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AN OPTIMAL CONTROL LAW BY EIGENVALUE ASSIGNMENT FOR IMPROVED DYNAMIC STABILITY IN POWER SYSTEMS

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Abstract - Whenever control laws are computed on the basis of linear optimal control theory and implemented on non-linear systems, such as power systems, the performance is not as good as expected because of saturation type non-linearities in the system components. The only way to ensure that a control law is adequate, short of actually testing the real system, is to observe the simulated behavior of the system. Departure between a calculated linear control law and the actual response can be minimized by (a) computing a feedback control law with gains of small magnitudes to achieve a pre-assigned set of eigenvalues for the closed loop system and by (b) judicious assignment of the eigenvalues to be achieved. This paper discusses a method for achieving such a control law.

INTRODUCTION

The application of the optimal control theory of linear systems to improve the dynamic stability of power systems is well known. There are two important criticisms against the use of this theory. One of them is that in computing the optimal control a quadratic cost function of state and control variables is assumed. The arbitrariness involved in the parameters of the cost function always affect the final choice of the control law. It can also be shown that any stable control law is optimal with respect to some positive definite quadratic cost function [1]. In this case, why should one bother with calculating an exact optimal control law with respect to an arbitrarily chosen cost function? One might, just as well, select some control law which stabilizes the system, and this control law is, of course, optimal in some sense. The second criticism in the application of optimal control theory to a linearized model of a power system is that even when a control law is designed to achieve very fast damping of the electromechanical oscillations, the performance of this control law may be very poor when the non-linearities are considered. So the only way to decide if a given control law is adequate and can be implemented practically is to simulate the behavior of the system and to observe (a) how well the oscillations are damped and (b) how large a transient displacement can be damped.

A good estimation of how well the oscillations are damped can be determined from the eigenvalues of the closed loop system. It would be advantageous to be able to compute a control law to achieve a pre-selected set of eigenvalues for the closed loop system. Methods exist for computing a linear constant control law by employing state feedback to achieve an arbitrary set of eigenvalues for a completely controllable and observable linear system [2-12]. Feedback control law gains required to achieve an eigenvalue with a large negative real part (to ensure a fast decay of oscillations) are usually large. The large control effort required to realize this feedback law may not be available from the controlling devices, because of saturation type non-linearities common to such devices. So even though such control laws can be computed, they cannot be implemented practically. Whereas this defect is inherent in all applications of linear control theory to non-linear systems such as power systems, its impact could be minimized if one were (a) able to achieve a pre-assigned set of eigenvalues with "minimum" possible magnitudes of feedback constants and (b) able to judiciously select the set of eigenvalues. In general, minimizing the magnitudes of the feedback constants reduce the effect of noisy state measurements. The purpose of this paper is develop a procedure and algorithm for selecting an optimal control law for a selected set of eigenvalues.

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STATEMENT OF THE PROBLEM

Given a set of linearized state equations for a power system in terms of the state variables as follows:

$$\dot{x} = Ax+Bu \tag{1}$$

where x is an n-vector of states and u is an m-vector of inputs and both A and B are constant matrices of appropriate dimensions, a linear constant feedback control law,

$$u = Fx \tag{2}$$

where F is an mxn constant matrix, is to be computed such that a diagonal matrix D of eigenvalues λ_i, i=1, 2, ... n can be realized for the closed loop system, while at the same time the sum of the squares of the elements of F is minimized.

In other words given equation (1) and (2), the values of F are computed such that for the objective function V given by

$$V = tr\{\frac{1}{2} F' R F\}$$

(where tr $\{\cdot\}$ is the sum of diagonal elements of $\{\cdot\}$ and $\{\cdot\}'$ is transpose of $\{\cdot\}$ and R is a diagonal matrix of appropriate weighting factors which is to be minimized subject to the constraint that

$$(A+BF)P-PD = 0$$

where P is the matrix of the eigenvectors of the closed loop system.

SOLUTION OF THE PROBLEM

Now the solution for F of this problem is that which minimizes the Lagrangian

$$\Omega = tr\{\frac{1}{2} F'RF+L'[(A+BF)P-PD]\} \tag{6}$$

where L is a matrix of Lagrange multipliers.

So for the optimal feedback matrix F the following is true

$$\frac{\partial\Omega}{\partial F} = 0 \rightarrow RF+B'LP' = 0$$

$$\rightarrow F = -R^{-1}B'LP' \tag{7}$$

$$\frac{\partial\Omega}{\partial L} = 0 \rightarrow (A+BF)P-PD = 0 \tag{8}$$

$$\frac{\partial\Omega}{\partial P} = 0 \rightarrow (A+BF)'L-LD = 0 \qquad (9)$$

Equation (9) implies that L is the matrix of eigenvectors of (A+BF). So

$$L = P'^{-1}Q \qquad (10)$$

where Q is a diagonal matrix of weighting factors which may assume any non-zero values. Substituting (10) in (7) results in

$$F = -R^{-1}B'P'^{-1}QP' \qquad (11)$$

So the optimal control law u = Fx is given by the solution of (8) and (11) with appropriate weighting factors Q.

The following algorithm is suggested to solve these equations.

Step 1. Assume any values for F.

Step 2. Modify the 1st row of F so as to realize eigenvalues given by D and find P to satisfy (8). (Proof that this step can be executed for almost any initial values of F follows from Theorem 4 in [5] and the procedure suggested in Appendix III).

Step 3. Substitute P and the new value of F in (11) and find the diagonal matrix Q such that the mean square error between the left and right hand sides of the equation is minimized.

Step 4. Substitute the computed value of Q on the right hand side of equation (11) to obtain a value of F_1.

Step 5. The new value of F is computed as

$$F = F+\alpha[F_1-F] \qquad (12)$$

where α is a scalar chosen appropriately to improve the convergence of the algorithm.

Step 6. Repeat Step 2.

NUMERICAL EXAMPLE

Consider a single synchronous generator shown in Figure 1, connected to an infinite bus through a transmission line. For simplicity the resistances of the generator and the transmission line are neglected. The machine and system parameters as well as the steady state operating values of the various variables are given in Table I.

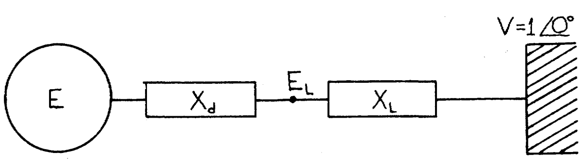

Figure 1. A Schematic Diagram of the Single Machine Power System

The state equations for the system (linearized about the steady state operating point) are found to be

$$\begin{bmatrix}\dot{\Delta E}\\ \dot{\Delta\delta}\\ \dot{\Delta\omega}\\ \dot{\Delta p}\end{bmatrix} = \begin{bmatrix}-0.3556 & 0.0 & .7467 & 0.0\\ 0.0 & 0.0 & 1.0 & 0.0\\ -30.0 & -14.5833 & -1.0 & 50.0\\ 0.0 & 0.0 & 0.0 & -1.0\end{bmatrix}\begin{bmatrix}\Delta E\\ \Delta\delta\\ \Delta\omega\\ \Delta p\end{bmatrix} + \begin{bmatrix}0.3556 & 0.0\\ 0.0 & 0.0\\ 0.0 & 0.0\\ 0.0 & 1.0\end{bmatrix}\begin{bmatrix}U_1\\ U_2\end{bmatrix} \qquad (13)$$

See Appendix I for the derivation of these state equations.

Table I
System Parameters
(See Appendix II)

M = 0.02 p.u.	D = 0.02 p.u.
x_d = 1.0 p.u.	x_d' = 0.3 p.u.
x_1 = 0.6 p.u.	T_g = 1.0 sec.
T_d = 5.0 sec.	V = 1.0 /0° p.u.
E_o = 1.6667 p.u.	P_o = 1.0 p.u.
δ_o = 73.74°	E_1 = 1.0 p.u.
	δ_1 = 36.87°

For comparison with the proposed method an optimal control F_1 using a standard approach is computed. The optimal control $u = F_1x$ which minimizes a quadratic cost function of the states and the control variables is computed [13]. The cost function is assumed to be

$$J = \frac{1}{2}\int_{t=0}^{\infty}(x'Q_1x+u'R_1u)dt \qquad (14)$$

where

$$Q_1 = \begin{bmatrix}10 & 0 & 0 & 0\\ 0 & 10 & 0 & 0\\ 0 & 0 & 10 & 0\\ 0 & 0 & 0 & 10\end{bmatrix} \text{ and } R_1 = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad (15)$$

The solution for F_1 is found to be

$$F_1 = \begin{bmatrix}-4.0746 & 0.2896 & 0.4463 & 2.7672\\ 7.7818 & 1.5464 & -2.5261 & -14.9984\end{bmatrix} \qquad (16)$$

The eigenvalues of the closed loop system with this control are -0.7626, -1.7961, $-8.1221\pm j.9076$. When the method proposed in this paper is used to compute a control law $u = F_2x$ which realize the same eigenvalues for the closed loop system F_2 is found to be

$$F_2 = \begin{bmatrix} -3.6813 & -0.0603 & -1.6292 & 1.1672 \\ 3.4625 & 1.9686 & -2.8846 & -15.1382 \end{bmatrix}$$

The behavior of the non-linear system incorporating these three different control laws was simulated with a three phase fault applied at the terminals of the machine. The critical clearing time with $u = F_1x$ is found to be 0.059 sec., with $u = F_2x$ it is 0.071 sec., and for the uncontrolled system it is 0.091 sec. The cost function J was computed for all three cases with the fault applied for 0.05 sec. The value of the cost function with $u = F_1x$ was 6.290, with $u = F_2x$ it was 6.124, and for the uncontrolled system it was 13.917. Figures indicating the response of the states are shown in Figures 2-7.

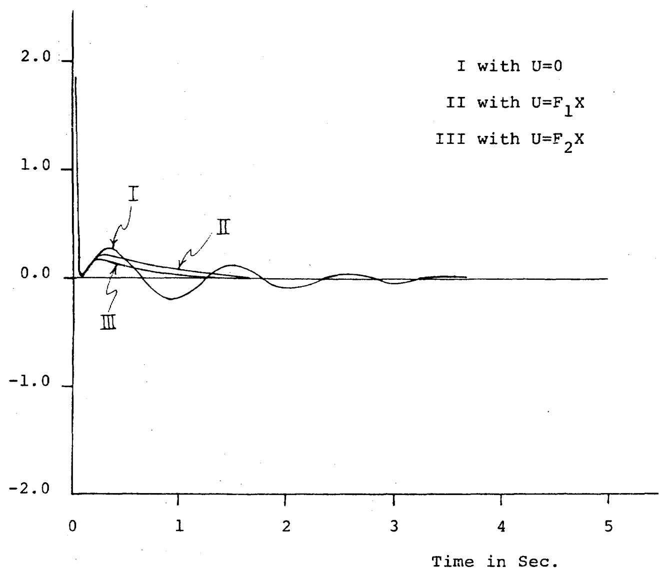

Figure 2. Response of the Machine to a Three Phase Short Circuit at its Terminals, Cleared in 0.05 Sec. - ΔE vs. Time

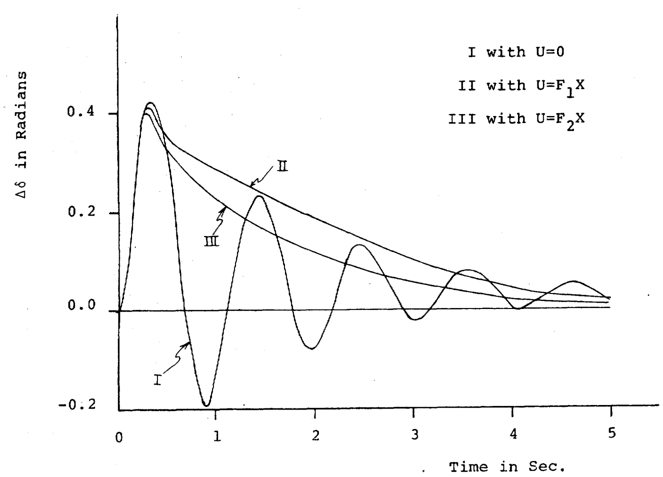

Figure 3. Response of the Machine to a Three Phase Short Circuit at its Terminals, Cleared in 0.05 Sec. - $\Delta\delta$ vs. Time

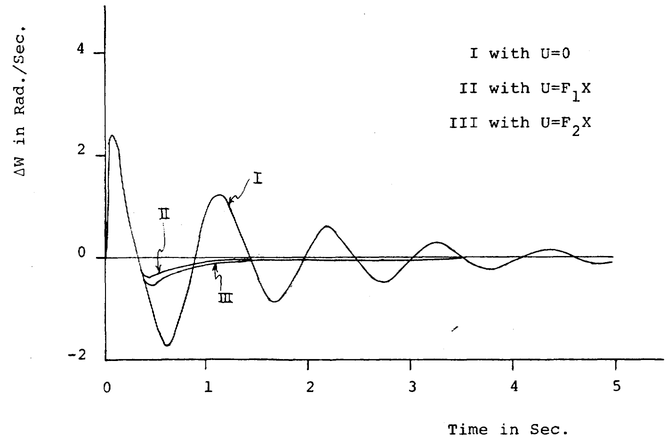

Figure 4. Response of the Machine to a Three Phase Short Circuit at its Terminals, Cleared in 0.05 Sec. - ΔW vs. Time

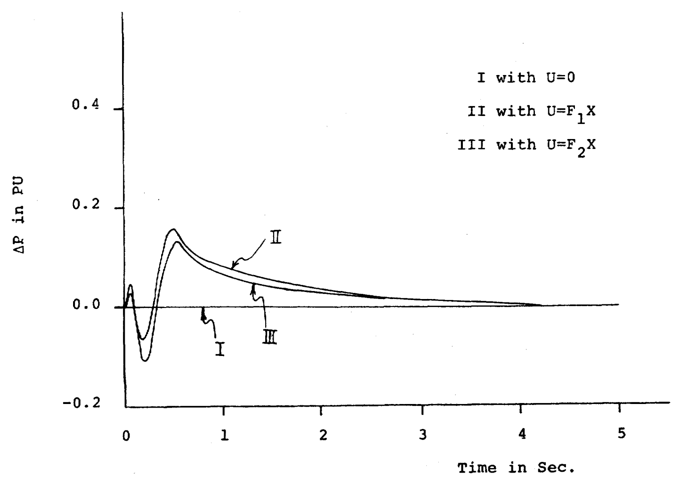

Figure 5. Response of the Machine to a Three Phase Short Circuit at its Terminals, Cleared in 0.05 Sec. - ΔP vs. Time

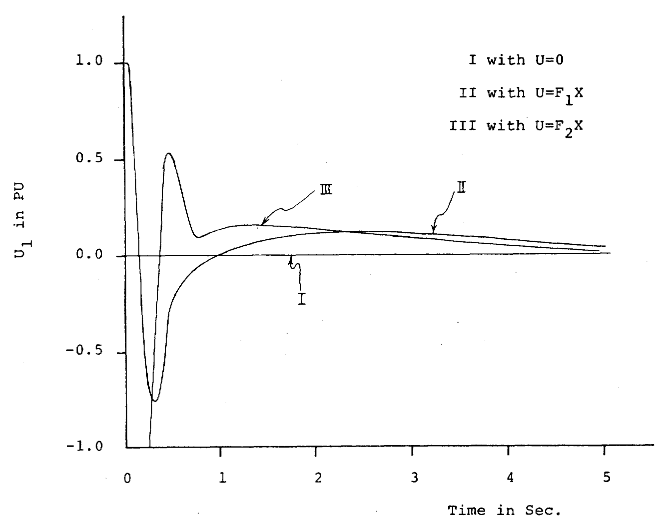

Figure 6. Response of the Machine to a Three Phase Short Circuit at its Terminals, Cleared in 0.05 Sec. - U_1 vs. Time

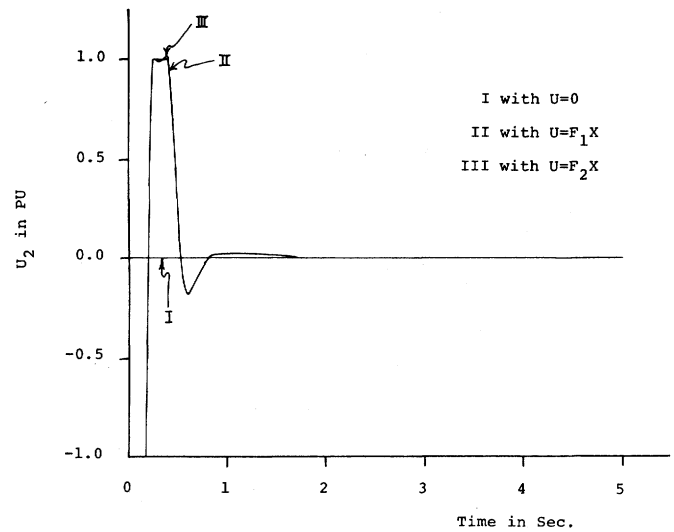

Figure 7. Response of the Machine to a Three Phase Short Circuit at its Terminals, Cleared in 0.05 Sec. - U_2 vs. Time

The computations requirements per iteration for an nth order system by the new method proposed are of the order of n^4 whereas conventional optimal control theory requires computations of the order of n^6.

CONCLUSIONS

A method is presented for computing an optimal linear constant feedback law to achieve arbitrarily pre-assigned eigenvalues for a closed system. The computational requirements of this method are much smaller than those of conventional linear optimal control theory. With the same eigenvalues for the closed loop system, The conventional optimal control and the optimal control proposed in this paper result in about the same kind of response. So whenever a fast calculation of a good control law is required, the method proposed here is very appropriate.

APPENDIX I
THE STATE VARIABLE FORMULATION OF THE SINGLE MACHINE POWER SYSTEM MODEL

For the single machine system, the system differential equations are

$$M\Delta\ddot{\delta} + D\Delta\dot{\delta} + \Delta P_e = P \tag{17}$$

This is the equation of electromechanical oscillation where

$$Pe = \frac{EV}{(x_d+x_1)} \sin\delta$$

and

$$\Delta Pe = C_1\Delta\delta + b_1\Delta E$$

where

$$C_1 = \frac{\partial Pe}{\partial\delta} = \frac{EV}{(x_d+x_1)} \cos\delta$$

and

$$b_1 = \frac{\partial Pe}{\partial E} = \frac{V}{x_d+x_1} \sin\delta$$

Further we know that

$$\Delta\dot{\delta} = \Delta\omega \tag{18}$$

By neglecting the time lag in the exciter, we have

$$\Delta E + \frac{d}{dt}[T_{do}\Delta E'] = u_1 \tag{19}$$

where

$$E' = E-(x_d-x_d')I_d$$

$$I_d = \frac{E-V\cos\delta}{(x_d+x_1)}$$

and u_1 is the deviation of exciter voltage from the steady state value in per unit. Therefore

$$\Delta E' = C_2\Delta\delta + b_2\Delta E \tag{20}$$

where

$$C_2 = \frac{\partial E}{\partial\delta} = -\frac{V(x_d-x_d')}{(x_d+x_1)} \sin\delta$$

and

$$b_2 = \frac{\partial E'}{\partial E} = 1 - \frac{(x_d-x_d')}{(x_d+x_1)}$$

This is substituted in (19) to get

$$\Delta\dot{E} = -\frac{1}{T_{do}b_2}\Delta E - \frac{C_2}{b_2}\Delta\omega + \frac{1}{T_{do}b_2}u_1 \tag{21}$$

Further

$$\Delta P + \frac{d}{dt}(T_g\Delta P) = u_2 \tag{22}$$

where u_2 is the deviation from steady state, of the input valve opening for the turbine, in per unit. Equations 17, 18, 21 and 22 are rewritten in the form of state equations to get

$$\begin{bmatrix} \Delta\dot{E} \\ \Delta\dot{\delta} \\ \Delta\dot{\omega} \\ \Delta\dot{p} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{do}b_2} & 0 & -\frac{C_2}{b_2} & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{b_1}{M} & -\frac{C_1}{M} & -\frac{D}{M} & \frac{1}{M} \\ 0 & 0 & 0 & \frac{-1}{T_g} \end{bmatrix} \begin{bmatrix} \Delta E \\ \Delta\delta \\ \Delta\omega \\ \Delta p \end{bmatrix} + \begin{bmatrix} \frac{1}{T_{do}b_2} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_g} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{23}$$

Substituting the values of various parameters from Table I yields the state equations (13).

APPENDIX II
NOMENCLATURE

M	Inertia constant of the machine
D	Damping coefficient

x_d	Direct axis synchronous reactance
x_d'	Direct axis transient reactance
T_{do}	Time constant of direct axis field
T_g	Time constant for turbine inlet valve
x_1	Reactance of the transmission line
V	Reference bus voltage
E_1/δ_1	Terminal voltage of the machine
E	Open circuit voltage of the machine
E'	Voltage behind transient reactance of the machine
δ	Rotor angle of the machine in electrical radians
ω	Angular velocity
P	Mechanical power input
P_e	Electrical power input
Δ	Incremental operator
I_d, I_v	Direct and quadrature axes currents of the machine
$[\cdot]_o$	Steady state operating value of $[\cdot]$

APPENDIX III

In recent years there has been an increasing interest in the design of multivariable feedback systems to achieve a desired behavior of the closed loop system, where this behavior is defined by a set of pre-selected eigenvalues. Many different algorithms are available to compute the feedback gains and realize the arbitrarily assigned eigenvalues for the closed loop system. However, these methods involve computation of certain canonical forms for the given system equations [10, 11]. In this paper a new algorithm, which does not require the computation of such canonical forms is presented.

Statement of the Problem

For a linear time invariant system described by the state equations (1) and (2) in the paper

$$\dot{x} = (A+BF)x \tag{24}$$

The eigenvalues of the matrix (A+BF) determine the characteristics of the transient response of the closed loop system.

The problem addressed in this paper is to determine an F such that the closed loop system will have any pre-assigned set Λ of eigenvalues $\lambda_1, \lambda_2, \ldots \lambda_n$. Where the λ's are distinct and for any complex value of λ there exists a $\lambda_j = \lambda_i^*$ in the set Λ.

The following theorem is well known. Theorem 1: For every choice of the set Λ, there is a matrix F such that A+BF has Λ for its set of eigenvalues if and only if the system $\dot{x}$ = Ax+Bu is controllable (1). Therefore only controllable systems are considered

Solution of the Problem
Case 1: Systems With Single Input:

An nth order linear system with only one input can be described by:

$$\dot{x} = Ax+bu$$

and

$$u = fx$$

where b is the column vector $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ and b_1 is a non-zero scalar and b_2 is an (n-1) vector. Under the assumption that the given system is controllable, $[b]$ contains at least one non-zero element which can be denoted by b_1. Note: the system equations can always be arranged such that b occupies the 1st row of b. Requiring b to have a non-zero value for the first element therefore does not restrict the class of systems considered.

There exists a constant (1xn) matrix f such that (A+bf) has λ_i, i=1,2,...n as its eigenvalues and p_i as the corresponding eigenvectors, i.e.,

$$(A+bf)p_i = \lambda_i p_i \tag{25}$$

so

$$bfp_i = (\lambda_i I-A)p_i = \bar{A}p_i \tag{26}$$

where I is a unit matrix of the proper dimension and $\bar{A} = (\lambda_i I-A)$. Let $\bar{A}$ be partitioned into $\begin{bmatrix} \bar{A}_1 \\ \bar{A}_2 \end{bmatrix}$ where $\bar{A}_1$ is the 1st row of $\bar{A}$. Then,

$$b_1 fp_i = \bar{A}_1 p_i \tag{27}$$

and

$$b_2 fp_i = \bar{A}_2 p_i \tag{28}$$

From (27) and (28) it follows that

$$b_1^{-1}\bar{A}_1 p_i = fp_i \tag{29}$$

$$(\bar{A}_2 - b_2 b_1^{-1}\bar{A}_1)p_i = [0] \tag{30}$$

It is easy to see that a unique solution for f exists and also that the n equations resulting from (29) as i assumes values 1,2,...n, would be sufficient to solve for f provided that p_i can be solved for, using (30). There are, however, only (n-1) equations given by (30) to solve for the n elements of p_i, therefore, pi cannot be determined uniquely. However, p_i need not be unique because an eigenvector of any matrix can be scaled by any non-zero scalar. The ratios of the elements of p_i are, however, unique. So if one of the non-zero elements of p_i is scaled to be 1, then the other elements should be uniquely determined by (30). This leads to the following algorithm for calculating f.

Algorithm 1

Step 1. Augment equation (30) by a trivial equation

$$\begin{bmatrix}0 & 0 & 0 & \dots & 0\end{bmatrix} p_i = \begin{bmatrix}0\end{bmatrix}$$

and write

$$\begin{bmatrix}0 & 0 & \underset{\underline{A}}{0} & \dots & 0\end{bmatrix} p_i = \begin{bmatrix}0\end{bmatrix} \qquad (31)$$

where

$$\underline{A} = \bar{A}_2 - b_2 b_1^{-1} \bar{A}_1$$

Step 2. By a procedure similar to Gaussian Elimination rewrite the equations (31) in the form

$$\underline{\underline{A}} p = \begin{bmatrix}0\end{bmatrix} \qquad (32)$$

where the elements of $\underline{\underline{A}}$ above the diagonal and the first (n-k) diagonal elements are zero. All other diagonal elements are 1's, and k is the rank of the largest non-singular diagonal submatrix of $\underline{\underline{A}}$.

Step 3. Set the first (n-k-1) elements of p_i to zero and the (n-k)th element to 1 and solve for the remaining elements of p_i from (31).

Step 4. Repeat Step 1, 2, and 3 for $i=1,2,\dots n$.

Step 5. Substitute p_i's in (29) and solve for f.

Case 2: Systems With Multiple Inputs

It is assumed that the state equations of the system

$$\dot{x} = Ax+Bu \qquad (33)$$

are written such that B_{11} is non-zero. Without loss of generality, these equations could be rewritten as

$$\dot{x} = Ax+Bv+bw \qquad (34)$$

where b is the first column of B and w is a scalar variable. Let

$$v = \underline{F}x$$

and

$$w = fx$$

Then

$$u = Fx$$

where

$$F = \underline{F} + \begin{bmatrix}f\\0\end{bmatrix} \qquad (35)$$

Then equation (34) reduces to

$$\dot{x} = (A+B\underline{F})x+bw \qquad (36)$$

Now the following theorem is useful. Theorem 2: If the system given by equations (34) is controllable, then for almost all $\underline{F}$ the system described by (36) is also controllable. The set of values of $\underline{F}$ for which it is not true is either empty or a hypersurface in the parameter space of $\underline{F}$. (This is a special case of Theorem [4] in [3].)

Therefore for almost all $\underline{F}$, (36) describes a controllable single input system. Algorithm 1 can then be used to find an f such that w = f x would result in any pre-assigned set of eigenvalues for the closed loop system (36) and also for the system (33).

This leads to the following algorithm for arbitrary eigenvalue assignment in multivariable systems with multi-inputs.

Algorithm 2

Step 1. Assume any values for $\underline{F}$.

Step 2. Use Algorithm 1 for eigenvalue assignment for the system (36) to compute f.

Step 3. Calculate $F = \underline{F} + \begin{bmatrix}f\\0\end{bmatrix}$.

The preceding algorithms 1 and 2 are useful only when the desired eigenvalues are distinct. It is easy to modify the procedure to include repeated eigenvalues. However, assigning repeated eigenvalues is generally to be avoided because it only increases the computations required. For example, if an eigenvalue is repeated twice, then the 2n elements of the two corresponding eigenvectors have to be evaluated simultaneously rather than independently. Assigning repeated eigenvalues does not necessarily improve the closed loop system behavior appreciably.

Numerical Example

Consider the system

$$\dot{x} = Ax+Bu$$

where

$$A = \begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\1 & 0 & 1\end{bmatrix} \text{ and } B = \begin{bmatrix}1 & 0\\0 & 1\\0 & 0\end{bmatrix}$$

It is easy to show that this system is controllable when both controls are present. If only one control is used, the system is not controllable.

Now write

$$\dot{x} = (A+B\underline{F})x+bw \qquad (36)$$

and

$$w = fx$$

where $\underline{F}$ is arbitrarily chosen to be $\begin{bmatrix}0 & 0 & 0\\0 & 0 & 1\end{bmatrix}$. Then with this $\underline{F}$ it can be shown that system (36) is controllable. If it is not controllable, $\underline{F}$ has to be chosen differently. (Theorem 2 guarantees that proper values for $\underline{F}$ can be chosen almost arbitrarily.)

Now for system (36), with desired eigenvalues λ_i, the form of equation (31) is

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & (\lambda_i-1) & -1 \\ -1 & 0 & (\lambda_i-1) \end{bmatrix} \begin{bmatrix} p_{1i} \\ p_{2i} \\ p_{3i} \end{bmatrix} = [0]$$

Assuming the desired eigenvalues are to be 3, 2, 1, respectively (usually eigenvalues with positive real parts are undesirable, however, in this example the eigenvalues are intentionally chosen to be 3, 2, and 1 to illustrate what the procedure is when the first element of the eigenvector cannot be scaled to be 1) for $\lambda_1 = 3$ equation (31) is

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -1 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{22} \\ p_{33} \end{bmatrix} = [0]$$

By proper scaling and addition of the rows of the above matrix, one gets the following form for equation (32)

$$\begin{bmatrix} 0 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{22} \\ p_{33} \end{bmatrix} = [0]$$

So if p_{1i} is scaled to be 1, the other two elements of the eigenvalue are uniquely determined. So the eigenvector is

$$\begin{bmatrix} 1 \\ 0.25 \\ 0.5 \end{bmatrix}$$

For $\lambda_2 = 2$, equation (10) becomes

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \end{bmatrix} = [0]$$

Equation (11) is then

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \end{bmatrix} = [0]$$

And the eigenvector is found to be

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda_3 = 1$ equation (31) becomes

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \end{bmatrix} = [0]$$

Equation (32) is

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \end{bmatrix} = [0]$$

In this case scaling p_{13} to be 1 does not uniquely determine the other two elements of the eigenvector. So p_{13} cannot be scaled to 1. This happens when p_{13} is zero. So p_{13} is set to zero and p_{23} is scaled to 1. This uniquely determines the remaining elements (in this case only one element). $p_{33} = 0$. So the eigenvector is

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

By substituting the values of eigenvectors and eigenvalues in equation (29), the following equation is obtained

$$\begin{bmatrix} 1.5 & 0 & 0 \end{bmatrix} = f \begin{bmatrix} 1 & 1 & 0 \\ .25 & 1 & 1 \\ .5 & 1 & 0 \end{bmatrix}$$

or

$$f = (3 \;\; 0 \;\; -3)$$

so

$$F = F + \begin{bmatrix} f \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$u=Fx$ is a control law which assigns the eigenvalues 3, 2, and 1 to the closed loop system.

This method of eigenvalue assignment involves computations of the order of n^4 for the nth order system. This is similar to the requirement for the method based on the Jordan Canonical form [30]. However, unlike the latter, the method proposed here yields the eigenvector matrix for the closed loop system. The method based on the normal form [31] requires only computations of the order of n^3 [32]. However, this method does not yield any information about the eigenvectors for the closed loop system.

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Discussion

Nelson Martins (CEPEL Rio De Janeiro, Brazil): Control methods which allow the assignment of closed loop systems eigenvalues can be useful if applied judiciously. The authors have obtained unrealistic results since the eigenvalues specified for the closed loop system are excessively damped, which caused the effective gain of the governor control loop to be very high. The extremely large control effort required by the proposed controller cancels out rotor oscillatiors completely, while a desirable rotor angle response would be one just a little more damped than that of the uncontrolled system! Additionally, power system stabilization should be obtained through excitation control rather than governor control, since the latter is sluggish and results in mechanical wear. The discussor would like to make the following remarks:

1. The problem of stabilizing a single machine - infinite bus system is trivial. The main concern nowadays is in locating the best points in the multi-machine system for placing the damping effort, and also in finding suitable tuning for the stabilizing signals.

2. A problem with pole assignment techniques is that one has no control over the zeros of the closed loop system which have a fundamental influence on the system transient response. One may design a control which assures that all the closed-loop poles are equal to, say -5. + j0. This type of design results in a well damped response but may result in large excursions of the state variables [A]. Another point to mention is that a closed loop system having only real eigenvalues may show an oscillatory response. In order to clarify this point, consider the state $x_i(t)$ where:

$$x_i(t) = K_1 e^{\lambda 1 t} + K_2 e^{\lambda 2 t} + \ldots + K_n e^{\lambda n t}$$

and assume that the constants K_i have alternating signs. With proper choice of λ's and K's it is evident that the state x_i may show an oscillatory behaviour.

As a final question, do the authors foresee any problems resulting from the application of a controller designed for a low order system to the actual large order system?

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A. B. R. Kuman and **E. F. Richards:** We thank the discussor for his interest in our paper. The intent of the example presented in the paper was to illustrate (a) a new method of eigenvalue assignment which yields both the feedback gains and the closed loop system eigenvectors simultaneously and (b) to use the method in synthesizing an optimal control that is different from the usual quadratic cost function of state and control variables.

We agree that (1) the eigenvalues chosen for the example are excessively damped (2) Our single machine-infinite bus system example is rather trivial, but we feel sufficient to illustrate the method and (3) the method can be extended to a multimachine system in a very straight forward manner.

We share the concern of the discussor about the lack of control over the zeros of the closed loop system which is very important. However, we believe that any control strategy will be tested thoroughly by simulation, before being implemented in a real power system. The method actually produces alternative candidates for such evaluation studies through simulation.

We do not expect any special problems in the application of a controller designed for a low order model to an actual large order system, except those problems resulting from lack of control over the system zeroes.

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