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K. W. Schneider

William H. Tranter
Missouri University of Science and Technology

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EFFICIENT SIMULATION OF MULTI-CARRIER DIGITAL COMMUNICATION SYSTEMS IN NONLINEAR CHANNEL ENVIRONMENTS

Kevin W. Schneider

ADTRAN, Inc.
4955 Corporate Drive
Huntsville, AL 35805

William H. Tranter

Department of Electrical Engineering
University of Missouri-Rolla
Rolla, Missouri, 65401

ABSTRACT Although computer based simulation has become an important tool for the design and analysis of digital communication systems, the use of simulation has been limited due to the long runtimes that are often necessary. For systems that contain both frequency division multiplexing and bandpass nonlinearities, the high sampling rate that is necessary for accurate simulation contributes to the long runtimes. A partial sum of products (ParSOP) method is proposed to reduce the sampling rate of such systems. Simulation results are given that indicate that an order of magnitude reduction in the sampling rate is possible for some systems.

I. INTRODUCTION

Over the past few years, computer simulation has become an important tool for both the analysis and design of complex communication systems. The use of simulation is often limited by the long run times that are necessary for the determination of the system performance, especially the bit error rate (BER). This problem becomes more acute if the transmitted signal contains a multiplex of channels because the large resulting bandwidth necessitates a high sampling frequency in the simulation. If, in addition, the channel contains nonlinear elements, intermodulation distortion further increases the bandwidths of the signals present in the system. These effects demand an even higher sampling frequency which leads to simulation run times that are proportionally longer.

When simulating a system that uses frequency-division multiplexing (FDM), one usually concentrates on a single channel within the system for conducting a performance analysis. Using lowpass decomposition, the center frequency of the bandpass channel of interest is usually translated to zero frequency. The other channel signals in the composite FDM signal are only important if they give rise to distortion in the channel of interest. If the channel spacing is sufficiently wide, and if the system is linear, the other channels can usually be neglected and therefore need not be included in the simulation. If the channel is nonlinear, however, intermodulation distortion results in significant energy from the signals in the other channels being folded into the channel of interest. In this case, these other channels must be included in the simulation since they give rise to errors in the channel of interest.

In order to reduce the sampling rate for simulations of FDM digital communication systems where the multiplexed signal passes through a nonlinear device, we present a new simulation method which we call the Partial Sum of Products (ParSOP) method. The ParSOP method involves the decomposition of the signal at the output of a nonlinear device into a sum of products. Each product is composed of baseband versions of the individual modulated subcarriers that constitute the undistorted FDM signal. The "partial" portion of the sum includes only those products that have significant energy within the bandwidth of the single subcarrier of interest. If the channel spacing is sufficiently wide, the partial sum will include only those products that are centered in the channel of interest. The resulting reduction in the simulation bandwidth is shown in Figure 1. With the ParSOP method, the simulation sampling rate need only be large enough to accurately model the intermodulation products falling within the channel of interest rather than having to be large enough to simulate the entire bandwidth of the composite signal. As a result, a ParSOP simulation can show a significant reduction in the simulation runtime when compared to a conventional simulation.

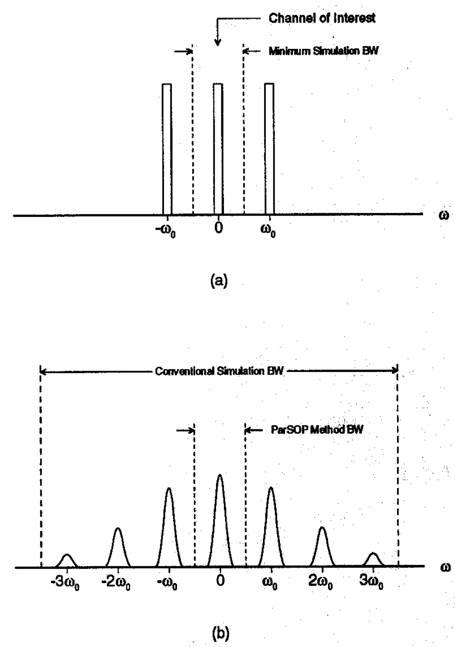

Figure 1. (a) Spectrum of a sample three-channel FDM signal. (b) Spectrum of third-order intermodulation products.

II. SYSTEM MODEL

The digital communication system being considered is modeled as shown in Figure 2. In this model, a frequency-division multiplexed (FDM) signal is the input to a bandpass nonlinear device. The nonlinear device is modeled by a nonlinearity followed by a zonal bandpass filter. The zonal filter is assumed to pass the fundamental component of the signal, centered around ω_c (the input carrier frequency), without distortion while rejecting all other harmonics of the signal [1]. After the addition of noise, the signals are de-multiplexed and each modulated subcarrier is fed to its respective demodulator. As mentioned in the introduction, it is usually desirable to demodulate and observe only one subcarrier in a given simulation. We will refer to this observed subcarrier as the kth subcarrier and the frequency band surrounding this subcarrier will be referred to as the channel of interest.

In the model of the assumed communication system of Figure 2, the signal $x(t)$, a frequency-division multiplexed signal consisting of

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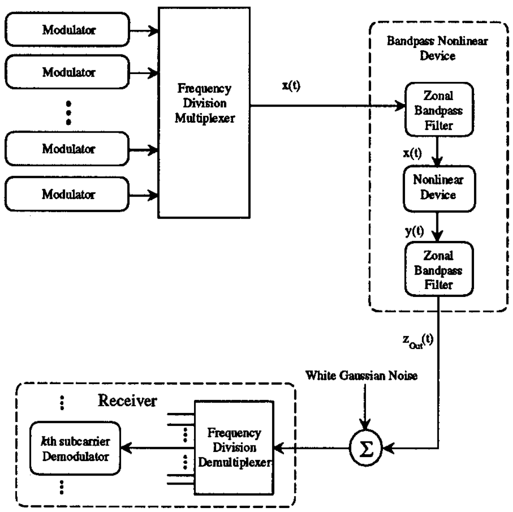

Figure 2. Block diagram of the digital communication system model.

N modulated subcarriers, can be written in the form

$$x(t)=\sum_{i=1}^{N} s_i(t)\cos[(\omega_c+\omega_i)t+\phi_i(t)] \qquad (1)$$

where ω_c is the channel carrier frequency, $s_i(t)$ is the ith subcarrier envelope, ω_i is the ith subcarrier frequency and $\phi_i(t)$ is the ith subcarrier phase. The complex envelope of $x(t)$ can be written as

$$\tilde{x}(t)=\sum_{i=1}^{N} \tilde{s}_i(t)e^{j\omega_i t} \qquad (2)$$

where

$$\tilde{s}_i(t)=s_i(t)e^{j\phi_i(t)} \qquad (3)$$

is the complex envelope of the ith subcarrier. The complex envelope of the signal $x(t)$ can also be written

$$\tilde{x}(t)=V(t)e^{j\theta(t)} \qquad (4)$$

where the envelope, $V(t)$, of the composite signal, $x(t)$, can be found from the expression

$$[V(t)]^2 = \tilde{x}(t)\tilde{x}^*(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{s}_i(t)\tilde{s}_j^*(t)e^{j(\omega_i-\omega_j)t} \qquad (5)$$

The phase $\theta(t)$ is given by

$$\theta(t)=\tan^{-1}\left(\frac{\sum_{i=1}^{N}\sin[\omega_i t+\phi_i(t)]}{\sum_{i=1}^{N}\cos[\omega_i t+\phi_i(t)]}\right) \qquad (6)$$

A bandpass nonlinear device is typically characterized by the amplitude-modulation-to-amplitude-modulation (AM/AM) and amplitude-modulation-to-phase-modulation (AM/PM) conversion characteristics [2]. For our purposes, it is more convenient to characterize the bandpass non-linear device by a complex function, $h[V]$, that operates on the input envelope and can be obtained from the AM/AM and AM/PM characteristics. The complex envelope of the output of the bandpass nonlinear device, $\tilde{z}_{Out}(t)$, is then given by

$$\tilde{z}_{Out}(t) = h[V(t)]e^{j\theta(t)} \qquad (7)$$

where $V(t)$ is the signal envelope at the input to the nonlinearity. Various forms for the nonlinear characteristic $h[V]$ have been used to model nonlinear amplifiers such as those found in satellite communication systems. [3] [4] [5]. Of these forms, the complex power series model lends itself particularly well to a direct derivation of the ParSOP method. When using this model, the complex function consists of a complex power series that includes only odd-order terms. In any practical implementation of this model, the series must be truncated to a finite number of terms. The result is a complex polynomial of degree $2R + 1$, where the value of R is selected as a trade-off between model complexity and model accuracy. This polynomial model is written in equation form as

$$h[V(t)]=\sum_{r=0}^{R} C_r V(t)^{2r+1} \qquad (8)$$

where the coefficient C_r is in general complex, but is real for all r if the nonlinearity contained in the bandpass device is memoryless.

III. DERIVATION OF THE ParSOP METHOD

Having defined the system to be investigated, we now consider the derivation of the ParSOP method. This derivation is accomplished by writing the output signal in terms of a sum of products (SOP) composed of only the complex envelopes of the modulated subcarriers. Once the SOP is obtained, the selection of products to be retained in the partial sum can be made.

As an initial step toward the formation of the SOP, we wish to express the output of the nonlinear device in terms of the modulated subcarriers. To do this, we use (7) and (8) to obtain

$$\tilde{z}_{Out}(t)= h[V(t)]e^{j\theta(t)} = \left[\sum_{r=0}^{R} C_r[V(t)]^{2r}\right]\tilde{x}(t) \qquad (9)$$

Substituting (5) for $[V(t)]^2$ and (2) for $\tilde{x}(t)$ in (9) results in

$$\tilde{z}_{Out}(t)=\sum_{r=0}^{R} C_r \left[\sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{s}_i(t)\tilde{s}_j^*(t)e^{j(\omega_i-\omega_j)t}\right]^r \left[\sum_{m=1}^{N} \tilde{s}_m(t)e^{j\omega_m t}\right] \qquad (10)$$

We next expand the expression in (10), using i with an odd subscript to replace each of the r occurrences of the index i, and i with an even subscript to replace the index j. This yields

$$\tilde{z}_{Out}(t) = \sum_{r=0}^{R}\sum_{m=1}^{N}\sum_{i_1=1}^{N}\sum_{i_2=1}^{N}\cdots\sum_{i_{2r-1}=1}^{N}\sum_{i_{2r}=1}^{N} C_r\tilde{s}_m(t)\tilde{s}_{i_1}(t)\tilde{s}_{i_2}^*(t)\ldots\tilde{s}_{i_{2r-1}}(t)\tilde{s}_{i_{2r}}^*(t)e^{j\omega_d t} \qquad (11)$$

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where

$$\omega_d = \omega_m + \left(\omega_{i_1} - \omega_{i_2}\right) + \ldots + \left(\omega_{i_{2r-1}} - \omega_{i_{2r}}\right) \qquad (12)$$

Each term in the sum represents a unique intermodulation product (IMP). The r=0 term is the undistorted input signal passed through the nonlinearity to the output. The other terms in the sum represent the intermodulation distortion (IMD) except for the case in which all of the subcarriers have constant envelopes (pure phase modulation). For this case, some of the terms for which $r \neq 0$ will be scaled versions of the undistorted input signal and thus will not constitute distortion.

To realize the reduction in the simulation sampling rate, and thus the improvement in simulation performance, that is possible with the ParSOP method, we approximate (11) by a partial sum that includes only those IMPs that have significant energy within the bandwidth of the subcarrier of interest (the kth subcarrier). If the subcarriers are sufficiently spaced in frequency, the IMPs included in the partial sum will be only those centered in the channel of interest. If the kth subcarrier is the subcarrier corresponding to the channel of interest, we only retain the IMPs that are centered at ω_k. These are the IMPs for which $\omega_d = \omega_k$. The approximation to the output signal within a frequency band around ω_k is denoted $\tilde{z}_{Out,\omega_k}(t)$ and is given by

$$\tilde{z}_{Out,\omega_k}(t) = \sum_{r=0}^{R} \sum_{m=1}^{N} \sum_{i_1=1}^{N} \sum_{i_2=1}^{N} \cdots \sum_{i_{2r-1}=1}^{N} \sum_{i_{2r}=1}^{N} C_r \tilde{s}_m(t) \tilde{s}_{i_1}(t) \tilde{s}^*_{i_2}(t) \ldots \tilde{s}_{i_{2r-1}}(t) \tilde{s}^*_{i_{2r}}(t) e^{j\omega_d t} \mathrm{H}\left[\omega_d, \omega_k\right] \qquad (13)$$

where ω_d is given by (12) and $\mathrm{H}[\omega_d, \omega_k]$ is an indicator function defined by

$$\mathrm{H}\left[\omega_d, \omega_k\right] = \begin{cases} 1, & \omega_d = \omega_k \\ 0, & otherwise \end{cases} \qquad (14)$$

which specifies which terms of the sum are to be included in the partial sum.

By examining (13) we can see the benefits that the ParSOP method brings to simulation. First, the IMPs can be formed as the product of lowpass signals. Specifically, these signals are the complex envelopes of the modulated subcarriers. If we choose ω_c so that $\omega_k = 0$ (which can be done without loss of generality), then the sum of the non-zero IMPs in (13) is also a lowpass signal. Since (13) consists of only terms that are centered at zero frequency, it can be sampled at a rate much lower than can (11). This reduction in the sampling rate is what makes the ParSOP method attractive as a simulation tool.

IV. USE OF ParSOP IN ANALYSIS

The bit error rate of digital communication systems that contain nonlinear elements cannot typically be evaluated only through theoretical analysis. However, by employing the ParSOP method, an analytical estimate of the bit error rate by can be obtained for systems that do not exhibit inter-symbol interference. When the ParSOP method is used in this way, it is referred to as the Analytic ParSOP Method.

This example application of the Analytic ParSOP method utilizes the system model of Figure 2, with only three subcarriers, whose complex envelopes are denoted by, $\tilde{s}_1(t)$, $\tilde{s}_2(t)$ and $\tilde{s}_3(t)$. The subcarrier of interest is assumed to be $\tilde{s}_1(t)$, which is located in the subchannel in which we are conducting the performance analysis. The signals $\tilde{s}_2(t)$ and $\tilde{s}_3(t)$ represent subchannels that are located at frequencies that are ω_D above and below $\tilde{s}_1(t)$, respectively, corresponding to the frequency spacing shown in Figure 1(a). In this system, the complex envelope of the signal at the input to a band-pass nonlinear device is given by

$$\tilde{x}(t) = \tilde{s}_1(t) + \tilde{s}_2(t) e^{j\omega_D t} + \tilde{s}_3(t) e^{-j\omega_D t} \qquad (15)$$

We will assume a third-order nonlinearity defined by the bandpass characteristic

$$\mathrm{h}\left[V(t)\right] = C_0 V(t) + C_1 \left[V(t)\right]^3 \qquad (16)$$

Using (13), the ParSOP method can be applied and the output signal within the bandwidth of interest can be written as

$$\tilde{z}_{Out,\omega_1=0}(t) = C_0 \tilde{s}_1(t) + C_1 \Big(\left|\tilde{s}_1(t)\right|^2 \tilde{s}_1(t) + 2\left|\tilde{s}_3(t)\right|^2 \tilde{s}_1(t) + 2\left|\tilde{s}_2(t)\right|^2 \tilde{s}_1(t) + 2\tilde{s}^*_1(t)\tilde{s}_2(t)\tilde{s}_3(t) \Big) \qquad (17)$$

Let the modulated subcarriers in the input signal $\tilde{x}(t)$ be of the same modulation type and bit rate. These signals can then be expressed as

$$\tilde{s}_1(t) = \sum_{i=-\infty}^{\infty} a_i \, p\left(t - iT + \Delta_1\right) \qquad (18\text{-a})$$

$$\tilde{s}_2(t) = \sum_{i=-\infty}^{\infty} b_i \, p\left(t - iT + \Delta_2\right) \qquad (18\text{-b})$$

$$\tilde{s}_3(t) = \sum_{i=-\infty}^{\infty} c_i \, p\left(t - iT + \Delta_3\right) \qquad (18\text{-c})$$

where $p(t)$ represents the pulse shaping function and the random sequences $\{a_i\}$, $\{b_i\}$, and $\{c_i\}$ are obtained from the modulating data. The delays Δ_1, Δ_2, and Δ_3, allow for a variation in the bit timings between the signals. By substituting (18) into (17), we obtain

$$\tilde{z}_{Out,\omega_1=0}(t) = \sum_{l=-\infty}^{\infty} p\left(t - lT + \Delta_1\right)\left[C_0 a_l + C_1 w_l(t) \right] \qquad (19)$$

where

$$\begin{aligned} w_l(t) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \Big[& a_l a_i a^*_j p\left(t - iT + \Delta_1\right) p\left(t - jT + \Delta_1\right) \\ & + 2 a_l b_i b^*_j p\left(t - iT + \Delta_2\right) p\left(t - jT + \Delta_2\right) \\ & + 2 a_l c_i c^*_j p\left(t - iT + \Delta_3\right) p\left(t - jT + \Delta_3\right) \\ & + 2 a^*_l b_i c_j p\left(t - iT + \Delta_2\right) p\left(t - jT + \Delta_3\right) \Big] \end{aligned} \qquad (20)$$

Although more complicated modulation formats can be analyzed, for simplicity we assume binary phase reversal keying (BPRK) with signaling pulses that are a constant amplitude A over a bit period and zero otherwise. In other words $p(t) = A\Pi(t/T)$, where $\Pi(t/T)$ represents a square pulse centered at zero and having width T. It is also assumed that each modulated subcarrier has the same delay which we will set equal to zero without loss of generality.

The probability of bit error for this system can be easily determined (for an AWGN channel) by using the analytic ParSOP method. We begin by finding $\tilde{z}_{Out,\omega_1=0}(t)$ as defined by (19). Since BPRK modulation is assumed, the variables a_i, b_i and c_i are real and take on the values of -1 and 1 with equal probability.

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Because the modulating pulses are non-zero only for a single non-overlapping bit period, the cross-product terms in (20) are all zero and (19) reduces to

$$\tilde{z}_{Out,\omega_1=0}(t) = \sum_{k=-\infty}^{\infty} \Big[C_0 a_k A\,\Pi(t-kT) + C_1(5a_k + 2a_k b_k c_k) A^3\,\Pi(t-kT) \Big] \tag{21}$$

The signal at the output of the integrate and dump receiver, at the dump time, is given by

$$D = \left[A C_0 + \left(5 + 2b_k c_k\right) A^3 C_1 \right] a_k T \tag{22}$$

An error is made when the noise at the output of the integrator has a magnitude that exceeds the magnitude of D and a sign that is opposite to that of D. For a linear BPRK system, the error probability is given by [6]

$$P_E = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{A^2 T}{N_0}}\right) \tag{23}$$

We can use this result to find the error probability for our system by conditioning the error probability on the value of the product $b_k c_k$. When using BPRK, the product $b_k c_k$ is -1 or 1 with probability 1/2, so the error probability can be found by substituting the magnitude of (22) in place of A in (23), for each value of the product $b_k c_k$, and multiplying the result by 1/2. The resulting expression for the error probability is

$$P_E = \sum_{i=-1,1} \frac{1}{4}\,\mathrm{erfc}\left(\sqrt{\frac{\left(A C_0 + (5+2i) A^3 C_1\right)^2 T}{N_0}}\right) \tag{24}$$

To compare this result to the probability of error for any other system, we must make the result a function of the ratio E_B/N_0, where E_B is the average received signal energy for one bit interval. The average received signal energy for one bit interval is

$$E_B = \left(C_0^2 + 10A^2 C_0 C_1 + 29A^4 C_1^2\right) A^2 T \tag{25}$$

The probability of bit error is then found by making a substitution in (24) for A^2T in terms of the bit energy. The resulting expression is

$$P_E = \sum_{i=-1,1} \frac{1}{4}\,\mathrm{erfc}\left(\sqrt{\frac{E_B/N_0 \left(C_0 + (5+2i) A^2 C_1\right)^2}{\left(C_0^2 + 10A^2 C_0 C_1 + 29A^4 C_1\right)}}\right) \tag{26}$$

Equation (26) has been evaluated and compared with the result given by a conventional simulation for the case of A = 0.3, C_0 = 1.77188 and C_1 = -0.835248. The parameter A was selected to keep V(t) in the interval [0,1] and C_0 and C_1 were selected to fit TWT AM/AM data over that interval. The comparison between the ParSOP and conventional simulation results is shown in Figure 3, (conventional simulation results are shown for several values of ω_D) from which it can be seen that the result from the ParSOP based analysis agrees very will with the conventional simulation results. In addition, results of the ParSOP simulation, to be discussed in the next section, agree with both the analytic ParSOP method results and the conventional simulation results.

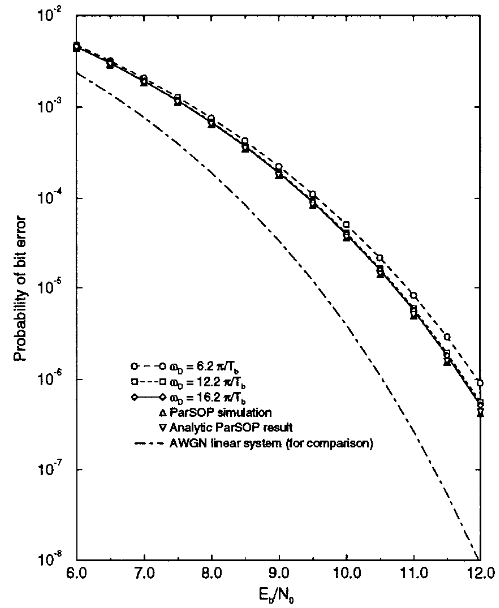

Figure 3. Conventional and ParSOP based bit error rate curves for the Analytic ParSOP Method example.

The Analytic ParSOP Method can be used on systems in which a FDM signal is passed through a bandpass nonlinear device. When the spacing between the subcarriers is sufficiently wide to prevent significant overlap in the intermodulation products, the Analytic ParSOP Method will produce excellent results.

V. ParSOP SIMULATION RESULTS

As was previously discussed, the simulation of bandpass systems is usually carried out through the use of equivalent lowpass models. A basic simulation model can be developed as shown in Figure 4. In this model, the modulated subcarrier in the channel of interest is assumed to be centered at zero frequency as is customary when using lowpass models. Simulations using this model are referred to as conventional simulations. A simulation model for the ParSOP method is shown in Figure 5, and is based on (13) with ω_k set equal to zero. Both the conventional simulation and ParSOP simulation models were used to generate the simulation results that appear in this chapter.

Since the purpose of the ParSOP method is to reduce the simulation execution time (runtime) by reducing the sampling rate that is necessary to produce accurate results, it is necessary to compare the accuracy of the bit error rate (BER) estimates obtained through the use of ParSOP simulation with those obtained through the use of conventional simulation. Although all simulation techniques involve approximations and some degree of modeling error, the ParSOP method introduces additional error when compared to conventional simulation. The errors in the BER estimate, when using the ParSOP simulation technique, occur for two reasons. First, the intermodulation products (IMPs) that are not centered on the subcarrier of interest are neglected. In addition, changes in the simulation sampling frequency cause changes in the discrete time model of the system. The error due to the neglected IMPs is the

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residual error that is incurred through the use of the ParSOP method. This error can be isolated and quantified in simulations by running the ParSOP simulation at the same sampling frequency used by the conventional simulation. The additional error due to the changes in the system model resulting from reductions in the sampling frequency can then be identified by reducing the sampling rate of the ParSOP simulations.

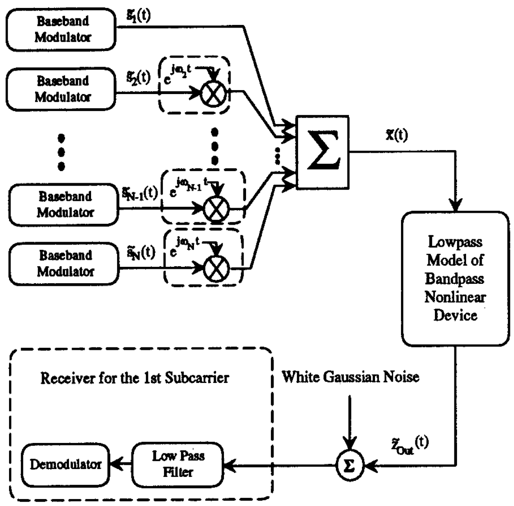

Figure 4. Block diagram of the conventional simulation model.

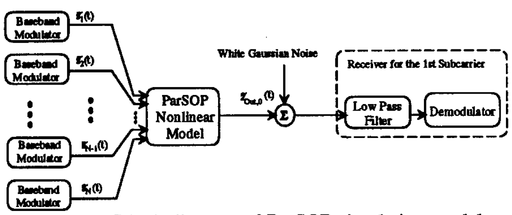

Figure 5. Block diagram of ParSOP simulation model.

Two system configurations were simulated to evaluate the performance of the ParSOP method. Both systems included 5 subcarriers. The baseband modulators in the models shown in Figure 4 and Figure 5 were each composed of an ideal BPRK baseband modulator followed by a lowpass filter. The only difference between these two systems was the choice of filters and the channel spacing. In System 1, the filters in the modulators and the receivers were all fifth-order Butterworth lowpass filters with 3-dB bandwidths of 1.5 times the bit rate. The frequency spacing between the subcarriers was 4 times the bit rate for System 1. In System 2, the frequency spacing between subcarriers was 8 times the bit rate, the modulator filters were fourth-order Chebyshev lowpass filters with 3-db bandwidths of 1.83 times the bit rate and a passband ripples of 0.1 dB, and the lowpass filter in the receiver was a fourth-order Chebyshev with a 3-dB bandwidth of 4.14 times the bit rate and 0.2 dB passband ripple. In both systems, the ideal modulators used an amplitude of 0.2 and the nonlinear characteristic was represented by $h[V] = 1.77188V - 0.835248V^3$ which was obtained by making a minimum mean-square-error fit between a third-order model and 20 sample points obtained from the AM/AM portion of a TWT characteristic over the interval $0 \le v \le 1$, which is the range of the input signal. Although the ParSOP method can be used with any BER estimation technique, semi-analytic (sometimes referred to as quasi-analytic) BER estimation [7] was used to produce the results presented here. In these results, the sampling rates and the simulation runtimes have been normalized with respect to the runtime of a conventional simulation operating with a sampling frequency large enough to accurately simulate the system.

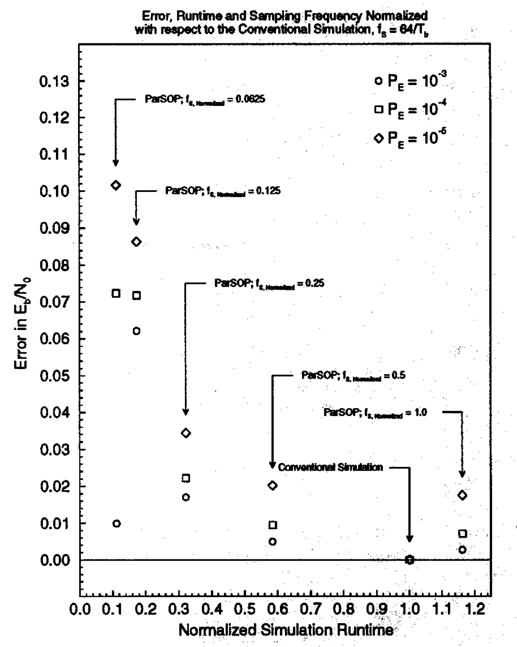

Figure 6. Plot of the error in the BER estimate vs. the runtime of the ParSOP simulation for System 1.

The result that the ParSOP method has on the simulation runtime of System 1 is shown in Figure 6. This figure illustrates the error in the BER estimate (in terms of the E_b/N_0 that was required to produce that BER) as a function of the simulation runtime for three values of the BER. Figure 6 shows that the simulation runtime can be reduced by an order of magnitude while only introducing 0.1 dB of error in E_b/N_0. The residual error that is due solely to the neglected IMPs is shown as the error associated with the ParSOP method at a normalized sampling frequency of 1.0. This error component is clearly negligible in the cases considered here. Figure 7 shows similar results for System 2. In this case the error, in terms of the E_b/N_0 that produced the BER estimate, is less than 0.04 dB while the sampling frequency is reduced by a factor of 8. In both cases the ratio of simulation execution times for the ParSOP based simulation and the conventional simulation is very near to the ratio of their respective sampling frequencies.

To better assess the effect of the subcarrier spacing on the performance of the ParSOP method, the residual error of the ParSOP method was measured for several subcarrier spacings, f_D. Figure 8 shows the residual error as a function of the frequency spacing between the modulated subcarriers for System 1. As the frequency spacing between the subcarriers was reduced, the residual error increased due to increased adjacent channel interference.

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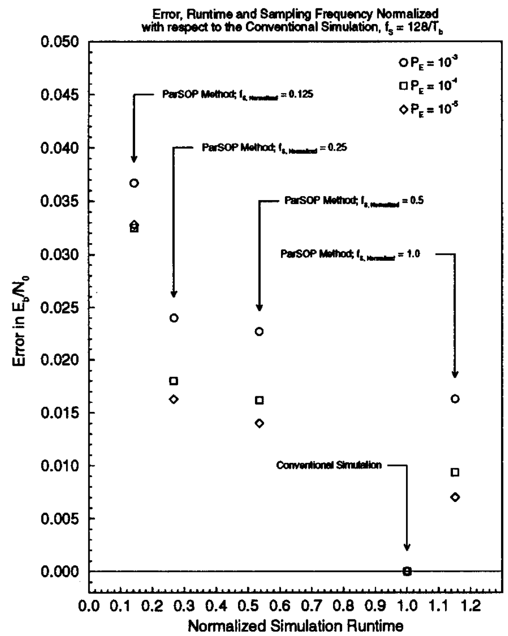

Figure 7. Plot of the error in the BER estimate vs. the runtime of the ParSOP simulation for System 2.

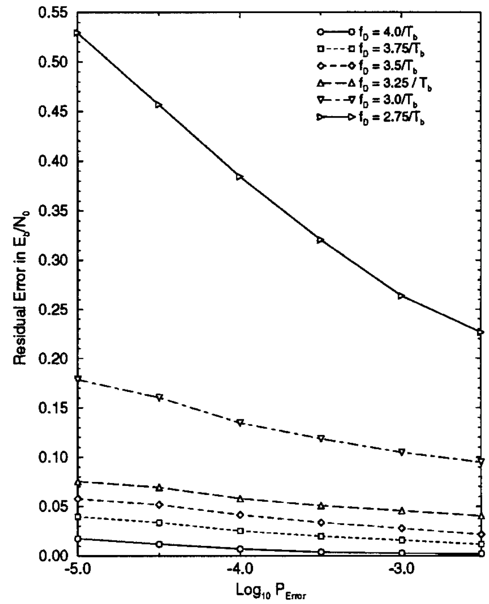

Figure 8. Residual error of the ParSOP method as a function of subcarrier spacing for System 1.

VI. CONCLUSIONS

In the conventional simulation of frequency-division multiplexed systems operating over channels containing nonlinear devices, a large sampling frequency must be used to accurately represent the signals involved. The ParSOP method allows the simulation sampling frequency to be reduced by only generating those intermodulation products that are centered in the bandwidth of the single observed modulated subcarrier. The direct derivation of the ParSOP method is limited to systems where the bandpass characteristic of the nonlinearity can be represented by a complex power series.

Simulation results, obtained by using the complex power series nonlinear model on a filtered BPSK system, showed that a reduction in the simulation runtime by an order of magnitude is possible through the use of the ParSOP method on systems involving third-order nonlinearities. In these simulations, the reduction in runtime paralleled the reduction in the sampling rate, although as the model order is increased, and the number of intermodulation products increases, this may not always be the case. The residual error, as expected, was shown to increase as the spacing between the subcarriers was reduced.

In conclusion, the ParSOP method has a demonstrated ability to reduce the simulation runtime of simulations of multi-carrier digital communication systems that operate over nonlinear channels. Because of this ability, the ParSOP method is a useful addition to the family of simulation tools used by communications engineers.

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