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REDUCED ORDER ROBUST CONTROLLERS FOR AN EXPERIMENTAL FLEXIBLE GRID

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ABSTRACT

The design and implementation of control strategies for large, flexible structures presents challenging problems. One of the difficulties in controller design arises from the incorrect knowledge of the structural parameters. This paper describes a procedure for designing and actual implementation of reduced order robust controllers for active vibration control on an experimental flexible grid structure. The experimental structure consists of a 5' X 5' grid made up of 2" wide, 1/8" thick aluminum strips. The grid hangs vertically down, being cantilevered at the top to a large I-beam anchored to a cinder block wall. The grid structure is represented by 75 degrees of freedom, finite element model and the controller uses three piezoelectric accelerometer sensors to command three non-collocated DC motor torquers. The modified balance-truncation model reduction method is used to derive a control synthesis model. These reduced order models preserve stability, controllability, observability and have a good frequency response match at low frequencies. A 10-mode mathematical representation of the experimental grid structure has non-minimal phase zeros. The effects of non-minimum phase zeros on the performance of a closed loop LQG/LTR controller are investigated. The reduced order controllers are implemented on the structure using an ISI Max 100 computer. Experimental closed loop performance of the grid is obtained for various parameter variations.

I INTRODUCTION

In recent years, an increased amount of research effort has been directed toward the design of controllers for large space structures. The control objectives are vibration damping, mode shape requirements and attitude control. The design and implementation of control strategies for large space structures presents challenging problems. One is the presence of a large number of closely-spaced, lightly-damped coupled modes. Another difficulty in controller design arises from the presence of unmodelled dynamics as well as incorrect knowledge of the structural parameters, which can cause instability. A control strategy which can guarantee stability and provide satisfactory performance in the presence of model uncertainties, is called a robust controller. These uncertainties may include modelling error between the control synthesis model and the actual system, parameter variations and the effects of various disturbances on system performance. Among the various design methods for robust controllers, the linear quadratic gaussian with loop transfer recovery (LQG/LTR) design procedure has many advantages. This methodology will result in control systems with excellent stability robustness, command following, disturbance rejection and sensor noise suppression properties.

The LQG/LTR design methodology is particularly suitable for the control of large flexible structures due to the considerable modelling inaccuracy that inherently exist in the mathematical models. Sundararajan, et al [1], Joshi and Armstrong [2], Yadavalli [3], have successfully applied LQG/LTR methodology for the design of robust controllers for flexible structures. The computational requirements of the method are excessively high for large flexible structures. In this paper a procedure is developed to reduce the computational and implementation requirements by designing LQG/LTR controllers based on reduced order models of the structures.

The balance-truncation model reduction method has many advantages [4]. This representation is based on controllability and observability (location of actuators and sensors) considerations of the structural system. The frequency response characteristics of this reduced model are such that the spectral norm of the model reduction error is large at low frequencies and small at high frequencies. Prakash and Rao [5], have developed a modified reduced order model which has a good frequency response match at low frequencies. These reduced order models preserve stability, controllability and observability. An expression for error due to model reduction is also developed. The reduced order models are used to design linear quadratic regulators (LQR) and LQG/LTR controllers for the Astronautics Laboratory, Air Force Systems command (AL) experimental grid structure.

The mathematical representation of the experimental grid structure has non-minimum phase (NMP) zeros. The influence of NMP zeros on the closed loop system is investigated in this paper. A loop transfer recovery procedure for systems with NMP zeros is also discussed in the paper.

A reduced order robust controller is implemented on the structure using Integrated system Max 100 controller. The closed loop performance of the grid is recorded to illustrate the effects of parameter variations and sensor noise on the results.

DESCRIPTION AND STRUCTURAL MODELLING OF EXPERIMENTAL GRID

An experimental grid structure is built at the Astronautics Laboratory, Air Force System Command (AL) to develop a simple ground test bed for future large flexible structures. The principal objective of this research facility has been to achieve satisfactory agreement between theoretical results and experimental measurements [6].

The two-dimensional experimental structure shown in Fig. 1 consists of a 5' x 5' grid made up of 2" wide, 1/8" thick aluminum strips. At every point where the vertical and horizontal strips cross each other, they are connected by four rivets, thus effectively removing any play at the joints. The grid hangs vertically down, being cantilevered at the top to a large I-beam anchored to a cinder block wall.
The structural vibrations are monitored using high sensitivity, low mass piezoelectric accelerometers. Permanent magnet DC motor torquers are used as actuators for the grid. The grid can be exited by an electrodynamic shaker or a quartz impulse hammer.

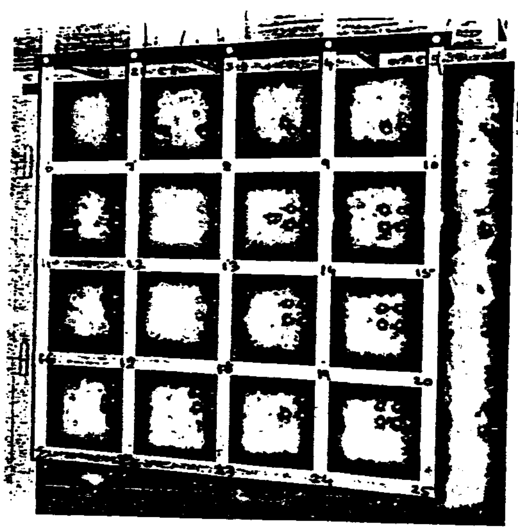

Fig. 1. A L Experimental Grid.

Structural Modelling

A model of the structure which relates inputs and outputs is needed for control design purposes. A theoretical mathematical model of the structure is developed using finite element methods. The natural frequencies of the Nastran finite element model are compared with the experimentally obtained natural frequencies of the grid.

In order to design robust controllers for the structure we need a state variable representation of the grid. A procedure for realization of a state variable model for finite element model is given below.

The experimental grid structure is represented by 75 degrees of freedom (DOF) finite element-model. Each grid node has three DOFS: translation in z direction and rotations about horizontal and vertical axes. Equation of motion of the grid are given by

$$[m]\ddot{q}+[c]\dot{q}+[k]q=F(t) \quad \ldots(1)$$

The vector q(t) represents the physical coordinates of the structure.

A 10-mode model representation of the grid is evaluated by using a linear transformation and truncation

$$q=\Phi\eta \quad \ldots(2)$$

where η = Modal coordinates of the structure
Φ = eigenvectors

Modal representation of the grid structure is given by

$$\ddot{\eta}+\mathrm{diag}(2\zeta\omega_1\ldots 2\zeta\omega_{10})\dot{\eta}+\mathrm{diag}(\omega_1^2\ldots\omega_{10}^2)\eta=\Phi^T\mathcal{B}u(t) \quad \ldots(3)$$

where F(t) = u(t)
$\mathcal{B}$ = A matrix contains information about location and type (x - torquer / y - torquer) of actuators
ζ = A uniform damping of 0.0025 is assumed for every mode.

A state variable model corresponding to Eq. (3) is given by

$$\dot{x}=\begin{bmatrix}0 & I\\ -\mathrm{Lambda} & -\mathrm{Zetamat}\end{bmatrix}x+\begin{bmatrix}0\\ \Phi^T\mathcal{B}\end{bmatrix}u \quad \ldots(4)$$

$$\dot{x}=Ax+Bu \quad \ldots(5)$$

where $x=\begin{bmatrix}\eta\\ \dot{\eta}\end{bmatrix}$

$\mathrm{Lambda} = \mathrm{diag}(\omega_1^2,\omega_2^2,\ldots\omega_{10}^2)$

$\mathrm{Zetamat} = \mathrm{diag}(2\zeta\omega_1,\ldots 2\zeta\omega_{10})$

Norris et al [7] have presented a procedure for determining the dynamic effect of gravity on low frequency accelerometer measurements. These gravity effects are incorporated in the output equation as given below:

The output of the ith accelerometer is given by

$$a_i=\ddot{q}_i+g(\cos\alpha_i)q'_i \quad ,i=1,2,\ldots m. \quad \ldots(6)$$

where α_i is the angle between vertical and the tangent to the structure in equilibrium at accelerometer location p_i.

From eq (3) and (4)

$$\ddot{\eta}=-\mathrm{diag}(\omega_1^2\ldots\omega_{10}^2)\dot{\eta}-\mathrm{diag}(2\zeta\omega,\ldots 2\zeta u_\omega)\eta+\Phi^T\mathcal{B}u \quad \ldots(7)$$

$$\ddot{\eta}=[0 \quad I]\begin{bmatrix}0 & I\\ -\mathrm{Lambda} & -\mathrm{Zetamat}\end{bmatrix}\begin{bmatrix}\eta\\ \dot{\eta}\end{bmatrix}+[0 \quad I]\begin{bmatrix}0\\ \Phi^T\mathcal{B}\end{bmatrix}u$$

$$\ddot{\eta}=[0 \quad I][Ax]+[0 \quad I]Bu \quad \ldots(8)$$

From eq (2)

$$\ddot{q}=\Phi\ddot{\eta}=\Phi[0 \quad I]Ax+\Phi[0 \quad I]Bu \quad \ldots(9)$$

consider the accelerometer reading given by

$$a_i = \ddot{q}_i + g(\cos\alpha_i)q'_i$$

the angle $\alpha_i = 0$ for grid structure

$$q'_i = \Phi\eta' = [\Phi \quad 0]x \qquad ...(10)$$

Hence output equations given by

$$y = SUVMAT*a \qquad ...(11)$$

where SUVMAT = standard unit vectors for accelerometers.

From equations (9) through (11),

$$y = SUVMAT\{\Phi[0 \quad I]Ax + \Phi[0 \quad I]Bu + [\Phi \quad 0]x\}$$

$$y = SUVMAT\{\Phi[0 \quad I]Ax + [\Phi \quad 0]x\} + SUVMAT\{\Phi[0 \quad I]Bu\} \qquad ...(12)$$

$$y = Cx + Du \qquad ...(13)$$

$$\text{where } C = SUVMAT\left\{\Phi[0 \quad I]A + \begin{bmatrix}\Phi & 0\\ 0 & 0\end{bmatrix}\right\}$$

$$D = SUVMAT\{\Phi[0 \quad I]B\}$$

The transfer function of the grid structure is given by

$$G(s) = C(sI - A)^{-1}B + D \qquad ...(14)$$

The first ten-natural frequencies of the grid determined by using NASTRAN finite element model and experimental values are given in Table 1.

TABLE 1 Comparison of Natural Frequencies

Mode #	NASTRAN (Hz)	Experiment (Hz)
1	0.7784	0.762
2	1.9093	1.810
3	4.1641	4.110
4	4.9618	5.150
5	6.3587	6.220
6	10.7318	10.750
7	10.8196	11.05
8	11.3227	11.50
9	13.8646	13.80
10	16.092	16.6

REDUCED ORDER MODELLING METHODS

A large number of procedures are available in the literature for deriving reduced order models [8]. Moore [4] has introduced a balanced-truncation model reduction method based on controllability/observability consideration. This reduced order modelling technique is appropriate for large space structure problems. However the frequency response characteristics of their reduced model are such that the spectral norm of the model reduction error is large at low frequencies and small at high frequencies. In order to design LQG/LTR controllers based on reduced order modes, it is desirable to obtain a reduced order model which offers a good match a low frequencies. Prakash and Rao [5] have introduced modifications to balanced-truncation procedure and these models are used to design LQG/LTR controllers in this paper.

DESIGN OF ROBUST CONTROLLERS

Among the various design methods for robust controllers the linear quadratic Gaussian with loop transfer recovery (LQG/LTR) design procedure has many advantages. This methodology is particularly suitable for the large flexible structures. The LQG/LTR design procedure for system with minimum zeros with the loop broken at output [9] can be summarized as follows:

(i) Determine a nominal state variable model of the structure from NASTRAN finite element model. To reduce the computational/implementation requirements of the controller, obtain a reduced order model of the structure using balance-truncation procedure. Augment pure integrators to the plant for zero steady state error.
(ii) Define the desired performance loop shape by singular value plots.
(iii) Design a Kalman filter with state noise covariance matrix $Q_e = \Gamma\Gamma^T$ and sensor noise matrix $Re = \mu I$. Choose the matrix Γ to meet the robustness characteristics at high and low frequencies.
(iv) Design a linear quadratic regulator for recovering the stability margins of the closed loop system. The regulator weighting matrices are $Q_r = q^2C^TC$ and $R_r = \rho I$ plot singular value plots of $G(s)K(s)$ and vary q^2 until a desirable level of recovery is achieved.
(v) Plot singular value plots of sensitivity function $[I+G(s)K(s)]^{-1}$.

One limitation of the above LQG/LTR design procedure is that it can obtain arbitrarily good recovery only for minimal phase plants. If the plant is non-minimum phase, the LTR technique can not recover the state feedback loop arbitrarily well. The original 10-mode state variable model of the experimental grid structure has 3-pairs of non-minimum phase zeros. Hence the effects of NMP zeros on the closed loop system performance and techniques for loop transfer recovery for system with NMP zeros is investigated in this paper.

The objective of a LTR procedure is to recover guaranteed stability margins by selecting proper linear quadratic regulator (LQR) gains

$$Q_r = H^TH + q^2C^TC \qquad ...(15)$$

$$R_r = \rho I \qquad ...(16)$$

As $q^2 \to \infty$, the eigenvalues of closed loop system will move towards the plant transmission zeros. Since q^2C^TC is at least positive semidefinite, the closed loop eigenvalues will never lie in the right half plane for any value of q^2. Hence the closed loop eigenvalues do not move towards the non-minimal phase zeros as $q^2 \to \infty$. Unlike the minimal phase case, we can not see any pattern on the poles and zeros of the compensator for various values of q^2. The effects of non-minimal phase zeros on the closed loop system performance can be summarized as follows:

(a) If the NMP zeros lie well outside the required bandwidth, the recovery procedure will give satisfactory performance. If any of them lie within the required bandwidth, the closed loop system may become unstable near NMP zero frequencies.

(b) If the plant is minimum phase the recovery procedure will cancel all undesired dynamics of the plant i.e., as $q \to \infty$, $K(s) \to G^{-1}(s)C\Phi(s)K_f$. In the case of non-minimum phase systems, the recovery process can not cancel all undesired dynamics. These unwanted dynamics will tend to aggravate the compensator.
This phenomenon is observed in the grid structure at 13.85 Hz (87 rad/sec). The singular value plot of the target feedback loop and G(s)K(s) is shown in Fig 2. The sensitivity plots are given in Fig 3.

(c) If the recovery procedure introduces additional poles in the vicinity of the desired closed loop poles, an additional gain and a phase shift of (-180°) is introduced in the loop and closed loop system will be unstable at that frequency. This phenomenon has been observed at a frequency 13.85 Hz for a value of $q^2 = 500$.

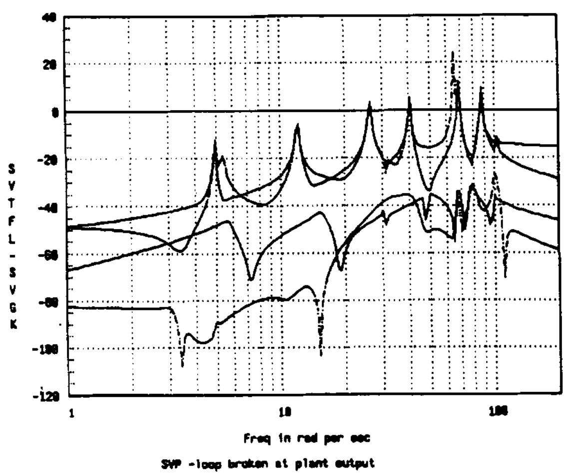

Fig. 2. Singular Value Plots of TFL and G(s)K(s).

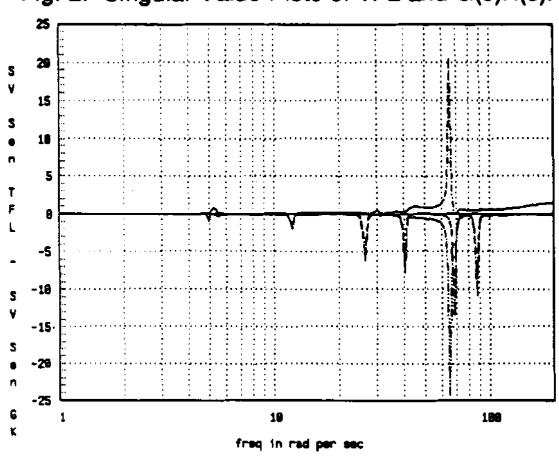

Fig. 3. Sensitivity Plots of TFL and G(s)K(s).

In order to avoid the problems associated with NMP systems, Stein and Athans [10], Zhang and Freudenberg [11], and Sogarrd-Anderson and Niemann [12] have developed methods using minimum phase equivalent of NMP systems. Recently Misra [13] has presented a method for converting non-minimum phase plant into an augmented minimum phase plant. We are currently investigating the applicability of these methods to experimental grid structures.

EXPERIMENTAL RESULTS

A block diagram representation of experimental set up is given in Fig 4. The accelerometer were mounted at node numbers 14, 21, and 25 on the grid. The x-axis torquers are located at node numbers 22 and 24 and y-axis torquer is placed at node 18 on the grid. The Max-100 computer is used for the implementation of the controller. The Max 100 computer was loaded with real time code generated using Matrixx (software package) capabilities.

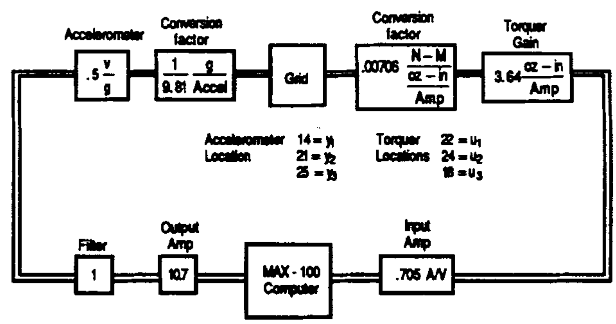

Fig. 4. Block Diagram Representation of Experimental Setup.

Several LQG/LTR controllers were designed and implemented on an overlapped experimental grid structure. A 20-th order model of the grid is determined by using NASTRAN finite element model. To minimize the computational and implementation requirements of the controller, a 12th order reduced model is derived by using balance-truncate procedure.

To minimize the computational requirements, a linear transformation is performed on the LQG/LTR compensator to convert into modal form. A discrete equivalent of this controller is obtained with a sampling frequency of 80 Hz.

The 12th order LQG/LTR controller exhibited high loop gain at mode #9 (13-85 Hz). In addition a (-180°) phase shift in the controller output is observed and the closed loop system was unstable at that mode.

We repeated the controller design procedure and implementation using 10th order, 8th order and 6th order models. In the 10th order compensator system, the mode at 11.5 hz become unstable. In the 8th order system the mode at 11.05 has become marginally stable. The 6th order compensator gave stable performance at all modes.

Open and closed loop responses with 6th order controller for mode #5 is given in Fig 5. To illustrate the robustness properties of the controller, the following plant perturbations were performed.

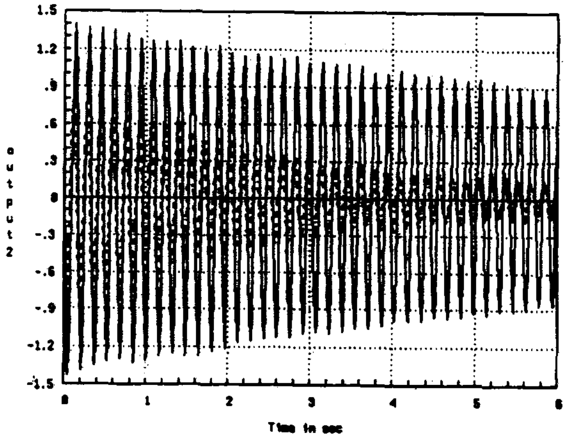

Fig. 5. Open and Closed Loop Response of Mode #5.

Perturbation #1: 70g masses were removed from structure nodes 7 and 9.
Perturbation #2: Two more 70g masses were removed from structure nodes 16 and 20.
Perturbation #3: A mass of 278g is added at node 17 of the structure.
Perturbation #4: A mass of 278g is added at node 11 (flexing mode) of the structure.

The structural mode shape for 5th mode is given in Fig 6. The initial condition responses with these perturbations are presented in Figs 7 and 8.

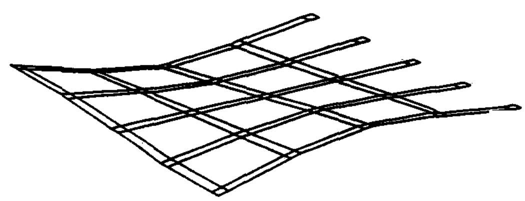

Fig. 6. The Structural Mode Shape for 5th Mode.

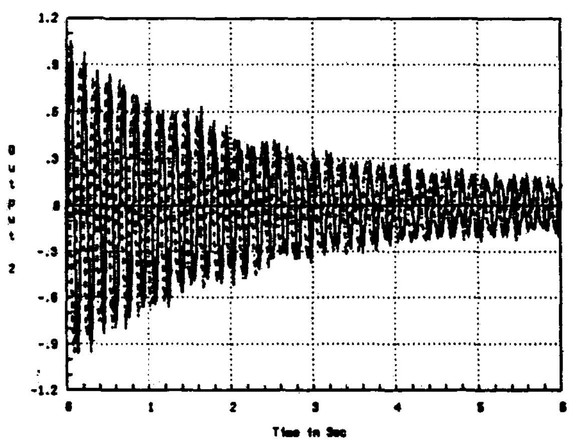

Fig. 7. Closed Loop Response of Mode #5 with Perturbations 1 and 2.

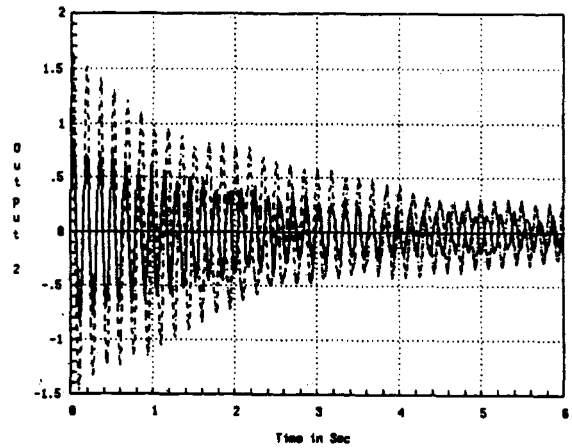

Fig. 8. Closed Loop Response of Mode #5 with Perturbations 3 & 4.

CONCLUSIONS

Robust control design methodologies were tested on an experimental grid structure. The reduced order models are employed to design LQG/LTR controllers. The controllers were implemented on the structure using Max 100 computer and closed loop system responses were recorded. The grid structure has non-minimum phase zeros and the presence of these zeros aggravated the controller action at certain frequencies. An unstable system response was also observed. The effects of non-minimum zeros on the closed loop system were identified and reported in this paper. The design of controllers using loop transfer recovery methods for non-minimum phase system is under investigation.

The closed loop system performance under various plant perturbations is satisfactory for a sixth order controller.

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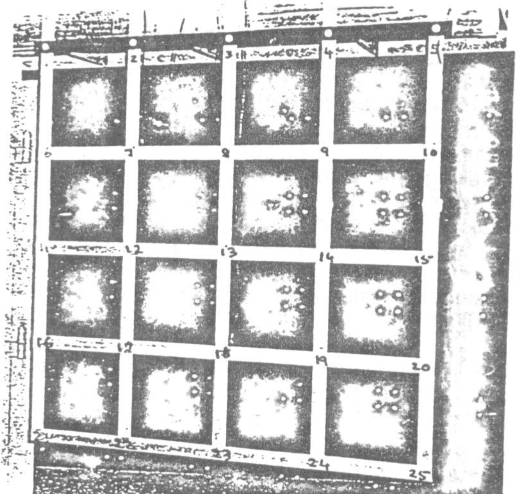

Fig. 1. A L Experimental Grid.