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1 Abstract

Regulating machining forces is a common process control technique used to increase productivity and quality. Model parameters vary significantly during a normal operation, thus, adaptive techniques have predominately been used. However, model-based techniques that carefully account for changes in the force process have again been examined due to the reduced complexity afforded by such techniques. In this paper, the effect of model parameter variations on the closed-loop stability for two model-based force controllers is examined.

It was found that the stability boundary in the process parameter space can be exactly determined for force control systems designed for static force processes. For force control systems designed for first-order force processes, it was found that the stability boundary is sensitive to the estimate of the discrete-time pole. The analysis was verified via simulations and experimental studies.

2 Introduction

Force control technology can significantly impact the economics of machining operations by increasing productivity and part quality. These controllers are challenging to develop as there are many nonlinearities which the designer must consider: a nonlinear relationship between the output (force) and the input (feed), input saturation, and a saturation in the force process due to the tool leaving the workpiece. Masory and Koren (1980) noted that fixed-gain controllers can become unstable given typical changes in cutting conditions (e.g., feed, depth-of-cut). Normal machining phenomena also effect the force process (e.g., tool wear typically increases the process gain) and will deteriorate the controller performance, sometimes to the point of instability. Thus, adaptive controllers have traditionally been employed to regulate machining forces (e.g., Ulsoy *et al.*, 1983; Fussell and Srinivasan, 1988). Adaptive approaches adjust controller gains as the process changes; however, these techniques suffer from the fact that they are complex and, thus, are difficult to analyze, tune, and maintain. With the recent advances in the modeling of machining processes, model-based methodologies have again been explored (Harder, 1995; Landers and Ulsoy, 1996). Model-based controllers are simpler than adaptive techniques and directly incorporate process nonlinearities; however, their performance is limited due to model parameter variations.

In this paper, the effect of model parameter variations on the stability of the closed-loop system for the model-based machining force controller presented in Landers and Ulsoy (1996) is analyzed. This control approach explicitly accounts for the force-feed process nonlinearity in such a way that traditional linear control design techniques may be utilized, thus, providing for a simple design which is easy to analyze, tune, and maintain. The first property (i.e., ease of analysis) is utilized to explore the effect of model parameter variations on the closed-loop system stability.

3 Machining Force Control: Static Process

3.1 Process Model

The structure for the static force processes is

$$F = Kd^{\beta}V^{\gamma}f^{\alpha} \tag{1}$$

where F is the machining force, K is the process gain, d is the depth-of-cut, V is the cutting speed, f is the feed, and α, β, and γ are coefficients describing the nonlinear relationships between the machining force and the process inputs (i.e., f, d, and V). The four variables which must be calibrated for each tool-workpiece combination are K, α, β, and γ. Typically, the feed is adjusted on-line to regulate the machining force and, therefore, the force process gain may be seen as $\theta = Kd^{\beta}V^{\gamma}$ which is sensitive to the cutting conditions; namely, the depth-of-cut and cutting speed. Static models are used when considering a force *per spindle revolution* such as a maximum or average force.

3.2 Controller Design

A change of variable is made and the control variable is defined as

$$u_c \equiv f_t^{\overline{\alpha}} \tag{2}$$

where $\overline{\alpha}$ is an off-line estimate of α. The static force process model used for controller design is

$$F(z) = \overline{\theta}u_c(z) \tag{3}$$

where z is the discrete-time forward operator and $\overline{\theta} = \overline{K}d^{\overline{\beta}}V^{\overline{\gamma}}$ is an estimate of the force process gain. Therefore, the force process gain estimate is a function of off-line estimates of model parameters (i.e., $\overline{K}$, $\overline{\beta}$, and $\overline{\gamma}$) and cutting conditions (i.e., d and V). Now a variety of simple, linear control techniques may be utilized for controller design. The change of variable made in equation (2) allows the designer to directly incorporate the force-feed nonlinearity. Further, the force process gain estimate may be changed on-line as the cutting conditions change (e.g., a new spindle speed reference may be selected to suppress chatter). This allows the force controller to adjust for changing cutting conditions and directly incorporate these process nonlinearities.

An integral state is added to the plant and the Model Reference Control (MRC) technique is used to design the following *implemented* control law

$$u_c(z) = \frac{1}{z-1}\frac{1+b_0}{\bar{\theta}}\left[F_r(z) - F(z)\right] \tag{4}$$

where the discrete-time polynomial $z + b_0 = 0$ defines the desired closed-loop dynamical characteristics. The commanded feed/tooth (f_t) at the kth iteration is found through the inverse of equation (2) and is

$$f_t(k) = \exp\left\{\frac{\ln\left[u_c(k)\right]}{\bar{\alpha}}\right\} \tag{5}$$

The control variable (u_c) must be positive to perform the transformation in equation (5); therefore, the control variable is bounded from below by

$$u_{\min}(k) = f_{\min}^{\bar{\alpha}}(k) > 0 \tag{6}$$

where f_{min} is selected to be an arbitrarily small, positive number to ensure that the inverse in equation (5) is possible. A maximum feed/tooth (f_{max}), typically selected from machining handbooks or some machining process criteria such as tooth chippage, is also set; therefore, the control variable is bounded from above by

$$u_{\max}(k) = f_{\max}^{\bar{\alpha}}(k) \tag{7}$$

The *applied* control variable (u) is

$$u(k) = \begin{cases} u_{\min}(k) & \text{if} \quad u_c(k) < u_{\min}(k) \\ u_c(k) & \text{if} \quad u_{\min}(k) \le u_c(k) \le u_{\max}(k) \\ u_{\max}(k) & \text{if} \quad u_c(k) > u_{\max}(k) \end{cases} \tag{8}$$

A block diagram of the complete machining force control system is shown in Figure 1.

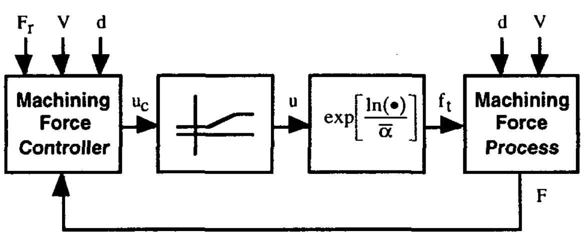

Figure 1: Block Diagram of Model-Based Machining Force Control System.

3.3 Stability Analysis and Verification

The closed-loop dynamics of the controlled static force process are given by equations (1), (2), and (4). Combining these equations yields

$$f_t^{\bar{\alpha}}(k) = f_t^{\bar{\alpha}}(k-1) + \frac{1+b_0}{\bar{\theta}}\left[F_R - \theta f_t^{\alpha}(k-1)\right] \tag{9}$$

where F_R is the value of the constant reference machining force. Using traditional linearization techniques for this system, and ignoring saturation, one equilibrium point is found where $F = F_R$. The eigenvalue of this equilibrium is

$$\lambda = 1 - \left(1 + b_0\right)\frac{\theta}{\bar{\theta}}\frac{\alpha}{\bar{\alpha}}\frac{f_e^{\alpha}}{f_e^{\bar{\alpha}}} \tag{10}$$

where f_e is the feed/tooth corresponding to $F = F_R$. The system will become marginally stable if the eigenvalue (λ) is located at 1 or -1. In the case where $\lambda = 1$, the closed-loop pole would have to be located at 1 or the model parameters would have to be selected to be infinite. Since these cases are not of practical interest, the case where λ = -1 is explored. The following relationship for the stability boundary in the parameter space is found to be

$$\frac{\theta}{\bar{\theta}} = \frac{2}{\left(1+b_0\right)}\frac{\bar{\alpha}}{\alpha}\frac{f_e^{\bar{\alpha}}}{f_e^{\alpha}} \tag{11}$$

A plot of the stability boundary for a specific set of model parameters is shown in Figure 2. The results demonstrate the excellent prediction of the linearization analysis; however, there is slight error in the region $0.5 \le \bar{\alpha} \le 1.0$. Simulation studies reveal a different dynamical response at the stability boundary for this region as compared to outside this region (Figure 3). In this region, the system becomes unstable in a manner such that the magnitudes of the error between the reference and actual force are equal at every time step. For this situation, the stability boundary is given by

$$\bar{\theta} = \frac{\left(1+b_0\right)\left(F_R - \theta f_0^{\alpha}\right)}{\exp\left[\left(\frac{\bar{\alpha}}{\alpha}\right)\ln\left(\frac{2F_R}{\theta} - f_0^{\alpha}\right)\right] - f_0^{\bar{\alpha}}} \tag{12}$$

where f_0 is the initial feed/tooth. If the system saturates at the second time step in this region, then the magnitudes of the error between the reference and the actual force are equal at the second and subsequent time steps. The stability boundaries when the system experiences minimum and maximum saturation, respectively, are given by

$$\bar{\theta} = \frac{\left(1+b_0\right)\left(F_R - \theta f_{\min}^{\alpha}\right)}{\exp\left[\left(\frac{\bar{\alpha}}{\alpha}\right)\ln\left(\frac{2F_R}{\theta} - f_{\min}^{\alpha}\right)\right] - f_{\min}^{\bar{\alpha}}} \tag{13}$$

$$\bar{\theta} = \frac{\left(1+b_0\right)\left(F_R - \theta f_{\max}^{\alpha}\right)}{\exp\left[\left(\frac{\bar{\alpha}}{\alpha}\right)\ln\left(\frac{2F_R}{\theta} - f_{\max}^{\alpha}\right)\right] - f_{\max}^{\bar{\alpha}}} \tag{14}$$

The "constant error" analysis is plotted in Figure 2 for the specific case given in that figure. The results demonstrate that together, the linearization and "constant error" analyses may be combined to perfectly predict the stability boundary in the parameter space. The analysis is also verified via experimentation (Figure 4). These analytic techniques provide the designer with powerful tools which may be used to select the controller parameters to ensure the force controller will remain stable given a range of variations in the model parameters.

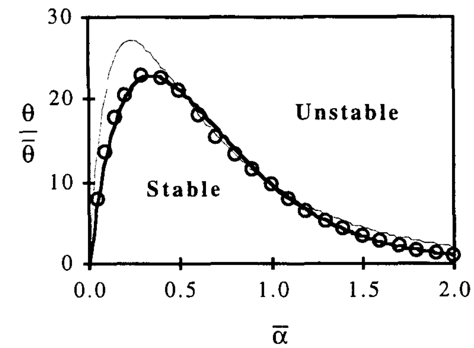

Figure 2: Stability Borderline for Machining Force Control System for Static Force Process. Linearization (medium line) and "Constant Error" Analysis (thin line) are combined to predict the borderline as compared to the simulations (circles). Simulation parameters: F_R = 0.2 kN, f_0 = 0.2 mm/tooth, α = 0.5, θ = 0.8705, and b_0 = -0.9048.

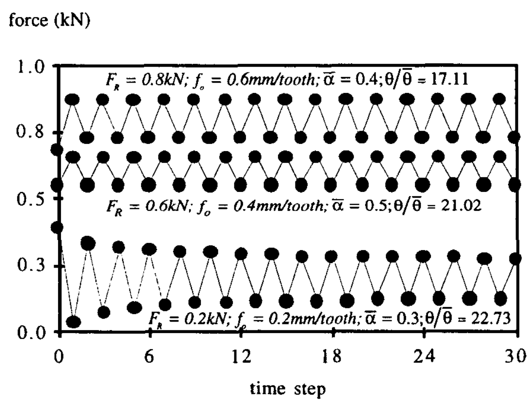

Figure 3: Various Types of Marginally Stable Force Responses. Top (upper saturation), middle (constant error), and bottom (typical). Simulation parameters: α = 0.5, θ = 0.8705, and b_0 = -0.9048, f_{max} = 1.0 mm/tooth.

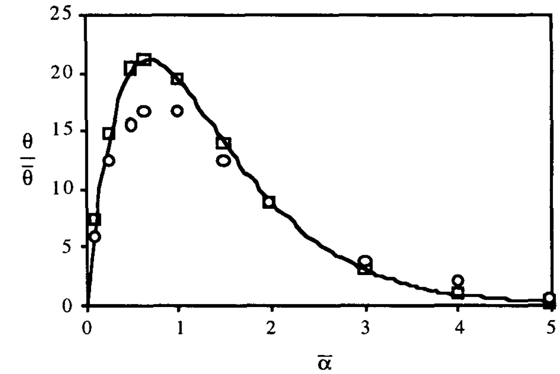

Figure 4: Stability Borderline for Machining Force Control System for Static Force Process. Analysis (medium line), simulations (boxes), and experimentation (circles). Parameters: F_R = 0.3 kN, f_0 = 0.1 mm/tooth, α = 0.63, θ = 0.76, and b_0 = -0.9048.

4 Machining Force Control: First-Order Process

4.1 Process Model

The structure for the first-order force processes is

$$F = Kd^{\beta}V^{\gamma}\frac{1+a}{z+a}f^{\alpha} \quad (14)$$

where the parameter a is the discrete-time pole which depends upon the process time constant and the sample period. In addition to the other model parameters, the parameter a must be calibrated for each different tool-workpiece combination. First-order models are typically employed when considering an instantaneous force that is sampled several times per spindle revolution.

4.2 Controller Design

The control variable is defined as in equation (2) and the first-order force process model used for controller design is

$$F(z) = \bar{\theta}\frac{1+\bar{a}}{z+\bar{a}}u_c(z) \quad (15)$$

where $\bar{a}$ is an off-line estimate of a. Again, an integral state is added to the plant and the MRC technique is used to design the *implemented* control law

$$u_c(z) = \frac{z}{z-1}\left[\frac{1+b_1+b_0}{\bar{\theta}(1+\bar{a})}F_r(z) - \frac{(b_1-\bar{a}+1)}{\bar{\theta}(1+\bar{a})}F(z) - \frac{(b_0+\bar{a})}{z\bar{\theta}(1+\bar{a})}F(z)\right] \quad (16)$$

where the discrete-time polynomial $z^2 + b_1 z + b_0 = 0$ defines the desired closed-loop dynamical characteristics. The commanded feed/tooth (f_r), minimum control variable (u_{min}), maximum control variable (u_{max}), and applied control variable (u) are again given by equations (5), (6), (7), and (8), respectively, and the force control system is represented by Figure 1.

4.3 Stability Analysis and Verification

The closed-loop dynamics of the controlled first-order force process are given by equations (2), (14), and (16). Again, there is one equilibrium point where $F = F_R$. Linearization about this point yields the following relationship for the system eigenvalues

$$z^2+\left[a-1+\eta\frac{\theta}{\bar{\theta}}(b_1-\bar{a}+1)\right]z+\left[-a+\eta\frac{\theta}{\bar{\theta}}(b_0+\bar{a})\right]=0 \quad (17)$$

where

$$\eta=\frac{(1+a)}{(1+\bar{a})}\frac{\alpha}{\bar{\alpha}}\frac{f_e^{\alpha}}{f_e^{\bar{\alpha}}} \quad (18)$$

There are four distinct conditions for marginal stability.

Condition 1

The closed-loop poles are located at −1 and q where

$$q=\frac{a(b_1-b_0+1)-2(b_0+\bar{a})+2ab_0}{b_1-b_0+1-2\bar{a}} \quad (19)$$

For this condition, the stability boundary is given by

$$\frac{\theta}{\bar{\theta}}=\frac{1}{\eta}\frac{2(1-a)}{b_1-b_0+1-2\bar{a}} \quad (20)$$

Simulation and experimental studies verify this solution is valid for $-1\le\bar{a}<\bar{a}^*$ where

$$\bar{a}^*=\frac{b_1-b_0+1}{2} \quad (21)$$

Condition 2

The closed-loop poles are located at $q_1 \pm q_2\sqrt{-1}$ where

$$q_1=\frac{a-1}{2}+\frac{1+a}{2(b_0+\bar{a})}[b_1-\bar{a}+1] \quad (22)$$

and

$$q_1^2+q_2^2=1 \quad (23)$$

For this condition, the stability boundary is given by

$$\frac{\theta}{\bar{\theta}}=\frac{1}{\eta}\frac{1+a}{b_0+\bar{a}} \quad (24)$$

Simulation and experimental studies verify that this solution is valid for $\bar{a}^*\le\bar{a}\le 0$. When $\bar{a}=\bar{a}^*$, both poles are located at 1.

Condition 3

The closed-loop poles are located at 1 and q. For this condition, the stability boundary is given by

$$\frac{\theta}{\bar{\theta}}=0 \quad (25)$$

This condition is only encountered when the model parameters are selected to be infinite.

Condition 4

The closed-loop poles are located at 1 and -1. For this condition, the stability boundary is given by

$$\frac{\theta}{\bar{\theta}}=\frac{1}{\eta}\frac{a-1}{b_0+\bar{a}} \quad (26)$$

This condition is only encountered when $b_0 = -(1+b_1)$. Under this condition, the desired closed-loop dynamics would be marginally stable; therefore, this condition is not practical.

The analysis was verified through simulation studies (Figure 5). To verify the analysis experimentally, studies were performed on a machine tool axis with the following structure

$$v=\theta\frac{1+a}{z+a}u_v \quad (27)$$

where v is the axis linear velocity in mm/s, u_v is the digital input to the D/A converter, θ = 0.2835 (mm/s)/digital number, and a = -0.8338. Note that α = 1.0. Experimental and simulation studies (Figures 5-7) were conducted with the following parameters: commanded axis velocity of 10 mm/s, initial axis velocity of 0 mm/s, b_0 = 0.8681, and b_1 = -1.863. For these parameters, $\bar{a}^*=-0.86348$. The studies show excellent correlation with the analysis.

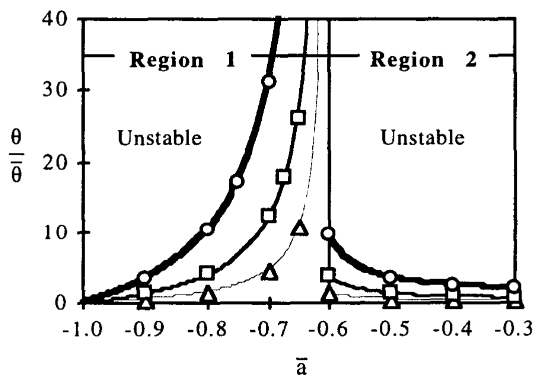

Figure 5: Stability Borderline for Machining Force Control System for First Order Force Process. Linearization analysis (thick line [$\bar{\alpha}=0.6$], medium line [$\bar{\alpha}=0.04$], and thin line [$\bar{\alpha}=1.5$]) predicts the borderline well as demonstrated by the simulations (circles [$\bar{\alpha}=0.6$], squares [$\bar{\alpha}=0.04$], and triangles [$\bar{\alpha}=1.5$],). Simulation parameters: F_R = 0.3 kN, f_0 = 0.05 mm/tooth, α = 0.6, θ = 2.0, a = -0.8206, b_1 = -1.562, and b_0 = 0.6413. $\bar{a}^*$ = -0.6017.

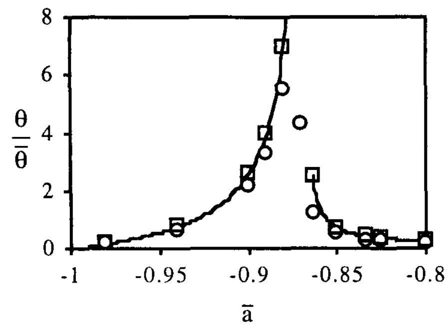

Figure 6: Analytical (line), Simulation (boxes), and Experimental (circles) Results for Axis Control System with $\bar{\alpha}$ = 0.5.

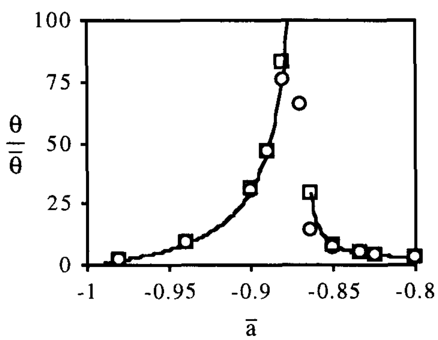

Figure 7: Analytical (line), Simulation (boxes), and Experimental (circles) Results for Axis Control System with $\bar{\alpha}$ = 1.0.

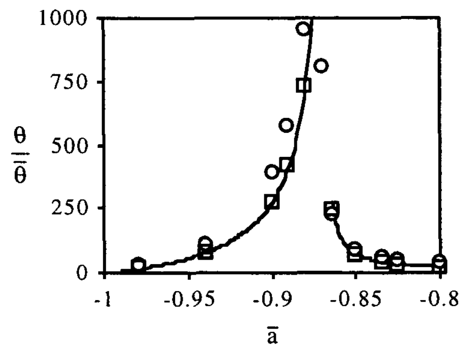

Figure 8: Analytical (line), Simulation (boxes), and Experimental (circles) Results for Axis Control System with $\bar{\alpha}$ = 1.5.

Note the linearization analysis provides excellent prediction over a wide range of $\bar{\alpha}$. Again, the analytic technique provides the designer with a powerful tool to select controller and model parameters. The results show that one may select $\bar{a}$ as a function *solely* of controller parameters to obtain a very large stability margin with respect to the force process gain variation.

5 Summary and Conclusions

The effect of parameter variations on the stability of the closed-loop system for two model-based machining force controllers has been analyzed. For the force controller designed for static force processes, the stability boundary in the parameter space can be exactly determined by a combination of the linearization and the developed "constant error" techniques. For the force controller designed for first-order force processes, the linearization technique was used to determine the stability boundary. In this case, it was found that a large stability margin with respect to the system gain may be obtained by properly choosing the estimated discrete-time pole as a function of the user-selected controller parameters. The analysis techniques for both the static and first order systems were verified via simulations and experimentation. For both systems, these analytic techniques allow the designer to select controller and model parameters to ensure closed-loop system stability given expected variations in the model parameters.

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