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ANALYTICAL GUIDANCE LAWS AND INTEGRATED GUIDANCE/AUTOPILOT
FOR HOMING MISSILES

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ABSTRACT

An approach to integrated guidance/autopilot design is considered in this study. It consists of two parts: 1) recognizing the importance of polar coordinates to describe the end game in terms of problem description and measurement acquisition, the terminal guidance problem is formulated in terms of polar coordinates; 2) a. through the use of the state transition matrix of the intercept dynamics, a closed form solution for the transverse command acceleration is obtained; b. through a commonly used approximation on time-to-go and a coordinate transformation, a family of proportional navigation optimal guidance laws is obtained in a closed form. A typical element of such a guidance law is combined with the autopilot dynamics to result in a feedback control law in terms of output variables. Numerical simulations are in progress.

I. INTRODUCTION

The most popular homing missile guidance is based on a control law called proportional navigation [1]. The basic notion is that if the line-of-sight rate is annulled, then (for a non-maneuvering, constant velocity target) the missile is on a collision course. If the target is considered smart or maneuvering, then variations to the proportional navigation have been shown to result in better miss distances. These variations have been given optimal control foundations through linear quadratic Gaussian (LQG) formulations [2-5].

There are, however, a few problems with the use of such guidance laws. First is that the measurements in an end game are nonlinear (bearing angle, range, and range rate) in cartesian coordinates. As a consequence, there is linearization in the filtering update process. The measurements are linear in a polar coordinate based state space. However, the propagation between the measurement updates in this case leads to nonlinear equations. Therefore, the states used in the guidance law are suboptimal. The second problem lies with the guidance law which is formulated assuming separability of the guidance (control) law and the estimators which do not hold. It is usually formulated in cartesian coordinates for linearity [3, 4, 6]. The third problem is that the autopilot is usually designed independent of the estimator and the guidance law. Whether it is designed based on classical control or modern control theory (LQG/LTR, H_∞), linearized dynamics are assumed at different operating points and the autopilot design really does not take the guidance law into account. As a result, there is considerable scope for research in improving the missile performance in terms of estimator, guidance, and autopilot in an intercept scenario [7].

The research in this study is focused on obtaining improvements with a properly posed controller for guidance and its use in an integrated guidance/autopilot design. A few studies have been presented in the area of integrated design of guidance and autopilot [8-10]. The difference here is that we approach the problem from proper formulation of the intercept kinematics. Such a view will enable us to integrate the estimator in the loop in an optimal way and help us address the three problems mentioned earlier in an integrated manner. The central idea here is that the polar coordinates present natural coordinate system for a missile engagement. In order to obtain a closed form solution for the commanded accelerations, the radial and transverse coordinates are decoupled. The decoupling of the coordinates leads to a two point boundary value problem with linear time-varying coefficients. However, with a nonlinear transformation, a class of closed form solutions are obtained which yield several proportional guidance laws.

The rest of the paper is organized as follows: the optimal guidance problem is developed in polar coordinates in Section II. It is further shown to decompose into two decoupled optimal control problems where a closed form control solution is available in the radial direction and a time-varying linear dynamic system has to be solved for control in the transverse direction. In Section III the state transition matrix of the intercept dynamics is used to produce a closed form solution for the transverse command acceleration. A commonly used approximation for time-to-go and a transformation are shown to lead to a class of proportional navigation-type feedback guidance laws in Section IV. In section V, a feedback control law is shown to result by combining the kinematics of the optimal guidance law with the dynamics of the autopilot. The conclusions are summarize in Section VI.

II. OPTIMAL GUIDANCE IN DECOUPLED POLAR
COORDINATES

The dynamics of a target-intercept geometry are expressed by a set of coupled nonlinear differential equations in an inertial polar coordinate system as (Figure 1)

$$\ddot{r} - r\dot{\theta}^2 = a_T - a_M \quad (1)$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = a_T - a_M \quad (2)$$

In Eqs. (1) and (2) r is the relative range between the target and the missile, θ is the bearing angle and \mathbf{a}_T and \mathbf{a}_M are respectively the target and missile accelerations in the line-of-sight (radial) direction. Similarly, $\mathbf{a}_{T\theta}$ and $\mathbf{a}_{M\theta}$ represent the target and missile accelerations respectively in the transverse directions. Dots denote differentiations with respect to time. Note that if the analysis is carried out in three dimensions, there will be another equation involving elevation angle.

Line-of-Sight (Radial) Commanded Acceleration

It can be easily observed that Eq. (1) and (2) are coupled. In order to decouple the dynamics, a pseudo-control in the radial direction, $\mathbf{a}_{M,r}$ is defined as

$$\mathbf{a}_{M,r} = \mathbf{a}_M - r\dot{\theta}^2 \quad (3)$$

This definition decouples the radial coordinate from the transverse coordinate. It facilitates a state space, \mathbf{y} , in the line-of-sight direction as $\mathbf{y} = [r, \dot{r}, \mathbf{a}_{T,r}]^T$ and describe their dynamics as

$$\dot{y}_1 = y_2 \quad (4)$$

$$\dot{y}_2 = y_3 - \mathbf{a}_{M,r} \quad (5)$$

$$\dot{y}_3 = -\lambda_r y_3 \quad (6)$$

where λ_r is the time constant associated with target acceleration.

The optimal guidance law in the radial direction is obtained as a solution to minimizing the performance index, J_r , where

$$J_r = \frac{1}{2} S_f y_{1,r}^2 + \frac{1}{2} \int_0^{t_f} \gamma \mathbf{a}_{M,r}^2 dt \quad (7)$$

In Eq. (7), $y_{1,r}$ the value of the relative range (miss-distance) at the final time, t_f , S_f is the weight on the miss distance, and γ is the weight on the pseudo-control effort. The final time, t_f ,

which is the time-to-go is approximated as r/\dot{r} . The minimizing control is

$$\mathbf{a}_{M,r}(t) = \frac{t_f}{\gamma} \lambda_1(t) \quad (8)$$

and

$$\lambda_1(t) = \frac{S_f(y_1(t) + t_f y_2(t) + (1/\lambda_r^2) \mathbf{a}_{T,r}(\exp(-\lambda_r t_f) + \lambda_r t_f - 1))}{(1 + t_f^3 S_f / 3\gamma)} \quad (9)$$

In Eqs. (8) and (9), λ_1 is a Lagrangian multiplier which adjoins the state in Eq. (5) to the performance index in Eq. (7). The actual missile acceleration can be obtained from Eq. (3) as

$$\mathbf{a}_{M,r}(t) = \mathbf{a}_{M,r}(t) + r(t) \dot{\theta}^2(t)$$

The instantaneous values of the relative range, $r(t)$, and relative range rate, $\dot{r}(t)$, can be solved for by integrating Eqs. (4)-(6).

Transverse Acceleration

The equation of motion in the transverse direction in Eq. (2) can be rewritten as

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} + \frac{1}{r} \mathbf{a}_{T\theta} - \frac{1}{r} \mathbf{a}_{M\theta} \quad (10)$$

Note that since r and \dot{r} are known from Eqs. (4)-(6) they can be treated as functions of time. Consequently, Eq. (10) can be expressed as a time-varying linear differential equation as

$$\ddot{\theta} = f(t) \dot{\theta} + g(t) \mathbf{a}_{T\theta} - g(t) \mathbf{a}_{M\theta} \quad (11)$$

where $f(t) = -\frac{2\dot{r}}{r}$ and $g(t) = \frac{1}{r}$.

With a first-order dynamics for target acceleration, Eq. (11) can be expressed in a state space $\mathbf{z} = [\theta, \dot{\theta}, \mathbf{a}_{T\theta}]^T$ as

$$\dot{z}_1 = z_2 \quad (12)$$

$$\dot{z}_2 = f(t) z_2 + g(t) z_3 - g(t) \mathbf{a}_{M\theta} \quad (13)$$

and

$$\dot{z}_3 = -\lambda_\theta z_3 \quad (14)$$

where λ_θ is the time-constant associated with the transverse target acceleration.

A performance index, J_θ , similar to Eq. (7) for the transverse direction is

$$J_\theta = \frac{1}{2} S_f z_{2,t}^2 + \frac{1}{2} \int_0^{t_f} (\gamma_1 z_2^2 + \gamma_2 \mathbf{a}_{M\theta}^2) dt \quad (15)$$

where S_f , γ_1 , and γ_2 are the weights.

The optimization process to yield the controller minimizing Eq. (15) leads to a two-point boundary value problem:

$$\begin{bmatrix} \dot{z}_2 \\ \dot{\lambda}_2 \end{bmatrix} = \begin{bmatrix} f(t) & -\frac{g^2(t)}{\gamma_2} \\ -\gamma_1 & -f(t) \end{bmatrix} \begin{bmatrix} z_2 \\ \lambda_2 \end{bmatrix} + \begin{bmatrix} g(t) z_{30} \exp(-\lambda_\theta t) \\ 0 \end{bmatrix} \quad (16)$$

where z_{30} is assumed known and $\lambda_{2,t} = S_f z_{2,t}$. In Eq. (16), λ_2 represents the Lagrangian multiplier which adjoins Eq. (13) to the performance index, J_θ . This system can be solved either numerically by techniques such as the shooting method or analytically if functional forms of $f(t)$ and $g(t)$ are known. The minimizing control in the transverse direction is given by

$$\mathbf{a}_{M\theta}(t) = \lambda_2(t) g(t) / \gamma_2 \quad (17)$$

III. A GUIDANCE LAW SOLUTION USING STATE TRANSITION MATRIX

In this section a solution to the two point boundary value problem in Eq. (16) will be accomplished by using the state transition matrix of the intercept dynamics. Without target acceleration Eq. (16) can be rewritten as

$$\dot{x}(t) = A(t)x(t) \quad (18)$$

where $x(t) = [z_2(t) \lambda_2(t)]^T$ and $A(t)$ is

$$A(t) = \begin{bmatrix} f(t) & -g^2(t) \\ \gamma_2 & \\ -\gamma_1 & -f(t) \end{bmatrix} \quad (19)$$

Note that the particular solution with target acceleration can be added easily.

The solution to this homogeneous differential equation is

$$x(t) = \phi(t,0)x(0) \quad (20)$$

where $\phi(t,0)$ is the state transition matrix which can be written in matrix form as

$$\phi(t,0) = \begin{bmatrix} \phi_{11}(t,0) & \phi_{12}(t,0) \\ \phi_{21}(t,0) & \phi_{22}(t,0) \end{bmatrix} \quad (21)$$

The state transition matrix can be found by solving the following equation;

$$\dot{\phi}(t,0) = A(t)\phi(t,0) \quad , \quad \text{with } \phi(0,0) = I \quad (22)$$

where $A(t)$ comes from Eq. (19) and I is the identity matrix.

Using the assumption that $r(t) = r(t_f - t)$, $f(t)$ and $g(t)$

can be made into functions of time as $f(t) = -\frac{2}{(t_f - t)}$ and $g(t) = \frac{t_f}{r_o(t_f - t)}$. Using the time functions of $f(t)$ and $g(t)$ in $A(t)$ produces

$$A(t) = \begin{bmatrix} -\frac{2}{(t_f - t)} & -\frac{F}{(t_f - t)^2} \\ -\gamma_1 & \frac{2}{(t_f - t)} \end{bmatrix} \quad (23)$$

where $F = \frac{t_f^2}{\gamma_2 r_o^2}$.

Equation (22) can now be expanded to result in four scalar equations as

$$\dot{\phi}_{11}(t,0) = -\frac{2}{(t_f - t)}\phi_{11} - \frac{F}{(t_f - t)^2}\phi_{21} \quad (24)$$

$$\dot{\phi}_{12}(t,0) = -\frac{2}{(t_f - t)}\phi_{12} - \frac{F}{(t_f - t)^2}\phi_{22} \quad (25)$$

$$\dot{\phi}_{21}(t,0) = -\gamma_1\phi_{11} + \frac{2}{(t_f - t)}\phi_{21} \quad (26)$$

$$\dot{\phi}_{22}(t,0) = -\gamma_1\phi_{12} + \frac{2}{(t_f - t)}\phi_{22} \quad (27)$$

These equations must now be solved for $\phi_{11}(t,0)$, $\phi_{12}(t,0)$, $\phi_{21}(t,0)$, $\phi_{22}(t,0)$. In order to solve Eq. (24) for $\phi_{11}(t,0)$ we must first differentiate Eq. (24) with respect to time. This results in the following equation

$$\ddot{\phi}_{11}(t,0) = -\frac{2}{(t_f - t)}\left[\dot{\phi}_{11} + \frac{\phi_{11}}{(t_f - t)}\right] - \frac{F}{(t_f - t)^2}\left[\dot{\phi}_{21} + \frac{2\phi_{21}}{(t_f - t)}\right] \quad (28)$$

The resulting equation must be manipulated to be a function of only ϕ_{11} . Eq. (24) and (26) can be used to produce ϕ_{21} and $\dot{\phi}_{21}$ as function of ϕ_{11} . Substituting the solutions for ϕ_{21} and $\dot{\phi}_{21}$ into (28) yields

$$\ddot{\phi}_{11}(t,0) = 2\frac{\dot{\phi}_{11}}{(t_f - t)} + \frac{(F\gamma_1 + 6)\phi_{11}}{(t_f - t)^2} \quad (29)$$

Similarly for $\ddot{\phi}_{12}$, $\ddot{\phi}_{21}$, $\ddot{\phi}_{22}$ we get

$$\ddot{\phi}_{12}(t,0) = 2\frac{\dot{\phi}_{12}}{(t_f - t)} + \frac{(F\gamma_1 + 6)\phi_{12}}{(t_f - t)^2} \quad (30)$$

$$\ddot{\phi}_{21}(t,0) = \frac{(F\gamma_1 + 6)}{(t_f - t)^2}\phi_{21} \quad (31)$$

$$\ddot{\phi}_{22}(t,0) = \frac{(F\gamma_1 + 6)}{(t_f - t)^2}\phi_{22} \quad (32)$$

If we assume a solution for ϕ_{21} of the form

$$\phi_{21}(t,0) = K(t_f - t)^b \quad (33)$$

$$\dot{\phi}_{21}(t,0) = -Kb(t_f - t)^{b-1} \quad (34)$$

$$\ddot{\phi}_{21}(t,0) = Kb(b-1)(t_f - t)^{b-2} \quad (35)$$

Then Eqs. (33) - (35) can be put into Eq. (31) to find a solution for b , by solving the quadratic equation below.

$$b^2 - b - (F\gamma_1 + 6) = 0 \quad (36)$$

The resulting solution for b is

$$b = \frac{1}{2} \pm \frac{1}{2}D \quad (37)$$

where

$$D = \sqrt{25 + 4F\gamma_1} \quad (38)$$

Replacing F as it was defined earlier results in D becoming

$$D = \sqrt{25 + \frac{4t_f^2 \gamma_1}{r_0^2 \gamma_2}} \quad (39)$$

where all of the variables are known. If the ratio of $\frac{\gamma_1}{\gamma_2}$ is 1000 (i.e. the weight on the line-of-sight rate is two orders of magnitude larger than the weight on the command acceleration) then D can be approximated as 5 for many typical intercept scenarios.

We now assume a solution for $\phi_{21}(t,0)$ to be of the form

$$\phi_{21}(t,0) = A_1(t_f-t)^{\frac{1}{2}(1-D)} + A_2(t_f-t)^{\frac{1}{2}(1+D)} \quad (40)$$

From Eq. (22) we know that $\phi_{21}(0,0) = 0$, so Eq. (40) can be solved for A_1 in terms of A_2 .

$$A_1 = -A_2 t_f^D \quad (41)$$

We will now use Eq. (26) to obtain $\phi_{11}(t,0)$ as a function of ϕ_{21} and $\dot{\phi}_{21}$.

$$\phi_{11}(t,0) = \frac{1}{\gamma_1} \left[\frac{2}{(t_f-t)} \phi_{21}(t,0) - \dot{\phi}_{21}(t,0) \right] \quad (42)$$

$\dot{\phi}_{21}$ can be found by taking the derivative of Eq. (40) with respect to time.

$$\dot{\phi}_{21}(t,0) = -\left[\frac{A_1}{2}(1-D)(t_f-t)^{-\frac{1}{2}(1+D)} + \frac{A_2}{2}(1+D)(t_f-t)^{-\frac{1}{2}(1-D)} \right] \quad (43)$$

Substituting Eq. (40) and (43) into Eq. (42) produces a solution for $\phi_{11}(t,0)$ as a function of A_1 and A_2 .

$$\phi_{11}(t,0) = \frac{1}{\gamma_1} \left[\frac{A_1}{2}(5-D)(t_f-t)^{-\frac{(1+D)}{2}} + \frac{A_2}{2}(5+D)(t_f-t)^{-\frac{(1-D)}{2}} \right] \quad (44)$$

Since we know from Eq. (22) that $\phi_{11}(0,0) = 1$, and since A_1 is known as a function of A_2 from Eq. (41), we can solve for A_2 using Eq. (44). The resulting solution for A_2 is

$$A_2 = \frac{\gamma_1 t_f^{\frac{1}{2}(1-D)}}{D} \quad (45)$$

Substituting Eq. (45) into Eq. (41) yields the solution for A_1 as

$$A_1 = \frac{-\gamma_1 t_f^{\frac{1}{2}(1+D)}}{D} \quad (46)$$

With A_1 and A_2 , we can find $\phi_{11}(t,0)$ and $\phi_{21}(t,0)$ at any time t.

If we assume the same type of solution for $\phi_{22}(t,0)$

$$\phi_{22}(t,0) = A_3(t_f-t)^{\frac{1}{2}(1-D)} + A_4(t_f-t)^{\frac{1}{2}(1+D)} \quad (47)$$

A_3 and A_4 can be determined by using the same process that was used to determine A_1 and A_2 . The resulting solutions for A_3 and A_4 are

$$A_3 = \frac{(5+D)}{2Dt_f^{\frac{1}{2}(1-D)}} \quad (48)$$

$$A_4 = \frac{(D-5)}{2Dt_f^{\frac{1}{2}(1+D)}} \quad (49)$$

With A_3 and A_4 , we can find $\phi_{12}(t,0)$ and $\phi_{22}(t,0)$ at any time t. The solution for $\phi(t,0)$ is now complete.

In order to solve Eq. (20) for the states at time t, we must be able to determine the states at the initial time. We will do this by evaluating the states at the final time, t_f .

$$\lambda_2(t_f) = \phi_{21}(t_f,0)z_2(0) + \phi_{22}(t_f,0)\lambda_2(0) \quad (50)$$

$$z_2(t_f) = \phi_{11}(t_f,0)z_2(0) + \phi_{12}(t_f,0)\lambda_2(0) \quad (51)$$

From the terminal term in the performance index, $\lambda_2(t_f)$ is found to be

$$\lambda_2(t_f) = S_{f_6} z_2(t_f) \quad (52)$$

By substituting Eq. (51) into Eq. (52) we can solve for $\lambda_2(t_f)$ as a function of $\lambda_2(0)$ and $z_2(0)$.

$$\lambda_2(t_f) = S_{f_6} [\phi_{11}(t_f,0)z_2(0) + \phi_{12}(t_f,0)\lambda_2(0)] \quad (53)$$

We can now set Eq. (53) equal to Eq. (50) and solve for $\lambda_2(0)$ as a function of $z_2(0)$.

$$\lambda_2(0) = \frac{[S_{f_6}\phi_{11}(t_f,0) - \phi_{21}(t_f,0)]}{[\phi_{22}(t_f,0) - S_{f_6}\phi_{12}(t_f,0)]} z_2(0) \quad (54)$$

If the terminal term of the performance index is left out, $z_2(t_f)$ is zero and only Eq. (51) is used to solve for $\lambda_2(0)$.

$$\lambda_2(0) = -\frac{\phi_{11}(t_f,0)}{\phi_{12}(t_f,0)} z_2(0) \quad (55)$$

We can now solve for the minimizing control in the transverse direction at the current time using Eq. (17), which is repeated here for convenience.

$$a_{M_0} = \frac{\lambda_2(0)g(0)}{\gamma_2} \quad (56)$$

IV. A CLASS OF PROPORTIONAL NAVIGATION GUIDANCE LAWS THROUGH TRANSFORMATIONS

Although Eqs. (16) and (7) represent a general decoupled solution, interesting analytical solutions for the terminal guidance problem and a feedback guidance law can be obtained through a transformation of coordinates. For comparison with existing results, the target acceleration is assumed zero.

Note that the final time (time-to-go) calculation involves an assumption that the closing velocity (relative range rate) is constant. This assumption can be translated to

$$\dot{r}(t) = \dot{r} (t_f - t) \quad (57)$$

where t is the current time. In a feedback rule, this assumption is not very restrictive since \dot{r} is updated at each instant. By using Eq. (57) in Eq. (10), we get

$$\frac{d}{dt} \dot{\theta} = \frac{2\dot{\theta}}{t_f - t} - \frac{a_{M_0} t_f}{r_0 (t_f - t)} \quad (58)$$

This equation is difficult to integrate numerically since $(t_f - t)$ appears in the denominator. Hence, define a variable u as

$$u = (t_f - t)^2 \dot{\theta} \quad (59)$$

The differential equation for u is (after some algebra)

$$\dot{u} = -a_{M_0} \frac{t_f}{r_0} (t_f - t) \quad (60)$$

Note that Eq. (60) is devoid of any expression in u on the right hand side and $(t_f - t)$ in the denominator.

The optimal control problem is now solved through the use of the new variable u . Consider a performance index, J_{01} , given by

$$J_{01} = \int_0^{t_f} \frac{1}{2} \gamma_2(t) a_{M_0}^2 dt \quad (61)$$

Although this performance index seems simpler than J_0 in Eq. (15), it is shown later that Eq.(61) can accommodate a variety of designs by assuming different functional representations for the weight $\gamma_2(t)$.

The Hamiltonian, H , of this system is given by

$$H = \frac{1}{2} \gamma_2(t) a_{M_0}^2 - \lambda_2 a_{M_0} \frac{t_f}{r_0} (t_f - t) \quad (62)$$

The propagation of the Lagrangian multiplier, λ , is governed by

$$\dot{\lambda}_2 = 0 \quad (63)$$

Hence, λ_2 is a constant. The optimality condition leads to

$$a_{M_0} = \frac{t_f(t_f - t)}{r_0 \gamma_2(t)} \lambda_2 \quad (64)$$

By using Eq. (64) in the propagation equation for u in Eq. (60), we get

$$\dot{u} = \frac{-t_f^2 (t_f - t)^2}{r_0^2} \frac{\lambda_2}{\gamma_2(t)} \quad (65)$$

We will now derive a family of proportional-navigation laws. Let

$$\gamma_2(t) = (t_f - t)^{-k} \quad (66)$$

where k is a positive integer. The implication of this time-varying weight is that the control effort should achieve most of the trajectory shaping before the time-to-go reaches the last second.

With this expression for $\gamma_2(t)$, Eq. (65) can be integrated. The Lagrangian multiplier consequently can be solved for as

$$\lambda_2 = \frac{r_0^2 \dot{\theta}_0 (k + 3)}{t_f^{k+3}} \quad (67)$$

With Eq. (67) we can solve for $u(t)$ from Eq. (65) as

$$u(t) = u(0) + \frac{\dot{\theta}_0}{t_f^{k+1}} [(t_f - t)^{k+3} - t_f^{k+3}] \quad (68)$$

The control acceleration, $a_{M_0}(t)$ and the line-of-sight rate, $\dot{\theta}(t)$ can be obtained as explicit functions of time as

$$a_{M_0}(t) = (k+3) \dot{r}_0 \dot{\theta}_0 (1 - (t/t_f))^{k+1}, \quad k \neq -3 \quad (69)$$

and

$$\dot{\theta}(t) = \dot{\theta}_0 (1 - t/t_f)^{k+1} \quad (70)$$

By varying k ($\neq -3$), we can obtain a family of proportional navigation guidance laws. In particular, let $k = 0$, in Eq. (69) and (70). We get

$$a_{M_0}(t) = 3\dot{r}_0 \dot{\theta}_0 (1 - t/t_f) \quad (71)$$

and

$$\dot{\theta}(t) = \dot{\theta}_0 (1 - t/t_f) \quad (72)$$

If we assume $t = 0$ as the current time, we get

$$a_{M_0} = 3\dot{r}_0 \dot{\theta}_0 \quad (73)$$

This is the standard proportional navigation guidance law. By assuming non-zero values for k which are greater than zero, we can get the line-of-sight rate decreased sooner as desired.

V. INTEGRATED GUIDANCE LAW/AUTOPILOT DESIGN

As indicated earlier, an avenue to improve the homing missile performance is to consider an integrated design of the guidance law and autopilot instead of separate designs. In this manner, the kinematics of the engagement geometry used in the guidance law development and the dynamics of the airframe as reflected in the linearized equations of motion and used in the autopilot design can be brought together.

Consider the geometry of the engagement in Figure 1 and the relationship between the flight path angle and the pitch angle and angle of attack as shown in Figure 2.

One way to formulate the integrated approach is to relate the guidance law to the dynamics of the airframe directly and solve for the commanded control surface deflection.

For this purpose, let us assume a conventional proportional navigation guidance law given by

$$a_{M_0}(t) = k(1 - t_f) \quad (74)$$

where $k = 3\dot{\theta}_0 \dot{r}_0$

and $t_f = t/t_f$

This transverse acceleration can be approximated to be perpendicular to the flight path of the missile so that

$$V_m \dot{\gamma} = k(1 - \gamma) \quad (75)$$

where V_m = missile velocity

and γ = flight path angle.

The flight path angle is related to the pitch angle and angle of attack as

$$\gamma = \sigma - \alpha \quad (76)$$

or

$$\dot{\gamma} = \dot{\sigma} - \dot{\alpha} - q - \dot{\alpha} \quad (77)$$

where q = pitch rate.

The linearized dynamics of the airframe in a short period mode is described by

$$\dot{\alpha} = Z_\alpha \alpha + q + Z_\delta \delta_c \quad (78)$$

$$\dot{q} = M_\alpha \alpha + M_q q + M_\delta \delta_c \quad (79)$$

where Z_α , Z_δ , M_α , M_q , and M_δ are the dynamic stability derivatives of an airframe.

From Eqs. (75)-(79), a feedback law for the control surface deflection can be obtained as

$$\delta_c = (C_1 + C_2)q - C_1 \dot{\alpha} + C_3 \dot{q} \quad (80)$$

or

$$\delta_c = C_1 \frac{3\dot{\theta}_o \dot{\alpha}}{V_m} + C_2 q + C_3 \dot{q} \quad (81)$$

$$\text{where } C_3 = \frac{Z_\alpha}{Z_\alpha M_\delta - M_\alpha Z_\delta} \quad (82)$$

$$C_2 = -M_q C_3 \quad (83)$$

$$\text{and } C_1 = \frac{M_\alpha}{Z_\alpha M_\delta - M_\alpha Z_\delta} \quad (84)$$

Note that the variables in Eq. (81) can be picked up through measurements. Numerical experiments of these controls are being conducted.

VI. CONCLUSIONS

A closed form solution for the transverse acceleration has been derived through the use of the state transition matrix of the intercept dynamics. A class of proportional navigation guidance laws have been derived through an approximation of time-to-go and a transformation of state variables. A closed form feedback autopilot control law has also been derived by the use of the guidance law in the dynamics of the airframe.

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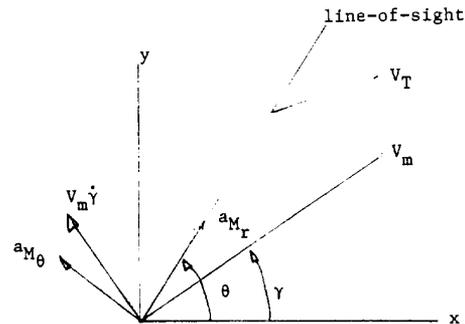


Figure 1. Engagement Geometry.

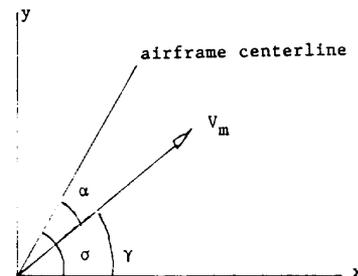


Figure 2. Schematic of Angular Relationship.