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Eddie R. Fowler

Missouri University of Science and Technology

Rao Yarlagadda

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A State-Space Approach to *RLCT* Two-Port Transfer-Function Synthesis

EDDIE R. FOWLER, MEMBER, IEEE, AND RAO YARLAGADDA, MEMBER, IEEE

Abstract—This paper presents an interesting procedure for the synthesis of an *RLCT* two-part transfer function. An *RLC* ladder, consisting of *n* reactive elements and two resistors, is derived by using a tridiagonal matrix developed by Navot. The entries in this matrix are expressed in terms of the element values of the ladder network. Two voltage drivers are introduced into the ladder network to obtain a desired short-circuit transfer-admittance function numerator degree, using the classical theorems on transmission zeros. If the numerator degree of the transfer function is *i* (*i* < *n*), then, in general, (*i*) ladder networks need to be derived. The final network, corresponding to this transfer function, is obtained by paralleling the ladder networks (with transformers if necessary). Extensions to general short-circuit transfer admittance, open-circuit transfer impedance, and voltage transfer functions are briefly discussed.

I. Introduction

The synthesis of two-port transfer functions with complex frequency-domain procedures is presently done with a number of synthesis schemes based upon the characteristics of the given transfer function. Examples of these various synthesis procedures are the Dasher [1], Guillemin [2], and Lucal [3] methods of realizing transfer functions with two-port *RC* networks and Ho's [4], [5] *RLC* realization procedure. Karni [6], Balabanian [7], and Weinberg [8] have written texts which present various transfer-function synthesis procedures in the frequency domain.

Most of the transfer-function synthesis procedures use classical synthesis techniques and usually vary for each of the different types of transfer functions; this is especially true when a resistance termination is required. This paper presents a unified state-space synthesis procedure by which a two-port s-domain transfer function yields an *RLC* network with or without transformers and equations for determining the network component values. The synthesis procedure is unified in the sense that the procedure is the same regardless of type or characteristic of the transfer function.

The main part of this paper is concerned with the synthesis of a short-circuit transfer-admittance function $Y_{12}(s)$ defined by

$$Y_{12}(s) = \left.\frac{i_{a_1}(s)}{V_{a_2}(s)}\right|_{V_{a_1}(s)=0} = \frac{P(s)}{Q(s)} \tag{1}$$

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E. R. Fowler is with the Department of Electrical Engineering, University of Missouri, Rolla, Mo. 65401.

R. Yarlagadda is with the School of Electrical Engineering, Oklahoma State University, Stillwater, Okla. 74074.

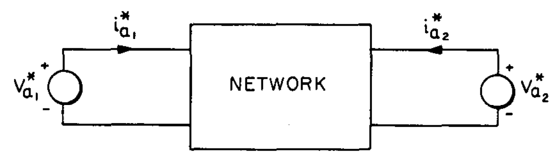

Fig. 1. Network driver configuration for $Y_{12}(s)$.

where the voltage and current variables are defined as shown in Fig. 1; $P(s)$ and $Q(s)$ are polynomials in s with real coefficients and their degrees are n_1 and n, respectively; the degree n_1 is assumed to be, at most, equal to $(n+1)$ [7]. Further, it is assumed that $Q(s)$ is a Hurwitz polynomial. The synthesis procedure is based upon finding an *RLC* network which has the characteristic polynomial $Q(s)$. The synthesis is first considered for a strictly Hurwitz polynomial and later extended for $Q(s)$. The synthesis is achieved by first finding a tridiagonal matrix by Navot's method [9]. In [10] and [11] synthesis procedures using tridiagonal matrices have been considered. The tridiagonal matrix is modified to relate to Bashkow's A matrix [12]. A state model which is realizable by n reactive elements and two resistors is derived. The synthesis of this model is rather simple [10], [13]. The element values are given in terms of simple equations, which are very easy to solve. Two voltage drivers are introduced to obtain a desired numerator degree in the transfer function using the classical theorems on transmission zeros [14]. This gives rise to an interesting synthesis technique for classical low-pass filters, such as Butterworth, Chebyshev, etc., especially when one of the terminating resistances is arbitrary. In classical synthesis techniques, reflection and transmission coefficients are used in obtaining this result [6]–[8].

Synthesis of short-circuit transfer-admittance functions, the main part of this paper, is discussed in Section IV. The method does not require the knowledge of one of the short-circuit driving-point functions, like some of the transfer-function synthesis techniques [7]. A detailed example is presented to illustrate the procedure. Extensions to other types of transfer functions are briefly discussed, illustrating the versatility of the synthesis method.

II. Navot's Method of Obtaining a Tridiagonal Matrix and its Modifications

Given a strictly Hurwitz polynomial, called the primary polynomial,

$$H_n(s) = s^n + h_{n-1}s^{n-1} + \cdots + h_0 \tag{2}$$

which is related to a secondary polynomial

$$G_n(s) = s^n + g_{n-1}s^{n-1} + \cdots + g_0$$

by

$$G_n(s)G_n(-s) = H_n(s)H_n(-s) - C \tag{3}$$

where

$$0<C\le m \quad \text{and} \quad m=\min\{\,|H_n(j\omega)|^2;\, 0\le\omega<\infty\}. \tag{4}$$

A tridiagonal matrix

$$K_1=\begin{bmatrix} -f_0 & -f_1 & & & & & \\ 1 & 0 & -f_2 & & & & \\ & \cdot & \cdot & \cdot & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & 1 & 0 & -f_{n-1} \\ & & & & & 1 & -f_n \end{bmatrix} \tag{5}$$

is generated such that $|sI-K_1|=H_n(s)$. The entries in the K_1 matrix are obtained from the continued fraction expansion

$$\frac{H_n(s)-G_n(s)}{(h_{n-1}-g_{n-1})[H_n(s)]}=\cfrac{1}{f_0+s+\cfrac{f_1}{s+\cfrac{f_2}{s+\ddots+\cfrac{f_{n-1}}{s+f_n}}}} \tag{6}$$

where the f_i are positive. Since C is not unique, it follows that the f_i are not unique.

III. Synthesis of a Strictly Hurwitz Polynomial by a Passive Network

Consider a strictly Hurwitz polynomial $D(s)=H_n(s)$. A matrix K_1 in (5) can be generated such that $|sI-K_1|=D(s)$. Further, K_1 can be transformed into a matrix

$$K_2 = \Lambda K_1 \Lambda^{-1} \tag{7}$$

where Λ is a positive diagonal matrix. This implies that a state model can be generated:

$$\frac{d}{dt}X = K_2 X. \tag{8}$$

Obviously, the characteristic polynomial of this state model is $D(s)$. The matrix K_2 in (8) can be realized as an A matrix of a ladder network terminated with resistors [12], [13]. Since a ladder network is a planar network, the dual of the realized network has also the same A matrix. Furthermore, any other realization of K_2 is two isomorphic with one of these dual networks.

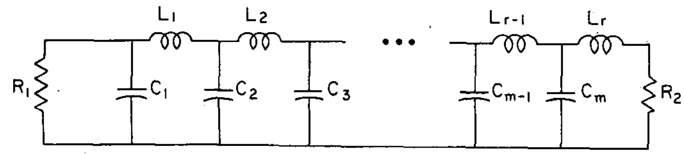

Fig. 2. Synthesized network for $D(s)$ when n is even.

Since the procedure is straightforward, only the results are presented. Further, these results are presented in terms of one ladder network terminated by resistors. Dual networks follow. When n is even, the network will have the form shown in Fig. 2. The element values of this network can be related to the entries in the K_1 matrix in (5). These equations are

$$\begin{aligned} f_0 &= (R_1C_1)^{-1} & f_n &= R_2L_r^{-1} \\ f_1 &= (C_1L_1)^{-1} & f_2 &= (C_2L_1)^{-1} \\ f_3 &= (C_2L_2)^{-1} & &\cdots \\ f_{n-2} &= (C_mL_{r-1})^{-1} & f_{n-1} &= (C_mL_r)^{-1}. \end{aligned} \tag{9}$$

It can be seen that there are $(n+1)$ equations with $(n+2)$ unknowns. By selecting one variable arbitrarily, one can solve these equations. It can be shown that the system is consistent and the element values are positive.

When n is odd, the network will have the same general form as in Fig. 2, except that L_r will be a short circuit. The element values of this network are related to the K_1 matrix by

$$\begin{aligned} f_0 &= (R_1C_1)^{-1} & f_n &= (R_2C_m)^{-1} \\ f_1 &= (C_1L_1)^{-1} & f_2 &= (C_2L_1)^{-1} \\ f_3 &= (C_2L_2)^{-1} & &\cdots \\ f_{n-2} &= (C_{m-1}L_{r-1})^{-1} & f_{n-1} &= (C_mL_{r-1})^{-1}. \end{aligned} \tag{10}$$

Again, it can be seen from the above set of equations that there are $(n+1)$ equations $(m+r-1=n)$ involving $(n+2)$ unknowns. By selecting one variable arbitrarily, say $C_1\neq 0$, the remaining variables can be found. It can easily be shown that the system is consistent and the element values are positive.

IV. Short-Circuit Transfer-Admittance Function—Classifications and Synthesis

Consider the transfer admittance $Y_{12}(s)$ defined in (1). In this section $Y_{12}(s)$ is restricted to a proper function. Further, it is assumed that $Q(s)=D(s)$ and is strictly Hurwitz. There are four classifications which includes all possible proper transfer functions. These are the following.

Case 1	n_1 even	n even
Case 1, special	$P(s)$ constant	n even
Case 2	n_1 even	n odd
Case 2, special	$P(s)$ constant	n odd
Case 3	n_1 odd	n even
Case 4	n_1 odd	n odd

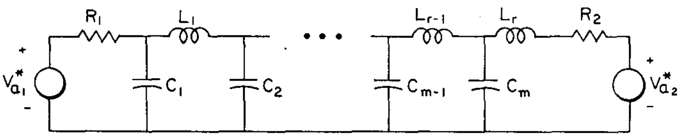

Fig. 3. Network corresponding to Case 1, special: $m+r=n$.

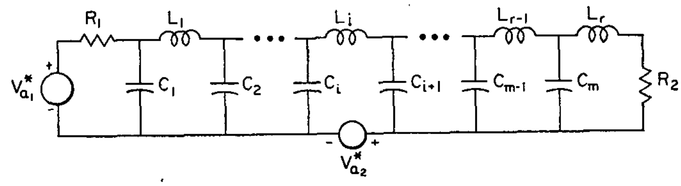

Fig. 4. Network corresponding to Case 1: $i=(n-x_1)/2$, $n=m+r$, $m=r$.

As can be seen from the above classifications, special cases of 1 and 2 are particularly mentioned. These are important because they correspond to the classical low-pass filters—Butterworth, Chebyshev, etc. These will be considered first as they are simple, important, and form the basis for the other cases.

Note: As is well known for all cases, all of the numerator coefficients will be negative for the orientation of the drivers, as shown in Fig. 1. By reversing the orientation of one driver, one can obtain all positive numerator coefficients.

Case 1, Special

The transfer-function admittance of this case is

$$Y_{12}(s) = \frac{a_0}{s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}, \qquad a_0 < 0, \quad n \text{ even}. \tag{11}$$

The ladder network corresponding to this case is shown in Fig. 3. The degree of the numerator in $Y_{12}(s)$ for this network follows, as the transmission zeros of a ladder network are contained among the zeros of the admittance functions of the series arms and the impedance functions of the shunt arms [14]. Now to solve for the element values, one needs to consider (9) and another constraint, which is given by

$$\frac{-1}{R_1 + R_2} = \frac{a_0}{b_0}. \tag{12}$$

The constraint in (12) can easily be seen by considering $s=0$. Element value C_1 can be solved from (9) and (12) and is

$$C_1 = \frac{-a_0[(f_{n-1}f_{n-3}\cdots f_3 f_1) + f_0 f_n(f_{n-2}f_{n-4}\cdots f_4 f_2)]}{b_0 f_0(f_{n-1}f_{n-3}\cdots f_3 f_1)}. \tag{13}$$

The remaining variables can be solved from (9). Note that a_0 is a negative number.

Case 2, Special

The transfer admittance of this case is the same as shown in (11) except that n is odd. The ladder network corresponding to this case has the same form as in Fig. 3, except that $L_r=0$. The degree of the numerator in $Y_{12}(s)$ for this network can easily be justified by again considering the classical theorems on transmission zeros of a ladder network. In addition to the equations given in (10), the constraint of (12) also applies. The element value of C_1 can be solved from (10) and (12) and is

$$C_1 = \frac{-a_0[f_n(f_{n-2}f_{n-4}f_{n-6}\cdots f_5 f_3 f_1) + f_0(f_{n-1}f_{n-3}f_{n-5}\cdots f_6 f_4 f_2)]}{b_0 f_0 f_n(f_{n-2}f_{n-4}f_{n-6}\cdots f_5 f_3 f_1)}. \tag{14}$$

The remaining variables can be solved from (10) and can be easily shown to be real and positive. Note that a_0 is a negative number.

Perhaps a comment is worthwhile on the synthesis of Butterworth, Chebyshev filters, etc. The above procedure gives the element values, and by scaling, one of the resistor values can be selected as the given source or load resistance, but both of these cannot be satisfied. When both source and load resistances R_1 and R_2 are specified and are not identically zero, C takes a value $C=4a_0^2R_1R_2$, where a_0 is given in (11) [17]. With $C=m$ there will be only one resistor. Note that C must be bounded by the limits in (4).

Case 1

The transfer admittance of this classification is

$$Y_{12}(s) = \frac{a_{x_1}s^{x_1} + a_{x-1}s^{x_1-1} + \cdots + a_1 s + a_0}{s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0} \tag{15}$$

where $a_i<0$, $x_1=\text{even}\neq 0$, and n is even. The ladder network corresponding to this case is shown in Fig. 4, with $i=(n-x_1)/2$ representing the mesh consisting of the elements C_i, L_i, C_{i+1}, and in which the driver $V_{a_2}{}^*$ must be located to yield the desired numerator degree of $Y_{12}(s)$ in (15). There are only $(n+2)$ elements in the network as in the previous case. Therefore only one of the coefficients in the numerator can be satisfied. The selected coefficient here is a_{x_1}. The remaining coefficients in the numerator of $Y_{12}(s)$ are not arbitrary but will be fixed by the network. To find the element values for this network, use the constraint in (12) with the numerator coefficient a_0 replaced by some arbitrarily chosen nonzero value a_0'. Using the equations given in (9), solve for the element values. Find the transfer-admittance function corresponding to Fig. 4. Scale the network (magnitude scaling) such that the coefficient of s^{x_1} is equal to a_{x_1}. The remaining a_i are fixed by this procedure.

Case 2

The transfer admittance of this case is the same as shown in (15) except that $x_1=\text{even} \neq 0$ and n is odd. The ladder network corresponding to this case has the same form as Fig. 5, except that $L_r=0$ with $i=(n-x_1+1)/2$. Again as in Case 1, this procedure satisfies only one numerator coefficient and the numerator coefficient selected is a_{x_1}. The procedure to

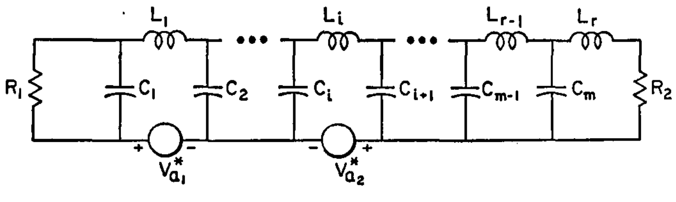

Fig. 5. Network corresponding to Case 3: $i=(n-x_1+1)/2$, $n=m+r$, $m=r$.

determine the network element values is identical to that of Case 1.

Case 3

The transfer admittance of this case is the same as shown in (15) except that x_1 is odd and n is even. The ladder network corresponding to this case is shown in Fig. 5 with $i=(n-x_1+1)/2$. Again as in Case 1, this procedure satisfies only one numerator coefficient in (15) and the coefficient selected is a_{x_1}. The procedure to determine the network element values is identical to that of Case 1.

Case 4

The transfer admittance for this case is the same as shown in (15) except that $x_1=\text{odd}\neq 0$ and n is odd. The ladder network corresponding to this case has the same form as in Fig. 4, except $L_r=0$, with $i=(n-x_1)/2$. Again as in Case 1, this procedure satisfies only one numerator coefficient in (15) and the coefficient selected is a_{x_1}. The procedure to determine the network element values is identical to that of Case 1.

V. Synthesis of $Y_{12}(s)$ with Numerator Degree Greater than Zero

In the last section, synthesis of all possible classes of proper functions are considered, where in each case the ladder network satisfies only one coefficient in the numerator polynomial of $Y_{12}(s)$. Therefore if the numerator degree is (i), then in general there will be (i) ladder networks paralleled (with transformers if necessary to satisfy Brune's conditions [16]) in the resulting network that synthesizes the transfer admittance. This procedure is illustrated in the following example.

VI. Example

Consider the Case-3 short-circuit transfer admittance of

$$Y_{12}(s) = \frac{-2s^3-5s^2-3s-9}{s^4+2s^3+10s^2+10s+17}. \tag{16}$$

The synthesis of this transfer admittance will be done by synthesizing the four transfer admittances given below, which correspond to the four coefficients in the $Y_{12}(s)$ numerator:

$$Y_{12}^{(1)}(s) = [a_3^{(1)}s^3+a_2^{(1)}s^2+a_1^{(1)}s+a_0^{(1)}]/D(s) \tag{17}$$

$$Y_{12}^{(2)}(s) = [a_2^{(2)}s^2+a_1^{(2)}s+a_0^{(2)}]/D(s) \tag{18}$$

$$Y_{12}^{(3)}(s) = [a_1^{(3)}s+a_0^{(3)}]/D(s) \tag{19}$$

$$Y_{12}^{(4)}(s) = [a_0^{(4)}]/D(s) \tag{20}$$

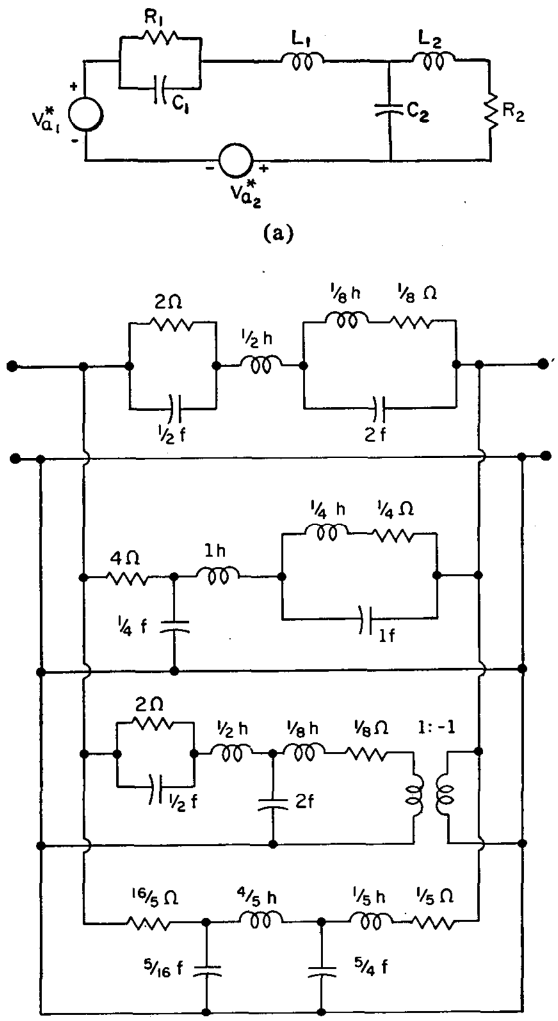

Fig. 6. (a) Network for $Y_{12}^{(1)}(s)$ in Example. (b) Network for $Y_{12}(s)$ in Example.

where $D(s)=s^4+2s^3+10s^2+10s+17$ and $Y_{12}(s)=Y_{12}^{(1)}(s)+Y_{12}^{(2)}(s)+Y_{12}^{(3)}(s)+Y_{12}^{(4)}(s)$. Therefore the constraints on $a_j^{(i)}$ are $a_3^{(1)}=-2$; $a_2^{(1)}+a_2^{(2)}=-5$; $a_1^{(1)}+a_1^{(2)}+a_1^{(3)}=-3$; $a_0^{(1)}+a_0^{(2)}+a_0^{(3)}+a_0^{(4)}=-9$. Each of the transfer admittances $Y_{12}^{(i)}$ will be realized by the techniques discussed in Section IV. Then the ladder networks are connected in parallel (with transformers if necessary).

Synthesis of $Y_{12}^{(1)}(s)$

This is a Case-3 transfer-admittance function. The ladder network corresponding to this function is shown in Fig. 6(a). The following steps are used in the synthesis of $Y_{12}^{(1)}(s)$: 1) the element values of Fig. 6(a) are calculated for some arbitrary $a_0^{(1)}$; 2) the transfer function is derived for the above element values; and 3) the network is scaled such that $a_3^{(1)}=-2$.

Let $a_0'=-1$, which implies that $1/(R_1+R_2)=1/17$. Also a K_1 matrix in (5) of order 4 derived by Navot's method yields $f_0=1$, $f_n=1$, $f_1=4$, $f_2=1$, and $f_3=4$. It can be easily seen that it has $D(s)$ as the characteristic polynomial. Cal-

culate C_1' by (13). (The prime is added since the $a_3^{(1)}$ derived will not be the correct value.)

$$C_1' = \frac{-(-1)[(4)(4)+(1)]}{17(4)(4)} = \frac{1}{16}. \tag{21}$$

The remaining network component variables are calculated from (10) and are

$$L_1' = 4 \quad C_2' = 1/4 \quad L_2' = 1 \quad R_1' = 16 \quad R_2' = 1. \tag{22}$$

The above network component values applied to the network of Fig. 6(a) yield a transfer admittance of

$$Y_{121}^{(1)}(s) = \frac{-\frac{1s^3}{4} - \frac{1s^2}{2} - 10s - 1}{s^4 + 2s^3 + 10s^2 + 10s + 17}. \tag{23}$$

Now the above network is scaled such that $a_3^{(1)} = -2$. Since $Y_{121}^{(1)}(s)$ is an admittance function, multiply the capacitor values by 8 and divide the inductor and resistor values by 8 to scale the network as desired. The element values are now $C_1 = \frac{1}{2}$, $C_2 = 2$, $L_1 = \frac{1}{2}$, $L_2 = \frac{1}{8}$, $R_1 = 2$, and $R_2 = \frac{1}{8}$. The new transfer admittance function is

$$Y_{12}^{(1)}(s) = \frac{-2s^3 - 4s^2 - 10s - 8}{s^4 + 2s^2 + 10s^2 + 10s + 17}. \tag{24}$$

Upon comparing (24) with (17), the $a_i^{(1)}$ coefficients can be obtained and are $a_3^{(1)} = -2$, $a_2^{(1)} = -4$, $a_1^{(1)} = -10$, and $a_0^{(1)} = -8$.

In a similar manner $Y_{12}^{(2)}(s)$, $Y_{12}^{(3)}(s)$, and $Y_{12}^{(4)}(s)$ can be found and synthesized. The transfer functions $Y_{12}^{(2)}(s)$, $Y_{12}^{(3)}(s)$, and $Y_{12}^{(4)}(s)$ are given below:

$$\begin{aligned} Y_{12}^{(2)}(s) &= (-s^2 - s - 4)/D(s) \\ Y_{12}^{(3)}(s) &= -(8s + 8)/D(s) \\ Y_{12}^{(4)}(s) &= -5/D(s). \end{aligned}$$

The resultant network which has the $Y_{12}(s)$ of (16) is shown in Fig. 6(b). A transformer is added to the network corresponding to the transfer function $Y_{12}^{(3)}(s)$ to satisfy the Brune's Condition. Note that by adding a transformer with the proper turns ratio to each network, the same set of network component values can be used for each $Y_{12}^{(i)}(s)$.

VII. General *RLC* Transfer-Admittance-Function Synthesis

Previously, the synthesis procedure had been limited to proper transfer-admittance functions that have strictly Hurwitz characteristic polynomials. Here the characteristic polynomials which have simple roots on the imaginary axis in addition to the roots on the left half-plane are considered. Further, the transfer function may in general have a pole at infinity. $Y_{12}(s)$ can always be written as

$$\begin{aligned} Y_{12}(s) &= \frac{P(s)}{Q(s)} = (a_1 s + a_2) + \frac{P_1(s)}{D_1(s)} + \frac{P_2(s)}{D_2(s)} \\ &= Y_{121}(s) + Y_{122}(s) + Y_{123}(s) \end{aligned} \tag{25}$$

where $Y_{122}(s)$ and $Y_{123}(s)$ are proper functions, $D_1(s)$ is a strictly Hurwitz polynomial, and $D_2(s)$ has roots only on the imaginary axis. Synthesis of $Y_{122}(s)$ is presented in Section IV. To synthesize $Y_{123}(s)$, the polynomial $D_2(s)$ is realized using the principles discussed in [13]. Drivers are inserted in the network according to the principles discussed in Section IV. Classical synthesis procedures can be used for the synthesis of $Y_{121}(s)$. Now to complete the synthesis of $Y_{12}(s)$, parallel the networks obtained for $Y_{121}(s)$, $Y_{122}(s)$, and $Y_{123}(s)$. Use transformers if necessary to satisfy Brune's conditions. The only restrictions on the transfer functions to be synthesized are that the characteristic polynomial $Q(s)$ in (25) be Hurwitz, that the transfer function numerator degree be no more than 1 greater than the denominator degree, and that the numerator coefficients be real.

VIII. Extensions to Open-Circuit Transfer-Impedance and Voltage Transfer Function $T(s)$

Synthesis of $Z_{12}(s)$ and $T(s)$ is very similar to that of $Y_{12}(s)$. The duality property can be used in obtaining the synthesis procedures for $Z_{12}(s)$, as it is dual to $Y_{12}(s)$. The synthesis of $T(s)$ is a little different from that for $Y_{12}(s)$ and $Z_{12}(s)$ functions, as the drivers will include both types, i.e., current and voltage drivers. But the procedure is very similar; i.e., first synthesize the characteristic polynomial and then insert the drivers to achieve the proper degree in the numerator. The synthesis of $Z_{12}(s)$ and $T(s)$ will not be discussed here any further; see [15] for details.

IX. Conclusions

In this paper a systematic procedure is developed to synthesize a short-circuit transfer-admittance function. The only restrictions on the transfer functions to be synthesized are that the characteristic polynomial be Hurwitz, that the transfer-function numerator degree be no more than 1 greater than the denominator degree, and that the numerator coefficients be real. Extensions to other types of transfer functions, namely open-circuit transfer impedance and voltage transfer functions are discussed briefly.

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Synthesis of $(n+2)$-Node Resistive n-Port Networks

P. SUBBARAMI REDDY AND K. THULASIRAMAN

Abstract—Certain properties of the network of departure N_d and the padding network N_p of an $(n+2)$-node resistive n-port network containing no negative conductances are established. Based on these properties, some necessary conditions and a sufficient condition for the realizability of the Y matrices of $(n+2)$-node resistive n-port networks are obtained. A new proof for the supremacy condition is given. Also, a necessary and sufficient condition for the realization of $(n+2)$-node n-port networks having specified Y and K matrices is given.

I. Introduction

THE PROBLEM of realization of a real symmetric matrix Y, as the short-circuit conductance matrix of a resistive n-port network having more than $(n+1)$ nodes, has been a subject of research for more than a decade. Guillemin was the earliest to suggest a method of solution when the port configuration T of the required n-port network is specified [1]. His approach is based on the determination of 1) the unique network of departure N_d [5] with respect to the given matrix Y having the specified port configuration; and 2) a suitable padding n-port network N_p so that the parallel combination of N_d and N_p contains no negative conductances [5]. Later approaches [2]–[5] differ from Guillemin's only in the procedure used to generate padding n-port networks. The procedures given in [1], [2], and [5] to generate padding networks are general and applicable to the generation of all padding n-port networks having more than $(n+1)$ nodes. The procedure given in [3] is applicable to $(n+2)$-node networks only and the one given in [4] can generate only a class of n-port networks whose potential factors are related in a special way. Frisch and Swaminathan [2] have also obtained a significant result, viz., the formulation of the supremacy condition, which is necessary for the realizability of the Y matrices of $(n+2)$-node n-port networks.

In this paper we consider the synthesis of $(n+2)$-node resistive n-port networks containing no negative conductances. Unless stated otherwise, we follow the notation used in [5]. In this section we restate certain results discussed in [5].

Consider an $(n+2)$-node resistive n-port network N containing no negative conductances. Permitting edges with zero conductances, the linear graph of N may be assumed to be complete. Let the two connected parts T_1 and T_2 of the port configuration T of N be linear trees. Let the vertices of any linear tree T_0 of N, of which T_1 and T_2 are subgraphs, be numbered consecutively starting from one end vertex of T_0. Let the first $(m+1)$ vertices be in T_1 and the remaining vertices in T_2. Let the set of vertices of $T_1(T_2)$ be denoted by $V_1(V_2)$. Let N_d and N_p denote the network of departure and the padding network of N, respectively. For all $i \in V_1(V_2)$ and $j \in V_2(V_1)$, let $S_i = \sum_j (g_{ij})_p = \sum_j g_{ij}$

$$S_{i0} = \sum_j \left| (g_{ij})_d \right| \qquad \text{for all } j\text{'s for which } (g_{ij})_d \leq 0 \quad (1)$$

and

$$S = \sum_{i \in V_1} S_i = \sum_{i \in V_2} S_i$$

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The authors are with the Department of Electrical Engineering, Indian Institute of Technology, Madras, India.
K. Thulasiraman is now on leave with the Department of Electrical Engineering, Sir George Williams University, Montreal, Que., Canada.