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Concentration by sorting : Feasibility determination by radial distribution analysis

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CONCENTRATION BY SORTING:

FEASIBILITY DETERMINATION BY RADIAL DISTRIBUTION ANALYSIS

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ABSTRACT

A method was developed for determining certain factors for assessing the feasibility of sorting fragments of an inhomogeneous solid to concentrate a species. These factors are the typical size of zones in which the grade exceeds a selected value, and the recovery potentially achievable by sorting fragments of such size.

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I. INTRODUCTION

This report is to present a method for determining two essential factors needed to assess the sortability of a material, the sizes of regions of some selected target grade and the potential recovery of the target phase from an inhomogeneous solid crushed to a size to permit sorting to such a target grade. When these factors are coupled with cost data in relation to the average grade of the solid, the feasibility of sorting the material can be established.

As mines go deeper and ores grow leaner, new methods and improvements on old ones are necessary in order to meet needs for mineral products at acceptable real costs. The availability of modern electronic circuits has facilitated new approaches, such as high-speed automatic sorting, to minerals up-grading. Actually, automatic sorting is not new; magnetic separation and flotation are examples of sorting, typically of quite small rock fragments. However, with modern electronics, sorting action is not limited to such a direct response to the property upon which the sort is based. If a species in a rock fragment is detectable by methods or properties such as induction bridge coupling, X-ray fluorescence, color, or radioactivity, an electronic system can actuate a separate, powered sorting mechanism, such as a lever, vacuum cutoff, or air blast, to separate the selected fragment from the feed stream at a suitable place and time in the system. As with any method of beneficiation, sortability depends on the presence of significant grade differences from fragment to fragment. Optimum application of sorting depends in detail on the nonuniformity of distribution of the valuable species within the rock, and on breakage to fragments of a size to "liberate" adequately rich regions. Because of the applicability of the liberation analogy, and the characteristics of beneficiation in general, standard mathematical methods for computing recoveries and losses (1) apply. Nonuniformity, as used in this report, does not apply to the microscopic nonuniformity that is of concern for liberation for, say, flotation, nor to the gross nonuniformities of mineral deposits treated elsewhere (2, 3, 4). The method presented in this report is complementary to such analyses of deposits.

High-speed automatic sorting with separate detection and sorting functions is presently in use in a number of areas (5, 6). Peas or corn kernels are sorted by color shade at 400 pieces per second (7); carrot cubes, diced apples, pecan pieces, lemons, and other food items are color-shade sorted (8); and peeled potatoes are sorted for defects at 1,000 tons per day (9). Silver coins are sorted from clad on the basis of their different natural frequencies of vibration (10) and insects are electronically sorted by sex (11).

Limestone (12, 13) and marble chips (14) are color-sorted at high rates (15) and diamonds are sorted from gravels by X-ray methods (16, 17). Uranium ore pieces are analyzed for total U_3O_8 content by scintillation, size-measured, graded, and sorted without critical presorting by size; all this occurs while the fragment is falling a few feet. Because most pieces are of acceptable grade, only the reject pieces are forced from the stream (18). Native-copper ores were formerly hand-sorted to provide 8 to 30 percent of the yield (19), and are subject to automatic sorting (20). Poor rock from old native-copper waste dumps is being automatically sorted (21). Automatic sorting dates back to 1927 (22), and continued growth of sorting in the minerals industries has been predicted (23, 24).

It appears entirely feasible to weigh automatically each fragment, measure several properties, have a computer decide whether the fragment should be classed as concentrate, middlings, or tails, and control the physical sorting device. For well-sized fragments to be subjected to sorting, a much simpler system, even one in which only the amount of the valuable component is measured rather than the fraction, is feasible.

The deep, underground mines for native-copper deposits in the Upper Peninsula of Michigan (25, 26) have been closed, despite large copper reserves, because of increasing costs and insufficient average grade to sustain conventional processing. Significant among the cost sources are hoisting, grinding, and waste disposal. Because the copper distribution is nonuniform, and because the metallic copper is instrumentally detectable in the rock fragments, sorting has been applied (20, 21). Underground sorting, if feasible, could minimize hoisting, grinding, and waste disposal costs (20). The importance of fragment size on sortability of ore has been recognized and the effect researched (20).

A concentrate from conglomeritic native-copper ore was used for the experimental part of the research here reported.

The concept of analyzing specimens of fixed size (27) presupposes, if applied to this problem, foreknowledge of the appropriate specimen size. Analyses of nonuniformity of distribution in other research fields have been accomplished by the use of clumping (28, 29) or clustering (30, 31) techniques. However, these techniques involve the use of a data matrix which would, for a sortability analysis, include the size of each of the copper particles and the distance from it to each of the other copper particles of the specimen. For n particles, the resulting n x n / 2 array of data would have been impossibly large to handle in computer core, even for relatively small specimens, and would have included much more data than are economically processible or needed for practical results. Therefore, an alternative approach, a radial distribution analysis that permitted essentially vectorial treatment of the data, was developed. Vectorial treatment enabled the use of a standard utility sort routine in the computer.

It has been shown (32, 33) that grade analyses made in two dimensions are statistically applicable to three dimensions.

To visualize the radial distribution analysis, consider a polished section of the ore, with a vanishingly small circle centered on an appropriate copper particle in the polished surface. The grade of copper within the circle is unity, because the circle lies entirely on the particle. As the radius of the circle is gradually increased, other copper particles, and areas of rock devoid of copper, are enveloped; the overall grade of copper within the circle generally declines. The rate of decline depends on the relative rates of inclusion of copper and lean rock, and, with due allowance for areas of the circle extending beyond the edges of the specimen, the grade within the circle ultimately reaches the average grade for the specimen, possibly only after declining to values less than the average. The largest radius at which the grade within the circle is equal to a chosen target grade, taken to be greater than the average, is readily determined from an empirical equation relating the grade to the radius. The fraction of total specimen copper within that radius is the potential recovery for the target grade chosen. The most appropriate central particle to choose is the one that leads to the highest indicated recovery at the target grade chosen, consistent with an acceptable fit of the equation to the data.

The radii for selected concentrate grades and potential recoveries from fragment to fragment should be statistically normal or skewed normal variables, and thus amenable to treatment by standard statistical methods.

II. PROCEDURE

A supply of conglomeritic native-copper ore that had been crudely concentrated by testing each piece for copper by an induction bridge coupling method and retaining those for which the presence of metal was indicated was obtained. The use of raw ore would have required too much data collection. The pieces were sectioned by diamond sawing and the resulting plane faces were polished. By use of a microscope with a stage vernier and a grid reticle, data on the size and location, relative to an arbitrary origin, of each copper particle in the plane were obtained and recorded, as were data on the location of the perimeter of the specimen and of any pores therein. In the nine-specimen sample of rock, totaling 7,260 mm^2, the plane area and location of 34,660 copper particles were measured and recorded. In addition, 15,562 groups of one or more (mean about 5.5) small particles (10^{-5} to 10^{-4} mm^2) were logged; for each such group the number of particles and position of the group were recorded. The data on the small particles were not used in the effort here reported, but could be used to evaluate the tendency for small particles to be associated with regions of high copper concentration, as suggested by Carlson (20).

The microscope, with a nominal 10X objective and wide-angle oculars, provided a measured 91.3X magnification. One eyepiece was equipped with a 10 x 10 square grid reticle, so that each small square enclosed 0.00834 mm^2 of the specimen, and each setting of the verniers allowed the viewing of 0.834 mm^2. The sizes of copper particles no larger than one small square were estimated and logged; for larger particles the amount of copper in each small square was recorded, with coding to permit the computer to determine the two-dimensional center of gravity and size of the particle.

A set of computer programs was prepared as follows: An edit program located possible inconsistencies in the data and determined certain totals needed for subsequent programming and processing. The corrected raw data, consisting of coding, particle-size information, and reticle and vernier location numbers, were converted to the needed size-and-location form and the locations of specimen perimeter and pore perimeter points. Next, the data were sorted according to descending copper particle size for long-term disk file storage. Then, the specimen was mapped to show edges, pores, and the locations of the 111 largest particles of copper. The grade-versus-radius data for the radial distribution analyses (about appropriate "central" particles) were developed, and a forward stepwise linear multiple regression analysis routine (34) was modified and used to provide the required empirical relationship for the radial distribution. For each specimen, the final program is used to tabulate the radius and potential recovery for each of the selected concentrate grades (2 to 12 percent copper), and to plot the radial distribution curve.

The central particle for each specimen was chosen from among the 50 or so largest copper particles in the specimen. Because computation charges depended upon the square of the number chosen, it was impractical to opt for a larger number of candidate particles. The overall copper grade within a certain distance of each such particle was determined, and the particle chosen to be the central one was the one that led to the highest grade within such distance. Two distances were tried for each specimen and each such particle, half the distance to the most remote such particle and a fixed 6 mm. In most cases, the two distances led to different central particles; the one chosen was the one that led to the better indicated potential recovery. The 6 mm distance was found better only for some of the lean, probably reject, specimens and the half-maximum distance for the richer ones.

The model used for the radial distribution relationships was

$$Y = \sum_{i=0}^{7} B_i x_i + \varepsilon, \text{ where}$$

$Y = \ln$ (grade within circle of radius r)

B_i = i-th regression coefficient

$x_0 = 1$

x_1 = radius r

$x_2 = {x_1}^2$

$x_3 = {x_1}^3$

$x_4 = 1/x_1$

$x_5 = {x_4}^2$

$x_6 = {x_4}^3$

$x_7 = \ln (x_4)$, and

ε = random error

The data were weighted inversely to the radius to account for the greater area increment represented by a unit radius increment at large radii. A grade - versus - radius point was calculated for each copper particle in the specimen, not just for the 50 or so largest.

III. RESULTS AND DISCUSSION

Characterizations of the sample are presented in Table I and Fig. 1. In Table I are shown the absolute and relative rock and copper contributions of each specimen to the sample, and selected totals and means. Most of the copper is contained in two rich specimens which aggregate 65 percent of the copper in 24 percent of the rock. The overall sample assay is 2.29 percent copper, and the mean specimen area is 807 mm^2 (1.25 in^2).

Fig. 1 consists essentially of beneficiation curves for hypothetical arrangements of the nine whole specimens comprising the sample, in order of decreasing copper content. For Fig. 1A the specimens are included in the descending order of their total copper contents and for Fig. 1B they are included in the descending order of their grades or assays. The curves thus bear relationships to sorting systems that discriminate on the basis of total or absolute amount of the target phase (Fig. 1A) in a fragment (18) or on the basis of the grade (Fig. 1B) of the fragment (14). Owing largely to the two rich specimens, concentrates with 5 percent copper and fairly high recovery are shown to be possible from sorting the already concentrated sample without further crushing.

The results from fitting the data to the model are shown in Table II. For no specimen did all the terms of the model prove significant. The multiple regression analysis computer routine (34) includes only the significant terms in the final expression. Trends in the results, if any, are obscured by the interplay of positive and negative terms.

TABLE I. - Characteristics of Specimens Comprising Sample

Specimen*	Area, mm^2	Cu, mm^2	Cu, %	Fraction of sample		Mean sizes of measured particles, mm^2
				Cu	Total area	
1	946.2	73.7	7.79	0.442	0.130	0.52
2	757.9	33.9	4.48	.204	.104	3.10
3	671.3	13.6	2.02	.081	.092	.31
4	250.4	3.4	1.37	.021	.034	.37
5	1365.1	15.7	1.15	.094	.188	.24
6	1230.7	13.3	1.08	.080	.170	.72
7	655.5	6.0	0.927	.036	.090	.22
8	772.4	6.7	0.868	.040	.106	.21
9	610.8	0.3	0.049	.001	.084	.03
Totals	7260.3	166.5	19.734	1.	1.	
Means	806.7	18.5	2.193**	.111	.111	

* Arranged in order of descending grade.
** Weighted mean is 2.29 percent.

FIG 1. SAMPLE CHARACTERIZATION: ARRANGEMENTS OF WHOLE SPECIMENS

A. BY DECREASING TOTAL COPPER CONTENT

B. BY DECREASING COPPER ASSAY

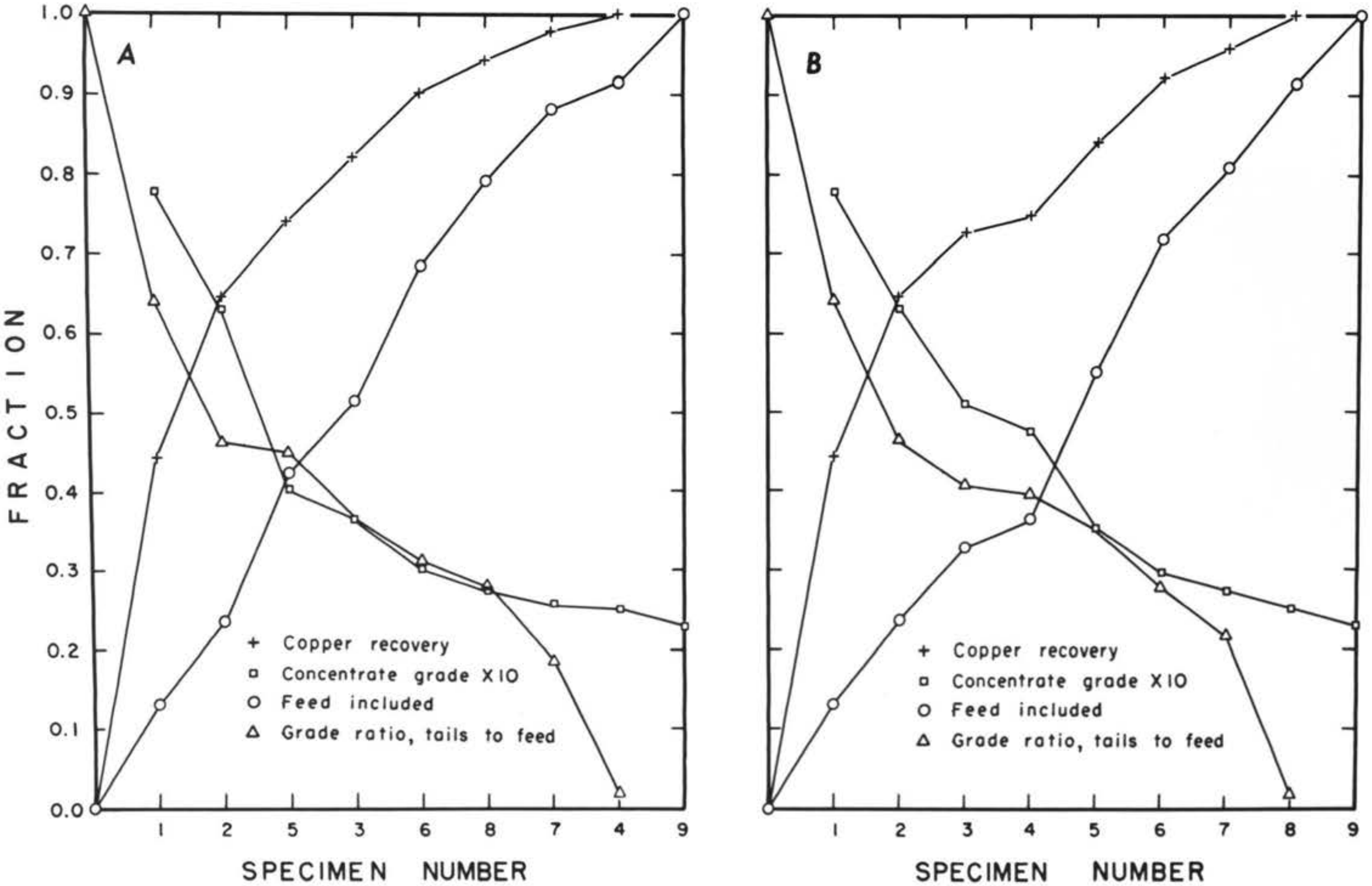
A
B
FRACTION
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1 2 5 3 6 8 7 4 9
1 2 3 4 5 6 7 8 9
SPECIMEN NUMBER
SPECIMEN NUMBER
Copper recovery
Concentrate grade X10
Feed included
Grade ratio, tails to feed

FIG. 1

TABLE II. - Regression Coefficients for Radial Distribution Analyses

Specimen	B_0	B_1	B_2	B_3	B_4	B_5	B_6	B_7
1	0	.1264	-.002411	0	-7.182	12.92	-4.518	1.282
2	-6.641	-.1498	.001529	0	.6413	0	-.4110	-1.892
3	-1.295	.2818	0	-.0003636	-.7251	0	.03197	1.923
4	-5.174	0	-.02293	.0006941	1.662	-.1290	.003799	-1.440
5	-2.808	-.1418	.0006901	.00001493	0	1.066	-.2246	-.5543
6	-4.325	-.1327	0	.00004640	2.729	0	-.1263	-.6920
7	-2.400	-.2431	.02220	-.0004837	.3041	-.02414	0	.7902
8	-.3243	.1267	0	-.00002151	-1.533	0	-.1561	2.238
9	-5.157	-.02899	.008445	-.0001936	0	-.02255	.0008141	1.282

The resulting radii for a target concentrate grading 5 percent Cu, recoveries, and a measure of the accuracy of the fit of the data to the model are presented in Table III. Three specimens are seen to have regions containing 5 percent Cu and of radius not less than 12 mm. Specimen 5, ranked fifth in overall grade, ranks third on the basis of radius for a 5-percent target concentrate.

Crushed to a 12- to 15-mm radius range, the sample should be sortable to a concentrate with a grade well over 5 percent and with recovery of about 62 percent of the copper in about 18 percent of the feed (with raw ore these results would have been better). All of specimen 1 would be included in a 5-percent concentrate, even without further crushing. Crushed pieces from specimens 6, 7, and 9, aggregating 34 percent of the feed, would be rejected if the value of the copper requires a concentrate grading 5 percent, on the basis of the low potential recovery at that grade, and the fragments from specimens 3 and 8 would probably also be rejected on the same basis. Fragments such as specimen 4 would be rejected if the cost of crushing and sorting at a 6-mm radius is uneconomic.

As examples of the radial distribution analysis, detailed tabular results are presented for two specimens. For specimen 2 (758 mm^2, of which 4.48 percent is copper) these results are presented in Table IV. The recovery falls off abruptly from 56 percent to 2 percent upon changing the target concentrate from 5 percent to 6 percent, and strongly suggests that the specimen contains two or more rich regions each of which could be acceptable in the 5-percent concentrate but not in a 6-percent concentrate. The most remote copper particle is about 39 mm from the central one; this suggests that the central one is not

TABLE III. - Results from Radial Distribution Analyses

Specimen	(Correlation coefficient)^{2}*	Radius for 5% target concentrate, mm	Recovery, percent
1	.999	32.5**	100.00**
2	.886	15.4	56.4
3	.983	3.8	17.7
4	.954	6.0	57.6
5	.946	12.0	71.4
6	.991	2.7	8.6
7	.991	1.7	6.2
8	.987	3.3	20.9
9	.998	0.1	2.1

* A measure of the degree of fit of the data to the model.
** Specimen graded about 8%.

TABLE IV. - Radial Distribution Analysis for Specimen 2

Target concentrate grade, percent	Radius for target grade, mm	Recovery at this radius, percent
5	15.4	56.4
6	2.7	1.9
7	2.3	1.9
8	2.1	1.5
9	1.9	1.2
10	1.8	1.2
11	1.7	1.2
12	1.6	1.2

near the center of the specimen, i.e., that fracture occurred through or near the richest copper region. Parallel results for specimen 5 (1,365 mm^2, of which 1.15 percent is copper) are presented in Table V. Here the recovery falls off slowly when the target concentrate grade is increased from 2 to 7 percent, while the radius drops from about 25 mm to about 7 mm. Thus, much of the specimen copper is in the clump that includes the central particle. The most remote copper particle is about 45 mm from the central one, again showing that the central one is not centered on the specimen face.

The sizes of rich regions naturally differ; statistically, the size distribution curve is probably skewed normal; the great majority of regions that grade 5 percent copper are quite small. Sorting serves to reject fragments that contain only small copper-rich regions; fragments, such as specimen 1, that contain large copper-rich regions (from the upper tail of the size distribution curve) would lead to concentrates richer than the target grade. In a very similar manner, skewness with many particles of the target phase in the lower tail was found to bias against working a conglomeritic gold deposit (4). The variance of the radii for the target grade, as developed from the radial distribution analysis of sufficiently large samples, would enable estimation of the degree of the effect, as would the determination of the grade of the concentrate after sorting. The fragment size sorted, or the sorter sensitivity, could then be increased to improve recovery while diluting the concentrate to the target grade.

The factor of shape of rich regions may be important (4), especially if rock breakage occurs in a manner to provide fragments that are strongly non-equiaxed. The shape factor is not treated in this report.

TABLE V. - Radial Distribution Analysis Results for Specimen 4

Target concentrate grade, percent	Radius for target grade, mm	Recovery at this radius, percent
2	25.2	94.7
3	18.9	91.2
4	15.0	83.4
5	12.0	71.4
6	9.4	67.7
7	7.0	51.5
8	3.8	10.5
9	1.6	3.8
10	1.3	3.6
11	1.1	3.6
12	1.0	1.8

There appears to be no simple index of sortability, nor should there be. Electronic sortability is a function of the distribution of the valuable species in the rock, the size to which the rock is broken, the relationship between the costs of breakage and sorting and the value of the phase sought, and the recovery that is reasonable. Another important factor is the tendency of the rock to fracture through regions rich in, or largely free of, the valuable species, to separate rich from lean regions, or to fracture randomly. This factor is implicitly treated in the method described in this report to the extent that the rock sample is a fracture sample. Accordingly, the entire polished section through a specimen should be analyzed - to its edges.

The method of radial distribution analysis enables the determination of key factors needed for judging the feasibility of sorting a solid. However, manual recording of the voluminous data proved to be too tedious to permit working enough specimens to perform statistical analyses and provide confidence limits on the results. Thus, it is strongly recommended that for implementation, the data be recorded by means of a computerized microscope (35, 36, 37) with the computer controlling the stage vernier drives, although problems have been recognized (38). Such apparatus is entirely within the state of the art. Further, with a computerized microscope, it would be practical to simply assay each square or hexagonal field of view, record its position, and develop a data point for each such field rather than for each particle. This approach, which is not feasible with manual data acquisition, would simplify programming and reduce computer memory requirements, without degrading the results.

IV. CONCLUSIONS

The method of radial distribution analysis provides data that are useful in judging whether or not a mineral deposit lends itself to sorting as a method of concentration or preconcentration. These data are the rock fragment size at which sorting is optimum for a selected target grade, and the potential recovery at that grade. Rich fragments, when present, lead to actual concentrates richer than the target grade; the effect could be measured in a given case, statistically evaluated, and adjustment to provide coarser fragments at the input to the sorter or greater sorter sensitivity could be made to enhance recovery.

For the conglomeritic native copper concentrate used in an experimental analysis, it is indicated that sorting fragments of about a 12- to 15-mm radius range could lead to a new concentrate containing appreciably more than 5 percent copper in 18 percent of the feed with recovery of about 62 percent.

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VITA

Robert Martin Doerr was born in St. Louis, Missouri on November 5, 1927. His primary education was in St. Louis, Cleveland, Ohio, and Wichita, Kansas; his secondary education was in St. Louis. He served with the Army Air Force and was further educated at South Dakota State College, Brookings, South Dakota, and St. Louis and Washington Universities, St. Louis, Missouri. The Bachelor of Science degree in physics was awarded him by St. Louis University in September 1950. In 1951 he married Vivian Bokel; they have five daughters.

American Steel Foundries, Granite City, Illinois, employed Mr. Doerr as an analyst from 1951 to 1958. Employed as a research physicist by the U.S. Bureau of Mines since June 1958, he has been enrolled on a part-time basis in the Graduate School of the University of Missouri - Rolla, Rolla, Missouri, since September 1965.