

01 Jun 2005

Hierarchical Optimal Force-Position-Contour Control of Machining Processes. Part I. Controller Methodology

Yan Tang

Robert G. Landers

Missouri University of Science and Technology, landersr@mst.edu

S. N. Balakrishnan

Missouri University of Science and Technology, bala@mst.edu

Follow this and additional works at: https://scholarsmine.mst.edu/mec_aereng_facwork



Part of the [Aerospace Engineering Commons](#), and the [Mechanical Engineering Commons](#)

Recommended Citation

Y. Tang et al., "Hierarchical Optimal Force-Position-Contour Control of Machining Processes. Part I. Controller Methodology," *Proceedings of the American Control Conference (2005, Portland, OR)*, Institute of Electrical and Electronics Engineers (IEEE), Jun 2005.

The definitive version is available at <https://doi.org/10.1109/ACC.2005.1470706>

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in Mechanical and Aerospace Engineering Faculty Research & Creative Works by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact scholarsmine@mst.edu.

HIERARCHICAL OPTIMAL FORCE-POSITION-CONTOUR CONTROL OF MACHINING PROCESSES: PART I – CONTROLLER METHODOLOGY

Yan Tang, Robert G. Landers, and S.N. Balakrishnan
Department of Mechanical and Aerospace Engineering
University of Missouri–Rolla
Rolla, Missouri 65409–0050
{ytnm4,landersr,bala}@umr.edu

ABSTRACT

There has been a tremendous amount of research in machine tool servomechanism control, contour control, and machining force control; however, to date these technologies have not been tightly integrated. This paper develops a hierarchical optimal control methodology for the simultaneous regulation of servomechanism positions, contour error, and machining forces. The contour error and machining force process reside in the top level of the hierarchy where the goals are to 1) drive the contour error to zero to maximize quality and 2) maintain a constant cutting force to maximize productivity. These goals are systematically propagated to the bottom level, via aggregation relationships between the top and bottom-level states, and combined with the bottom-level goals of tracking reference servomechanism positions. A single controller is designed at the bottom level, where the physical control signals reside, that simultaneously meets both the top and bottom-level goals. The hierarchical optimal control methodology is extended to account for variations in force process model parameters and process parameters.

INTRODUCTION

There has been a tremendous amount of research in machine tool servomechanism control, contour control, and machining force control; however, to date these technologies have not been tightly integrated. These three areas have been researched separately in laboratory settings. However, there is no general methodology for combining these areas and, thus, integrating these technologies is a complex task. This paper will develop a hierarchical optimal control methodology that generates one controller that simultaneously regulates servomechanism positions, contour error, and machining forces.

The majority of machining force control methodologies used adaptive techniques [e.g., Ulsoy *et al.*, 1983]. In the adaptive machining force control methodology, model parameters are estimated on-line and control gains adjusted to maintain stability over a wide range of parameter variations. Another popular method is robust machining force control [e.g., Rober *et al.*, 1997] where,

given bounds on model parameter variations, robust control techniques are utilized. The robust methodology was extended in Kim *et al.* [2003], which decreased model uncertainty by directly accounting for known process parameter variations. Other types of machining force control techniques include log transform [e.g., Harder, 1995], nonlinear with process compensation [e.g., Landers and Ulsoy, 2000], neural network [e.g., Tang *et al.*, 1994], and fuzzy logic [e.g., Hsu and Fann, 1997]. A review of model-based techniques is given in Landers *et al.* [2004]. In a recent work by Pandurangan *et al.* [2004], hierarchical optimal control techniques were used to integrate machining force and servomechanism position control in a turning operation. However, contour control was not incorporated into the methodology and only simple contours were considered.

The control of single axes has been well-researched for many decades. A technique known as Zero Phase Tracking Error Control (ZPTEC) was applied to single-axis systems [Tomizuka, 1987]. In the ZPTEC technique, feedback and feedforward controllers are utilized to achieve good tracking and zero phase error between the reference and the output. This technique has been applied to complex contours [Tomizuka *et al.*, 1987] and was extended to time-varying, uncertain systems via the integration of adaptive techniques [Tsao and Tomizuka, 1987]. The issues involved in servomechanism motion control are reviewed in Ellis and Lorenz [1999]. For many applications, including machining, it is more important to drive the contour error to zero than it is to drive the individual axis errors to zero. The idea of contour error was presented in Poo *et al.* [1972] and the need for contour control led to the development of cross coupling control [Koren, 1980; Srinivasan and Kulkarni, 1990; Koren and Lo, 1991; Koren and Lo, 1992]. In this methodology, an additional algorithm is added to the control architecture that, based on the contour error, calculates offsets for each servomechanism control signal. Typically, cross coupling control design does not take the individual servomechanism controllers into account. In a recent work by Landers and Balakrishnan [2004], hierarchical optimal control techniques were utilized to integrate servomechanism and contour control for two-axis motion control systems.

In this paper, a hierarchical optimal control methodology is introduced that simultaneously regulates machining force processes, contour error, and servomechanism position errors in machining operations. In roughing operations, it is important to regulate contour and position errors to minimize thickness variations in the subsequent finishing operation. In finishing operations, it is important to regulate forces to minimize structural deflections. The next section presents the control methodology and the following section extends the methodology to account for variations in force process model parameters and process parameters.

HIERARCHICAL OPTIMAL CONTROL METHODOLOGY

A control methodology that simultaneously regulates forces, position errors, and contour error in machining operations is now presented. A multi-axis machine tool is conceptualized as a hierarchical system (Figure 1). The contour error and machining force are located at the top level and the servomechanisms are located at the bottom level. While the top level has physical outputs (i.e., contour error and cutting force), it does not contain physical control signals. The bottom level consists of a number of axes whose coordinated motion allows the machine tool to produce complex contours. This level consists of physical outputs (e.g., position, velocity) as well as physical control signals (e.g., voltages, currents). The top-level goals are to maintain zero contour error and a constant machining force, and the bottom level goal is to maintain zero servomechanism position errors. Since the top level does not contain physical control signals, the goals of zero contour error and a constant machining force must be realized via the bottom level control signals. Thus, the control methodology presented in this paper will propagate the top-level goals to the bottom level where a controller will simultaneously meet the top and bottom-level goals.

The hierarchical optimal force-position-contour control methodology will propagate the top-level goals of zero contour error and a constant machining force to the bottom level via an aggregation relationship between the contour error/machining force and the servomechanism position errors. A single optimal controller will be constructed that is capable of simultaneously addressing three objectives: zero contour error, constant machining force, and zero servomechanism position errors. Therefore, the increased complexity of additional contour and force control algorithms is avoided. The methodology developed below provides an intuitive means for the designer to weight the relative importance of the three objectives.

The state space representation of the servomechanism dynamics is

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \quad (1)$$

$$\mathbf{y}(t) = G\mathbf{x}(t) \quad (2)$$

The servomechanism states are next transformed into error states. The reference tool path is approximated as a circular arc at each instant. Therefore, the reference axial positions satisfy

$$\ddot{x}_r(t) + \omega_r^2 x_r(t) = 0 \quad (3)$$

Using the error-space approach in Franklin *et al.* [1994] and extending it to MIMO systems, the servomechanism system is transformed into

$$\begin{aligned} \dot{\mathbf{x}}_{bot}(t) &= A_{bot}\mathbf{x}_{bot}(t) + B_{bot}\mathbf{u}_{bot}(t) \\ &= \begin{bmatrix} E_{bot} & G_{bot} \\ 0_{(n)(2p)} & A \end{bmatrix} \begin{bmatrix} \mathbf{e}(t) \\ \dot{\mathbf{x}}(t) \\ \boldsymbol{\zeta}(t) \end{bmatrix} + \begin{bmatrix} 0_{(2p)(m)} \\ B \end{bmatrix} \mathbf{u}_{bot}(t) \end{aligned} \quad (4)$$

where

$$\mathbf{e}^T(t) = [e_1(t) \quad \cdots \quad e_p(t)] \quad (5)$$

$$\boldsymbol{\zeta}(t) = \ddot{\mathbf{x}}(t) + \omega_r^2 \mathbf{x}(t) \quad (6)$$

$$E_{bot} = \begin{bmatrix} 0_{(p)(p)} & I_p \\ -\omega_r^2 I_p & 0_{(p)(p)} \end{bmatrix} \quad G_{bot} = \begin{bmatrix} 0_{(2p)(n)} \\ G \end{bmatrix} \quad (7)$$

$$\mathbf{u}_{bot}(t) = \ddot{\mathbf{u}}(t) + \omega_r^2 \mathbf{u}(t) \quad (8)$$

where $\mathbf{e}(t) = \mathbf{s}(t) - \mathbf{r}(t)$ and the subscript *bot* refers to the bottom level. Equation (4) describes the bottom level dynamics in the hierarchical optimal control methodology.

The next step in the controller formulation is to determine the top-level goals and an aggregation relationship between the goals at the top and bottom levels such that the top-level goals are propagated to the bottom level. One of the top-level goals is to maintain zero contour error, which is the minimum distance between the actual tool position and the desired tool path. The contour error is related to the individual axis errors and may be expressed as

$$\varepsilon(t) = \mathbf{c}_1(t) \mathbf{x}_{bot}(t) \quad (9)$$

where $\mathbf{c}_1(t)$ depends upon the tool path. It is assumed that all axis positions are measured. The other top-level goal is to maintain a constant machining force. Machining forces depend upon the feed, depth-of-cut, and cutting speed and are related to these parameters by the following

control-oriented nonlinear relation [Landers and Ulsoy, 2000]

$$F(t) = Kf^\alpha(t)d^\beta(t)V^\gamma(t) \quad (10)$$

A reference feed is calculated based on equation (10) to maintain a specified machining force. This reference feed is then translated into a reference velocity, which is input to the interpolator. The force process parameters (i.e., K , α , β , and γ) and the process parameters (i.e., V and d) in equation (10) are nominal values and may change during the machining operation. The structural vibrations are assumed to be small as compared to the feed and the cutting tool angles are assumed to be constant. Also, effects due to tool wear and cutting temperature are assumed to be reflected in the force process gain.

Linearizing equation (10) about the operating conditions yields

$$\Delta F(t) = [KV^\gamma d^\beta \alpha f_r^{\alpha-1}] \Delta f(t) = \Theta \Delta f(t) \quad (11)$$

where $\Delta F = F_r(t) - F(t)$ and

$$\Delta f(t) = f_r(t) - f(t) = \text{func}[\dot{e}(t)] \quad (12)$$

The force error is related to the individual axis error derivatives by linearizing equation (12). This relationship may be expressed as

$$\Delta F(t) = c_2(t) \mathbf{x}_{bot}(t) \quad (13)$$

The contour and force errors are related to the individual servomechanism errors and their derivatives and may be expressed through the aggregation relation

$$\mathbf{x}_{top}(t) = \begin{bmatrix} \varepsilon(t) \\ \Delta F(t) \end{bmatrix} = \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} \mathbf{x}_{bot}(t) = C(t) \mathbf{x}_{bot}(t) \quad (14)$$

The next step in the controller formulation is to create and solve an optimal tracking control problem [Lewis and Syrmos, 1995] where the bottom level control signals seek to simultaneously track the top-level goals (i.e., zero contour error and constant cutting force) and the bottom-level goals (i.e., zero servomechanism tracking error). The cost function at the bottom is

$$J_{bot} = \frac{1}{2} [E(t_f)]^T S_{bot} [E(t_f)] + \int_0^{t_f} L_{bot}(t) dt \quad (15)$$

where $E(t_f) = C(t_f) \mathbf{x}_{bot}(t_f) - \begin{bmatrix} \varepsilon_r(t_f) \\ \Delta F_r(t_f) \end{bmatrix}$ and

$$L_{bot}(t) = \frac{1}{2} \left\{ \begin{aligned} & [E(t)]^T Q_{bot} [E(t)] \\ & + \mathbf{u}_{bot}^T(t) R_{bot} \mathbf{u}_{bot}(t) + \mathbf{x}_{bot}^T(t) Q \mathbf{x}_{bot}(t) \end{aligned} \right\} \quad (16)$$

Note that the top-level goals require $\varepsilon_r(t) = 0$ and $\Delta F_r(t) = 0$. The first term in equation (15) ensures the bottom-level states track the top-level states at the final time. The first term in equation (16) ensures the aggregation relationship between the top and bottom levels is met. In effect, this term is used to send commands from the top level to the bottom level to ensure the top-level objectives are met. The second term in equation (16) penalizes bottom-level control signal usage, where the physical control signals reside. The third term in equation (16) penalizes deviations in the bottom-level states. In effect, this term is used to ensure the bottom-level objectives are met. The Hamiltonian at the bottom level is

$$H_{bot}(t) = L_{bot}(t) + \lambda_{bot}(t) [A_{bot} \mathbf{x}_{bot}(t) + B_{bot} \mathbf{u}_{bot}(t)] \quad (17)$$

where the Lagrange multiplier is of the form

$$\lambda_{bot}(t) = P_{bot}(t) \mathbf{x}_{bot}(t) + \mathbf{k}_{bot}(t) \quad (18)$$

The optimal control law is found by taking the partial derivative of equation (17) with respect to $\mathbf{u}_{bot}(t)$ and equating to zero

$$\begin{aligned} \mathbf{u}_{bot}(t) &= -R_{bot}^{-1} B_{bot}^T \lambda_{bot}(t) \\ &= -R_{bot}^{-1} B_{bot}^T [P_{bot}(t) \mathbf{x}_{bot}(t) + \mathbf{k}_{bot}(t)] \end{aligned} \quad (19)$$

The elements of the matrix $P_{bot}(t)$ and the vector $\mathbf{k}_{bot}(t)$ are found by solving, respectively, the differential equations

$$\begin{aligned} \dot{P}_{bot}(t) &= -P_{bot}(t) A_{bot} - A_{bot}^T P_{bot}(t) \\ &+ P_{bot}(t) B_{bot} R_{bot}^{-1} B_{bot}^T P_{bot}(t) - C^T(t) Q_{bot} C(t) - Q \end{aligned} \quad (20)$$

$$\begin{aligned} \dot{\mathbf{k}}_{bot}(t) &= -A_{bot}^T \mathbf{k}_{bot}(t) + P_{bot}(t) B_{bot} R_{bot}^{-1} B_{bot}^T \mathbf{k}_{bot}(t) \\ &+ C^T(t) Q_{bot} \mathbf{x}_{top}(t) \end{aligned} \quad (21)$$

These differential equations must be solved backward in time. The boundary conditions for equations (20) and (21), respectively, are

$$P_{bot}(t_f) = C^T(t_f) S_{bot} C(t_f) \quad (22)$$

$$\mathbf{k}_{bot}(t_f) = C(t_f) S_{bot} \mathbf{x}_{top}(t_f) \quad (23)$$

The matrix $P_{bot}(t)$ is used for regulation and the vector $\mathbf{k}_{bot}(t)$ is used for tracking. However, the top-level objectives are $\varepsilon_r(t) = 0$ and $\Delta F_r(t) = 0$. Therefore, $\mathbf{k}_{bot}(t_f) = 0$ and $\mathbf{k}_{bot}(t)$ is unforced and, thus, $\mathbf{k}_{bot}(t) = 0$. Simulations of equation (20) reveal that the elements of $P_{bot}(t)$ are

constant except near $t = t_f$; therefore, the steady-state solution is utilized. This greatly aids the stringent real-time computational demands required by machining processes. Note that $\mathbf{u}_{bot}(t)$ is a vector of dummy control signals. The physical control signals are found by solving

$$\ddot{\mathbf{u}}(t) + \omega_r^2 \mathbf{u}(t) = -R_{bot}^{-1} B_{bot}^T P_{bot}(t) \mathbf{x}_{bot}(t) \quad (24)$$

ROBUSTNESS TO PARAMETER VARIATIONS

The force model given by equation (10) includes model parameters (i.e., K , α , β , and γ) that must be determined empirically and process parameters (i.e., d , f , and V) that are functions of the machine tool's linear axis and the spindle motions. The controller derived above assumed no variation in these parameters; however, these parameters naturally vary during a machining operation. For example, the model gain K strongly depends on the tool wear and cutting temperature. Also, the depth-of-cut depends on the part geometry. When a model parameter varies, monitoring techniques must be used to determine the amount of variation, while process parameter variations may be determined from the part drawing and sensing the machine variables. When there is parameter variation, the linearized relation given by equation (11) is not valid. In this section, the hierarchical optimal force-position-contour controller is modified to account for uncertainties in the fore process model gain and depth-of-cut. This method may be extended to account for uncertainties in other force process model and process parameters.

Expanding the force-feed relation given by (11) in a Taylor series expansion about the reference feed, nominal force process model gain, and nominal depth-of-cut

$$\begin{aligned} \Delta F(t) \cong & [K_0 \alpha f_r^{\alpha-1} d_0^\beta V^\gamma] \Delta f(t) + [f_r^\alpha d_0^\beta V^\gamma] \Delta K(t) \\ & + [K_0 \beta f_r^\alpha d_0^{\beta-1} V^\gamma] \Delta d(t) + [\alpha f_r^{\alpha-1} d_0^\beta V^\gamma] \Delta f(t) \Delta K(t) \\ & + [K_0 \alpha \beta f_r^{\alpha-1} d_0^{\beta-1} V^\gamma] \Delta f(t) \Delta d(t) \\ & + [\beta f_r^\alpha d_0^{\beta-1} V^\gamma] \Delta d(t) \Delta K(t) \\ & + \frac{1}{2!} [K_0 \beta (\beta - 1) f_r^\alpha d_0^{\beta-2} V^\gamma] \Delta d^2(t) \\ & + \frac{1}{2!} [K_0 \alpha (\alpha - 1) f_r^{\alpha-2} d_0^\beta V^\gamma] \Delta f^2(t) \end{aligned} \quad (25)$$

where $\Delta K(t) = K(t) - K_0$ and $\Delta d(t) = d(t) - d_0$. Assuming that the second order term in $\Delta f(t)$ in equation (25) is negligible

$$\begin{aligned} \Delta F(t) - [f_r^\alpha d_0^\beta V^\gamma] \Delta K(t) - [K_0 \beta f_r^\alpha d_0^{\beta-1} V^\gamma] \Delta d(t) \\ - \frac{1}{2!} [K_0 \beta (\beta - 1) f_r^\alpha d_0^{\beta-2} V^\gamma] \Delta d^2(t) \\ = [K(t) \alpha f_r^{\alpha-1} d_0^\beta V^\gamma + K_0 \alpha \beta f_r^{\alpha-1} d_0^{\beta-1} V^\gamma \Delta d(t)] \Delta f(t) \end{aligned} \quad (26)$$

The term $-[f_r^\alpha d_0^\beta V^\gamma] \Delta K(t) - [K_0 \beta f_r^\alpha d_0^{\beta-1} V^\gamma] \Delta d(t) - \frac{1}{2!} [K_0 \beta (\beta - 1) f_r^\alpha d_0^{\beta-2} V^\gamma] \Delta d^2(t)$ can be regarded as a bias to the top-level goal of a constant machining force. The effective goal propagated from the top level of the hierarchy is now $\Delta F_{eff}(t) = 0$ where

$$\begin{aligned} \Delta F_{eff}(t) = & \Delta F(t) - [f_r^\alpha d_0^\beta V^\gamma] \Delta K(t) \\ & - [K_0 \beta f_r^\alpha d_0^{\beta-1} V^\gamma] \Delta d(t) \\ & - \frac{1}{2!} [K_0 \beta (\beta - 1) f_r^\alpha d_0^{\beta-2} V^\gamma] \Delta d^2(t) \\ = & [K(t) \alpha f_r^{\alpha-1} d_0^\beta V^\gamma + K_0 \alpha \beta f_r^{\alpha-1} d_0^{\beta-1} V^\gamma \Delta d(t)] \Delta f(t) \end{aligned} \quad (27)$$

The effective aggregation matrix is

$$C_{eff}(t) = \begin{bmatrix} c_x(t) & c_z(t) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0_{(1)(4)} \\ -\frac{60(K(t)\alpha f_r^{\alpha-1} d_0^\beta V^\gamma + K_0 \alpha \beta f_r^{\alpha-1} d_0^{\beta-1} V^\gamma \Delta d(t))}{N_s} & 0 & 0_{(1)(4)} \end{bmatrix} \quad (28)$$

The cost function to minimize at the lower level is given by equations (15) and (16) where $C_{eff}(t)$ and $\Delta F_{eff}(t)$ are substituted for $C(t)$ and $\Delta F(t)$, respectively. The controller given by equation (24) is implemented where the steady-state solution for $P_{bot}(t)$ is utilized and the controller gains are updated, based on $C_{eff}(t)$, each time the model gain and depth-of-cut change. The vector $\mathbf{k}_{bot}(t)$ is again identically zero. Note that when only the force process model parameter changes, equation (26) reduces to

$$\begin{aligned} \Delta F(t) - [f_r^\alpha d_0^\beta V^\gamma] \Delta K(t) \\ = [K_0 \alpha f_r^{\alpha-1} d_0^\beta V^\gamma + \alpha f_r^{\alpha-1} d_0^\beta V^\gamma \Delta K(t)] \Delta f(t) \\ = [K(t) \alpha f_r^{\alpha-1} d_0^\beta V^\gamma] \Delta f(t) \end{aligned} \quad (29)$$

Similarly, when only the depth-of-cut changes, equation (26) reduces to

$$\begin{aligned} \Delta F(t) - [K_0 \beta f_r^\alpha d_0^{\beta-1} V^\gamma] \Delta d(t) \\ - \frac{1}{2!} [K_0 \beta (\beta - 1) f_r^\alpha d_0^{\beta-2} V^\gamma] \Delta d^2(t) \\ = [K_0 \alpha f_r^{\alpha-1} d_0^\beta V^\gamma + K_0 \alpha \beta f_r^{\alpha-1} d_0^{\beta-1} V^\gamma \Delta d(t)] \Delta f(t) \end{aligned} \quad (30)$$

SUMMARY AND CONCLUSIONS

A hierarchical optimal control methodology was developed in this paper for the simultaneous regulation of servomechanism position errors, contour error, and machining forces. The hierarchy contained two levels: the top level where the machining forces and contour error

resided and the bottom level where the servomechanism position errors resided. The requirements of a constant machining force and zero contour error were propagated to the bottom level via aggregation relationships between the machining force and contour errors and the servomechanism position errors. An optimal control problem was formulated and solved to construct a control law at the bottom level that simultaneously regulates the machining force, contour error, and servomechanism position errors. The hierarchical optimal control methodology was extended to account for variations in force process model parameters and process parameters. In Part II, the hierarchical optimal control methodology is applied to a two-axis turning operation and simulations of three different operations were conducted to verify the developed methodology.

NOMENCLATURE

A	$n \times n$ axis system state matrix
B	$n \times m$ axis system input matrix
C	aggregation matrix
C_{eff}	effective aggregation matrix
d	depth-of-cut [m]
d_0	nominal depth-of-cut [m]
e	p -dimensional error vector [m]
\dot{e}	p -dimensional error derivative vector [m/s]
e_x, e_z	x and z -axis position errors [m]
\dot{e}_x, \dot{e}_z	x and z -axis velocity errors [m/s]
f	feed [mm]
f_r	reference feed [mm]
F	machining force [kN]
F_r	reference machining force [kN]
G	$l \times n$ axis system output matrix
I_j	identity matrix with j rows and j columns
K	force process model gain
K_x, K_z	x and z -axis velocity gains [$(m/s)/V$]
K_0	nominal force process model gain
l	number of axis position measurements
m	number of servomechanism inputs
n	number of servomechanism states
N_s	spindle speed [rpm]
p	number of axes
Q	bottom level weighting matrix
Q_{bot}	aggregation weighting matrix
r	p -dimensional axis reference position vector [m]
r_x, r_z	x and z -axis reference positions [m]
\dot{r}_x, \dot{r}_z	x and z -axis reference velocities [m/s]
R_{bot}	dummy control signal weighting matrix
s	p -dimensional axis position vector [m]
s_x, s_z	x and z -axis positions [m]
S_{bot}	final time aggregation weighting matrix
t	time [sec]
t_f	final time [sec]
u	m -dimensional axis control input vector [V]
u_x, u_z	x and z -axis control inputs [V]
V	cutting speed [km/min]

x	n -dimensional axis state vector
\dot{x}	n -dimensional axis state derivative vector
x_r	generic axis reference position [m]
x_x, x_z	x and z -axis positions [m]
\dot{x}_x, \dot{x}_z	x and z -axis velocities [m/s]
\ddot{x}_x, \ddot{x}_z	x and z -axis accelerations [m/s^2]
$\ddot{\ddot{x}}_x, \ddot{\ddot{x}}_z$	x and z -axis jerks [m/s^3]
X_c, Z_c	x and z -contour centers [m]
X_{cc}, Z_{cc}	x and z -contour instantaneous centers of curvature [m]
y	l -dimensional axis measurement vector
α, β, γ	machining force model constant
ΔF_{eff}	effective perturbed reference machining force [kN]
ΔF_r	perturbed reference machining force [kN]
ε	contour error [m]
ε_r	reference contour error [m]
ρ	contour instantaneous radius of curvature [m]
τ_x, τ_y	x and z -axis time constants [sec]
ϕ	contour polar angle [rad]
ω_r	contour reference angular velocity [rad/s]
ξ	n -dimensional error space state vector
$0_{(i)(j)}$	zero matrix with i rows and j columns

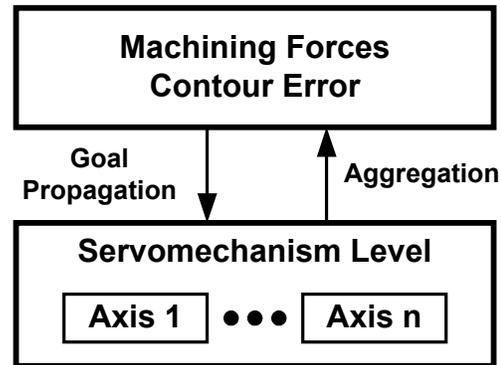


Figure 1: Hierarchical Representation of a Machining Operation for Force-Position-Contour Control.

REFERENCES

- Ellis, G. and Lorenz, R.D., 1999, "Comparison of Motion Control Loops for Industrial Applications," *IEEE Industrial Applications Conference*, Vol. 4, pp. 2599–2605.
- Franklin, G.F., Powell, J.D., and Emami-Naeini, A., 1994, Third Edition, *Feedback Control of Dynamic Systems*, Addison Wesley.
- Harder, L., 1995, "Cutting Force Control in Turning – Solutions and Possibilities," Ph.D. Dissertation, Department of Materials Processing, Royal Institute of Technology, Stockholm.
- Hsu, P-L. and Fann, W-R., 1997, "Fuzzy Adaptive Control of Machining Processes with a Self-Learning

Algorithm,” *ASME Journal of Manufacturing Science and Engineering*, Vol. 118, No. 4, pp. 522–530.

Kim, S.I., Landers, R.G., and Ulsoy, A.G., 2003 “Robust Machining Force Control with Process Compensation,” *ASME Journal of Manufacturing Science and Engineering*, Vol. 125, No. 3, pp. 423–430.

Koren, Y., 1980, “Cross-Coupled Biaxial Computer Control for Manufacturing Systems,” *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 102, No. 4, pp. 265–272.

Koren Y. and Lo, C-C., 1991, “Variable Gain Cross Coupling Controller for Contouring,” *Annals of the CIRP*, Vol. 40/1, pp. 371–374.

Koren, Y. and Lo, C-C., 1992, “Advanced Controllers for Feed Drives,” *Annals of the CIRP*, Vol. 41/2, pp. 689–698.

Landers, R.G. and Balakrishnan, S.N., 2004, “Hierarchical Optimal Contour-Position Control of Motion Control Systems,” *ASME International Mechanical Engineering Congress and Exhibition*, Anaheim, California, November 13–19 (on CD ROM).

Landers, R.G. and Ulsoy, A.G., 2000, “Model-Based Machining Force Control,” *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 122, No. 3, pp. 521–527.

Landers, R.G., Ulsoy, A.G., and Ma, Y-H., 2004, “A Comparison of Model-Based Machining Force Control Approaches,” *International Journal of Machine Tools and Manufacture*, Vol. 44, No. 7–8, pp. 733–748.

Lewis, F.L. and Syrmos, V.L., 1995, *Optimal Control*, Second Edition, John Wiley and Sons, New York.

Pandurangan, B., Landers, R.G., and Balakrishnan, S.N., 2004, “Hierarchical Optimal Force-Position Control

of a Turning Process,” *IEEE Transactions on Control System Technology* (to appear).

Poo, A.N., Vollinger, J., and Younkin, G.W., 1972, “Dynamic Error in Type 1 Contouring Systems,” *IEEE Transactions on Industrial Applications*, Vol. IA-8, No. 4, pp. 477–484.

Rober, S.J., Shin, Y.C., and Nwokah, O.D.I., 1997, “A Digital Robust Controller for Cutting Force Control in the End Milling Process,” *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 119, No. 2, pp. 146–152.

Srinivasan, K. and Kulkarni, P.K., 1990, “Cross-Coupled Control of Biaxial Feed Drive Servomechanisms,” *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 112, No. 2, pp. 225–232.

Tang, Y.S., Hwang, S.T., and Wang, Y.S., 1994, “Neural Network Controller for Constant Turning Force,” *International Journal of Machine Tools and Manufacture*, Vol. 34, No. 4, pp. 453–460.

Tomizuka, M., 1987, “Zero Phase Error Tracking Algorithm for Digital Control,” *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 109, No. 1, pp. 65–68.

Tomizuka, M., Chen, M.S., Renn, S., and Tsao, T-C., 1987, “Tool Positioning for Noncircular Cutting with Lathe,” *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 109, No. 2, pp. 176–179.

Tsao, T-C. and Tomizuka, M., 1987, “Adaptive Zero Phase Error Tracking Algorithm for Digital Control,” *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 109, No. 4, pp. 349–354.

Ulsoy, A.G., Koren, Y., and Rasmussen, F., 1983, “Principle Developments in the Adaptive Control of Machine Tools,” *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 105, No. 2, pp. 107–112.