Design Of Robust Controllers For Gas Turbine Engines

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ABSTRACT

This paper describes robust controller design methodologies for gas turbine engines. A linear state variable model for the engine is derived using partial derivatives. The Linear Quadratic Gaussian with Loop Transfer Recovery (LQG/LTR) and the Parameter Robust Linear Quadratic Gaussian (PRLQG) robust controller design methodologies have been used to design a controller for gas turbine engines. A new method is proposed by combining the features of LQG/LTR and PRLQG methods and yields good robustness properties with respect to both unstructured uncertainties in the frequency domain and structured parameter variations in the time domain. The new procedure is illustrated with the help of an aircraft gas turbine engine model.

Nomenclature

LQG/LTR	Linear Quadratic Gaussian with Loop Transfer Recovery
PRLQG	Parameter Robust Linear Quadratic Gaussian
LQR	Linear Quadratic Regulator
KF	Kalman Filter
SISO	Single Input Single Output
MIMO	Multi-input Multi-Output
TFL	Target Feedback Loop
PR	Parameter Robust Controller
ARE	Algebraic Ricatti Equation
G(s)	Nominal transfer function of the plant
K(s)	Transfer function of the controller
$\hat{G}(s)$	Transfer function of the perturbed model
L(s)	Multiplicative frequency domain representation of model uncertainties
Φ	$(sI - A)^{-1}$
I	Identity matrix
x	Vector of state variables
y	Output vector
u	Input vector
n	Number of states in the model
m	Number of inputs in the model
p	Number of outputs in the model
N_e	Engine speed
N_p	Propeller speed
N_1	Propeller speed referenced to engine speed
Q_L	Load torque
Q_1	Torque delivered to the propeller
T_5	Temperature at the turbine inlet
ω_f	Fuel flow
β	Propeller blade angle
Δ	Perturbation parameter
ζ,η	Gaussian white noise processes
GM	Gain margin
PM	Phase margin

I. INTRODUCTION

In recent years, an increased amount of research effort has been directed toward the design of controllers for gas turbine engines. The control system is designed to meet specified steady-state performance requirements at different operating points and to have a safe and rapid transient response when moving from one operating point to another. A control strategy which can guarantee stability and provide satisfactory performance in the presence of model uncertainties, is called a robust controller. The robustness problem has been studied in the single-input single-output (SISO) case for many years and much of the classical control theory deals with this in one way or another. These SISO methods are being extended to the multi-input multi-output (MIMO) cases. Among the various design methods for robust controllers, the linear quadratic Gaussian with loop transfer recovery (LQG/LTR) design procedure has many advantages [1]. This methodology will result in control systems with excellent stability robustness, command following, disturbance rejection and sensor noise properties.

The linear quadratic Gaussian with loop transfer recovery (LQG/LTR) methodology [1,2] is a design method for both SISO and MIMO linear systems, which produces

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controllers that are robust with respect to unstructured uncertainties. In many systems, however, information is available about the structure of the variations which can be expected in the model. In that case, the information should be used to improve the performance robustness of the system.

The Parameter Robust Linear Quadratic Gaussian (PRLQG) design method [3,4] has been proposed to deal with parameter variations where information about the structure is available. In this method, the robustness properties are described in the time domain, while for the LQG/LTR, they are given in terms of the frequency domain. By itself, the PRLQG method has several limitations.

This paper presents a variation of the LQG/LTR method, incorporating aspects of the PRLQG method. The resulting procedure considers the robustness problem with respect to both the frequency domain with unstructured uncertainties, and the time domain with parameter variations of known structure.

Controller design for aircraft gas turbine engines is an area where structural information about the parameter variations is available. These variations arise due to changes in the engine operating environment corresponding to changes in ambient conditions, altitude, or airspeed. The effects of these environmental conditions on engine behavior should be used when designing a controller.

The modified PRLQG method is demonstrated by using an aircraft gas turbine engine system. The results are compared with a standard LQG/LTR design to show the types of improvements which can be obtained.

The paper is arranged in seven primary sections. Section II provides a brief description of gas turbine engines and then describes the derivation of the model for the Allison T56 engine. Section III reviews the basic LQG/LTR design methodology. The PRLQG method is described in Section IV. A new procedure by combining the PRLQG with the LQG/LTR method is developed and presented in Section V. Section VI contains robust controller design methods for gas turbine engines. A comparison of system response between proposed method and LQG/LTR method are also given in Section VI. Conclusions are given in Section VII.

II. MODELING OF GAS TURBINE ENGINES

The Allison T56 engine is a single-spool turbo-shaft engine and is used to power several types of propeller driven aircraft. Due to the changes in engine behavior as the operating conditions vary, gas turbine engines are nonlinear systems. A linear model framework can be developed at an operating point using partial derivatives [5]. The partial derivatives can be evaluated for each operating condition, through the use of a nonlinear simulation program on a digital computer.

The manipulative input variables are fuel flow (ω_f) and propeller blade angle (β). The output variables are engine speed (N_e) and torque to the output shaft (Q_1).
The linear model of the engine incorporating partial derivatives, is shown
Fig. 1. An equivalent electrical circuit for the shaft, gearbox, propeller system is shown in Fig. 2, where all quantities are referenced to engine speed.

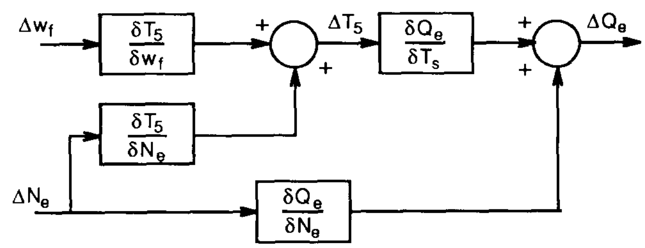


Fig. 1. Linear Model of the Engine

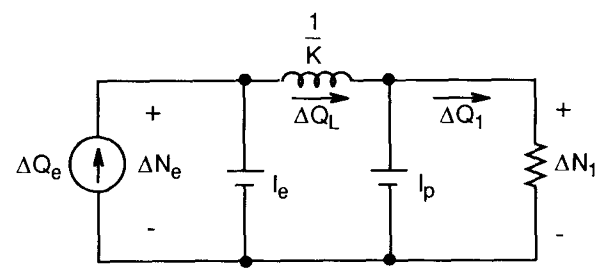


Fig. 2. Equivalent Electrical Circuit of the Shaft Gearbox and Propeller

The state equations of the engine and propeller system are given by

$$\dot{\mathbf{x}}_p = \mathbf{A}_p \mathbf{x}_p + \mathbf{B}_p \mathbf{u}$$

where

$$\mathbf{y} = \mathbf{C}_p \mathbf{x}_p + \mathbf{D}_p \mathbf{u} \qquad (1)$$

$$\mathbf{x}_p = \begin{bmatrix} \Delta N_e \\ \Delta N_1 \\ \Delta Q_L \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} \Delta w_f \\ \Delta\beta \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} \Delta N_e \\ \Delta Q_1 \end{bmatrix} \qquad (2)$$

and Δ represents change around the nominal operating point.

In matrix form the linear model can be summarized as follows:

$$\mathbf{A}_p = \begin{bmatrix} \mathbf{X}_1 & 0 & \mathbf{C}_1 \\ 0 & \mathbf{X}_2 & \mathbf{C}_2 \\ \mathbf{K} & \mathbf{-K} & \mathbf{0} \end{bmatrix} \qquad \mathbf{B}_p = \begin{bmatrix} \mathbf{X}_3 & 0 \\ 0 & \mathbf{X}_4 \\ 0 & 0 \end{bmatrix} \qquad (3)$$

$$\mathbf{C}_p \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathbf{X}_5 & 0 \end{bmatrix} \qquad \mathbf{D}_p = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{X}_{10} \end{bmatrix} \qquad (4)$$

where the C's and K represent constants that are independent of the operating point.
The X's represent elements which are functions of partial derivatives and therefore change as the operating conditions change. In order to obtain numerical values for the model, the partial derivatives were calculated at a given operating point using data from a nonlinear simulation program. Fourteen such operating points were selected and a linear model was evaluated for each. It is desirable to have one linear model as the nominal model to describe system operation over the entire flight envelope. Since linear models have been derived and validated at thirteen other points encompassing most of the flight

envelope, the types and magnitudes of the parameter variations which can be expected in the nominal model can be assessed quite easily.

III ROBUST CONTROLLER DESIGN METHODOLOGY

A controller is said to be robust if the performance characteristics of the closed loop system remain satisfactory even when the system is subjected to uncertainties. Among the various design methods for robust controllers, the linear quadratic Gaussian with loop transfer recovery (LQG/LTR) design procedure [1] has many advantages. This methodology will result in systems with excellent stability robustness, command following , disturbance rejection and noise suppression properties. A brief review of this methodology is given in this section.

Consider a system represented by

$$\dot{x} = Ax + Bu + \Gamma\xi \tag{5}$$

$$y = cx + Du + \eta \tag{6}$$

where ξ and η are uncorrelated, Gaussian white noise processes.

The LQG/LTR procedure essentially consists of designing a target feedback loop (TFL) and synthesizing the compensator K(s) so that the loop transfer function matrix is close to that of TFL. The output version of the procedure can be summarized as follows:

(1) Define a nominal model of the gas turbine engine. Augment pure integrators to the plant for zero steady state tracking error. Define the stability and performance barriers in the frequency domain.

(2) For the loop broken at output, design a Kalman filter by adjusting the weighting matrices to meet the robustness specifications at high and low frequencies.

(3) Design a linear quadratic regulator for recovering the stability margins of the closed loop systems.

(4) Verify the closed loop performance of the system.

The LQG/LTR controller methodology was successfully employed to design robust controllers for gas turbine engines [6,7]. We have also used this methodology to design controllers for the Allison T56 engine and the results are presented in Section VI.

IV. PARAMETER ROBUST LINEAR QUADRATIC GAUSSIAN METHODS

Tahk and Speyer [3,4] have presented the parameter robust linear quadratic Gaussian (PRLQG) controller design method for systems which are robust with respect to structured parameter variations in the state space model representations. These variations are represented in the time domain using an internal feedback loop (IFL) modeling technique. Many of the design steps in this method are very similar to those in the LQG/LTR methods, however the motivation is somewhat different. The salient features of this method are presented in this section.

Internal Feedback Loop (IFL) Modeling of uncertainties:

Consider a system represented in state space form

$$\dot{x} = Ax + Bu \tag{7}$$

$$y = Cx \tag{8}$$

where some of the elements of the A matrix are uncertain. The perturbed system can be described as

$$\dot{x} = (A + \Delta A)x + Bu = (A - MLN)x + Bu \tag{9}$$

where M, L, and N are matrices chosen to represent the information known about the uncertainties in A. Define an additional input w and output z such that

$$w = -Lz \tag{10}$$

$$z = Nx \tag{11}$$

then

$$\dot{x} = Ax + Bu + Mw \tag{12}$$

$$x = (sI - A)^{-1}[Bu + Mw] = \Phi Bu + \Phi Mw \tag{13}$$

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} C\Phi B & C\Phi M \\ N\Phi B & N\Phi M \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix} \tag{14}$$

Define an output feedback controller u(s) = r(s) - K(s)y(s) where r(s) is the reference input. The dependence on s will be dropped to simplify notation. Let G = $C\Phi B$. The resulting system

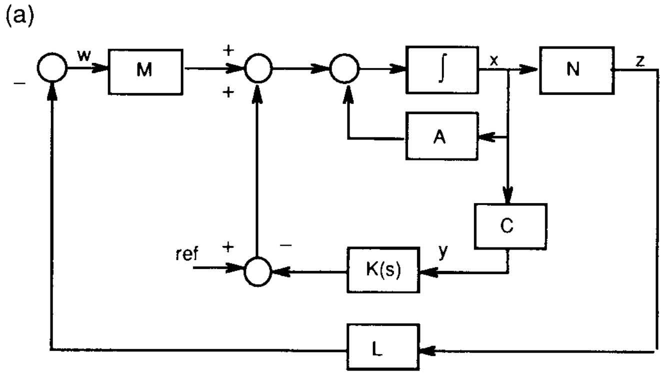
$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} G(I+KG)^{-1} & (I+GK)^{-1} \\ N\Phi B(I+KG)^{-1} & N\Phi M - (I+KG)^{-1}KC\Phi M \end{bmatrix} \begin{bmatrix} r \\ w \end{bmatrix} \tag{15}$$

is drawn in Figure 3 where the uncertainty is represented by the feedback loop [3,4]. Let G_{22} be the transfer function between z and w, and is given by

$$G_{22} = N\Phi M - N\Phi B(I+KG)^{-1}KC\Phi M \tag{16}$$

Tahk and Speyer [3,4] have shown that if Γ is column similar to M, then the closed loop system poles due to K_f will be insensitive to parameter variations in A, which are described by M, as $\mu \to 0$. By a dual procedure, it can be shown that if H is row similar to N, then the closed loop poles due to K_c will be insensitive to parameter variations described by N, as $\rho \to 0$. A simple procedure for the selection of the matrices M, L and N is given by Tahk and Speyer [4].

(a)



(b)

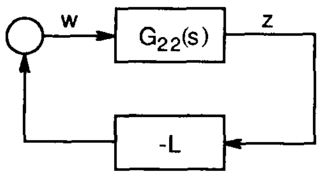


Fig. 3. Two Input-Two Output System with uncertainties represented by IFL.
(a) Detailed Block Diagram (b) Simplified Block Diagram

Limitations:. A controller designed using the procedure outlined above will produce a system which is insensitive to parameter variations. However this PRLQG method has a severe limitation. This limitation is evident from Kalman filter loop properties equation. For a small value of a design parameter v[3], the singular values of the target feedback loop (TFL) get very large, causing a large bandwidth. This causes large overshoots and rapid oscillations in the time response of the system. Many real system components such as pumps, actuators and other hardware have limited rate of change or maximum output capabilities.

The LQG/LTR method avoids this problem because the asymptotic procedure does not change the loop shape. This procedure assures that as the weighting matrix gets large the frequency domain properties of the total open loop system approach those of the TFL. It is suggested by Tahk and Speyer [3,4] that the PRLQG procedure be substituted for the recovery procedure of the LQG/LTR to decrease the sensitivity to parameter variations. This will make the regulator poles (for the case of output uncertainties) insensitive to parameter variations, however the frequency domain properties of the TFL are no longer preserved.

V. PROPOSED COMBINED METHOD

A combined robust controller design method utilizing the ideas of the PRLQG and LQG/LTR is developed in this section. This combination of the methods guarantees stability robustness with respect to unstructured perturbations, and performance robustness with respect to known parameter variations. This procedure considers the closed loop system performance in both time and frequency domains.

Many times it is necessary to augment integrators to the input of the plant in order to guarantee steady state matching between the reference inputs and the outputs of the system. These integrators are actually part of the controller but for design purposes they are assumed to be part of the plant model.

$$\begin{bmatrix}\dot{x}_i\\ \dot{x}_p\end{bmatrix}=\begin{bmatrix}0 & 0\\ B_p & A_p\end{bmatrix}\begin{bmatrix}x_i\\ x_p\end{bmatrix}+\begin{bmatrix}I\\ 0\end{bmatrix}u \qquad (17)$$

$$y=\begin{bmatrix}D_p & C_p\end{bmatrix}\begin{bmatrix}x_i\\ x_p\end{bmatrix} \qquad (18)$$

where A_p, B_p, C_p and D_p are the state space model matrices of the plant. The vector x_i represents the state variables of the integrators.

The uncertainties are assumed to occur at the output. Now the system performance will be determined by

$$\sigma[C\Phi K_f] \cong \frac{1}{\sqrt{\mu}}\sigma[C\Phi\Gamma] \qquad (19)$$

At low frequencies this becomes

$$C\Phi\Gamma \cong \frac{1}{s}\left[C_p\left(-A_p\right)^{-1}B_p + D_p\right]\Gamma_L = \frac{1}{s}[LFP] \qquad (20)$$

For high frequencies

$$C\Phi\Gamma \cong \frac{1}{s}\left[D_p\Gamma_L + C_p\Gamma_H\right] = \frac{1}{s}[HFP] \qquad (21)$$

In the standard matching procedure for the LQG/LTR, both LFP and HFP are set equal to the identity matrix. This eliminates much design flexibility. For performance robustness, assume HFP and LFP are diagonal matrices. For diagonal matrices, the singular values are equal to the magnitudes of the diagonal elements, which makes the frequency domain properties easy to see. This defines some relationships between elements in Γ which depend on the values of the unaugmented plant model matrices A_p, B_p, C_p, and D_p. Selecting Γ in this way assures that the TFL will have an approximately constant -20dB per decade slope for each singular value. In order to bring the maximum and minimum singular values together, the diagonal elements of HFP should be approximately equal and the diagonal elements of LFP should also be approximately equal. Thus we would like HFP and LFP should also be approximately equal to be scalar multiples of the identity matrix, however we do not restrict them to be exactly equal to the identity matrix. In general the high frequency behavior is most important for determining the bandwidth of the system and for meeting the stability robustness requirements set by the unstructured uncertainties L(s). Therefore it is usually more critical to have HFP approximate a scalar multiple of the identity matrix than it is for LFP to do so. The remaining elements of Γ are then selected, either by trial and error, or by relationships derived by setting diagonal elements of HFP or LFP equal to each other. The key to this method comes from the PRLQG development. For insensitivity to parameter variations Γ should be approximately column similar to M. In other words, the rows of A which are subject to variations should correspond to the rows of Γ which have the largest values. With Γ selected in this manner, K_f can be calculated using the algebraic Riccati equation where μ is chosen to raise or lower the TFL for bandwidth or stability robustness constraints. Then K_f should also be column similar to M, or at least have its largest values in the rows corresponding to the perturbed rows of the A matrix.

Using this approach, the TFL will have an approximately uniform slope of -20dB per decade, the bandwidth can be fixed by selecting μ, and the KF poles should be relatively insensitive to the parameter variations described by M. If the regulator gain K_c is calculated using the loop transfer recovery (LTR) procedure, that is if $K_c = (1/\rho)B^TP$ where P is the solution of algebraic Riccati equation

$$A^T P + PA + Q_{co} + q^2 C^T C - \frac{1}{\rho} PBB^T P = 0 \qquad (22)$$

and q is large, then the open loop system G(s)K(s) will have the same frequency domain properties as the TFL, except at high frequencies where $\sigma[G(s)K(s)]$ will exhibit -40 dB per decade rolloff. An example of this method will be given in the next section.

A similar procedure could be used to make the regulator poles insensitive to parameter variations in A by replacing C in with N and letting ρ get small. However if this is done, the frequency domain properties of G(s)K(s) will change and will no longer approximate those of the TFL. The control engineer should be aware of this since the C matrix can sometimes be changed by the selection of sensor locations in the physical system. If that were the case, from a robust controller point of view, the best configuration would make the C matrix row similar to N; or at least have nonzero elements in rows which correspond to the rows of A which are subject to variations. For uncertainties at the plant input, the recovery procedure is enhanced with respect to parameter variations if the B matrix is column similar to M. The structure of the B matrix is affected by locations of the actuators, that is the manipulative variables, of the system.

We note that K_f is only approximately similar to M in the sense that rows corresponding to perturbed rows of the A matrix have the larger entries. Also μ does not approach zero, but is just small enough to satisfy bandwidth requirements. For complete insensitivity to parameter variations, all the rows of K_f corresponding to rows of A which are definitely known would have to be identically equal to zero and μ would have to be very small. However insensitivity to parameter variations is not the only consideration in control system design. Nominal performance and noise rejection, for example, are extremely important and thus tradeoffs must be made. What this new procedure provides, is a way to include some consideration of parameter variations in the design of an LQG/LTR controller.

VI. ROBUST CONTROLLER DESIGN FOR GAS TURBINE ENGINES

The proposed method described in Section V is illustrated by designing a robust controller for a engine-propeller system. The results of the controllers are compared with a robust controller designed by using the standard LQG/LTR procedure.

The mathematical model of the engine-propeller system is described in Section II. The control strategy is to vary the output torque corresponding to the power level requests from the pilot or flight computer, while maintaining constant engine speed. Speed regulation is important for efficient propeller operation, and also for generators or accessories which are driven from the gearbox. The actuators in the propeller are limited according to the maximum rate of change of the blade angle, so the bandwidth of the controller is limited. Also temperatures in the engine are directly related to fuel flow, which means that overshoots in the fuel flow are associated with temperature overshoots, and therefore should be avoided if possible.

The linear models are fairly accurate from a transient point of view, but steady state errors will be present if the operating conditions change. For this reason, integrators will be augmented to the inputs to the plant to insure steady state matching of the reference and the feedback signals from the torque and speed sensors. The singular value plots of the augmented plant, are shown in Figure 4. Note that the maximum and minimum singular values are considerably different, which is a reflection of the differences between the dynamics associated with the input and output variables.

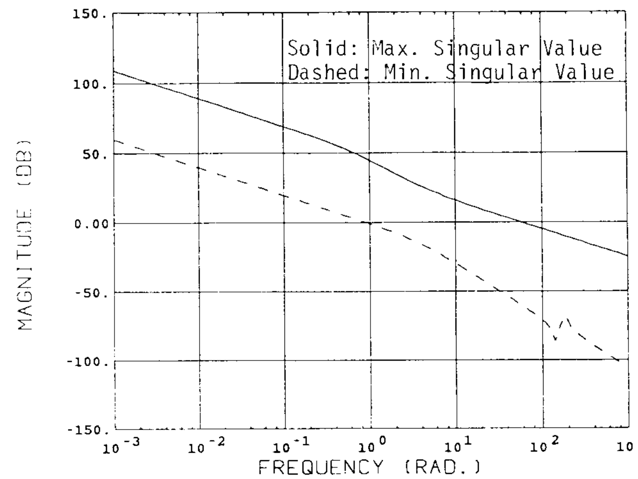


Fig. 4 Singular value plots of the plant with Integrators.

(a) Uncertainties Due to Operating Point Variations

Of the fourteen linear models available, one nominal model was selected to represent the plant characteristics. The other thirteen models are used to define variations or uncertainties in the nominal model.

$$\bar{\sigma}[L(s)] = \bar{\sigma}\left[\hat{G}(s)G^{-1}(s) - I\right] \qquad (23)$$

where G(s) is the nominal model and $\hat{G}(s)$ the perturbed model corresponding to the other operating points. The maximum singular values of L(s) calculated using linear models at thirteen different operating points in the flight envelope is shown in Fig. 5. From the figure it is concluded that the system bandwidth should be limited to less than 10 rad/sec.

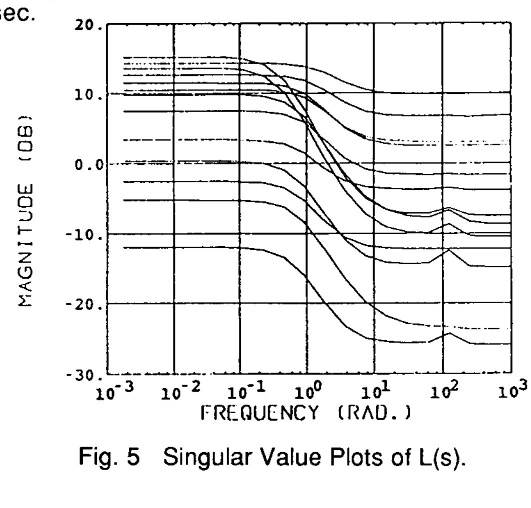


Fig. 5 Singular Value Plots of L(s).

(b) Standard LQG/LTR Design:

The first step in designing a LQG/LTR controller for uncertainties at the output, is to select Γ and μ such that the TFL has good frequency domain properties. The singular values of the plant are not close together, so the matching procedure is used. Setting $\mu = .1$ results in the TFL shown in Figure 6 and has an appropriate crossover frequency. The resulting matrices are

$$\Gamma = \begin{bmatrix} 0.0907 & 1.0404 \\ -0.0907 & 0.0171 \\ 1.0 & 0.0 \\ 1.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} \qquad K_f = \begin{bmatrix} 0.2877 & 3.2962 \\ -0.0068 & 0.0539 \\ 3.1653 & 0.0 \\ 3.1542 & -0.0023 \\ -0.0011 & 3.1618 \end{bmatrix} \tag{24}$$

The LQR gain, K_c is found using the standard recovery procedure. In this case q = 25 gave adequate recovery without requiring excessive gains in the LQR. The matrix K_c is given by

$$K_c = \begin{bmatrix} 11.7 & -3.0 & 15.18 & 5.64 & -0.0465 \\ -3.0 & 1471.0 & -2.11 & -6.55 & -0.0325 \end{bmatrix} \tag{25}$$

The resulting open loop singular values are shown in Fig. 7. The stability margins for the system are

$$\begin{gathered} -5.06\ \text{dB} < GM < 13.58\ \text{dB} \\ -46.6° < PM < 46.6° \end{gathered} \tag{26}$$

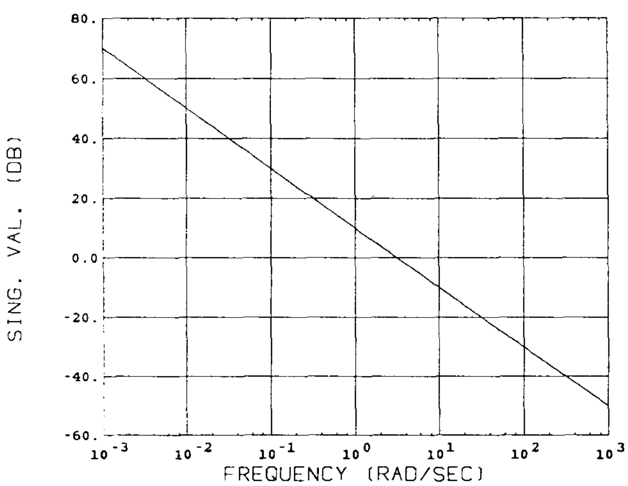


Fig. 6 Target Feedback Loop

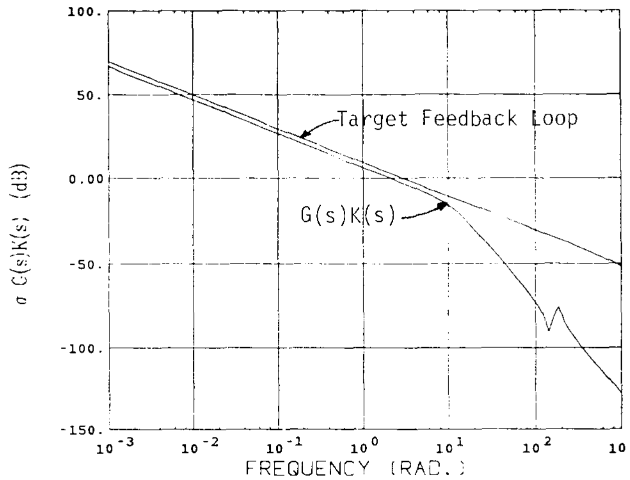


Fig. 7 Singular Value Plot of G(s)K(s) for the LQG/LTR Design

(c) Design Using Proposed Method

In this design method, the same system and models will be used, however the information about parameter variations will be utilized.

Consider the augmented system matrix A given by

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ X_3 & 0 & X_1 & 0 & C_1 \\ 0 & X_4 & 0 & X_2 & C_2 \\ 0 & 0 & K & -K & 0 \end{bmatrix} \tag{27}$$

where the X's are the elements which change as the operating conditions change. In this case, M should have zeros in rows one, two, and five, and N should have zeros in column five. For insensitivity to parameter variations, K_f should be approximately column similar to M, that is, K_f should have its largest entries in rows three and four. The K_f from the LQG/LTR design does not meet this requirement. By using the proposed method, the matrix Γ was determined such that the entries in rows three and four are large compared to the other rows while at the same time yielding a target feedback loop which has good frequency domain properties.

The matrix Γ was chosen to be

$$\Gamma = \begin{bmatrix} 0.3626 & 1.0294 \\ -0.0086 & 0.0168 \\ 2.0 & 0.0 \\ 4.0 & 8.0 \\ 1.0 & 2.0 \end{bmatrix} \tag{28}$$

The bandwidth requirements are satisfied with $\mu = 0.1$. The Kalman gain matrix K_f is given by

$$K_f = \begin{bmatrix} 2.4296 & 2.4513 \\ -0.0019 & 0.0598 \\ 16.0736 & -0.06595 \\ -4.3394 & 6.392 \\ -20.8338 & -1.7926 \end{bmatrix} \tag{29}$$

A plot of the target feedback loop for this design is shown in Fig 8.

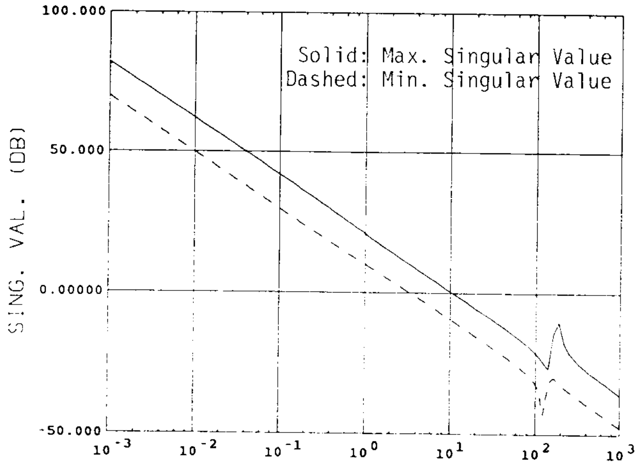


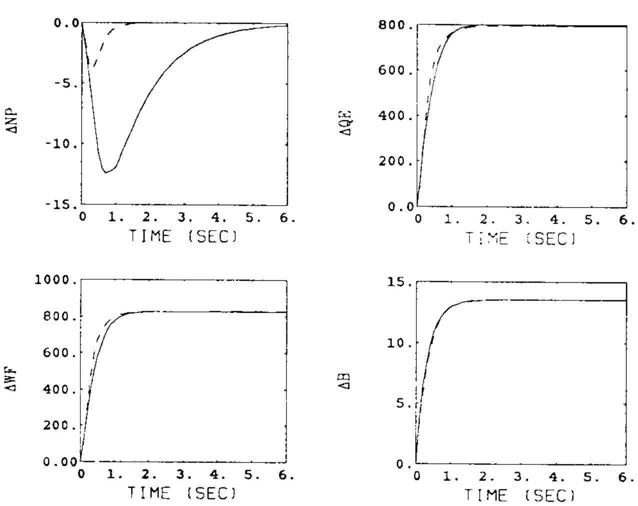
Fig. 8. Singular Value Plot of Target Feedback Loop for the PR Design

The LQR gain, K_c will be the same as in equation [25] since the recovery procedure guarantees that the frequency domain properties of G(s)K(s) will be similar to those of the TFL. The stability margins are

$$-4.69\ \text{dB} < \text{GM} < 10.9\ \text{dB} \qquad (30)$$
$$-42.0° < \text{PM} < 42.0°$$

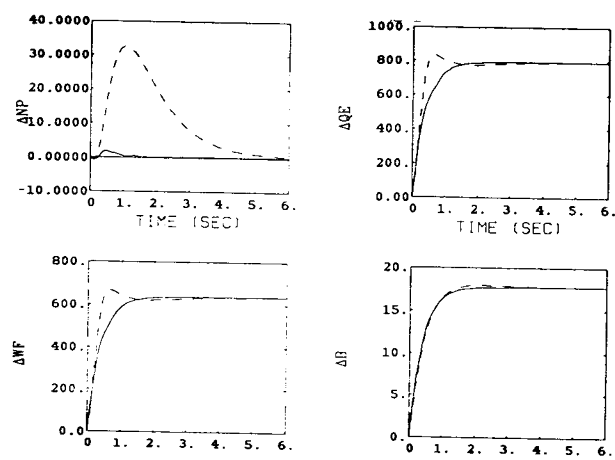
which are slightly worse than the LQG/LTR controller, but not significantly different.

Figure (9) shows the step response of the system with this new controller, and with the LQG/LTR controller, at the nominal operating point.



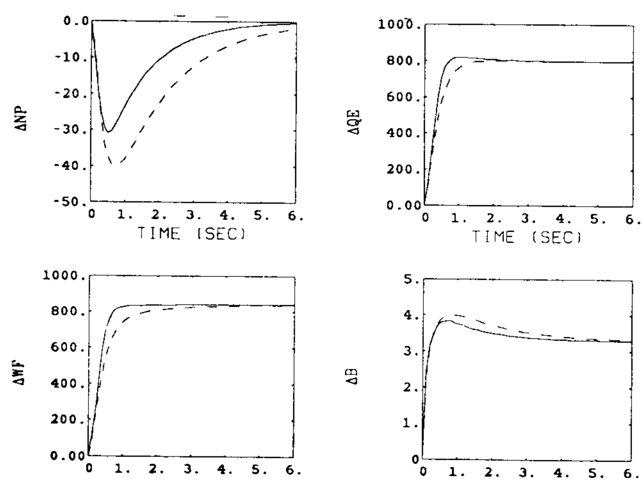
Solid = PR System Dashed = LQG/LTR

Fig. 9 Transient Response to a Step in Torque Reference at the Nominal Operating Point.



Solid = PR System Dashed = LQG/LTR

Fig. 10 Transient Responseto a Step in Torque Reference at Operating Point 232



Solid = PR System Dashed = LQG/LTR

Fig. 11. Transient Response to a Step in Torque Reference at Operating Points 214

To assess performance robustness, the time response to a step input in torque reference is computed when the controller remains fixed, but the plant model is changed to that of a different operating point. The results from the two operating points which exhibit the most deviation from the nominal point are shown in figures 10 and 11. Note that when the plant model and the model imbedded in the controller are not the same, the separation principle no longer applies, and the closed loop poles are not equal to the filter poles and the regulator poles. In the frequency domain, both controllers exhibit similar properties, as one would expect since the TFL is approximately that of an integrator in both cases. The time responses show that the parameter robust (PR) controller exhibits improved performance robustness properties in the presence of parameter variations. Although the speed change in the nominal case is slightly more for the PR controller, it is improved for both the off nominal points. Also the PR controller eliminates the overshoot in fuel flow, and corresponding temperature, that appears with the LQG/LTR design at point 232.

It should be noted that the PR procedure was not used in either case in the design of K_c, yet the results are good. The main reason for this can be seen by observing the C matrix of the augmented plant model.

$$C = \begin{bmatrix} 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & X_{10} & 0.0 & X_5 & 0.0 \end{bmatrix} \quad (31)$$

X_{10} is the largest entry, on the order of fifty, in the nominal model while X_5 is roughly 0.1. The largest variations in the nominal A matrix occur in X_4 which corresponds to the second column. The variations in X_3 and X_1, that is columns one and three, are relatively small. Consequently, the C matrix of the model is close to the N matrix which would have been chosen for the PR procedure for K_c. One would expect that if the C matrix had not been approximately similar to N, the performance robustness, especially for the LQG/LTR, would not have been as good. This illustrates the fact that selection of the C matrix, or the B matrix for input uncertainties, can have a significant effect on the robustness of the system.

VII. CONCLUSIONS

Robust controller design methodologies were applied to design controllers for gas turbine engines which are subject to both structured parameter variations and unstructured uncertainties.

The stability robustness properties of the LQG/LTR methodology with respect to unstructured uncertainties were discussed. In order to deal with both stability and performance robustness in the time domain, for structured parameter variations, the PRLQG methodology was described.

These two methodologies were compared in order to illustrate their respective strengths and weaknesses. Then a new a PR design procedure was suggested, which combines aspects of both LQG/LTR and the PRLQG. This new approach is particularly useful when the plant is subject to parameter variations of a known structure.

Gas turbine engines for aircraft are an example of a situation where structured parameter variations are well defined. A description of this type of system was given, as well as a specific example illustrating the modeling techniques.

Finally, the suggested PR controller design procedure was applied to an aircraft gas turbine engine example to illustrate the method and results. Comparisons were made between the PR Controller and a standard LQG/LTR controller for the same system. It was shown that the PR design method yielded improved performance robustness with identical controller structure and only a minimal increase in design computations.

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