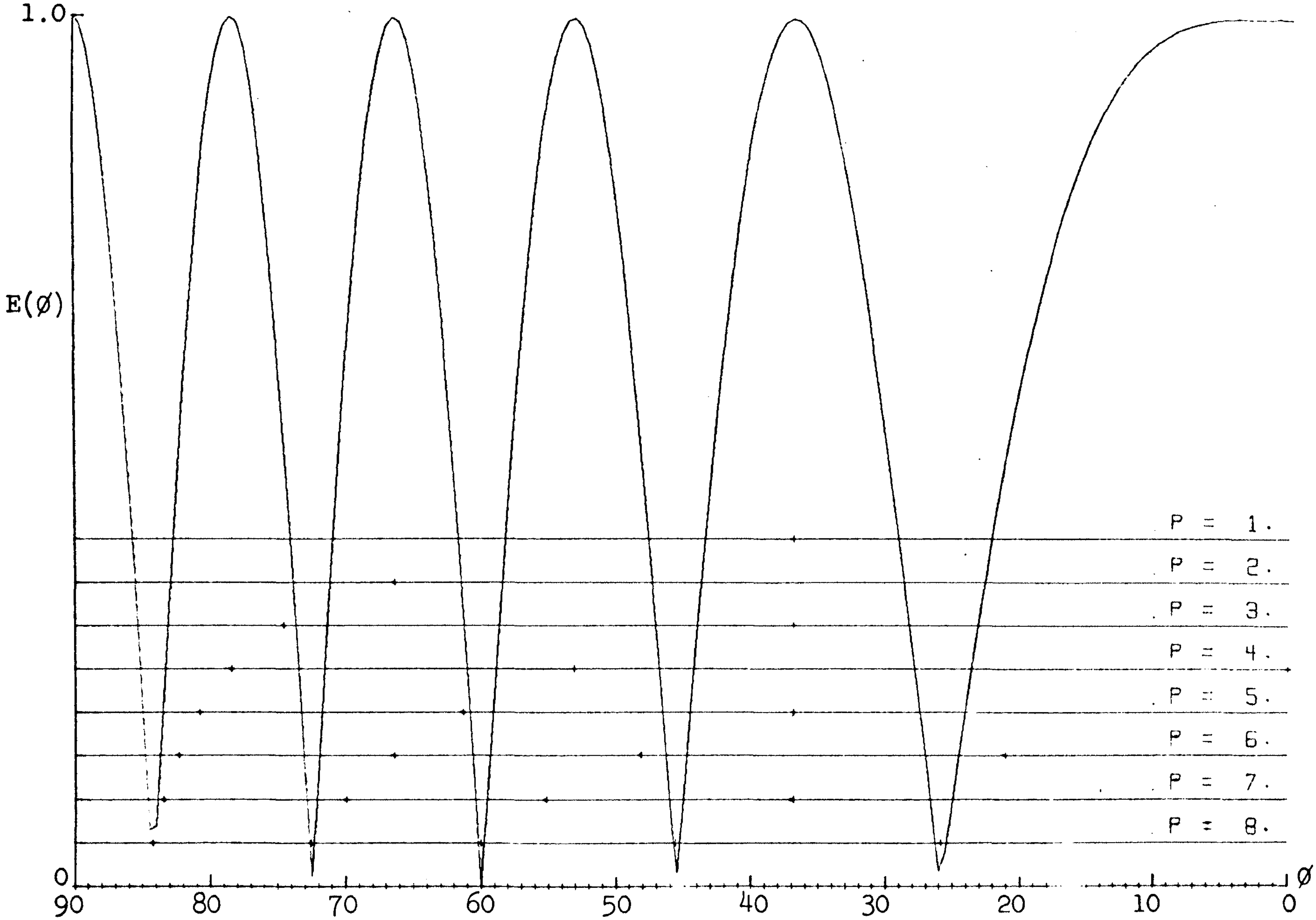


Figure 4.4. E(Ø) vs Ø, two antennas 5λ apart, with zeros for antennas added at P(5/8)λ.

of digitized spaces) will be given by $L = P_1+P_2+ \ldots +P_N$. The antennas will occupy positions whose numbers are obtained by taking the sum (or number) given by taking the N values of P one at a time, then two at a time, then three at a time, until finally they are taken N at a time to obtain the aperture. This is illustrated in the plot below, where the dots indicate antennas positioned by taking the values of P obtained above.

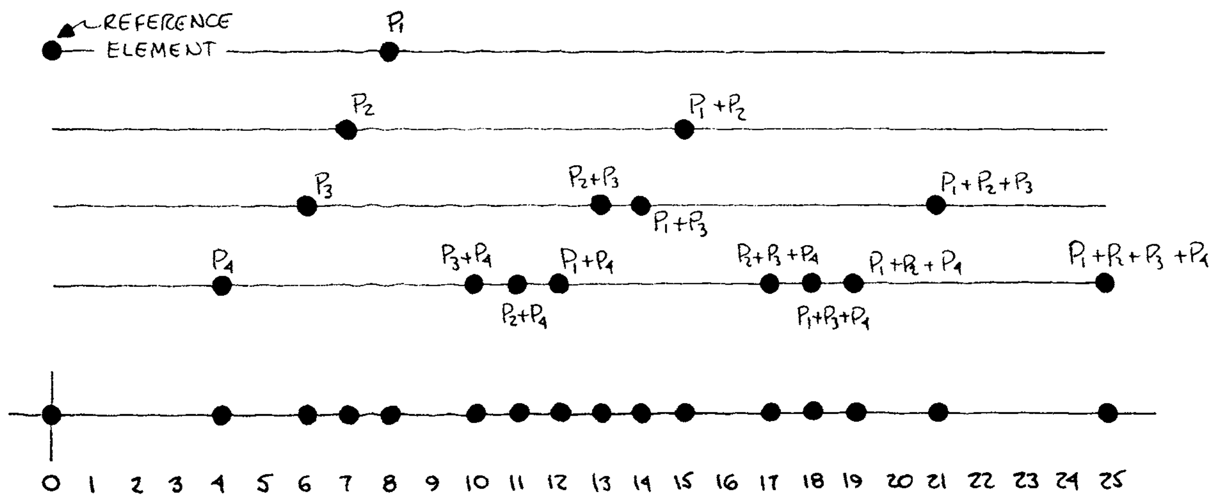


Figure 4.5. Linear array obtained from design parameters.

The bottom row indicates the completed array, for the values chosen. There is no coincidence of elements in this design, but it is seen that the choice of $P_4 = 5$ instead of $P_4 = 4$ would have resulted in two elements, or double current amplitude, in position #13. Mathematically, the conditions for coincidence are seen to be that the sum of one combination of the P_i's is equal to the sum of a different combination. The more P values used in any given design, the more opportunity there will be for coincidence. Thus, even though the number of individual "elements" increases as 2^N, where N is the number of values of P used, the higher rate of coincidence for large N will reduce the total number of antennas considerably.

Plots of the field patterns for the array designed above, and for a uniform linear array with the same number of elements, appear on the following pages for comparison. It is seen that the non-uniform array has a distinct advantage in terms of maximum sidelobe level, and also in suppression of the minor lobe immediately adjacent the main beam. Plots for several additional examples are presented in Chapter V, along with a short summary of possible digital computer design methods.

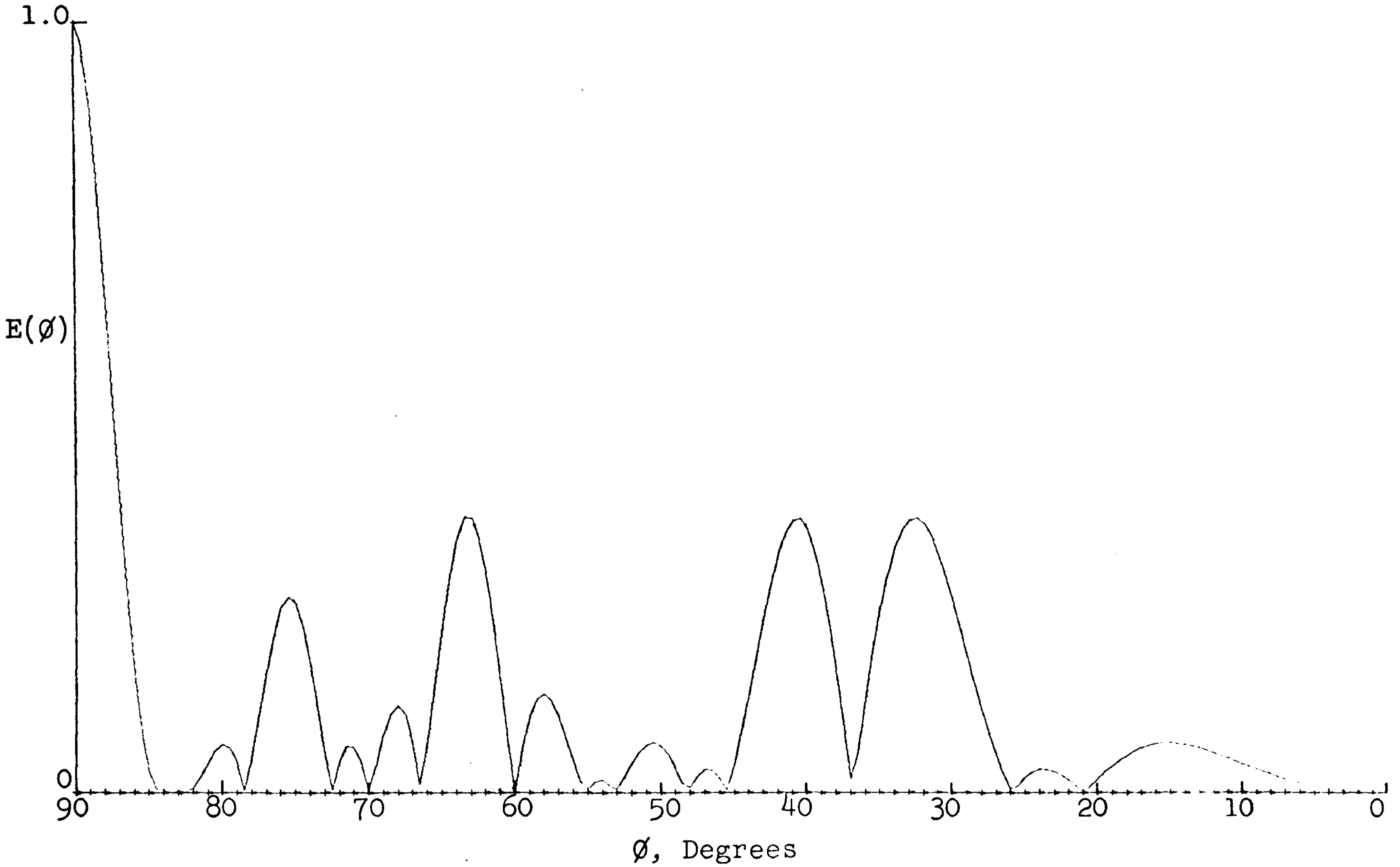


Figure 4.6. Field pattern for non-uniform array; 16 elements with 25λ aperture.

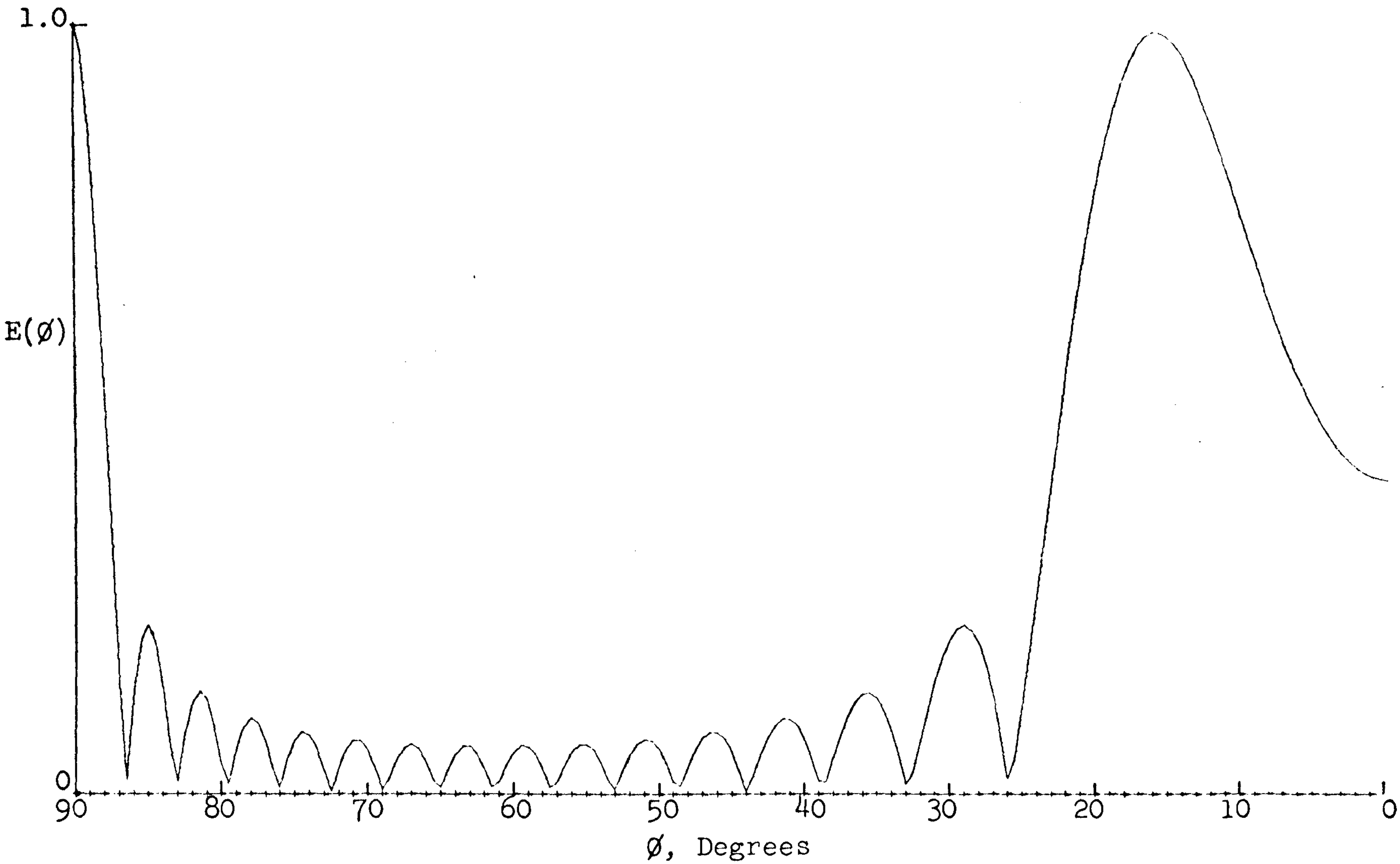


Figure 4.7. Field pattern for uniform linear array; 16 elements with 25λ aperture.

CHAPTER V

USE OF THE DIGITAL COMPUTER IN ARRAY DESIGN

A. Programming the Computer

One possible method for designing an array with unequal spacings is total enumeration (10). In this approach all possible combinations of spacings are examined, the radiation pattern computed for each combination, and the best pattern selected. Although it is possible in principle to implement such a brute-force procedure, it is generally not practical except in the simplest of cases. If each of the N elements of an array can occupy any of m possible positions, there are a total of m^N combinations that must be examined. Ten elements, each capable of occupying ten different possible positions, would result in ten billion combinations for which patterns would have to be computed. Thus, a programming scheme which allows for many of the alternatives to be discarded before they are examined completely must be devised for computer analysis and design to be a practical approach.

Since the sidelobes of the field pattern of an array are significantly dependent on the arrangement of elements in the array, it seems reasonable to establish the criteria for selection on the basis of the sidelobe properties. One criterion might be to make all sidelobes of equal amplitude, similar to the Dolph-Tchebycheff (2) method of conventional antenna design. This is probably not possible, however, because of the lack of sufficient degrees of freedom (elements) in a "thinned" array of unequally spaced antennas (13) to specify the radiation pattern at a large number of coordinates. The criterion used in this study is an attempt to make the sidelobes as uniform as possible. This is done by programming the computer so as to select the design whose highest sidelobe over the specified interval is less than the highest peak of any other pattern, and by choosing values of the design parameters which will force nulls nearest the maximum of any minor lobe which is to be suppressed.

As an example of the computer program, the following review of the design example of Chapter IV will show the logic involved in the determination of the optimum design for a given set of conditions on main lobe and spacing.

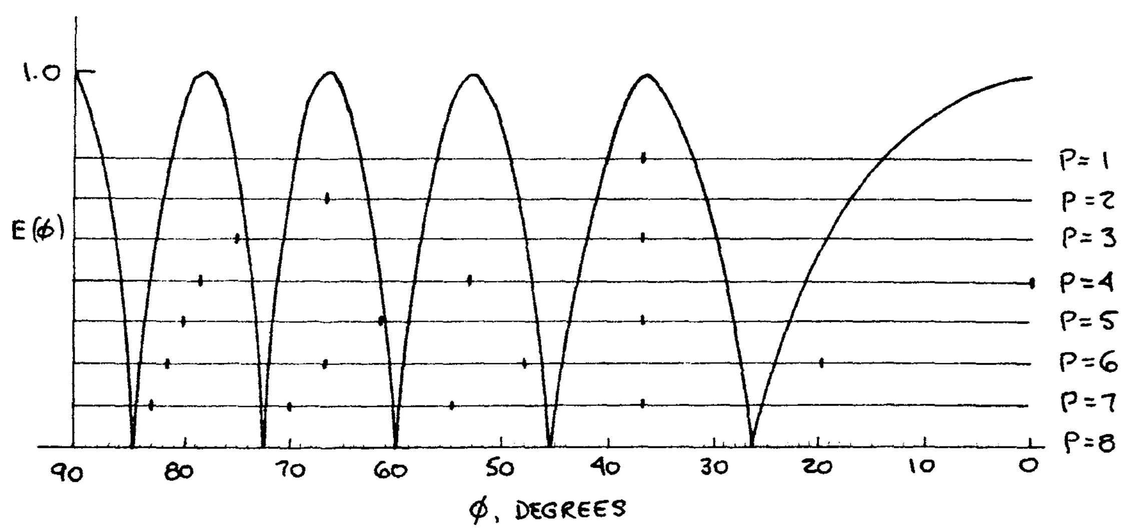


Figure 5.1. Superimposed plots of design parameters.

The reduction of the grating lobes involves forcing zeros at or near the sidelobe maxima. The procedure should start with the end-fire lobe and move toward the main beam null. In this case there is only one possibility for a null at $\phi = 0^\circ$, so $P = 4$ is selected. The examination then moves to the next maximum, where it is found that either $P = 1, 3, 5$ or 7 may be chosen. When two or more values are available the largest one will be selected since more "free" nulls will be obtained. $P = 7$ is then chosen to provide the zero at the second maximum. The third maximum, at $\phi = 53.1^\circ$ in this case, has already been suppressed by the choice of $P = 4$ and without the use of additional elements. The next maximum may be suppressed by choosing $P = 2$ or $P = 6$, and again the higher value is chosen because of the correspondingly higher number of "free" zeros. The last sidelobe has already been suppressed, with $P = 4$, so the first selection of P's is complete and the pattern may be computed.

Before proceding to the calculation of the field pattern it should be noted here that selection of $P = 1, 2, 4$ and 8 would result in a uniform linear array of the same number of elements as the array chosen. This will always be the case: If there are N values of P chosen for an array design, there will be 2^N elements in the completed array, some of which may be coincident. A uniform linear array of the same number of elements may always be realized by choosing $P = 2^k$, $k = 0,1,2...(N-1)$, and using $P = 2^{(N-1)} = P_{max}$ for the uniform array field pattern. In general it will not be true that zeros will be available at each of the P's corresponding to the uniform array, as they are in this example.

This is just a coincidence, due to the values ϕ_1 and Q chosen, and is taken advantage of for this example.

An investigation of the symmetry evident in the array plotted in Figure 4.5 shows that this symmetry will appear in every array produced by this method. The two end terms will always be the reference and the term given by $L = P_1 + P_2 + P_3 + \ldots + P_N$. The next two elements in from each end will always be the reference plus the smallest value of P and the last term minus the smallest value of P_1, etc. There will always be an even number of terms, with 2^N elements including coincidence, and with symmetry about $\frac{L}{2} = \frac{P_1 + P_2 + \cdots + P_N}{2}$, so the field pattern may be calculated using a summation of cosine terms. For the 16 element array there will be 8 cosine terms, the first of which will be due to the outside pair of elements and is given by

$$\begin{aligned} \bar{E}_1(\phi) &= \cos\left(\frac{\beta d}{2}\cos\phi\right) \\ &= \cos\left\{\beta\left[\frac{(P_1+P_2+P_3+P_4)}{2}\frac{1}{Q}\lambda\right]\cos\phi\right\} \\ &= \cos\left[(P_1+P_2+P_3+P_4)\frac{\pi}{Q}\cos\phi\right]. \end{aligned} \tag{5.1}$$

The distance between the next pair of elements is $(P_1+P_2+P_3)-(P_4)$, between the next pair $(P_1+P_2+P_4)-(P_3)$, etc. If a "distance factor" M_i is substituted into equation (5.1), such that

$$\bar{E}_1(\phi) = \cos\left[\left(\frac{M_1\pi}{Q}\right)\cos\phi\right] \tag{5.2}$$

$$M_1 = P_1+P_2+P_3+P_4 \quad ,$$

the total field pattern for the array of the example may be written

$$\bar{E}(\phi) = \frac{1}{2^{(N-1)}} \sum_{i=1}^{2^{(N-1)}} \cos\left[\left(\frac{M_i\pi}{Q}\right)\cos\phi\right] , \tag{5.3}$$

where N = the number of P terms selected and the M_i = all possible sign combinations in the expression $(P_1 \pm P_2 \pm P_3 \pm \ldots \pm P_N)$. The magnitude of $\overline{E}(\emptyset)$ is then plotted, to obtain the usual relative field pattern for the array. The relative power pattern may be obtained from the square of the relative field pattern. Directivity, which is a function of the total power, may be found from the area under the power curve, and several other parameters of interest may be determined from either or both of the relative patterns.

The study for this thesis concluded with the comparison of sidelobe level between arrays designed by the method presented and uniform linear arrays of the same number of elements of the same aperture. Several computer plots, with completely descriptive titles, are presented on the following pages, and the advantages of the technique developed here are clearly demonstrated.

B. Analysis of Computer Design Examples

The computer plots on the following four pages are representative of those that would be obtained in a design procedure. The first one, Figure 5.2, is a plot of the relative field pattern for two antennas for which the broadside beam null is at an angle $\emptyset_1 = \cos^{-1}(.10)$. Superimposed on this plot are markers indicating the angles at which nulls may be obtained when antennas in an array are restricted to positions which are multiples of 1.25λ from the reference element. The second plot is for the same initial element

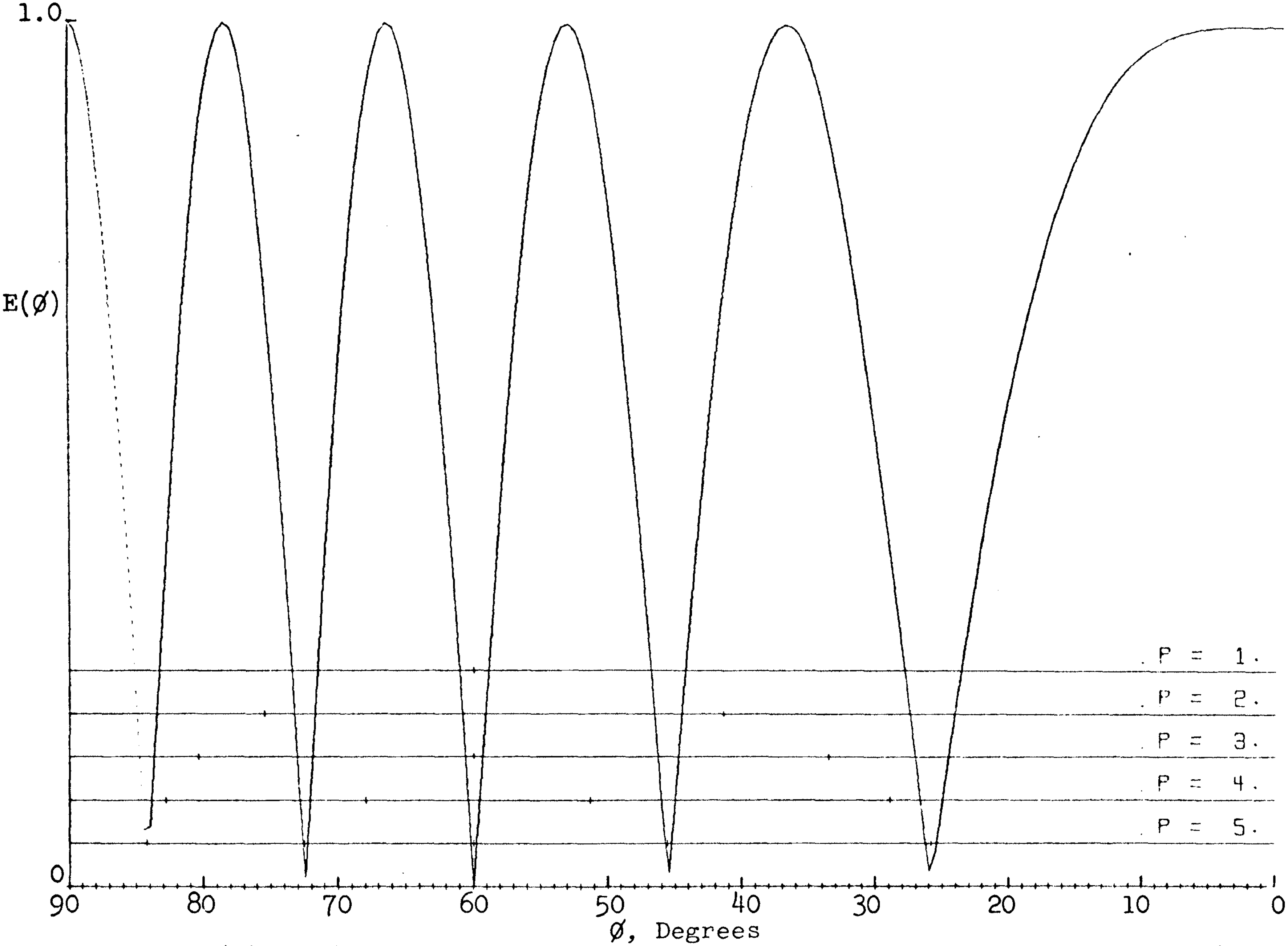


Figure 5.2. E(Ø) vs Ø, two antennas 5λ apart, with zeros for antennas added at (1λ)P.

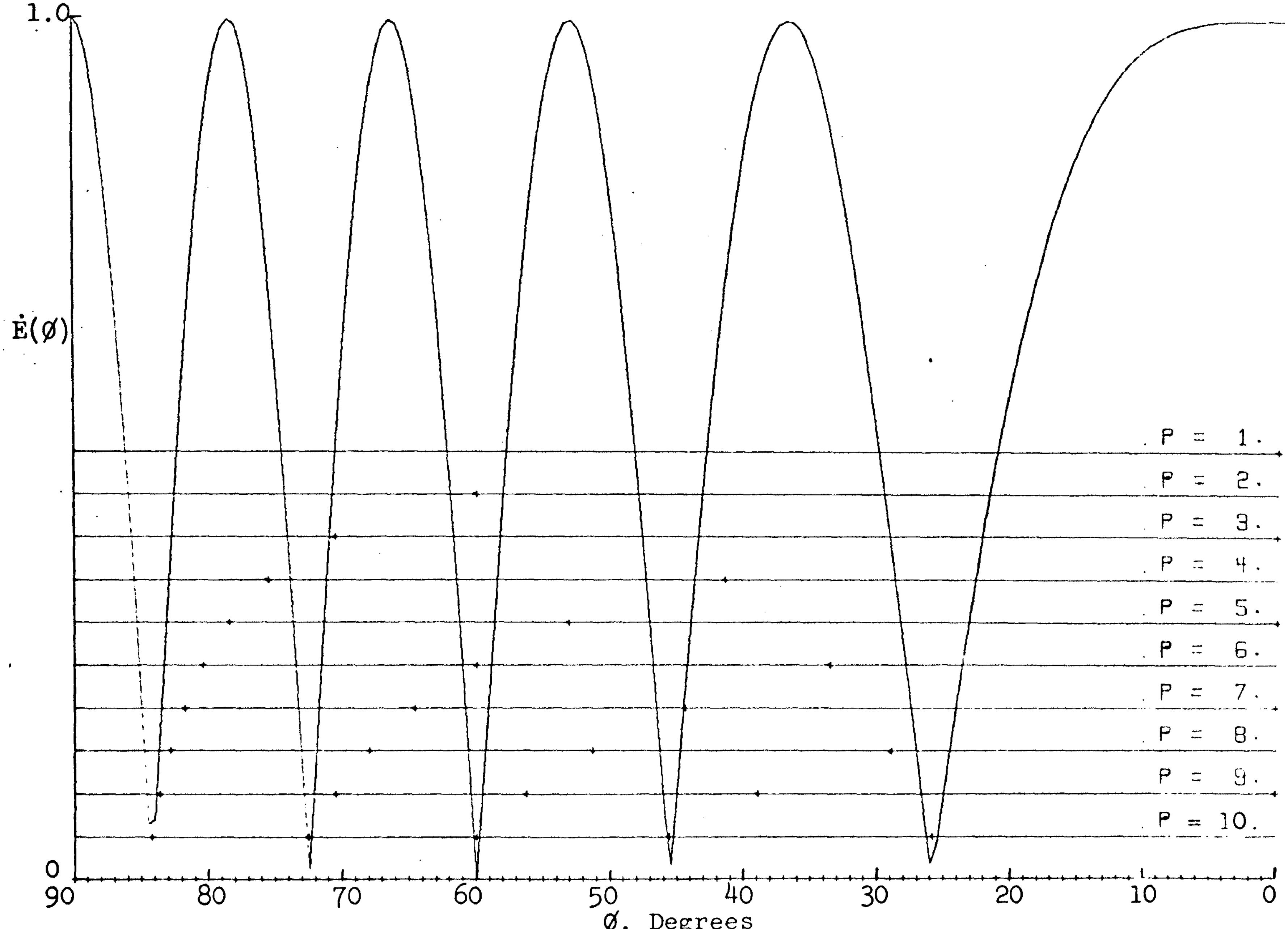


Figure 5.3. E(Ø) vs Ø two antennas 5λ apart with zeros for antennas added at (½λ)P.

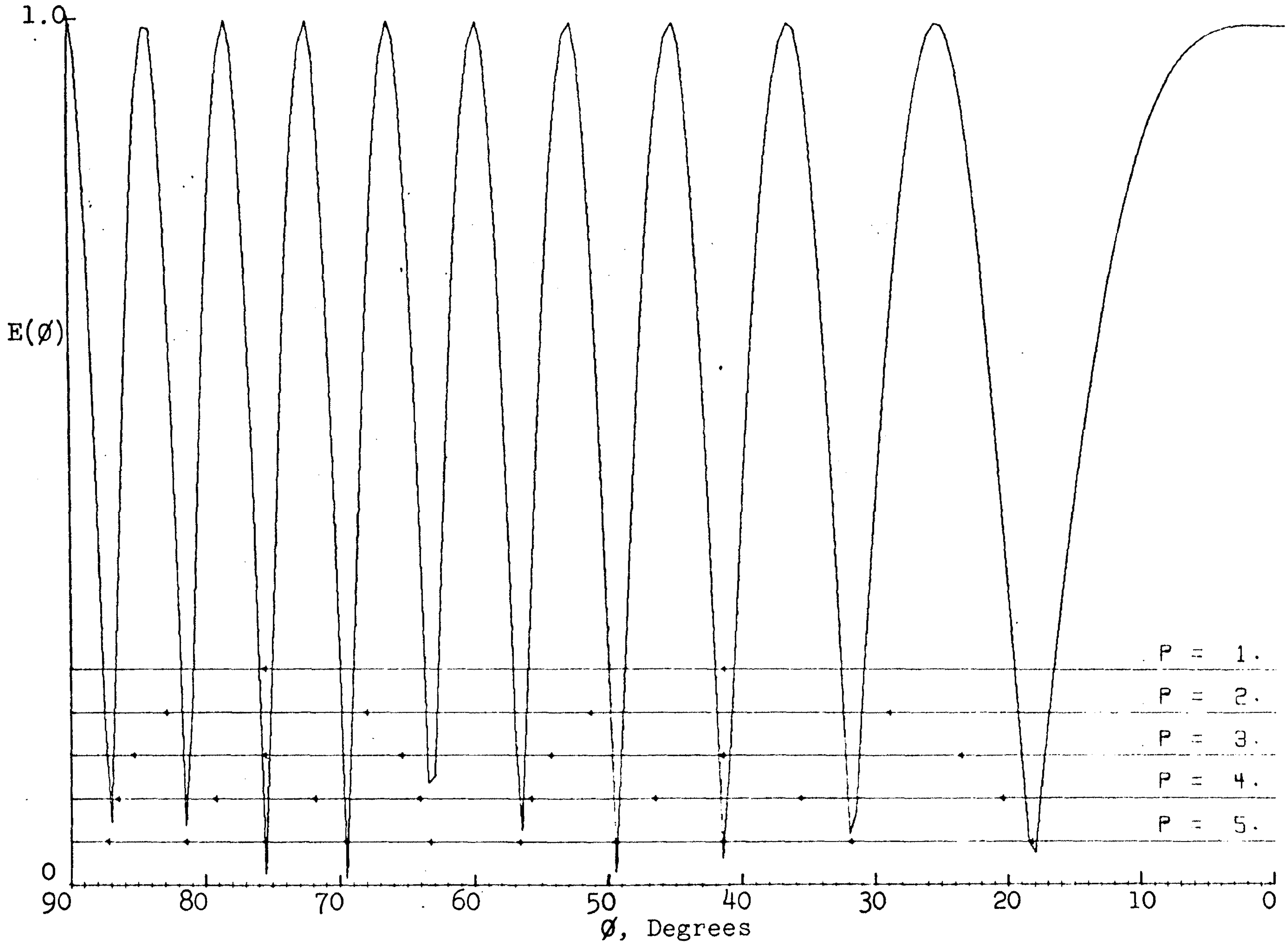


Figure 5.4. E(Ø) vs Ø, two antennas 10λ apart, with zeros for antennas added at (2λ)P.

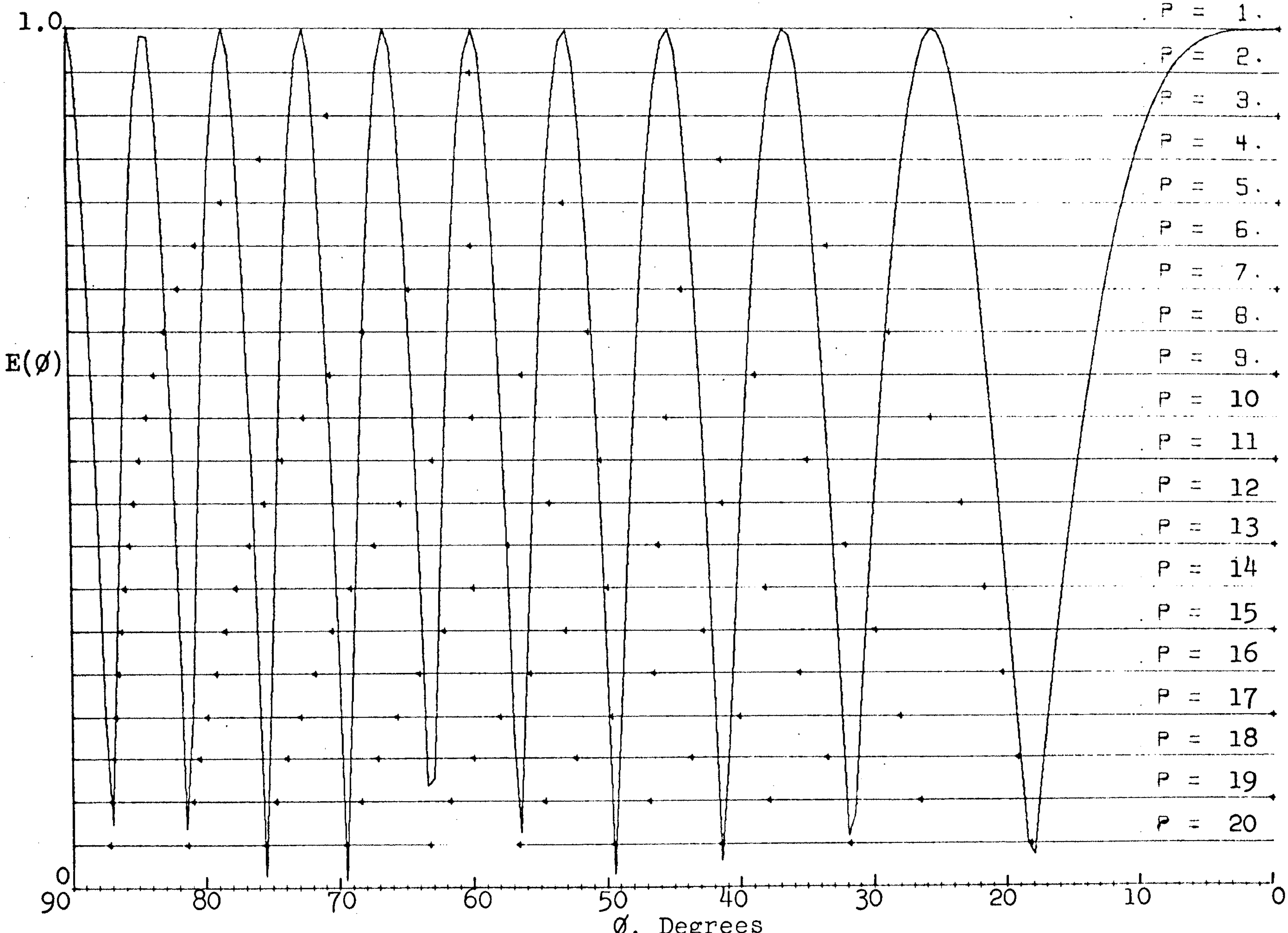


Figure 5.5. E(Ø) vs Ø, two antennas 10λ apart, with zeros for antennas added at (½λ)P.

spacing, but indicates zeros available when antennas are forced into positions which are in $\frac{1}{2}\lambda$ increments. The next two plots are for antennas with broadside beam null at an angle $\phi = \cos^{-1}(.05)$ and for position increments of 2λ and $\frac{1}{2}\lambda$ respectively.

It is readily seen that smaller increments in spacing yield more versatile arrays. That is, there are many more available field pattern nulls for the $\frac{1}{2}\lambda$ spacings than for either of the larger spacings. In many cases there are combinations of zeros near the top of the chart which may be produced by selection of a single value near the bottom. In Figure 5.5, for example, the nulls available with the selection of P = 1, 3, and 5 are also available with the selection of the single value P = 15. In an array of 2^N elements, where N is the number of values of P chosen for the design, the substitution of one value for three values would mean a saving of $(2^N - 2^{N-2})$ elements without any change in field pattern nulls.

Figure 5.6 is a plot of the field pattern for eight elements occupying digitized positions $(5/6)\lambda$ apart. Two very strong minor lobes are evident in the pattern. When compared with the pattern of eight equally spaced elements distributed in the same 11.67λ aperture, as given in Figure 5.7, the suppression of the lobe nearest the main beam, and the reduction of the grating lobe, in the first pattern are evident. The pattern of Figure 5.8 shows the effect of adding another set of zero-producing elements to the array.

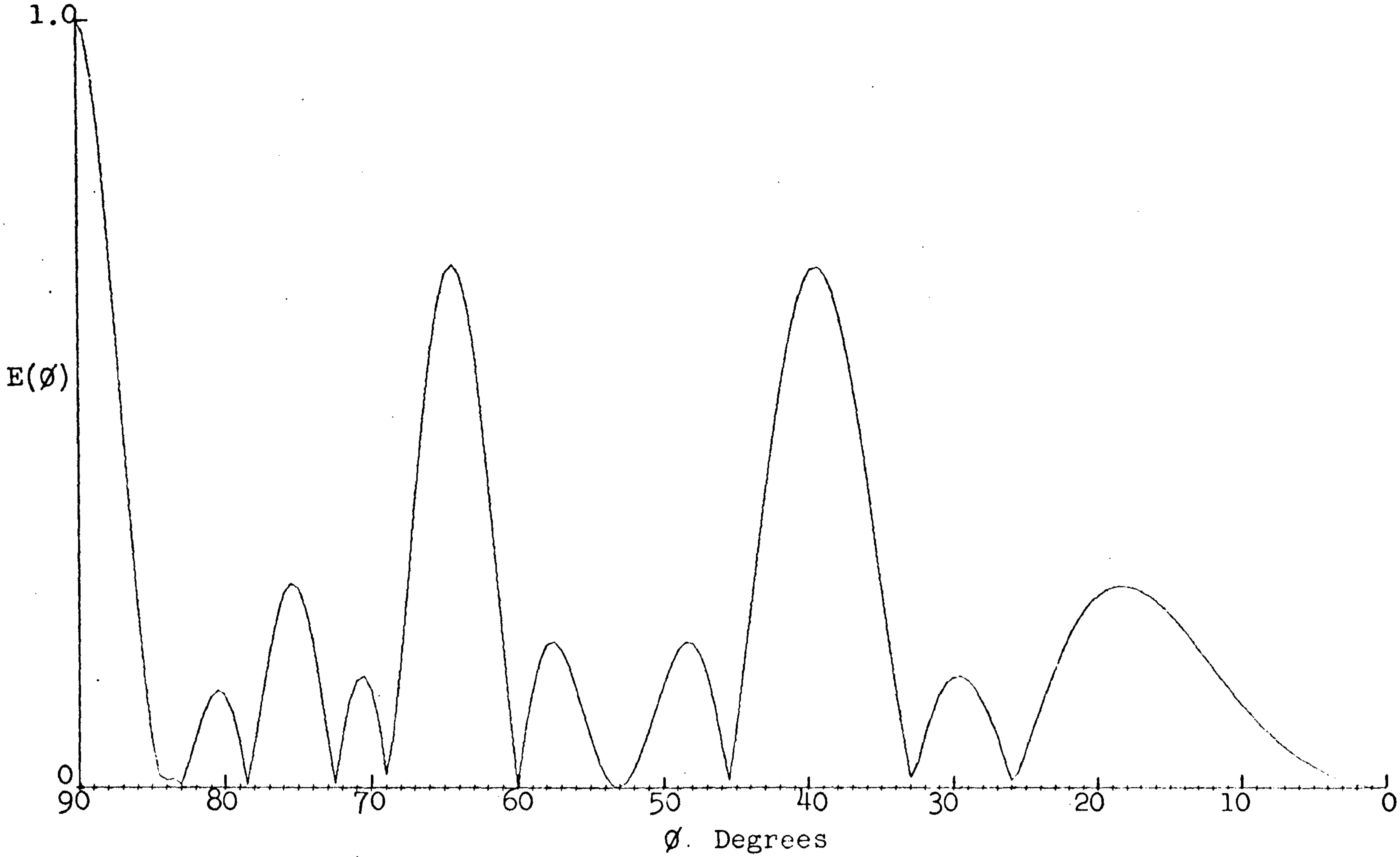


Figure 5.6. Pattern for 8 element array, elements in (5/6)λ digitized positions.

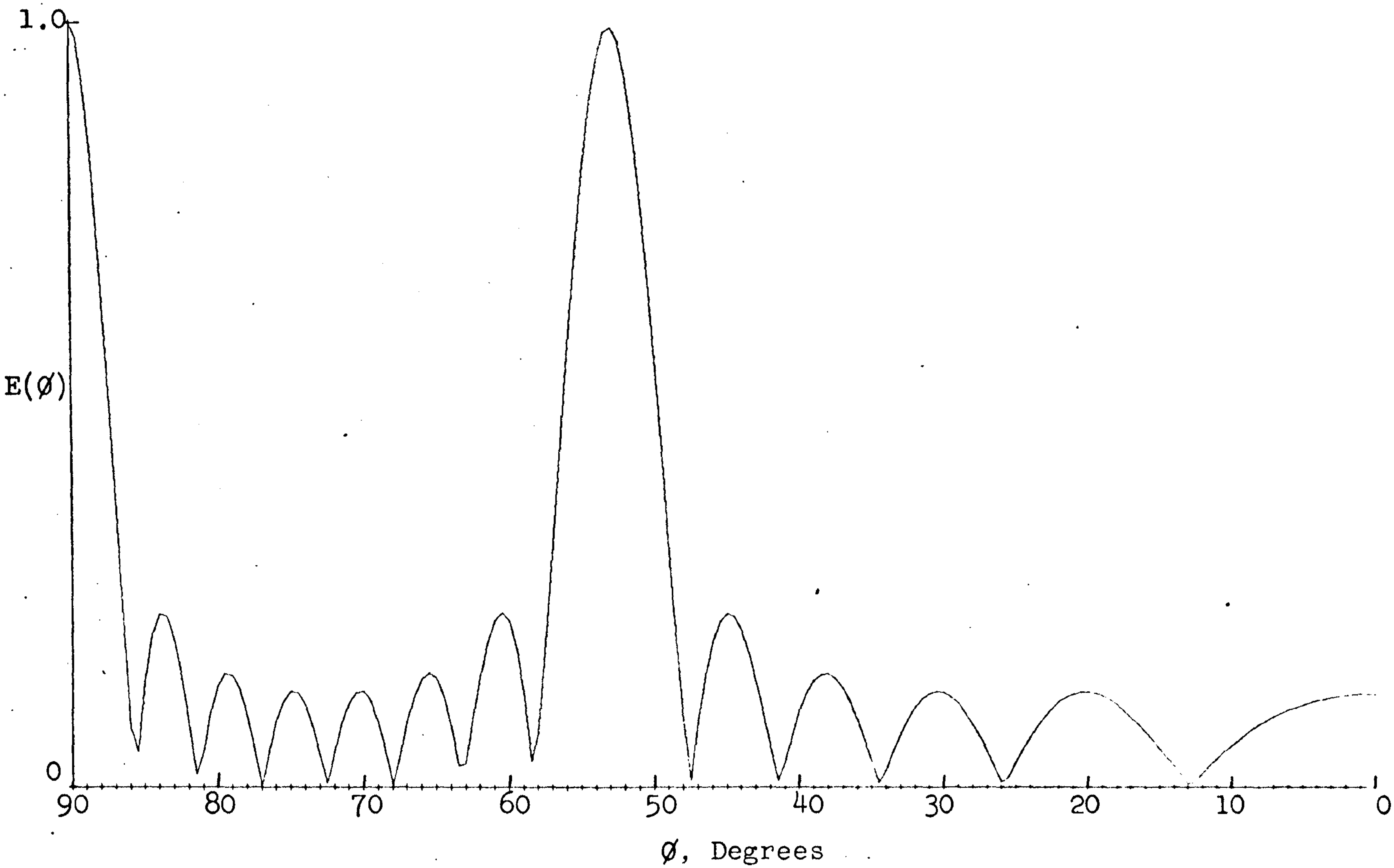


Figure 5.7. E(Ø) vs Ø for uniformly spaced 8 element array with 11.67λ aperture.

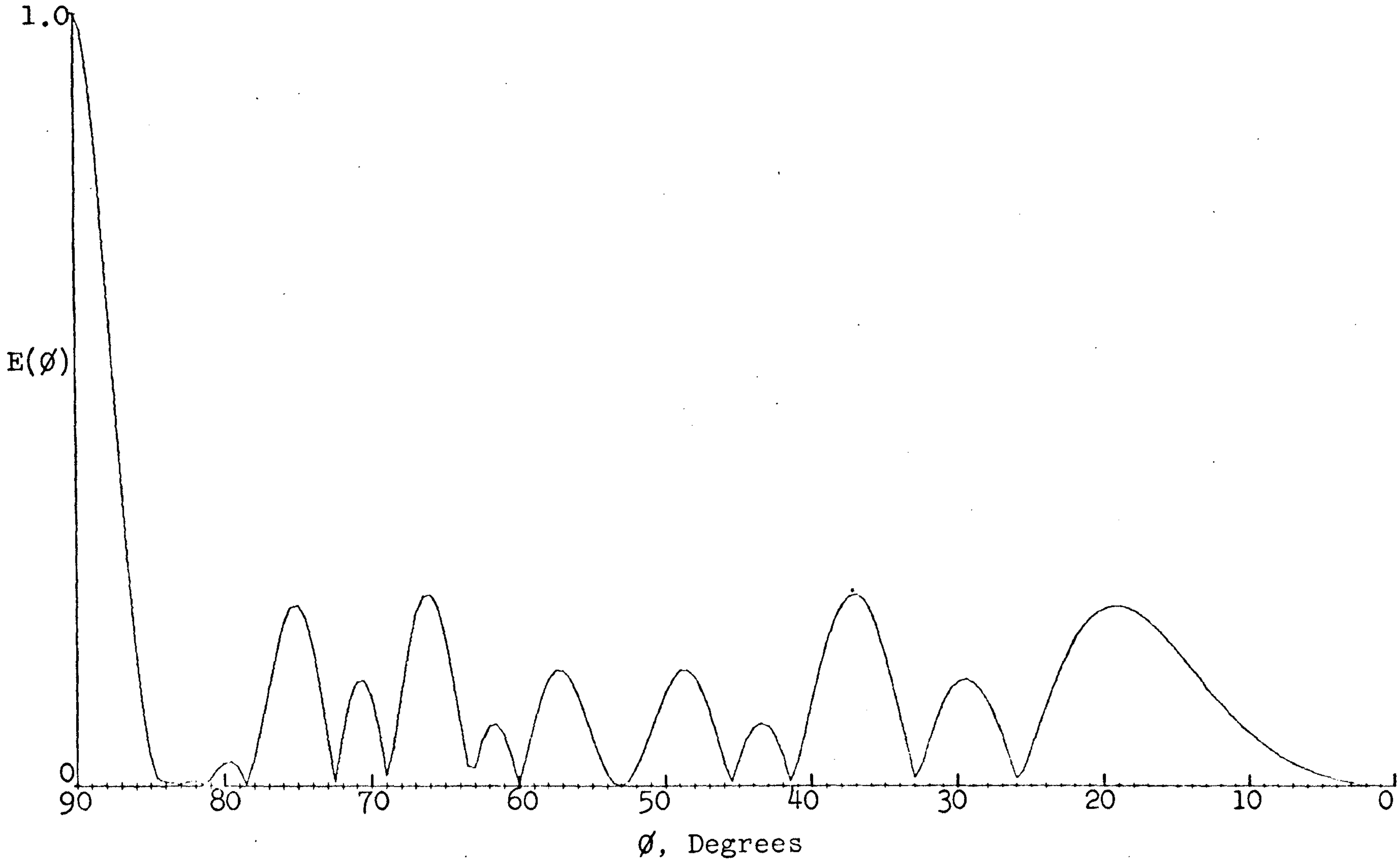
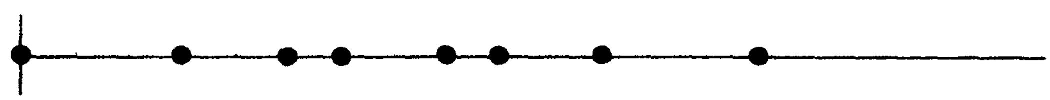


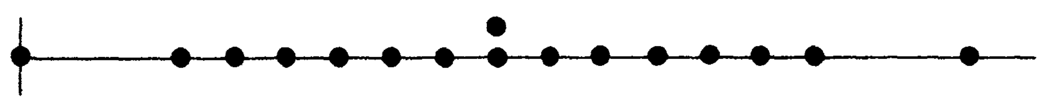
Figure 5.8. Field pattern for 16 element non-uniform array with 15λ aperture.

The total number of antennas required for this pattern is 16, and the aperture is 15λ. The pattern for this array is significantly better than for an equally-spaced array of the same 15λ aperture, for the uniform array would result in equal 1λ spacings and would show the characteristic very strong end-fire, or on-axis, minor lobe.

The location of the elements in the arrays for Figures 5.6 and 5.8 are plotted below. The coincidence of elements



(a.) 8-element array for pattern of Figure 5.6

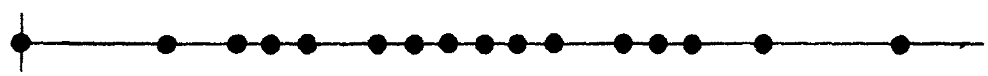


(b.) 16-element array for pattern of Figure 5.8

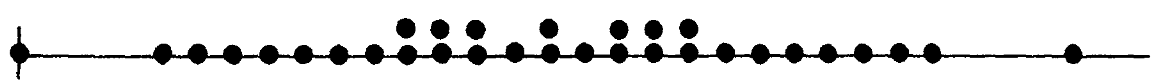
Figure 5.9. Antenna arrays with (5/6)λ spacing.

in position 9 of the array of Figure 5.9b corresponds to doubling the current amplitude to that element, and results in reduction of the number of required elements. This particular feature of the design method becomes more important, and is more evident, in the design of larger arrays, where it may represent a real saving in space and costly antennas.

As an additional example of element coincidence, the example problem of Chapter IV will be re-examined. When zeros corresponding to the value P = 5 in Figure 4.4 are added to the field pattern of Figure 4.6, the sidelobes are significantly lowered, and the main beam width is slightly reduced because of the new zero forced immediately adjacent to the first null. Figures 5.11 and 5.12 show the patterns for the 16 element array of Chapter IV and the 32 element array pattern with the added zeros. The 32 element array, plotted in Figure 5.10 below, shows coincidence in 7 positions and may be constructed with only 25 antennas. This is a reduction of more than 20 per cent in the number of elements required in the array, and represents a considerable saving.



(a.) 16 element array for pattern of Figure 5.11



(b.) 32 element array for pattern of Figure 5.12

Figure 5.10. Antenna arrays with $(5/8)\lambda$ spacing.

Figures 5.13 through 5.18 were obtained for a design requirement that the main beam null angle be $\phi_1 = \cos^{-1}.05$, so that the half-power beam width would be approximately 3^o. Plots similar to Figures 5.4 and 5.5 were used to manually obtain values of P from which to construct arrays with position increments of $\frac{1}{2}\lambda$ and 1.25λ. Figure 5.13 gives the field pattern for 32 elements, occupying digitized positions $\frac{1}{2}\lambda$ apart, in a total aperture of 25. There are six instances of element coincidence in this design,

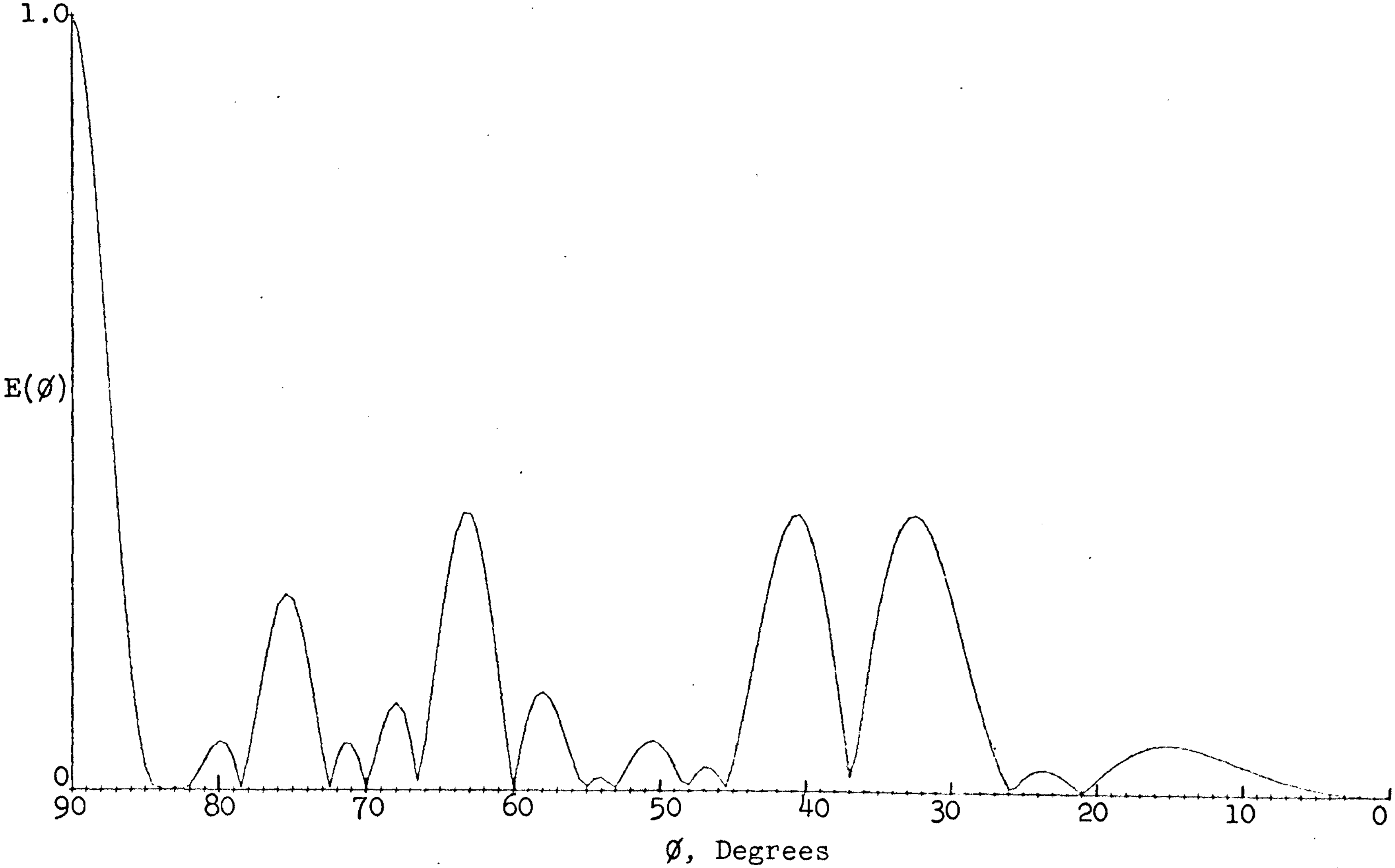


Figure 5.11. Pattern for 16 element array with (5/8)λ digitized positions.

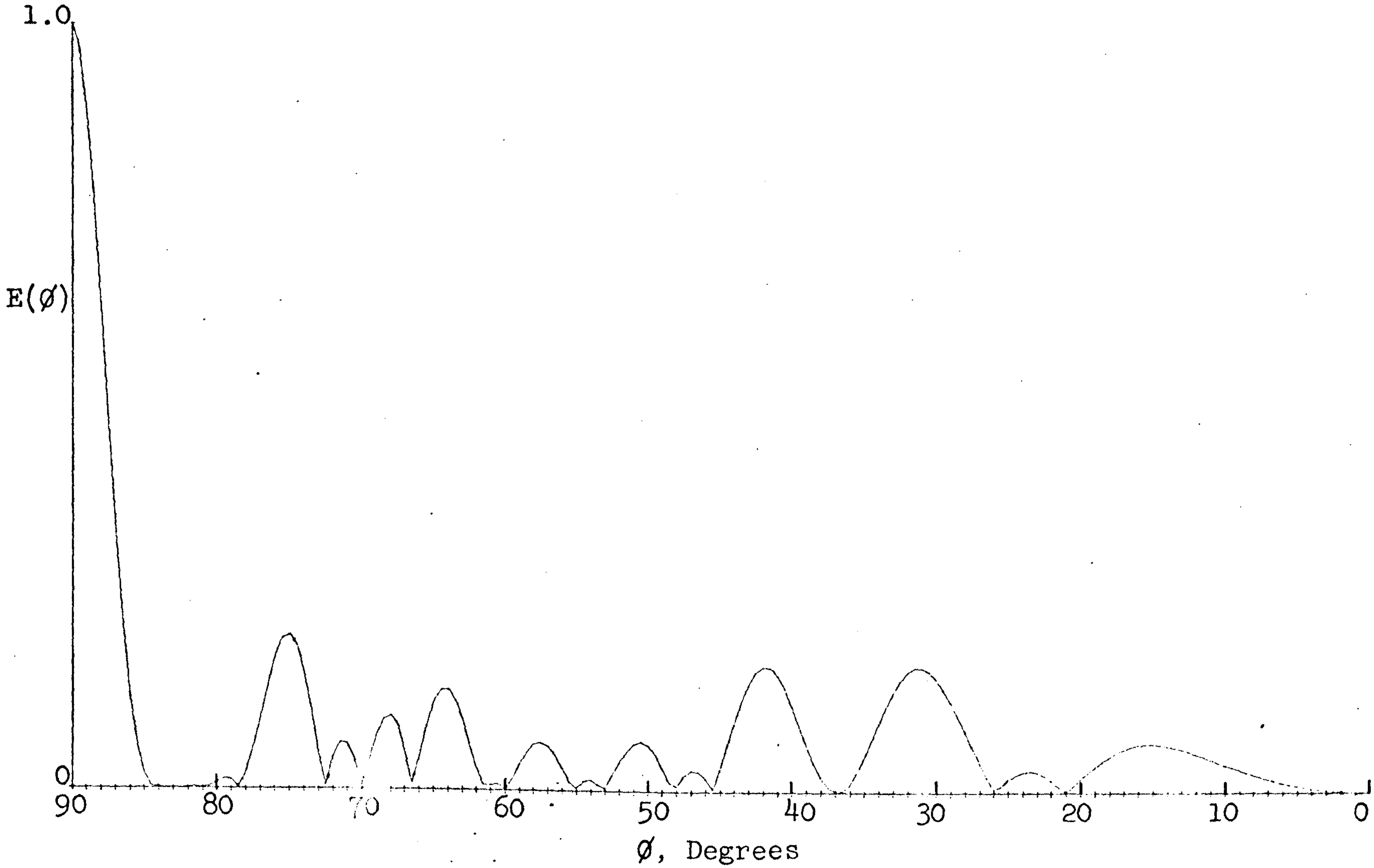


Figure 5.12. Field pattern for 32 element array with 6 coincident elements.

reducing the number of actual antennas from 32 to 26 in the array. When compared with the field pattern for equally-spaced elements in the same aperture, Figure 5.14, it is seen that there are no sidelobes in the non-uniform array pattern any higher than those for the uniform case, the grating lobe does not exist in the non-uniform case, and the minor lobe nearest the main beam has been suppressed in the non-uniform case. A problem does exist in that the large minor lobe has been moved nearer the main beam in the non-uniform array pattern. In an attempt to reduce this large minor lobe several different spacing increments were used, with the same elements as above. Figure 5.15 gives the field pattern obtained for digitized positions .40λ apart with the same array. In this case the large lobe near the main beam is completely suppressed, and only the lobe near the axis of the array has increased in size. This is a much improved design, but has been obtained manually, and has not been optimized in any sense.

Figure 5.16 gives the field pattern obtained from an array of 16 elements, in positions which are multiples of 1.25λ from the reference element, with a total aperture of 31.25λ. The narrow beam width and characteristic suppression of the minor lobe nearest the main lobe are evident. The presence of a grating lobe in this design illustrates the very basic problem with arrays involving spacing between elements of more than one wavelength: when the element positions are equally spaced, in increments larger than one wavelength, one or more grating lobes will appear in the pattern. This is true whether the positions are occupied or not, and cannot be avoided for a broadside array with in-phase currents fed to the elements.

Figure 5.17 gives the pattern obtained with the same 16 elements spaced uniformly over the same 31.25λ aperture. In this case there are two grating lobes, because the distance between elements exceeds 2λ. The non-uniform array is "better" than the uniform array in the sense that both yield the same beamwidth with the same number of elements, but the non-uniform array pattern contains only one grating lobe in the range of variables under consideration. Figure 5.18 concludes this section, and is the plot of the pattern obtained from the same array as Figure 5.16, except with spacing increments reduced to $(2/3)\lambda$. This design shows the reduction of the grating lobes when spacing is reduced, and is not intended to represent an optimum array pattern.

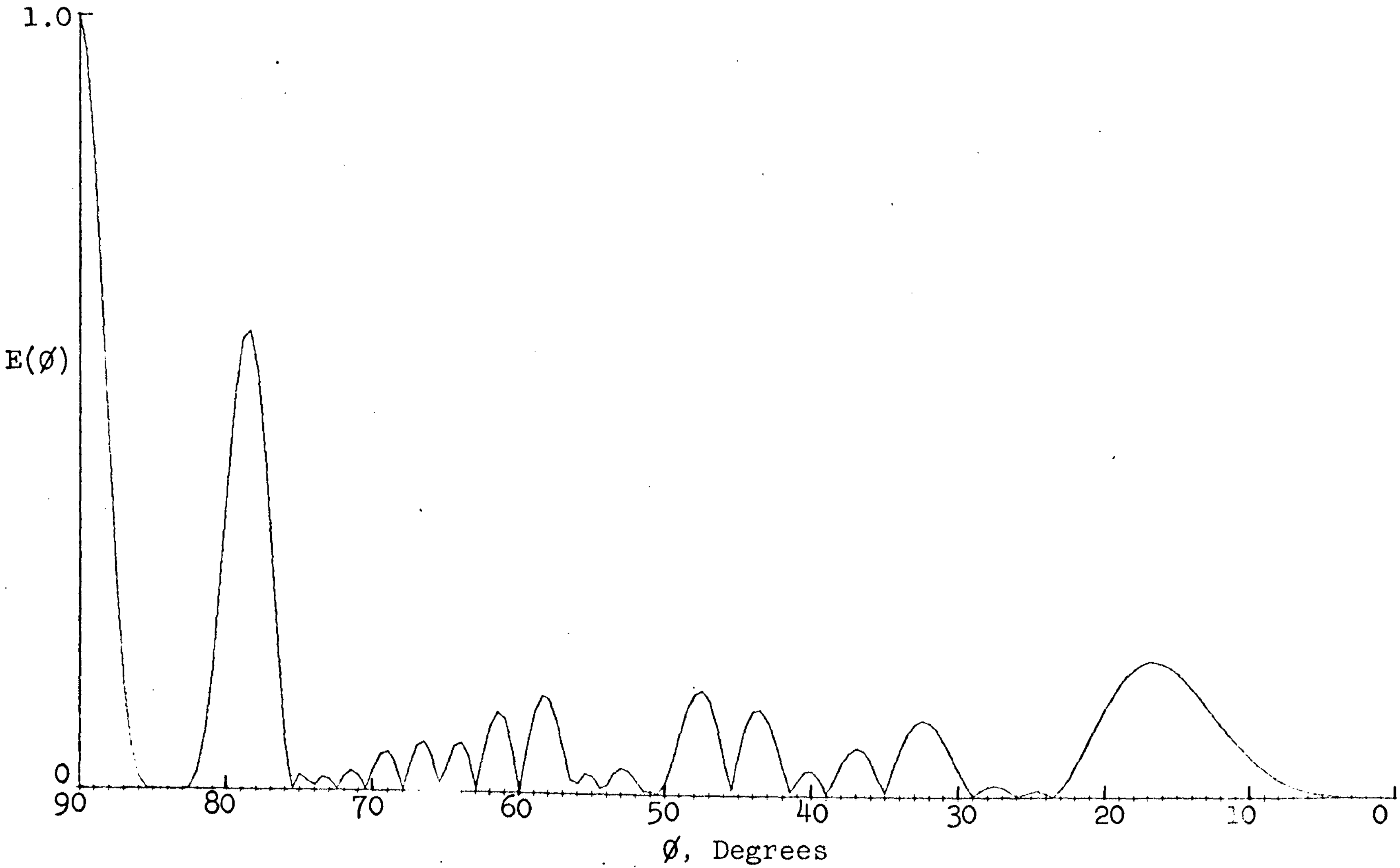


Figure 5.13. Field pattern for non-uniform array with ($\frac{1}{2}\lambda$) position increments.

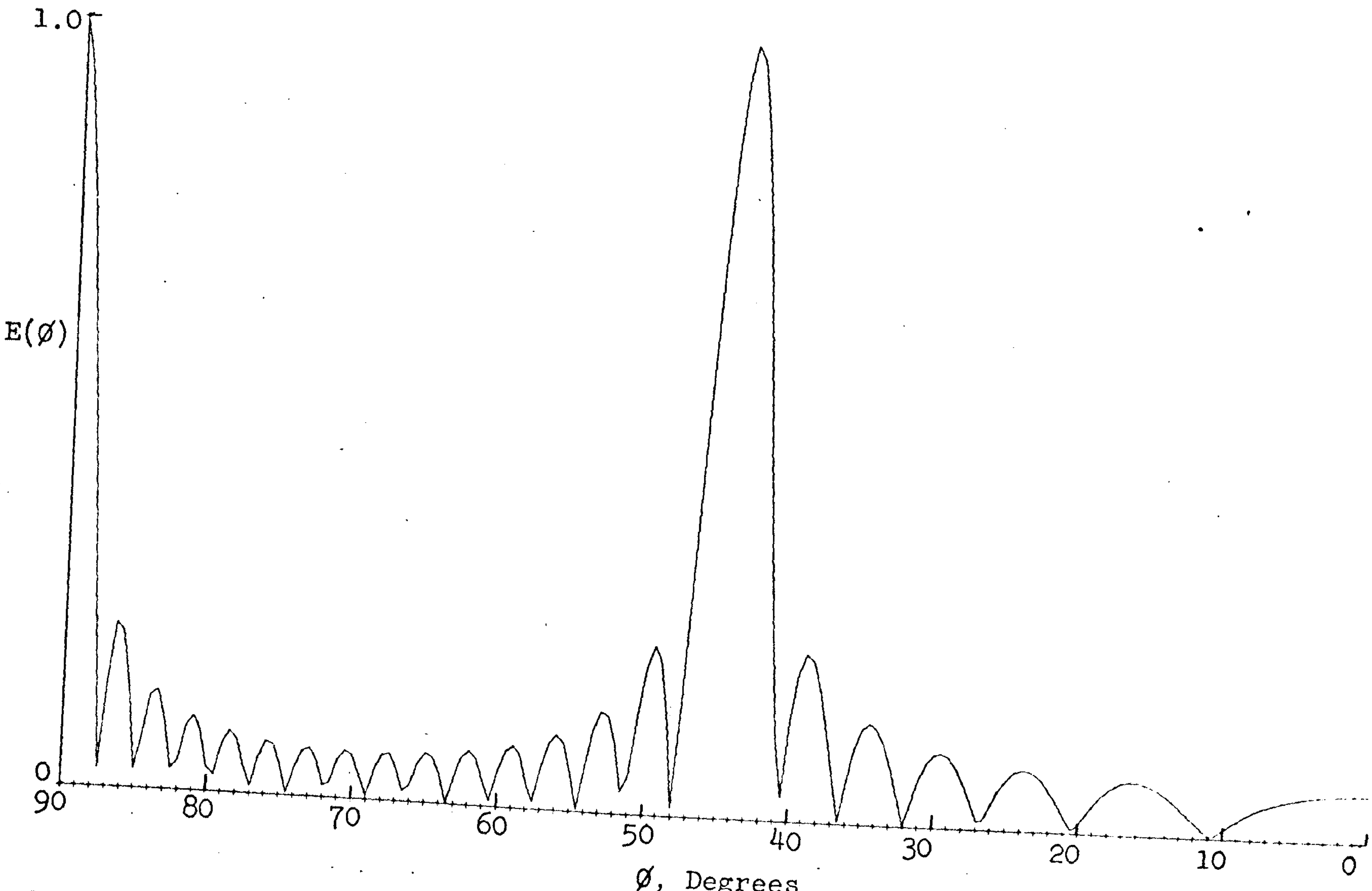


Figure 5.14. Pattern for uniform array with 25λ aperture of Figure 5.13 array.

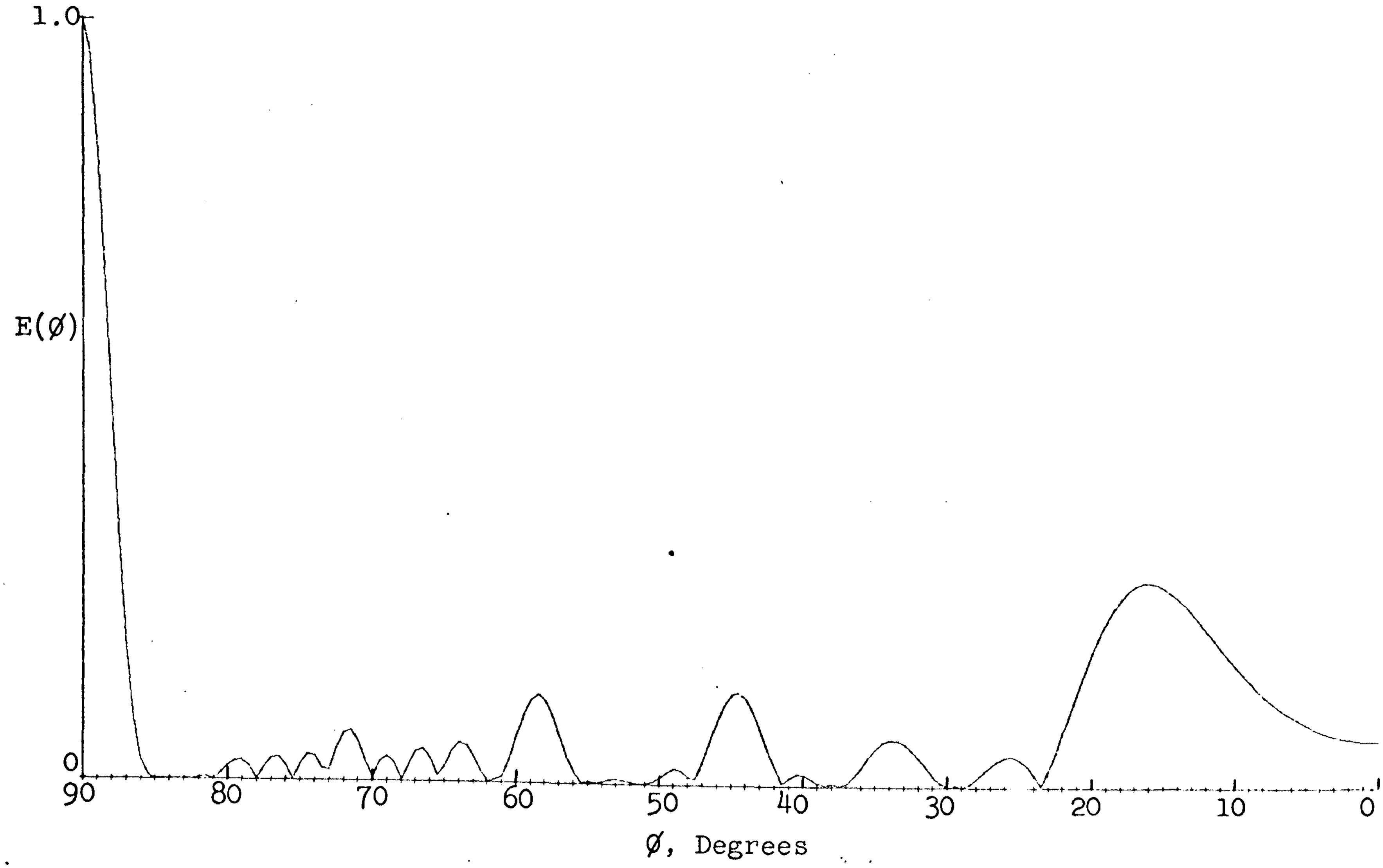


Figure 5.15. Pattern for array of Figure 5.13 with spacing adjusted to .40λ increments.

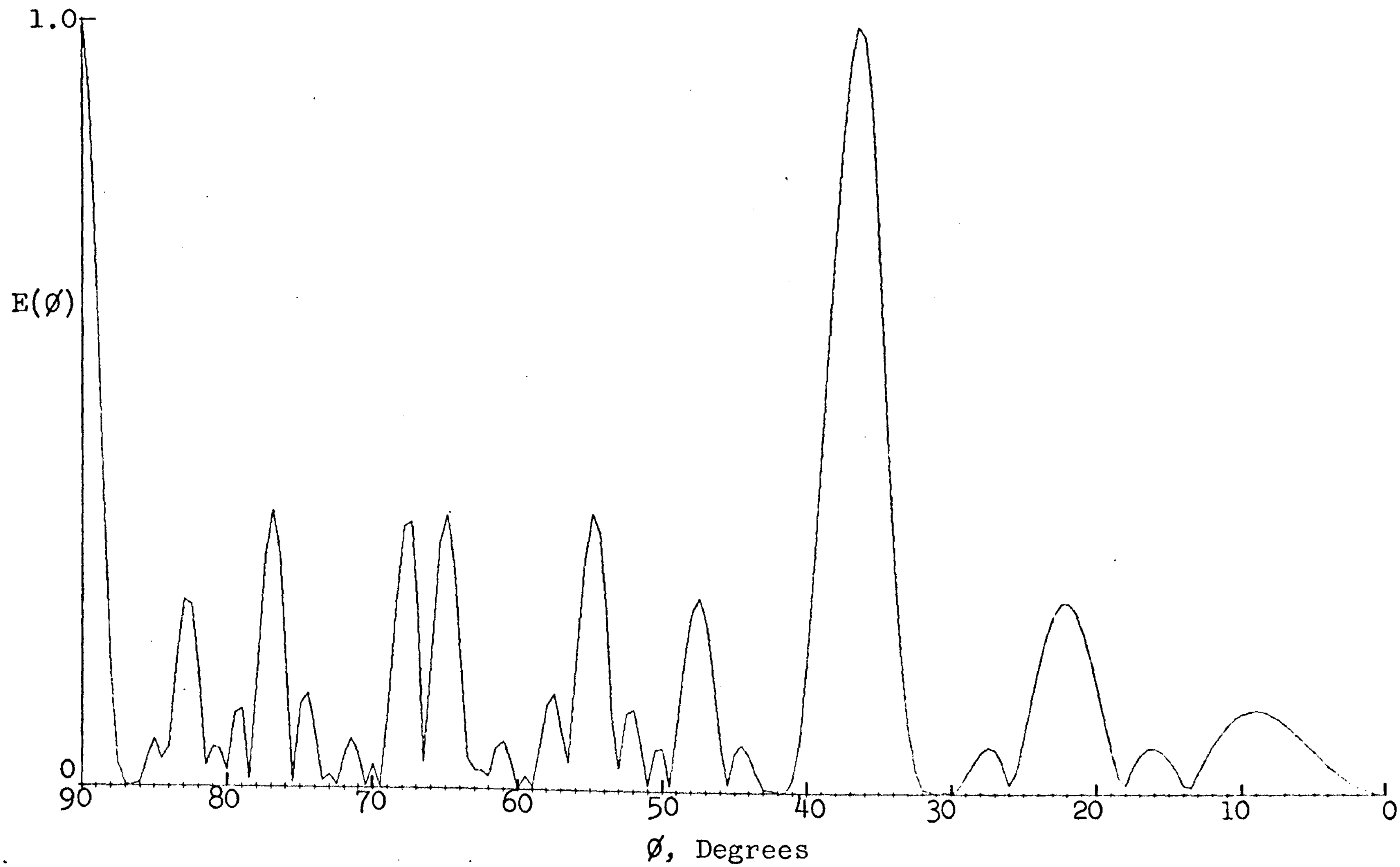


Figure 5.16. E(Ø) vs Ø for 16 element array, 1.25λ increments, 31.25λ aperture.

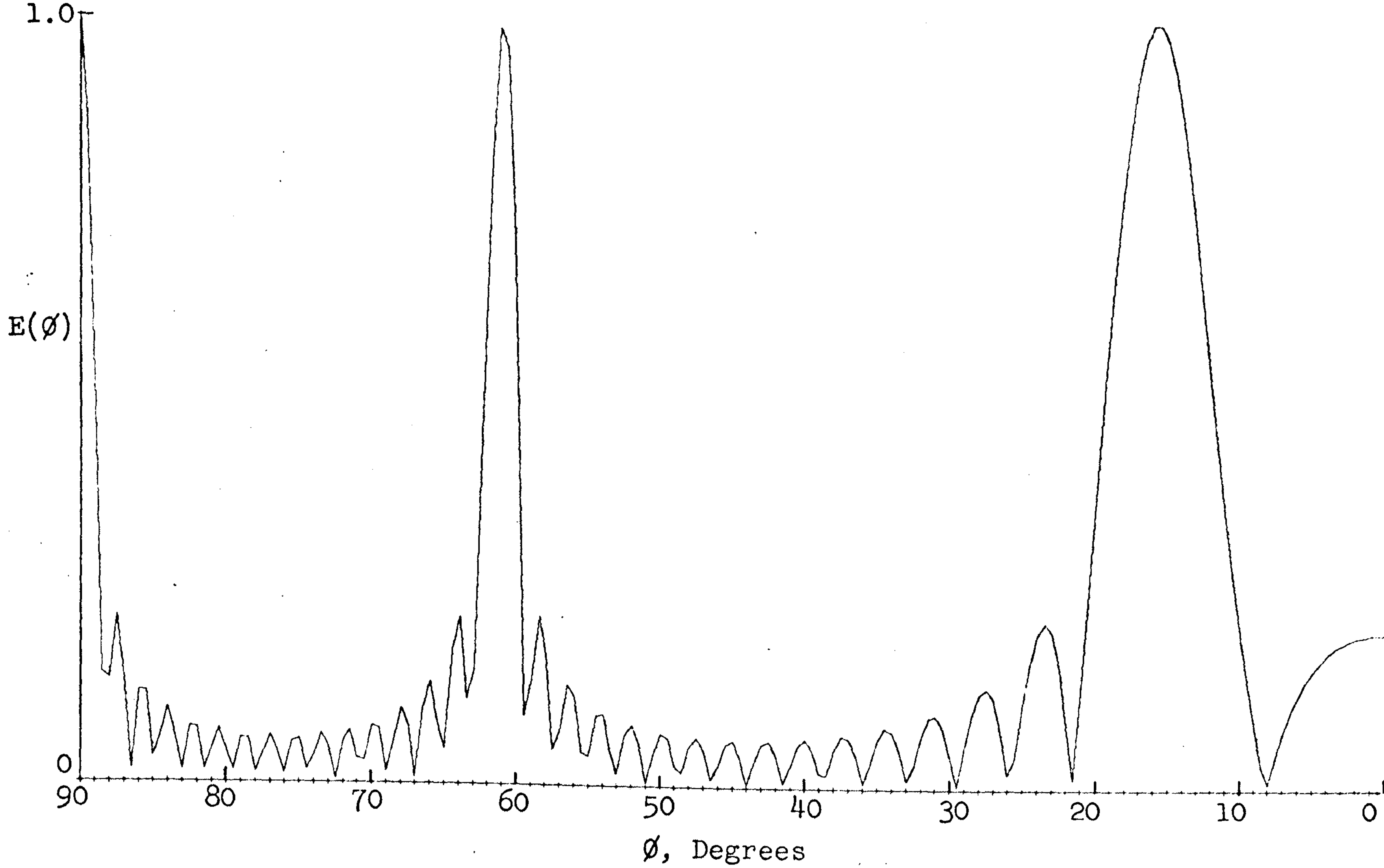


Figure 5.17. Pattern for 16 elements uniformly spaced over a 31.25λ aperture.

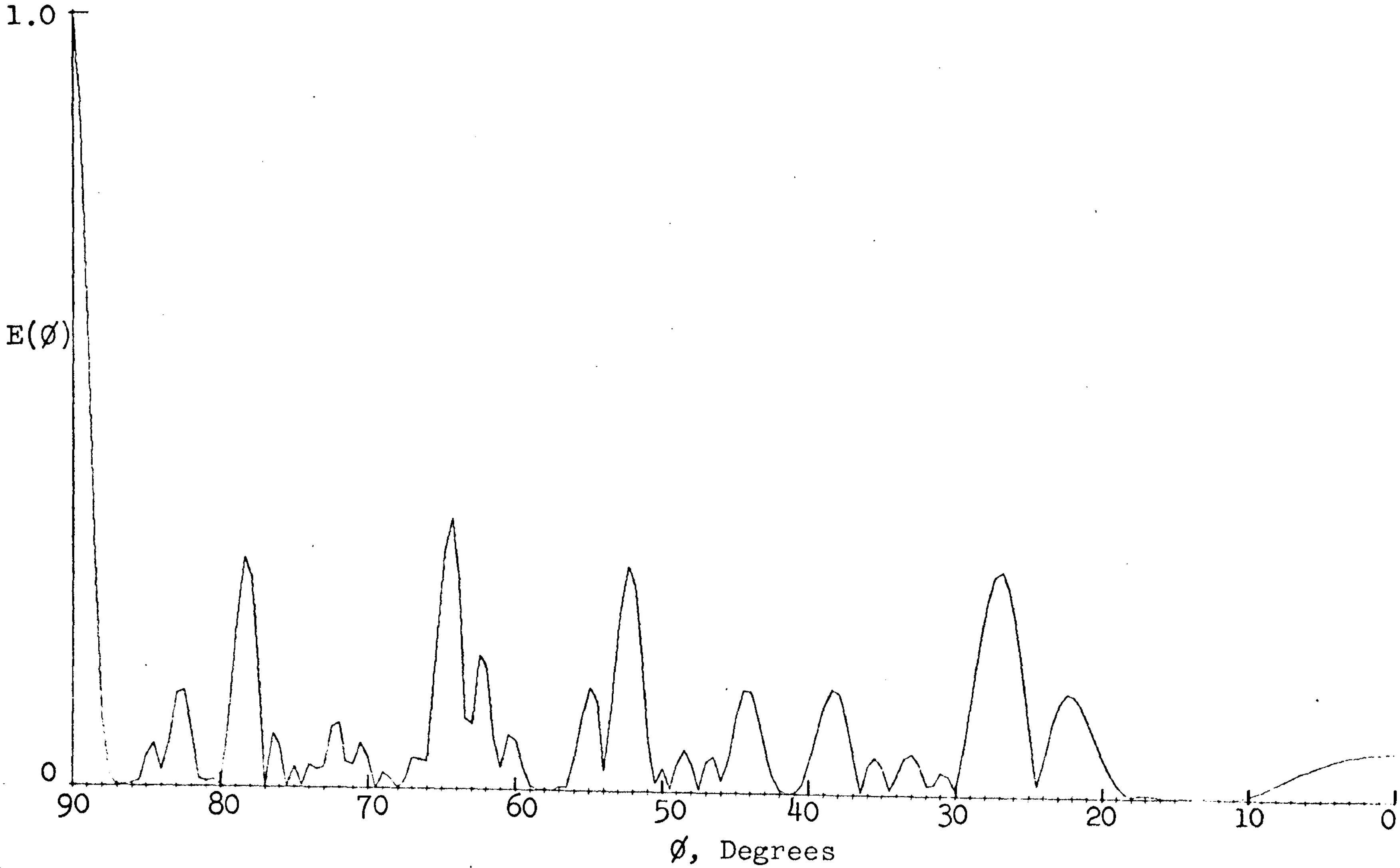


Figure 5.18. Pattern for non-uniform array of Figure 5.16, spacing reduced to $(2/3)\lambda$.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

A. Summary and Conclusions

Summarizing the knowledge gained from the computation of the field pattern of symmetric linear arrays with constant in-phase excitation and non-uniform inter-element spacing, it has been found that, (a) for a moderate sidelobe level, these arrays can be designed with many fewer elements than uniform linear arrays with the same sidelobe level, beam width, and overall aperture, (b) the 3 db beam width of the main lobe depends primarily on the length of the array, and the sidelobe level depends primarily on the number of elements in the array, and (c), the problem of synthesizing an array with non-uniform inter-element spacing to reduce sidelobe level does not have a unique solution; instead, there are numerous solutions, with different sidelobe levels. These conclusions are in agreement with the consensus of the literature in the field of non-uniform arrays, but still merit further explanation as applied to this study.

In almost every case that was examined it was found that the uniform linear array with the same number of elements and same spacing as the non-uniform array produced a field pattern which was lower in sidelobe level but much wider in beam width, than the non-uniform array. When compared on an aperture basis, however, the non-uniform array designed by the method of this thesis is in general capable of lower sidelobes, elimination of grating lobes, suppression of minor lobes nearest the main beam, and a beam width at least as narrow as that for the uniform array. Because of the great slope of the main beam, and the large number of zeros immediately adjacent the main beam null, arrays designed by this method should be capable of higher resolution than those designed by other methods. The gain in the broadside direction for an array of isotropic point-sources, fed in phase with equal amplitude currents, is a simple function of the number of elements in the array, and does not require further study.

This attempt to apply computer programming to array design has indicated its potential for determining improved element spacings of "thinned" arrays, or arrays in which not all of the available positions are occupied (12). Computational difficulties might be encountered, using the digital computer, if the number of elements becomes too large. Other design techniques suffer from the same limitation, however, and the computer program that generated the results reported here can certainly be extended and made more efficient for enlarging the scope of investigation.

Computer analysis may be used to explore the properties of antenna arrays by systematically varying the input parameters, examining the results, and making the proper deductions as to array behavior. Optimization programs may be utilized, where results are compared against a predetermined standard and input parameters varied until an optimum design, in some particular sense, is achieved. Computer design does not yield closed-form solutions, as may be obtained by analytical techniques in a few special cases, but it has the important advantage that it can supply useful answers where other more elegant techniques fail to provide practical solutions.

B. Recommendations for Further Study

Essentially only two problems were examined here in order to exemplify the design method. These were the investigation of the effect of different spacing increments on the available field pattern nulls, and the comparison of sidelobe levels between non-uniform arrays and uniform arrays of the same aperture. There are many possibilities for further research in the computer-designed array area, including research to extend the usefulness of the method proposed in this thesis. With sufficient time available on the computer, a full-scale optimization program might be accomplished for a large array. Sidelobe power comparisons and directivity comparisons might be made utilizing the computer.

Another possibility for future work is the extension of this method using a technique developed by Dickey (13) in his thesis work on the analog computer. This would involve plotting the field pattern as a function of some derived quantity, such as $\Psi(\phi)$ or $K(\phi)$, and redefining the variable over an interval on which the pattern has desirable characteristics. The pattern of Figure 5.15, out to about 22°, is a good example of an application of this scheme.

There are many criteria by which an antenna array may be measured against other arrays, and the computer is an ideal device for exploration in a field for which a general theory has not been developed. The methods used in this study can be applied to larger arrays, to planar arrays, and perhaps to non-planar apertures. Computer design has proven to be a useful tool for the design of one class of antennas, and should also be of value in the solution of other problems in this field.

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