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BY

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SYNOPSIS

Considerable attention has been given to conductor vibration of suspended transmission lines during the past 25 years. Even so, the engineers of the public utilities feel that they have only started to get a solution to the vibration problems. Extensive research programs are being conducted at the present time by manufacturers of conductor cable and ground wires and by numerous public utilities themselves.

Aeolian vibrations are the chief concern because they occur whenever a steady wind, of from one mile per hour to hurricane velocity, blows across the suspended line. Thus, millions of flexures, however small or large in amplitude, may be produced in a relatively short period of time. Each flexure will contribute to fatigue, especially if its affects are concentrated at a suspension point.

Stockbridge dampers and armor rods are two popular means of combating aeolian vibrations. Heretofore most analysists have considered these means of reducing conductor vibrations as being energy absorbers or dissipaters. By introducing the electrical analog it is easily shown that Stockbridge dampers, armor rods and similar dampers do not function as energy absorbers. They are mechanical impedance matching devices, which introduce reflected waves of the proper amplitude to cancel the unwanted waves. Dampers of the Stockbridge type are tuned mechanical impedance matching devices, and armor rods and "festoons" are untuned impedance matching devices. Proper design and installation of these dampers (impedance matching devices) can make them much more effective.

Ten public utilities and manufacturers of cable recently reported vibration data and experiences. Each of the reports contributes considerable to the analysis of conductor vibration, although only two of them have been given publication as technical papers. The valuable data and experiences of these ten reports are reviewed and correlated in this paper, Analysis of Conductor Vibration.

The limitations for stressing overhead high-strength shield or grounding cable should be lowered to 15% of ultimate strength at 60°F. as compared with the present National Electrical Safety Code of 25% of ultimate at 60°F. Flexible suspension clamps should be used.

The conductors of high voltage transmission lines should be sagged to carry a tension of not to exceed 20% of ultimate strength at 60°F. Stockbridge or similar dampers should best be installed at all supports.

It has been found desirable in many cases to also install armor rods at each flexible suspension support and dead end. It is known that armor rods effectively suppress vibration and give added protection. However, the collected data indicated that properly installed dampers of the Stockbridge type, and others, should sufficiently reduce aeolian vibrations.

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I INTRODUCTION

A study of the textbooks on vibration leads one to believe that the vibration problems as regards suspended transmission lines are generally understood, and that these problems have been analyzed with appropriate mathematics.(1,2,3) Also, in the past 20 years many technical discussions of the practical aspects have been published in the journals. However, the engineers of the public utilities feel that they have only started to get a solution to the vibration problems. Extensive research programs are being conducted at the present time by manufacturers of conductor cable and ground wires and by numerous public utilities themselves.

This report describes aeolian vibrations and galloping or dancing conductors. Basic electrical and mechanical vibrating systems are discussed. The electrical analog is used to explain the operation of dampers and armor rods.

Ten public utilities and manufacturers of cable recently reported vibration data and experiences. These reports are here reviewed one by one. A study of the data indicates certain limitations which are helpful in vibration prevention. These limitations are given as recommendations in the concluding section.

A number of other papers have been studied in preparation for this analysis. The references are listed in the section Bibliography.

II BASIC DEFINITIONS AND VIBRATING SYSTEMS

There are two categories of vibrations which have been found to cause breakage of suspended lines near the suspension supports. The two categories are called aeolian vibration and galloping or dancing conductors. It is important to note that each type of vibration has caused conductor failure. To be sure, the accumulative affects of both hurry rupture of conductors.

Aeolian vibrations are those of the natural frequencies of the suspended span, which are stimulated by very steady winds of 1 to 30 miles per hour.(4,5,6) It is the periodically varying eddy turbulence on the leeward side of the conductor that produces the excitation. The frequencies range from 1 to possibly 100 cycles per second, and the amplitudes normally range from a fraction of an inch to several inches.

However, aeolian vibration amplitudes of five feet and more have been observed. The frequencies and amplitudes of aeolian vibrations are functions of the wind velocity, the span length, the distance between nodes, the tension in the conductor, the diameter of the conductor, and its weight per unit length. On short spans the vibration is of extremely small amplitude, and it is evident only by the humming sound produced, as the "singing" of telephone lines on clear cold winter mornings.

Stockbridge dampers and armor rods are two popular means of combating aeolian vibrations. The Stockbridge damper has been found quite effective. It is but a double-cantilever vibrating element, which is suspended in the middle. These dampers are clamped to the conductor span at a distance of one to six feet from the suspension support. One or more is placed on each side.

Armor rods are used chiefly on ACSR conductors. They are constructed by using a layer of aluminum conductors which are somewhat larger in diameter than the outer strands of the ACSR conductor. The layer of aluminum armor is wound with the lay of the outer strands of the conductor for a distance of two to six feet each side of the support. The support clamp is placed on the outside of the armor, and two smaller clamps are used to hold the ends of the armor rod layer to the cable. Armor rods are only about 10 per cent as effective as Stockbridge dampers, but are quite effective on smaller diameter ACSR conductors with short spans.

Galloping or dancing conductors are large-amplitude low-frequency vibrations—up to 10 feet at about one cycle per second. Galloping is caused by strong gusty winds blowing across irregularly ice-covered conductors. No definite means has been found for damping these vibrations. The only known way of eliminating the phenomenon is to prevent the ice from forming or to melt it off as quickly as possible after it forms and before damage occurs. The critical diameter of cable for ice forming is about 1.4 inches—i.e., cable of 1.4 inches in diameter or larger will collect very little ice in heavy storms at freezing temperatures!

Consideration of the electric analog for mechanical systems is important because of the relative ease in setting-up an equivalent electrical system and of the greater accuracy of electrical measurements on a given electrical system.(7,8,9) Three basic vibrating systems, of one degree of freedon, are illustrated in figures 1, 2, and 3. An examination of the behavior of the mechanical spring system and of the mechanical torsional system reveals that the same differential equation governs their behavior as does that for the LRC electrical system.

There is then one to one correspondence for the basic mechanical and electrical elements. At this time interest is chiefly in the electrical and mechanical linear-spring system. The analogies which arise are: emf or voltage is like a force; inductance is like mass; charge is like distance or displacement; current is like velocity; resistance is like damping; one over capacitance, 1/C, is like spring constant. Further, electromagnetic energy is analogous to kinetic energy; stored energy in a capacitor is analogous to potential energy stored in a

compressed spring; time-rate of electrical energy loss is analogous to time-rate of frictional (damping) energy loss. The analogs tabulated symbolically are:

Electrical Term			Mechanical Term
voltage or emf	$E(t)$	$F(t)$	force
inductance	L	M	mass
charge	Q	x	distance
current	$i = \frac{dQ}{dt}$	$v = \frac{dx}{dt}$	velocity
resistance	R	c	damping
reciprocal of capacitance (elastance)	$\frac{1}{C}$	k	spring constant (elastance)
magnetic energy	$M.E. = \frac{1}{2}Li^2$	$K.E. = \frac{1}{2}Mv^2$	kinetic energy
potential energy	$P.E. = \frac{1}{2}\frac{Q^2}{C}$	$P.E. = \frac{1}{2}kx^2$	potential energy
power loss	$i^2 R$	$c\,v^2$	power loss

Equivalent two degrees of freedom systems are illustrated in figures 4 and 5. There the mutual coupling is elastance (capacitive). It is entirely possible to have either capacitive, inductive or resistive coupling or to have any combination thereof. The significant point to note here is that each degree of freedom of the mechanical system is analogous to a mesh of the electrical circuit, where a mesh is a circuit loop.

Consideration of the basic definitions and vibrating systems as reviewed in this section lend to understanding of the ten reports which are reviewed in the following sections of this paper.

The heading of each of the sections will carry the corresponding report title and bibliography reference number.

III VIBRATION AND FATIGUE LIFE OF STEEL STRAND (10)

As early as 1928 samples of vibration failure were reported.(20) Remedial measures accuring from experiences with conductors since then have resulted in a pronounced lengthening of conductor life. The recent increase of failures produced by aeolian vibration of higher-strength-type of strands of overhead shield wires indicated that the materials were

being worked beyond their endurance limits. Hence, the American Steel and Wire Company decided to evaluate the mechanical properties, construction practices, and other factors which might now be affecting the performance.

Many reversed bending tests of the center wires of different 7-wire strands were made to obtain the endurance limits of each. Endurance limit is defined as the maximum stress that a metal will withstand without failure during a specified large number of cycles of stress. The endurance limit is generally expressed in terms of the endurance ratio---e.g., 15 per cent of ultimate strength at 60° F.

The reversed bending tests were followed by vibration fatigue tests on the complete strands held in conventional line hardware and under tension. The data were plotted on curves as stress versus millions of cycles and amplitude versus millions of cycles.

The plots indicate that the usual tension of 25% of ultimate at 60° F. as specified by the current National Electrical Safety Code is excessive for the materials now used for overhead shield or ground strands. The logical remedial measure is to lower the endurance ratio. The data indicate that the use of 20% of ultimate at 0° F. would be a more appropriate limitation for the extra high strength materials used.

The tabulated data show that an increase of sag up to 50% is obtained when the limitations are reduced to the more appropriate level.

Extensive tests were also made on stranded cable equipped with preformed armor rods, and held under tension in conventional line hardware. For a given amplitude level the armored strand had a life more than ten times that of the unarmored strand. Hence, it is recommended that armor be used at all points of support in locations where vibration is liable to be a hazard.

IV USE OF TECHNIQUES FROM ELECTRICAL ANALOGUES IN THE STUDY OF TRANSMISSION LINE VIBRATION (11)

According to the well known formula, $v = f\lambda$ - i.e., velocity of propagation equals frequency of wave times wave length equals the constant $300 (10)^6$ meters per second - it is easily ascertainable that electrical waves of a few centimeters to several meters in length may be readily obtained by VHF and UHF radio electrical systems.(21) The effects of terminating radio transmission lines in impedances which do not match those of the electrical line itself are strictly analogous to a mechanical system formed by a cable span and its support clamps, insulators and towers. Radio transmission over cables is extremely faulty if not impossible when the termination is not matched for electrical impedance. When there is not proper termination the electrical waves are reflected, and standing waves result with only a small energy input. The standing waves of aeolian mechanical vibration

on a cable span are analogous. Hence the techniques of radio transmission are very helpful in furthering the study of aeolian vibrations. It is to be noted, too, that the techniques of physical measurement are difficult on a light physical span without disturbing the system.

By using the electrical analog the Southern California Edison Company verified that Stockbridge dampers and armor rods are not true dampers at all as Den Hartog and others have analyzed.(1,2,3) There is very little energy dissipated in these devices.

The Stockbridge damper is analogous to tuned impedance matching devices. The damper introduces a reflected wave into the system at a point one-quarter wave length from troublesome reflection points so that the introduced wave is 180 degrees out of phase with the unwanted reflection, and it has the proper amplitude to almost exactly cancel the unwanted wave. Hence for most effectiveness, multiple dampers (2,4,6 or 8 in number) are mounted near each suspension support to cancel the first harmonic and several higher harmonics of vibration.

Armor rods are like untuned matching devices as tapered horns of a microwave guide. They introduce reflections all along the length of the taper. Hence, armor rods were found to be much more effective when they, too, are tapered. A somewhat similar effect is obtained by using very long tapered support clamps.

V BPA EXPERIENCE WITH CONDUCTOR VIBRATION (12)

During the last nine years the Bonneville Power Administration (BPA) has accumulated valuable field experience, and has made many conductor vibration studies on their 33 transmission lines, in the Pacific Northwest. Most of these lines are of 230 kv steel-tower designs. The Aluminum Company of America and the American Steel and Wire Company have been most cooperative and helpful in supporting the research program.

The BPA has extensive plans for continuing their studies. In addition, they have given Washington State College a considerable grant of test equipment for conductor vibration study, as well as an initial two-year contract to insure a research program.

30,000 vibration dampers have been installed or are at present planned on 4,567 conductor spans and 216 overhead ground wire spans. All of these dampers will have been mounted with the aid of a simple hot-line tool on the energized 230 kv lines. This prevents outages, and is also more economical in promoting safety than on de-energized lines whereon it is necessary to install temporary grounds on either side of the section of line being worked. Also, all vibration studies and tests were made on energized lines.

Data are given which again show that two or three Stockbridge dampers on each side of the support clamp are most desirable to eliminate the harmful vibrations. The actual number of dampers used depends upon the span length. For example, one damper is used on each end of spans up to 1200 ft.; two dampers on each end of spans 1200 ft. to 2200 ft.; three dampers on each end of spans 2200 ft. to 3200 ft., etc., on 500 MCM expanded copper conductors as well as on 795 MCM ASCR conductors. In order to obtain the greatest effectiveness over a wide band of frequencies the spacing of the dampers at one end of the span are "staggered" with respect to those at other end by using, for example, 4 feet and 3½ feet spacing at the respective span ends.

All of the ACSR conductors on the BPA lines were originally installed with tapered armor rods. No signs of fatigue failures have been found on the ten-year old line. However, numerous breakages of 1 to 8 of the outer copper strands have been discovered on a 500 MCM high-strength copper conductor at 23½% of the towers in a 100-mile section of a 230 kv line which did not have armor rod or dampers. Special low-magnetic preformed armor rods are now being applied to repair the 500 MCM cable to its original strength, and also to combat further aeolian conductor vibration.

The investigators confirm that it is highly desirable or necessary to encourage the present trend towards improvement of the design of suspension clamps, armor rods and other conductor hardware and accessories, particularly in the transmission of power at the higher voltages of 230 kv and above.

VI FACTORS IN TRANSMISSION LINE DESIGN RELATED TO CONDUCTOR VIBRATION (13)

The Department of Water and Power, City of Los Angeles, has had 15 years of experience operating circuits of the 287.5 kv Boulder Transmission Lines. Initially dampers, of torsional design, were applied in locations which were regarded as particularly bad from a vibration standpoint. About 10% of the line was equipped with these dampers. Not until late 1949 was any trouble experienced due to vibration. Soon several other breaks were located. Then a complete line inspection was made, tower by tower. A total of 9 segement cracks were found. The startling discovery was that vibration was being experienced far in excess of any ever measured in the first two years of service. A 180-foot length of the Type HH conductor was cut from the line at one of the vibrating points, and shipped to the Ryan High Voltage Laboratory where the first laboratory measurements had been made. It was found that much less energy was required to vibrate the cable after its 15 years in service than when it was new. Also, it was exposed that the torsional dampers were far less effective than when the cable was new. By oiling the cable so that oil lubricated the dove-tail joints of the Type HH conductor a condition was aroused wherein the required energy to produce vibrations was greater than that required when new. Also, it was found that the torsional dampers were as effective with the oiled conductor as with the new. Hence, it was deduced that the increased friction between the sections of the conductor became so large

after 15 years' service that sliding was no longer present. Thus absorption of the energy which produces the vibration was reduced sufficiently that severe vibrations developed.

It has been found that either the Stockbridge damper or the Swedish damper is highly effective in reducing the vibrations in the 287.5 kv line to a safe level. It is anticipated that total application of dampers of one of these types will be necessary to insure the desired 50-year life with a margin of safety. It is to be noted that thru the use of lower tensions and meticulous clamp design an economical line was originally built with excellent vibration characteristics which obviated the need for total application of dampers. Without doubt this desirable condition would have existed for the life of the line had the vibration characteristics not changed.

The ground wires of the Boulder Transmission Lines are of 1/2 inch extra high strength galvanized strand having a breaking strength of 18,000 pounds. A normal tension of 1800 pounds at 60° F. is used--i.e., 10% of ultimate. Although special suspension clamps of the trunnion type are used, no vibration dampers have ever been used on the overhead ground wires. At this time there has been no reason to consider their use.

It is realized that this lower tension permits greater sag which must be compensated by higher supports at the towers. However, this design is economically desirable.

The Department of Water and Power has also designed a 258-mile 230 kv single-circuit steel-tower line which is now under construction. It is being shielded with the same $\frac{1}{2}$ inch extra high strength wire. A normal tension of 2100 pounds at 60° F. is being used--i.e., 12% of ultimate. No dampers are considered.

A very careful investigation was made for the new 230 kv line as regards the main conductor tensions as they affect the cost of the steel towers. It was found that it is economical as well as practical to utilize fairly low tensions in both the conductors and the ground wires. The new line is being constructed with 954,000 CM ACSR conductors in 1100-foot spans at a normal tension of 4600 pounds at 60° F. and a maximum tension of 10,000 pounds with ice and wind loading. The rated strength of the conductor is 34,200 pounds which gives a tension of only 13.5% of ultimate at 60° F.

Control of possible aeolian vibrations is considered of sufficient importance that armor rods and Stockbridge vibration dampers are being installed at all supports on the new 230 kv line.

VII FAILURES OF OVERHEAD GROUND WIRES CAUSED BY AEOLIAN VIBRATION (14)

Experience with several lines in the Southwest, of The Texas Power & Light Company, has shown that rigid suspension clamps and high tensions are sure to facilitate the fatigue failure in less than one year. Tensions of 25 to 35% of ultimate strength at 60° F. are considered high.

One overhead extra high strength shield line supported by semi-flexible clamps, at 18% of ultimate strength and 60° F., developed breaks in less than four years as a result of aeolian vibration. The tension was then reduced to 14.5%, and there was no evidence of further trouble for seven years.

Several overhead ground-wire lines having rigid supports and carrying tensions of 10% have evidenced no vibration difficulties in over 30-years' service.

In 1947 a line was built between Jewett and Lufkin, Texas. The shield wire was equipped with preformed armor rod and flexible supports, and was installed with a tension of 20% of ultimate at 60° F. There have been no vibration difficulties.

VIII AEOLIAN VIBRATION—EXPERIENCE WITH AND REMEDIAL MEASURES FOR FAILURES IN OVERHEAD GROUND WIRES (15)

The Oklahoma Gas and Electric Company has four relatively new transmission lines which developed severe aeolian vibrations on the overhead shield wires. Each carries shield wires of extra high strength cable. The oldest, which was built with a tension of 25% of ultimate strength at 60° F., experienced extreme vibration fatigue in less than 20 months. At one end of the line, the shield wires at 61 towers were inspected. It was found that one or more strands were broken at 100 of 122 suspension clamps. Immediate overhaul of the line was initiated. It was resagged for 15% tension, and armor rods were installed at each support.

The three other lines had tensions of 25%, 25% and 21.5%, respectively. Severe vibrations were observed, and a number of broken strands were found in the shield wires after the lines had been in service for only 5 months. Each has been resagged for a tension of 15%. Also, armor rods have been installed at each support.

Two other lines, originally sagged for 15%, have been operating in this area for 20 years. No shield wire difficulties have been found with them. However, it could not be ascertained with certainty that fatigue would not develop. It is also to be noted that the company had made experiments with armor rods following the siege of trouble. The engineers observed the reduction of vibration produced by armor rods. Hence, it was easy for them to decide to add the armor rods at the supports of the damaged shield wires on the four lines in addition to resagging to 15%.

IX VIBRATIONS--REVIEW OF EXPERIENCES (16)

A brief review of the field experience of the Public Service Company of (Denver) Colorado during the last 41 years indicates that armor rods should be used at all supports for main conductors of ACSR transmission lines. Still more desirable vibration characteristics have been exhibited on these lines when the line tension is limited to 20% of ultimate at 60° F. and dampers are used at all dead ends. Dampers are also used on all spans over 1000 feet.

X THE TORSIONAL DAMPER FOR CONDUCTORS--SERVICE EXPERIENCE AND FURTHER EXPERIMENTAL WORK (17)

The information discussed in this section was obtained from tests and field experiences of the Hydro-Electric Power Commission of Ontario, Canada.

An extensive investigation was made on ACSR conductors of sizes 3/0 to 605,000 CM on spans of 650 to 1400 feet to determine the effect of conductor tension on vibration. The results demonstrate that there is little tendency to vibrate below a tension of about 8% of the ultimate strength. As the tension is increased above 8% the maximum deflection increases rapidly up to 15% tension, after which the maximum deflection remains constant for farther increase of tension. The number of cycles of vibration per day above any given deflection increases almost linearly with tension--i.e., frequency is proportional to tension. Further, the span length does not seem to be a major factor in vibration.

The Commission has a Type HH copper-conductor 220 kv line similar to the 287.5 kv Boulder Transmission Line. This line is 42 miles in length, has a double circuit, and is rated 220 kv. The conductor is of the 500,000 CM Type HH construction. The spans are 880 feet, and the conductors are sagged for 20% tension at 60° F. The line has been in service for nine years, and the spans are undamped. Complete inspections have indicated no vibration difficulties whatsoever.

Since 1940 the Commission has adopted the practice of omitting armor rods completely and using dampers exclusively to prevent damage by aeolian vibration on all high voltage circuits. One damper is installed on each side of each suspension support. The mounting distance is six feet from the support for 795,000 CM ACSR conductor, and is less for smaller conductors. More than two dampers are arbitarily used at each support for spans over 1200 feet. Line tensions of 15 to 19% at 60° F. are used.

The following two lines were built in 1940: (1) a 110 kv line, 477,000 CM ACSR, average span 1000 feet, 60° F. tension 15.5%, and 103 miles long; and (2) a 220 kv line, 795,000 CM ACSR, average span 1150 feet, 60° F. tension 19%, and 271 miles in length. Both were equipped with only dampers, one third of them are of the T&B torsional type and

the other two thirds are of the Stockbridge type.(22,23) The ten-year inspection showed no wear or breakage due to aeolian vibration.

A trial attempt was made to control galloping conductors in a high voltage 2200-foot river span, which had given trouble on three occasions. Torsional dampers were installed, 8 per span per conductor. No claims were made for this as a cure for galloping. However, it is noteworthy that the span has been free of galloping for approximately four years.

XI CONDUCTOR FAILURES (MECHANICAL) DUE TO GALLOPING, TO AEOLIAN VIBRATION FATIGUE, OR TO COMBINATIONS OF THESE TWO TYPES OF FATIGUE (18)

The aluminum strands of ACSR conductors may be broken by flexure fatigue introduced by either galloping or aeolian vibration. The structural patterns at the breaks are peculiarly characteristic to the kind of vibration. Breaks in aluminum and other wires which are caused by galloping have a course fracture such as is obtained when small soft wires are broken in reverse bending by hand. Failures due to aeolian vibration are characterized by a glassy fracture which is generally accompanied by a peculiar "S" curve across the diameter.

The breaks, which have been discovered on the lines of the Hydro-Electric Power Commission of Ontario, Canada, have generally been at dead-end clamps of an old design on ACSR conductor lines which have tensions greater than 10% of ultimate at 60° F. In each case the structural patterns on the severed aluminum strands indicated both galloping and aeolian vibration fatigue. The steel cores broke within the cable at a distance of 3 to 4 feet from the ends of the broken aluminum strands. At this point the heat of current flow in the steel core could not be carried away fast enough, and the metal became hot enough to flow. In each case the steel core had been drawn out to about one half size, and showed the usual cupping as when wires are pulled to their ultimate strength.

XII PROGRESS IN CONDUCTOR VIBRATION INVESTIGATIONS (19)

The Aluminum Company of America has maintained a laboratory for the study of conductor vibration since 1928 at Massena, New York. The facilities consist of a number of indoor test spans of reduced length and of outdoor lines which simulate actual spans up to 1500 feet in length. There is continual effort to improve test techniques and the associated apparatus. Although it is extremely difficult to correlate laboratory data and field experience, there is little doubt concerning the value of the assistance that the laboratory investigations give. Means are now being devised to measure accurately the amounts of input energy required to excite vibrations of various amplitudes for sundry cable conditions.

Quantitative tests have been made which show that the ALCOA armor rods contribute considerably more damping than has been formerly recognized.

Further, the damping ability increases with higher effective wind velocities.

Tests also show that auxiliary cables of steel-wire rope or multiple-strand fully-annealed aluminum cable used as "festoons" at the supports are very effective as dampers on 1000-foot spans of 397,500 CM ACSR, 30/7.

Application of a single Stockbridge damper at one end of a 1000-foot test span of a large ACSR conductor, which was vibrating vigorously in the wind, quickly and completely damped the vibration. The test was repeated many times to insure the results.

XIII CONCLUSION

There is need for continued research and study of the means available for matching the mechanical impedance of a conductor span at its supports. The use of the electrical analog has proven to be quite helpful in the study, and the indications are that it will continue to contribute valuable information.

The Stockbridge damper, armor rods and several other dampers are means of mechanical impedance matching. By proper design and installation these dampers (impedance matching devices) can be made much more effective.

The limitations for stressing overhead high-strength shield or grounding cable should be lowered to 15% of ultimate at 60° F. as compared with the present code of 25% at 60° F. Flexible suspension clamps should be used.

The conductors of high voltage transmission lines should be sagged to carry a tension of not to exceed 20% of ultimate strength of 60° F. Stockbridge or similar dampers should best be installed at all supports.

It has been found desirable in many cases to also install armor rods at each flexible suspension support and dead end. It is known that armor rods effectively suppress vibration and give added protection. However, the collected data indicate that properly installed dampers of the Stockbridge type, and others, should sufficiently reduce aeolian vibrations.

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* References 10 through 19 are technical papers presented at the Conductor Vibration Symposium of the October 1950, AIEE Fall General Meeting, Oklahoma City, Oklahoma. TP 50-237 and TP 50-278, references 10 and 11, will be published in the 1950 AIEE Transactions. The eight papers, references 12 through 19, were conference papers for this meeting, and are not available through the AIEE.

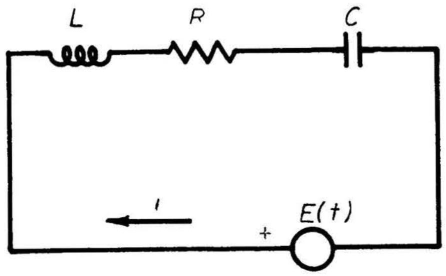

Figure 1 LRC electrical system of one degree of freedom. The differential equation is

$$L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} = E(t), \text{ where } \frac{dQ}{dt} = i$$

is the current in amperes or time rate of change of charge Q is charge in coulombs, R is resistance in ohms, C is capacitance in farads, L is inductances in henrys, and E(t) is the applied voltage in volts

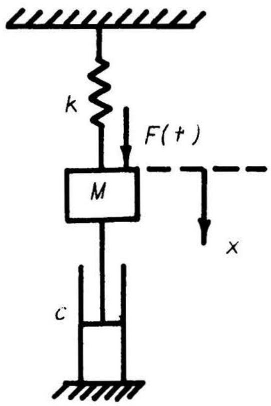

Figure 2 Mechanical spring system of one degree of freedom The differential equation is

$$M \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F(t), \text{ where } \frac{dx}{dt} = v$$

is the velocity in inches per second or time rate of change of distance x is distance (displacement) in inches, c is dynamic damping constant in pounds-seconds per inch, k is linear spring constant in pounds per inch, M is mass in pound-seconds squared per inch, and F(t) is the applied force in pounds.

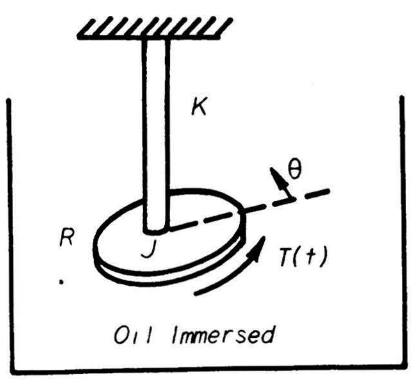

Figure 3 Mechanical torsional system of one degree of freedom The differential equation is

$$J \frac{d^2\theta}{dt^2} + R \frac{d\theta}{dt} + K\theta = T(t), \text{ where } \frac{d\theta}{dt} = w$$

is the angular velocity in radians per second of time rate of change of angular displacement, θ is angular displacement in radians, R is dynamic damping constant in pound-inch-seconds per radian, K is torsional spring constant in pound-inches per radian, J is mass moment of inertia in pound-inch-seconds squared per radian, and T(t) is the applied torque in pound inches.

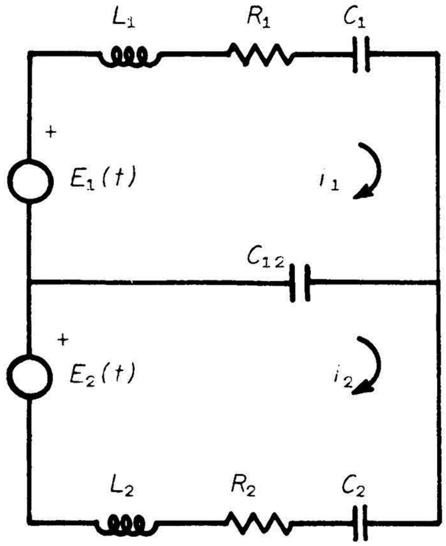

Figure 4. Electrical system of two degrees of freedom. The two differential equations which describe the activity are:

$$L_1 \frac{d^2Q_1}{dt^2} + R_1 \frac{dQ_1}{dt} + \frac{C_1 \; C_{12}}{C_1 + C_{12}} Q_1 - \frac{1}{C_{12}} Q_2 = E_1(t)$$

$$L_2 \frac{d^2Q_2}{dt^2} + R_2 \frac{dQ_2}{dt} + \frac{C_{12} \; C_2}{C_2 + C_{12}} Q_2 - \frac{1}{C_{12}} Q_1 = E_2(t)$$

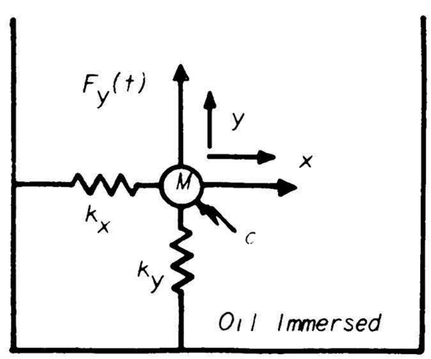

Figure 5. Mechanical syscem of two degrees of freedom. The two differential equations which describe the activity are.

$$M \frac{d^2x}{dt^2} + c_x \frac{dx}{dt} + k_x \; x + k_y \; f(x) = F_x(t)$$

$$M \frac{d^2y}{dt^2} + c_y \frac{dy}{dt} + k_y \; y + k_x \; f(y) = F_y(t)$$