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## Grouping WWW Image Search Results by Novel Inhomogeneous Clustering Method

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# An Error Analysis of the Multirate Method for Power System Transient Stability Simulation

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**Abstract--** This efficient application of any numerical integration method depends on accurate estimation of the local truncation error to govern step size control. In this paper, the local truncation error of the multirate method using the forward Euler integration is derived leading to the calculation of the optimal step ratio. The techniques of applying multirate method in power systems are discussed. By implementing the multirate method in a practical power system example, the multirate strategies are proved to be efficient compared with traditional simulation methods.

**Index Terms--** multirate method, numerical integration, power systems simulation

## I. INTRODUCTION

**D**YNAMIC time domain simulation is a crucial issue with respect to the operation of power systems. The dynamic simulation of power system may involve a time scale from seconds to minutes, even hours, it is necessary to combine short-term and long-term analysis in a single program [1]. In recent years, effort has been spent in this direction [2]-[4]. Traditional power system simulation methods focus on fixed or variable step methods, which are suitable for the simulation of systems that exhibit infrequent fast decaying transients. When integrating systems of differential equations whose components evolve and persist at different time scales, it is preferable to avoid unnecessary calculations on slowly changing solution components.

For power systems, the existence of FACTS devices and induction machine loads increases the response time scale. In most typical transients, only a small fraction of the variables in the system exhibit fast dynamics, therefore it is inefficient to simulate the entire power system with a small integration time step when most variables react slowly and accuracy constraints can be easily satisfied with a large step.

Multirate methods were first proposed by Gear [5] for systems with widely ranging time response behavior. This paper further extends the multirate method through error derivation and the step size adjustment. The Local Truncation Error (LTE) of the multirate method implemented by Forward Euler integration is first derived and is shown to be the basis for choosing the optimal step ratio. A numerical example is then presented to illustrate this relationship. The multirate method is then applied to a power system example.

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## II. DERIVATION OF LOCAL ERROR

The basic principle of the multirate method is to integrate the fast variables using a short integration time step while using longer integration time steps for slowly changing variables. The correct choice of integration time step is a step size that results in a local truncation error smaller than some predetermined bound. Methods of estimating the local truncation error for a wide range of integration methods have been well-established [5], but have not been developed for multirate methods. The local truncation error of the multirate method depends on three distinct aspects:

1. the numerical integration method utilized
2. the ratio between the slow and fast variables, and
3. the interpolation method of the slow variable.

To derive the local truncation error, suppose the system variables are already separated as fast changing variable  $x$  and slow changing variable  $y$ :

$$\dot{x} = f(t, x, y) \quad (1)$$

$$\dot{y} = g(t, x, y) \quad (2)$$

Assume  $x(t_i)$  and  $y(t_i)$  are the exact solutions at time  $t_i$  and  $\hat{x}(t_i)$ ,  $\hat{y}(t_i)$  are the calculated solutions at corresponding time. Let the slow variable integration step (macro step) be denoted as  $H$  and the fast variable integration step (micro step) as  $h$ . Let the ratio of macro step to the micro step be  $m$ . Consider the case of using Forward Euler integration method with:

$$m = 2, \quad (H = mh = 2h), \quad \text{and}$$

$$t_{i+2} = t_{i+1} + h = t_i + H$$

Expanding  $x(t_{i+1})$  and  $y(t_{i+1})$  about  $t_i$  using the Taylor series yields:

$$x(t_{i+1}) = x(t_i) + (t_{i+1} - t_i)\dot{x}\Big|_{t=t_i} + \frac{(t_{i+1} - t_i)^2}{2}\ddot{x}\Big|_{t=\xi_i}, \quad \xi_i \in (t_i, t_{i+1}) \quad (3)$$

$$x(t_{i+2}) = x(t_{i+1}) + (t_{i+2} - t_{i+1})\dot{x}\Big|_{t=t_{i+1}} + \frac{(t_{i+2} - t_{i+1})^2}{2}\ddot{x}\Big|_{t=\xi_{i+1}}, \quad (4)$$

$$\xi_{i+1} \in (t_{i+1}, t_{i+2})$$

$$y(t_{i+1}) = y(t_i) + (t_{i+1} - t_i)\dot{y}\Big|_{t=t_i} + \frac{(t_{i+1} - t_i)^2}{2}\ddot{y}\Big|_{t=\gamma_i}, \quad \gamma_i \in (t_i, t_{i+1}) \quad (5)$$

$$y(t_{i+2}) = y(t_i) + (t_{i+2} - t_i)\dot{y}\Big|_{t=t_i} + \frac{(t_{i+2} - t_i)^2}{2}\ddot{y}\Big|_{t=\eta_i}, \quad \eta_i \in (t_i, t_{i+2}) \quad (6)$$

The approximate value of the slow variable  $y$  at time  $t_{i+1}$  can be estimated using the Forward Euler integration method:

$$\hat{y}(t_{i+1}) = y(t_i) + (t_{i+1} - t_i)\dot{y}\Big|_{t=t_i} \quad (7)$$

Similarly,

$$\hat{x}(t_{i+1}) = x(t_i) + (t_{i+1} - t_i) \dot{x} \Big|_{t=t_i} \quad (8)$$

To calculate  $x$  at  $t_{i+2}$  for the fast integration time step  $h$ , both  $\hat{x}(t_{i+1})$  and the interpolation value  $\hat{y}(t_{i+1})$  are used:

$$\hat{x}(t_{i+2}) = \hat{x}(t_{i+1}) + (t_{i+2} - t_{i+1}) \dot{x}(t_{i+1}, \hat{x}(t_{i+1}), \hat{y}(t_{i+1})) \quad (9)$$

Combining equations (3)-(9) yields the following local truncation error for the fast variable  $x$  at time  $t_{i+2}$ :

$$\begin{aligned} \hat{x}(t_{i+2}) - x(t_{i+2}) &= \hat{x}(t_{i+1}) - x(t_{i+1}) - \frac{(t_{i+2} - t_{i+1})^2}{2} \ddot{x} \Big|_{t=\xi_{i+1}} \\ &\quad + (t_{i+2} - t_{i+1}) [f(t_{i+1}, \hat{x}(t_{i+1}), \hat{y}(t_{i+1})) - f(t_{i+1}, x(t_{i+1}), y(t_{i+1}))] \\ &= (t_{i+2} - t_{i+1}) [f(t_{i+1}, \hat{x}(t_{i+1}), \hat{y}(t_{i+1})) - f(t_{i+1}, x(t_{i+1}), y(t_{i+1}))] \\ &\quad - \frac{(t_{i+2} - t_{i+1})^2}{2} \ddot{x} \Big|_{t=\xi_{i+1}} - \frac{(t_{i+1} - t_i)^2}{2} \ddot{x} \Big|_{t=\xi_i} \\ &\quad \xi_i \in (t_i, t_{i+1}) \quad \xi_{i+1} \in (t_{i+1}, t_{i+2}) \quad (10) \end{aligned}$$

By combining the local truncation error at  $t_{i+1}$  and the local truncation error at  $t_{i+2}$ , the total local truncation error in  $x$  becomes:

$$\hat{x}(t_{i+2}) - x(t_{i+2}) = -\frac{(t_{i+1} - t_i)^2}{2} \ddot{x} \Big|_{t=\xi_i} - \frac{(t_{i+2} - t_{i+1})^2}{2} \ddot{x} \Big|_{t=\xi_{i+1}} + O(h^3) \quad (11)$$

Similarly, the local truncation error of  $y$  becomes

$$\hat{y}(t_{i+2}) - y(t_{i+2}) = -\frac{(t_{i+2} - t_i)^2}{2} \ddot{y} \Big|_{t=\eta_i} + O(h^3) \quad (12)$$

If  $(t_{i+2} - t_{i+1}) = (t_{i+1} - t_i) = h$ , then

$$\begin{aligned} \hat{x}(t_{i+2}) - x(t_{i+2}) &= -h^2 \ddot{x} \Big|_{t=\tau} + O(h^3) \quad \tau \in (t_i, t_{i+2}) \\ \hat{y}(t_{i+2}) - y(t_{i+2}) &= -\frac{h^2}{2} \ddot{y} \Big|_{t=\tau} + O(h^3) \quad \tau \in (t_i, t_{i+2}) \end{aligned} \quad (13)$$

From the expression above, we can see both fast variable  $x$  and slow variable  $y$  have an accuracy on the order of  $h^2$ .

When the step size ratio is  $m$ , the LTEs (13) can be expanded as the following:

$$LTE_x = -\frac{m}{2} h^2 \max_{\tau \in [t_i, t_i + mh]} (\ddot{x}) + O(h^3) \quad (14)$$

$$LTE_y = -\frac{m^2}{2} h^3 \max_{\tau \in [t_i, t_i + mh]} (\ddot{y}) + O(h^3) \quad (15)$$

### III. RELATIONSHIP OF ERROR AND $m$

In the last section, the LTE for the forward Euler multirate has been calculated. To satisfy a given LTE boundary, the largest possible slow-fast step ratio  $m$  should be used for maximum computational efficiency.

In traditional integration methods, the integration step size for the entire system and all of the states is the same; it is chosen small enough to meet the error limit of the fastest variable. At the same time, the slow variables error is typically far less than the error limit, thus a common (one-rate) step

size method calculates the slow variables at a greater frequency than is necessary.

In a multirate method, the LTE for fast and slow variables is given in equations (14) and (15). If the LTE is limited by an upper bound  $B_{LTE}$ , the fast step  $h$  can be chosen according to  $B_{LTE}$ , and the largest (or optimal) ratio of the slow variable step to the fast variable step is

$$m = \frac{\ddot{x}}{\ddot{y}} \quad (16)$$

To illustrate the error difference between the fast and slow variables, consider an example system of fast and slow:

$$\begin{aligned} \dot{x} &= -1.0x + 0.5y \\ \dot{y} &= 0.01x - 0.1y \end{aligned}$$

with initial value  $x(0)=3$  and  $y(0)=43$ . The exact solution to this system is:

$$\begin{aligned} x(t) &= -20.67e^{-1.01t} + 23.67e^{-0.095t} \\ y(t) &= 0.23e^{-1.01t} + 42.77e^{-0.095t} \end{aligned}$$

In this small system, the variable  $x$  can be regarded as the fast variable and  $y$  as the slow variable.

The error for both the slow variable ( $y$ ) and fast variable ( $x$ ) is given in Figure 1 when applying the fixed step Forward Euler method with one integration step of  $h=0.0002$ . From this figure it is obvious that the fast variable error is much greater than the slow variable error. It is the largest during the initial period where the fast variable  $x$  dominates. As the fast transients die out, the slow variable  $y$  begins to dominate and the errors in the two states coalesce.

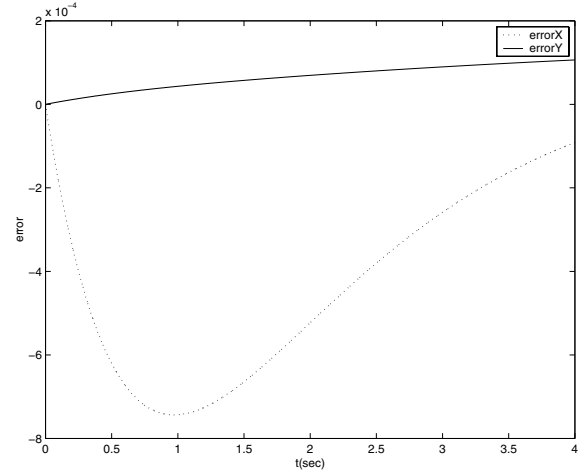


Fig. 1 fixed step error for both fast and slow variables

Figure 2 shows a comparison of the error of the fast and slow variables as  $m$  increases. The error for  $x$  stays relatively constant over the range of  $m$ , whereas the error in  $y$  increases nearly linearly. This result supports the basic premise behind the multirate method that the error in system integration is dominated by the fast state and therefore the fast and slow can be integrated with different stepsizes and still maintain the required simulation accuracy.

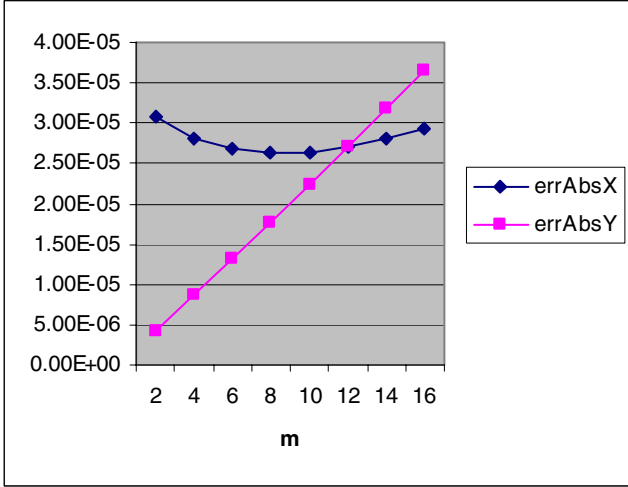


Fig. 2 Fast variable (x), slow variable (y) with different ratios

For an upper bound of  $B_{LTE} \leq 3.25 \times 10^{-5}$  the accuracy is maintained until about  $m=15$ . Thus the best computational efficiency and accuracy will be achieved for the case when the slow system is integrated with a step size 15 times larger than the fast system.

The optimal step ratio  $m=15$  can be related to  $\frac{\ddot{x}}{\ddot{y}}$ , which is shown in Figure 3. Figure 3 shows the ratio of the second derivatives and indicates that a step size difference of 15:1 is valid until approximately one second into the simulation. This is consistent with the results of Figure 1.

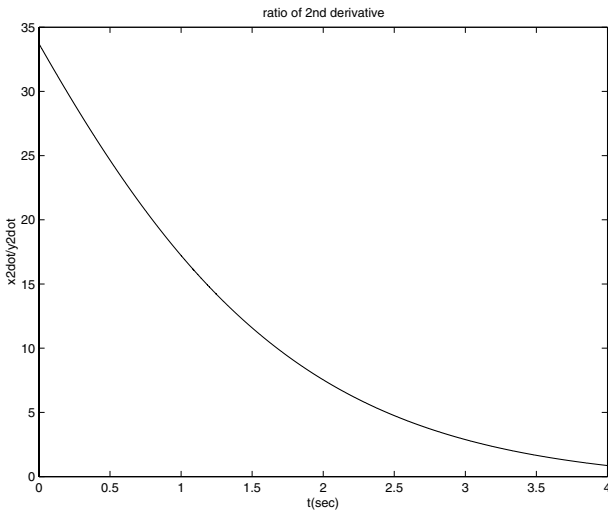


Fig. 3 Ratio of  $\frac{\ddot{x}}{\ddot{y}}$  versus time

This example also illustrates that after about 4 seconds, the errors in  $x$  and  $y$  are similar since the fast transients have died out and the multirate method is no longer required. This also indicates that the multirate method is best applied to systems that have sustained fast dynamics, such as systems that contain FACTS devices or induction motor loads.

#### IV. NONLINEAR SYSTEM--NR IMPLEMENTATION

In the previous section, a small linear system was used to illustrate the relationship between the integration step size and the local truncation error. In this section, the multirate method will be applied to a small power system example to illustrate that a similar relationship between error and ratio  $m$  exist even for nonlinear systems. The first application of the multirate method to power systems was reported in [6]-[7].

Consider a system of  $n$  nonlinear differential equations:

$$\dot{y}(t) = f(t, y), \quad y(t_0) = y_0$$

Suppose these states are divided into sets of states representing two time scales: the fast states and slow states. The ratio of macro step to micro step is  $m$ .

$$\dot{y}_f(t) = f_f(t, y_s, y_f), \quad y_f(t_0) = y_{f0}$$

$$\dot{y}_s(t) = f_s(t, y_s, y_f), \quad y_s(t_0) = y_{s0}$$

In a linear system, it is possible to find the matrix  $M$  which relates  $y(t+mh)$  to  $y(t)$  for both fast and slow variables [7]. This type of closed form relationship is not possible in a nonlinear system.

In the nonlinear case, a predictor-corrector method must be used for slow states. The slow variables must be predicted at the end of macro step and then interpolated to provide approximations for each micro step interval. Typically a linear interpolation is used. Once the slow variables are approximated at the each fast interval, the fast variables can be found by numerical integration at each micro step. The entire system is solved at each macro step. The updated values of the slow variables are compared to the predicted values. If they are within the specified tolerance, the time step is advanced to the next time interval; otherwise the step is repeated using the updated values to provide better interpolated values. The multirate procedure is summarized in the following.

- 1) predict the slow variable values at time  $t+H$ :

$$y_{t+H,s}^p = y_{t,s} + H * \dot{y}_{t,s}$$

the superscript 'p' refers to the predicted value

- 2) integrate the fast components at every micro step for  $i=1,2,\dots,m-1$ :

$$\dot{y}_f(t+hi) = f_f(\hat{y}_s(t+hi), y_f(t+hi))$$

with integration time step  $h$  where  $\hat{y}_s(t+hi)$  is the interpolated value of  $y_s$  at  $(t+hi)$ . For linear interpolation:

$$\hat{y}_s(t+hi) = \frac{i}{m}(y_{s,t+H}^p - y_{s,t}) + y_{s,t}$$

Note that since this is a nonlinear function, that if an implicit numerical integration method is used, then the discretized equation must be solved iteratively using a Newton-Raphson-type nonlinear solution.

3) integrate both fast and slow components at the macro time step (which is the same as the final micro step). Note that the fast subsystem will be integrated with integration time step  $h$  and the slow subsystem will be integrated with time step  $mh (=H)$ .

4) compare the calculated slow value with predicted value

if  $|y_s^p - y_s| > \mathcal{E}$ , set  $y_s = y_s^p$ , go to step 1)

otherwise  $t = t + H$ , go to step 1)

To make the multirate method more efficient, several programming techniques can be incorporated.

1. There is an outer loop iteration in which the entire system is solved at time  $t+H$ . If the slow system has not converged, then the integration is restarted to  $t$  and repeated. It is desirable to minimize the number of outer loop iterations. Since the iteration is repeated if the predicted and corrected slow values are not within some predefined tolerance  $\mathcal{E}$ , one method of decreasing the number of iterations is to improve the accuracy of the predicted values of the slow variables at  $t+H$ . To achieve sufficient accuracy, a high order explicit method can be used to predict the slow variables. If the predicted slow value is accurate within  $\delta$ , then the interpolated values will have error of less than  $\delta/m$ , further improving the accuracy of the fast variable calculations.
2. Within the fast calculations, each fast system of equations must be solved at every time step  $t_i$  in the interval  $(t, t+H)$ . If the fast system is nonlinear, this requires a Newton-Raphson (NR) solution of the discretized integration equations. This approach is illustrated in Figure 4. The NR iteration will converge more rapidly if the initial guess is sufficiently close to the solution. One method of insuring that the initial guess is relatively close to the solution is to predict the solution using an explicit integration method during the first macro (outer loop) iteration. Then in subsequent macro iterations, the converged solution at  $t_i$  from the previous macro iteration can be used as the predicted value for the fast system iteration.

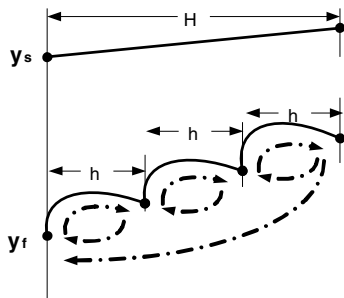


Fig. 4 Multirate method illustrating fast and slow system iterations

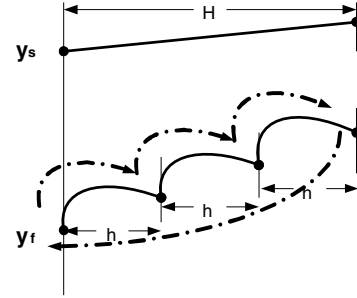


Fig. 5 strategy of iterating on every large step

3. The number of iterations can be further reduced if only one NR iteration is performed at every micro step. There are several arguments to justify this approach. First, if the slow system prediction is poor, the interpolated values will also be poor and considerable effort is expended to find accurate fast values based on incorrect data. Secondly, if it is assumed that there will be more than one macro iteration and the previous iteration values are used as predictions for the fast system, then this process mimics a “decoupled over time” NR iteration at each time step  $t_i$ .

## V. POWER SYSTEM ILLUSTRATION

In addition to differential equations, power systems models also contain algebraic equation constraints. These algebraic variables must be partitioned between the slow and fast subsystems. This issue was addressed in [7]. In this section, the small 6-bus power system shown in Figure 6 is studied. This system contains a two-axis model generator, an IEEE DC-I type exciter/AVR [9], an IEEE Basic Model I SVC [10], a third order induction machine [11], and a constant PQ load.

Three different multirate methods are implemented: 1) repeat the iteration on every small step as in Figure 4; 2) repeat the iteration on every large step as in Figure 5; and 3) compare linear and quadratic interpolation for the algebraic variables. The computation time and error for these three methods are compared in Figures 7 and 8.

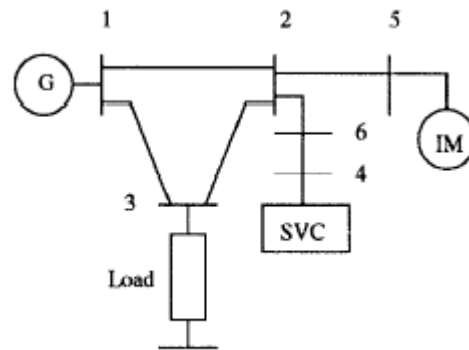


Fig. 6 Example power system

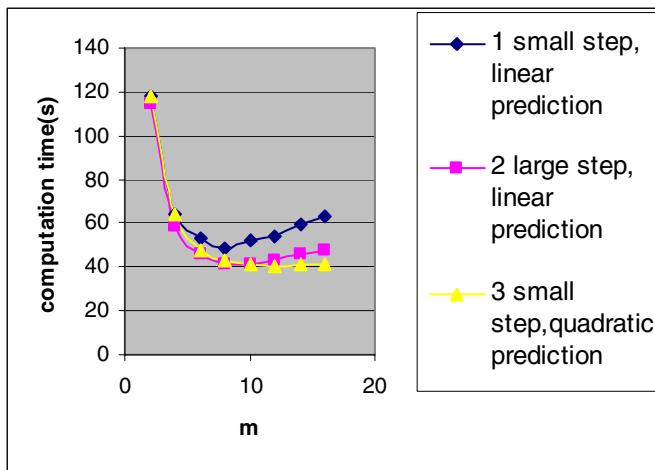


Fig. 7 computation time of different iteration method

Figure 7 illustrates the impact of the three different approaches on computation time as a function of the fast:slow ratio  $m$ . This figure is best interpreted by comparing cases 1 and 2, and cases 1 and 3. In both cases, as  $m$  is increased, the computation time decreases. However, this decrease tends to “saturate” at roughly at 10:1 ratio. Comparing cases 1 and 2 indicate that for larger  $m$  it is more computationally efficient to only iterate the fast steps once per macro step (as shown in Figure 5). Comparing cases 1 and 3 indicates that for larger  $m$  it is more computationally efficient to use a more accurate interpolation method. This is because for larger step sizes, the nonlinearity of the slow system becomes increasingly apparent.

## VI. CONCLUSION

In this paper the multirate method is further studied. The LTE of multirate method integration is derived first, and the calculation of the optimal step ratio is derived. Related techniques of applying the multirate method to power systems are also discussed. At last, by implementing multirate method in a practical power system example, the multirate strategies are proved to be efficient compared with traditional simulation methods.

Future study will extend and generalize the local truncation error results of Section II to other integration methods. The derivation of the local truncation error will also be further generalized to systems of differential-algebraic equations.

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## VIII. BIOGRAPHIES

**Jingjia Chen** obtained his B. S. and M. S. degrees from Wuhan University in 1999 and 2002 respectively. He is currently a Ph. D. candidate in the Electrical & Computer Engineering department of the University of Missouri-Rolla. His interests are in power system modeling, simulation and control.

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**Badrul H. Chowdhury (M’1983, SM’1993)** obtained his M.S. and Ph.D. degrees in Electrical Engineering from Virginia Tech, Blacksburg, VA in 1983 and 1987 respectively. He is currently a Professor in the Electrical & Computer Engineering department of the University of Missouri-Rolla. From 1987 to 1998 he was with the University of Wyoming’s Electrical Engineering department. Dr. Chowdhury’s research interests are in power system modeling, analysis and control.

**Levent Acar** obtained his M.S. and Ph. D degrees in Electrical Engineering from Ohio State University, Columbus, OH in 1984 and 1988 respectively. He is currently as Associate Professor in the ECE department at UMR. His research interests are in intelligent control, large-scale systems and decentralized and distributed control.