01 Jan 1984

# Correction to “Two-Dimensional Markov Representations of Sampled Images”

John A. Stuller

*Missouri University of Science and Technology*, stuller@mst.edu



## Recommended Citation

J. A. Stuller, "Correction to “Two-Dimensional Markov Representations of Sampled Images”," *IEEE Transactions on Communications*, vol. 32, no. 6, p. 744, Institute of Electrical and Electronics Engineers, Jan 1984.

The definitive version is available at https://doi.org/10.1109/TCOM.1984.1096115

$r_1 = r_2$, we have found that the optimal $p$ that minimizes the total average delay is always 1. From the steady-state condition, it is easy to see that when $r_1 = r_2 = r$, the total arrival rate $\gamma\,(\gamma = 2r)$ should be less than 2/3 for steady state. In Fig. 5, the minimal total average delay (i.e., $p = 1$) is plotted versus $\gamma$ when $r_1 = r_2$ for the access scheme of this paper (the acknowledgment-based access scheme), as well as for the random access scheme analyzed in [1]. As expected and as is shown in Fig. 5, the network performs much better with the access scheme analyzed in this paper. This latter result is actually correct for all values of arrival rates.

### References


[1] M. Sidi and A. Segall, "Two interfering queues in packet-radio networks," *IEEE Trans. Commun.*, vol. COM-31, Jan. 1983.

[2] H. Kobayashi and A. G. Konheim, "Queueing models for computer communications system analysis," *IEEE Trans. Commun.*, vol. COM-25, pp. 2–28, Jan. 1977.

[3] E. T. Copson, *Theory of Functions of a Complex Variable.* London, England: Oxford Univ. Press, 1948.

[4] J. D. C. Little, "A proof for the queueing formula $L = \lambda W$," *Oper. Res.*, vol. 9, pp. 383–387, 1961.

[5] L. Kleinrock and Y. Yemini, "Interfering queueing processes in packet-switched broadcast communication," presented at the IFIP Congr., Tokyo, Japan, 1980.

[6] T. N. Saadawi and A. Ephremides, "Analysis, stability, and optimization of slotted ALOHA with finite number of buffered users," *IEEE Trans. Automat. Contr.*, vol. AC-26, pp. 680–689, June 1981.

[7] F. A. Tobagi and L. Kleinrock, "On the analysis and simulation of buffered packet radio systems," in *Proc. 9th Hawaii Int. Conf. Syst. Sci.*, Honolulu, HI, Jan. 1976, pp. 42–45.


## Correction to "Two-Dimensional Markov Representations of Sampled Images"

J. A. STULLER

A representation for a sequence $\{x_i\}$ of $N^2$ arbitrarily scanned image intensity samples was sought in the above paper[1] such that each sample $x_i$, $1 \leqslant i \leqslant N^2$ is given as a linear combination of the preceding $x_{i-1}, x_{i-2}, \cdots, x_1$ plus an orthogonal random variable (r.v.) $w_i$. That is,

$$x = Lx + w \tag{1}$$

and

$$E\{x_i w_j\} = 0, \qquad j < i \tag{2}$$

where $x$ is the $N^2$ element vector of the $x_i$, $w$ is the $N^2$ element vector of the $w_i$, and $L$ is strictly lower triangular (entries on and above the principal diagonal equal zero). The paper related this one-sided representation to a two-sided representation in which each $x_i$, $1 \leqslant i \leqslant N^2$ is given as a linear combination of $x_j$, $j \neq i$, $1 \leqslant j \leqslant N^2$ plus an orthogonal r.v. $u_i$. That is,

$$x = Hx + u \tag{3}$$

and

$$E\{x_i u_j\} = 0, \quad j \neq i \tag{4}$$

where $u$ is the $N^2$ element vector of the $u_i$ and $H$ has a principal diagonal zero. An error occurred in the assumption that $\{w_i\}$ and $\{u_i\}$ are necessarily stationary. For an *arbitrary* scanning path over the $N \times N$ image raster, (9) and (13) of the above paper[1] should read

$$K_{xu} = E\{xu^T\} = D_u{}^2 \tag{5}$$

and

$$K_w = E\{ww^T\} = D_w{}^2 \tag{6}$$

where $D_u = \operatorname{diag}\,[\sigma_{u_i}]$ and $D_w = \operatorname{diag}\,[\sigma_{w_i}]$. This correction can be easily incorporated into the above paper[1] by representing $u$ and $w$ throughout as $u = D_u u_0$ and $w = D_w w_0$, with $E\{u_0 u_0{}^T\} = E\{w_0 w_0{}^T\} = I$, the $N^2 \times N^2$ identity matrix. Equations (10) and (11) of the above paper[1] then become, respectively,

$$K_u = (I - H)D_u{}^2 \tag{7}$$

and

$$\begin{aligned} K_x &= (I - H)^{-1} D_u{}^2 \\ &= D_u{}^2 K_u{}^{-1} D_u{}^2. \end{aligned} \tag{8}$$

On combining (7) and (8) above, one has

$$(I - H) = D_u{}^2 K_x{}^{-1}. \tag{9}$$

All entries on the principal diagonal of $H$ are zero. Therefore, all elements on the diagonal of $D_u{}^2 K_u{}^{-1}$ are unity. This implies that

$$\sigma_{u_i}{}^2 = \frac{1}{\gamma_i}, \qquad i = 1, 2, \cdots, N^2 \tag{10}$$

where $\gamma_i$ is the $i$th element on the principal diagonal of $K_x{}^{-1}$. The orthogonality condition (4) implies that $H$ is the LLMSE interpolator and $\sigma_{u_i}{}^2$ is the interpolation error.

Similarly, (16) of the above paper[1] becomes

$$BK_xB^T = D_w{}^2 \tag{11}$$

and a unique lower triangular $B \triangleq I - L$ can be determined with unit diagonal [1]. Since $L$ is strictly lower triangular, the orthogonality condition (2) implies that it is the LLMSE predictor and $\sigma_{w_i}{}^2$ is the interpolation error.

The relation between the matrices $H$ and $L$ in the general case is

$$H = I - D_u{}^2 (I - L)^T D_W{}^{-2} (I - L). \tag{12}$$


### Acknowledgment

The comments of a very competent reviewer resulted in an improvement in this correspondence.


### References


[1] D. M. Young and R. T. Gregory, *A Survey of Numerical Mathematics.* Reading, MA: Addison-Wesley, 1973.



Paper approved by the Editor for Signal Processing and Communication Electronics of the IEEE Communications Society for publication without oral presentation. Manuscript received December 7, 1982; revised January 24, 1984.



The author is with the Department of Electrical Engineering, University of Missouri, Rolla, MO 65401.


[1] J. A. Stuller and B. Kurz, *IEEE Trans. Commun.*, vol. COM-24, pp. 1148–1152, 1976.