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OPTIMIZATION OF AN INVESTMENT STRATEGY
FOR THE McDONNELL DOUGLAS
SALARIED SAVINGS PLAN

BY

RICHARD CHARLES SEIFERT, 1952-

A THESIS

Presented to the Faculty of the Graduate School of the

UNIVERSITY OF MISSOURI-ROLLA

In Partial Fulfillment of the Requirements for the Degree

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

1981

Approved by

Herbert A. Crosby (Advisor) E.C. Bestnoll

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ABSTRACT

An automated technique is developed to maximize the return on investments in the McDonnell Douglas Salaried Savings Plan. Continual guidance is provided to the participant as to the currently preferred type of investment within the Plan, and to the strategic timing of withdrawals from the Plan. The decision criteria within the modelled system address the tradeoffs which result from the participant's past investment decisions, his current tax situation, alternative market conditions, withdrawal penalties and other Plan characteristics. Decisions are made on the premise that historic performance patterns will continue, though adaptive procedures are included to react to changing conditions.

To test the benefit of the technique, a simulation employing its decisions is compared to simulations of twelve arbitrary or blind investment strategies. Also a control of not participating in the Savings Plan is considered, all simulations being performed under an identical set of conditions. It is shown that while most any pattern of participation in the Plan will benefit the employee, a structured approach to investment decisions yields a substantial additional return on his savings. A further advantage of stability in the growth of the holdings is noted.

PREFACE

The author wishes to express his appreciation to Dr. Herbert Crosby for his advice and criticism in the preparation of this paper. Thanks are also extended to Bruce Meng for his assistance with word processing equipment, and to McDonnell Douglas Corporation for providing a valuable employee benefit in the Salaried Savings Plan.

McDonnell Douglas Corporation has not approved or authorized the preparation of this document. Nor has McDonnell Douglas, Bankers Trust Company or the Internal Revenue Service confirmed the validity or accuracy of any statement made herein. This paper is purely an academic exercise, and recommendations for actual investment decisions are not implied.

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I. INTRODUCTION

Salaried employees of the **McDonnell Douglas Corporation** (McDonnell) with six months tenure are eligible to participate in the **Employee Savings Plan of McDonnell Douglas Corporation - Salaried Plan** (Plan). An employee may invest a portion of his salary in the Plan through payroll deductions. McDonnell contributes to the Plan an amount equal to 75% of the employee's payment. Certain options are presented to the participant with regard to the investment of his contributions and to withdrawals from the Plan. This paper seeks a technique for determining the optimal decisions for the purpose of maximizing the total long term return on the investment. Prior to the start of research, a goal was set of realizing a 20% improvement over the return of any arbitrary investment strategy.

A. SAVINGS PLAN SUMMARY

All features and rules of participation in the Plan are found in Reference 1. A summary of those items considered relevant to this paper is presented in the following paragraphs.

The Plan is structured in the form of three mutual funds (Funds). Fund "units" are purchased by the employee's and McDonnell's contributions in amounts determined by the "unit values" of the Funds. Upon withdrawal from the Plan, the units are redeemed according to their then current unit values. The three investment vehicles are designated: Fund A, a fixed income fund; Fund B, an equity fund; Fund C, a McDonnell common stock fund. Unit values for the Funds are established monthly

and equal the total market value of their assets at the close of the previous month's last business day, divided by the total number of units owned by all participants. Since 1969 the Trustee of the Plan has been the **Bankers Trust Company** (Trustee).

The participant may invest 2%, 4%, 6% or 8% of his base salary in the Plan. He may direct that his contributions be put entirely into any one of the Funds, or he may specify two Funds which each receive half that amount. All McDonnell contributions, equal 75% of the employee's, are currently placed in Fund C. Changes in the participant's rate of contribution or choice of Funds may only be made twice a year, in February or August, effective the following month.

The employee may withdraw amounts from the Plan resulting from his contributions and that portion of McDonnell contributions that are vested. Vested units are those purchased by McDonnell more than 24 months prior to the withdrawal. Those units purchased with McDonnell contributions within the two preceeding years cannot be withdrawn, but remain credited to the participant's Plan account. Upon making a withdrawal, all contributions are suspended for the six months following; that is, payroll deduction stops and McDonnell contributions are forfeited for that period. Partial withdrawals may be made from Funds A and B, but all vested assets of Fund C must be withdrawn. The participant may also suspend his contributions at any time without withdrawing. He of course foregoes McDonnell's contributions during that period. Restart of payments after a suspension may only be arranged in February or August.

All of the participant's own contributions are taxed according to his personal income tax conditions prior to the payroll deduction. However, McDonnell contributions and any gains or losses realized on either source are tax-sheltered within the Plan. Upon making any withdrawal, the employee is liable for income taxes on the full value of his account that could be withdrawn, minus any amounts therein previously subjected to taxation. The resulting value is treated as ordinary income. (Actual tax liability is more complicated, sometimes involving small long term capital gains, but this paper will consider the difference negligible.) Suspending payments without withdrawing does not trigger a tax charge.

B. INTENT OF AN INVESTMENT STRATEGY

The goal of this study will be to maximize the total value of an employee's savings over the long term. In contemplating an investment in the Savings Plan, the employee is faced with three primary decisions. The first is whether it is beneficial to him to participate. Considering that McDonnell will put \$.75 into the Plan for every dollar he contributes, it would seem logical that it is to his advantage. Even so, our strategy of investment should test this assumption continually.

Secondly, having decided on participation, which Fund provides the "best" means of investment? A look at Figure 1 shows that Funds A, B and C have behaved very differently from one another. Our investment strategy must choose the most favorable Fund(s) for his contribution each February and August. McDonnell of course dictates that its matching funds go to Fund C.

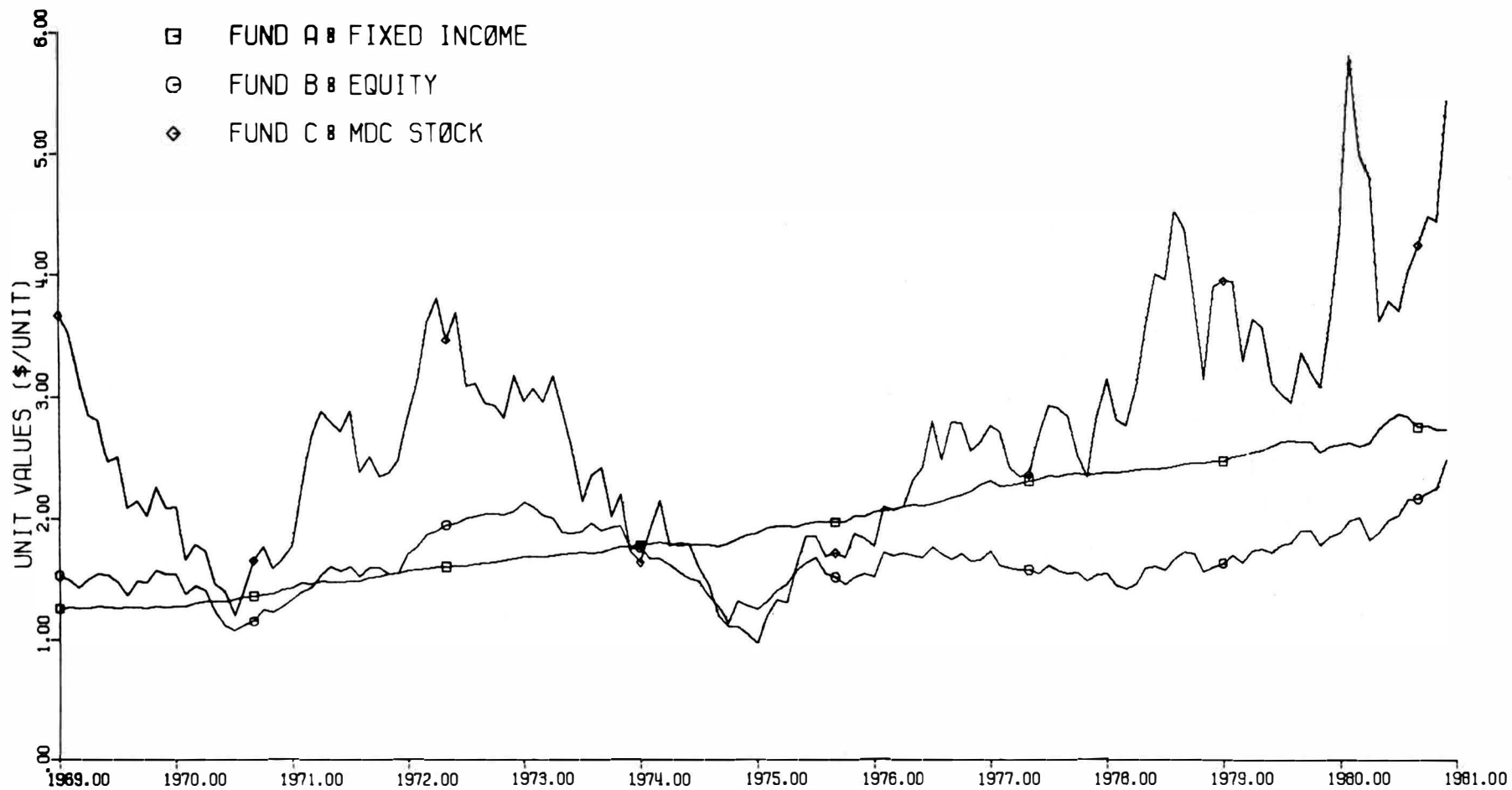


Figure 1 Plan Unit Value Performance - 1969 to 1980

The third, and most complicated decision involves a series of tradeoffs: Is it ever economically advantageous to withdraw from the Plan? Obviously to maximize the dollar amounts put into the Plan, the employee would always contribute (until retirement), never forfeiting any McDonnell contributions. But noting the volatility of Fund C, is it ever possible McDonnell contributions when unit values are very high, could be more than offset by their subsequent drop? Furthermore, could building up large tax-sheltered gains over many years actually be counterproductive, when the huge tax liability is eventually realized at very high rates? The investment strategy must address these questions.

C. APPROACH TO OPTIMIZATION

Having identified the three basic investment decisions which must periodically be made, criteria are sought for making them toward the goal of maximizing the total of savings. Further, a means of testing the effects of decisions and the validity of the decision criteria is desirable. A three-fold approach to these ends was undertaken: Past performance data of the three Funds were analyzed; A computer model of the Savings Plan was developed, permitting comparisons of competing decisions; A series of simulations were run to compare the results of the "optimal" strategy to that of arbitrary decisions under identical conditions.

The statistical analysis of the Funds' unit value history is the subject of Section II of this paper. A long term trend was derived for each Fund, and statistics were gathered on their actual deviations from the trend values. The Fund unit values are viewed as generally

following the trend over time, but corrupted by a stochastic noise component. The analysis was conducted because the Funds' behavior patterns would influence the timing and utility of any investment in the Plan.

The Comprehensive Savings Plan Model (Model), discussed in Section III, includes the relevant rules and features of the Plan and various external considerations. It also serves as an ongoing accounting system, reflecting the effects of all past decisions. Inputs are current conditions and unit values, entered monthly. Also a part of the model are the decision criteria. Decisions are made by considering all alternatives, extrapolating the effects of each into the future according to future expectations, and choosing the course with the best results. Expectations of the future are involved, but these are not specific predictions. For example, while a Fund unit value may be very high or low at present, one would expect it to eventually reapproach its trend (zero noise) value. The Model may also be set up to ignore the "optimal" decisions and use those input by the operator.

To test the value of the decision criteria, simulations were run to represent a 36 year career of a McDonnell employee. A history of his salary and income was established to drive the Model. Also a data base of Fund unit values was created reflecting the statistical nature of the actual Fund data. Section IV describes how a simulation making use of the optimal investment decisions compared to twelve alternate investment strategies under identical conditions. Also, a control case of not participating in the Plan was simulated.

II. STATISTICAL ANALYSIS

An important part of making meaningful investment decisions with regard to Plan participation is an understanding of the nature of the component Funds. Standard statistical techniques were applied to the available data on past performance: Fund unit values for the years 1969 through 1980. This section documents the analyses performed in efforts to identify long term trends of each Fund, as well as any short term characteristics. The results herein serve two purposes: First, the calibration and optimization of the Comprehensive Model's decision criteria; Second the creation of an extensive, statistically consistent data base with which the Model will be tested. Reference 2 provides a thorough review of statistical analysis techniques with an emphasis on their application to financial/economic data.

A. FUND TRENDS AND NOISE

The thrust of this study of past performance data was the identification of a basic trend that would characterize each Fund over time. The Fund unit values are viewed in this paper as reflecting some long term trend, but distorted by a stochastic noise component. Three types of trends were considered and statistical measures were applied to each. The three candidate trends were a constant, a linear rate (over time), and a logarithmic rate. Other possible trend models such as higher order polynomials or trigonometric functions were rejected outright. Theoretical justification for polynomials could not be established and the Fund behaviors were not believed to be purely periodic (Note Figure 1).

The constant trend is straightforward and simple. The expected Fund value at any time would be some constant value plus the noise component. This trend value was established as being the arithmetic mean of all available data. Subtracting the mean from each data point would leave the set of residual noise components. Were this trend to be chosen as the best for any of the Funds, the simulation mechanics would indeed be simplified, but the implication would be disappointing. Most every financial investment is made with the prospect of some return or growth. A constant trend indicates there generally is no return on this investment.

To facilitate the concept of growth, a constant rate trend was tested. A slope-intercept equation was derived from the historic data, with time as the independent variable, employing the linear least squares (LLS) method. (The desirability of LLS for curve fitting is a common topic in statistics literature, including Reference 2, and won't be discussed here.) Said linear equation portrays a Fund which increases by a constant amount over each time period, except of course for the deviations due to a noise component.

The third trend model, a logarithmic rate, is intuitively more satisfying for economic applications. It represents a constant rate of growth wherein all returns are continually reinvested or compounded. Indeed, all interest, dividends and gains realized within the Plan are reinvested, though their importance varies between the different Funds. The trend equations were determined by taking the logarithm of each data point and subsequently using LLS to produce the slope-intercept

formulation. All statistical measures are then performed in this logarithmic domain. The symbol "L\$" will be used to distinguish these cases from those in the linear "\$" domain. As the discussion to follow will indicate, the logarithmic rate produced the best results for all three Funds, and was therefore chosen to model each.

Having established the three candidate trend equations for each of the three Funds, statistical measures were applied to the residual stochastic noise components. Table I presents the nine trend equations and a summary of the statistical results. Each is the product of 144 data points from the years 1969 through 1980. Included in the Table are the following: Trend equation (approximate); Indicated annual growth; Standard deviation of the noise component about the trend; Autocorrelation time constant of the noise; Number of times the data crosses the trend line (or zero crossings in the residual noise). Mean of the noise components are zero by definition.

1. Fund A Fund A is designated a fixed income fund where growth is achieved principally through the reinvestment of the interest on the securities held. Reference 3 indicates this Fund owns a variety of corporate bonds and notes having fixed income (coupon), plus a number of government obligations. Their maturities are mostly under ten years. Historically, Fund A has displayed consistent and steady growth. The few instances of a noticeable drop in its unit value correspond directly to sharp rises in market interest rates. Debt instrument market values generally fall so their effective yields will rise with market interest rates.

TABLE I
STATISTICAL SUMMARY OF CANDIDATE TRENDS

FUND	TREND TYPE	TREND EQUATION	ANNUAL TREND GAIN	STANDARD DEVIATION ABOUT TREND	TIME CONSTANT (MONTHS)	ZERO CROSSINGS
A	Constant	\$ = 1.9352	0	\$.4689	36.8	1
A	Linear	\$ = .1346*T - 8.1524	\$.1346	\$.0500	6.7	15
A	Logarithmic	L\$ = .0308*T - 2.0377	7.36%	L\$.0096 \$.0463	8.5	19
B	Constant	\$ = 1.6394	0	\$.2540	10.1	13
B	Linear	\$ = .0291*T - .5400	\$.0291	\$.2332	9.9	22
B	Logarithmic	L\$ = .0079*T - .3829	1.84%	L\$.0617 \$.2329	9.2	26
C	Constant	\$ = 2.6672	0	\$.9392	13.0	14
C	Linear	\$ = .1388*T - 7.7336	\$.1387	\$.8069	10.9	12
C	Logarithmic	L\$ = .0209*T - 1.1689	4.93%	L\$.1414 \$.7914	11.0	18

Time is in units of years past the turn of the century. e.g. T=87.5833 indicates August 1987

Modelling Fund A with a constant is obviously a poor choice. As one can see from Figure 2, the Fund shows continual growth and only crosses its mean once. Figure 3 displays graphically some of the statistics found in Table I. At the top is a bar graph of the distribution of the actual unit values for the Fund, centered about its mean. Overlaying the bars is a normal (Gaussian) distribution curve having the same mean and standard deviation as the Fund data. Above it are arrows placed at intervals equal to one standard deviation, also centered about the mean (μ). The lower portion of the Figure is a plot of the autocorrelation of these twelve years of Fund data. Lines within the graph show where the function reaches the value $1/e = .3679$, yielding its time constant. Discussion of this specific autocorrelation technique is available in Reference 4. As the reader can see, the raw Fund A data makes a poor match to the Gaussian function. Its time constant is longer than three years suggesting long term errors in this trend model.

The linear trend in Figure 4 is seen to be a much better model for Fund A than its mean. With a slope of .1346 (\$/yr) this trend line and the actual data cross 15 times. The distribution of the residual noise components (Figure 5) matches the Gaussian curve very nicely and yields a standard deviation about one tenth that of the constant trend. Note that the bin intervals will change from plot to plot but are rounded to exactly one fourth the interval of the numbers along the axis (.25, .1, .05, .0125, or .0025). The autocorrelation function with the linear trend removed drops more quickly than the constant's, and it includes a wavy pattern implying some higher frequency content. The resulting

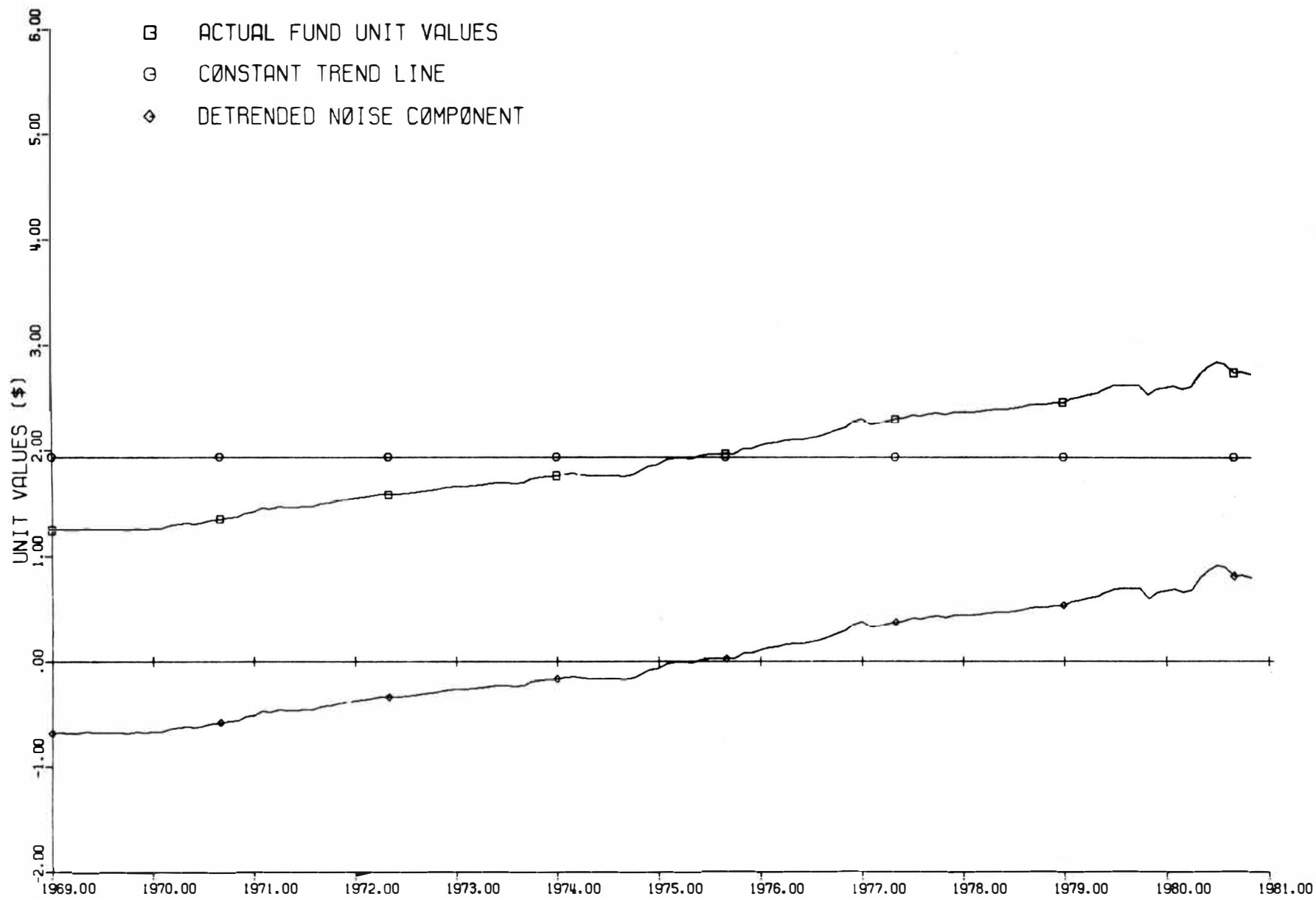


Figure 2 Fund A History - Mean Removed

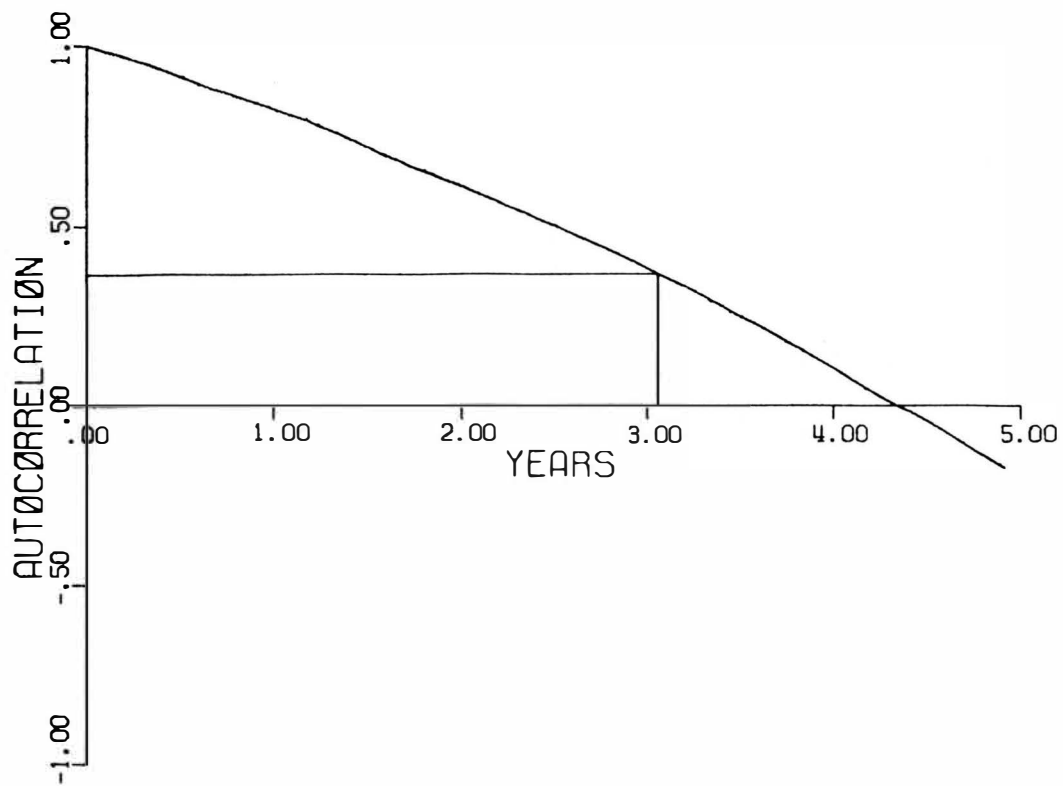
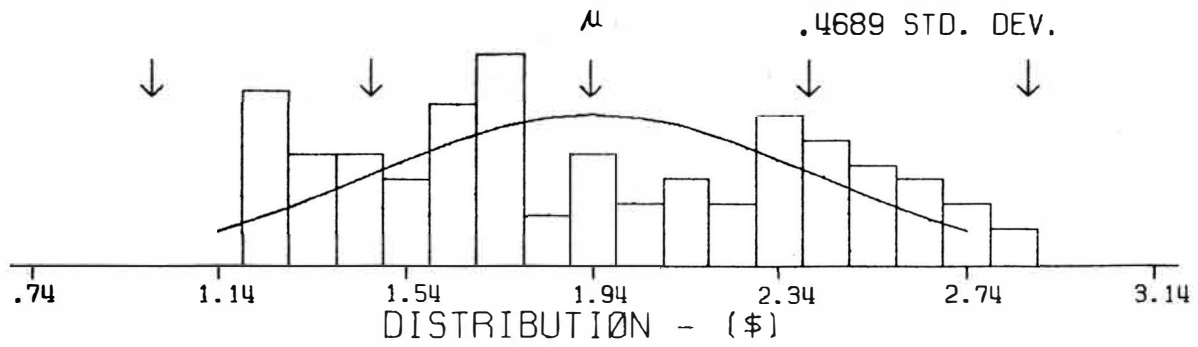


Figure 3 Fund A Statistics

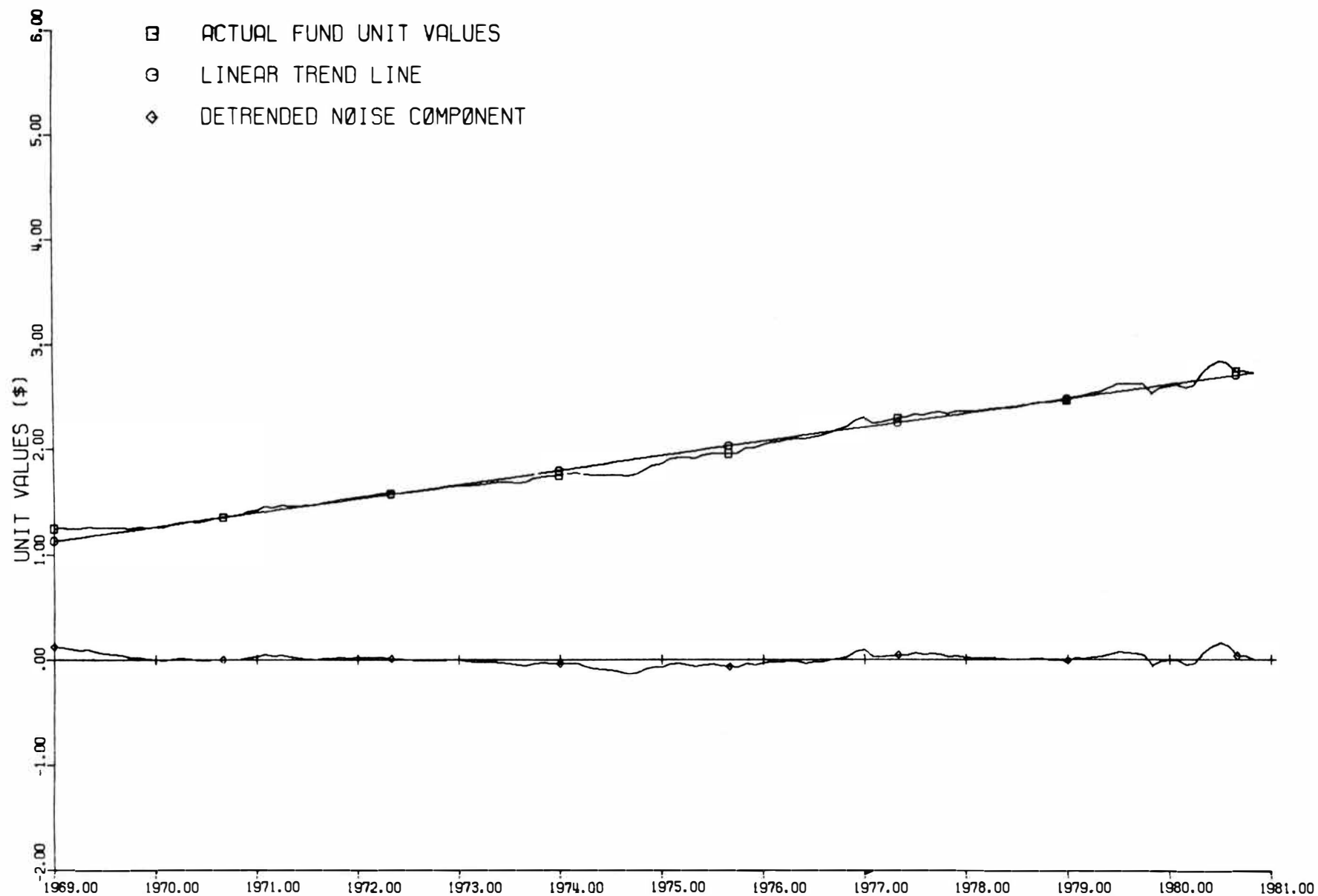


Figure 4 Fund A History - Linear Trend Removed

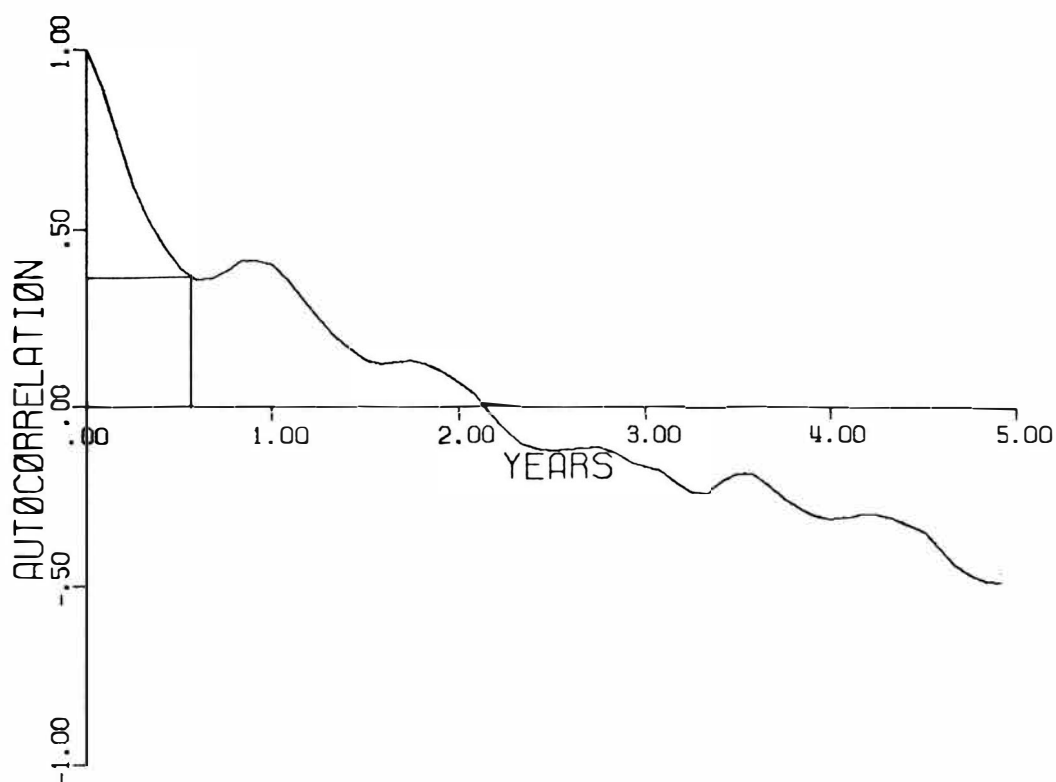
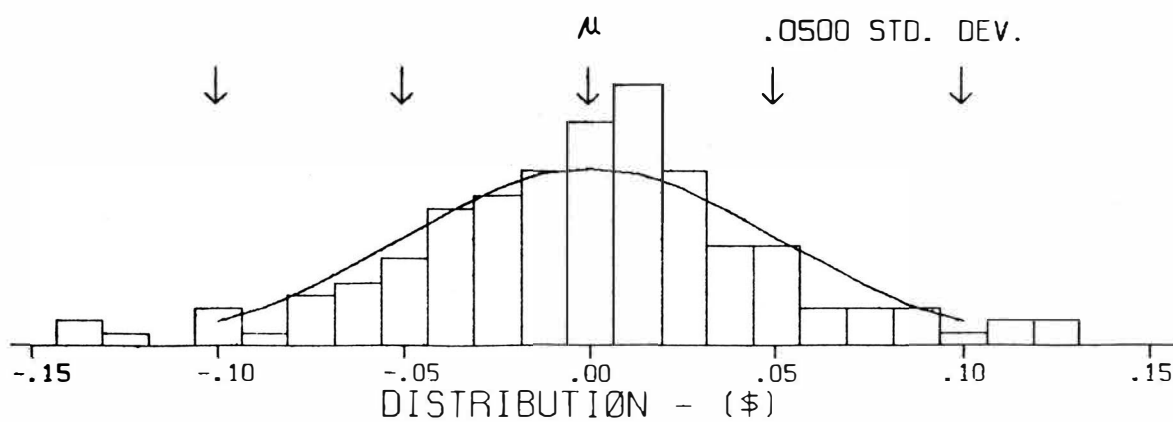


Figure 5 Fund A Statistics - Linear Trend Removed

time constant is 6.7 months, although were the initial dip and rise in the function not so pronounced, a 12 month time constant would have seemed more realistic.

The third trend, and again the one chosen to represent this Fund in the Comprehensive Plan Model, is the logarithmic rate. The derived log equation for Fund A has an annual aggregate gain of 7.36%. In Figure 6 it is seen to be an even better match than the linear trend, possessing curvature similar to the true performance set, and crossing that data 19 times. In viewing the plots of Figure 7, the reader is reminded that these statistics were gathered in the logarithmic domain. The distribution is somewhat less pleasing than the linear trend's when compared to their Gaussian equivalents, but it is definitely superior to the constant trend. It should be noted that there are only 144 data points, and with up to 25 distribution bins available in the plot, some coarseness is to be expected. The autocorrelation function with the log trend removed shows a faster drop than with the other trends. Its 8.5 month time constant should probably be compared to the 12 month value discussed above, and to the 36.8 month time constant of Figure 3. Figure 7's plot also suggests the presence of a dominant frequency component in the noise residuals, having a total period of about 5 years. (Consider the autocorrelation of a pure sinusoid: It oscillates between +1 and -1, crossing zero at the 90° and 270° phase shifts.) Also, less high frequency content is visible in this plot. In order to compare the standard deviation of noise about the log trend to that of the other trends, the trend values were converted point by point to the linear domain (as pictured in Figure 6), subtracted from the original

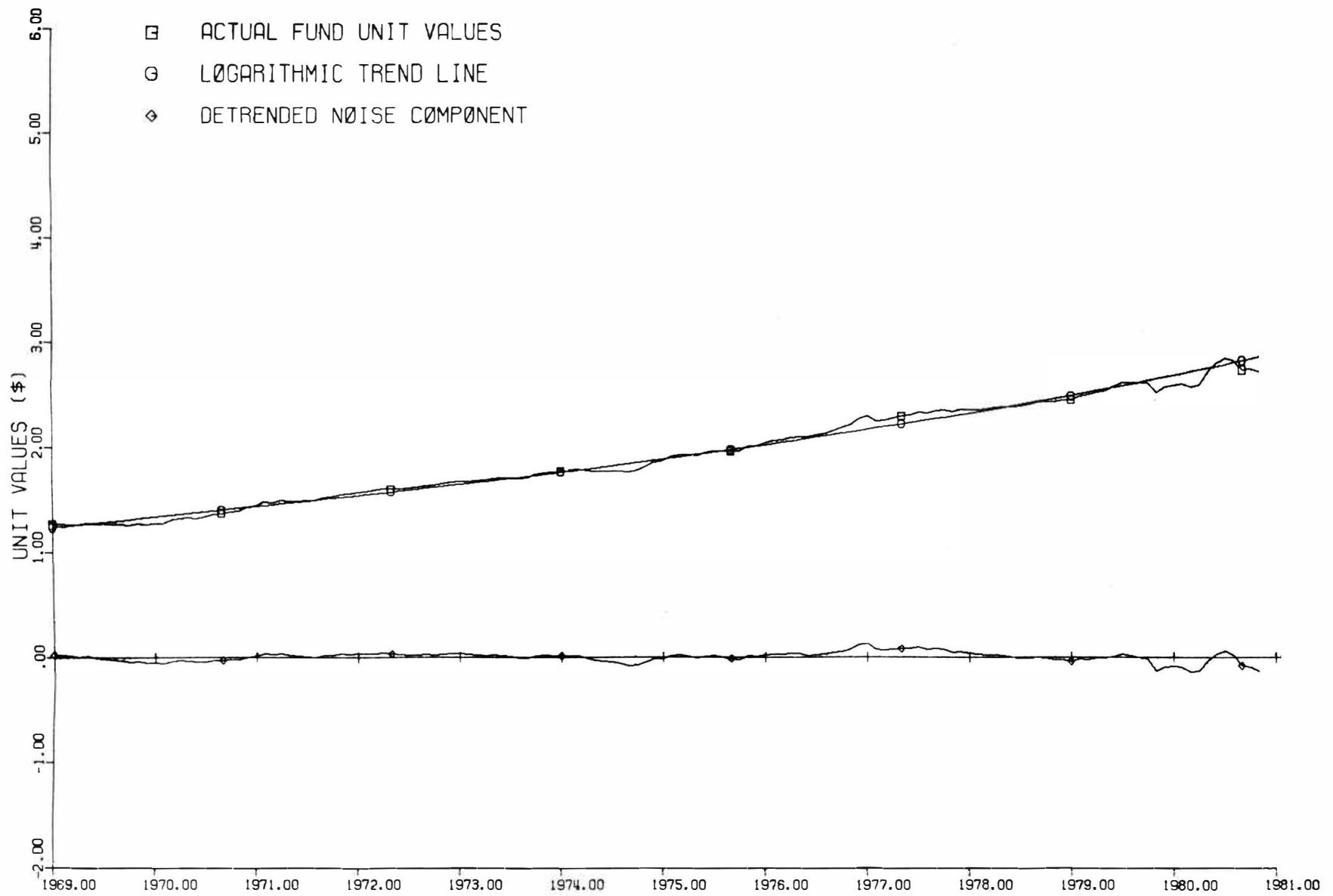


Figure 6 Fund A History - Logarithmic Trend Removed

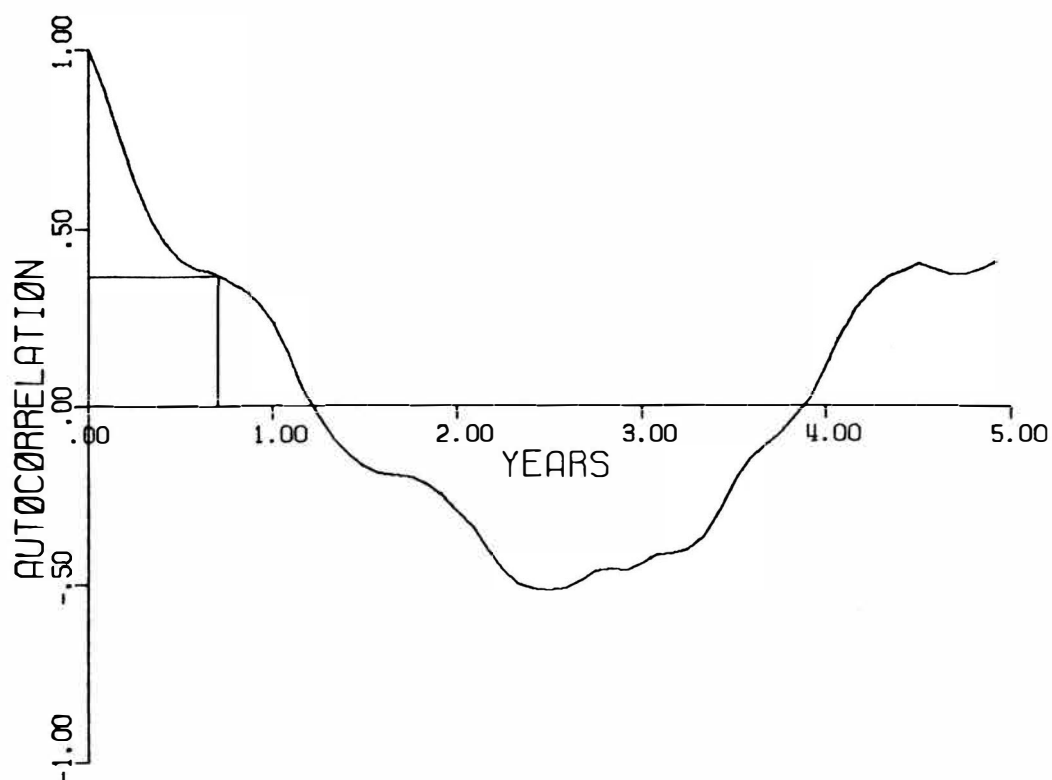
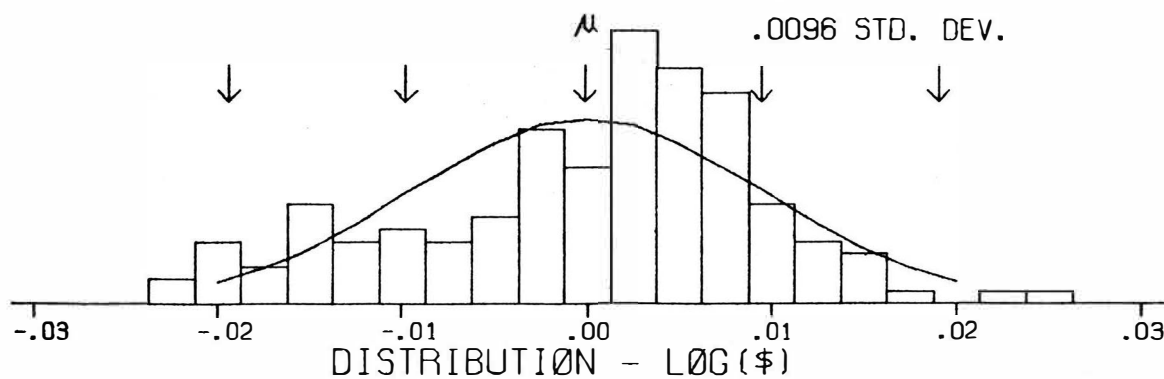


Figure 7 Fund A Statistics - Logarithmic Trend Removed

data set, and the statistic then taken; thus the double entry on Table I. The standard deviation of \$.0463 is even lower than the linear trend's \$.0500 . A simple computation of the antilog of .0096 would not have been meaningful.

2. Fund B Fund B is an equity fund, its holdings a diversified portfolio of common stocks of perhaps 75 to 100 companies. Specifics as of November 30, 1980 may again be found in Reference 3. The unit value reflects the current market values of the component securities; dividends are continually reinvested. As might be expected with equity holdings there are often noticeable changes in their worth month to month, both up and down. With its diversification, though, Fund B is not extraordinarily volatile.

As in the analysis above, equations for the three candidate trends were derived and they were compared to the historic Fund B data (Figures 8, 10 and 12). In this case however, the differences in results between the trends were not great. The slope of the linear trend is only .0291 (\$/yr), and the logarithmic trend yields an annual growth rate of only 1.84%. (This speaks poorly of the Trustee's management of the Fund.)

The logarithmic trend was again chosen for Fund B because its statistics were the best. Referring to Table I, its noise's standard deviation of \$.2329 (linear domain) was lowest by a slight margin. The 9.2 month time constant is the shortest, though all are within a one month range. The logarithmic trend follows the actual data best, crossing the same 26 times. Viewing Figures 9, 11 and 13, one sees that

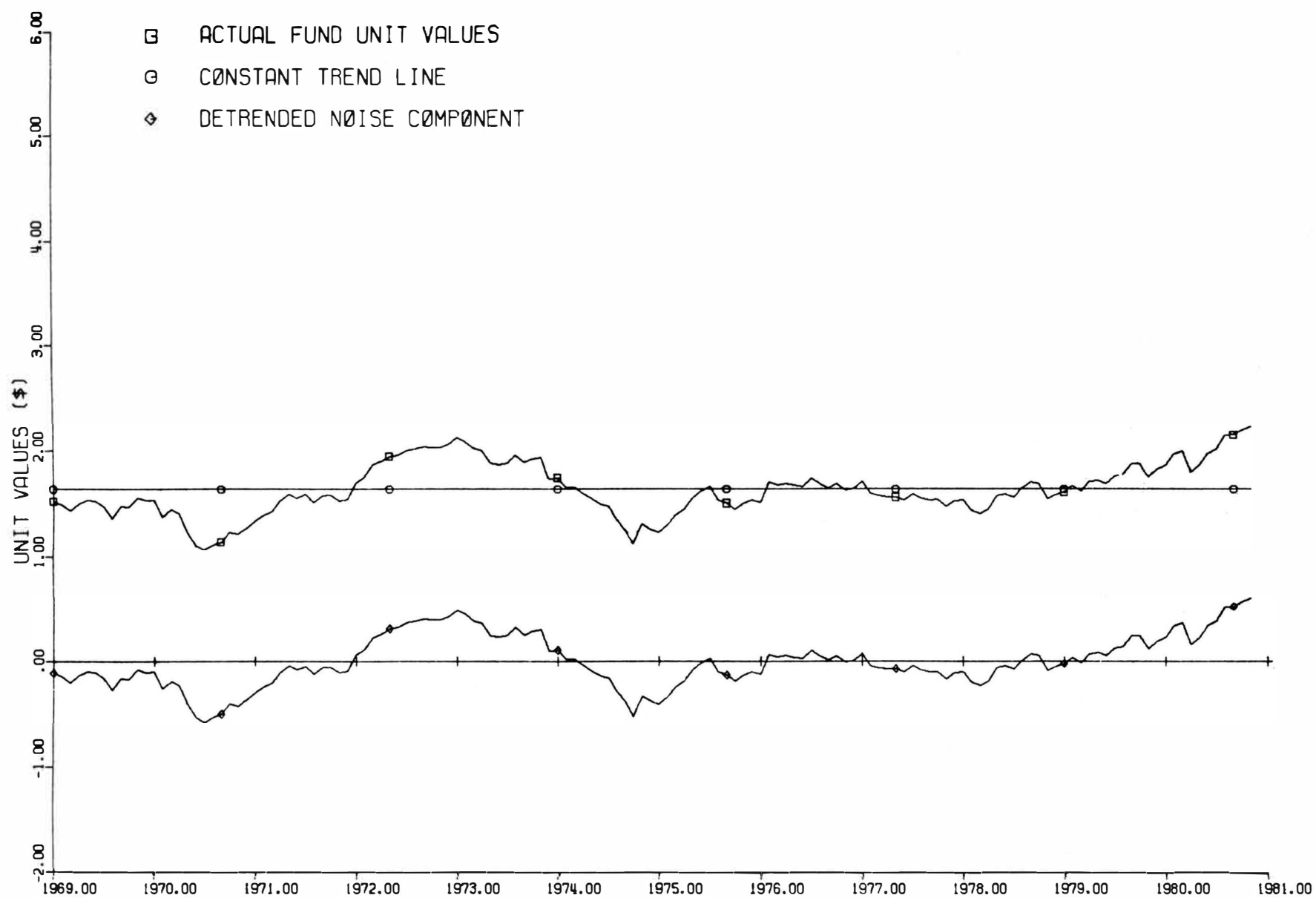


Figure 8 Fund B History - Mean Removed

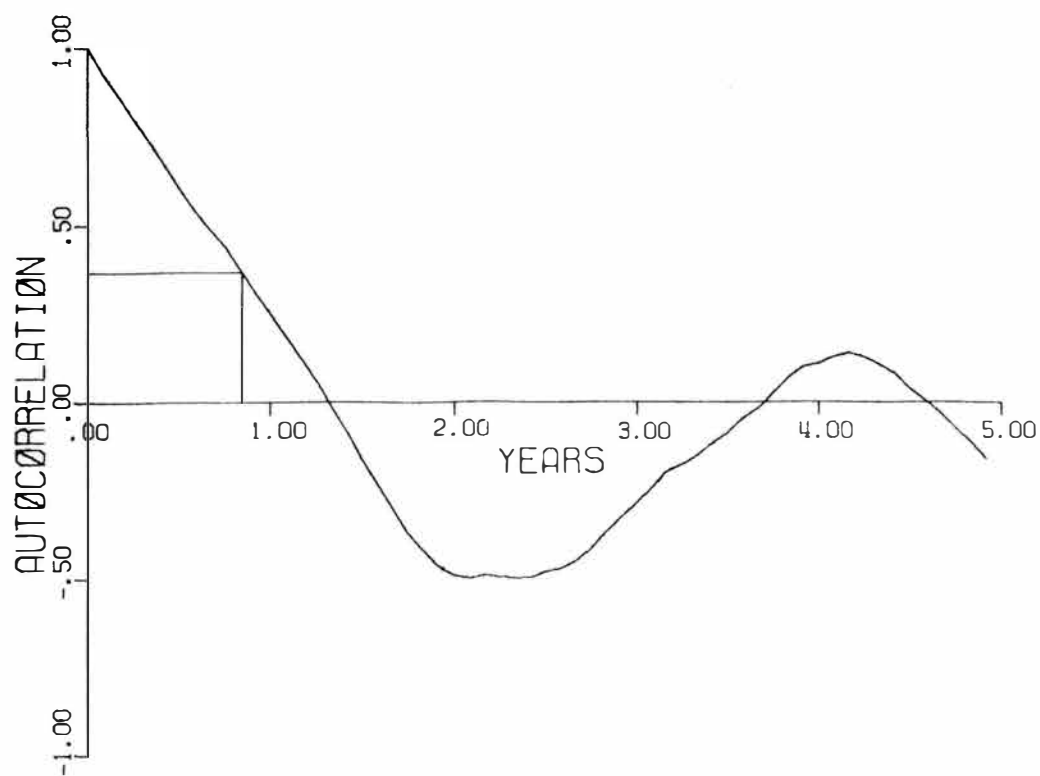
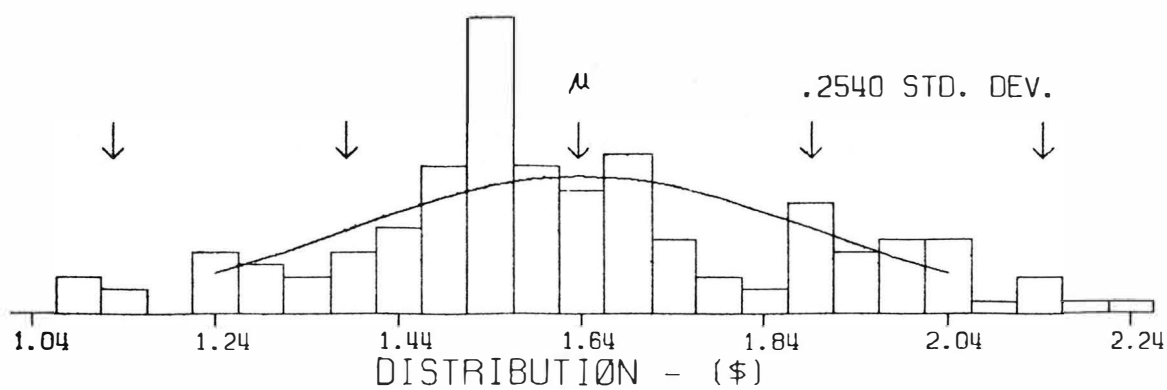


Figure 9 Fund B Statistics

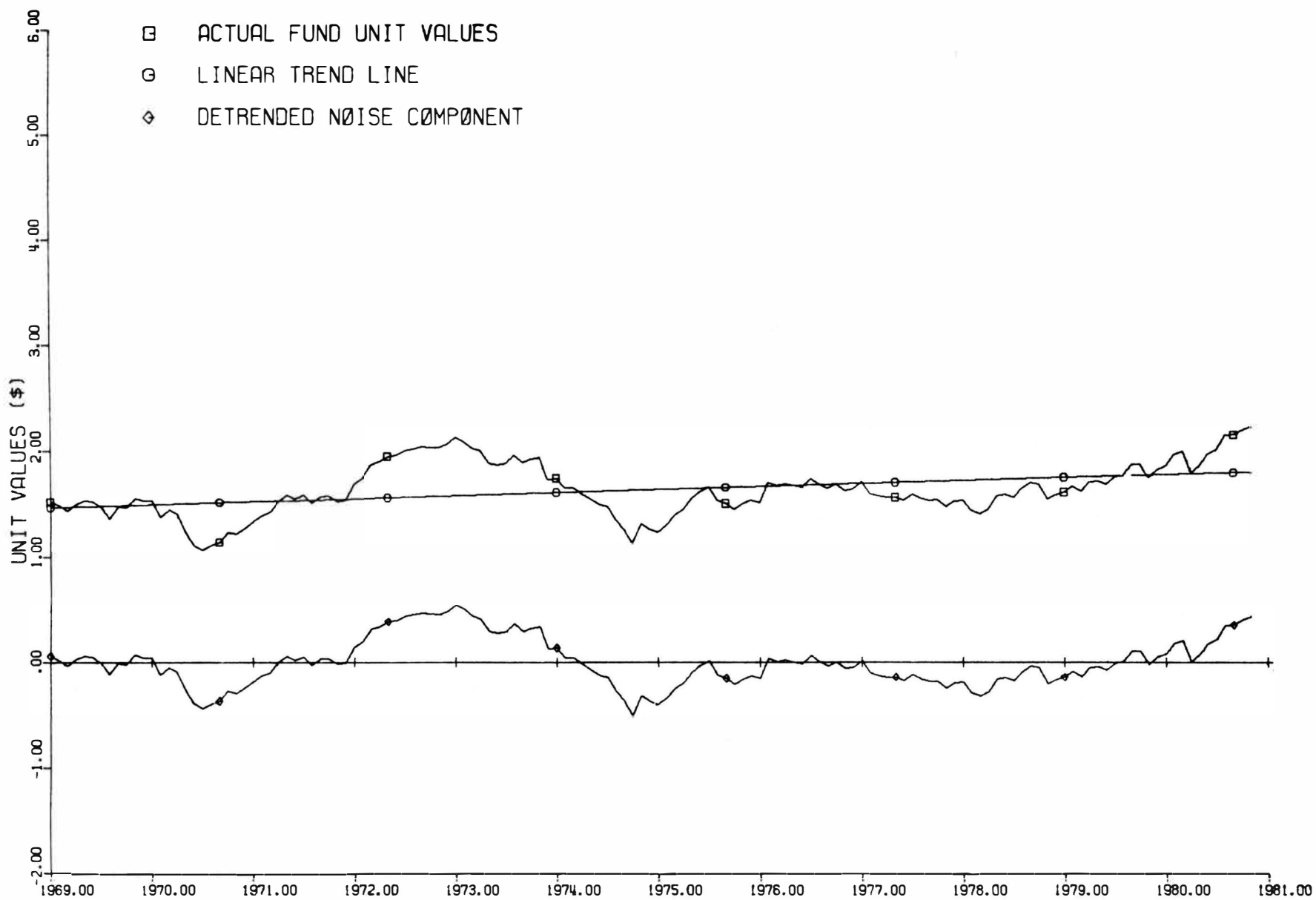


Figure 10 Fund B History - Linear Trend Removed

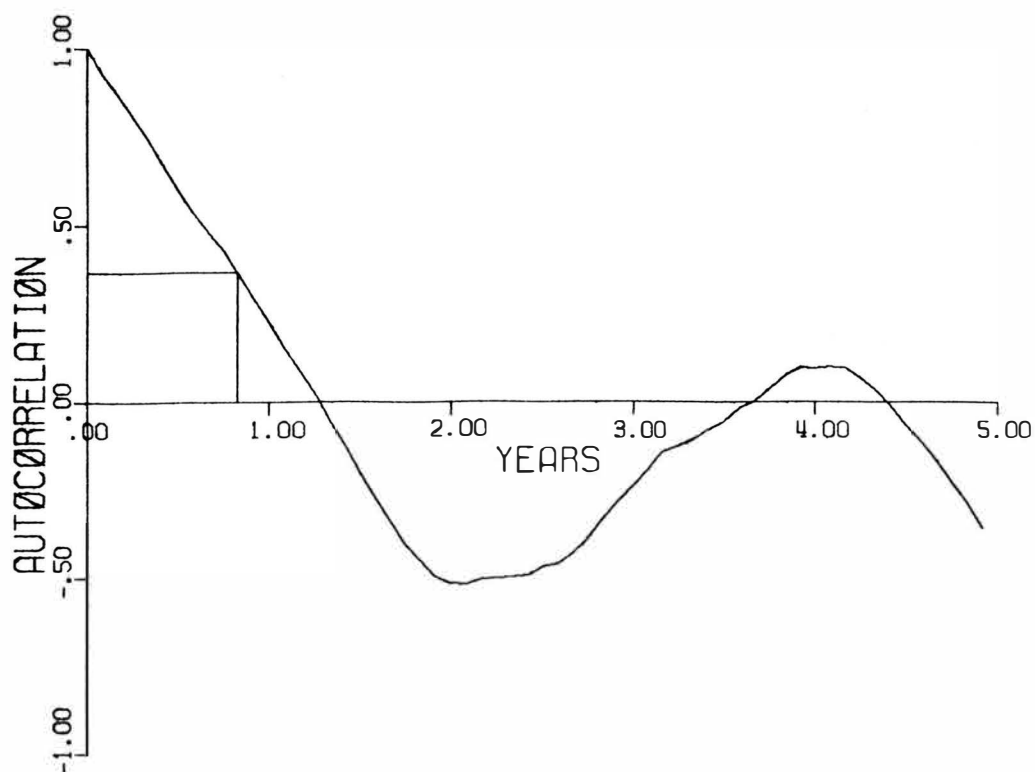
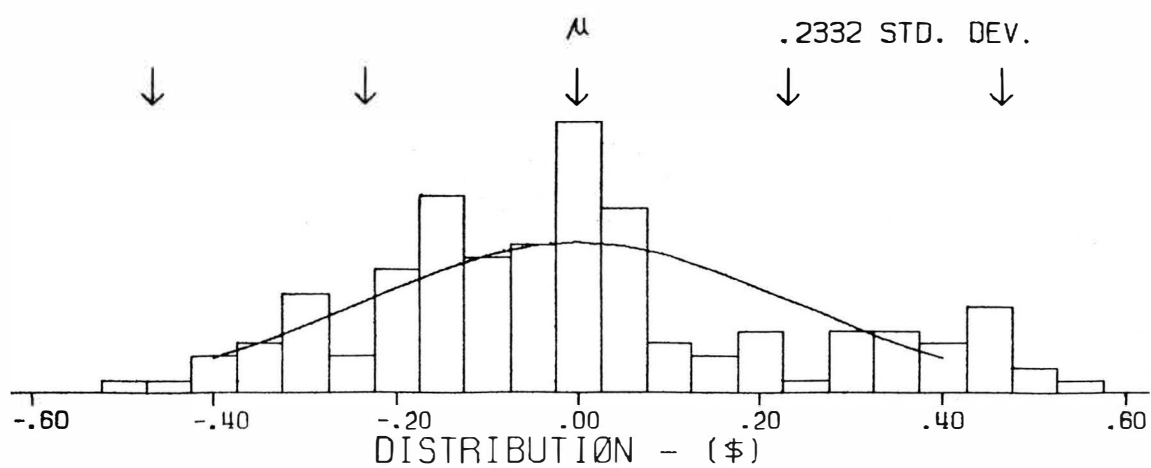


Figure 11 Fund B Statistics - Linear Trend Removed

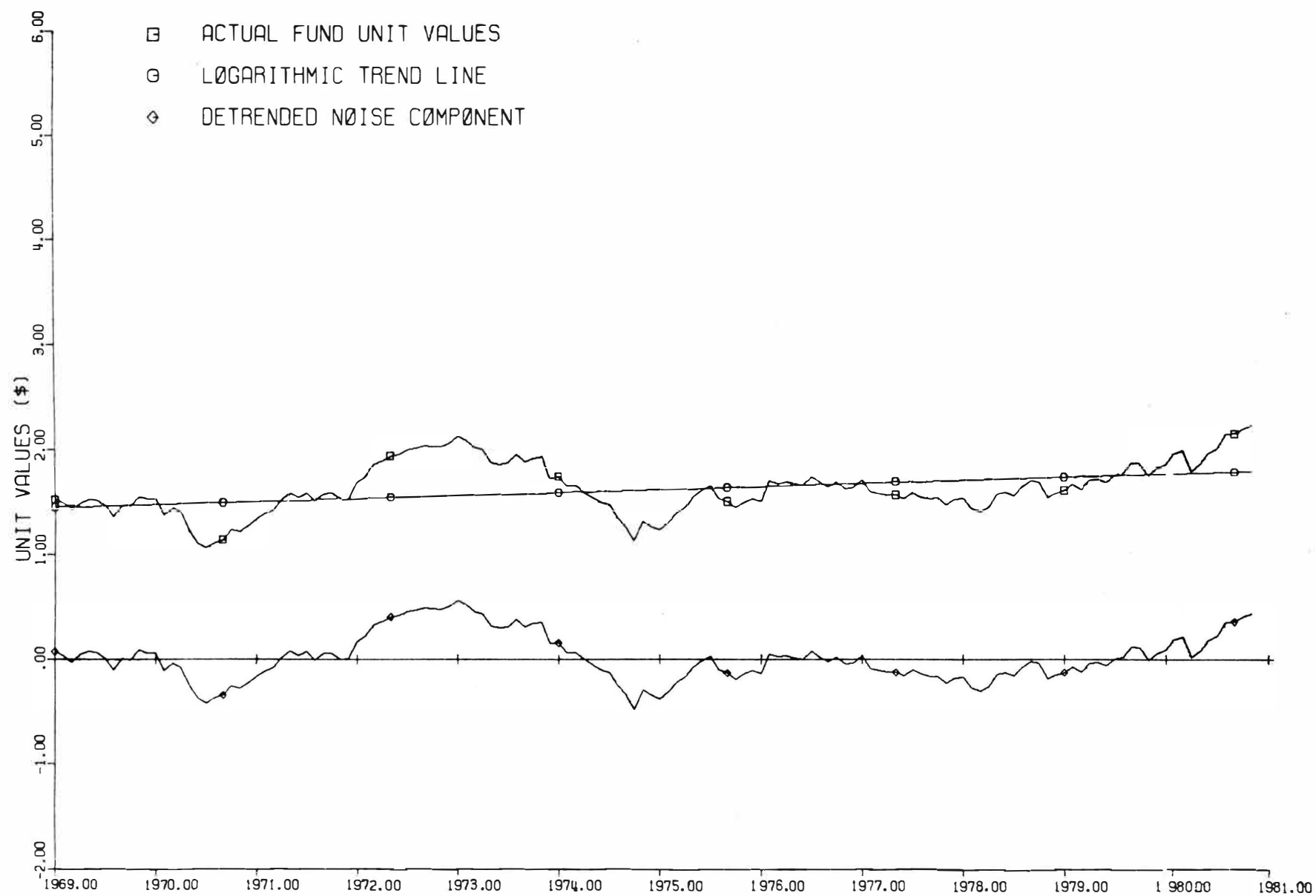


Figure 12 Fund B History - Logarithmic Trend Removed

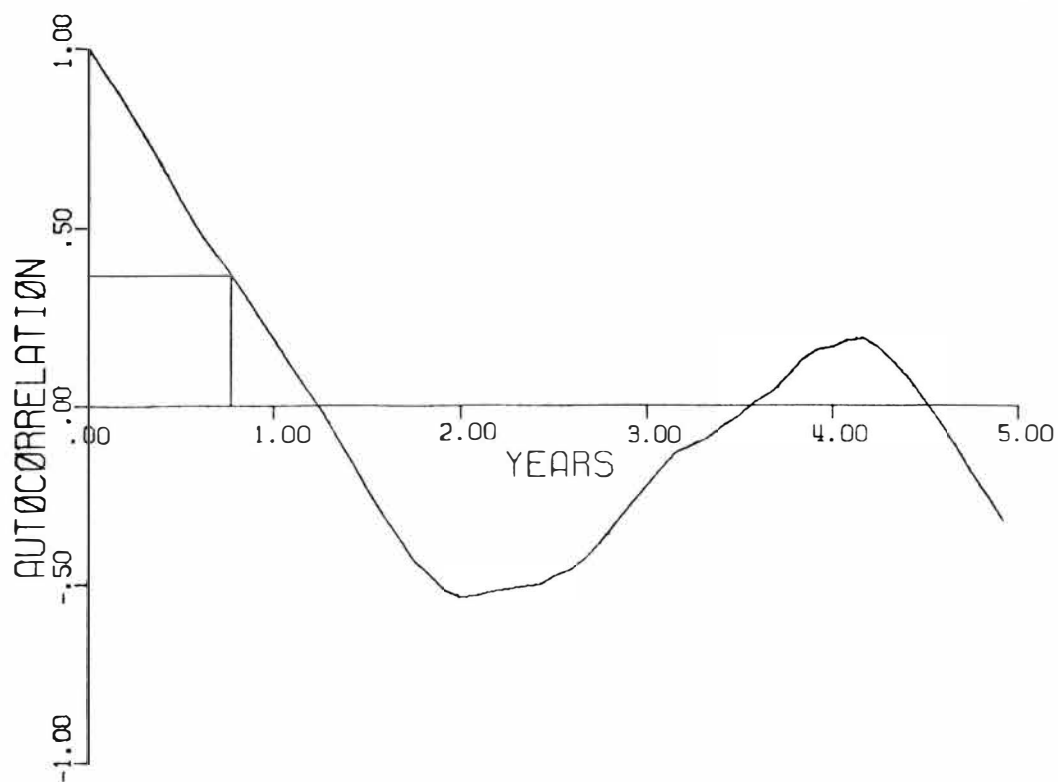
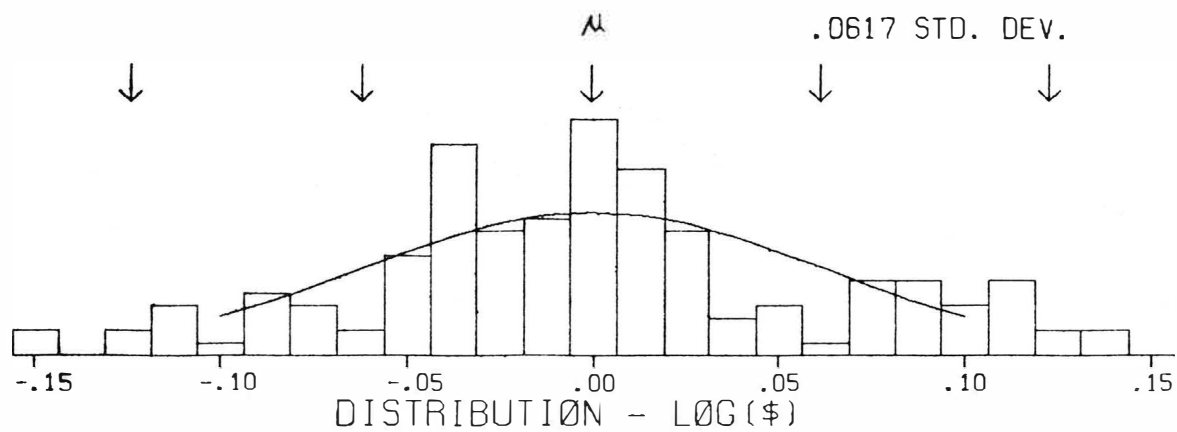


Figure 13 Fund B Statistics - Logarithmic Trend Removed

none provides an outstanding match to the Gaussian distribution. All three indicate an inherent noise frequency component with about a four year period.

3. Fund C Fund C is dedicated strictly to the accumulation of McDonnell Douglas Corporation common stock. Also managed as a mutual fund, with dividends reinvested and expenses deducted, Fund C's unit value closely follows the directions of the stock's market price. A ratio of 8.5 units per share is typical. Dividend yields are traditionally low and thus are not a large factor in the Fund's performance. A look back at Figure 1 shows that Fund C is quite volatile, at least in comparison to A and B.

As one can see from Figure 14, Fund C's volatile "noise" component is the dominant characteristic of its long term behavior. With a mean of under 2.67 (\$/unit), it once climbed to almost 6, while at another time dipped below 1. The mean value was crossed 14 times. The distribution of the raw Fund unit values seems to be Gaussian (Figure 15) and a 13.0 month time constant is indicated. Its standard deviation is of course the largest yet at \$.9392 .

Some long term growth is indicated with the linear rate (Figure 16) yielding somewhat better statistics than the constant, and the logarithmic rate (Figure 18) provides a slight improvement over the linear trend line. Their respective standard deviations, \$.8069 and \$.7914, are noticably lower than the raw data's statistic, but they are high in comparison to Funds A and B's results. The linear rate's

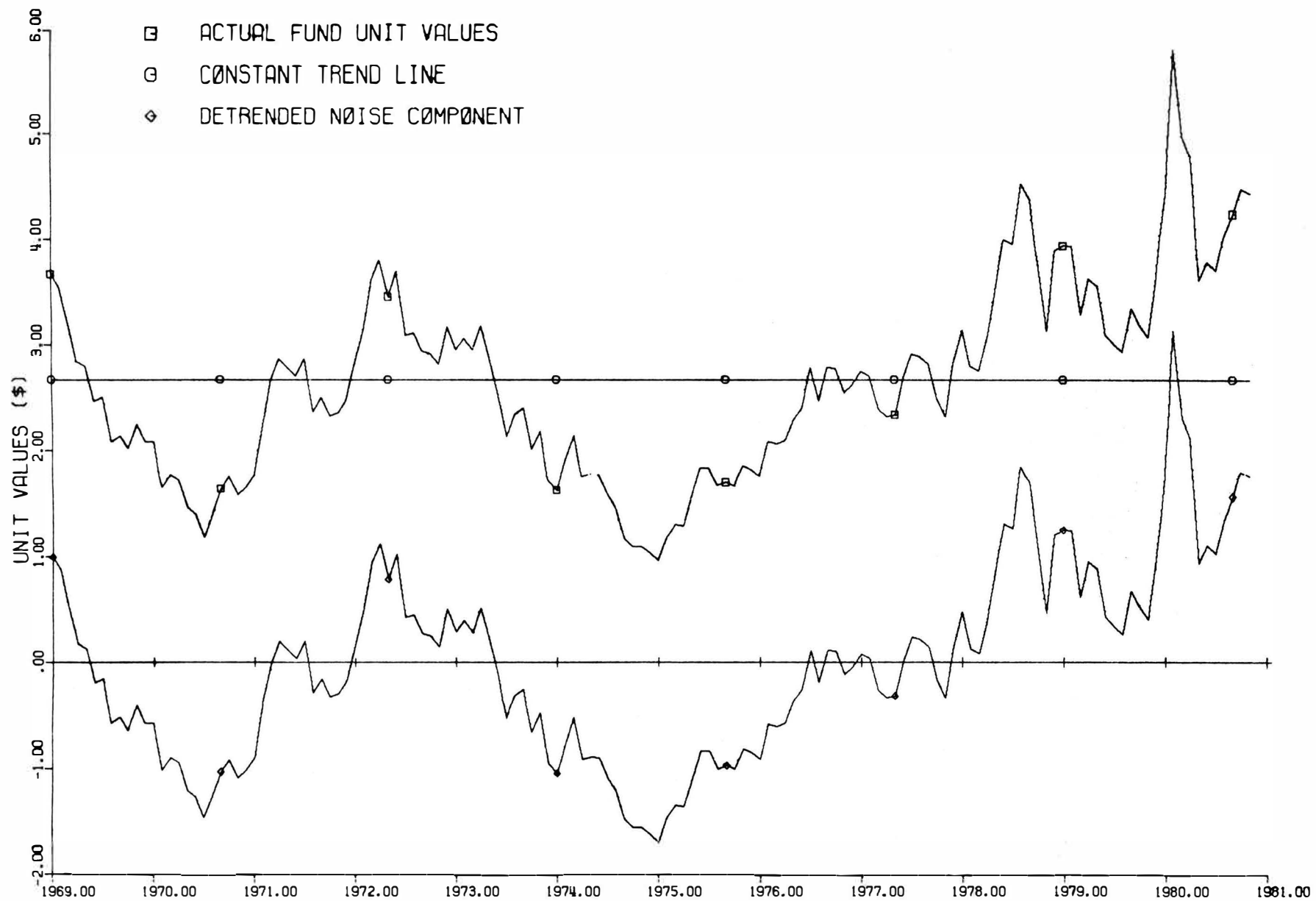


Figure 14 Fund C History - Mean Removed

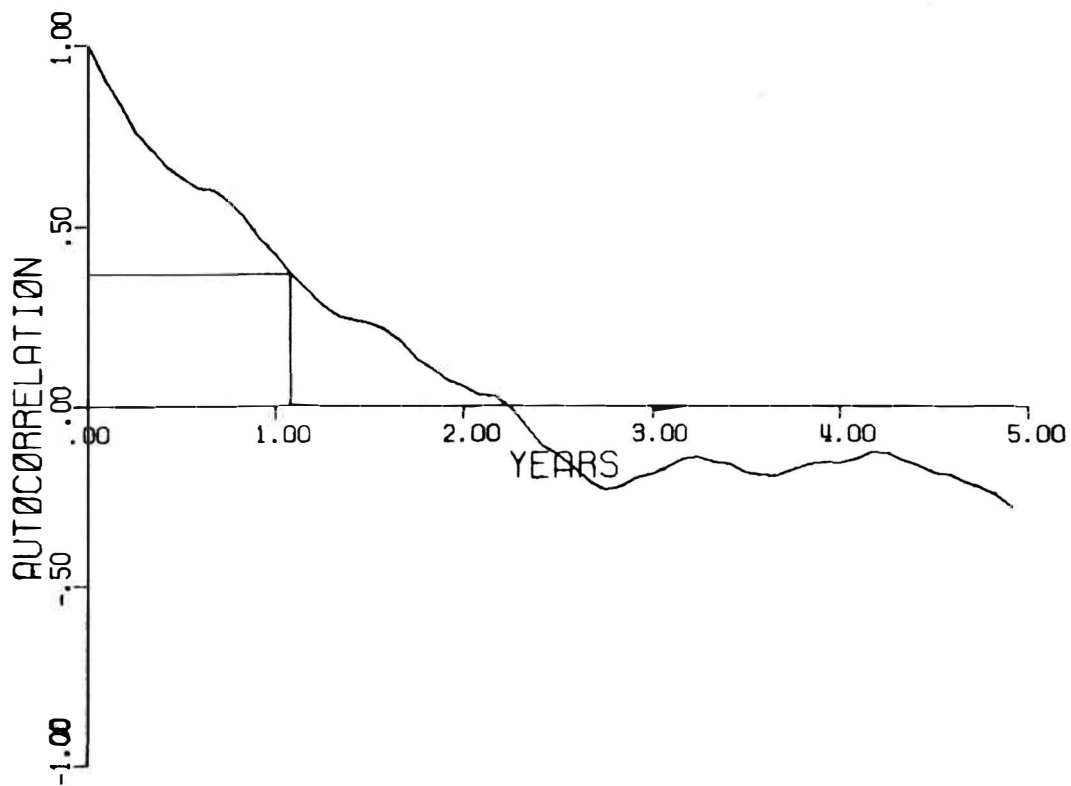
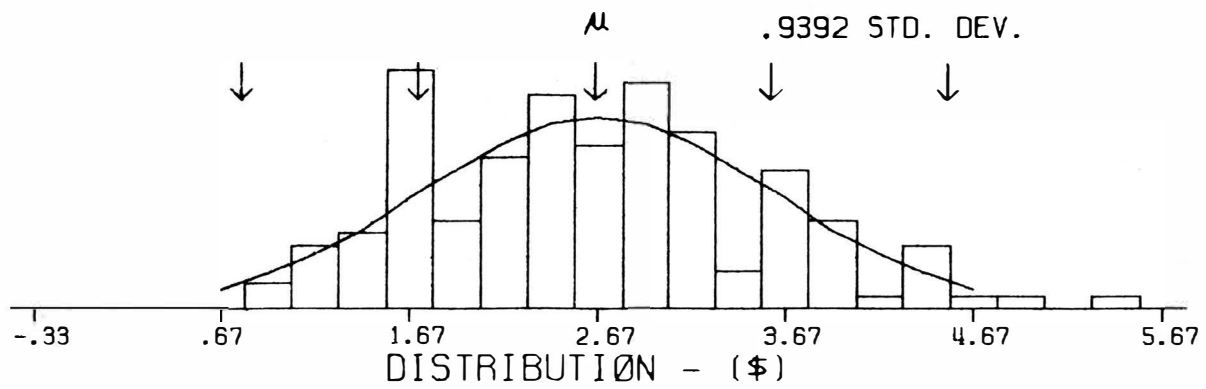


Figure 15 Fund C Statistics

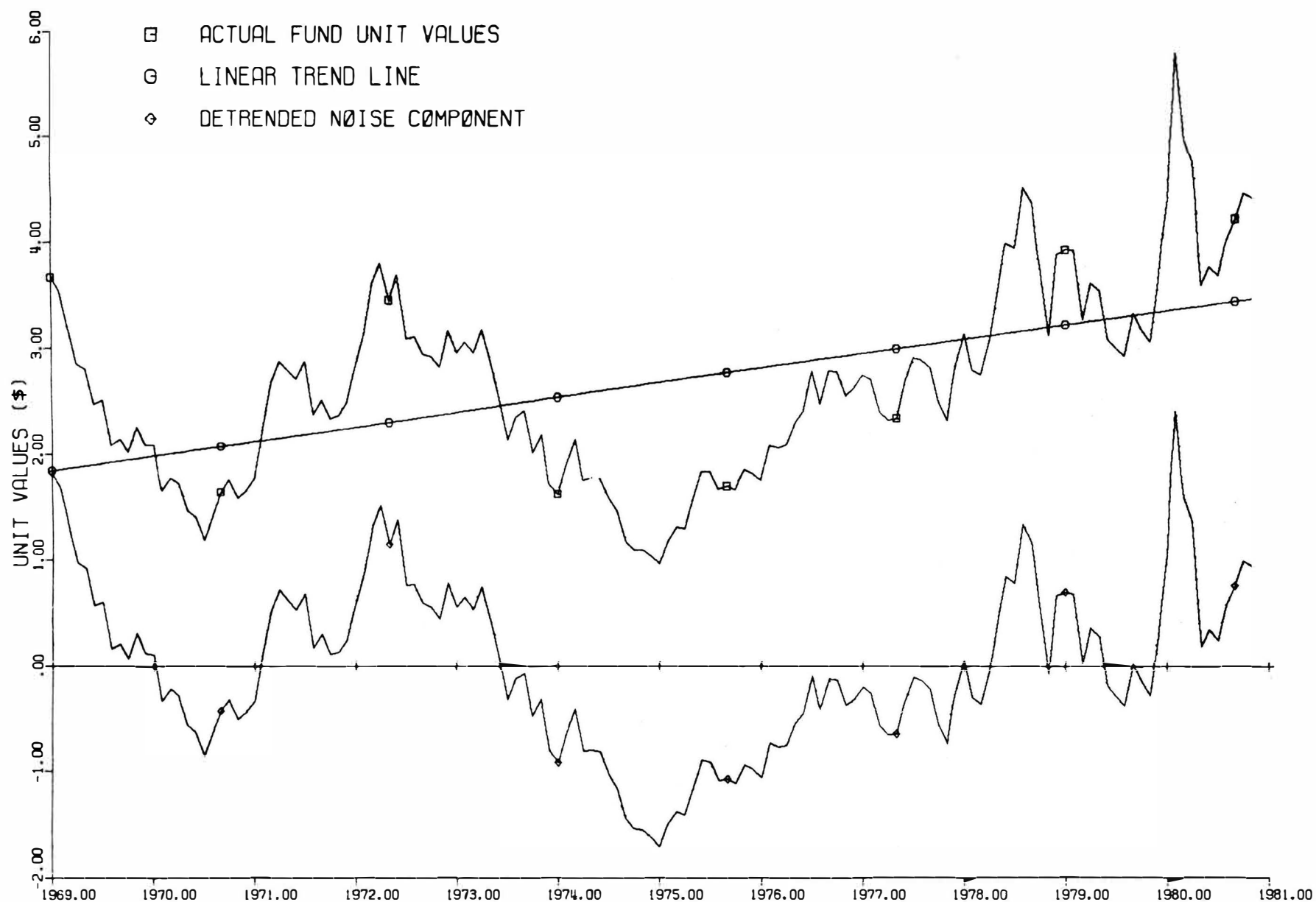


Figure 16 Fund C History - Linear Trend Removed

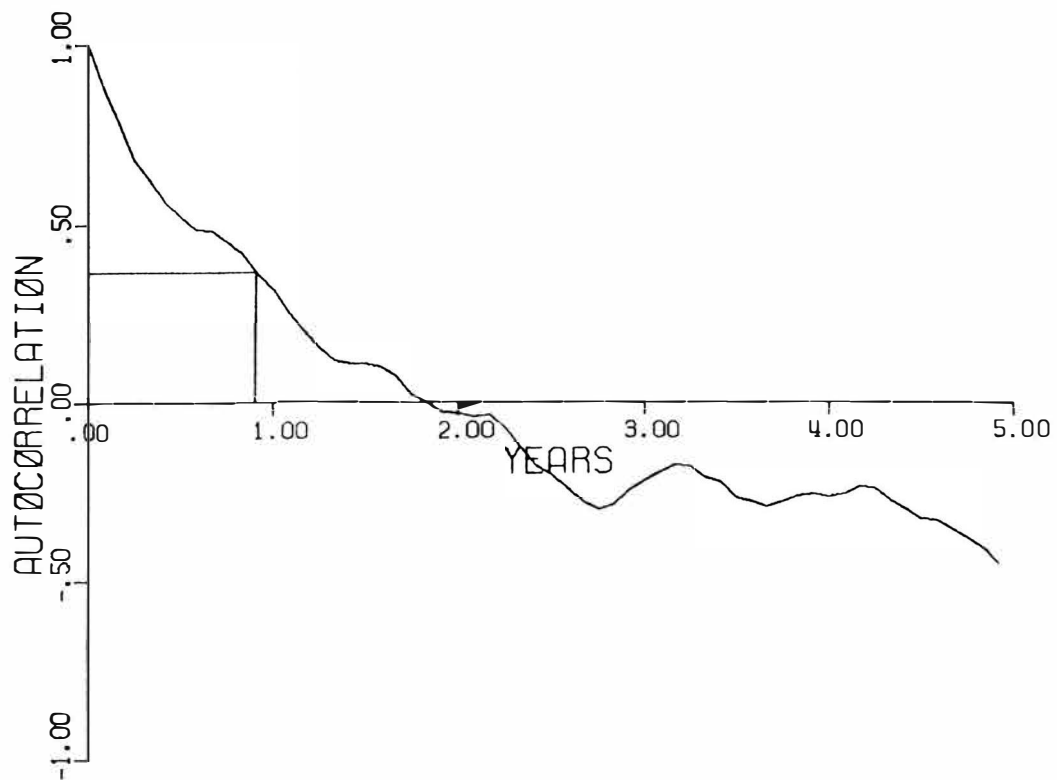
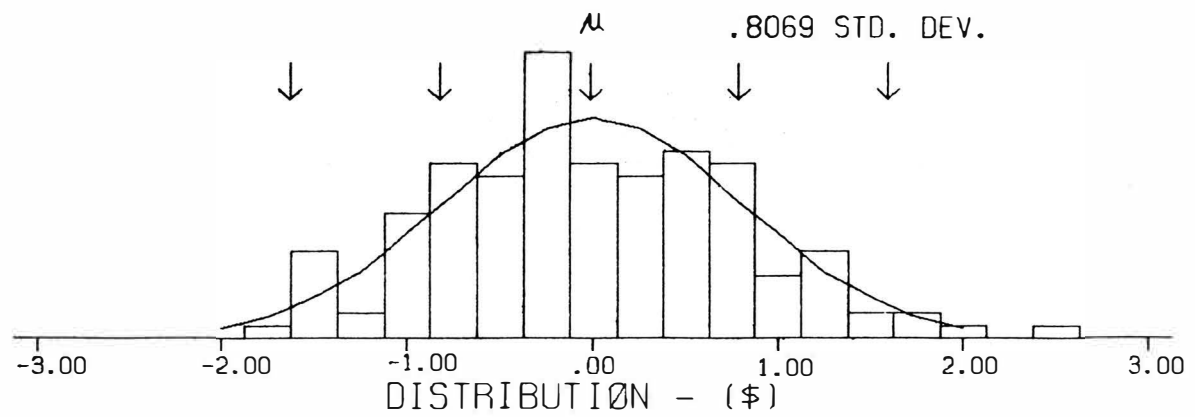


Figure 17 Fund C Statistics - Linear Trend Removed

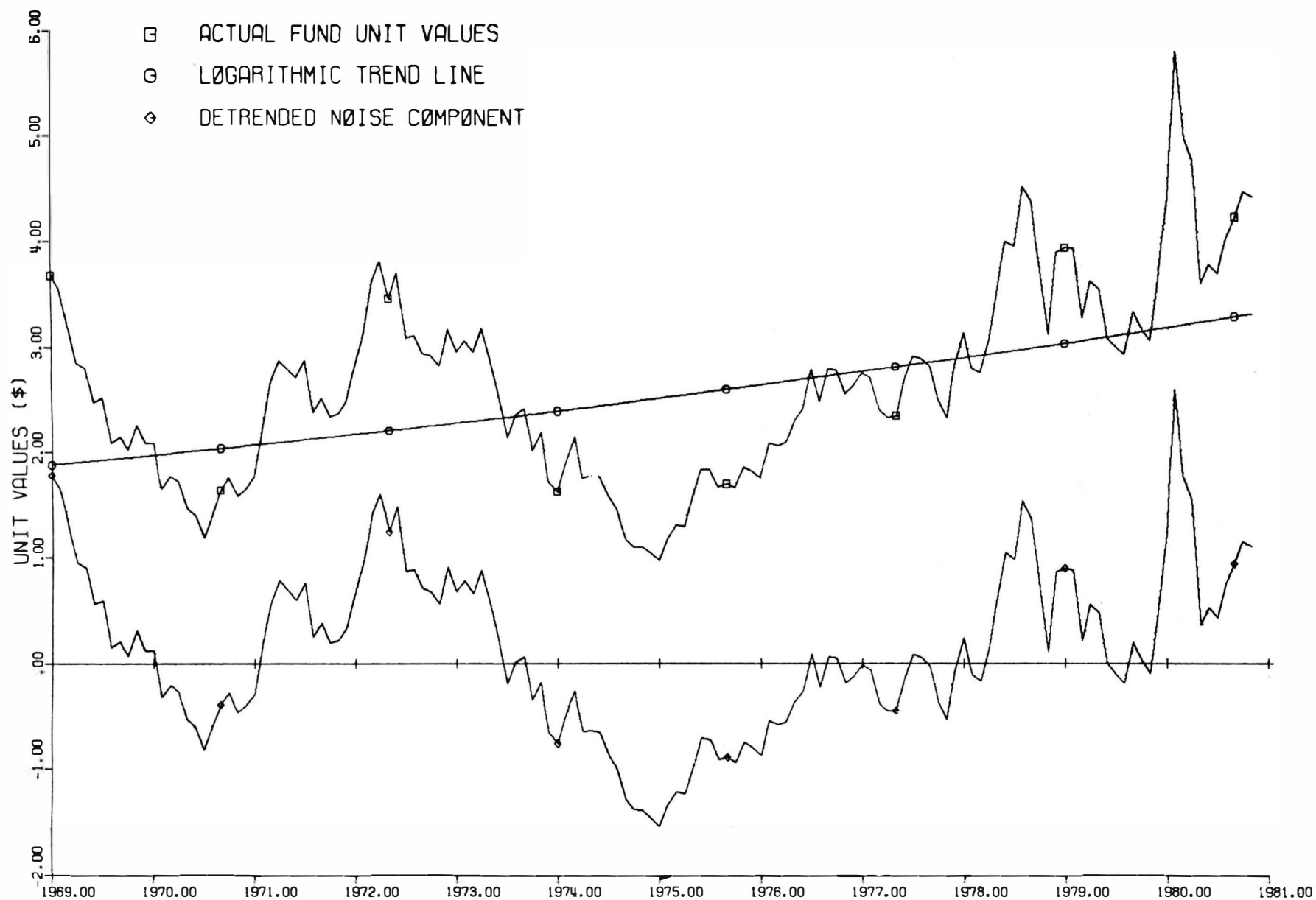


Figure 18 Fund C History - Logarithmic Trend Removed

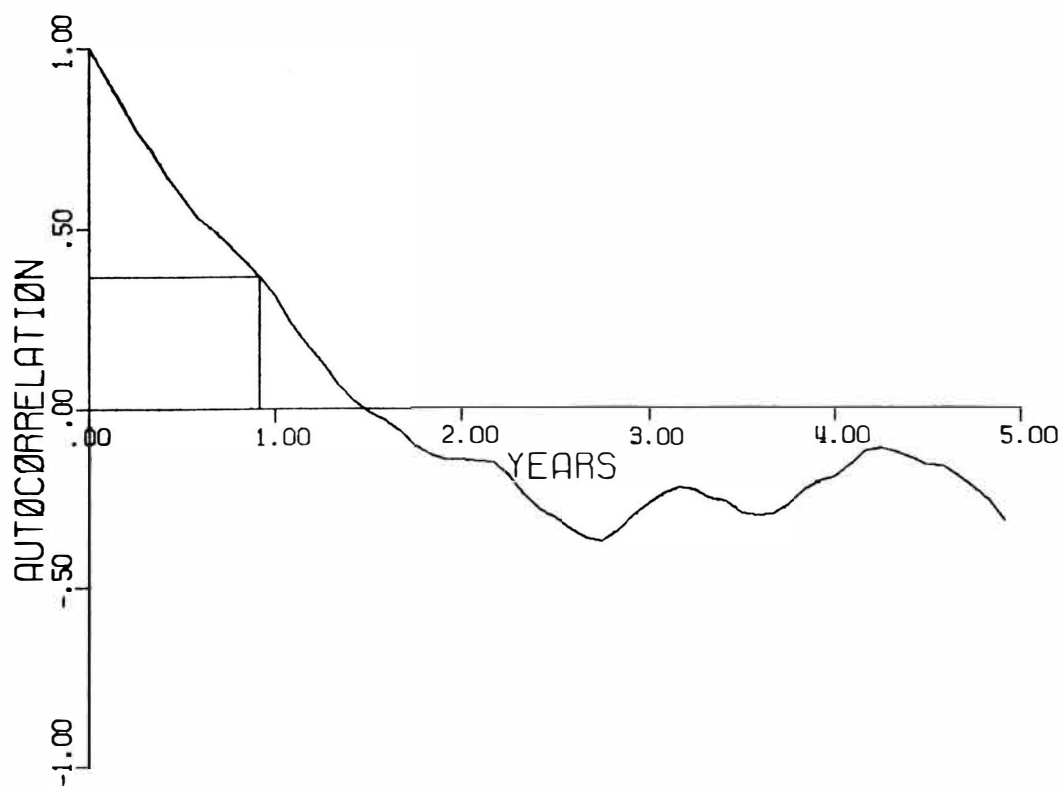
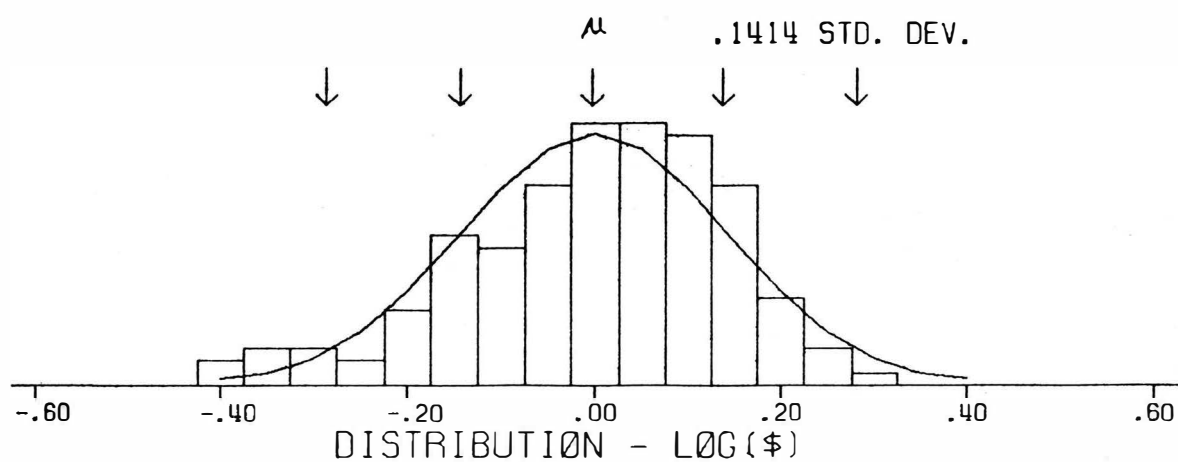


Figure 19 Fund C Statistics - Logarithmic Trend Removed

indicated growth of .1388 (\$/yr) is slightly higher than that of Fund A. The logarithmic trend's annual growth rate of 4.93% is however significantly below Fund A's. Still this Fund appears to have performed much better than Fund B.

The distributions of Figures 17 and 19 show pleasing matches to their overlayed Gaussian curves. Time constants of about 11 months are seen in both autocorrelation functions. There does not appear to be one dominating frequency component in this Fund. Refer to the discussion above where the autocorrelations of Funds A and B (with logarithmic trends removed) appeared to be somewhat cyclic. Fund C's noise component is by no means random (uncorrelated) however.

4. Statistics Overview The conclusions drawn from the statistical analysis may be summarized as follows. Funds A, B and C are best viewed as having constant rates of growth over the long term. The derived annual increases of the Funds are 7.36%, 1.84% and 4.93% respectively. In the short term the actual unit values of each are seen to deviate about their "trend" some amount, ranging from very small movements in Fund A to extremely large distortions in Fund C. These noise components are considered to be Gaussian in nature, but not entirely random. The autocorrelation time constants of the noises are seen to be 8 to 11 months. Furthermore, Funds A and B include some cyclic element, with periods of about 5 and 4 years respectively. Finally one assumption will be made, that the noise is time-invariant; that is the noise statistics will not change with time.

One common procedure in time-series analysis is to investigate the cross-correlation of two or more variables. This has not been done in the case of the three Funds' unit value histories for the simple reason that it will not be of interest in our simulation. One objective of the Comprehensive Model's operation will be to choose the "best" of the three Funds at any point in time, but it will not matter whether or not their individual levels of investment desirability move in tandem. The Model is to react to circumstances as they are encountered with no intentions of specific prediction or control.

B. SIMULATION DATA BASE

To fully test the Comprehensive Plan Model, a data base of Fund unit values is necessary which spans the intended 36 year simulation. It is important that this data base be realistic yet largely independent of the historic data with which the Model's decision criteria were developed. Simple repetition of the historic Fund unit values would of course be improper. In fact where noticable growth occurred (Funds A and C), the repetition would itself introduce discontinuities which would markedly alter the nature of the Funds. To be realistic it was decided the Simulation Data Base should reflect both the long term trends and the statistical contents of the noise components measured earlier. Obviously this an application of the time-invariance assumption made above.

The first step in structuring the simulated unit values was employment of the selected trend equations. These equations may be found in Table I though the actual coefficients have been rounded off

for display purposes. The time period chosen for simulation was the years 1975 through 2010. Each trend equation was solved for each month of those years and entered into the Data Base. Thus there are 432 data points for each Fund. Since the logarithmic rate trends were chosen, the trend equations yield values in the logarithmic domain.

The stochastic noise components were next added to the Data Base's trend values. Generation of random numbers to represent the noise was rejected. Even with filtering it is difficult to produce a data set that truly reflects all of the desired statistical properties. Instead, the sets of actual noise components were retrieved from the years 1969 through 1980. Subsets of this historic noise were then reapplied to trend values, but the noise components were inverted (multiplied by -1), reversed (in time sequence) or both. Care was taken to choose subsets with matching endpoints so no discontinuities would be introduced. Each Fund was processed individually with a different pattern of inversions and reversals. Thus any correlations between the Funds should have been eliminated.

Finally the combined trend and noise sets were converted to the linear domain, point by point. The resulting Simulation Data Base is displayed in Figure 20. Comparison may be made to Figure 1, but note that both plotting scales have changed. The reader may have observed that the apparent noise gyrations get larger in the later years of the Data Base. This is because the noise components were applied in the logarithmic domain, so their deviations as a percentage of the trend values remain consistant.

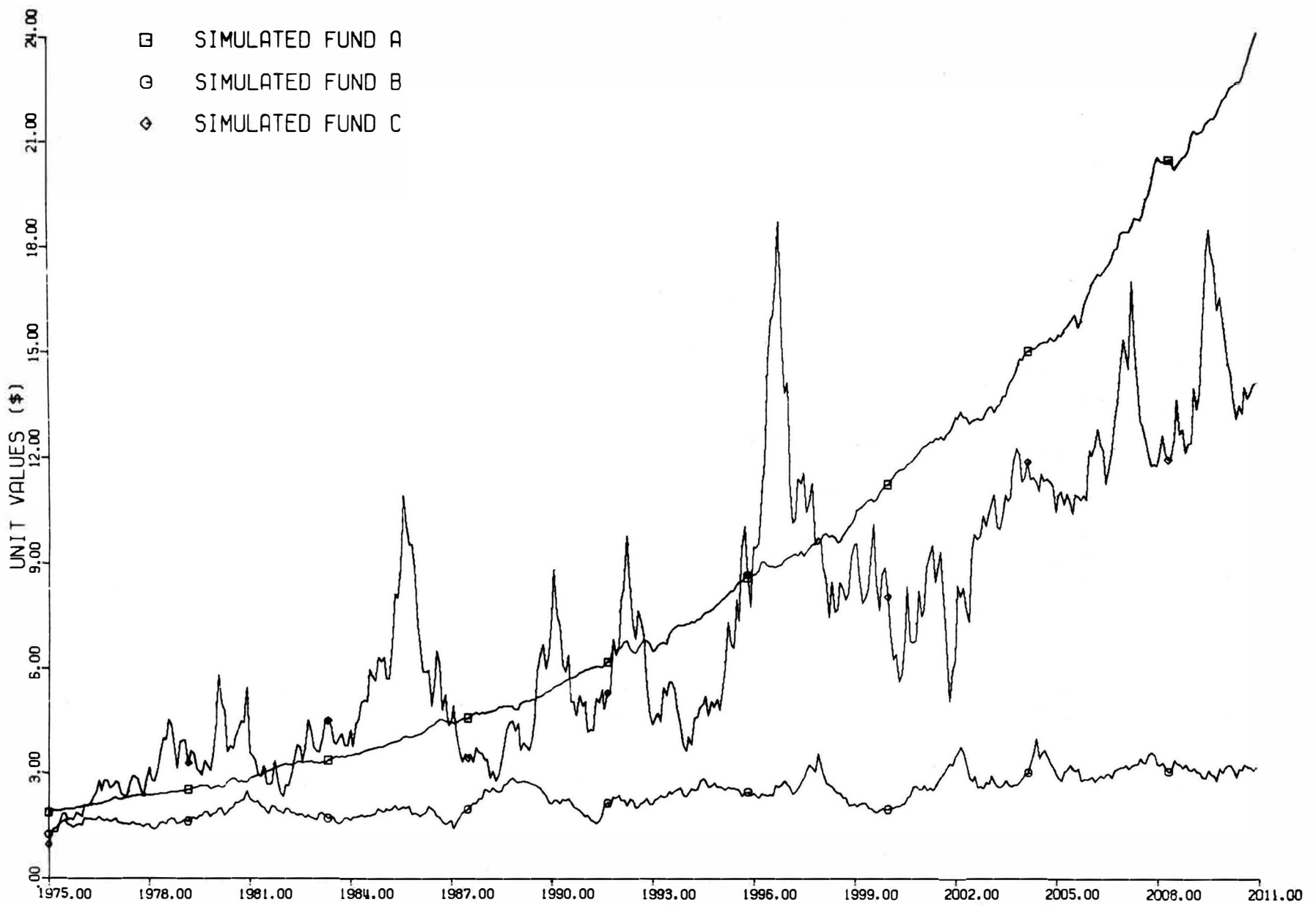


Figure 20 Simulation Data Base - 1975 to 2010

The full complement of statistical measures were not again applied to the Simulation Data Base. However an adaptive procedure in the Comprehensive Plan Model which provides an ongoing check on certain parameters showed the Fund statistics to be stable. More discussion on this will appear in Section III.

III. SAVINGS PLAN MODEL

The vehicle for determining the optimal investment decisions is the Comprehensive Savings Plan Model. The Model, a FORTRAN computer program named DECIDE, is actually a simulation of both the relevant Plan rules and its environment. That is it keeps account of funds within the Plan and those removed from it, plus aggregate tax liabilities.

This section presents both the derivation and implementation of the component parts of the Model. The discussion follows the sequence of the program itself. For those readers who wish to follow the actual computer code in the Appendix, the text will include occasional references to the page numbers of this document as say [68], or to a program line designator as say <50>. Specific variables will be written in all capital letters. Five portions of the Model have been identified as follows: Initialization and data input; Performance expectations; Investment decisions; Application of decisions to the Plan Model; Subroutines. All of the preceeding processes except initialization are simulated at a monthly rate, so the program forms an endless loop, continuing until the input data runs out.

A. INITIALIZATION AND INPUT

Most of the initialization process [67-68] is self-explanatory. A number of program variables must be set equal to zero. Input files (device numbers 10 and 11) are set at their beginnings. The summary printout files (numbers 50, 55, 60 and 65) are then given headers. One

variable parameter, ICYC, remains for the operator to input; Its explanation will come in a later paragraph. The first call to Subroutine Trend is a further initialization; Discussion of it will also appear later.

The first input file <10> provides seven variable values. The first two, MON and IYR, are month number and year (where an 82 represents 1982). IWK and IWD are flags which indicate investment decisions if they are to be inputs to the program. However, for simulations where the optimal decisions are to be applied, these inputs are always zero. Variable RAT is the current annual interest rate available outside the Plan, from say a savings account. SALA is the employee's weekly base salary, while TINC is his total taxable income expected that year (excluding Fund withdrawals). Each of these variables is redefined for usage in the program (TIME, SAL, JWK, JWD, TIN, RATE) and inputs of zero imply no change from previous values. The participant's weekly payroll deduction, CONT, is recomputed from SAL twice a year <16>; the maximum 8% contribution is assumed so to maximize benefits of participation. The call to Subroutine Caln produces the number of contributions to be made that month (NWK), always four or five.

The second input file <20> introduces the Fund unit values A, B, and C. Time must be matched with the other input to be meaningful. Variable TIM2, called extrapolation time, is computed from TIME and ICYC. Explanation will be in the following subsection.

B. PERFORMANCE EXPECTATIONS

Each month the employee must decide whether it is in his best interests to participate or not, and to withdraw or not. In support of these decisions, certain information needs to be gathered as to the effects of each possible course of action. The four possibilities are simulated to some future time (TIM2) in accordance with the expected performance of the Funds and alternatives [68-69].

1. Effects of Not Withdrawing To measure the value of the employee's current Plan holdings at some time in the future, one needs an estimate of future unit values. The call to Subroutine Trend, with its call index set to 2 and TIM2 an input, yields expected unit values ATT, BTT and CTT at that future withdrawal time. The implications of these variables are addressed in the discussion of optimization in the next subsection. To determine the future value (FV) of his Plan assets, the Model multiplies each future unit value by the number of units currently owned (AU, BU, CU) and sums. (In the first month of a simulation, no units are yet owned so a jump is made to <40>.) Since a withdrawal must eventually be made, any gains (GAIN) over amounts previously taxed (STAXP) must then face taxation. The call to Subroutine Taxtab, inputting annual income TIN and the marginal increase GAIN, outputs the marginal tax liability TAX. Final return on the future withdrawal (FRA) is the future value minus tax then due.

2. Withdrawal Effects The opposite of the above decision is to withdraw from the Plan immediately and invest the proceeds externally. A simple savings account is assumed. Again, assets equal the sum of the

products of the numbers of units owned and now the current unit values (A, B, C). Taxes owed on any gain must be subtracted to find the true current value (VAL). To make a fair comparison with the non-withdrawal situation, the external savings need to be extrapolated to the same future point in time. The loop to <30> permits savings growth at the RATX monthly interest rate, being taxed along the way. ICYC is the number of months in the extrapolation period. Final return for this case is FRB.

Because Plan rules impose a withdrawal penalty of six months suspension, the FRB calculation should be lessened by the six months of McDonnell's contribution which is forfeited. Actually it was more convenient to increase FRA by that amount. This is proper because the later usage of FRA and FRB will be in the difference of the two variables. (Note that $19.5 * \text{CONT} / \text{C}$ is three fourths of 26 weeks contributions converted to Fund C units.)

3. Contribution Effects Assuming no withdrawal is made, the participant should decide each month whether further contributions are to his benefit. Extrapolations are performed on one month's amount to time TIM2, similar to the calculations of FRA. McDonnell matching funds are included. FRC is the final return on this month's contribution to the Plan. Because this information will be used to choose the "best" Fund in February and August, all three Funds are simulated (FRCA, FRCB, FRCC). JWD is the flag which directs actual contributions to the selected Fund(s).

4. Effects of Not Contributing As a measure for judging FRC, the equivalent of one month's investment is simulated in an external savings account (FRD). Interest is paid and taxes are due along the way. The mechanics are a repetition of the extrapolation in FRB's computation. Finally, TOTAL1 and TOTAL2 are defined as the expected dollar differences of withdrawing or not, and contributing or not, respectively.

C. INVESTMENT DECISIONS: OPTIMIZATION

At this point discussion is in order on optimization of the Plan Model's decision criteria. The goal has already been established: Maximize the overall long term return on savings. The philosophy for achieving this is taken from a traditional investment axiom: Buy low and sell high. The Model must therefore interpret current data in terms of relative value and establish expectations of future performance in making investment decisions.

1. Withdrawal Decision It was stated earlier that a decision to withdraw one's assets from the Plan is the most complicated one, involving changing tax liabilities and volatile unit values. It must also be the first decision because a withdrawal suspends participation for six months, making other decisions meaningless. As part of the process we draw on the statistical information on past Fund performance gathered in the preceeding section of this paper.

The analysis of the unit values' historic data led to the approximation of each Fund by a logarithmic trend line of a determined

rate of growth. Actual Fund data continually varied about the trends, and these "noise" deviations were seen to be roughly Gaussian in distribution. It was concluded that in forming performance expectations, Fund unit values can at some future time be expected to return to its trend line value. The past Fund data noise content was also seen to be correlated over time; All three Funds had similar autocorrelation time constants and two of them displayed periodic components of about 4 and 5 year cycles. Therefore it was deduced that a period of "high" unit values should be expected to occur, on the average, 2 to 2.5 years in the future.

The structure of the decision was subsequently established as follows: First, consider withdrawal only if the Fund assets are expected to produce a greater return outside the Plan than within. If such is the case, the second test is whether unit values are sufficiently high to justify a withdrawal (the sell high philosophy). Third, after a withdrawal decision has been made, determine which Funds warrant the removal (Plan rules require the withdrawal of Fund C assets; Funds A and B are discretionary).

A judgement as to what constitutes a "high" unit value is needed. Having described Fund performances as following trends, though distorted by noise components, a comparison of a unit value to its concurrent trend value should be informative. A call is made to Subroutine Trend [69], with its call index set to 3 and three dummy variables (PA, PB, PC) inputting the current unit values. Upon return, these variables represent the percentile levels of said unit values, relative to their

corresponding trend values. Stated differently PA, PB and PC become the probabilities of not exceeding the stated unit values, assuming the derived Gaussian distributions about the current trend values. For example, if unit value A happened to equal the Fund A trend value, PA would be set in the subroutine to .5000 (50%ile). Meanwhile, if unit value B was found to be one standard deviation greater than Fund B's corresponding trend, PB would return as .8413 (84%ile). A fourth probability rating PTOT is defined as an average of the three Fund readings, but each is weighted according to the assets held therein.

In following the procedure of the withdrawal decision [69- 70] the listing's comment cards should be helpful. If a withdrawal was made in the last six months, it should not be considered again. The participant is waiting out the six month exclusion period. Secondly, withdrawal should be considered only if the assets are expected to grow more outside the Plan than within; that is, if TOTAL1 is negative.

Assuming withdrawal is still being considered, a check is made as to whether unit values are high enough to justify withdrawal this month. An arbitrary probability threshold of .80 (80%ile) was initially set for this decision. It represented a compromise. Surely a threshold above .50 should be adopted because half the time one could expect higher values a month later. On the other hand too high a threshold, say .99 , might never be reached and valuable withdrawal opportunities might be missed. It was further decided that both the overall PTOT measure and Fund C's PC should be tested, because rules require that C's assets be included in any withdrawal. An element of flexibility was added so a

withdrawal is made when both PTOT and PC are .75 or higher, and the average of the two is .80 or above.

If the decision was made to withdraw from the Plan this month, the Funds involved must be specified. As noted above Fund C must be included. The probability measures of the other two Funds, PA and PB, are merely tested against the .80 threshold. The variable IWD is then set to reflect the withdrawal decision: negative implies a withdrawal, while the ones, twos and fours bits indicate Funds A, B and C respectively.

2. Fund Choice Decision Each February and August, Plan participants may choose the Fund(s) into which their contributions are placed, applicable the following month. Although rules permit splitting the investment between two Funds, this program always chooses one, with the intent of maximizing savings by concentrating on the "best" Fund. The decision is straightforward <60> in that the maximum of FRCA, FRCB and FRCC is selected. These variables, discussed earlier, represent the expected final return on this month's contribution for each Fund over the extrapolation period. Consideration was given to current unit values, tax liabilities and Fund trends. Variable JWD is coded with the Fund choice in the manner of IWD.

3. Participation Decision This final decision is almost trivial: It is rarely seen to not be affirmative. An employee may at any time suspend his contributions without withdrawing and without triggering a tax liability. He may only be reinstated after February and August. A

decision to suspend participation is only made when the expected return for his month's contribution invested outside the Plan (FRD) exceeds the expected returns on his and McDonnell's payments within the Plan (FRC). Should this be the case, variable IWK is set negative.

D. INVESTMENT SIMULATION

Now that the month's investment decisions have been established, either by the decision criteria just discussed or as a program input, they are to be applied to a model of the Savings Plan and its environment [71-72]. Certain portions of the program code will of course be bypassed when inappropriate for the current monthly cycle. Discussion will continue to follow the sequence of the program listing found in the Appendix.

First, if the employee is participating and contributing to the Plan this month, new Fund units are purchased. Variable JWD directs his contribution to the designated Fund or Funds. The weekly contribution $CONT$, times NWK number of weeks this month, is divided by the current unit value, say A . These newly purchased units are added to the units already owned (AU). The McDonnell contribution, equal 75% of the employee's, is converted to Fund C units but placed in the last element of array $CMAC$. This array shields the company-bought units from any withdrawal for 24 months. Variable $STAX$, defined as the sum of Plan assets that have already been taxed, is then increased by the amount of the employee's contribution.

In the special case where a withdrawal decision has just been made, the unit purchases above were bypassed in favor of redemption code <70>. Counter IWDR is set to 6 to number the coming months of nonparticipation required by Plan rules. Units are converted to dollars by multiplying the number owned, say CU, times the unit value (C). Variable IWD indicates which Funds are redeemed. WDR is the dollar total being taken out. A tax liability has now been induced <72> raising the employee's taxable income by TAXED. Internal Revenue Service (IRS) rules require that all assets that could be withdrawn be taxed, whether taken out or not. Thus variable TAXP is defined as being any amount left in the Plan after withdrawal, already subjected to taxation.

If a withdrawal has taken place in the past six months, no contributions can be made to the Plan. As variable IWDR counts down the months <75>, the money the employee would have put into the Plan (8% of salary) is placed into an auxiliary savings account AUX <76>. This account is an important feature in making fair comparisons between competing investment strategies. An identical amount will always be set aside by the employee regardless of decisions made. Each month interest is paid on the savings account <80> according to the current RATE. Interest paid is also an addition to taxable income.

To prepare for next month's transactions, the elements of the CMAC array are shifted one position <85>. The units of Fund C purchased by McDonnell 24 months earlier, CMAC(1), are now put under the control of the employee.

Each December (MON equal 12), the additional taxable income related to savings, withdrawals and interest (TAXED) is totaled . A call to Subroutine Taxtab yields the marginal tax due on this marginal income. The tax due (TAX) is then charged against the auxiliary account.

A permanent record of all decisions and conditions used in the simulation is maintained in output device 12, usually a disc file. Note that the format <1000> is identical to input device 10, allowing the operator to repeat a given simulation later, or test any specific decision changes.

The final call to Subroutine Trend, with its call index at 4 and inputting current unit values, allows the subroutine to update its statistics. More on this later. The next month's simulation cycle may now begin <10>. When all input data has been exhausted <100>, the output file is ended and the operator may request a new simulation <5>.

E. SUBROUTINES

1. Subroutine Trend Subroutine Trend [73] handles all of the statistical information relating to the Fund trend equations and distributions. It is called for four purposes as indicated by its call index NN. Also passed into the subroutine are the time of interest (T) and three unit values (A, B, C).

During the first call to Trend the statistics are initialized <10> by reading raw data from input file 13. Specifically these are the sums, sums of squares, sums of cross products and numbers of data points

for Fund performance for 1969 through 1980. Below <50> these items are used to define the initial trend equations (slope-intercepts) and distribution standard deviations. All are maintained in the logarithmic domain.

The second call to Trend <20> provides the expected high unit values at the extrapolation time T. The trend equations are solved for that time and increased by .842 of a standard deviation. The 80%ile level equals 84.2% of one standard deviation above the mean in a Gaussian distribution. Conversion of the expected unit values to the linear domain is then made before the return.

Call sequence three <30> provides the probability levels of unit values relative to their trends. Values are converted to the logarithmic domain, trend equations are solved for the corresponding time and calls to Subroutine Erf test them against an error function table.

The fourth call to Trend is to update the subroutine's statistics. Said equations are structured in their recursive formulations, providing an adaptive feature to the program. Each month's unit values are gathered for the summations but they are applied only once a year. This was to insure that numeric precision was not sacrificed by adding too small of numbers to large ones.

2. Subroutine Taxtab This subroutine [75] provides the additional (or marginal) tax due (TAX) on an additional amount of income (TAXED) over some base taxable income (TIN). It uses a table lookup for IRS Schedule X, which is for single taxpayers and applies to the writer. Array A defines the bracket endpoints, array B gives the base taxes for the brackets, while array C has the corresponding marginal tax rates. Additional Missouri state income taxes are then computed. While the Federal tax will be exact for all incomes, the Missouri tax equation assumes that the top bracket of \$9000 has been reached.

3. Subroutine Erf This subroutine is a simple interpolated table lookup of the normalized error function. Outputs are accurate to a few thousandths. This is quite sufficient because the program is not considered sensitive to this parameter.

4. Subroutine Caln Subroutine Caln provides the number of Plan weeks in the specified month. Using a perpetual calender, its output effectively equals the number of Sundays in the month. The calender is not valid before the year 1900, but then neither was the Savings Plan.

IV. SIMULATIONS AND RESULTS

A means of judging the Comprehensive Savings Plan Model and its decision methodology was finally needed. The test of time would provide conclusive results, but was hardly practical. Simulations were instead chosen to compare the Model's results to an array of alternate investment strategies under controlled conditions. A 36 year career of a McDonnell employee was created to drive the simulations, in conjunction with the unit value Data Base discussed in Section II. The alternate strategies were designed to be blind to investment and market conditions, but still to investigate the merits of all three Funds (separately) and consider a wide range of withdrawal intervals.

A. CAREER MODEL

The conditions of the employee's career were formulated for the years 1975 through 2010, matching that of the Simulation Data Base. The inputs required were salary, income and outside interest rate. The other input variables discussed in Section III were not entered at this point because the Model is to first choose the "optimal" decisions. The selection of the input values was somewhat arbitrary. Generally a very simple pattern was maintained.

Weekly salary was required to establish the contribution levels to the Plan. The Model assumes that 8% is always set aside for savings, so as to maximize benefits from the Plan. The employee was started at \$230 per week, typical for an entry level engineer at McDonnell in 1975. He was then given annual increases of 10%, but the amount was rounded to

the nearest \$5 quantity. Any allocation of these raises between inflation and career advancement is at the reader's discretion. It does not influence the operation of the simulation.

Annual taxable income was input to determine the employee's marginal tax liabilities. The initial income was set to \$12000, matching his annual salary. Increases in income however, were 11% per year to suggest some savings or investments unrelated to the Plan.

Outside interest rate is used to compound growth of the assets withdrawn from the Plan. It was initialized at 5.5% per year. In 1985 the interest rate was raised to 6.0%, then to 6.5% in 1995, but lowered to 6.0% in the year 2002. It was tempting to use much higher interest rates (at this writing money market funds return about 17.1%). However, this input is also used as a part of the decision criteria's expectation calculations. Therefore the operator should use an interest rate that he expects can be maintained for a long period. Withdrawal decisions are irrevokable.

B. OPTIMAL SIMULATIONS

The simulation using the optimized decisions could now be run but for one parameter; the extrapolation period was never finalized. Discussion in Section III suggested that a value of 2 to 2.5 years might be appropriate. That would represent about one half cycle of the inherent periodicity observed in Funds A and B. Instead it was decided to derive an empirical solution for this optimization parameter.

The full simulation was hence executed six times, varying the extrapolation period run to run, from 18 months to 48 months, by 6 month intervals. Table II presents a summary of the results of this exercise. Samples of total values of savings are given at five year intervals. These amounts will be termed the "layoff values" and represent the best parameter to judge a simulation's ongoing worth. Plan rules stipulate that an employee who retires or is laid off will receive the proceeds from the full assets of his Plan account, vested or not. (An employee who is fired or quits is penalized according to a formula which is different from vesting and not modelled in this study.) As with a withdrawal, a tax liability is triggered upon layoff. So the layoff value equals his total Plan assets, minus tax that would be due, plus funds already in his auxiliary savings account. Table II shows that the 36 month extrapolation period gave the best results most of the time; Thus it was chosen as the optimal case. Happily 36 months is not substantially different from the 24 to 30 months suggested by analytical means above. Furthermore, the Model is not highly sensitive to this parameter. The simulations all yield layoff values within a few percent of each other.

A closer look at this optimal simulation will be of interest. First, with regard to the choice of Funds, there were 64 opportunities to make Fund selections: twice a year except when contributions have been suspended. Each Fund was chosen at some time. Fund C was the most popular by far, being selected 56 times. Fund A was the choice for 5 six month periods, while the employee's investments went into Fund B during only 3 of the time segments.

TABLE II
SIMULATION RESULTS: OPTIMIZATION

EXTRAPOLATION PERIOD (MONTHS)	LAYOFF VALUES (\$) AT YEAR'S END						
	1980	1985	1990	1995	2000	2005	2010
18	16793	37950	69210	119333	186761	291467	487139
24	16793	37950	69210	119333	186761	291467	487139
30	16793	37950	69511	119691	186486	291174	460003
36 *	16793	38001	69736	120136	193303	298939	469268
42	16793	38001	69610	119938	193085	298702	470101
48	16793	37851	69450	119790	184697	290383	462452

* Chosen as optimal.

Over the 36 years the Model induced withdrawals 9 times, though they were not evenly spaced. In one period 3 withdrawals were made within two years, yet several times more than three years would pass in which there was no withdrawal. Three times the assets were taken out of Funds A and C, while six times only C was redeemed (though Funds A and B were empty two of those times). The small amounts in Fund B were held to retirement. At no time were contributions voluntarily suspended without withdrawing.

If conclusions can be drawn as to how the Model behaves, they might be as follows. Fund C is most often favored because its volatile nature offers great leverage. In periods where Fund C's unit values are especially high (relative to its trend), contributions are directed to Funds A and B. It is also during these periods that most withdrawals take place. Obviously when Fund C holds most of the assets, its unit value has the most influence over the timing of liquidations.

C. ALTERNATE STRATEGIES

While the layoff values produced in the optimal simulation looked enormous, some perspective was needed. The control simulation was quite simple. The same 8% of salary that would have been invested in the Savings Plan was instead placed directly into the auxiliary savings account. The current rate of interest was paid to the account and taxes due were deducted. As one might expect, the control case could nowhere near match the optimal simulation, with its substantial McDonnell contributions. Table III includes the control's results as will Figures 21 to 23.

The question remains though, was it merely participation in the Savings Plan that increased the employee's savings so substantially, or did the decision criteria produce tangible benefits? A series of twelve simulations of participation was conducted, applying different sets of investment decisions (optimal decisions were bypassed). Each simulation restricted employee contributions to a single Fund. Four simulations concentrated on each Fund. Because the Funds were believed to be independent of one another, it was concluded that any random switching between Funds, blind to market conditions, should normally yield results between the extremes of single Fund simulations. One run with each Fund group tested the effects of never making a withdrawal. Periodic withdrawals were then considered, with intervals of 10 years, 5 years and 2.5 years.

Figure 21 displays the results of the Fund A investment simulations, plotting layoff values over time. Because the dollar amounts covered such a large range, the vertical direction uses a logarithmic scale. A given vertical distance therefore represents a constant ratio of two values. Table III presents periodic samples of each line plotted. As one can see from the plot, the optimal simulation is usually a noticable distance above the Fund A results. In fact only the non-withdrawal case ever exceeds the optimal run, that in the last years and shortly in 1996. The volatility of that simulation can be explained by the McDonnell contributions that built up in Fund C. The other runs include withdrawals which transfer assets to the stable auxiliary savings account.

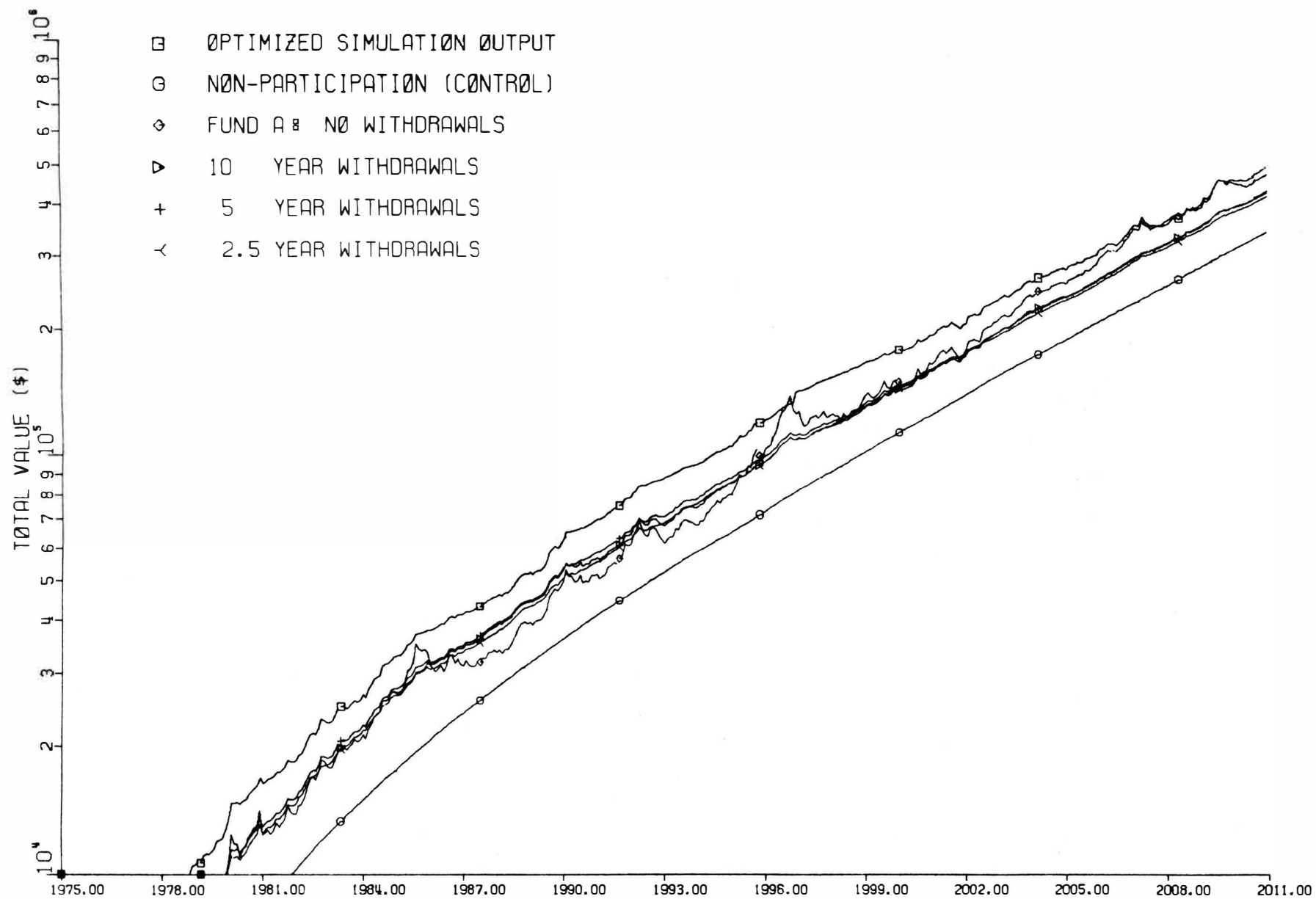


Figure 21 Simulation Results - Fund A Investments

TABLE III**SIMULATION RESULTS: ALTERNATIVE STRATEGIES**

STRATEGY: FUND & PERIOD OF WITHDRAWALS	LAYOFF VALUES (\$) AT YEAR'S END						
	1980	1985	1990	1995	2000	2005	2010
Optimized	16793	38001	69736	120136	193303	298939	469268
Control	8049	20494	40792	72849	124228	206636	340081
A 36 Years	14076	33775	51411	97873	164987	284314	490421
A 10 Years	14076	31869	56193	95108	157155	261260	427256
A 5 Years	13495	32254	57974	97761	158764	260066	424937
A 2.5 Years	13040	31090	54971	95206	156991	255975	415893
B 36 Years	15291	31558	44406	85224	139253	236377	395914
B 10 Years	15291	30333	52243	90980	150911	250565	409259
B 5 Years	13643	31846	56615	96261	153506	252167	412761
B 2.5 Years	13043	30579	53801	93709	154875	252411	412245
C 36 Years	16972	40135	47518	95344	147919	257860	435761
C 10 Years	16972	35193	58953	94655	153140	255729	416355
C 5 Years	15636	36530	65434	104292	162660	243488	429279
C 2.5 Years	14230	33123	58168	98922	160603	260090	420470

Corresponding investments in Funds B and C are plotted in Figures 22 and 23. The optimized and control runs are repeated for comparison purposes. Fund B apparently fared less well than Fund A over the long term. Concentrating investments in Fund C exposes the participant to enormous value fluctuations. Thinking back to Section II, these observations should come as no surprise.

Finally a test was made to verify that computer precision was not a factor in the simulation results. All programming for this paper was performed on a Harris 135 computer. It is a 24 bit machine, where floating point variables are normally allotted a 39 bit mantissa and an eight bit exponent (2 words); A compiler option permits faster run times by cutting all mantissas to 23 bits. A cursory review of program code suggested there should be no difficulty with precision. The only point of concern was in Subroutine Trend where large statistic summations may accumulate. There, preparations had been made by gathering data monthly, but only applying it to the recursive formulas annually. To test the situation, the optimized simulation was run with both the normal and the reduced precision levels. The two runs yielded identical decisions, and all dollar values matched to the penny. It was concluded that computer precision was not a factor in this study. Actual overflows of real or integer variables would have caused fatal run errors, so obviously they did not occur.

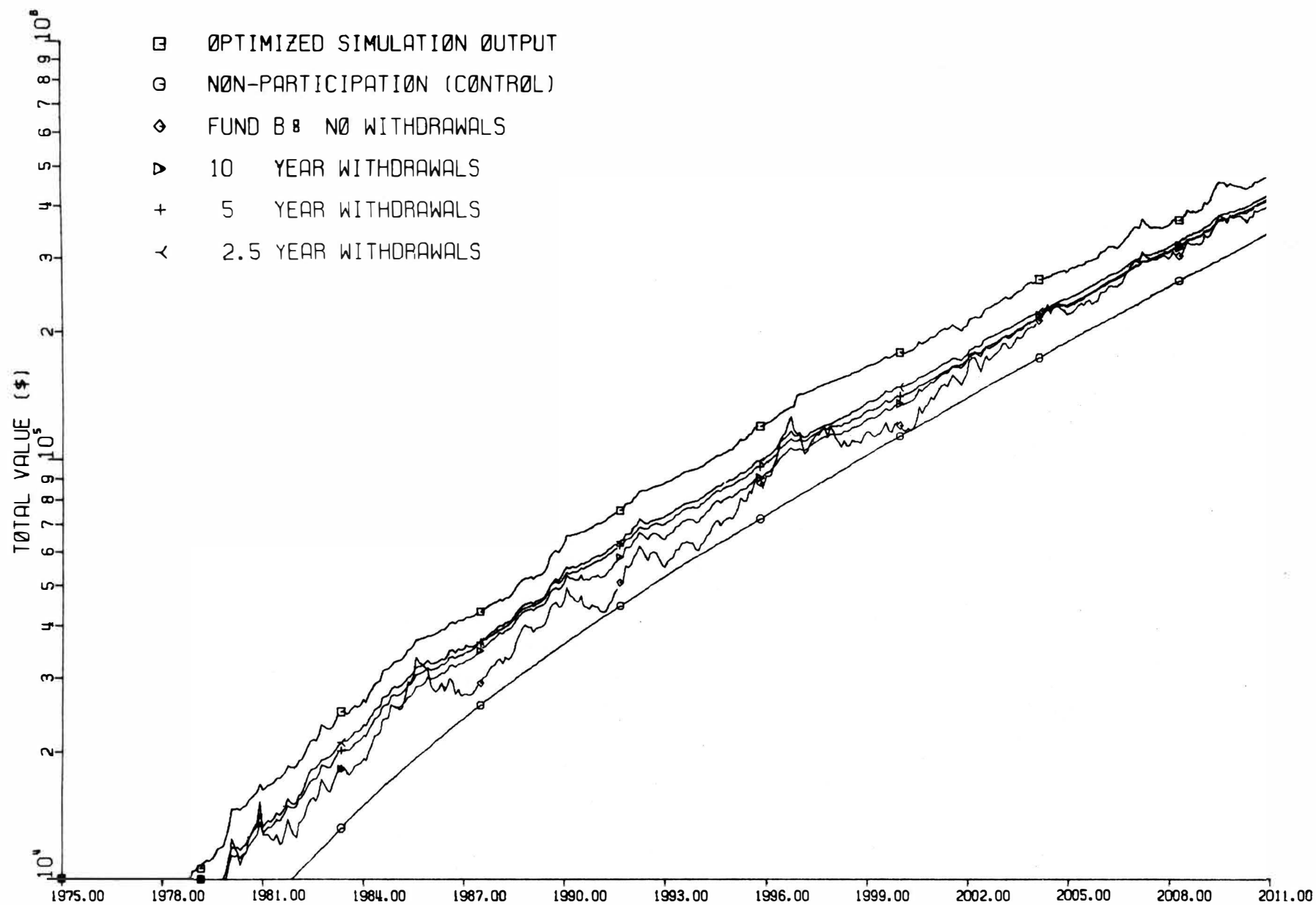


Figure 22 Simulation Results - Fund B Investments

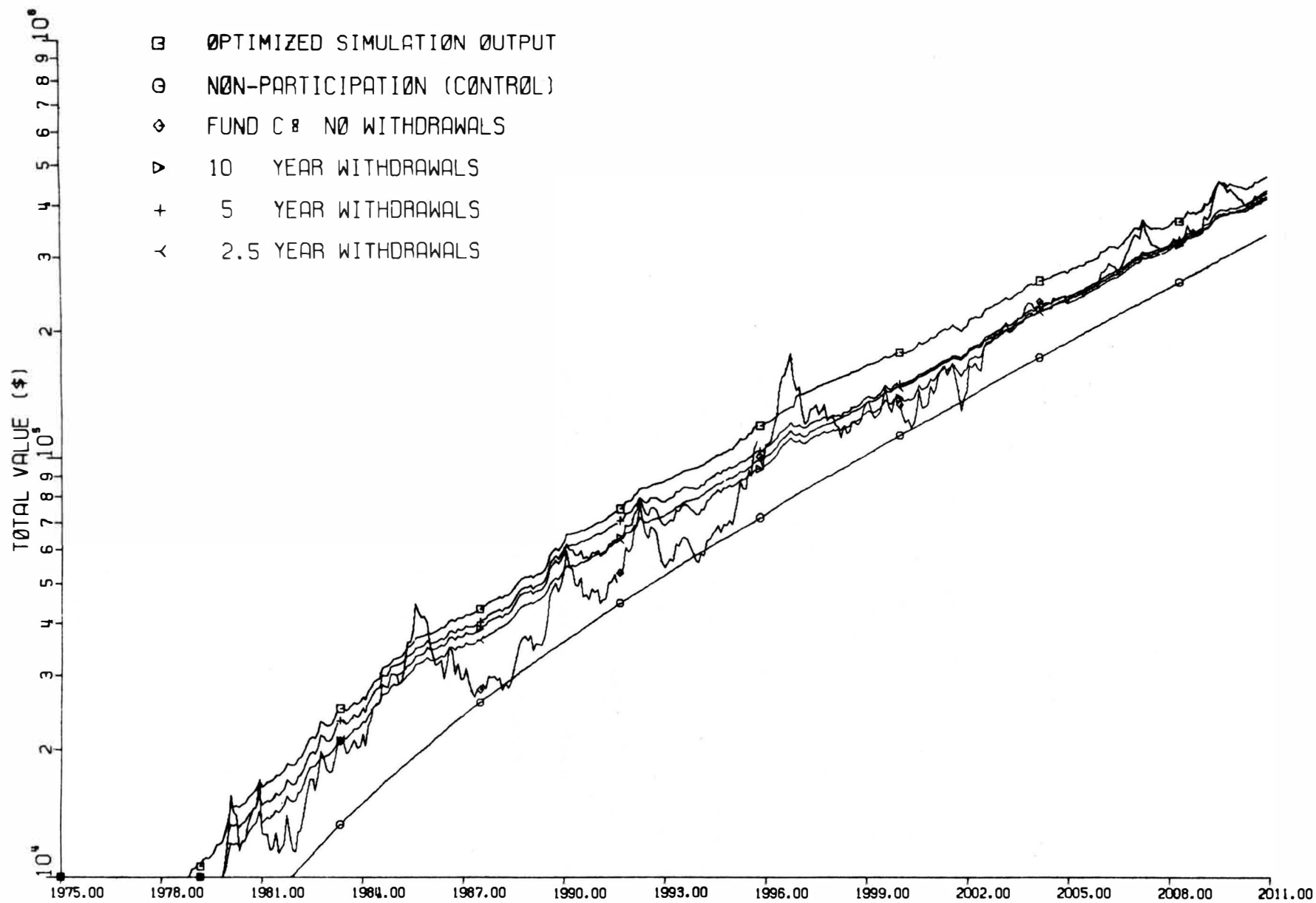


Figure 23 Simulation Results - Fund C Investments

V. CONCLUSIONS

As mentioned above, at the outset of research a goal was set to develop an investment system which would produce a 20% improvement in total savings over any arbitrary strategy within the Plan. In running the various simulations it became apparent that merely testing the goal would be a matter of judgement. Although one can see from Table III that said improvement is not achieved over every simulation at every point in time, the writer believes that the spirit of the goal has been realized. Of the 91 entries for alternate strategies in the Table, the corresponding optimal value exceeds 47 of them by more than 20%. The improvement is greater than 10% on an additional 30 entries. In only 4 cases does an alternate actually exceed the optimal: 2 in 1980 and 1 each in 1985 and 2010. In no case does an alternative consistently remain comparable to the optimal. Because of the periodic withdrawals inherent in the optimized technique, a degree of stability in total assets results through liquidations of the volatile Fund C.

In reviewing further the results presented in Table III and Figures 21 to 23, one notices that during the middle years of the simulation series, say 1985 to 2000, the benefit of the optimized strategy is greatest. Advantages of 25% are typical. It is understandable that the early years appear as they do. It takes time for the effects of investment decisions to be realized in total values. The seeming loss of the optimized strategy's gains in the later years though, is a curious situation. The Model seems to run out of steam in the last decade or so.

The author would like to suggest as one possible explanation, that the Model was hobbled by inflation and tax policy. Consider that the Simulation Data Base was developed on the experiences of the 1970's, a very inflationary period. Furthermore, the Career Model also extrapolated the experience of the late 1970's with the 10% annual salary increases (McDonnell starting salaries reflect that rate since 1975, though such things as engineering demand are also factors). Yet throughout the simulation the tax tables went unchanged. Looking back at the simulation output data, it was found that the optimal strategy induced no withdrawals in the last 14 years. The employee's total income had reached the maximum tax rate (70%) in 1997. Apparently strategically timed withdrawals were no longer important. One had to merely maximize income by never forfeiting the McDonnell contributions.

In retrospect, it seems that making no adjustments to tax rates or making no compensation for inflation may represent flaws in the Model. Indeed, at this writing a tax bill has been signed into law which will ultimately index taxation rates to inflation. However, at the time the Model was being developed, less than one year earlier, such notions as indexation seemed far-fetched.

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VITA

Richard Charles Seifert was born on June 8, 1952 in St. Louis, Missouri. He received his primary and secondary education in St. Louis, graduating from Lutheran High School South in 1970. He received his undergraduate college education at the University of Missouri-Rolla in Rolla, Missouri. In December, 1974 he graduated with a Bachelor of Science degree in Electrical Engineering. More recently he has attended the University's Graduate Extension Center in St. Louis as a candidate for a Master of Science degree, also in Electrical Engineering.

In January, 1975 he joined the Guidance and Control Mechanics Department of McDonnell Douglas Astronautics Company - East. He has worked primarily on the development of the Cruise Missile Guidance System. As a Senior Engineer, his principle assignment concerns the application of Terrain Correlation as a means of enhancing missile navigation.

APPENDIX

COMPREHENSIVE SAVINGS PLAN

MODEL PROGRAM LISTING

NAME DECIDE

```

C*****
C***** PROGRAM DECIDE IS THE COMPREHENSIVE SAVINGS *****
C***** PLAN MODEL IT INCLUDES THE CRITERIA FOR *****
C***** INVESTMENT DECISIONS AND APPLIES THEM TO A *****
C***** MODEL OF THE PLAN AND ITS ENVIRONMENT *****
C*****

```

```

      DIMENSION CMAC(24),FUNDS(3)

```

```

C      INITIALIZE PROGRAM AND REQUEST EXTRAPOLATION PERIOD
C      OVER WHICH TO BASE DECISION STATISTICS

```

```

5      REWIND 10
      REWIND 11
      IWDR=0
      JWD=0
      AU=0.
      BU=0.
      CU=0.
      AUX=0.
      CONT=0.
      STAX=0.
      TAXP=0.
      TINT=0.
      TAXED=0.
      STAXP=0.
      DO 6 I=1,24
6      CMAC(I)=0.
      FUNDS(3)=1HC
      WRITE(20,1007)
      READ(20,1007) ICYC
      IF(ICYC.LE.0) CALL EXIT
      PRINT 1008, ICYC
      PRINT 1009, ICYC
      WRITE(50,1008) ICYC
      WRITE(55,1008) ICYC
      WRITE(60,1008) ICYC
      WRITE(65,1008) ICYC
      WRITE(50,1010)
      WRITE(55,1011)
      WRITE(60,1012)
      WRITE(65,1023)
      CALL TREND(0.,0.,0.,0.,1)

C      READ CURRENT INCOME INFORMATION AND INTEREST RATE
C      ZEROES INDICATE NO CHANGE
10     READ(10,1000,END=100,ERR=10) MON,IYR,IWK,IWD,RAT,SALA,
      TINC
      TIME=1900.+IYR+(MON-1)/12.+.0001
      CALL CALN(MON,IYR,NWK)
      IF(SALA.EQ.0.) GO TO 11
      SAL=SALA
11     IF(TINC.EQ.0.) GO TO 12
      TIN=TINC
12     IF(IWK.EQ.0) GO TO 13
      JWK=IWK
13     IF(IWD.LE.0) GO TO 14
      JWD=IWD
14     IF(RAT.LE.0.) GO TO 15
      RATE=RAT
      RATX=RATE/12.+1.
15     IF(CONT.EQ.0.) GO TO 16
      IF(MON.EQ.3 .OR. MON.EQ.9) GO TO 16

```

```

16      GO TO 20
      CONT=FLOAT(INT(.08*SAL+.5))
C      READ UNIT VALUES FOR THE CURRENT MONTH
20      READ(11,1001,END=100,ERR=20) M,N,A,B,C
      TIM=1900.+N+(M-1)/12.+0001
      IF(TIM.GT.TIME) GO TO 10
      IF(TIM.LT.TIME) GO TO 20
      TIM2=TIME+ICYC/12.

C*****
C*****      GATHER STATISTICS ON EXPECTED PERFORMANCE      *****
C*****

C*** PART A - PREDICT THE EFFECTS OF NOT WITHDRAWING      ***
C      PRESENT HOLDINGS FOR SPECIFIED TIME PERIOD

      CALL TREND(TIM2,ATT,BTT,CTT,2)
      IF((AU+BU+CU).LE.0.) THEN
        WRITE(50,1020) M,N
        GO TO 40
      ENDIF
      CUT=CU+19.5*CONT/C
      FV=AU*ATT+BU*BTT+CUT*CTT
      GAIN=FV-STAXP
      CALL TAXTAB(TIN,GAIN,TAX)
      FRA=FV-TAX
      WRITE(50,1020) M,N,AU,A,ATT,BU,B,BTT,CU,C,CTT,STAXP,
      .      TAX,FRA

C*** PART B - EFFECTS OF WITHDRAWING THIS MONTH ***
C      CONVERT UNITS TO CASH AND PAY TAX DUE NOW

      VAL=AU*A+BU*B+CU*C
      GAIN=VAL-STAXP
      CALL TAXTAB(TIN,GAIN,TAX)
      VAL=VAL-TAX
      TAXT=TAX

C      EXTRAPOLATE SAVINGS ACCOUNT OVER SPECIFIED TIME PERIOD

      DO 30 I=1,ICYC
      VALP=VAL
      VAL=RATX*VAL
      GAIN=VAL-VALP
      CALL TAXTAB(TIN,GAIN,TAX)
      TAXT=TAXT+TAX
30      VAL=VAL-TAX
      FRB=VAL
      WRITE(50,1021) FRB,TAXT

C*** PART C - PREDICT EFFECTS OF CONTRIBUTING THIS MONTH ***
C      PURCHASE NEW UNITS WITH MONTH'S CONTRIBUTION

40      AM=NWK*CONT/A
      BM=NWK*CONT/B
      CM=NWK*CONT/C

C      EXTRAPOLATE EACH FUND OVER SPECIFIED TIME PERIOD

      FV=AM*ATT+.75*CM*CTT
      GAIN=FV-CONT*NWK

```

```

CALL TAXTAB(TIN,GAIN,TAX)
FRCA=FV-TAX
FV=BM*BTT+.75*CM*CTT
GAIN=FV-CONT*NWK
CALL TAXTAB(TIN,GAIN,TAX)
FRCB=FV-TAX
FV=1.75*CM*CTT
GAIN=FV-CONT*NWK
CALL TAXTAB(TIN,GAIN,TAX)
FRCC=FV-TAX
FRC=0.
IF (JWD.EQ.1) FRC=FRCA
IF (JWD.EQ.2) FRC=FRCB
IF (JWD.EQ.4) FRC=FRCC
IF (JWD.EQ.3) FRC=.5*(FRCA+FRCB)
IF (JWD.EQ.5) FRC=.5*(FRCA+FRCC)
IF (JWD.EQ.6) FRC=.5*(FRCB+FRCC)

```

C*** PART D - EFFECTS OF NOT CONTRIBUTING THIS MONTH ***

C EXTRAPOLATE TO TERMINATION TIME

```

VAL=CONT*NWK
DO 50 I=1,ICYC
VALP=VAL
VAL=RATX*VAL
GAIN=VAL-VALP
CALL TAXTAB(TIN,GAIN,TAX)
50 VAL=VAL-TAX
FRD=VAL
WRITE(55,1022) M,N,CONT,FRC,FRD,FRCA,FRCB,FRCC

TOTAL1=FRA-FRB
TOTAL2=FRD-FRD
WRITE(60,1004) M,N,TOTAL1,TOTAL2
IF (STAXP.GT.0.) THEN
    FRA=FRA/STAXP
    FRB=FRB/STAXP
    FRC=FRC/CONT
    FRD=FRD/CONT
    WRITE(60,1006) FRA,FRB,FRC,FRD
ENDIF

```

```

C*****
C***** MAKE THE INVESTMENT DECISIONS *****
C*****

```

C DETERMINE FUND PROBABILITIES RELATIVE TO EXPECTED VALUES

```

PA=A
PB=B
PC=C
CALL TREND(TIME,PA,PB,PC,3)
A1=AU*A
B1=BU*B
C1=CU*C
T1=A1+B1+C1
PTOT=0.
IF (T1.GT.0.) PTOT=(PA*A1+PB*B1+PC*C1)/T1
PSUM=PTOT+PC
WRITE(65,1015) M,N,PTOT,PA,PB,PC

```

C HAVE YOU WITHDRAWN IN THE LAST 6 MONTHS ?

```

IWD=0

```

```

IF(IWDR.GT.0) THEN
    WRITE(65,1013)
    GO TO 75
ENDIF

C    SHOULD WITHDRAWAL BE CONSIDERED THIS MONTH ?

IF(TOTAL1.GT.0.) THEN
    WRITE(65,1014)
    GO TO 60
ENDIF

C    IS THIS A GOOD MONTH TO WITHDRAW ?

IF(PTOT.LT..75 .OR. PC.LT..75 .OR. PSUM.LT.1.6) THEN
    WRITE(65,1016)
    GO TO 60
ENDIF

C    IF WITHDRAWING, FROM WHICH FUNDS ?

IWD=-4
FUNDS(1)=1H
FUNDS(2)=1H
IF(PA.GE..8) THEN
    IWD=-5
    FUNDS(1)=1HA
ENDIF
IF(PB.GE..8) THEN
    IWD=IWD-2
    FUNDS(2)=1HB
ENDIF
WRITE(65,1017) FUNDS
GO TO 70

C    CHOOSE THE BEST FUNDS IN FEBRUARY AND AUGUST
60  IF(M.EQ.3 .OR. M.EQ.9 .OR. JWD.EQ.0) THEN
    IWK=0
    IF(FRCA.GT.FRCB) THEN
        I=1
        FRC=FRCA
        FUNDS(1)=1HA
    ELSE
        I=2
        FRC=FRCB
        FUNDS(2)=1HB
    ENDIF
    JWD=I
    IF(FRCC.GT.FRC) THEN
        I=3
        JWD=4
        FRC=FRCC
    ENDIF
    WRITE(65,1018) FUNDS(I)
ENDIF

C    SHOULD CONTRIBUTION BE MADE THIS MONTH ?

IF(FRC.LT.FRD) THEN
    IWK=-1
    WRITE(65,1019)
ENDIF
IF(IWK.LT.0.) GO TO 76

```

```

C*****
C*****      APPLY THE DECISIONS TO THE MODEL      *****
C*****      *****

C      PURCHASE NEW UNITS OF THE APPROPRIATE FUNDS

      IF (JWD.EQ.1) AU=AU+NWK*CONT/A
      IF (JWD.EQ.2) BU=BU+NWK*CONT/B
      IF (JWD.EQ.4) CU=CU+NWK*CONT/C
      IF (JWD.EQ.3 .OR. JWD.EQ.5) AU=AU+.5*NWK*CONT/A
      IF (JWD.EQ.3 .OR. JWD.EQ.6) BU=BU+.5*NWK*CONT/B
      IF (JWD.EQ.5 .OR. JWD.EQ.6) CU=CU+.5*NWK*CONT/C
      CMAC(24)=.75*NWK*CONT/C
      STAX=STAX+NWK*CONT
      GO TO 80

C      WITHDRAW FROM FUNDS AS INDICATED
C      (FUND C MUST BE WITHDRAWN)

70      IWDR=6
      WDR=CU*C
      CU=0.
      IF (IWD.EQ.-4) GO TO 72
      IF (IWD.EQ.-6 .OR. IWD.EQ.-2) GO TO 71
      WDR=WDR+AU*A
      AU=0.
71      IF (IWD.EQ.-5 .OR. IWD.EQ.-1) GO TO 72
      WDR=WDR+BU*B
      BU=0.
72      TAXED=WDR+AU*A+BU*B-TAXP-STAX
      TAXP=AU*A+BU*B
      STAX=0.

C      WAIT OUT SIX MONTHS AFTER A WITHDRAWAL - USE AUXILIARY
C      SAVINGS ACCOUNT

75      IWDR=IWDR-1
76      AUX=AUX+WDR+NWK*CONT

C      PAY INTEREST TO THE AUXILIARY SAVINGS ACCOUNT

80      IF (IWDR.EQ.0) XINT=AUX*RATE/12.
      IF (IWDR.GT.0 .OR. IWK.LT.0) XINT=(AUX-WDR-NWK*CONT)
                                   *RATE/12.
      WDR=0.
      AUX=AUX+XINT
      TINT=TINT+XINT
      STAXP=STAX+TAXP

C      CYCLE DOWN THE SHIELDED MCDONNELL CONTRIBUTIONS

      CU=CU+CMAC(1)
      CT=0.
      DO 85 I=1,23
      CT=CT+CMAC(I+1)
85      CMAC(I)=CMAC(I+1)
      CMAC(24)=0.

C      PAY TAXES (IN DECEMBER) & TOTAL NET WORTH OF SAVINGS

      IF (MON.NE.12) GO TO 90
      TAXED=TAXED+TINT
      CALL TAXTAB(TIN,TAXED,TAX)
      AUX=AUX-TAX
      TINT=0.

```

```

TAXED=0.

90  A1=AU*A
    B1=BU*B
    C1=CU*C+CT*C
    T1=A1+B1+C1
    T2=T1+AUX
    C2=CU+CT
    TAXTST=TAXED+TINT+T1-TAXP-STAX
    CALL TAXTAB(TIN,TAXTST,TAX)
    T3=T2-TAX
    PRINT 1002, MON,IYR,A1,B1,C1,T1,AUX,T2,AU,BU,C2,T3

C   OUTPUT DECISIONS TO A SAVE FILE

    JWW=JWD
    IF(IWD.NE.0) JWW=IWD
    IF(IWK.LT.0) NWK=-NWK
    WRITE(12,1000) M,N,NWK,JWW,RATE,SAL,TIN

C   UPDATE TREND STATISTICS

    CALL TREND(TIME,A,B,C,4)
    GO TO 10

100  END FILE 12
     GO TO 5

1000  FORMAT(1X,I2,3I3,F6.3,F6.0,F8.0)
1001  FORMAT(1X,I2,I3,3F7.4)
1002  FORMAT(1X,2I4,6F9.2,3F9.2,20X,F9.2)
1003  FORMAT('1MONTH YR $ A $ B $ C $TOTAL',
           'SAUXIL $TOTAL A B C UNITS')
1004  FORMAT(1X,2I4,40X,2F10.2)
1005  FORMAT(1X,F8.3,3F10.3,F9.2)
1006  FORMAT(1H+,8X,4F10.2)
1007  FORMAT(' ENTER EXTRAPOLATION TIME PERIOD (MONTHS) ')
1008  FORMAT('1 EXTRAPOLATION TIME: I5, MONTHS/')
1009  FORMAT(14X,'SA',7X,'SB',7X,'SC',3X,'SUBTOTAL AUXILIARY',
           'TOTAL',3X,'A UNITS' B UNITS' C UNITS',21X,
           '$LAYOFF')
1010  FORMAT(17X,'AU',4X,'$ A',4X,'$ ATT',6X,'BU',4X,'$ B',
           '4X,$ BTT',6X,'CU',4X,'$ C',4X,'$ CTT',5X,'$ TAXP',6X,
           'TAXA',7X,'FRA',7X,'FRB',6X,'TAXB')
1011  FORMAT(13X,'CONTRIB FRC FRD',24X,'FRCA FRCB',
           'FRCC')
1012  FORMAT(15X,'FRA',7X,'FRB',7X,'FRC',7X,'FRD',4X,
           'FRA-FRB',3X,'FRC-FRD')
1013  FORMAT(1H+,99X,'WAITING 6 MONTHS AFTER WITHDRAWAL')
1014  FORMAT(1H+,79X,'DON'T WITHDRAW',6X,'HOLDINGS WORTH',
           'MORE IN THE PLAN')
1015  FORMAT(1X,2I4,3X,F8.4,2X,3F8.4)
1016  FORMAT(1H+,79X,'DON'T WITHDRAW',6X,'WAIT FOR ',
           'HIGHER UNIT VALUES')
1017  FORMAT(1H+,79X,'WITHDRAW FROM',3(1X,1A1))
1018  FORMAT(1H+,47X,'CHOOSE FUND',1A1)
1019  FORMAT(1H+,61X,'DON'T CONTRIBUTE')
1020  FORMAT(2I4,3X,3(F10.3,2F7.4),3F10.2)
1021  FORMAT(1H+,112X,2F10.2)
1022  FORMAT(2I4,3X,3F8.2,20X,3F8.2)
1023  FORMAT(14X,'P(ALL)',5X,'P(A)',4X,'P(B)',4X,'P(C)')
1024  FORMAT(/53X,'$',F9.2,' AFTER TAXES')
END

```



```

SUBROUTINE TREND(T,A,B,C,NN)

C  SUBROUTINE TREND DETERMINES FUND TRENDS AND STATISTICS
C  OVER TIME AND UPDATES DATA EACH YEAR

  DIMENSION AA(12),BB(12),CC(12)

  GO TO (10,20,30,40), NN

C  INITIALIZE STATISTICS
10  REWIND 13
    WRITE(70,1002)
    READ(13,1000) NA,SA,SSA,SAT,ST,SST
    READ(13,1000) NB,SB,SSB,SBT
    READ(13,1000) NC,SC,SSC,SCT
    GO TO 50

C  DETERMINE EXPECTED 80%ILE FUND VALUES AT TIME T
20  A=A1*(T-1900.)+A2+.842*ASIG
    B=B1*(T-1900.)+B2+.842*BSIG
    C=C1*(T-1900.)+C2+.842*CSIG
    A=10.**A
    B=10.**B
    C=10.**C
    RETURN

C  DETERMINE PROBABILITIES OF CURRENT FUND VALUES
30  AM=A1*(T-1900.)+A2
    BM=B1*(T-1900.)+B2
    CM=C1*(T-1900.)+C2
    A=ALOG10(A)
    B=ALOG10(B)
    C=ALOG10(C)
    CALL ERF(A,AM,ASIG)
    CALL ERF(B,BM,BSIG)
    CALL ERF(C,CM,CSIG)
    RETURN

C  GATHER FUND PERFORMANCE DATA AND
C  UPDATE STATISTICS ONCE A YEAR
40  IF(T.LT.1981.) RETURN
    N=N+1
    AA(N)=ALOG10(A)
    BB(N)=ALOG10(B)
    CC(N)=ALOG10(C)
    AS=AS+ALOG10(A)
    BS=BS+ALOG10(B)
    CS=CS+ALOG10(C)
    TS=TS+T-1900.
    TSS=TSS+(T-1900.)**2
    AST=AST+ALOG10(A)*(T-1900.)
    BST=BST+ALOG10(B)*(T-1900.)
    CST=CST+ALOG10(C)*(T-1900.)
    IF(N.LT.12) RETURN

```

```

NA=NA+N
SA=SA+AS
SB=SB+BS
SC=SC+CS
SAT=SAT+AST
SBT=SBT+BST
SCT=SCT+CST
ST=ST+TS
SST=SST+TSS

50  A1=(SAT-SA*ST/NA)/(SST-ST**2/NA)
    A2=SA/NA-A1*ST/NA
    B1=(SBT-SB*ST/NA)/(SST-ST**2/NA)
    B2=SB/NA-B1*ST/NA
    C1=(SCT-SC*ST/NA)/(SST-ST**2/NA)
    C2=SC/NA-C1*ST/NA

    IF(NN.EQ.1) GO TO 70
    DO 60 I=1,12
    AEXP=A1*(T-1901.+I/12.)+A2
    BEXP=B1*(T-1901.+I/12.)+B2
    CEXP=C1*(T-1901.+I/12.)+C2
    ASS=ASS+(AA(I)-AEXP)**2
    BSS=BSS+(BB(I)-BEXP)**2
60  CSS=CSS+(CC(I)-CEXP)**2

    SSA=SSA+ASS
    SSB=SSB+BSS
    SSC=SSC+CSS

70  ASIG=SQRT(SSA/NA)
    BSIG=SQRT(SSB/NA)
    CSIG=SQRT(SSC/NA)
    WRITE(70,1001) NA,A1,A2,ASIG,B1,B2,BSIG,C1,C2,CSIG

    N=0
    AS=0.
    BS=0.
    CS=0.
    TS=0.
    TSS=0.
    ASS=0.
    BSS=0.
    CSS=0.
    AST=0.
    BST=0.
    CST=0.
    RETURN

1000 FORMAT(I12,5E16.8)
1001 FORMAT(I10,3(F14.7,' * T +',F11.7,' ;',F7.4))
1002 FORMAT(1H1,' NO. PTS',13X,' TREND(A)',15X,' ASIG',13X,
.      ' TREND(B)',15X,' BSIG',13X,' TREND(C)',15X,' CSIG',/)
    END

```

```

SUBROUTINE TAXTAB(TIN,TAXED,TAX)
C   TAX TABLE FROM IRS SCHEDULE X
    DIMENSION A(16),B(16),C(16)
    DATA A /2300.,3400.,4400.,6500.,8500.,10800.,12900.,
.           15000.,18200.,23500.,28800.,34100.,41500.,
.           55300.,81800.,108300./
    DATA B /0.,154.,314.,692.,1072.,1555.,2059.,2605.,3565.,
.           5367.,7434.,9766.,13392.,20982.,37677.,55697./
    DATA C /.14,.16,.18,.19,.21,.24,.26,.30,.34,.39,.44,
.           .49,.55,.63,.68,.70/

    TTIN=TIN+TAXED
    DO 10 I=1,15
    IF(TTIN.LT.A(I+1)) GO TO 20
10   CONTINUE
    I=16
20   TTAX=B(I)+C(I)*(TTIN-A(I))
    IF(TTAX.LT.0.) TTAX=0.

    DO 30 I=1,15
    IF(TIN.LT.A(I+1)) GO TO 40
30   CONTINUE
    I=16
40   BTAX=B(I)+C(I)*(TIN-A(I))
    IF(BTAX.LT.0.) BTAX=0.
    TAX=TTAX-BTAX

C   MISSOURI STATE INCOME TAX
C   6% MARGINAL RATE : FEDERAL TAX DEDUCTABLE
C   ASSUMES TAXABLE INCOME OF $9000 OR MORE

    TAX=TAX+.06*(TAXED-TAX)

    RETURN
    END

```

```

SUBROUTINE ERF(A,AM,ASIG)
C   NORMALIZED ERROR FUNCTION TABLE
    DIMENSION E(16)
    DATA E / 0.,.0793,.1554,.2258,
.           .2881,.3413,.3849,.4192,
.           .4452,.4641,.4772,.4861,
.           .4918,.4953,.4974,.4987/

    DEV=ABS(A-AM)/ASIG
    DO 10 I=2,16
    IF(DEV.LT..2*(I-1)) GO TO 20
10   CONTINUE
    AA=.5
    GO TO 30

20   AA=E(I-1)+(E(I)-E(I-1))*(5.*DEV+2.-I)
30   IF(A.GE.AM) AB=.5+AA
    IF(A.LT.AM) AB=.5-AA
    A=AB
    RETURN
    END

```

```

SUBROUTINE CALN(MONTH,IYEAR,NWKS)
C  SUBROUTINE DETERMINES NUMBER OF PLAN WEEKS IN THE MONTH
C  EQUALS THE NUMBER OF SUNDAYS : FROM A PERPETUAL CALENDER

DIMENSION NMON(12)
DATA NMON/31,28,31,30,31,30,31,31,30,31,30,31/

IYR=IYEAR+1900
I=28
IF(MOD(IYR,4).EQ.0) I=29
IF(MOD(IYR,100).EQ.0) I=28
IF(MOD(IYR,400).EQ.0) I=29
NMON(2)=I
NWKS=0
NMO=NMON(MONTH)
J=0
DO 10 I=1,MONTH
J=J+NMON(I)
10 CONTINUE
M=IYEAR+100
M=MOD(M+(M-1)/4-M/100+(IYR-1)/400,7)
IF(M.EQ.0) M=7
M=M+J-NMO
DO 20 I=1,NMO
K=M+I-1
KK=K-7*(K/7)
IF(KK.EQ.1) NWKS=NWKS+1
20 CONTINUE
RETURN
END

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