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Electric Field Investigations and a Model for Electrical Liquid Spraying[1]

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The equation generally used for determining the electric field at the liquid tip in electrical spraying of liquids was originally derived for determining the electric field around a metal tip in the extraction of electrons from metals. In the present paper experimental data were taken at the instant of spraying to determine the validity of using this equation for liquid tips. The results indicate that the value of electric field given by this equation needs to be increased by about 20% in the normal range of operation. A model for the spraying mechanism of the liquid is also presented.

INTRODUCTION

Electrical spraying of liquids results from liquids emerging from a small capillary under a high potential. This paper deals with a dc potential only; however, Vonnegut and Neubauer (1) state that a 60-cycle ac potential gives results similar to that produced by direct current. The relationship for the electric field between a plate and the capillary comes from work done by Eyring, Mackeown, and Millikan (2). In their work a dc potential was established between a metal point shaped in the form of a hyperboloid of revolution and a plate in order to extract electrons from metals under the influence of electric fields.

Their equation for the electric field was later simplified by Loeb *et al.* (3) to the form:

$$E = \frac{2V}{N \ln (4d/N)}, \qquad [1]$$

where d is the spacing between the tip and ground plate in meters, N is the radius of curvature of the tip in meters, and V is the applied potential in volts. The stress produced by this electric field at the tip is:

$$\tfrac{1}{2}\epsilon_0 E^2 = \frac{2\epsilon_0 V^2}{N^2[\ln (4d/N)]^2}, \qquad [2]$$

where ϵ_0 is the permittivity of free space.

Consider an electric field acting on a spherical liquid drop in free space. If the dielectric stress is neglected, i.e., the liquid is a perfect conductor, then the condition for instability of the liquid surface as given by Zeleny (4) is:

$$P_E = P_\gamma - P, \qquad [3]$$

where P_E is the electrostatic pressure, P_γ is the pressure due to surface tension, and P is the net pressure. When the liquid emerges from the tip of a capillary, the hydrostatic pressure P_h forcing the liquid out must be added to the electrostatic pressure or:

$$P_E + P_h = P_\gamma - P. \qquad [4]$$

This paper deals with spraying of glycerine, a viscous liquid, through a small capillary when subjected to limited liquid heads. The hydrostatic pressure for this case was found to be negligible in comparison to the electrostatic and surface tension pressures. Although the following equations will have the term P_h omitted, the hydrostatic pressure does influence a change in the radius of curvature with respect to time. Equation

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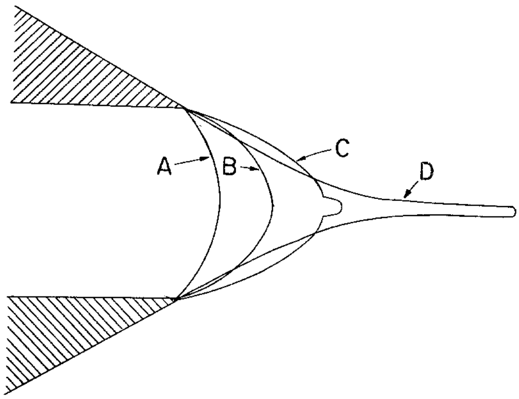

FIG. 1. Model of spraying mechanism: A and B—no spraying; C—spraying begins; D—spraying continues.

[4] becomes:

$$P = \frac{2\gamma}{N} - \tfrac{1}{2}\epsilon_0 E^2, \quad [5]$$

where N is the radius of curvature of the liquid tip and γ is surface tension in dynes/meter.

LIQUID MODEL

The model used for the liquid tip in calculating the radius of curvature is shown in Fig. 1. From time $t = 0$ at surface A to some time later at surface B, the pressure due to the liquid head has forced more liquid out of the capillary. The radius of curvature continues to decrease as time increases until surface C is reached where the electrostatic pressure is equal to the surface tension pressure and spraying begins. If the applied potential is limited to prevent the brush effect described by Hendricks (5), the spraying will consist of a large number of droplets directed axially from surface D to the plate. As spraying continues, the volume of the liquid tip decreases to surface A, where spraying ceases, and the process is then repeated.

By assuming that the liquid tip is in the form of a prolate spheroid, the equation used to approximate its surface would be:

$$\frac{x^2}{R^2} + \frac{y^2}{R^2} + \frac{z^2}{(FR)^2} = 1, \quad [6]$$

where F in the third term is a time varying quantity. The product FR represents the z directed semiaxis of the spheroid. The volume of the liquid tip is given by:

$$\tau = \int_0^{FR} \pi r^2\, dz = 2/3\pi R^3 F. \quad [7]$$

Differentiating with respect to time gives:

$$\frac{d\tau}{dt} = \tfrac{2}{3}\pi R^3 \frac{dF}{dt}. \quad [8]$$

Poiseuille's equation for the change in volume with respect to time of an incompressible viscous liquid in a long narrow tube of circular cross section is:

$$\frac{d\tau}{dt} = \frac{\pi \Delta P R^4}{8\eta L} = \frac{\pi \Delta P R^4}{8\rho\nu L}, \quad [9]$$

where ΔP = pressure difference to overcome viscosity, dynes/cm^2;

R = tube radius, cm;
L = tube length, cm;
ρ = density, gm/cm^3;
η = dynamic viscosity, gm/cm sec;
ν = η/ρ = kinematic viscosity, cm^2/sec.

Assuming no appreciable effect on the mass flow rate due to the influence of the electric field and substituting Eq. [9] into Eq. [8] gives:

$$\frac{dF}{dt} = \tfrac{3}{16}\frac{R\Delta P}{\eta L} = \tfrac{3}{16}\frac{R\rho g H}{\eta L} = \tfrac{3}{16}\frac{RgH}{\nu L}, \quad [10]$$

where H is the pressure head. F is then given by:

$$F = A + \left(\tfrac{3}{16}\frac{RgH}{\nu L}\right) t = A + Bt. \quad [11]$$

The constant B has the dimension reciprocal of time. The dimensionless constant A is independent of time but is a function of pressure, applied voltage, etc. Observations indicate that if the value of A decreases linearly from approximately unity to zero as applied voltage is increased from the minimum spraying potential V_m to a critical value V_c for which the spraying pulses

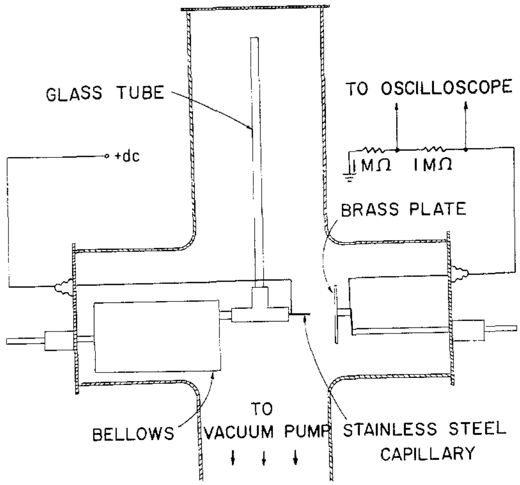

FIG. 2. Experimental apparatus.

cease, then it can be assumed that:

$$A \cong \frac{V_c - V}{V_c - V_m}. \qquad [12]$$

The dependence of A on pressure, liquid constants, geometry, etc., is automatically accounted for in the measured values of V_m and V_c.

The radius of curvature in the r-z plane is:

$$N = \left| \frac{[1 + (dz/dr)^2]^{3/2}}{d^2z/dr^2} \right|.$$

If we use $z = F(R^2 - r^2)^{1/2}$, the radius of curvature at the tip, where $r = 0$, is given by:

$$N = \frac{R}{F} = \frac{R}{A + BT_c}, \qquad [13]$$

where T_c is the time from the end of one pulse to the beginning of the next. Although Taylor (6) indicates some effect from the internal pressure it will be assumed here that when spraying commences the net pressure equals zero and Eq. [5] becomes:

$$E\Big|_{T_c} = \left[\frac{4\gamma}{\epsilon_0 N}\right]^{1/2}\Big|_{T_c}, \qquad [14]$$

where the electric field and radius of curvature are evaluated at T_c. Thus, with the use of experimental measurements, a value for the electric field can be obtained from Eq. [1] to compare with the value of electric field as given by Eq. [14]. Data were obtained by experimental methods to determine the various parameters with the spacing d measured from the tip of the capillary since the length of the liquid tip was negligible compared to the normal spacings used.

EXPERIMENTAL METHODS

Most of the data were taken in a vacuum of about 10^{-5} torr in the system shown in Fig. 2. The bellows was filled with glycerine and it could be compressed by an external knob to change the glycerine level in the vertical glass tube, thus varying the pressure head at the capillary. An external adjustment was also used to change the spacing between the plate and the capillary. Temperature was monitored by a thermometer attached to the vertical glass tube. Owing to ripple in the high-voltage power supply, a bank of four 7.5 μf capacitors connected in parallel were charged by the HV power supply and then used as the source of dc potential. A screen cage was built around the system to reduce noise. The current produced by the movement of the charged droplets was passed through a one-megohm resistor and the resulting voltage was monitored by a differential input oscilloscope. Another one-megohm resistor was inserted between the oscilloscope input and ground to give some isolation. An oscilloscope plug-in unit of 1 mv sensitivity was used since the voltages ranged from 2 mv to 10 mv.

Data were taken at various liquid pressures, voltages, and spacings, the latter being varied while the other parameters were held constant. This method gave more sensitivity and greater flexibility than varying the voltage. The data consisted of the width of the output pulses, the period between pulses, and the spacing at which the pulsing mode changed to a dc mode. The voltage was varied from 3.0 kv to 5.0 kv in steps of 0.5 kv and the height of liquid pressure was varied from 18 inches to 30 inches using 2-inch steps. The spacing was changed in steps of 1 mm from the point at which the dc level was established to a maximum of 24 mm. The capillary was mounted in a Teflon holder and was 1 inch long and had

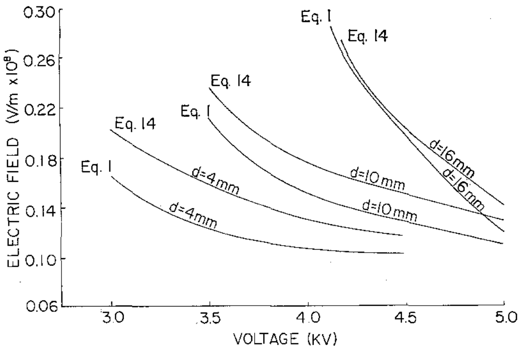

FIG. 3. Comparison of the electric field calculated by Eq. [1] and Eq. [14] versus applied voltage.

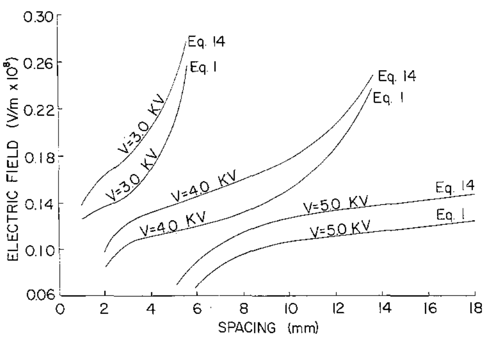

FIG. 4. Comparison of the electric field calculated by Eq. [1] and Eq. [14] versus spacing.

an inside diameter of 0.01 inch. It was made of stainless steel and the tip was cone shaped by an electrochemical etching process.

The temperature varied about ±2°F from its normal value of 74°F. The viscosity is the only major temperature-dependent parameter but does not cause a significant change over this small temperature range.

The viscosity was measured by the Saybolt Furol method giving a value of 6.8 cm²/sec, and the surface tension was measured by the Du Noüy Ring method giving a value of 60 dynes/cm.

EXPERIMENTAL RESULTS

Computer programs were written to calculate the values of electric fields given by Eq. [1] and Eq. [14]. Figures 3 and 4 show these two values of electric field versus parameters of applied voltage and spacing, respectively.

The logarithmic plot of Fig. 5 shows the value for electric field from Eq. [14], as a straight line, and the values of electric field from Eq. [1] as a scattering of points, versus the radius of curvature.

In order to determine the radius of curvature as given in Eq. [13], the values of V_c and V_m had to be found before computing a value for A. The value for B was determined from physical constants. V_m was found by making graphs of the period T_c versus applied voltage for each height and each spacing as shown in Fig. 6.

The value of V at which the period approached infinity for each spacing was then used as the value for the minimum spraying potential V_m. Thus for each height and each spacing a different value for V_m was obtained. It should be noted that this value

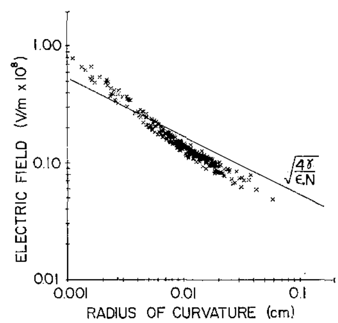

FIG. 5. Comparison of the electric field calculated by Eq. [1] (shown as individual points) and Eq. [14] (shown as a straight line) versus radius of curvature.

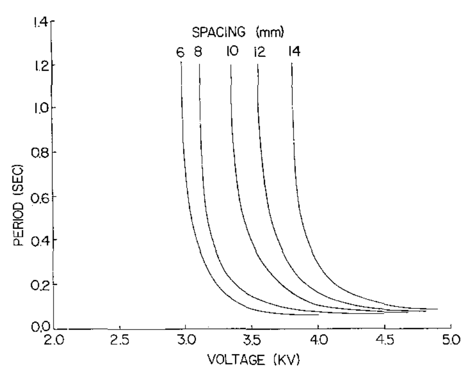

FIG. 6. Period versus applied voltage, with spacing as parameter.

for V_m was higher than that given by Hendricks, Carson, Hogan, and Schneider (7).

The value of V_c was determined by plotting the applied voltage versus the spacing at which the dc level was established for each height as shown in Fig. 7. From this plot the value for V_c could be read directly for any height or any spacing.

A few peculiarities were noticed while the experiment was being conducted. The first was the action of the dc level, which has been mentioned previously. As the spacing between the plate and the capillary was decreased, at a fixed voltage, the pulsing changed from the normal pulse as shown in Fig. 8*a* to a continuous jet emitting from the capillary. This established a dc level of a few millivolts on the oscilloscope. As the spacing was then increased the dc level remained at the same value until pulsing resumed. By visual observation of the jet, it was noticed that as the spacing increased, the radius of the liquid filament also increased. This indicates some sort of constant resistance or constant current mechanism in the filament.

Another point of interest occurred when the spacing between the capillary and the plate became so great that the period between pulses exceeded about 0.5 second. At this point the normal pulse shape shown in Fig. 8*a* began to exhibit a peak at a point slightly below its maximum amplitude, 8*b*. Figure 8 shows the sequence of events, *a* through *f*, as the spacing is increased. This indicates an entirely different spraying mechanism from the one considered in this paper, possibly one of a large drop followed by a series of smaller drops.

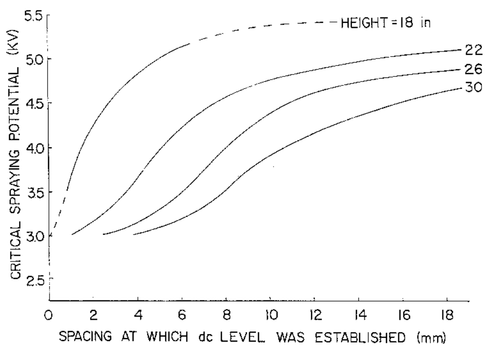

FIG. 7. Critical spraying potential versus spacing at which a dc level was established, with height as parameter.

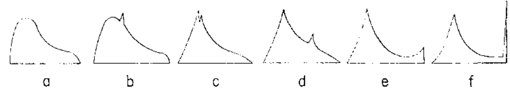

FIG. 8. Pulse shape as spacing is increased (*a*-*f*) beyond the "normal" operating range.

CONCLUSION

The variation of the electric field, at the onset of spraying, with respect to the voltage (Fig. 3) and spacing (Fig. 4), shows a deviation between the two methods of finding the electric field; however, the general trends of the curves are similar. Curves showing these same trends can be obtained at various heights as long as the pressure due to the liquid head is negligible. Although Eq. [1] seems to indicate a direct relationship between electric field and voltage and an inverse logarithmic relationship between electric field and spacing, it must be remembered that the radius of curvature is a function of voltage and spacing.

Figure 5 shows a logarithmic plot of about 260 values of electric field evaluated at T_c versus radius of curvature. These points should all lie on the line marked as $\sqrt{4\gamma/\epsilon_0 N}$ if Eq. [14] is true. Those points which correspond to a radius of curvature less than 0.005 cm may be neglected since the spraying mechanism changes to that described in Fig. 8. This value for the radius of curvature is determined from the period T_c, which was usually around 450 to 500 milliseconds when the peak began.

Of the approximately 230 points remaining, an average error of -20% exists. If all previous assumptions and approximations are valid, there are two possible sources of this error: (1) Experimental error; this is unlikely because the instruments involved had good accuracy and even this would have to have been all negative to give an error of -20%. (2) Error in the equation for electric field; this is a more likely candidate since the equation was derived for metal points

in the shape of a hyperboloid of revolution. Thus to apply to spraying of liquids it appears Eq. [1] should be increased by 20% in the normal range of operation.

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