ESTIMATION OF OCEAN WAVE WAVENUMBER AND PROPAGATION DIRECTION FROM LIMITED SYNTHETIC APERTURE RADAR DATA.

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Estimation of Ocean Wave Wavenumber and Propagation Direction from Limited Synthetic Aperture Radar Data

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Abstract—**A method for estimating wavenumber and propagation direction for the dominant wave component in an ocean wave field from a few scans of synthetic aperture radar data is described and analyzed. The use of just a few radar scans rather than a complete image reduces data storage requirements significantly. The analysis shown uses actual synthetic aperture radar data and provides parameter tradeoffs and statistical performance results. While reasonable estimates of wavenumber and propagation direction are achieved in some cases, the estimates are not sufficiently consistent to be satisfactory over a wide range of cases. The primary problem is one of low signal-to-noise ratio of the radar scan data.**

I. Introduction

THE determination of the wavenumber and propagation direction of the dominant wave component in an ocean wave field from remotely sensed data is of interest. It can be used to aid understanding of ocean wave phenomena and for generation of data for wave climate prediction at locations of interest.

An aircraft or satellite-borne synthetic aperture radar (SAR) can be used as the remote sensor to generate images of the ocean surface [1]. The theory and modeling of such SAR images of the ocean surface has been considered by a number of investigators [2]–[5]. If the image signal-to-noise ratio is adequate, then the ocean wave directional spectrum at a location of interest can be obtained by performing a two-dimensional Fourier transform of the portion of the image at the location of interst [6]–[10]. The wavenumber and propagation angle of the dominant wave component visible on the image can be estimated from the directional spectrum. These will be referred to as the indicated wavenumber and propagation angle of the dominant wave component and are very nearly equal to the actual dominant wave component wavenumber and propagation angle for images obtained from satelites. A correction is needed for images obtained from aircraft since the indicated wave propagation angle on the image is different than the actual wave propagation angle. This difference results from wave motion during the generation of the image which is significant since the ratio of aircraft velocity to wave velocity is not large enough. The propagation angle determined gives the propagation direction except for an 180° ambiguity which must be resolved by using other data.

In order to reduce data storage requirements for airborne-synthetic aperture radars used as remote sensors of the dominant wave component parameters, it is desirable to store only as much data at each sensing location as is needed. Thus the feasibility of obtaining estimates of the wavenumber and propagation angle of the dominant wave component in an ocean wave field from a few digitized, focused synthetic aperture radar data scans has been investigated and is reported here.

Brief descriptions of the methods defined to estimate the wavenumber as measured in the radar scan direction (along-scan wavenumber) and the indicated wave propagation angle from the few data scans are presented in later sections. For data scans generated from satellite, the actual propagation angle is essentially equal to the indicated propagation angle and an estimate of the wavenumber of the dominant wave component can be easily computed using the indicated angle estimate and the along-scan wavenumber estimate. For data scans generated from an aircraft, the vehicle velocity and ocean depth (if not in deep water) are also required and the solution for the actual propagation angle must be obtained iteratively [11]. The computed actual propagation angle can then be used with the along-scan wavenumber to compute the wavenumber of the dominant wave component.

Performance of the along-scan wavenumber and indicated propagation angle estimators and parameter tradeoffs for them are also presented in later sections. These performance results were obtained by analyzing the estimators using synthetic aperture radar image data. The estimates of along-scan wavenumber and indicated propagation angle are not consistently reasonable for all cases considered. Therefore, they cannot be expected to lead to consistently reasonable results for estimates of the actual wavenumber and propagation direction. For this reason, performance results were left in terms of the along-scan wavenumber and the indicated propagation angle.

Data and Methods Used in Analysis

The data used in the analysis of the along-scan wavenumber and indicated propagation angle estimators consist

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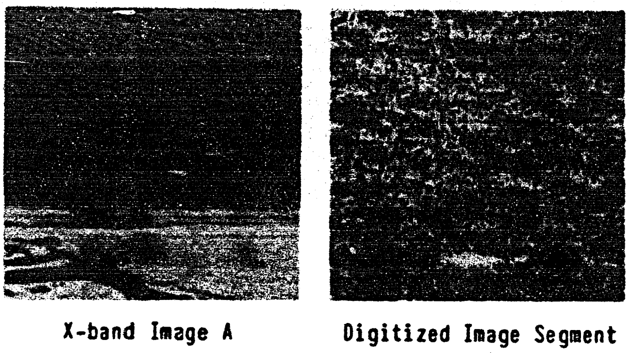

Fig. 1. Example of SAR ocean wave image.

of several digitized synthetic aperture radar images. These images were generously supplied by the Environmental Research Institute of Michigan (ERIM) and were taken off the coast of Marineland, Florida in December 1975.

All of the results presented here were taken from one of the *X*-band images supplied (referred to as Image A). A segment of this image and the digitized version of a portion of the segment are shown in Fig. 1. This image was selected since ground truth information is available for one location on it from a pitch-and-roll bouy (bouy is leftmost bright spot off the coast in the nondigitized image and the bright spot near the bottom center of the digitized image). Also, of all the images supplied, Image A has the most well-defined wave structure. The image has a resolution of 3 m in range and 4.5 m in azimuth. Digital data was supplied for locations on a 1.5 m grid.

The image was rescanned at several different angles to provide scan data with waves at different propagation angles with respect to the scans. This resulted in scan data with different along-scan wavenumbers. Also, in some cases, the image data was smoothed by averaging values for a 5×5 square of data points to obtain the data value for each location.

Analysis of the along-scan wavenumber and indicated propagation angle estimators were made using the Monte Carlo technique to obtain data to compute sample means and sample standard deviations of estimates obtained with the estimators. In most analyses, twenty different combinations of scans from the same region were used to generate estimates and the mean and standard deviations were computed from these twenty estimates. Groups of scans were interleaved so all twenty cases could be as close as possible to one location. This means that the individual cases are not statistically independent. However, the resulting individual case estimates of along-scan wavenumber and indicated propagation angle exhibit enough independence when the complete set of cases at each analysis area is considered to indicate that the sample statistics computed are reasonable indications of performance.

Along-Scan Wavenumber Estimation

The basic concept used in estimating the along-scan wavenumber is to compute smoothed periodograms for each of several scans, average the periodograms, and determine the wavenumber estimate from the location of the peak of the resulting smoothed and averaged periodogram. In computing the periodogram of each scan, smoothing is provided by using a Parzen window with length equal to three-fourths of the sample auto correlation function length. Several different window functions and window function widths were tried [11]. The width selected provides reasonable smoothing while still retaining reasonable resolution. Results were not very sensitive to window shape. Fig. 2 shows an example data scan, its nonsmoothed periodogram, and the average of the smoothed periodogram for this scan and six neighbors.

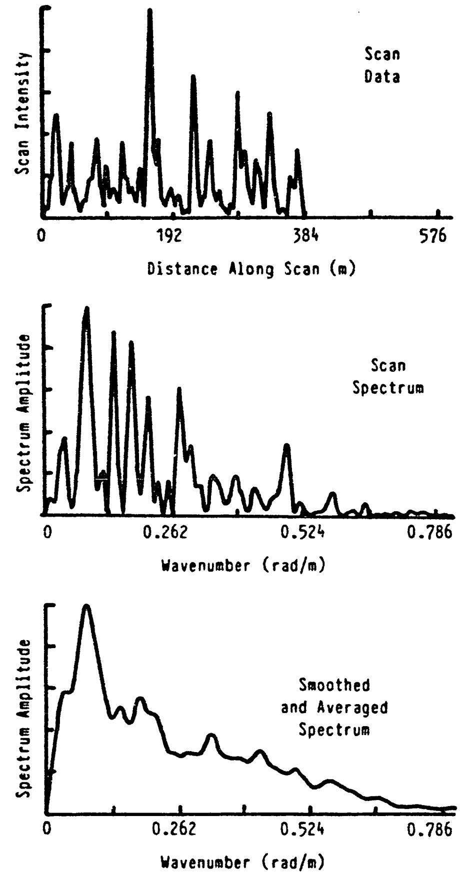

Fig. 2. Example data and spectra.

Initial analysis of the estimates obtained with the along-scan wavenumber estimator indicated that some scans in a set being used gave smoothed periodograms with peak locations far from the majority of the peak locations obtained. This is due to the noisy nature of the data and tended to give spurious results. thus the basic estimation concept was modified to eliminate smoothed periodograms whose peak locations were far from the majority of peak locations before averaging the periodograms. In practice it was found that data from approximately 70 percent of the scans in a set were used.

Another modification incorporated is the computation of estimates for increasing numbers of scans up to the number to be used to determine if the estimates converge.

If they do not, the along-scan wavenumber is said to be non-estimable in that case. The final modification incorporated is to use the average of the estimate obtained with the number of scans selected and the estimates obtained with one and two fewer scans. This modification is used to smooth the estimate further.

In summary, the method used to estimate the along-scan wavenumber consists of the following steps:

1) Compute a smoothed periodogram for each scan considered.
2) Compute wavenumber estimate from the peak location of each smoothed periodogram.
3) Start with the first scan and form subsets of scans by adding one scan at a time.
4) For each subset of scans, compute the mean and standard deviation of the wavenumber estimates obtained with each scan in the subset and then average periodograms for all scans whose peak locations are within plus or minus one standard deviation of the mean.
5) For each subset of scans, compute the wavenumber estimate from the peak location of the averaged periodogram.
6) If the wavenumber estimate converges as more scans are added, compute the final wavenumber estimate as the average of the estimates obtained with the three largest subsets of scans. If convergence does not occur as more scans are used, declare the wavenumber to be non-estimable.

An analysis area just above the bouy location on the images shown in Fig. 1 was designated as image area A-1 and used for initial along-scan wavenumber estimator analyses and parameter tradeoffs. Partial results from which number of scans, scan length, and separation selections were made are shown in Figs. 3–5. Actually families of curves were obtained so the effect of varying more than one parameter could be investigated [11]. However, for clarity, only the curve of each family corresponding to the selected value for other parameters is shown here. The results are presented in terms of the mean μ and standard deviation σ of the estimates obtained with the twenty cases considered for each set of parameters. The ground truth value of the along-scan wavenumber for image area A-1 as determined from the bouy data is also shown in the figures for comparison with the estimate mean values.

Fig. 3 is plotted for a scan length of 480 m and a scan separation of 15 m. From it, the number of scans selected as required is 14 based on the standard deviation of the estimates. Fig. 4 is plotted for 14 scans and a scan separation of 15 m. It is apparent from this figure that the scan length should be at least 480 m or 5.5 wavelengths of the wave being measured. Fig. 5 is plotted for 14 scans of length of 480 m. The range of scan separations considered is up to just slightly more than one-half the wavelength of the wave being measured. The figure shows that the estimation performance is not very sensitive to scan spacing changes in the range considered. A scan spacing of 15 m was selected to keep the scans in a compact area to minimize wave field variability. In the parameter tradeoff analyses, a total of 520 cases for scan lengths greater than or equal to 480 m were considered. Of these, 10 or 1.9 percent were declared non-estimable.

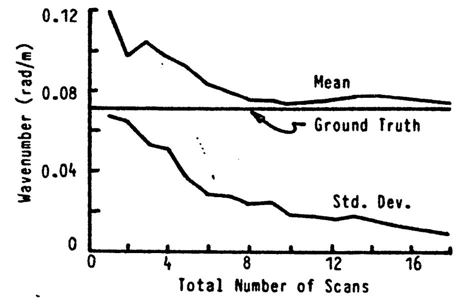

Fig. 3. Number of scans effect on wavenumber estimate.

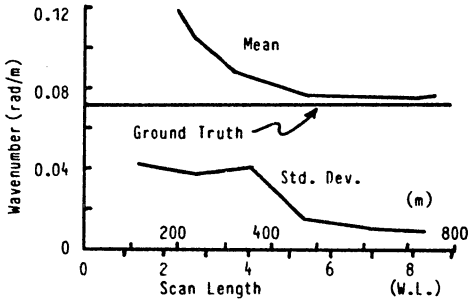

Fig. 4. Scan length effect on wavenumber estimate.

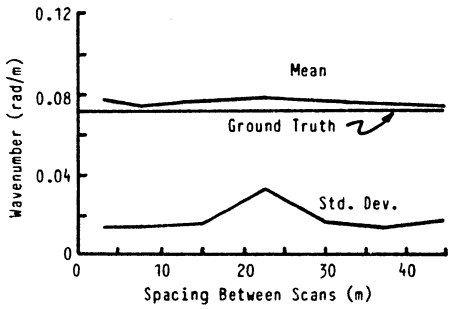

Fig. 5. Scan spacing effect on wavenumber estimate.

The performance of the along-scan wavenumber estimation using the above selected parameters was evaluated by obtaining estimates for several analysis areas. Results from two analysis areas of image A are presented below. The first area (designated A-3) overlaps area A-1 but was shifted slightly to move it farther from the image edge and thus permit rotated scans of reasonable length to be obtained. Fig. 6 shows the mean and standard deviation of the estimates obtained for scans rotated by 0°, 23°, and 35° as a function of scan length. Scan length was varied so the convergence of the estimates could be observed. Similar estimates were also obtained using image data smoothed by 5 × 5 averaging.

Results are summarized for both the nonsmoothed and smoothed data in Table I where the bias and standard

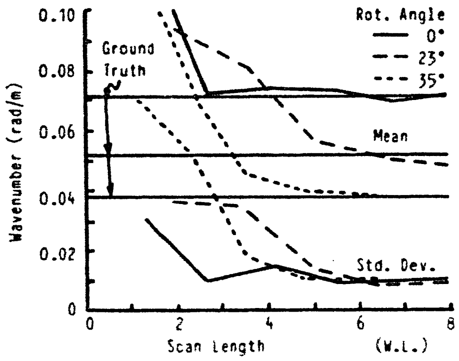

Fig. 6. Wavenumber estimate statistics for various scan angles at best analysis area.

TABLE I
SUMMARY OF ALONG-SCAN WAVENUMBER ESTIMATE PEFORMANCE FOR ANALYSIS AREA A-3

Scan Rotation Angle θ_r	Ground Truth Wavenumber k_y	Non Estimable Cases	Relative Average Bias	Relative Average Std. Dev.
Non-Smoothed Data				
0°	0.071	7.5%	1.8%	15.1%
23°	0.052	17.5%	-2.7%	19.3%
35°	0.038	5.0%	1.8%	26.7%
Smoothed Data (5x5 Ave)				
0°	0.071	3.8%	3.0%	13.9%
23°	0.052	7.5%	-6.0%	16.7%
35°	0.038	0%	-4.2%	24.2%

deviation of the estimate are indicated as percentage of the ground truth value and averaged over all cases with scan lengths greater than that required for convergence. The percent of these cases which were declared non-estimable is also shown. The relative average standard deviation is larger than is desirable. It also increases for smaller along-scan wavenumbers (larger along-scan wavelengths). The percent of non-estimable cases is larger than desirable for nonsmoothed data. The major effect of the smoothed data is to reduce the number of cases which are non-estimable.

Table II shows results obtained for area A-4 on image A. In this case, the estimator used with the smoothed data was further modified to eliminate any peaks at wavenumbers corresponding to wavelengths of 400 m or greater (called LWC). This was done as an attempt to improve performance by eliminating the portion of the spectrum for which it is known that results are not good since the scan length encompasses too few wavelengths to obtain a reliable estimate of wavenumber.

In general, the along-scan wavenumber estimator performance for analysis area A-4 is considerably poorer than for analysis area A-3. In this case, both the relative average bias and the relative average standard deviation increase as

TABLE II
SUMMARY OF ALONG-SCAN WAVENUMBER ESTIMATE PEFORMANCE FOR ANALYSIS AREA A-4

Scan Rotation Angle θ_r	Ground Truth Wavenumber k_y	Non Estimable Cases	Relative Average Bias	Relative Average Std. Dev.
Non-Smoothed Data				
0°	0.071	15.0%	2.7%	18.9%
23°	0.052	15.0%	29.6%	31.4%
35°	0.038	35.0%	24.9%	60.3%
Smoothed Data (5x5 Ave) & LWC				
0°	0.071	1.7%	1.3%	14.4%
23°	0.052	7.5%	28.9%	25.6%
35°	0.038	20.0%	33.0%	47.4%

along-scan wavenumber decreases. Once again, the major effect of using smoothed data is to reduce the number of non-estimable cases.

Along-scan wavenumber estimates were also obtained for areas on other images supplied by ERIM [11]. In general, these images did not have as well a defined wave structure apparent and estimator performance was considerably poorer. Thus while the along-scan wavenumber estimator gave reasonable estimates in a number of cases, data noise precluded obtaining consistently satisfactory estimates over a wide range of cases.

INDICATED PROPAGATION ANGLE ESTIMATION

Several different techniques for estimating the indicated propagation angle of the dominant wave component were postulated and evaluated [11]. Techniques using averaged cross correlations between pairs of scans did not give useful results. Therefore, the estimation technique used consisted of a partial two-dimensional cross-correlation with a reference function. The steps in this technique are:

1) Compute the along-scan wavelength estimate from the along-scan wavenumber estimate.
2) Select the longest segments containing an integer number of estimated wavelengths from the radar scans used for wavenumber estimation and remove the mean value from each scan segment.
3) Construct a two-dimensional square wave function with along-scan wavelength equal to the estimate and propagation angle φ spanning the area encompassed by the scan segments selected as shown in Fig. 7.
4) Compute a two-dimensional cross-correlation value for the radar scan data and the square wave function with propagation angle $\varphi = 0°$ and offset $\Delta B = 0$ by adding all scan data values at locations corresponding to shaded portions of the square wave function.
5) Increase the offset ΔB in steps of one data point and repeat step 4). The values obtained in this step give a cross-correlation function for along-scan shifts be-

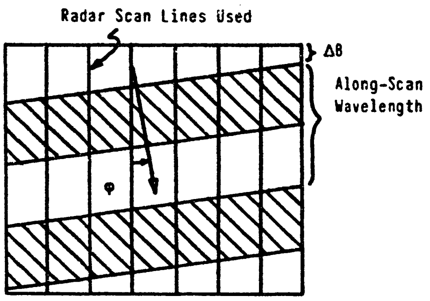

Fig. 7. Two-dimensional square wave for correlation with radar scan data.

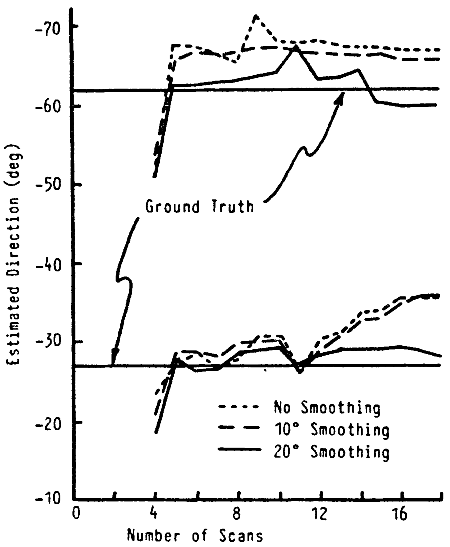

Fig. 8. Wave propagation direction estimate for example case.

tween the radar scans and the square wave function with a propagation angle of 0°. The peak value of this cross-correlation function is computed.

6) Steps 4) and 5) are repeated for other values of square wave propagation angle φ to produce the peak cross correlation as a function of propagation angle. Provision is included to use a moving average over 10° and 20° increments to smooth this function. The propagation angle at which the resulting function has its peak value is the estimate of the indicated propagation angle.

Examples of indicated propagation angle estimates obtained with one set of smoothed data (5 × 5 average) scans taken from image area A-1 at two different rotation angles are shown in Fig. 8 as a function of the number of scans used in the estimate. Plots are shown for estimates using 10° and 20° moving window averaging. It can be seen that moving window averaging can improve performance and that the number of scans selected for the along-scan wavenumber estimator (i.e., 14) is adequate for the indicated propagation angle estimator.

The same set of analysis cases used in evaluating the along-scan wavenumber estimator with data from image area A-3 were used to evaluate the indicated propagation angle estimator and perform parameter tradeoffs. It was

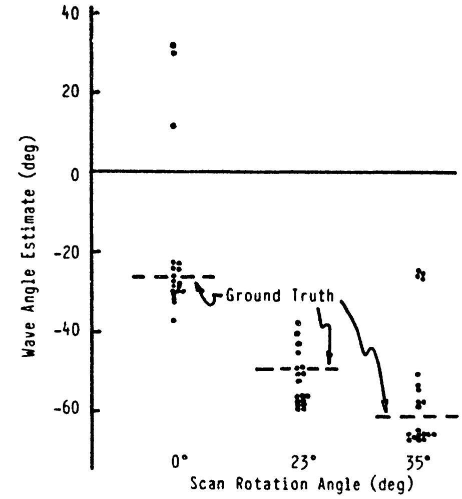

Fig. 9. Individual case indicated propagation angle estimate for three different wave angles for anlaysis area A-3.

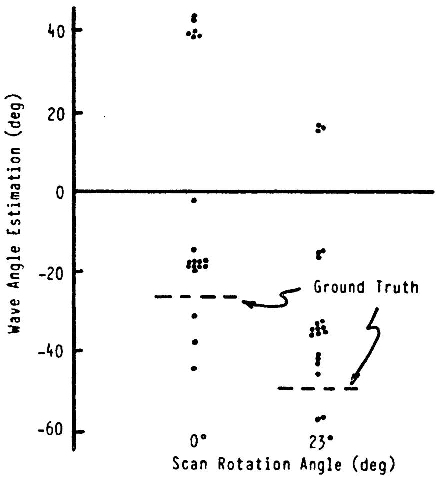

Fig. 10. Individual case indicated propagation angle estimates for two different wave angles for analysis area A-4.

determined that the scan lengths should be somewhat longer (8 wavelengths was selected), the 15 m scan spacing previously selected is adequate, a 10° moving window average provides sufficient smoothing, and smoothed image data (5 × 5 average) is needed for best performance. The use of 14 scans was also confirmed as a reasonable choice.

Fig. 9 shows the estimates obtained for the 20 cases at each scan rotation angle used for image area A-3. Clustering of the estimates in the neighborhood of the indicated propagation angle ground truth value is apparent for all three angles with only a few estimates far from the true value. Estimates obtained for two scan rotation angles for image area A-4 are shown in Fig. 10. In this case there are clusters but they are not clustered in the immediate vicinity of the ground truth values. Recall that this image area also gave poorer along-scan wavenumber estimates.

Table III summarizes the performance of the indicated propagation angle estimator for the two image areas considered. Performance is stated in terms of the percen-

TABLE III
PERCENTAGE OF PROPAGATION ANGLE ESTIMATES WITHIN ±10° AND ±5° INTERVALS OF GROUND TRUTH

Anal. Area	A-3		A-4	
Angle Limits	±10°	±5°	±10°	±5°
Scan Rotation Angle				
0°	75%	70%	50%	5%
23°	80%	30%	30%	5%
35°	75%	60%		

tage of estimates which are within ±10° and ±5° of the ground truth value. For image area A-3, 77 percent of the estimates are within ±10° and 53 percent are within ±5° for all angles considered. These numbers drop to 40 percent within ±10° and 5 percent within ±5° when the data from image area A-4 is used.

Indicated propagation angle estimates were not obtained for any additional image areas since the along-scan wavenumber estimates for other areas considered were not very good and thus the indicated propagation angle estimator could not be expected to perform very well. As for the along-scan wavenumber estimator, it is apparent that better signal-to-noise ratio is needed to provide consistently reasonable estimates.

SUMMARY AND CONCLUSIONS

Techniques for estimating the wavenumber and propagation direction of the dominant wave component in an ocean wave field from a few scans of synthetic aperture radar data were developed and evaluated. The feasibility of the techniques and the effect of various parameters was evaluated by using several locations on digitized synthetic aperture radar images of the ocean surface.

The best performance of the along-scan wavenumber estimator was obtained for an image location which had a visible, fairly well-defined wave structure. In this case, the number of non-estimable cases ranged from 0 to 17.5 percent, the estimate bias ranged from 1.8 to 6.0 percent and the estimate standard deviation ranged from 13.9 to 26.7 percent as the dominant wave component propagation direction was varied over a 35° range.

It was found that most estimates of the dominant wave component propagation direction clustered in the neighborhood of the correct propagation direction when radar scan data from the image location which gave the best wavenumber estimates was used. Approximtely 77 percent of the estimates were within ±10° of the correct propagation direction. Only a few are far from it.

The prime factor which limits the performance of the along-scan wavenumber and propagation direction estimators in the best case indicated above is the relatively large amount of noise on the digitized radar scans used. Other image locations considered contained even more noise with the result that poorer and inconsistent performance was achieved. It is encouraging that wavenumber and propagation direction estimates which were reasonable were achieved in a number of cases even though the noise on the digitized radar scan was relatively large. However. methods for reducing this noise are necessary before consistent estimates can be achieved in all cases.

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