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A COMPARISON OF THE EFFECT OF PVT DATA APPLICATION METHODS ON PREDICTED DEPLETION DRIVE RESERVOIR PERFORMANCE

BY

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A

THESIS

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Abstract

It is generally agreed that the evolution of gas from oil within a reservoir is most closely approximated by the differential liberation process, while gas liberation in the flow string, surface separator, and stock tank is most nearly represented by flash liberation. To conform with this assumed physical behavior, appropriate application of both flash and differential liberation data in the computation of reservoir performance by material balance methods is indicated.

In this work, simple depletion-drive oil and gas recoveries were calculated (1) using differential liberation data only, and (2) using differential liberation data adjusted to incorporate the results of flash separation tests. A comparison of the results obtained indicates that while the differences in depletion-drive recoveries so predicted may sometimes be unimportant from a practical standpoint, they can be appreciable where considerable differences exist between the flash and differential formation-volume-factors.

Utilization of differential liberation data which has been converted from a "residual oil" to a "stock tank oil" basis throughout the material-balance equations appears to be a simple, appropriate method of incorporating the effects of both liberation processes. This conversion may be accomplished by multiplying differential solubility and shrinkage values by the ratio of flash to differential bubble-point formation-volume-factor.

I. Introduction

The material balance equation (page 7) has been used for many years in reservoir engineering practice to predict gas-oil ratio variation and cumulative oil recovery from a reservoir as it produces gas and oil under various operational procedures.

One of the first quantitative applications of the pressure-volume-temperature (PVT) relationships of reservoir fluids to the study of reservoir behavior was presented in a paper by Coleman, Wilde and Moore (1)[1/]. This work described the results of investigations into the decline of reservoir pressure as related to the production of oil and gas, and presented a reservoir material-balance equation relating, as functions of reservoir pressure, the quantities of oil and gas produced and remaining in the reservoir, and the properties of the reservoir fluids based on ideal behavior at elevated pressures and temperatures. In 1933, Schilthius (2) presented a modified form of this relationship which utilized the application of actual laboratory determined PVT relationships of reservoir fluid samples, oil and gas production data, and reservoir pressure history to estimate oil in place.

This Schilthius equation or an equivalent form has since been utilized, in conjunction with fluid saturation and instantaneous gas-oil ratio relationships, by several

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[1/] Numbers in parentheses refer to references given in the bibliography at the end of this paper.

authors (3,4,5,6,7) in the formulation of computational techniques for forecasting the performance of depletion-drive reservoirs. (Depletion-drive, also referred to as solution gas-drive, denotes a primary recovery process whereby oil is displaced from reservoir rock by the energy of expansion of gas originally dissolved in the oil.) Since these calculation methods all involve the application of laboratory determined PVT data, however, the accuracy of recovery values as predicted by their use will depend in part on the degree of similarity between laboratory gas liberation and pressure depletion methods, and the actual liberation processes occuring in the producing formation, flow string and surface separators.

While it is generally acknowledged that little is known concerning the exact nature of gas liberation from a solution-gas drive reservoir, the evolution of gas from oil within the reservoir as pressure declines is generally conceded to approximate isothermal "differential liberation", while gas liberation in the flow string, surface separators, and storage facilities is generally agreed to approximate "flash liberation" conditions. Due to basic differences between these two liberation processes, gas solubility and oil shrinkage values as determined by them may differ significantly for the same reservoir fluid. The depletion-drive recoveries predicted using differential liberation solubility and shrinkage values only might therefore be expected to differ to some extent from recoveries predicted utilizing these same values, but incorporating appropriate flash separation data to approximate the results of the combination liberation sequence generally assumed to occur.

It is the purpose of this work to: (1) investigate the magnitude of the difference in depletion-drive oil recoveries as predicted with and without the application of appropriate flash separation data, (2) determine how this difference varies with certain physical properties of the reservoir oil (highly volatile type oils are not considered), and (3) discuss possible refinements in laboratory procedures or computational methods should they seem warranted by a considerable variation in the predicted recoveries.

II. Gas Liberation Processes

The term "flash liberation" denotes that type of gas liberation in which all vapors formed are allowed to remain in contact with the liquid phase until desired equilibrium conditions are reached, while "differential liberation" denotes a liberation process wherein vapors formed are continuously removed from contact with the liquid phase. The former process is therefore characterized by constant overall system composition, while the latter is characterized by a continuously changing system composition.

In the laboratory analysis of bottom-hole or recombined surface samples of reservoir fluids, it has been found that the total gas evolved and the residual oil volume resulting from a pressure depletion from reservoir to standard atmospheric conditions will usually differ depending upon whether a flash or differential liberation procedure is followed. This is readily understandable since, in addition to the basic differences discussed above, laboratory differential liberations are usually performed at reservoir temperatures, while in the flash tests the oil and evolved gas are separated after reaching equilibrium at the approximate surface temperatures encountered in the field. (It has become accepted practice in view of these differences to refer to the oil remaining after a differential depletion to standard atmospheric conditions as "residual oil", and to the oil remaining after flash liberation to the same conditions as "stock tank oil". This distinction will be made throughout this paper when referring to laboratory data.) The magnitude of the difference between solution gas content (solubility) and reservoir oil volume relative to atmospheric oil volume (formation-volume-factor) as determined by the two processes will depend primarily on the reservoir and surface separation temperatures and pressures, and on the overall system composition. In general, this difference increases with the volatility of the hydrocarbon system and the reservoir temperature, other factors remaining the same.

While gas liberation can be so controlled as to follow either the flash or differential process in the laboratory analysis of a reservoir fluid sample, the mechanism of oil production involves a complex composite of both types. The exact nature of this production liberation sequence has been adequately discussed in the literature (3,7,8,9) with the resultant opinion being that the evolution of gas from oil within a reservoir as production proceeds is most closely approximated by differential liberation at reservoir temperature, while gas liberation in the flow string, surface separators, and storage facilities is most nearly represented by flash liberation. In view of this composite liberation behavior during the course of production and of the possible difference in solubility and formation-volume-factor values as determined by the two laboratory liberation procedures, it would appear that material-balance calculations for the forecasting of reservoir performance should include the application of both flash and differential laboratory data in their appropriate places in the equations or result in erroneous predicted recoveries.

III. Predicting Depletion Drive Recoveries

Though there have been various computational techniques proposed for forecasting the performance of depletion-drive reservoirs, basically these methods are all similar in that they utilize a form of the material-balance equation in conjunction with gas-oil ratio and fluid saturation relationships to compute pressure decline and producing gas-oil ratio as a function of cumulative oil production.

In this work the general calculation procedure presented by Tarner (5) was followed, and only the case of simple depletion-drive production from a reservoir originally at bubble-point conditions was investigated. There was assumed to be no initial gas cap, no water encroachment or water production, and no vertical migration of liberated gas.

Physical and Mathematical Relationships

The computation of pressure decline and gas-oil ratio versus cumulative oil production for the conditions described above was accomplished by the trial and error solution of four simultaneous equations[1]:

The volumetric material-balance equation:

$$G_p = (N)\left[(R_{si}-R_s)-\frac{(B_{oi}-B_o)}{B_g}\right] - (N_p)\left(\frac{B_o}{B_g} - R_s\right) \ldots \text{ Eq. (1.)}$$

The fluid saturation equation:

$$S_L = \left(1 - \frac{N_p}{N}\right)\left(\frac{B_o}{B_{oi}}\right)(1-S_w) + S_w \ldots\ldots\ldots\ldots\ldots\ldots \text{ Eq. (2.)}$$

The instantaneous gas-oil ratio equation:

$$R = \frac{k_g}{K_o} \cdot \frac{\mu_o}{\mu_g} \cdot \frac{B_o}{B_g} + R_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ Eq. (3.)}$$

The cumulative gas production equations:

$$\Delta G_p = (\Delta N_p)(\bar{R}) = \left[(N_p)_2 - (N_p)_1\right]\left(\frac{R_1 + R_2}{2}\right)$$

$$G_p = \sum \Delta G_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ Eq's. (4.)}$$

---

[1] Derivations, nomenclature and discussions of any limiting assumptions are given in Appendix A.

These relationships are shown here in general form, i.e., with no distinction being made as to the liberation process utilized in the evaluation of solution gas content, R_s, or oil formation-volume-factor, B_o. The appropriate application of flash or differential liberation values for these terms will be discussed later in this paper.

General Calculation Procedure

The general calculation procedure as devised by Tarner for the calculation of a reservoir pressure P_2 and producing gas-oil ratio R_2 corresponding to an oil production increment $\left[(N_p)_2 - (N_p)_1\right]$ from previous conditions $(N_p)_1$, P_1 is as follows:

1. Assume that pressure P_2 prevails throughout the reservoir.
2. Estimate the incremental production of stock tank oil $\left[(N_p)_2 - (N_p)_1\right]$ that might result from the reservoir pressure drop $(P_1 - P_2)$.
3. Calculate the cumulative gas production corresponding to total oil production $(N_p)_2$ by means of Eq. (1.) using values for R_s, B_o, and B_g corresponding to pressure P_2.
4. Calculate total liquid saturation corresponding to total oil production $(N_p)_2$ by means of Eq. (2.). Evaluate $\frac{k_g}{k_o}$ at this liquid saturation from appropriate relative permeability-saturation relationships.

5. Calculate the instantaneous producing gas-oil ratio at P_2 by means of Eq. (3.).
6. Calculate the cumulative gas production corresponding to total oil production $(N_p)_2$ by means of Eq's. (4.) using the data from step 5 and the instantaneous producing gas-oil ratio from preceding pressure P_1.
7. If the cumulative gas productions as calculated in steps 2 and 6 agree, the assumed increment of oil production is taken as correct. Should they disagree, steps 2 through 6 must be repeated.

The above process is carried out for sufficiently numerous pressure decrements so that the assumptions made in the derivations of these equations (see Appendix A) are not invalidated.

IV. Application of PVT Data in Recovery Calculations

Tarner Method

In one of the first proposed solutions to the problem of forecasting the performance of depletion-drive fields, Tarner in his original paper (5) utilized differential liberation data only in his computations. Introducing the superscript d to denote those physical properties determined by differential liberation, preceding equations (1.) through (3.) as used by Tarner become:

$$G_p = (N)\left[(R^d_{si}-R^d_s) - \frac{(B^d_{oi}-B^d_o)}{B^d_g}\right] - (N_p)\left(\frac{B^d_o}{B^d_g} - R^d_s\right) \text{..Eq. (1a.)}$$

$$S_L = \left(1-\frac{N_p}{N}\right)\left(\frac{B^d_o}{B^d_{oi}}\right)(1-S_w) + S_w \text{ .... Eq. (2a.)}$$

$$R = \frac{k_g}{k_o} \cdot \frac{\mu_o}{\mu_g} \cdot \frac{B^d_o}{B^d_g} + R^d_s \text{ .... Eq. (3a.)}$$

The original "residual oil" in place is calculated in this method by dividing total reservoir hydrocarbon pore space by the differential liberation formation-volume-factor of the reservoir oil at its bubble point.

Muskat Method

Muskat (8), in his derivation of the material-balance equation, converts all differential liberation data from a residual oil to a stock tank oil basis. This is presumably accomplished by multiplying differential solubility and formation-volume-factor values by the ratio $\frac{B^f_{oi}}{B^d_{oi}}$, the superscript f denoting the flash liberation process. Applying this method to equations (1.) through (3.) results in the following relationships:

$$G_p = \left(\frac{B^f_{oi}}{B^d_{oi}}\right)(N)\left[(R^d_{si} - R^d_s) - \left(\frac{B^d_{oi}-B^d_o}{B^d_g}\right)\right]$$

$$- \left(\frac{B^f_{oi}}{B^d_{oi}}\right)(N_p)\left(\frac{B^d_o}{B^d_g} - R^d_s\right) \text{....... Eq. (1b.)}$$

$$S_L = \left(1-\frac{N_p}{N}\right)\left(\frac{B^d_o}{B^d_{oi}}\right)(1-S_w) + S_w \text{ .... Eq. (2a.)}$$

$$R = \left(\frac{B^f_{oi}}{B^d_{oi}}\right)\left(\frac{k_g}{k_o} \cdot \frac{\mu_o}{\mu_g} \cdot \frac{B^d_o}{B^d_g} + R^d_s\right) \ldots\ldots\ldots\ldots\ldots \text{Eq. (3b.)}$$

Noting that equation (2.) assumes the same form as in the Tarner method of data application, and that equations (1b.) and (3b.) both differ from (1a.) and (3a.) by the constant ratio $\frac{B^f_{oi}}{B^d_{oi}}$, it is evident that the use of either set of these equations in conjunction with equation (4.) and the calculation procedure previously outlined should result in the same per cent ultimate recovery of oil initially in place. The amount of stock tank oil initially in place is calculated in the Muskat application of data by dividing reservoir hydrocarbon volume by the bubble-point oil formation-volume-factor as determined by flash liberation, however. The total stock tank oil recovery as computed by using the Tarner method of data application should therefore differ from that computed by Muskat's method in the ratio $\frac{B^f_{oi}}{B^d_{oi}}$ for a given reservoir.

Similarly, total gas production as calculated using Muskat's method will differ from that computed by the Tarner method in the ratio $\frac{B^f_{oi}}{B^d_{oi}}$ for equal amounts of tank oil originally in place. However, since the oil in place as calculated by Muskat's method differs from that of Tarner in the ratio $\frac{B^d_{oi}}{B^f_{oi}}$, the calculated total gas produced for a given reservoir should be the same by either method.

Patton Method

Patton (7.) proposed still another method of applying flash and differential laboratory data in the material-balance equations. Based on the assumption that differential liberation prevails within the reservoir, he theorized that solubility and relative oil volume values which are applied to the oil initially in place, N, in the equations should be those determined in laboratory by the differential process. Assuming flash liberation to prevail in the flow string and surface separation facilities, he reasoned that values for the above properties which are applied to the produced oil, N_p, should be from laboratory flash liberation data. In accordance with these considerations, preceding equations (1.) through (3.) take the form:

$$G_p = (N)\left(\frac{B^f_{oi}}{B^d_{oi}}\right)\left[(R^d_{si}-R^d_s) - \left(\frac{B^d_{oi}-B^d_o}{B^d_g}\right)\right] - (N_p)\left(\frac{B^f_o}{B^d_g}-R^f_s\right) \ldots\ldots\ldots\ldots \text{Eq. (1c.)}$$

$$S_L = \left(1-\frac{N_p}{N}\right)\left(\frac{B^d_o}{B^d_{oi}}\right)(1-S_w) + S_w \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. (2a.)}$$

$$R = \frac{k_g}{k_o} \cdot \frac{\mu_o}{\mu_g} \cdot \frac{B^f_o}{B^d_g} + R^f_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. (3c.)}$$

The stock tank oil initially in place is calculated by Patton by dividing reservoir hydrocarbon pore space by the reservoir oil formation-volume-factor at bubble point conditions as determined by differential liberation. This procedure predicates that the conversion from reservoir to stock tank conditions should be made by differential liberation, which seems incompatible with the previously discussed assumption as to the liberation process prevailing as oil is produced to the surface. To be consistent, original stock tank oil in place was calculated in this work utilizing the flash liberation bubble-point formation-volume-factor; equation (1c.) above therefore takes a slightly different form than the material-balance equation as proposed by Patton. His principle of data application was otherwise adhered to.

To utilize the above relationships, solubility and relative oil volume data throughout the entire range of pressure depletion for both liberation processes is necessary. While normal laboratory analysis usually provides this information for the differential process, flash liberation data are usually available only for the original bubble-point fluid. Patton recommends that the required flash data be obtained by plotting the flash solubility and relative oil volume values at bubble-point pressure, then drawing the flash curves through these points and parallel to corresponding differential liberation curves which have been converted to a stock tank oil basis. Below the pressure at which the latter curves break away from a straight line and bend toward the pressure axis, the flash liberation data curves are obtained by drawing smooth curves having the same general shape as those obtained by differential liberation and terminating at the same end points. The differential liberation data is presumably converted to a stock tank oil basis through multiplication by the factor $\frac{B^f_{oi}}{B^d_{oi}}$.

Gas formation-volume-factor values as determined from differential liberation gas gravity values are used throughout the equations of necessity since these data are usually not available for the flash process. No appreciable difference in the calculated recoveries should result from this procedure, however, since there is negligible difference in the gravity (hence composition) of the gas liberated by either method at elevated pressures (3), and the variations in gravity which do occur in the low pressure range do not appreciably effect the gas compressibility factor values obtained from them.

V. Recovery Calculations

To determine the magnitude of the difference in depletion-drive recoveries as predicted utilizing the various methods of PVT data application discussed above, a hypothetical, 1,000,000 barrel volumetric reservoir containing a 40% connate water saturation, and possessing the relative permeability-liquid saturation properties shown graphically in Figure 1 was considered. An abandonment pressure of 100 psig was assumed in all calculations.

Since the methods of data application proposed by Tarner and Patton appeared likely to result in the most widely differing recovery predictions, they were utilized, in conjunction with the Tarner method of calculation, to forecast the performance of the above reservoir initially containing a reservoir fluid, which will be designated as Crude A, at its bubble-point conditions. The results of laboratory

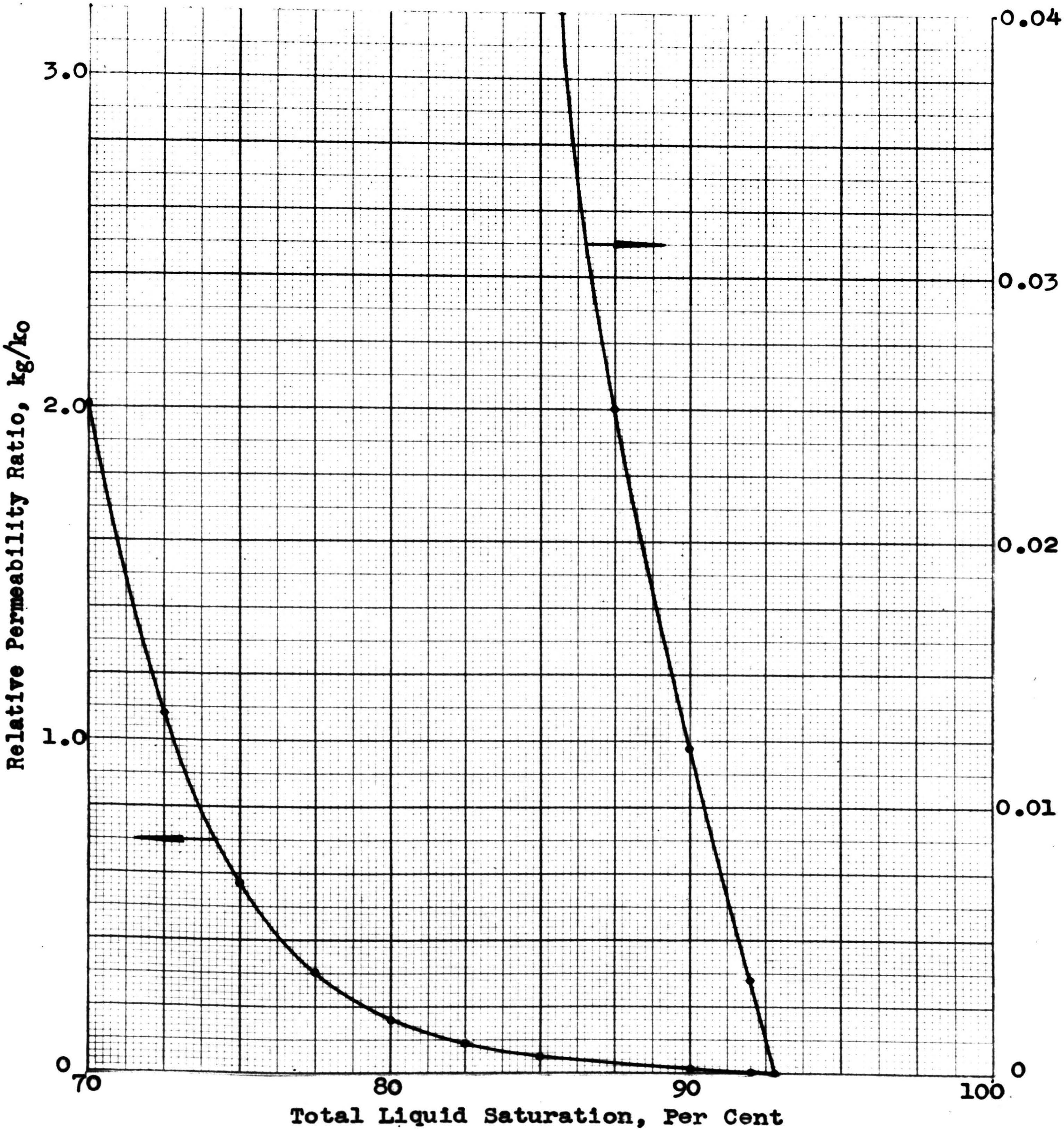

FIGURE 1

RELATIVE PERMEABILITY RATIO VS.
TOTAL LIQUID SATURATION

PVT tests originally performed on a subsurface sample of this fluid are presented on page 39, and shown graphically in Figures 2, 3, 4 and 5. Since there was so little difference in the Crude A recoveries as predicted utilizing the Tarner and Patton methods of data application, the Muskat method was not applied to these data.

All three methods of data application were used to predict the performance of the same reservoir initially containing a second reservoir fluid, Crude B. The physical properties of this fluid as determined by laboratory tests performed on a subsurface sample are tabulated on page 40, and shown graphically in Figures 2, 3, 4 and 5. Though reservoir temperatures were approximately the same for both fluids, the bubble-point formation-volume-factor and solution gas content, and the tank oil gravity (API) of Crude B were considerably higher than were the corresponding properties of Crude A.

The results of all reservoir performance calculations are summarized in Table I, page 24, and shown graphically in Figures 6 and 7. Calculated data used to construct the typical performance curves of Figures 6 and 7 are tabulated on pages 41 and 42.

VI. Discussion of Results

For all practical purposes, predicted depletion-drive reservoir performance using Crude A PVT data was the same by both the Tarner and Patton methods of data application (see

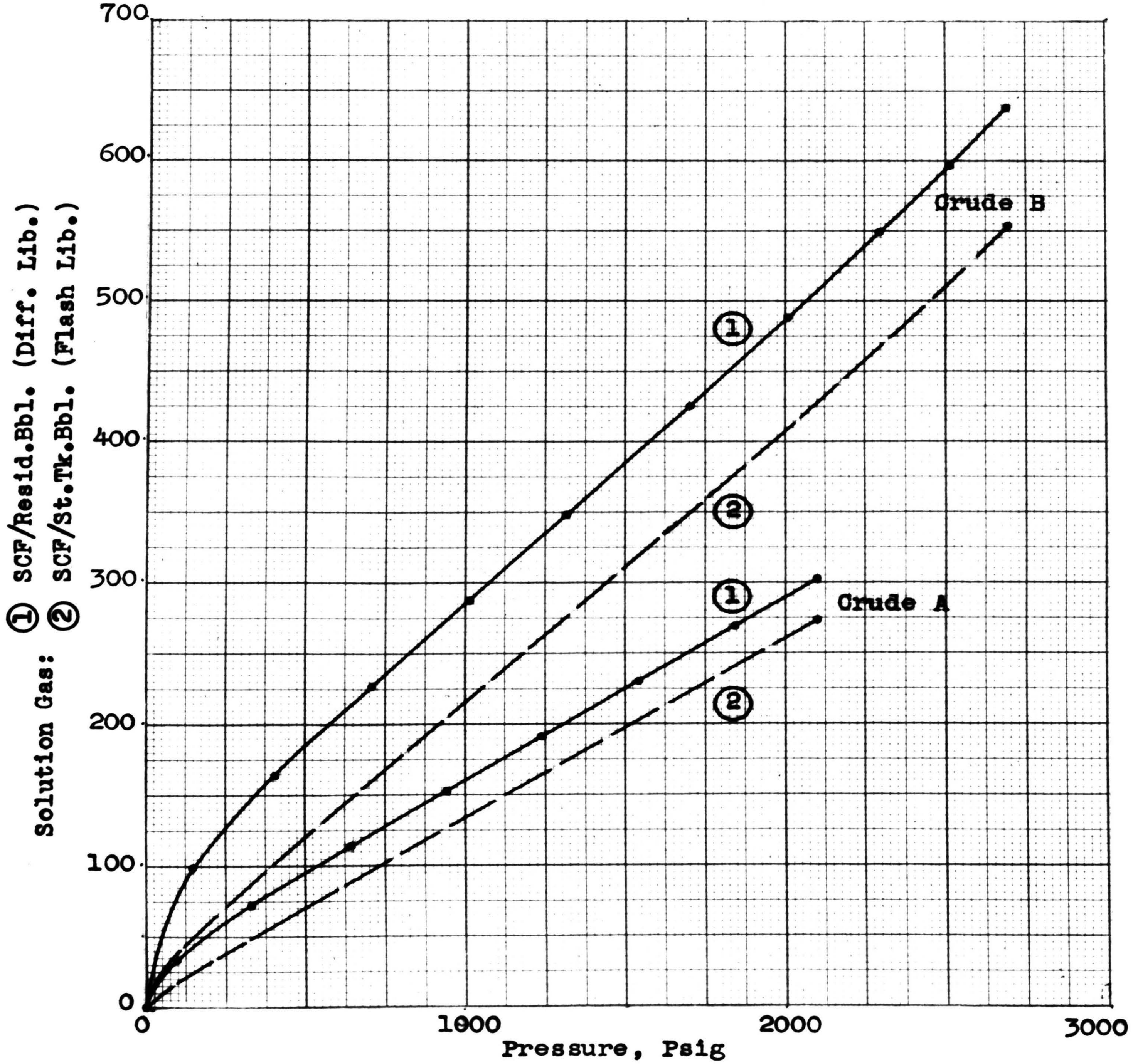

FIGURE 2

GAS SOLUBILITY VS. PRESSURE,
CRUDES A AND B

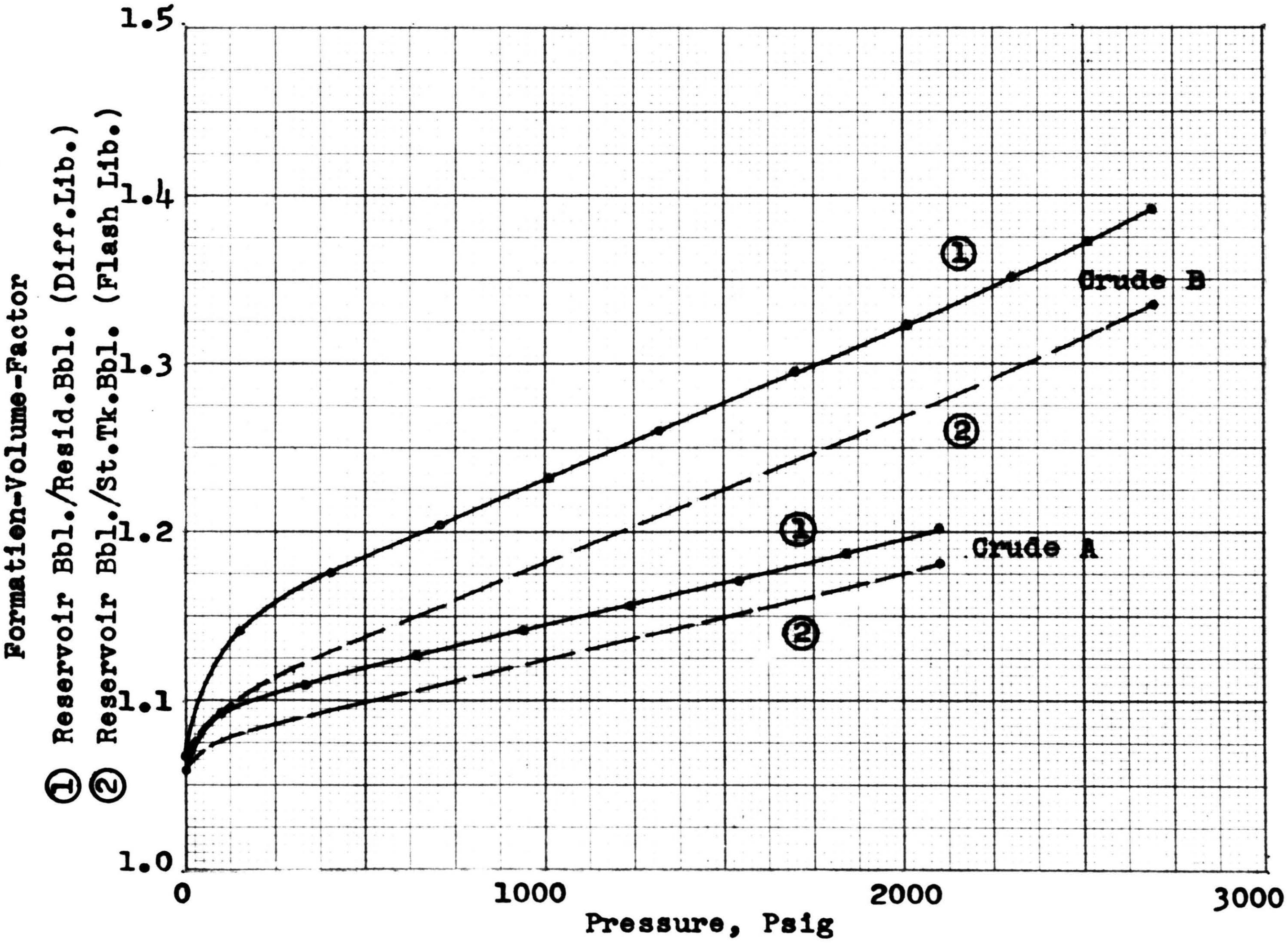

FIGURE 3

FORMATION VOLUME FACTOR VS.
PRESSURE, CRUDES A AND B

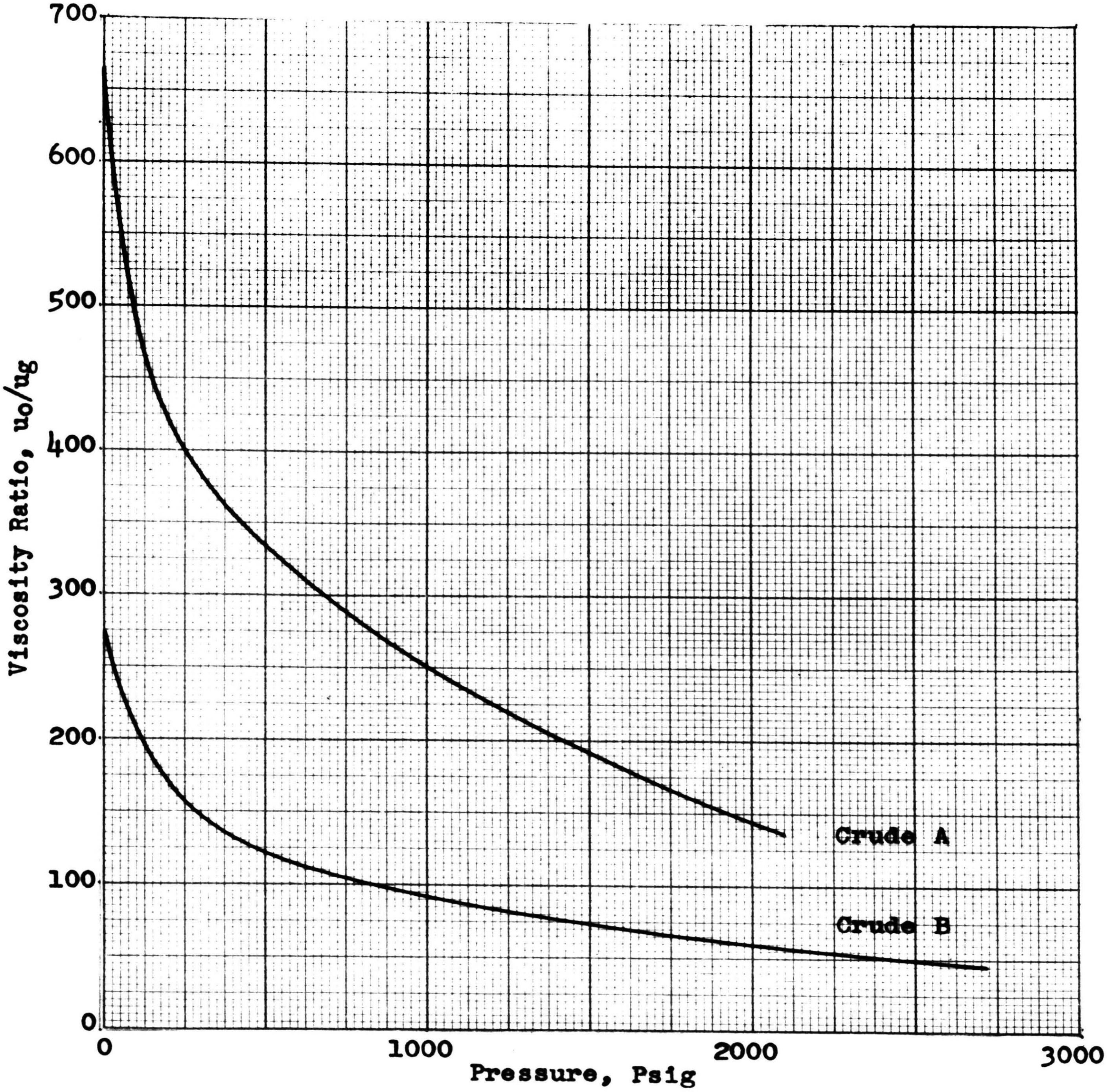

FIGURE 4

VISCOSITY RATIO VS. PRESSURE,
CRUDES A AND B

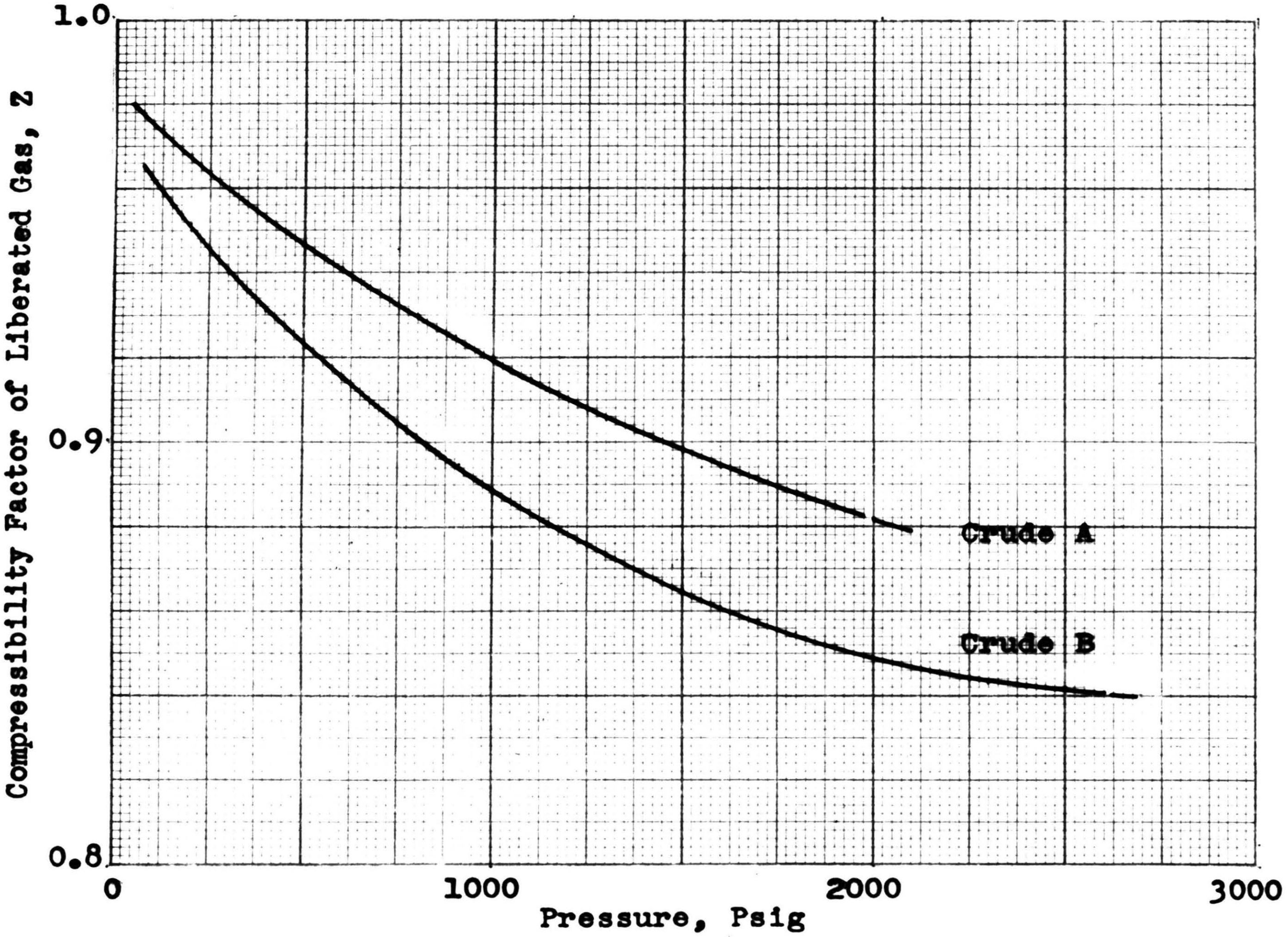

FIGURE 5

GAS COMPRESSIBILITY FACTOR VS.
PRESSURE, CRUDES A AND B

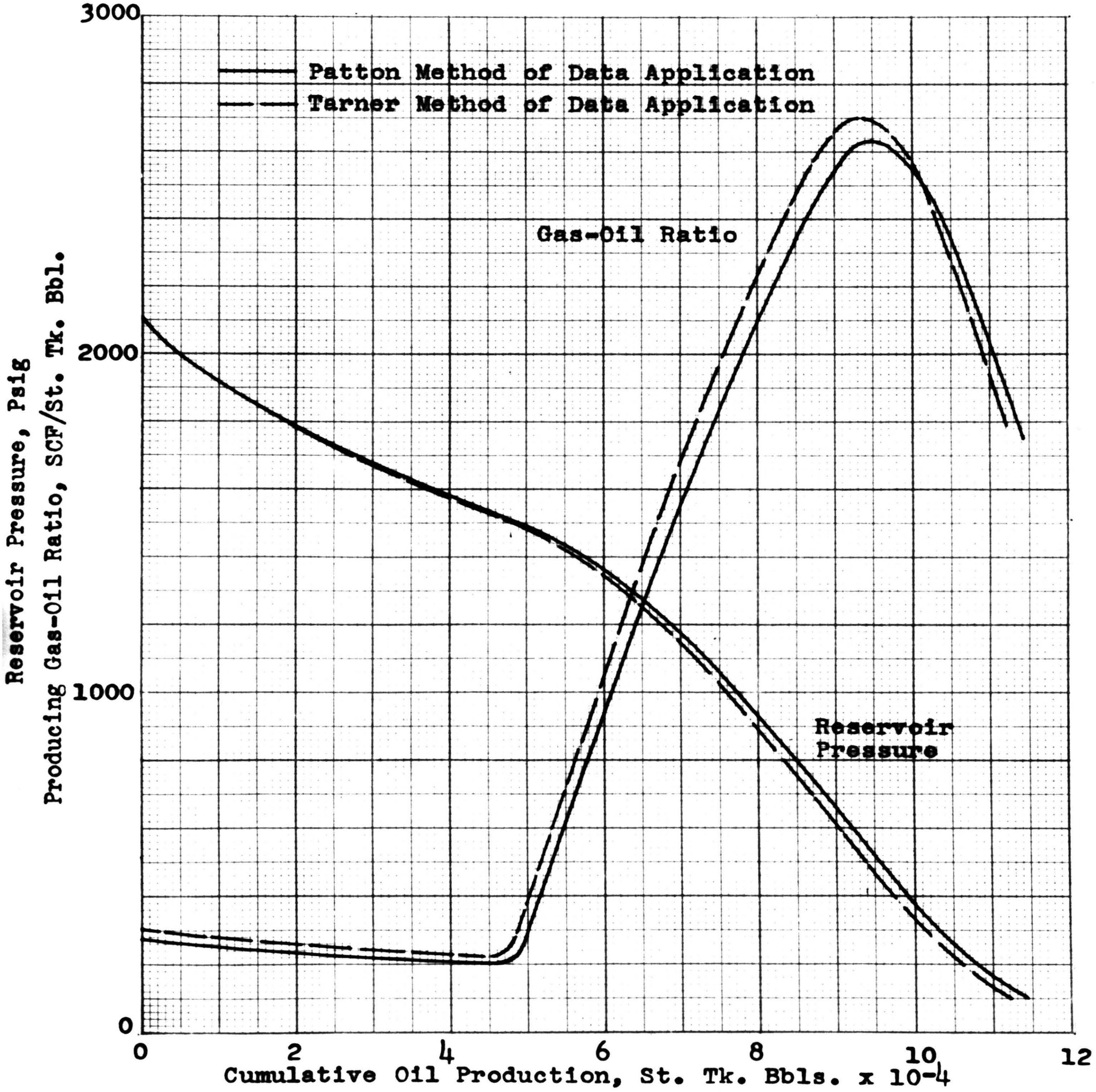

FIGURE 6

PREDICTED RESERVOIR PERFORMANCE,
CRUDE A PVT DATA

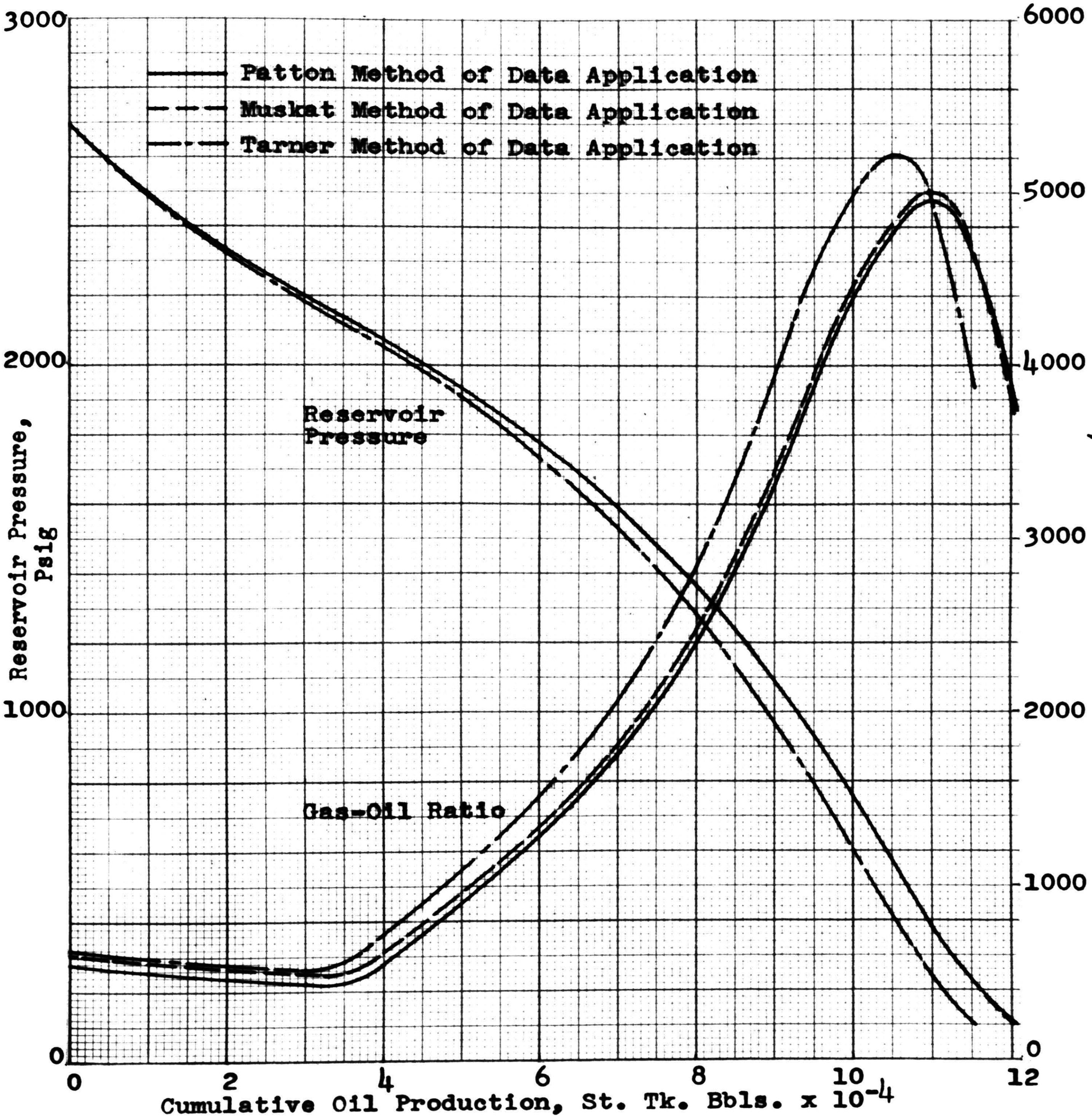

FIGURE 7

PREDICTED RESERVOIR PERFORMANCE,
CRUDE B PVT DATA

Table I). Oil recovery values in terms of per cent of initial oil in place were essentially identical. Total gas production values differed by approximately 1 per cent, total tank oil recovery by approximately 2 per cent. Since the differences of approximately 2 per cent in formation-volume-factor and 10 per cent in solubility as determined by flash and differential liberation for Crude A are felt to approach the maximum variation which will be encountered for a 22° API crude, the above agreement in predicted recoveries indicates that for low gravity crude oils the method of data application employed in depletion-drive performance calculations is relatively unimportant.

Since the gravity (31.9° API) and solution gas content of Crude B was considerably higher than that of Crude A while separator conditions and reservoir temperature were essentially the same, greater differences were found to exist in the bubble-point formation-volume-factor and solubility values as determined for Crude B by laboratory flash and differential liberation. Slightly greater differences were also found in depletion-drive recoveries calculated using the various methods of applying these data (Table I). Predicted oil recovery in terms of per cent of initial oil in place was the same using the Tarner and Muskat data application methods, and only slightly greater (0.1 per cent of oil in place) by the Patton method. Total gas production was also the same by the Tarner and Muskat methods, but approximately 2 per cent less by the

TABLE I

RESULTS OF DEPLETION DRIVE RESERVOIR PERFORMANCE CALCULATIONS

Method of Data Application	Reservoir Pore Vol. Bbl.	Initial Oil in Place St.Tk.Bbls.	**Oil Recovery St. Tk. Bbls.	**Oil Recovery %Oil in Place	**Total Gas Produced MMSCF
Crude A					
*Tarner	1,000,000	499,200	112,000	22.44	131.4
Patton	1,000,000	508,000	114,300	22.50	129.7
Crude B					
*Tarner	1,000,000	431,300	115,400	26.76	241.6
Muskat	1,000,000	449,400	120,300	26.76	241.6
Patton	1,000,000	449,400	120,700	26.86	237.2

*Oil in place and recovery values in "residual bbls".
**Produced to an assumed abandonment pressure of 100 psig.

Patton method. Total tank oil recovery as calculated by the Tarner method (differential liberation data only) was approximately 4 per cent less than by the Muskat or Patton methods (flash and differential data). The above differences resulted from a variation of approximately 4 per cent in formation-volume-factor and 15 per cent in solubility as determined by the two liberation processes.

Higher API gravity, more volatile crude oil systems would no doubt exhibit greater differences in flash and differential solubility and shrinkage values for the same separator conditions and reservoir temperature. However, it is felt that the magnitude of these differences for Crude B as stated above are in the range of the maximum deviations which will be encountered for normal crude oils (highly volatile systems have been excepted throughout this work) being produced through the usual single stage, low pressure separator facilities. Regardless, general conclusions to be drawn from an analysis of the calculations performed using the PVT data for Crude B will also apply to crude oil systems exhibiting larger differences in the magnitude of these properties.

Assuming flash liberation to prevail as reservoir fluid moves through the tubing, flow lines, and separator to the stock tank, it would appear that volumetric calculation of initial oil in place should be made utilizing the flash formation-volume-factor of the bubble-point oil. This procedure is followed in the Muskat method of data application, and was also utilized in the Patton method as applied in this work. Recoveries predicted using these methods are therefore felt to be more nearly correct than those calculated by the Tarner method in which oil in place is calculated using the differential liberation formation-volume-factor. All three methods of application result in essentially the same predicted <u>per</u> <u>cent</u> recovery of oil in place.

Though predicted total gas production will be the same using either the Tarner or Muskat method, gas-oil ratios at a given pressure by the Muskat method will differ from those by the Tarner method in the ratio of the flash to the differential formation-volume-factor. Total gas production and individual gas-oil ratios calculated by the Patton method will differ slightly from those calculated by the Muskat method depending upon the magnitude of the difference between flash and differential solubility values.

In view of usual uncertainties as to the complete validity of reservoir rock and fluid samples, and of the many limiting assumptions made in the derivation and application of the equations involved in calculating reservoir performance, differences in predicted depletion-drive recoveries as calculated using the Muskat or Patton method of PVT data application appear unimportant from a practical standpoint. The data application procedure utilized by Muskat does seem somewhat simpler to employ.

VII. Conclusions

It is generally assumed that the evolution of gas from oil within a reservoir is most closely approximated by the differential liberation process, while gas liberation in the flow string, separator and stock tank is most nearly represented by flash liberation. To conform with this physical behavior, appropriate application of both flash and differential PVT data in computing reservoir performance by material balance methods is indicated.

The results of simple depletion-drive reservoir performance calculations carried out using differential liberation data only were compared with similar values obtained by two methods of applying both flash and differential liberation data. This comparison resulted in the following observations:

1. Predicted oil recovery in terms of per cent of oil initially in place is essentially the same with or without the application of flash data.
2. If initial oil in place is calculated volumetrically (i.e., using reservoir volume and initial reservoir oil shrinkage), the volumes of recoverable oil predicted using differential data only and using both flash and differential data will differ in the ratio of the flash to the differential bubble-point formation-volume-factor.

3. Total gas production predicted using differential data only will compare closely (within 2 per cent) to that predicted utilizing both flash and differential data even if considerable difference (15%) exists between flash and differential bubble-point solubility values.

While differences in depletion-drive recoveries as predicted with and without the application of flash liberation may sometimes be unimportant from a practical standpoint, they can be sizeable where considerable differences exist between flash and differential formation-volume-factor values. Regardless, to conform with the assumed production-liberation sequence described above, both differential and flash liberation data should be incorporated in any reservoir performance calculation . The use of differential data which has been converted from a residual to a stock tank oil basis through multiplication by the ratio of flash to differential formation-volume-factor values throughout the material-balance equations appears to be a simple, appropriate method of data application.

VIII. APPENDIX A: DERIVATIONS AND NOMENCLATURE

Derivation of Equations

The derivations which follow are essentially the same as those given by Tarner (5). No distinction is made as to liberation processes prevailing during various phases of the production sequence or utilized in the laboratory determination of reservoir fluid physical properties. A list of nomenclature and specific units employed throughout this work is given in Table II, page 38.

The Volumetric Material Balance Equation

In a petroleum reservoir from which oil has been produced, the gas evolved from solution from oil remaining in the reservoir must equal the volume of excess gas produced plus the volume of gas occupying the net vacated space within the reservoir. Excess gas is defined here as that volume of gas produced with the oil but not originally in solution in the produced oil.

If R_{si} is the initial gas in solution, then $(N_p)(R_{si})$ is the amount of produced gas evolved from produced oil volume N_p. If G_p is the corresponding total gas production, then $[G_p - (N_p)(R_{si})]$ will be the excess gas produced in standard volume units.

Also, if B_{oi} is the initial reservoir oil formation-volume-factor, then $(N_p)(B_{oi})$ will be the reservoir space vacated by the produced oil, and $\frac{(N_p)(B_{oi})}{B_g}$ will be the standard volume of gas occupying this space at reservoir pressure P corresponding to oil production N_p.

If N is the total tank oil volume initially in place, then $(N-N_p)(B_o)$ will be the reservoir volume of the remaining oil, and $(N-N_p)(B_{oi}-B_o)$ will be the reservoir space vacated due to the shrinkage of the remaining oil. The standard volume of gas occupying this space at P will be $\frac{(N-Np)(Boi-Bo)}{Bg}$.

Since $(N-N_p)(R_{si}-R_s)$ will be the standard volume of gas evolved from the oil remaining in the reservoir during the production of oil volume N_p, then by the original hypothesis,

$$(N-N_p)(R_{si}-R_s) = G_p - (N_p)(R_{si}) + \frac{(N_p)(B_{oi})}{B_g} + \frac{(N-N_p)(B_{oi}-B_o)}{B_g}$$

Or, clearing and collecting terms,

$$G_p = (N)\left[(R_{si}-R_s) - \frac{(B_{oi}-B_o)}{B_g}\right] - (N_p)\left(\frac{B_o}{B_g} - R_s\right) \ldots\ldots\ \text{Eq. (1.)}$$

The above form of the material balance equation is applicable only in the case of primary production from a simple depletion-drive (volumetric) reservoir. It presumes the absence of an initial gas cap, no water encroachment or water production, and no fluid injection program. Other limiting assumptions made in its derivation are as follows:

1. The hydrocarbon containing reservoir may be represented as a constant volume container closed on all sides. Though there is a pressure decline, there is no connate water expansion, no rock expansion, and no geostatic compression.
2. Pressure equilibrium prevails throughout the reservoir at all times during the production

history. Laboratory determined fluid properties therefore apply throughout the reservoir at a given stage of depletion.

3. Reservoir fluid properties as determined in the laboratory are representative of actual fluid behavior.
4. No gas segregation takes place under the influence of gravity.
5. Fluid withdrawals are uniformly distributed throughout the reservoir.

The Fluid Saturation Equation

At any time the total liquid saturation, S_L, in a reservoir oil zone is equal to the sum of the volumes of the gas saturated oil plus the water, divided by the reservoir oil zone pore volume.

The reservoir volume initially occupied by saturated oil is given by $(N)(B_{oi})$. Assuming that the water saturation, S_w, does not change during the producing life of the reservoir, the total pore volume of the oil zone is given by $\frac{(N)(B_{oi})}{(1-S_w)}$.

The volume occupied by saturated oil remaining in the reservoir after the production of oil volume N_p is given by $(N-N_p)(B_o)$. Again assuming a constant water saturation, the total liquid saturation is given by

$$S_L = \frac{(N-N_p)(B_o)}{(N)(B_{oi})/(1-S_w)} + S_w$$

Or, rearranging,

$$S_L = (1-\frac{N_p}{N})(\frac{B_o}{B_{oi}})(1-S_w) + S_w \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. (2.)}$$

The Instantaneous Producing Gas-Oil Ratio Equation

Derivation of the instantaneous producing gas-oil ratio equation is based upon Darcy's Law which states that

$$v = \frac{k}{\mu} \cdot \frac{dP}{dr}$$

where: v = fluid flow velocity in the r direction.

μ = viscosity of the flowing fluid.

k = permeability constant depending on the texture of the porous medium.

$\frac{dP}{dr}$ = pressure gradient in the r direction.

An expression for the volumetric rate of fluid flow (q) is obtained by multiplying the flow velocity by the cross sectional area (A) through which flow occurs, or

$$q = vA = \frac{kA}{\mu} \cdot \frac{dP}{dr}$$

For radial flow, the area A is equal to $2\pi rh$, where r and h represent radius and thickness respectively. Substituting this value in the above expression and integrating between appropriate limits we have

$$q\int_{r_w}^{r_e}\frac{dr}{r} = \frac{2\pi hk}{\mu}\int_{P_e}^{P_w} dP,$$

or

$$q = \frac{2\pi hk(P_e-P_w)}{\mu \ln(r_e/r_w)}$$

where: r_e = effective radius of drainage.

r_w = radius of the well bore.

P_e = pressure at r_e.

P_w = pressure at r_w.

This is the equation for the isothermal, horizontal, steady state, radial flow of a homogenous, incompressible fluid through a porous medium.

Assuming reservoir oil to be an incompressible fluid, the above equation can be made the expression for the steady state, volume rate of oil efflux from the sand face in the well bore. Since petroleum reservoirs contain oil, water, and gas, it is necessary to substitute the effective permeability to oil, k_o, for absolute permeability constant, k. (Absolute permeability denotes the capacity of a porous medium to transmit any given fluid when 100 per cent saturated with that fluid. The effective permeability of a porous medium to a partially saturating fluid is dependent upon saturation state, and is always less than absolute permeability.) Also, to convert to volume rate of tank oil flowing, the reservoir oil formation-volume-factor must be employed so that

$$q_o = \frac{2\pi h k_o (P_e - P_w)}{\mu_o B_o \ln(r_e/r_w)}$$

where q_o is the volume rate of stock tank oil flowing.

To obtain a similar expression for the standard volume rate of gas flow q_g, corrections for temperature and pressure must be applied to the incompressible fluid flow equation since gas is a compressible fluid. Letting q_{gm} represent the volume rate of gas flowing at P_m, the average between reservoir and sand face pressures, and applying the equation of state for real gases we have,

$$q_{gm} \cdot \frac{P_e + P_w}{2 Z_m T_f} = q_g \cdot \frac{14.7}{520}$$

Solving this relationship for q_{gm} and substituting in the original radial flow equation,

$$q_g = \frac{\pi h k_g (P_e - P_w)(P_e + P_w)}{\mu_g \ln(r_e/r_w)} \cdot \frac{520}{(14.7) Z_m T_f}$$

This equation is an expression for the steady state, standard volume rate of flow of a compressible fluid through a porous medium.

By dividing the above equation for q_g by that for q_o, an expression for the gas-oil ratio (expressed in standard volumes per standard volume) in the porous medium is obtained.

$$\frac{q_g}{q_o} = \frac{k_g}{k_o} \cdot \frac{\mu_o}{\mu_g} \cdot \frac{P_e + P_w}{(2)(14.7)} \cdot \frac{B_o}{Z_m} \cdot \frac{520}{T_f}$$

If reservoir and well bore pressure may be assumed equal, the above equation becomes

$$\frac{q_g}{q_o} = \frac{k_g}{k_o} \cdot \frac{\mu_o}{\mu_g} \cdot \frac{P}{14.7} \cdot \frac{520}{T_f} \cdot \frac{B_o}{Z}$$

The assumption that there is no pressure differential imposed on the reservoir system due to fluid withdrawal conforms with the assumptions of uniform pressure and saturation distribution made in the derivation of the volumetric material-balance equation. Since theoretical depletion-drive recovery calculations performed by Loper and Calhoun (13) indicate that any variation in assumed pressure drawdown do not effect predicted reservoir behavior, the above simplification appears justified.

Converting this simplified reservoir gas-oil ratio expression to units of standard cubic feet per stock tank barrel, and adding the solution gas that is evolved from the oil upon being brought to atmospheric conditions, we have the instantaneous producing gas-oil ratio equation,

$$R = \frac{k_g}{k_o} \cdot \frac{\mu_o}{\mu_g} \cdot \frac{P}{14.7} \cdot \frac{520}{T_f} \cdot \frac{5.615}{Z} \cdot B_o + R_s,$$

or

$$R = \frac{k_g}{k_o} \cdot \frac{\mu_o}{\mu_g} \cdot \frac{B_o}{B_g} + R_s \text{ .... Eq. (2.)}$$

<u>The Cumulative Gas Production Equations</u>

If a cumulative oil production (N_p) is assumed to occur in j increments of oil production (ΔN_p), then the corresponding total gas production (G_p) is given by

$$G_p = \sum_{0}^{j} \Delta G_p$$

Providing that it may be assumed that the variation of producing gas-oil ratio (R) is linear, or nearly so, in any given production interval, the corresponding incremental gas production (ΔG_p) is given by

$$\Delta G_p = (\Delta N_p)(\bar{R}) = \left[(N_p)_2 - (N_p)_1\right] \left(\frac{R_1 + R_2}{2}\right)$$

The foregoing assumption of linearity is valid for all practical purposes providing the increments of oil production are sufficiently small (less than 2 or 3 per cent of the oil initially in place).

The above expressions appear as equations 4 in the text.

TABLE II

NOMENCLATURE

Symbol	Quantity	Units
<u>Symbols</u>		
B_g	Gas formation-volume-factor $(B_g = \frac{14.7}{P} \cdot \frac{T_f}{520} \cdot \frac{Z}{5.615})$	Reservoir bbl./SCF
B_o	Oil formation-volume-factor	Differential liberation: Reservoir bbl./residual bbl. Flash liberation: Reservoir bbl./stock tank bbl.
G_p	Cumulative gas produced	Standard cubic feet
ΔG_p	Gas produced during an interval	Standard cubic feet
k_g	Effective permeability to gas	Darcies
k_o	Effective permeability to oil	Darcies
N	Initial oil in place in reservoir	Barrels of stock tank oil
N_p	Cumulative oil produced	Barrels of stock tank oil
ΔN_p	Oil produced during an interval	Barrels of stock tank oil
P	Reservoir pressure	Psi absolute
R	Producing gas-oil ratio	SCF/stock tank barrel
R_s	Solution gas-oil ratio (gas solubility in oil)	Differential liberation: SCF/residual barrel Flash liberation: SCF/stock tank barrel
S_o	Oil saturation, fraction	Dimensionless
S_w	Water saturation, fraction	Dimensionless
T_f	Reservoir temperature	Degrees Rankine
μ_g	Gas viscosity	Centipoise
μ_o	Oil viscosity	Centipoise
Z	Gas compressibility factor or deviation factor	Dimensionless
<u>Subscripts</u>		
b	Bubble-point or saturation	
f	Formation	
g	Gas	
i	Initial value or condition	
L	Liquid	
o	Oil	
p	Cumulative produced	
s	Solution gas	
w	Water	
<u>Superscripts</u>		
d	Differential liberation	
f	Flash liberation	
-	Average	

IX. APPENDIX B: TABULAR DATA

TABLE III

RESERVOIR FLUID PROPERTIES, CRUDE A

Differential liberation @ 210°F

Pressure Psig.	Sol. GOR SCF per Resid. Bbl.	FVF, Reservoir Bbl. per Resid. Bbl.	Gas. Gravity (Air=1.0)	Viscosity, Cps.		**Gas Comp. Factor
				Oil	*Gas	
2101(P_b)	302	1.202		2.22		
1845	269	1.186	0.6685		0.0161	0.882
1730				2.61		
1540	230	1.171	.6679		.0154	.891
1430				2.94		
1240	191	1.156	.6703		.0147	.902
1115				3.35		
945	152	1.141	.6812		.0141	.915
790				3.78		
645	114	1.126	.7043		.0134	.930
533				4.23		
330	72	1.109	.7500		.0129	.948
230				4.85		
112				5.38		
95	34	1.092	.9038		.0119	.966
0	0	1.059	1.332	6.97	0.0105	0.980
0(60°F)		1.000				

Gravity of residual oil @ 60°F: 21.1° API

*Obtained using correlation by Carr et al (12).
**Obtained using correlations by Mathews et al (10), Standing and Katz (11).

Flash Separation Test, Bubble Point Oil

Separator Pressure, Psig.	Separator Temp., °F	Sep. GOR, SCF per St.Tk.Bbl.	St.Tk.GOR SCF per St.Tk.Bbl.	FVF, Reservoir Bbl. per St.Tk.Bbl.	St.Tk.Oil Gravity °API @ 60°F
100	100	249	25	1.181	22.0

TABLE IV

RESERVOIR FLUID PROPERTIES, CRUDE B

Differential Liberation @ 220°F

Pressure Psig.	Sol. GOR, SCF per Resid. Bbl.	FVF, Reservoir Bbl. per Resid. Bbl.	Gas Gravity (Air=1.0)	Viscosity, Cps.		**Gas Comp. Factor
				Oil	*Gas	
2695 (P_b)	638	1.391		0.880		
2560				.890		
2512	596	1.373	0.7595		0.0186	0.841
2300	549	1.351	.7591		.0179	.842
2008	488	1.323	.7596		.0171	.845
1960				.997		
1702	425	1.295	.7612		.0162	.854
1470				1.12		
1315	348	1.260	.7691		.0153	.865
1010	287	1.232	.7846		.0144	.880
940				1.30		
705	226	1.205	.8201		.0136	.898
450				1.57		
405	164	1.175	.8879		.0127	.918
150	99	1.141	1.091		.0116	.943
100				2.09		
0	0	1.066	1.445	2.87	0.0105	0.965
0 (60°F)		1.000				

Gravity of residual oil @ 60°F: 28.8° API

*Obtained using correlation by Carr et al (12).

**Obtained using correlations by Mathews et al (10), Standing and Katz (11).

Flash Separation Test, Bubble Point Oil

Separator Pressure, Psig.	Separator Temp., °F	Sep. GOR, SCF per St.Tk.Bbl.	St.Tk.GOR, SCF per St.Tk.Bbl.	FVF, Reservoir Bbl. per St.Tk.Bbl.	St.Tk.Oil Gravity °API @ 60°F
100	76	505	49	1.335	31.9

TABLE V

RESULTS OF RECOVERY CALCULATIONS, CRUDE A PVT DATA

Pressure, Psig.	Oil Recovery, St. Tk. Bbls.		Producing GOR, SCF/St.Tk.Bbl.	
	Method of Data Application		Method of Data Application	
	*Tarner	Patton	*Tarner	Patton
2101	0	0	302	274
2000	4,620	4,710	289	261
1900	10,950	11,140	276	248
1800	18,680	18,990	263	235
1700	26,780	27,240	251	223
1600	36,860	37,530	238	211
1500	48,360	49,240	281	253
1400	56,650	57,640	834	796
1300	62,500	63,600	1,220	1,176
1200	67,400	68,640	1,525	1,476
1100	71,800	73,100	1,781	1,727
1000	75,900	77,220	2,013	1,955
900	79,710	81,130	2,214	2,152
800	83,350	84,840	2,397	2,333
700	86,950	88,410	2,557	2,489
600	90,390	91,940	2,674	2,602
500	93,810	95,450	2,703	2,631
400	97,380	99,060	2,646	2,575
300	101,300	103,000	2,513	2,447
200	106,000	107,900	2,217	2,163
**100	112,000	114,300	1,791	1,754

Initial oil in place: Tarner method, N = 499,200 St. Tk. Bbls.
Patton method, N = 508,000 St. Tk. Bbls.

Total gas produced to 100 psig.: Tarner method, G_p = 131.4 MMSCF
Patton method, G_p - 129.7 MMSCF

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*Oil recovery and GOR in terms of residual bbls., and SCF/residual bbl. respectively.

**Assumed abandonment pressure.

TABLE VI

RESULTS OF RECOVERY CALCULATIONS, CRUDE B PVT DATA

Pressure, Psig.	Oil Recovery, St. Tk. Bbls. Method of Data Application			Producing GOR, SCF/St.Tk.Bbl. Method of Data Application		
	*Tarner	Muskat	Patton	*Tarner	Muskat	Patton
2695	0	0	0	638	612	554
2600	4,740	4,940	4,950	615	590	532
2500	9,450	9,850	9,850	593	569	511
2400	15,120	15,760	15,760	571	548	490
2300	21,500	22,400	22,400	549	527	469
2200	28,820	29,920	30,050	527	507	448
2100	36,730	38,250	38,300	609	565	497
2000	43,760	45,610	45,610	863	803	755
1900	50,340	52,440	52,440	1,116	1,041	998
1800	56,310	58,690	58,690	1,342	1,278	1,220
1700	61,620	64,250	64,250	1,572	1,519	1,451
1600	66,560	69,370	69,430	1,850	1,775	1,717
1500	70,990	73,990	73,990	2,144	2,058	2,000
1400	75,230	78,430	78,460	2,457	2,358	2,300
1300	79,160	82,510	82,510	2,762	2,659	2,601
1200	82,720	86,220	86,220	3,103	3,026	2,968
1100	86,060	89,690	89,720	3,496	3,355	3,297
1000	89,190	92,960	92,980	3,864	3,715	3,668
900	92,120	96,000	95,980	4,246	4,075	4,019
800	94,890	98,890	98,880	4,532	4,349	4,292
700	97,550	101,700	101,700	4,778	4,583	4,526
600	100,100	104,400	104,400	4,998	4,775	4,718
500	102,700	107,000	107,000	5,139	4,932	4,874
400	105,300	109,800	109,800	5,213	5,004	4,950
300	108,100	112,700	112,800	5,151	4,943	4,890
200	111,300	116,000	116,200	4,723	4,533	4,535
**100	115,400	120,300	120,700	3,876	3,720	3,738

Initial oil in place:
Tarner method, N = 431,300 St. Tk. Bbls.
Muskat method, N = 449,400 St. Tk. Bbls.
Patton method, N = 449,400 St. Tk. Bbls.

Total gas produced to 100 psig.:
Tarner method, G_p = 241.6 MMSCF
Muskat method, G_p = 241.6 MMSCF
Patton method, G_p = 237.2 MMSCF

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*Oil recovery and GOR values in terms of residual bbls. and SCF/residual bbl. respectively.

**Assumed abandonment pressure.

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XI. Vita

Frederick A. Klein, son of Adolph H. and Jessie Klein, was born July 3, 1927, in Richmond Heights, Missouri. He received his elementary and secondary education in St. Louis, Missouri.

He received his discharge from the U. S. Army in 1946, and enrolled at St. Louis University in 1947. After completing one year of undergraduate study there, he enrolled at the University of Missouri, School of Mines, from where he was graduated in January, 1952, with the degree of Bachelor of Science in Mining Engineering - Petroleum Engineering Option.

Thereafter he was employed as a petroleum engineer for three years by Phillips Petroleum Company, and for three years by Core Laboratories de Venezuela. He entered the Graduate School of the School of Mines in September, 1958.

He is a member of Tau Beta Pi and Phi Kappa Phi honor societies, and of the Society of Petroleum Engineers of the A. I. M. E..

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