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APPROXIMATE ANALYTICAL GUIDANCE SCHEMES FOR HOMING MISSILES

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Abstract

Closed form solutions for the guidance laws are developed using modern control techniques. The resulting two-point boundary value problem is solved through the use of the state transition matrix of the intercept dynamics. Results are presented in terms of a design parameter. The results of comparison with other guidance laws will be presented at the conference (for lack of space).

1. Introduction

Homing missile guidance is a guidance system which uses mainly the line-of-sight (LOS) rate to guide the missile towards its target. Proportional navigation guidance and its derivatives have been shown to be an effective LOS rate guidance system [1,3,5,6,8,9]. With the need for improved missile performance, new methods for missile guidance have been investigated using modern control techniques [2,7].

In this study an optimal homing missile guidance law will be developed in polar coordinates which are the natural coordinate system for a missile engagement since the measurements are bearing angle, range and range rate. Decoupling of the dynamic equations is accomplished by introducing a pseudo-control in the radial direction, which produces an optimal control problem in each direction. The closed form solution in the radial direction is found through the use of the pseudo-control and the closed form solution in the transverse direction is found by using the state transition matrix of the intercept dynamics.

2. Optimal Guidance Law in Decoupled Polar Coordinates [2,7]

The dynamics of a two dimensional target-intercept problem as shown in Figure 1, can be described in inertial polar coordinates by two coupled nonlinear differential equations as

$$\ddot{r} - r\dot{\theta}^2 = a_{T_r} - a_{M_r} \tag{1}$$

and

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = a_{T_\theta} - a_{M_\theta}. \tag{2}$$

In these equations r is the relative range between the target and the missile, θ is the bearing angle, a_{T_r} and a_{T_θ} are the target accelerations in the radial and transverse directions respectively, and a_{M_r} and a_{M_θ} are the missile commanded accelerations in the radial and transverse directions respectively. Dots denote differentiation with respect to time.

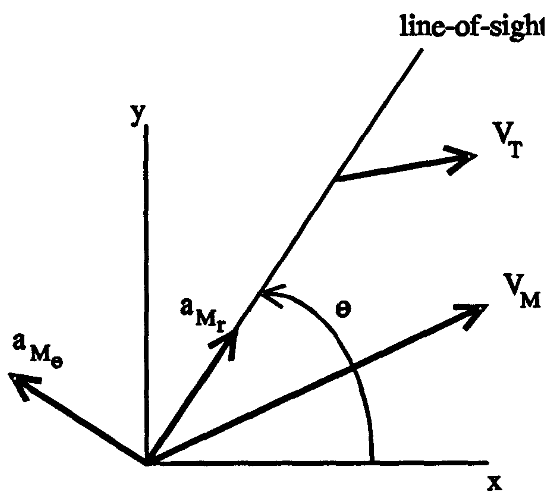

Figure 1: Engagement Geometry

In order to decouple the dynamics in the radial and transverse directions a pseudo-control is defined in the radial direction as

$$a_{M_{r1}} = a_{M_r} - r\dot{\theta}^2 . \qquad (3)$$

By introducing the pseudo-control, the dynamics in the radial and transverse directions are decoupled. This allows the commanded acceleration in each direction to be developed independent of the other. The performance index in the transverse direction can be written as

$$J_\theta = \frac{1}{2} S_{f_\theta} z_{2_f}^{2} + \frac{1}{2}\int_0^{t_f} (\gamma_1 z_2^2 + \gamma_2 a_{M_\theta}^{2}) dt \qquad (4)$$

$$\text{where} \quad z = [\theta, \dot{\theta}, a_{T_\theta}]^T,$$

is the corresponding state space. In Eq. (4), S_{f_θ} is the weight on the final line-of-sight rate and γ_1 and γ_2 are the weights on the line-of-sight rate and the transverse commanded acceleration respectively.

The optimization of Eq. (4) [4] results in a two-point boundary value problem in z_2 and λ_2

$$\begin{bmatrix} \dot{z}_2 \\ \dot{\lambda}_2 \end{bmatrix} = A(t) \begin{bmatrix} z_2 \\ \lambda_2 \end{bmatrix} + \begin{bmatrix} g(t) z_3(0) a(t)) \\ 0 \end{bmatrix}, \qquad (5)$$

$$\text{where} \quad A(t) = \begin{bmatrix} f(t) & -g^2(t)/\gamma_2 \\ -\gamma_1 & -f(t) \end{bmatrix}$$

$f(t) = 2\dot{r}/r$, $g(t) = 1/r$ and $a(t) = \exp[-\lambda_\theta t]$.

The term $z_3(0)a(t)$ represents the solution to the target acceleration by assuming a first-order model. λ_2 represents the Lagrange's multiplier corresponding to the LOS rate. The minimizing control, a_{M_θ}, in the transverse direction at any time is given by

$$a_{M_\theta} = g(t)\lambda_2/\gamma_2 . \qquad (6)$$

3. An Optimal Guidance Law Solution Using State Transition Matrix

This section deals with solutions to Eq. (5) with non-maneuvering and maneuvering targets. Since z_2 is known at the initial time and λ_2 at the final time, Eq. (5) represents a two-point boundary value problem. We approximate closing velocity so as to obtain closed-form solutions.

3.1. Non-Maneuvering Target

Without target acceleration Eq. (6) can be written in a state space form as

$$\dot{x}(t) = A(t)x(t) \qquad (7)$$

where $x(t) = [z_2(t) \ \lambda_2(t)]^T$.

The solution to Eq. (7) can be assumed as

$$x(t) = \phi(t,\tau)x(\tau) \qquad (8)$$

where $\phi(t,\tau)$ is the state transition matrix. It's elements are

$$\phi_{11}(t_1\tau) = -[A_1(D+3)b^{-a_2} - A_2(D-3)b^{a_1}]/2\gamma_1,$$

$$\phi_{21}(t_1\tau) = A_1 b^{-a_1} + A_2 b^{a_2}, \qquad (8)$$

$$\phi_{12}(t_1\tau) = -[A_3(D+3)b^{-a_2} - A_4(D-3)b^{a_1}]/2\gamma$$

$$\phi_{22}(t_1\tau) = A_3 b^{-a_1} + A_4 b^{a_2}, \text{ where}$$

$$a_1 = 1/2(D-1), \quad a_2 = 1/2(D+1),$$

$$b = t_f - t, \quad c = t_f - \tau,$$

$$F = 1/\dot{r}_0^2\gamma_2, \quad D = \sqrt{9 + 4F_{\gamma_1}}$$

$$A_1 = -\gamma_1 e^{a_2}/D, \quad A_2 = \gamma_1 e^{a_2}/D$$

$$A_3 = (D-3)/(2De^{-a_1}), \text{ and}$$

$$A_4 = (D+3)/(2De^{a_2}).$$

Note that we assume the closing velocity constant in Eq. (8). That is,

$$r(t) = -\dot{r}(t)(t_f - t) . \qquad (9)$$

The resulting solution to the homogeneous differential equation in Eq. (7) and hence, Eq. (6) are

$$\dot{\theta}(t) = (1 - \frac{t}{t_f})^{\frac{1}{2}(D-1)} \dot{\theta}_o$$

$$a_{M_\theta}(t) = -\frac{(D+3)}{2} \dot{r}_0 \dot{\theta}_0 (1 - \frac{t}{t_f})^{\frac{1}{2}(D-1)}, \qquad (10)$$

If $t = 0$ is assumed to be the current time, the minimizing control in the transverse direction with a non-maneuvering target becomes

$$a_{M_\theta} = -\frac{(D+3)}{2} \dot{r}_0 \dot{\theta}_0 . \qquad (11)$$

3.2. Maneuvering Target

The solution to the two-point boundary value problem for a maneuvering target can be obtained by adding the target acceleration to Eq. (7). The solution leads to adding $q_1(t)$ to $z_2(t)$ and $q_2(t)$ to $\lambda_2(t)$ where

$$q_i(t) = \frac{z_3(0) t_f}{r_0} \int_0^t \frac{\phi_{i1}(t,\tau) \exp(-\lambda_\theta \tau) d\tau}{(t_f - \tau)} \qquad i = 1,2 \qquad (12)$$

The resulting solutions to Eq. (5) are

$$\dot{\theta}(t) = e_1 \dot{\theta}_0 + \frac{2}{\dot{r}_0} [(3s_4 - s_5) - e_1 (3s_2 - s_3)] a(t), \qquad (13)$$

$$\lambda_2(t) = \frac{(D+3)}{2} \gamma_2 t_f \dot{r}_0^2 e_2 \dot{\theta}_0 + \left[\frac{6(D+3)}{D} r_0 \gamma_2 e_2\right] s_1 a(t) + \left[\frac{4\gamma_1 t_f}{\dot{r}_o}(1 - \frac{t}{t_f})\left(\frac{s_3}{D} e_1 - s_4\right)\right] a(t), \qquad (14)$$

where $e_1 = (1-t/t_f)^{(D-1)/2}$, $e_2 = (1-t/t_f)^{(D+1)/2}$, $a(t) = a_{T_\theta}(0) \exp(-\lambda_\theta t_f)$

$$s_1 = \left[\frac{1}{(D+1)} + \frac{\lambda_\theta t_f}{(D+3)} + \frac{\lambda_\theta^2 t_f^2}{2!(D+5)} + \ldots\ldots\right].$$

Note that s_1, s_2, s_3, s_4 and s_5 are all functions of the target acceleration model.

$$a_{M_\theta}(t) = -\frac{(D+3)}{2} \dot{r}_0 \dot{\theta}_0 e_1 - \frac{6(D+3)}{D} e_1 s_1 a(t) + (D^2 - 9)\left[s_4 - \frac{s_3}{D} e_1\right] a(t) \qquad (14)$$

The current time is assumed zero. The minimizing control in the transverse direction becomes

$$a_{M_\theta} = -\frac{(D+3)}{2} \dot{r}_0 \dot{\theta}_0 + (D+3) s_1 a(t). \qquad (15)$$

4. Design Parameter: D

In this section the expression for the design parameter, D, will be evaluated for various typical intercept scenarios. The effect of D on the line-of-sight rate, commanded acceleration and range will be analyzed.

The parameter D will always be a positive quantity since the second term under the square root is always greater than zero. If the second term under the square root is small compared to the first term under the square root, the lower limit of D can be approximated as 3. If the first term under the square root is small compared to the second term under the square root, D can be approximated as $2/\dot{r}_0\sqrt{\gamma_1/\gamma_2}$. An increasing value for D corresponds to controlling the level of the line-of-sight rate more than the commanded acceleration. A value of $D = 3$ corresponds to maintaining acceptable levels of both the line-of-sight rate and the commanded acceleration.

4.1. Approximations for D

For many typical intercept scenarios, D can be approximated as 3. If the weight on the line-of-sight rate, γ_1, is at most three orders of magnitude larger than the weight on the control effort, γ_2, then the approximation $D = 3$ holds, over the entire flight time, for intercept scenarios with initial ranges larger than 1000 feet.
If $t = 0$ is considered to be the current time and D can be approximated as 3, the commanded acceleration for the STM solution with a non-maneuvering target can be written as

$$a_{M_\theta} = -3\dot{r}_0\dot{\theta}_0 \qquad (16)$$

which is the standard proportional navigation equation. Similarly, the commanded acceleration for the STM solution with a maneuvering target can be written as

$$a_{M_\theta} = -3\dot{r}_0\dot{\theta}_0 + 6s_1 a_{T_\theta}(0)\exp(-\lambda_\theta t_f)\,. \qquad (17)$$

If the weight on the line-of-sight rate, γ_1, if more than three orders of magnitude or less than eight orders of magnitude larger than the weight on the control effort, γ_2, ($10^3 <$ rat $< 10^8$) then the previous approximations do not hold. This is because both terms under the square root become a significant part of the value of D. During an engagement, if the approximation does not hold, the full expression of D must be used and the value of D is larger than 3 over the entire flight time.

4.2. The Effects of the Design Parameter, D

Note that $\dot{\theta}$ can be written as

$$\dot{\theta} = \dot{\theta}_0 (r/r_0)^{\frac{1}{2}(D-1)}\,. \qquad (18)$$

The line-of-sight rate will always go to zero since the minimum value of D is 3.

A plot of Eq. (18), for D = 3; $\gamma_1 / \gamma_2 = 10^3$, D = 6; $\gamma_1 / \gamma_2 = 10^6$, D = 9; $\gamma_1 / \gamma_2 = 3 \times 10^6$, is shown in Figure 2.

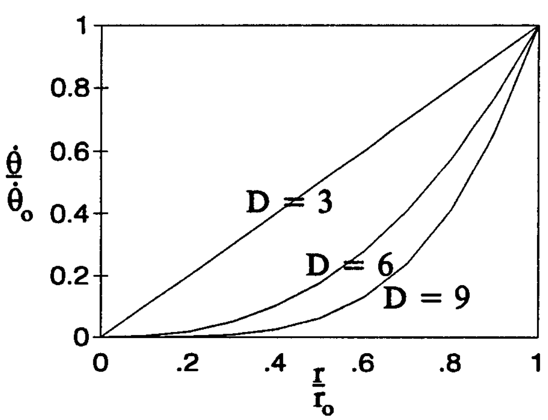

Figure 2: Line of Sight Rate vs. Range for Different D ($a_T = 0$)

From Figure 2 it can be seen that as D is increased, the line-of-sight rate goes to zero faster and earlier in the engagement when the range is still large. This means the heading error is corrected earlier in the flight with larger values of D.

Using the same substitution for $\dot{\theta}$ produces a plot of nondimensional acceleration for different values of D, as shown in Figure 3.

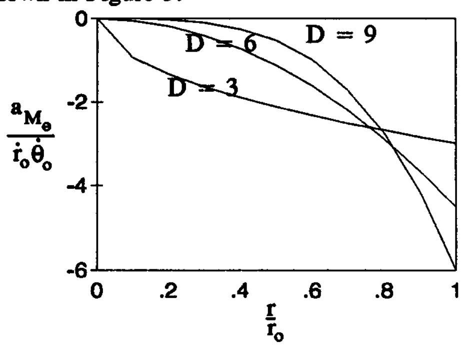

Figure 3: Commanded Acceleration vs. Range for Different D ($a_T = 0$)

It can be seen from Figure 3 that the initial commanded acceleration increases as D is increased but the commanded acceleration also goes to zero faster as D is increased.

A three-degree-of-freedom missile-target simulation was used to evaluate the effect D where the initial conditions: range 3,000 ft; altitude, 10,000 ft; aspect angle, 150 deg; off-boresight angle, 0 deg. The results for the range over the flight time, for D = 3, 6, 9, were all within ten feet of one another.
In order to observe the effects D has on the guidance law, with a maneuvering target, the equations for the line-of-sight rate (Eq. (13)) and commanded acceleration (Eq. (15)) will be used.

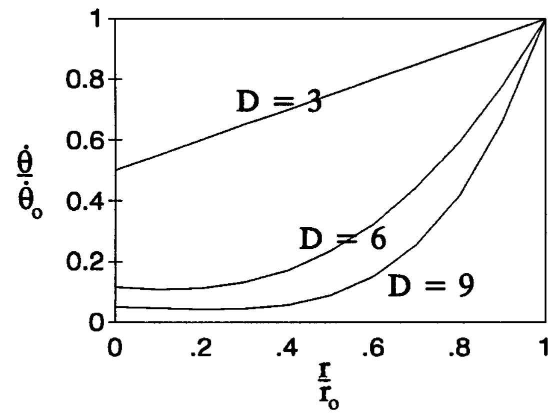

Figure 4: Line-of-Sight Rate vs. Range for Different D ($a_T \neq 0$)

From Figure 4 it can be seen that as D is increased, the line-of-sight rate reaches a minimum constant LOS rate faster and earlier in the engagement when the range is still large. The line-of-sight rate will never go to zero, regardless of the value of D, because of the target maneuver.

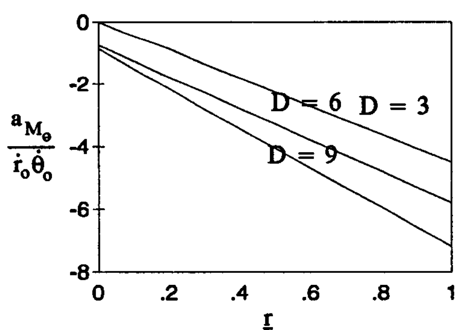

Figure 5: Commanded Acceleration vs. Range for Different D ($a_T \neq 0$)

It can be seen from Figure 5 that the initial commanded acceleration increases as D is increased but the commanded acceleration also settles to a constant faster as D is increased from 3 to 9. In the case where D = 3, the commanded acceleration goes to zero regardless of the target maneuver.

A three-degree-of-freedom missile-target simulation was used where the target performs a 5 g maneuver in the transverse direction. As was the case for the non-maneuvering target, the results for the range over the flight time, for D = 3, 6, 9, were all again within ten feet of one another so the plot is not presented.

5. Conclusions

An optimal guidance law has been developed in polar coordinates by introducing a pseudo-control to decouple the intercept dynamics.

Approximations for the state transition matrix solution were evaluated for typical intercept scenarios and it was found that the design parameter, D, can be approximated as 3 for intercept scenarios which have initial ranges of at least 1000 ft. and the weight on the line-of-sight rate is 3 orders of magnitude larger than the weight on the control effort. It was also determined that the minimum value of D was 3.

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