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A Stochastic Model for Dielectric Breakdown in Thin Capacitors

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A nontrivial two-dimensional stochastic model for dielectric breakdown within a parallel plate capacitor is presented for the first time. The model has been used to determine geometric properties of parallel plate discharges. Comparisons are made between these properties and known fractal properties of electrostatic discharges within cylindrical geometries. As the spacing between the plates of a capacitor increases, the value of the fractal dimension of the associated discharge structure increases from the minimum value of unity and approaches the limiting value corresponding to the case of infinite spacing. For any given spacing, this fractal exponent is equal to the exponent of first passage time for the discharge pattern to reach a given height. A study of various power law relationships governing the breakdown may provide insight into the breakdown mechanism and electrical insulating quality of various materials. The model is applicable to the breakdown of thin insulating layers of metal-oxide-semiconductor devices.

I. INTRODUCTION

Dielectric breakdown phenomena occur in a very wide range of materials and sizes. Lightning is a large-scale example of this, while the breakdown of the gate oxide of an metal-oxide-semiconductor (MOS) device is a small-scale example.[1,2] Whenever a high-voltage difference exists across a gaseous, liquid, or solid insulator, dielectric breakdown may occur. These discharge geometries branch into complicated stochastic patterns.[1] This randomness can be seen in lightning.

In a large-scale dielectric breakdown, the dimensions of the dielectric are much larger than the details of the branches of the discharge. Close structural similarity among each of the discharge branches can be observed.[3,4] Thus, the common feature of these structures is their dilation symmetry.

If a self-similar structure of the discharge exists, the size of the materials must be infinitely large so that the scale of discharge structure is infinitely renormalizable. An example is the surface discharges or Lichtenberg figures in compressed SF_6 gas.[1] In such a case, the relation between total length of all branches inside a circle of radius r and the radius r itself is shown to be a power law with fractal (noninteger) exponent D. This concept of fractal dimension D is due to Mandelbrot[4] and is the controlling quantity to describe the discharge phenomena.[5] The value of D in large-scale planar discharges is shown to be $D = 1.75$[1]

However, there are discharge phenomena where one of the dimensions is finite. Examples are MOS capacitors, where the oxide layers are as thin as 100 Å in thickness while the width and length are of the order of microns.[6] Because of finiteness in one dimension, the boundary will have an effect on the dilation symmetry and hence on the fractal exponent. The dielectric breakdown of such thin gate oxides usually starts at a trapped hole at the oxide-semiconductor interface. There are two main mechanisms.[7] One is avalanche breakdown which is caused by impact ionization. The other is the filament-heating transport which induces a destructive breakdown. The latter mechanism gives rise to a discharge pattern and the subsequent existence of molten materials near the breakdown region. Investigation of such electrical breakdown of thin oxides is very important, because the reduction of gate oxide thickness is necessary in order to achieve improved MOS device performance for future generations of integrated circuits. It is well known that the breakdown of thin gate dielectrics is the major cause of circuit failure, especially for large-scale dynamic random-access memories.[6] In addition, the rupture of the very thin tunnel dielectrics in nonvolatile E^2PROMs is the principal cause of E^2PROMs endurance (write/erase cycling) failures.[6–9] Thus, a good understanding of the discharge structure is needed.

There has been very limited theoretical investigation of discharge structures. Niemeyer *et al.*[1] studied radial discharges within cylindrical capacitors. Because of the cylindrical symmetry and the very large spacing between the two cylinders, the discharge structure possesses dilation symmetry. Wiesmann and Zeller[10] generalized the above model to include local dielectric breakdown effects. Turkevich and Scher[11,12] established the equivalence of the electrostatic problem associated with the above model to continuous-time random-walk theory of diffusion-limited aggregation. The relation between fractal dimension D and cluster-tip occupancy probabilities has been derived analytically. In this work, a study of thin parallel plate capacitors is made to model dielectric breakdown of thin oxides in MOS devices. Because of the very small spacing between the two parallel plates, it is expected that the discharges will possess different properties than in the large-scale case. Therefore, the discharge structures, the value of fractal dimension D, and hence the possession of dilation symmetry for the case of infinite spacing cannot be used to describe the dielectric breakdown of thin capacitors. Instead, the discharge structures will show transitional behavior when spacing of the

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capacitor starts to increase. This is discussed in Sec. III. In Sec. II, we describe the dielectric breakdown model. The effect due to the change of prescribed probability function for discharge between two neighboring lattice sites is also discussed.

II. THE DIELECTRIC BREAKDOWN MODEL

In order to obtain insight into the electrostatic discharge process, a computer program has been developed which imitates the phenomenon.[13,14] These discharges begin near one of the plates and continue toward the other plate, in a jagged pattern. The program attempts to produce structures which mimic these patterns so that their properties may be analyzed.

As the spark grows from one of the plates, the path that it follows depends upon the local electric field which surrounds it. The discharge is more likely to traverse those regions where the electric field is higher, since these regions are more likely to fail under the stress. This assumption governs how the program generates the discharge patterns.

The actual path which the algorithm chooses is a stochastic determination, based upon the electric field which exists in the region surrounding the discharge.

The program models a parallel plate capacitor in the two-dimensional case only. The horizontal dimension is taken to be much larger than the vertical dimension. This allows the model to approximate an infinite plate capacitor and, therefore, edge effects are neglected. The numerical accuracy of this calculation is discussed in a later section.

Each point in the capacitor array has associated with it a voltage potential. The potential along each of the edges of the array (the boundaries) is set to a specific value. The top and bottom rows of points represent the two plates of the capacitor. The upper plate is assigned a potential of 1 V. The actual gate voltage for thin oxide breakdown is about 20 V.[6] The lower plate is set at 0 V. The remaining two edges (the two end columns) are assigned a potential configuration which corresponds to the potential within a capacitor of infinite extent. That is, the potential along each end column increases linearly with distance from the bottom to the top.

For the algorithm to determine the path taken by the discharge, it must find the potential at all points in the interior of the capacitor. Thus the potential ϕ satisfies

$$\phi(i,j)=\tfrac{1}{4}[\phi(i+1,j)+\phi(i-1,j)+\phi(i,j+1)+\phi(i,j-1)], \quad (1)$$

where (i,j) indicates the coordinate of the point in the rectangular matrix and the computer algorithm which solves Laplace's equation [Eq. (1)] follows the simple relaxation method.[13]

The discharge pattern will expand from the $\phi=0$ structure. Initially, this includes only the lower capacitor plate, but as new points are added to the discharge, they become a part of the $\phi=0$ structure and are assigned a potential of 0 V. Potential differences which may exist along the discharge are ignored, and the entire discharge pattern is assumed to have the same potential of 0 V.

For each of the possible directions in which the discharge structure may extend, a probability is assigned. This probability is proportional to the electric field raised to some arbitrary power η. It will be of interest to study the effects this exponent has upon the geometry.

Recognizing that the potential of points (k,l) on the discharge is zero, the probability of branching from a point (k,l) on the discharge pattern to an adjacent point (i,j) is

$$p(k,l\rightarrow i,j)\sim\phi(i,j)^{\eta}. \quad (2)$$

Since the probabilities of all possible branch choices must add up to one, a final probability expression can be obtained:

$$p(k,l\rightarrow i,j)=\frac{\phi(i,j)^{\eta}}{\Sigma\phi(i',j')^{\eta}}. \quad (3)$$

The denominator sum is taken over all candidate branches. Our computer program uses this expression to determine a probability of branching for allowed growth directions from all points on the geometry. A branch is then randomly chosen according to the probability function given in Eq. (3). The new point is then added to the discharge geometry and is assigned a potential of 0 V. This change in the structure redefines the boundary conditions, which requires a new solution for the potential field.

This process of solving Laplace's equation [Eq. (1)] and randomly selecting new points to add to the growing discharge pattern is repeated until the discharge reaches the upper plate.

III. DISCUSSION

The size of the array is chosen to best approximate an infinite plate capacitor. The discharge geometry begins from the center of the plate and remains near the central region of the dielectric. Consequently, far away from the discharge geometry, the potential field is essentially unaffected by the discharge structure. By choosing a length dimension which is one order of magnitude greater than the spacing of the two parallel plates, it is insured that the infinite plate approximation can be achieved.

The results summarized here are based upon array sizes of 10×100, 20×200, 50×500, and 70×700. This choice meets the infinite plate approximation stated above, yet allows reasonable run times for the simulation. A discharge pattern of 70×700 lattice points is shown in Fig. 1.

The capacitor model has provided an insight into how the discharge expands through the region between the plates. Figure 2 shows the progress of a typical discharge of 50×500 lattice points with the branching probability linearly related to the electric field ($\eta=1$), when 50, 100, 250, and 519 (final) steps have been included in the pattern. It is important to note how the geometry grows. Its path is not direct and the structure contains several offshoots from the main branch but it cannot be infinitely renormalized in scale.

Figure 3 shows the undirected case ($\eta=0$) and the highly directed case ($\eta=2$). Note that for the undirected case, the discharge pattern growth is independent of the electric field (hence no Laplace solution is required) and that it spreads out very much with no tendency for moving toward the other plate. For this reason, Fig. 3(a) is deliberately shown at some intermediate stage. The highly directed case,

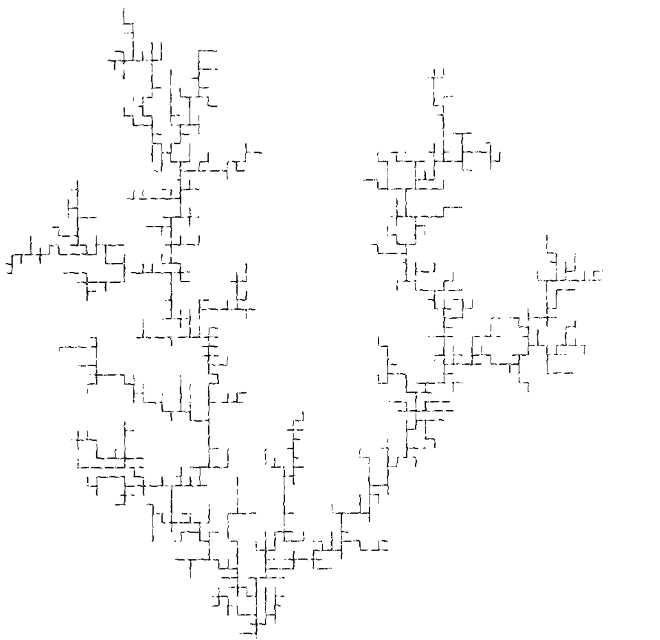

FIG. 1. A discharge pattern of 70×700 lattice points with branching probability linearly related to electric field [$\eta = 1$ in Eq. (3)]. The discharge begins at the bottom plate and extends toward the top plate.

on the other hand, is very narrow and the discharge pattern shoots rather quickly from one plate to the other [Fig. 3(b)].

One measurement which can be used to compare the discharges among each other is by counting the number of points within a distance r from the starting point of the geometry on the $\phi = 0$ plate. This number N should be a power law relationship given by

$$N(r) \sim r^D. \tag{4}$$

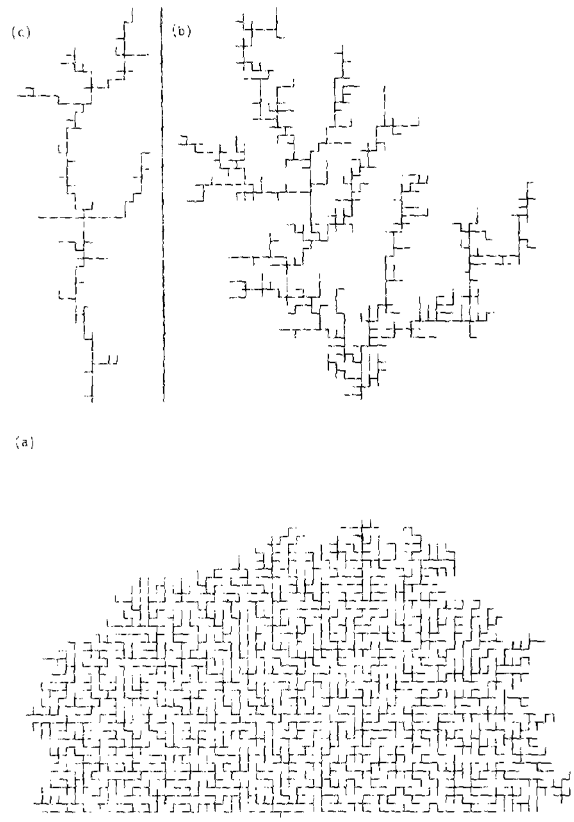

FIG. 3. Comparison of fractal discharge patterns for 50×500. (a) Undirected case: The branching probability is unrelated to the electric field, ($\eta = 0$); the figure is shown deliberately at some intermediate stage to indicate no tendency for moving toward the other plate. (b) Directed discharge: The branching probability is linearly related to the electric field ($\eta = 1$). (c) Highly directed discharge: The probability is related to the square of the electric field ($\eta = 2$).

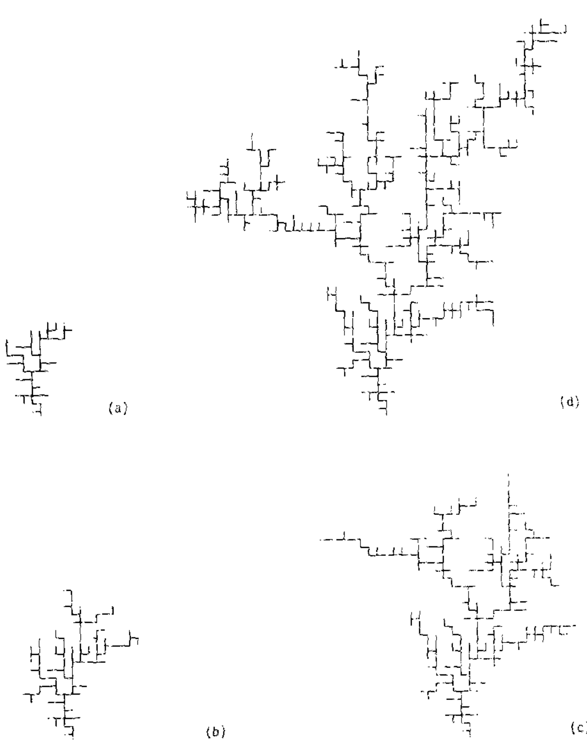

FIG. 2. A typical discharge sequence of 50×500 lattice points in $\eta = 1$ case. (a), (b), (c), and (d) are when 50, 100, 250, and 519 (final) steps have been included in the pattern, respectively.

The exponent D is a measure of how quickly the discharge moves away from the starting point and is calculated at breakthrough to the other plate.

From Niemeyer's work on radial discharges[1] it is known that $D = 1.75$ for the radial discharge pattern with $\eta = 1$. Turkevich and Scher's[11,12] analytical result proved that $D = 5/3$. Table I compares the values found for both radial discharges and for parallel plate discharges. In Table I, 50 runs were computed for the size of 20×200. The rest were evaluated with 10 runs each. In each size, the mean and standard deviation of the value D from those runs are indicated. The values found for a parallel plate discharge are lower than for the radial case. This makes sense if it is realized that in a radial discharge, where the distant cylinder surrounds the central electrode, there is lesser tendency for the discharge to move away from the central point. On the contrary, in a parallel plate capacitor, the field is directed toward the far plate and moves more quickly toward it. Thus, in a parallel plate discharge, the symmetry of branching is broken and the finite distance between the two plates affects the possible branchings which exist for the growing

TABLE I. Comparison of fractal dimension D for radial and parallel plate discharges. D is defined in Eq. (4) and P in the parentheses is defined in Eq. (5). In each size, the mean and standard deviation of D or P from many runs (see text) are indicated.

	Fractal dimension D				
	Radial discharge (infinite limit)		Thin parallel plate (Size = rows × columns)		
	Niemeyer *et al.*	Turkevich & Scher	10×100 $D,(P)$	20×200 $D,(P)$	50×500 $D,(P)$
$\eta = 0$	2.0	2.0	2,(2)	2,(2)	2,(2)
$\eta = 1$	1.75	5/3	$1.390 \pm 0.180,(1.409 \pm 0.190)$	$1.463 \pm 0.168,(1.445 \pm 0.267)$	$1.580 \pm 0.085,(1.634 \pm 0.120)$
$\eta = 2$	1.6	4/3	$1.203 \pm 0.155,(1.217 \pm 0.145)$	$1.248 \pm 0.126,(1.257 \pm 0.126)$	$1.319 \pm 0.054,(1.317 \pm 0.054)$

structure. Fewer branchings are required to reach the top plate. As a result, the fractal exponent is reduced.

In diffusion-limited aggregation problems, Turkevich and Scher[11,12] have shown that scaling relates the fractal dimension D to the cluster-tip occupancy probabilities and D does not depend on the cluster's random, irregular structure but rather on how it grows. In order to verify this, we calculate the number of steps required for the top of the discharge pattern to reach position n for the first time as a function of n. This "first passage time" $F(n)$ can be expressed as

$$F(n) \sim n^P. \tag{5}$$

The values of P are in the parentheses of Table I. The near equality of D and P agrees with the analytical derivation of Turkevich and Scher even in the finite discharge cases where the discharge structures are not infinitely renormalizable.

IV. CONCLUSIONS

A computer simulation of the electrostatic discharge within a parallel plate capacitor has been developed. This model has provided insight into the characteristics of finite discharges that have not been investigated previously. It was shown that because of finite spacing between two capacitor plates, the discharge structures are less self-similar as compared to the case of radial discharge and the reason for the reduction of fractal exponent can be explained. The fractal exponent D has the same value as the exponent P of first passage time of the top tip. Comparison of various power law relations for the branching probabilities also gives insight into the physical breakdown mechanism and electrical insulating quality of various types of materials.

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