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OPTIMIZATION OF ENERGY STORAGE SCHEDULING IN ELECTRICITY MARKETS

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OPTIMIZATION OF ENERGY STORAGE SCHEDULING IN ELECTRICITY MARKETS

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ABSTRACT

In the existing literature, merchants' trading actions are usually assumed not to affect market prices; however, a large-scale energy storage merchant’s actions can affect market prices. This work examined two electricity merchant scenarios: one with only energy storage and the other with both energy storage and renewable power plants. We approximated market impact via a linear function of the electricity traded by the merchant. This study began by applying dynamic programming to the optimal economic dispatch policy of electricity merchants and it considered the market impact, physical characteristics of storage systems, and the uncertainty of renewable energy sources. Then, this study evaluated the effect of the self-consumption demand on the co-optimization scheduling of prosumers with both energy storage and distributed renewable energy sources. Furthermore, this work investigated how the production tax credits (PTC) impacted merchants' co-optimization scheduling policy under two common PTC subsidy policies.

Finally, the time-coupling constraints require market participants to make decisions in advance based on forecasted electricity prices. However, independent system operators (ISOs) have the most comprehensive and detailed information regarding market operations, so they are more likely to generate more accurate pricing estimates than individual merchants. Therefore, this study analyzed whether allowing the ISO to schedule the generators and energy storage could bring economic benefits to the social-welfare maximizing ISO and the profit-maximizing electricity merchant or generators. This study found that if the ISO sends the cleared prices to the electricity merchant, a merchant will arrive at the same optimal scheduling decisions as those from the perspective of the ISO.

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1. INTRODUCTION

Governments and scientists prefer renewable energy to traditional fossil fuels because it produces clean electricity and few greenhouse gases or pollutants. Sustainable and renewable energy resources (solar, wind, etc.) were developed rapidly worldwide in the last two decades with the promise of no carbon emission, technology-driven cost reduction, and national/state-level regulations and targets (Liu et al., 2022a; Qi et al., 2015, Zhou et al., 2019). Renewable energy at a high penetration level, especially wind and solar energy, which are sensitive to weather changes and have a high level of uncertainty, and are intermittent and may lead to sudden and significant fluctuations in power generation and compromise the secure operation of the power system (Cory-Wright and Zakeri, 2020; Korpaas et al., 2003). The electricity supply and demand must be matched in real-time, so it is significant for grid operators to deal with electricity insufficiency and surpluses. Energy storage can rapidly respond to a control order and is expected to play an active role in alleviating renewable generation uncertainty. As the imbalance of electricity can be dealt with energy storage, storing surpluses for future resale is a common strategy for commodities (Ahmad et al., 2021; Lai et al., 2021).

Energy storage can provide many different types of services for ISO (Independent System Operator), utilities, electricity merchants, and end-users (Bo et al., 2021). Considering one aspect, the merchant can utilize energy storage to engage in energy arbitrage by purchasing electricity to store when prices are low and sell it to the market when prices are high to maximize the profit (Wu et al., 2012). Storing electricity for future resale is a typical approach of merchants that sell commodities (Williams and Wright, 1991). Conversely, when energy storage owners participate in the electricity market, the instability of the power system and the imbalance between supply and demand in the electricity market will be well solved. Energy storage, such as pumped storage hydro—having fast response abilities and high ramp rates—plays a significant role in mitigating fluctuations in power generation caused by increasing renewable energy sources (Kim and Powell, 2011; Liu et al., 2022a; Zhou et al., 2019). In the electricity market, an ISO (hereafter, he) clears energy and operates reserves, commits generators with the lowest price offers based on the required demand to minimize generation costs or maximize social welfare, and then generates the cleared locational marginal pricing (LMP) (Chen et al., 2011a; Soofi and Manshadi, 2022). With the control of storage, the electricity merchant can regulate the intermittency of the energy, meet the demand of the market, and reduce costs and maximize arbitrage gains by controlling the stored energy.

There are various energy storage technologies such as solar-thermal energy storage (Haslett, 1979), pumped storage hydropower (PSH) plant (Deane et al., 2010), battery storage (Cheng and Powell, 2018; Marcelino et al., 2019), and hydrogen energy storage (Feng and Menezes 2022). Hydropower has long remained the largest source of renewable electricity generation, accounting for roughly 40% of U.S. renewable electricity generation in 2018, remaining the most significant contributor to U.S. energy storage, with an installed capacity of 21.6 GW or roughly 95% of all commercial storage capacity in the United States (Water Power Technologies Office, 2020). As of 2021, renewable energy accounted for roughly one-quarter of global electrical supply (U.S. Energy Information Administration, 2021). With decreasing technology costs and the boosting of renewable deployment, energy storage is poised to be a valuable resource for future power grids. Will et al. (2021) reported that the energy storage would exceed 125 GW by 2050, more than a five-fold increase from the installed storage capacity of 23 GW in 2020. As a result, investigating the best energy storage scheduling policy in the electrical market benefits both electricity merchants and independent system operators.

Studies on electricity scheduling and operation for energy storage for the electricity merchant were conducted. For example, Zhou et al. (2016, 2019) concentrated on energy arbitrage approaches (i.e., buying electricity from the market at low prices and selling power to the market at high prices to maximize profit). Co-optimization of renewable power plants and energy storage (Garcia-Gonzalez et al., 2008; Liu and Ou et al., 2022; Zhou et al., 2019) can generate potential future values by mitigating the intermittent nature of renewable energy generation through storing electricity when the power demand is higher than the renewable power output. In this work, we discussed PSH or battery storage systems, both having their physical constraints, thus capacity of the energy storage, pumping/charging and generating/discharging capacities, and facility pumping/charging and generating/discharging efficiencies, must be considered when modeling (Liu et al., (2021a, 2022a); Steffen and Weber, 2016； Zhou et al., 2016). This work examined two electricity merchant scenarios: one with only energy storage and the other with both energy storage and renewable power plants.

1.1. THE MOTIVATION AND INNOVATION OF ENERGY STORAGE MERCHANT PROFIT MAXIMIZATION

When modeling energy storage, research into energy inventory has traditionally focused on the optimal policy or the optimal bidding decision. For example, Zhou et al. (2016) concentrated on energy arbitrage approaches (i.e., buying electricity from the market at low prices and selling power to the market at high prices to maximize profit). Most extant models (Kim and Powell, 2011; Zhou et al., 2019), assumed that the storage capacity was sufficiently small compared to the wholesale electricity market, so its charging and discharging decisions *did not* affect the electricity prices. Thus, given the price in each period, a merchant (hereafter, she) buys or sells a certain quantity of energy at a price that is not affected by her own operational decisions (i.e., in our terminology, *price-taker* merchant). However, the value of large-scale energy storage such as PSH facilities, is reflected in price arbitrage (Cruise et al., 2019; Felix et al., 2012, Newbery et al., 2015; Sioshansi 2010, 2014; Sioshansi et al., 2009), in which case the merchant's operating decisions affect electricity prices in the market (i.e., *price-maker* merchant).

Compared to the arbitrage value of energy storage for a *price-taker,* Sioshansi et al. (2009) analyzed the arbitrage value of a 1 GW of a storage device in PJM from 2002 to 2007 and showed that the price-smoothing effect reduced the arbitrage value by more than 20%. More specifically, the market load increased when a merchant bought electricity, thus leading to a rise in market prices; conversely, selling power increased the supply and reduced market prices. Therefore, large-scale electricity storage can reduce its energy arbitrage value by decreasing differences in sale prices on-peak load and purchase cost on off-peak periods. The first aim of this study is to develop a better understanding of the difference between the optimal policies under two perspectives (i.e., price impact vs. no price impact).

1.2. THE MOTIVATION AND INNOVATION OF CO-OPTIMIZATION ENERGY STORAGE MERCHANT PROFIT MAXIMIZATION

To manage the intermittency of renewable sources and create the flexibility for energy arbitrage, most wind plants owners embraced collocating electricity generation and grid-connected energy storage facilities such as PSH (Al-Masri et al., 2021), compressed air energy storage (Yu et al., 2021), and battery (Ahmad et al., 2021). One example is the Wilmot Energy Center that contained a 30-MW battery energy storage and a 100-MW solar array system (Tucson Electric Power, 2021). Co-optimization of grid-level storage (Garcia-Gonzalez et al., 2008; Zhou et al., 2019) with a wind farm created value by mitigating the intermittent nature of wind generation by pumping/charging electricity when the wind-generated power output mismatched power demand (i.e., PSH and storage benefited the environment also by reducing the wind generation curtailment), by storing wind generation and reselling in future when prices are low, and also by enabling the merchant to buy power for the future. The value of co-optimization of energy storage and wind power generation was reported in The Electric Power Research Institute (EPRI 2004) and the Department of Energy (DOE 2018).

This study analyzed how the market impact affected the co-optimization economic dispatch structure of merchants with co-located energy storage systems and renewable energy sources. As a result, considering the market impact, the profit-maximizing merchant's co-optimized scheduling policy depended not only on the traditional operational approach but also on the market impact of the merchant’s operational actions on prices and the uncertainty of forecasted wind generation. In this set-up, the merchant has four options: storing all renewable energy generation and also purchasing power to store; storing partial renewable energy generation and selling the rest of it; idle; discharging energy storage and selling all renewable energy generation to the market at the terminal period. Therefore, it is valuable to examine the co-optimized economic dispatch policy for electricity merchants who have large-scale energy storage facilities and wind plants and their market impacts on energy storage operations. Thus, this work aims to provide new insight into how optimal co-optimized scheduling policies differ for the merchant who has co-located energy storage systems and renewable power plants under these two scenarios (i.e., price impact vs. no price impact).

Prior research in this area and this study's work about energy storage merchant supposed that energy/power in storage was worthless in the last period (Liu et al., 2021a; Zhou et al., 2016, 2019). This assumption meant that the merchant should reduce the state of charge (SOC) down to the lower boundary of the energy storage capacity, so the choice is either discharging or remaining idle during the last period of optimization horizon. However, this study incorporated the value of water in the PSH at the terminal period (Liu et al., 2022b; Kim and Powell, 2011; Sánchez de la Nieta et al., 2015). In the long term, the residual energy in the storage has potential value for the future then influences the current actions, which is another innovation of this study.

1.3. THE MOTIVATION AND INNOVATION OF PROSUMER WITH ENERGY STORAGE AND CONSIDERING SELF-DEMAND

Distributed renewable energy was considered a primary solution for the issues regarding increased energy demands, fossil fuel depletion, and CO2 emissions (Matos et al.,2019; Gautam et al., 2020). Furthermore, with the development of smart grid technology, the reliability and economics of producing and distributing electrical energy were enhanced (Hamidi et al.,2010). More electricity end-users installed distributed energy resources

(DERs), such as small wind turbines and rooftop solar panels, which are grid-connected to generate electricity to balance their own demand and also to participate in the local electricity market (Parag et al., 2016; Morstyn et al., 2018). In this case, such end users were also called prosumers—energy consumer who are also producers. Renewable sources have limitations because production and consumption are not always simultaneous, as in a PV system that can only generate during daytime hours and will only produce optimally on long and cloudless days. This dynamic between the intermittent renewable energy generation and the dynamic demand of prosumer creates an imbalance between the supply and demand of energy. Thus, only a portion of DERs production can be used locally for prosumers, termed self-consumption rate (Gautier et al., 2018).

A high level of self-consumption rate benefits prosumers: however, due to renewable energy depending on environmental conditions, its power generation is highly sporadic and unpredictable irregularity (Qi et al., 2015). In such circumstances, a household owning a DER was used, and the consuming-electricity device with energy storage (e.g., battery) was connected to the local electricity market via transmission lines. The frequent energy flow to and from the grid affected the grid's stability and decrease prosumers' self-consumption (Jaszczur et al., 2020). Therefore, the energy storage systems are useful tools to improve self-consumption rate and balance the mismatch between demand and supply (Kerdphol et al., 2016) by storing surplus renewable energy when the renewable energy generation is larger than demand or discharging storage to satisfy the demand when local consumption exceeds her production.

The current economics dispatch studies mainly from profit-maximizing electricity merchants, but these studies assumed that these merchants would not consume energy during the optimization horizon (Kim and Powell, 2011; Zhou et al., 2019; Liu et al., 2022a; Liu et al., 2022b). In contrast to the merchant, the prosumers prioritized fulfilling their load and minimizing the total energy consumption cost (Kusakana, 2020). More specifically, the prosumer should first satisfy her load by generating renewable energy, discharging storage, or purchasing electricity from the power market. Thus, optimizing prosumers with storage enabled them to minimize the electricity bills by improving self-consumption rates of the renewable source generation (Keiner et al.,2019). Therefore, the aim of this work is to investigate how the prosumer's self-consumption demand affects the economic dispatch of energy storage.

1.4. THE MOTIVATION AND INNOVATION OF PRODUCTION TAX CREDIT IMPACT ON ECONOMICS DISPATCH FOR ELECTRICITY MERCHANT WITH ENERGY STORAGE AND WIND FARMS

For modeling energy storage, research on energy inventories traditionally concentrated on optimal bidding strategies or optimal policies. Most existent models (Avci et al., 2021, Kim and Powell, 2011; Lee, 2008; Qi et al., 2015; Richmond et al., 2014; Zhou et al., 2019) focused on joint energy arbitrage strategies (i.e., wind farm merchants have considered collocating electricity generation and grid-scale storage facilities); however, they neglect the government's economic subsidies from the U.S., such as the Production Tax Credit (PTC) for renewable power plants. Investing in U.S. wind farms spurred more than $143 billion in private investment over the last ten years because of PTC (Mai et al., 2016). These subsidies reduced U.S. wind power costs by 70% (Cullen, 2013; Siler-Evans et al., 2013). Furthermore, when a renewable subsidy—the production tax credit (PTC)—was provided to electricity merchants, the cost of wind power generation decreased, and the profit of selling electricity increased, then such merchants were encouraged to sell electricity rather than buy and store electricity.

Scholars recognized the importance of PTC in wind power generation. This work studied how the PTC affected the optimal economic dispatch of merchants and profits. In the traditional study, the profit of the electricity merchant was mainly based on selling power to the market and buying power from the market. In contrast, considering the PTC, the merchant's profit depended not only on the traditional selling and buying operation strategy but also on the government's subsidies by selling renewable power to the market. Thus, the PTC significantly affected the trading decisions of merchants who have both energy storage and renewable power plants. Therefore, the aim of this part is to develop a better understanding of optimal economic dispatch policy differences in relation to the two assumptions (PTC vs. no PTC).

1.5. THE MOTIVATION AND INNOVATION OF OPTIMAL DECISION RELATIONSHIP BETWEEN PROFIT MAXIMIZING ENERGY STORAGE MERCHANTS AND SOCIAL WELFARE MAXIMIZING ISO

Energy storage can provide many different types of services for electricity merchants and independent system operators (ISOs). On the one hand, the merchant can utilize energy storage to engage in energy arbitrage by purchasing electricity to store when electricity prices are low and selling it to the market when prices are high to maximize her profit (Wu et al., 2012). On the other hand, when energy storage owners participate in the electricity market, the instability of the power system and the imbalance between supply and demand in the electricity market will be well solved. Energy storage, such as PSH, which has fast response abilities and high ramp rates, is playing an increasingly significant role in mitigating fluctuations in power generation caused by increasing renewable energy sources (Kim and Powell, 2011; Liu et al., 2022; Zhou et al., 2019).

According to current rules, generators or market participants must submit, in advance, bids that consist of price-quantity pairs in the wholesale electricity market. In the Midcontinent Independent System Operator (MISO) day-ahead market, for example, the market bid window closes at 10:30 a.m., and the MISO market management system calculates and posts the trading results (including LMPs) at 1:30 p.m. As a result of time constraints, the merchant must determine whether to buy or sell power in quantities that reflect the ideal policy based on predicted pricing. In contrast, price is a result of market clearing based on minimizing system electricity generation costs and is (Hota et al. 2012, Sunar and Birge 2019), thus, difficult to predict accurately ahead of time.

Scholars examined the best economic dispatching approach for energy storage electricity merchants based on forecasted electricity prices (Bo et al., 2021; Gianfreda and Bunn, 2018; Shi et al., 2021; Özen and Yıldırım, 2021; Lehna et al. 2022). Although numerous approaches are utilized to forecast electricity prices, forecasted electricity prices differ from LMPs. ISOs, however, have the most comprehensive and detailed information about market operations, thus, they are likely to generate more accurate price forecasts than would any individual merchants. The ISO could clear the market to compute LMP and dispatch quantities for each generator based on the minimum dispatch cost (Hota et al., 2012; Sunar and Birge, 2019).

Schiro et al. (2016) and Hua and Baldick (2017) identified individual profit-maximizing decisions from generating companies that aligned with the welfare-maximizing solution of the ISO when he sent electricity prices to each generating *unit.* In other words, in ideal situations, both the generator and ISO arrived at the same optimal economic dispatch. Nevertheless, neither Schiro et al. (2016) nor Hua and Baldick (2017) considered PSH or battery storage in their studies, which caused non-convex constraints in modeling because the charging/pumping and discharging/generating cannot occur in the same period. As a result, it was evident that the scheduling problem with energy storage that the ISO and merchants encountered could not be addressed by a strong duality. Therefore, the aim of this work is to determine (a) whether it is economically beneficial to the merchant to be scheduled by an ISO directly and (b) whether a merchant with energy storage or generators will make less profit if she follows the ISO dispatch instead of seeking to maximize her own profit when she faces uncertainties.

1.6. THE OUTLINE

This research focuses on energy storage scheduling optimization, the impact of self-consumption on scheduling policy, and the renewable energy policy subsidy influence on economic dispatch of energy storage. This work uses dynamic programming, mixed integer linear programming, nonlinear optimization, duality theory, and Lagrange relaxation to investigate optimal economic dispatch strategies from the perspective of profit-maximizing electricity merchants.

This work was organized as follows: Section 1 summarized research motivation and contributions regarding energy storage electricity merchants in four different scenarios, and the relationship between optimal decisions made by profit-maximizing merchants and social-welfare maximizing ISOs. Section 2 was a review of the relevant literature. Section 3 discussed the optimal scheduling for profit maximization merchants with only energy storage, and it considered the market impact. Section 4 focused on the co-optimization economics dispatch strategy for electricity merchants who own both energy storage and renewable energy sources. Section 5 described the scheduling results when the self-consumption demand of prosumers with energy storage was considered. Section 6 investigated the impact of PTC subsidy policies on economic dispatch for electricity merchants with storage and wind farms. In Section 7, this study examined the optimal economic dispatch relationship between profit-maximizing electricity merchants and social welfare-maximizing ISOs. Finally, Section 8 concluded with a summary of the findings and some suggestions for future research.

This work verified the proposed results based on synthesis data and real data of electricity prices and wind generation from MISO, which is one of the ISOs in North America and runs one of the largest electricity markets in the world.

2. LITERATURE REVIEW

2.1. RENEWABLE SOURCE WITH ENERGY STORAGE

Renewable power generation (e.g., wind/solar electricity generation) has high uncertainty levels and is intermittent, and the forecast reliability is low (Liu et al., 2022a; Memarzadeh and Keynia, 2021), which significantly affects the operation of power systems (Golari et al., 2016; Papavasiliou and Oren, 2013; Parker et al., 2019). Because the demand and supply of electricity must be matched in real time, it is critical for grid operators to deal with electricity surpluses and insufficiency (Bo et al., 2021; Liu et al., 2022b). Energy storage systems (ESS) can solve this problem benefiting renewable energy market participation (Ding et al., 2014; Gomes et al., 2017; Luo et al., 2015; Zhang et al., 2018) and maintaining the stability of the power system (Liu et al., 2015). Li et al. (2022) and Liu and Du (2020) discussed the problem of renewable energy selection, and they proposed a novel PROMETHEE method to rank different types of renewable energies and to make a sensitivity analysis for decision results. Various energy storage technologies including battery storage (Cheng and Powell, 2018; Rehman et al., 2022) and PSH (Deane et al., 2010; Wang et al., 2021), were also discussed.

Previous scholars targeted renewable sources with energy storage. Liu et al. (2015) used the artificial neural network (ANN) to forest wind generation and LMP (locational marginal pricing) and to study the dispatch of wind farms with hybrid energy storage. Shi et al. (2018) optimized the generated scheduling of wind-storage systems by analyzing the link between wind power fluctuation and ESS based on quantization index (QI) clustering. Orsini et al. (2021) proposed a comprehensive computational framework for the optimal operation for a solar thermal plant with energy storage. Roslan et al. (2021) explored a day-ahead optimized scheduling controller for the optimal operation of distributed energy resources with energy in the microgrid. Li et al. (2021) studied the capacity design of an integrated energy system based on the active dispatch mode (ADM). Savolainen and Lahdelma (2022) solved the optimal dimension and operation of renewable energy with storage in the building based on a 15-minute power balance settlement.

Various methods were used by previous researchers to model the economic dispatch of energy storage, including approximate dynamic programming theory (Jiang and Powell 2015a, Zhou et al. 2016), convex relaxation method for energy arbitrage (Hashmi et al. 2019), stochastic optimization operation model (Bafrani et al., 2021), MILP (Heine et al. 2021, Koltsaklis and Dagoumas 2021), and hierarchical optimization algorithm (Shi et al. 2022). Yeh (1985) presented a general review of the mathematical models and simulations for reservoir operations. However, these studies followed the principle that buying occurs when electricity demand is low and electricity prices are low, and selling occurs when electricity demand is high, and electricity prices are high (Deane et al., 2010). Zhang and Wirth (2010) developed an online heuristic algorithm to smooth wind power variations with battery storage. Wang et al. (2008) and Dui et al. (2018) explored the optimal energy storage power and capacity for energy storage using second-order cone programming (SOCP). Huang et al. (2018) analyzed the operation of grid-level energy storage under three market mechanisms and found a modified mechanism that balanced social cost and owner’s profit. Al-Kanj et al. (2020) adopted approximate dynamic programming algorithms to optimize energy storage for arbitrage. Heine et al. (2021) employed the MILP model to investigate the design and dispatch problem of packaged cool thermal energy storage (CTES) in connected communities, based on annual cost minimization. Fan et al. (2022) analyzed the optimization of grid-scale energy storage in a day-ahead operation using a dynamic optimal power flow (DOPF)-based scheduling framework.

Aside from PSH systems, the battery is a conventional but more expensive form of energy storage with constraints. Considering the nature of the battery and other stochastic information like electricity prices, load demand, and regulation signals, Cheng and Powell (2018) proposed a dynamic programming to solve the operation problem of the battery to charge and discharge to maximize arbitrage gains. Considering battery life, Hashmi et al. (2018) proposed an optimal arbitrage algorithm to control the number battery operations cycles to maximize battery life and arbitrage return. Nguyen et al. (2018) focused on the characteristic of charge or discharge efficiency of energy storage and proposed nonlinear energy flow models based on nonlinear efficiency models and verify them by a Vanadium Redox Flow Battery (VRFB) system. To simplify this study, we consider charge or discharge efficiency as a constant value.

Another subsection of the literature concentrated on using storage to make better bidding decisions relative to periods in a market (e.g., Bathurst and Strbac, 2003; Kim and Powell 2011; Löhndorf et al., 2013; Jiang and Powell, 2015). In reference, this study assumed that any electricity offered to the market was accepted; thus, different from the above literature, this work did not consider bidding problems. Wiser and Bolinger (2015) determined that, in some U.S. electricity markets, wind generators are treated as "must-run" in normal conditions, and 38% of the wind capacity developed in the U.S. in 2009 was sold through merchant agreements involving no bidding; thus, this study assumption is acceptable.

In this work, unlike the previously discussed research, capacity optimization for energy storage and optimal coordination framework are not evaluated. Instead, from the profit-maximizing perspective, this study target how to get the analytically optimal economic dispatch policy of the electricity merchant with energy storage only and with both a wind farm and energy storage. Thus, the goal of the study is to find a policy that allows the electricity merchant to decide and compute the amount of energy (i.e., energy transaction quantity) to sell or buy based on the state of the environment period.

2.2. CO-OPTIMIZAION OF ENERGY STORAGE AND WIND FARM

U.S. DOE (2018) reported the value of co-located energy storage and wind plants. For the optimization of co-located energy storage and a wind plant system, Castronuovo and Lopes (2004) proposed a discrete optimization method to maximize daily profits and find the optimal daily operational strategy for a merchant with wind plants and hydroelectric power generation. Lee (2008) solved the short-term electricity scheduling problem by applying the MIPSO (multi-iteration particle swarm optimization) method on the combined wind farms and PSH system. Garces and Conejo (2010) studied the optimal bidding strategy for a price-taker producer in the day-ahead electricity market. Zhang et al. (2016) obtained the optimal day-ahead economic dispatch for a smart grid with renewable and storage device by a fully distributed algorithm. Ding et al. (2016), Kim and Powell (2011), Zhou et al. (2019) examined the optimal scheduling policy of a wind plant with a storage system. Levieux et al. (2019) discussed the complementary operation between an existing hydropower plant and a projected wind plant based on heuristic algorithm (HA). Bhoi et al. (2020) studied the optimal scheduling of Photovoltaic (PV) systems with a battery and incorporate the storage health and consuming cost. Taghikhani (2021) studied micro-grid optimal scheduling with renewable resources and storage considering uncertainty. He et al. (2022) proposed a multi-objective evolutionary algorithm with decision-making based on planning-operation co-optimization of renewable energy with storage. However, they all ignored the market impact of energy storage's operating activities on prices because these analyses considered the energy storage activities to be small, and merchant’s operational decisions did not influence electricity prices.

Various methods were used to model price-taker merchants; examples include the heuristic approach (Zhang and Wirth, 2010), mixed-integer linear programming (MILP) method (Wang et al., 2021), dynamic programming theory (Liu et al., 2022a; Xiao et al., 2021), Lagrangian relaxation technique (Cruise et al., 2019), stochastic optimization scheme (Powell and Meisel, 2016), and approximate dynamic programming algorithms to co-optimize energy storage for arbitrage (Al-Kanj et al., 2020). Parastegari et al. (2013) evaluated the best scheduling of wind plants and pumped-storage power plants in a joint operation and an uncoordinated operation, finding that the joint operation enhances the plant’s profit and risk value. Bruninx et al. (2016) investigated system operators’ co-optimization of PHS and controllable generation. Yang et al. (2020) used the PSO algorithm to investigate the best PV and BESS integrated generating system dispatch method. Zheng et al. (2020) researched the day-ahead optimal dispatch for an integrated energy system, and they considered the time-frequency characteristics of the predicted renewable energy source output. Liu et al. (2022a) investigate the impact of the PTC (production tax credit) on the optimal scheduling policy of energy storage and ignoring the market impact and the uncertainty of wind generation. Hou et al. (2022) investigated a data-driven economic dispatch model for islanded microgrid systems that included storage, wind power, diesel engines, and PVs, taking into account uncertainty and demand response. Lu et al. (2022a) developed the joint optimal scheduling model for wind-photovoltaic-hydropower-thermal-pumped storage.

In conclusion, an independent merchant with co-located energy storage and a wind plant could effectively enhance the stability of power system operation. Optimizing the energy arbitrage strategy could maximize its income in the real-time market. In Section 4, a series of physical constraints on the energy storage system, the maximum and minimum limits of the generating and pumping, the capacity of the energy storage system, the efficiency, and the residual value of water are considered. This study examines merchants' operation costs, which may be daily maintenance costs or battery self-discharge loss.

2.3. MARKET IMPACT OF ENERGY STORAGE

For merchants in the electricity market, most studies focused on assuming that the merchant's operational actions (i.e., pumping/charging and generating/charging) did not influence market prices, which is called price-taker. Notably, large-scale energy storage such as PSH, was reflected in energy arbitrage actions on the power market. This was because the merchant's trading actions (i.e., buying and selling) were sufficiently large to affect the electricity prices (Cruise et al., 2019; Felix et al., 2012).

Felix et al. (2012) offered a pioneering approach to storage valuation that incorporated the effect of a market impact. Along similar lines, Baslis and Bakirtzis (2011) used stochastic MILP to model how a hydropower company’s short-term profit maximization decisions affected its medium-term plans, which adopted an annual stochastic self-dispatching model. Steeger et al. (2018) studied the optimal bidding plan of a single hydropower company whose bidding behavior influenced the market price using Stochastic Dual Dynamic Programming (SDDP). Cruise et al. (2019) identified large-scale storage (e.g., PSH) trading decisions that affected the market price and addressed decoupling the optimization horizon through the Lagrangian approach. Habibian et al. (2020) employed Lagrangian methods to the optimal power purchase decision making of price-maker enterprises that consumed power.

Huang et al. (2018) compared the operation of grid-level energy storage under three market mechanisms and proposed a modified mechanism to balance social cost and owner's profit. Huang et al. (2019) analyzed the investment and operation for price-maker storage under the centralized market and deregulated mechanisms and explored the financial incentives for the cooperative operation of multiple grid-level storage devices. Chabok et al. (2019) focused on the influence of the energy storage system as a price-maker on the operation of the power system from the perspective of ISO and proposed a bi-level optimization problem. These works did not investigate the energy storage economic dispatch problem from the perspective of electricity merchants and did not specifically consider wind plants to be operated with energy storage. Liu et al. (2021a) investigated the optimal operational policy of merchants who only have energy storage and incorporated the market impact based on dynamic programming. Nasiri et al. (2021) examined the scheduling strategy for a multi-energy system as a price-maker player in the day-ahead wholesale market based on a hybrid robust-stochastic approach. Later, Nasiri et al. (2022) investigated the tactical response of a wind integrated MES in the wholesale electricity market (WEM) and the natural gas market (NGM) as a price setter via a bi-level optimization model.

Compared to the current study (Liu et al., 2022a; Jiang and Powell, 2015a; Zhou et al., 2019), note that the mode was non-trivial in achieving analytical results that employed dynamic programming approach when considering the market impact in the problem because it transformed the traditional study that considered only piecewise linear reward functions to nonlinear ones. The co-optimization policy of electricity merchants is quite different when uncertain renewable energy generation and the residual value of energy in the storage and the market impact are modeled.

2.4. CO-OPTIMIZATION OF PROSUMERS WITH ENERGY STORAGE

The widespread implementation of DERs and their integration with the utility grid added additional flexibility to the grid (Hong et al., 2022; Parizy et al.,2020). On the other hand, since the demand for electricity increases with the growth of the electric vehicle market and the increasing use of electrical equipment, the price of electricity gradually rises. These trends have incentivized consumers to install DERs, and then they can generate electricity to cover the self-demand and sell surplus energy to the market (Horta et al.,2017).

There are various approaches to model the prosumer: a robust virtual battery model describing the flexibility of the prosumer (Hu et al., 2021); the marginal utility functions (MUFs) expressing prosumers' trading willingness based on a stochastic approach (Ziras et al., 2021); a real-time rolling horizon energy management model considering the stochastic characteristics of PV consumers and the conditional value at risk based on the cooperative game theory (Ma et al., 2019); the Energy Cost Optimization via Trade (ECO-

Trade) algorithm for Peer-to-Peer (P2P) energy trading problem (Alam et al., 2019); and the Variational Equilibrium and Generalized Nash Equilibrium solutions for the P2P market design (Cadre et al., 2020).

With the emergence of prosumers, there are some studies on energy management and trading strategy optimization for prosumers. Liu et al. (2017) investigated the energy sharing management (ESM) model for microgrids with prosumers based on the Stackelberg game and obtained the optimal pricing model of ESM. Etesami et al. (2018) modeled the interactions between several prosumers and a utility company using stochastic game theory. With the number of prosumers in the smart grid increasing, Bitaraf and Rahman (2018) pointed to grouping prosumers into groups or coalitions by game theory and assessing the strategy using data sources. Kusakana (2020) developed the model for prosumer's optimal operation, including the residential prosumer and commercial prosumer in a P2P energy sharing scheme. Khorasany et al. (2021) proposed a framework for prosumers' joint economics dispatch and power trading. Gutiérrez et al. (2022) determined the ideal size of the photovoltaic solar kit that minimizes the average net billing energy cost over a finite planning horizon in order to satisfy the energy requirements for grid-connected photovoltaic renewable energy consumption with battery storage during each period.

Due to the uncertainty of renewable energy power and the mismatch between renewable energy generation and flexible electricity demand and to fully harvest the energy that the distributed generation can provide, storage technologies need to be utilized and improved. On this basis, Jaszczur et al. (2020) suggested that the development of storage technology, particularly battery storage, enables prosumers to maximize self-consumption rate and further reduce the total annual cost of energy.

For modeling prosumers with energy storage, Bruch and Muller (2014) conducted a long-term simulation of a household and reported that self-consumption rates were approximately 29%, 47%, and 51% for no energy storage, 2 kWh, and 4 kWh energy storage, respectively. Kiedanski et al. (2019) proposed a stochastic model of battery control by approximate dynamic programming (ADP), which significantly reduces the net energy exchanged with the grid and the monetary cost for production. In some countries, especially with high retail electricity prices, prosumers with energy storage become a valuable option for energy cost-saving (Keiner et al.,2019). Based on Monte Carlo simulation, Sha et al. (2020) pointed out that prosumers installing storage systems can significantly reduce their energy cost and the maximum load of the distribution network, benefiting both prosumer and distribution infrastructure. Faraji et al. (2020) considered the effect of the loss of life cost of the battery storage systems (BSSs) in the optimization and scheduling for the prosumer modeled as discrete nonlinear programming (DNLP) problem. Campana et al. (2021) indicated that introducing lithium-ion batteries as energy storage can shave the targeted peak, perform price arbitrage, and increase PV self-consumption for prosumers equipped with energy storage. A distributed two-stage reentrant hybrid flow shop bi-level scheduling model was developed by Dong and Ye in 2022 to minimize makespan, overall carbon emissions, and total energy costs. Therefore, this study considered a prosumer equipped with a battery to regulate the electricity demand.

Studies have assessed the operation for prosumers with energy storage and have emphasized the importance of batteries. The previously cited studies focused on prosumers' cooperative trading strategies and energy management, but they did not analyze the decision-making activities of prosumers with batteries, which will be discussed in this work.

The analytical solutions to quantify the optimal energy storage dispatch of prosumers based on dynamic programming were not achieved. In Section 5, this problem is formatted as Markov decision process, and solve it based on dynamic programming; it is then analyzed how the different load levels affect the optimal scheduling for prosumers.

2.5. PRODUCTION TAX CREDIT

Most existing models focus on joint energy arbitrage strategies; however, they neglect the government's economic subsidies from the U.S., such as the production tax credit (PTC) for renewable power plants. Production Tax Credit (PTC) helped reduce U.S. wind power costs by 70% (Cullen, 2013; Mai et al., 2016; Siler-Evans et al., 2013). Moreover, investing in U.S. wind farms spurred more than $143 billion in private investment over the last ten years because of PTC (see https://cleanpower.org/policy/tax-policy/ for details).

Scholars recognized the importance of PTC in wind power generation. Wiser et al. (2007) suggested that a long-term extension of the federal PTC would be beneficial to the boost of renewable energy growth. Barradale (2010) pointed out the negotiation dynamics of power purchase agreements (PPAs) facing the uncertainty of PTC that caused the U.S. wind industry’s fluctuating boom-and-bust pattern of investment. Xi et al. (2011) applied a spatial/GIS-based financial model to investigate the competitiveness and profitability of onshore wind power. They analyzed the quantified impacts of PTC on the competitive potential of wind power operations. Roach (2015) indicated that PTC was more effective at wind energy promotion in deregulated states than in regulated states using a structural supply model of wind power production. Eksioglu et al. (2014) developed the mixed-integer non-linear optimization model to capture the impact of production tax credit (PTC) on renewable electricity production. Shrimali et al. (2015) showed the effectiveness of the production tax credit that encouraged wind energy deployment in the U.S. by empirical examination. Goldfarb et al. (2016) pointed out that the public supported the extension of PTC to promote renewable energy development. The PTC played a significant role in increasing wind energy investments and supporting its growth in the electricity generation sector (Frazier et al., 2019; Esposito et al., 2021). The influences of PTC and investment tax credit (ITC) on reducing the costs of wind and solar technologies, the installed capacity of renewable generation, and the electricity market prices were reported in DOE (2016). Alizamir (2021) studied two types of subsidies for wind energy: the investment subsidy (ITC) and the production subsidy (PTC) and compared their roles.

Although PTC promotes wind energy development and reduces power generation costs as well as affects the reward functions, previous researchers focused on comparing it with other policies, such as ITC (investment tax credit). Unlike some of the previous studies, this study does not focus on renewable subsidy policy comparing. Instead, this study assess how PTC affects the economic scheduling of electricity merchants with energy storage and wind farm. In addition, two PTC subsidy policies were considered, and the superiority of both policies was compared and studied in Section 6.

2.6. ISO AND ELECTRICITY PRICE FORECASTING

The ISO is a central authority in the electric power industry, which controls both the transmission system and the spot electricity market. He is responsible for clearing energy and operating reserves reliably and economically within his market footprint (Chen and Li, 2011). Participants in the electricity market who want to buy from or sell electricity to the ISO must first submit their bids to the ISO. In the electricity market, an ISO clears energy and operates reserves, commits generators with the lowest price offers based on the required demand to minimize generation costs or maximize social welfare, and then it generates the cleared locational marginal pricing (LMP) (Chen and Li 2011, Soofi and Manshadi 2022).

Both day-ahead and real-time electricity markets have a market-clearing process modeled as a unit commitment and economic dispatch (UCED) problem. There are many methods employed in the ISO clearing market, such as benders decomposition (BD) algorithms, mixed-integer programming (MILP) and primal-dual algorithm (Madani and Vyve, 2015; Ye et al., 2020). MISO determines when each power plant should be on or off by MILP and establishes energy output levels and the energy trading prices by operations research methods (Carlson et al., 2012). The primal-dual strategy is a classical method to solve the bi-level market-clearing problem in the day-ahead electricity market (Ceyhan et al., 2022; Chatzigiannis et al., 2017). Foroud et al. (2011) studied the optimal bidding strategy for generation companies and distribution companies, taking into consideration the ISO's clearing market by a bi-level multi-objective optimization model. Ye et al. examined the non-convex generation operating characteristics in the market-clearing problem using bi-level optimization models. Soofi and Manshadi (2022) solved the ISO's market-clearing problem by employing the full AC Optimal Power Flow (ACOPF) problem formulation. A ISO price clearing methods were based on the concept of optimal shadow prices of energy balance constraints (i.e., supply meets demand), so the optimal dual variables or optimal Lagrangian multipliers associated with the demand constraint in the ISO clearing market problem (UCED problem) can usually be determined (Foroud et al., 2011; Hua and Baldick, 2017; Schiro et al., 2016).

Market participants decide when and how much electricity to buy or sell through price forecasting; therefore, accuracy in forecasting electricity prices is critical for wholesale electricity market participants. There are a wide range of studies on electricity price prediction. Gianfreda and Bunn (2018) proposed a four-parameter stochastic model for hourly market prices forecasting that considered the impact of the influx of renewable power on price. Shi et al. (2021) proposed a two-stage price forecasting method based on a deep neural network, which improves the prediction accuracy of spike electricity prices. Özen and Yıldırım (2021) applied the bagging approach to the electricity price forecasting. Peura and Bunn (2021) studied the effect of the intermittency of wind generation on electricity prices in the forward market by a game-theoretic market model. Tschora et al. (2022) investigated the ability of several machine learning algorithms to accurately predict power prices as well as the role of various features in model prediction. In Germany, Lehna et al. (2022) analyzed four techniques for forecasting electricity spot prices and showed that combining the two forecasting methods was superior. Using a multivariate logistic regression model, Liu and Bai et al. (2022c) studied the probability of day-ahead extremely low and high-power prices. Lu et al. (2022b) proposed a scenario modeling method to improve electricity price forecasting accuracy.

In traditional research, optimal scheduling strategies for energy storage and co-optimization scheduling optimization for storage and wind power were studied from the perspective of electricity merchants to maximize their own profits. Such research did not, however, examine the relationship between the ISO’s optimal scheduling and that of the electricity merchant. Unlike the previous studies, this study does not analyze the electricity price forecasting approach; instead, this study explores the relationship between merchants and ISOs' optimal dispatching decisions, and this work investigate whether the optimal scheduling decisions from energy storage owners or generators' profit-maximizing align with ISOs' social welfare-maximizing, which can bring economic benefits to both the system and the merchants. This study is based on the observations reported by Hua and Baldick (2017), Schiro et al. (2016), and Bo et al. (2021).

3. OPTIMAL SCHEDULING FOR PROFIT MAXIMIZATION ENERGY STORAGE MERCHANTS CONSIDERING MARKET IMPACT BASED ON DYNAMIC PROFRAMMING

3.1. OVERVIEW AND RESEARCH QUESTIONS

In this Section, we focus on the application context of a price maker PSH owner or a battery owner, namely an electricity merchant (to facilitate the exposition, hereafter, we use she when referring to the electricity merchant) using a storage strategy to manage electricity in the wholesale market. The storage facility's main features include the storage facility and pumping/generating capacity limits, a time-independent efficiency of energy inventoried in the storage facility dissipating during one period. We also include operational costs when trading electricity on the market. This work (Liu et al., 2021a) employs dynamic programming theory to investigate merchants' optimal economic dispatch considering the market impact and physical characteristics of storage systems.

With the motivations mentioned in Section 1, we aim to address the following two questions (1) What is the benefit of a price maker in electricity markets? Furthermore, (2) What is the difference in the optimal policy between both price taker and price maker? Toward that end, we first adjust the price by a linear function of the amount of the energy traded by the storage in the reward function to obtain the optimal policy considering the market impact. We investigate the electricity merchant's optimal decisions by employing the dynamic programming theory to maximize the profit according to the given available energy level/SOC in the storage, the current electricity prices, and the market impact. To the best of our knowledge, this is the first work to solve the storage problem from the respective price-maker using dynamic programming.

We organize the remaining work as follows: In Section 3.2, we highlight the main contribution of this work. We then formulate the model in Section 3.3. Considering the market impact of the trading decision on prices, we apply them to the objective profit functions and give the optimal solution in Section 3.4. Section 3.5 verifies the proposed results based on synthesis data and real data of electricity prices from MISO (Midcontinent Independent System Operator, USA, 2020), one of the Independent System Operators (ISOs) in North America and runs one of the largest electricity markets in the world. Finally, Section 3.6 summarizes our findings and some suggestions for future research.

3.2. THE PRINCIPLA CONTRIBUTIONS

Our study makes three principal contributions: First, for a price maker electricity merchant, the optimal trading policy at each decision time is deterministically determined by two optimal SOC (state of charge) reference points E_{t+1}^{p*} and E_{t+1}^{g*}, which depend on the available energy inventory or SOC E_t in the storage, the current power prices P_t, and the *market impact*. Considering the efficiency loss or operating cost, the feasible energy storage level or SOC can be divided into three regions: for the positive electricity prices, if there is less energy in the storage than the respective reference point (i.e., $E_t < E_{t+1}^{p*}$), the merchant should buy power from the market and bring the SOC up to E_{t+1}^{p*}, and if there is more energy in the storage than the respective reference point (i.e., $E_t > E_{t+1}^{g*}$), the merchant should sell power to the market and bring the SOC down to E_{t+1}^{g*} as close as possible. However, if the stored energy is within the boundary set forth by the two reference points (i.e., $E_{t+1}^{p*} \le E_t \le E_{t+1}^{g*}$), the merchant should do nothing (i.e., stay in the idle mode).

Under ideal condition, if both efficiency loss and operating costs are not considered, the feasible energy storage level or SOC can only be divided into two regions: buying-and-pumping and generating-and-selling.

Second, compared with the traditional study, when both price taker and price maker hold the same generating/pumping max capacity limits, the market impact will increase the cost of pumping, reducing the revenue generated in each period, which will reduce the optimal expectation profit. If the market impact is small, we will get similar optimal results, including unit commitment (UC) and economy dispatch (ED), as the scenario price-taker. When the market impact is large enough, the electricity merchant should reduce the power transaction quantity (i.e., amount of energy) at each period to lower the negative effect of market impact. The profit-maximizing merchant must, therefore, assay to perfectly balance the trade-off between the intensity of market impact and the power transaction quantity.

3.3. MODEL FORMULATION

We consider an electricity merchant (PSH or battery owner) to use a storage strategy to manage electricity in an electricity wholesale market and thus buy and sell electricity. We work in discrete time, in which the merchant makes operational and trading decisions periodically over a finite horizon in each period $t \in \{1, 2, \cdots, T\}$. We assume the PSH storage (i.e., upper reservoir) has the maximum energy capacity $\overline{E}$ (e.g., the total energy which could be stored) and the minimum energy level $\underline{E}$, where, $\overline{E} > \underline{E} \geq 0$, which means the storage capacity is finite. We also assume the PSH storage or battery has generating or discharging and pumping or charging capacity constraints. We denote $\overline{Q}^P$ and $\underline{Q}^P$ as the pumping/charging upper limit and lower limit that can be purchased from to market in each period, $\bar{Q}^g$ and $\underline{Q}^g$ as the generating/discharging upper limit and lower limit that can be sold to the market in each period, respectively. This quantity is also referred to as the pumping or generating power max capacity if one does not consider the energy loss when pumping or generating the PSH. To maintain our model's tractability, we adopt the conventional assumption (Jiang and Powell, 2015a) that $\underline{Q}^g = \underline{Q}^P = 0$.

Next, we will consider three types of efficiency with storage. The first of these is a fraction η_t, a time-independent efficiency, of energy inventoried in the storage facility, dissipates during one period, $1-\eta_t$ is the self-discharging rate of the battery, or the evaporation and leakage as well as spill rate of the PSH, equivalently, $\eta_t \in [0,1]$. The second of these are both α and β represent the efficiency of pumping mode and the efficiency of generating mode, where, $\alpha, \beta \in (0,1]$. Both $1-\alpha$ and $1-\beta$ represent the fraction of energy loss of pumping mode and generating mode, respectively. The third type of efficiency is ρ representing the fraction of transmission efficiency, that is, the ratio of electricity flowing out of the transmission line to that flowing into this line, so that $1-\rho$ is the line loss rate. Losses are incurred at the end of the transmission line in either direction, where, $\rho \in (0,1]$.

Based on the above discussion, we know that quantities $\bar{Q}^P/\alpha\rho$ and $\beta\rho \cdot \bar{Q}^g$ are the net pumping power capacity and gross generating power capacity. Different types of storage facilities can be modeled by varying the value of $\bar{Q}^P/\alpha\rho$ and $\beta\rho \cdot \bar{Q}^g$. If $\beta\rho \cdot \bar{Q}^g < \bar{E} - \underline{E}$ or $\bar{Q}^P/\alpha\rho < \bar{E} - \underline{E}$ represents slow storage, and the case $\beta\rho \cdot \bar{Q}^g \geq \bar{E} - \underline{E}$ or $\bar{Q}^P/\alpha\rho \geq \bar{E} - \underline{E}$ represents fast storage. In this work, we target the optimal policy structure/decision rule for slow storage (i.e., a storage facility that cannot be fully emptied and filled up in one decision period) (Cruise et al., 2019; Secomandi, 2010; Zhou et al., 2019). Fast storage is a special case for slow storage.

The electricity price in period t is denoted by P_t (dollars per unit energy). Both buying and selling prices at time t are shown by P_t conveniently for a price taker. The sequential levels of the price by a vector of $P = (P_1, P_2, \cdots, P_T)$. The decision for each period *t* is donated by q_t^g or q_t^p to represent the energy change (action) between period *t* and *t*+1 before accounting the efficiency loss. The quantity $q_t^g \cdot \beta\rho$ is the energy released from the storage to generate the power to sell to the market. The quantity $q_t^p / \alpha\rho$ is the energy/power bought from the market to pump the water to refill the upper reservoir of PSH or battery storage.

Following the assumption of (Cruise et al., 2019), if the storage decision has a market impact, which means when the trading decisions of the electricity merchant are sufficiently influential (i.e., large-scale storage) to have a market impact on power prices, the price at which the merchant buys or sells energy can be approximated by a linear function of the amount of the electricity traded by the storage merchant. The adjusted prices are shown as follows:

$$\hat{P}_t = \begin{cases} (P_t + \lambda P_t \dfrac{q_t^p}{\alpha\rho}) \ (q_t^p \geq 0) \\ (P_t - \lambda P_t q_t^g \beta\rho) \ (q_t^g \geq 0) \end{cases} . \tag{3.1}$$

In Eq. (3.1), the λP_t is a measure of the market impact of the storage on the price at time t. Parameter $\lambda \geq 0$ represents the *intensity of the market impact* of the merchant on power prices. If $\lambda = 0$, this special case corresponds to the situation of price-taker. $\hat{P}_t$ are the adjusted prices resulting from buying-and-pumping the storage by units of $q_t^p / \alpha\rho$ energy/power from the market and generating-and-selling the storage by units of $q_t^g \beta\rho$ energy/power to market, respectively.

During pumping/charging and generating/discharging, the electricity merchant needs to spend additional maintenance and operating costs. For the battery owner, the battery cycle life, a key issue when considering economic feasibility, varies between battery technologies and the operating conditions. In practice, the cost of maintenance and degradation is lower at first; after some point, costs increase much more rapidly. This work lets c (dollars per unit energy) denotes the maintenance and operating cost for PSH or the battery's degradation cost. Under the current practice of MISO, the operating cost of PSH is close to zero (Huang et al., 2020). To maintain the tractability of our model, in this work, we assume the energy storage has a linear operating cost of discharging/generating and pumping/charging.

Thus, the rewards function $R(q_t^p, q_t^g, \hat{P}_t)$ from performing decision pumping q_t^p and generating q_t^g, when the prices are $\hat{P}_t$ defined as follows from the respective of price-maker merchant:

$$R(q_t^p, q_t^g, \hat{P}_t) = \begin{cases} -(P_t + \lambda P_t \dfrac{q_t^p}{\alpha\rho}) \cdot q_t^p / \alpha\rho - c(q_t^p / \alpha\rho) \ (q_t^p \geq 0) \\ (P_t - \lambda P_t q_t^g \beta\rho) \cdot q_t^g \cdot \beta\rho - c(q_t^g \cdot \beta\rho) \ (q_t^g \geq 0) \end{cases} . \tag{3.2}$$

The first line in Eq. (3.2) represents the rewards when the electricity merchant releases the energy from storage to generate the power and sell to the market; for example, the $P_t \cdot q_t^g \cdot \beta \cdot \rho$ and $c \cdot q_t^g \cdot \beta \cdot \rho$ represent the revenue obtained and operating cost paid for her at time *t* for $q_t^g \cdot \beta \cdot \rho$ units of energy (power), respectively. The second line $P_t \cdot q_t^p / \alpha\rho$ indicates the cost when the electricity merchant buys power from the market to pump into the storage, and $c \cdot q_t^p / \alpha\rho$ shows the operating cost.

We denote E_t as the current energy/inventory in the storage or reservoir at the beginning of period *t*. The sequential levels of the storage by a $\hat{E} = (E_1, E_2, \cdots, E_T)$, where $E_i \in [\underline{E}, \overline{E}], \forall i \in \{1, 2, \cdots, T\}$. We define the feasible action/decision set based on the current energy level $E_t \in E$ as

$$\text{Action}(E_t) := \{(q_t^g, q_t^p) \in \mathbb{R} : 0 \le q_t^g \le \bar{Q}^g \cdot U_t^g,\ q_t^g \le E_t - \underline{E},\ 0 \le q_t^p \le \bar{Q}^p \cdot U_t^p,\ q_t^p \le \overline{E} - E_t\}. \quad (3.3)$$

Here, the Eq. (3.3) expresses the maximum amount of power/energy that can be generated and pumped. The first two constraints show the upper boundary generating due to the upper limit and available energy in the storage. The third and fourth constraints define pumping's upper boundary because of the upper limit and the storage space capacity, respectively. Both binary variables U_t^g and U_t^p mean the unit commitment of generating and pumping in $[t, t+1)$. Without loss of generality, we have $U_t^p + U_t^g \le 1$ where, $U_t^g \in \{0,1\}$ and $U_t^p \in \{0,1\}$, which means the PSH cannot pumping and generating at the same period. The merchant has three options, but at most, one of these decisions/actions is allowed. If the PSH unit at the mode of offline, there is $U_t^p + U_t^g = 0$, that means the merchant does nothing (i.e., idle or offline). A ternary pumped storage system can simultaneously operate both the pump and generate (ANL/DIS-13/07). However, this is a different problem and beyond the scope of this study.

At the beginning of period t, the merchant knows the storage level E_t and the price P_t, then she decides that the quantity of power $q_t^g \beta\rho$ to sell to the market or $q_t^p / \alpha\rho$ to buy from the market will get the rewards $R(q_t^p, q_t^g, P_t)$. At the end of the period t, the storage self-loss happens, so the storage level at the start of the *t*+1 equals $\eta_t(E_t + q_t^p - q_t^g)$. Thus, we can get the following equation, which represents the storage energy balance or state transition from period *t* to *t*+1.

$$E_{t+1} = \eta_t(E_t + q_t^p - q_t^g) \tag{3.4}$$

The price-taker and the price-maker differ in whether an electricity merchant can impact the market. Hence, we analyze the price maker scenario and find the optimal decision rules in the next Section.

3.4. OPTIMIZATION AND ANALYSIS OF THE PRICE-TAKER STORAGE

We first establish the objective profit functions in Section 3.4.1, Section 3.4.2 identifies the optimal solutions and insights from maximizing profit. Section 3.4.3 analyzes the effect of market impact for maximum expected profit.

3.4.1. Payoff Rewards and Objective Function. To maximize the profit, we assume for the merchant that all prices are known in advance so that the problem of controlling the storage is deterministic. The merchant makes operation and trading decisions periodically over a finite horizon in each period $t \in \{1, 2, \cdots, T\}$. Following the previous study (Zhou et al., 2016, 2019), in this work, we also model the merchant's storage strategy as a finite horizon Markov dynamic programming. Each stage of the Markov DP corresponds to one period. The state variables in each stage t are E_t and P_t, the state at time *t* is denoted by $S(t) = S_t(E_t, P_t)$. The merchant's goal is to find the optimal decision rule π that maximizes the value function at stage 1 (i.e., initial stage) during the horizon.

As a price-maker (PM) merchant, the objective function, is shown as follows:

$$\max_{\pi} \sum_{t=1}^{T} E\left[\left((P_t - \lambda P_t q_t^g \beta\rho)\cdot q_t^g \cdot \beta\rho - c(q_t^g \cdot \beta\rho) - (P_t + \lambda P_t \frac{q_t^p}{\alpha\rho})\cdot q_t^p / \alpha\rho - c(q_t^p / \alpha\rho)\right) | \, S(1)\right] \quad (3.5)$$

Subject to the capacity constraints $0 \le q_t^g \le \bar{Q}^g \cdot U_t^g;\ q_t^g \le E_t - \underline{E};\ 0 \le q_t^p \le \bar{Q}^p \cdot U_t^p;\ q_t^p \le \bar{E} - E_t$, the unit commitment constraints $U_t^p + U_t^g \le 1$, and the storage energy balance constraints $E_{t+1} = \eta_t (E_t + q_t^p - q_t^g)$, where, $t \in \{1, 2, \cdots, T\}$.

In this work, the function (3.5) ignores the discount factor, E is the expectation concerning E_1, P_1. Both E_1 and P_1 are the given initial level of the storage and the price in advance. Let $V_t(S(t))$ denotes the value function in period *t* and state $S(t) = S_t(E_t, P_t) \in \hat{E} \times P$. This function satisfies the Bellman equation. *Being a price-maker (PM) electricity merchant*,

$$V(S(t)) = \max_{Action(E_t)} [R(q_t^p, q_t^g, \hat{P}_t) + E\left(V_{t+1}(S(t+1) \,|\, S(t)\right)] \quad (3.6)$$

Following the previous study (Secomandi, 2010; Zhou et al., (2016; 2019)), in this Section, we assume that any electricity left in the storage is worthless in the terminal period $T+1$. In this way, we will get $V_{T+1}(S(T+1)) = V(E_{T+1}, \hat{P}_{T+1}) = 0$, which indicates the value of end energy (resp. water) in storage (resp. upper reservoir of PSH) equals zero. This assumption is interpreted as the merchant's only choice between generating-and-selling or doing nothing (offline or idle) in the last stage. E_{T+1} represents the energy level at the beginning of period T+1; it also equals the energy level at the end of period T.

The optimization problem of the merchant can be described to maximize $V(S(1))$. Eq. (3.5) is a typical binary integer Markov DP function that is too complex to obtain the closed-form optimal analytical solutions. In this work, we first replace the binary variables with an equivalent continuous decision variable and transfer the Eq. (3.5) to a traditional Markov DP to obtain the analytical optimal policy rule. Following Porteus (2002) and Zhou et al. (2016; 2019), we let $E_{t+1} = \eta_t(E_t + q_t^p - q_t^g) = \eta_t(E_t + \alpha_t)$ (i.e., $\alpha_t = q_t^p - q_t^g$) as the decision variable. We are using the action (decision) α_t for each period *t* to replace the previous decision q_t^g and q_t^p representing the energy storage level/SOC change between periods *t* to *t*+1 before accounting for the energy loss. Where $\alpha_t < 0$ represents the storage decrease due to the action of generating-and-selling, so the quantity of power sold to the market is $-\alpha_t \cdot \beta \cdot \rho$; $\alpha_t > 0$ indicates the storage increase due to the action of buying-and-pumping, so the quantity of power buy from the market is $\alpha_t / \alpha\rho$; $\alpha_t = 0$ shows the storage level does not change, or the merchant does nothing.

To obtain the optimal scheduling solutions, we also split the optimization in (3.6) into two sub-optimization problems corresponding to two different actions: buying-and-pumping from the market and the other generating-and-selling to the market in Eq. (3.1). Then, we find the optimal solution to each of these two sub-optimizations.

$$V_t(S(t)) = \begin{cases} V_t^p(S(t)) = \left\{ -P_t(1 + \dfrac{\lambda}{\alpha\rho}\alpha_t) \cdot \alpha_t - c \cdot \dfrac{\alpha_t}{\alpha\rho} + E[V_{t+1}(S(t+1) \mid S(t)] \right\} \ (\alpha_t \ge 0) \\ V_t^g(S(t)) = \left\{ -P_t(1 + \lambda\beta\rho\alpha_t) \cdot \alpha_t\beta\rho + c \cdot \alpha_t \cdot \beta\rho \ + E[V_{t+1}(S(t+1) \mid S(t)] \right\} \ (\alpha_t \le 0) \end{cases} \quad (3.7)$$

Obviously, if $\alpha_t = 0$, there is $V_t^p(S(t)) = V_t^g(S(t))$. By using $\alpha_t = E_{t+1}/\eta_t - E_t$, and let E_{t+1} as the decision variable, then, the $V_t(S(t))$ in Eq. (3.7) can be rewritten as follows:

$$\begin{cases} V_t^p(S(t)) = \left(E[V_{t+1}(S(t+1)\,|\,S(t)] - \frac{(P_t + c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) + \frac{(P_t + c)}{\alpha\rho}E_t - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right)(\alpha_t \ge 0) \\ V_t^g(S(t)) = \left(E[V(S(t+1)\,|\,S(t)] - (P_t - c)\cdot(\frac{E_{t+1}}{\eta_t})\beta\rho + (P_t - c)E_t \cdot \beta\rho - \lambda P_t \beta^2\rho^2(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right)(\alpha_t \le 0) \end{cases} \tag{3.8}$$

The optimization problem can be segmented into both $\max_{E_{t+1}} V_t^p(S(t))$ and $\max_{E_{t+1}} V_t^g(S(t))$ subject to $\max\{-\bar{Q}^g, \underline{E} - E_t\} = -\bar{q}_t^g = \underline{\alpha}_t \le \alpha_t \le \bar{\alpha}_t = \bar{q}_t^p = \min\{\bar{Q}^p, \bar{E} - E_t\}$ in (3.8).

Maximizing Eq. (3.8) can be approached by obtaining the optimal results to the Eq. (3.9) by removing the given state $S(t)$ (i.e., the given value E_t and P_t). Then, $V_t(S(t)), \forall 1 \le t \le T$ should have the following results based on the Bellman equation (Puterman,1994).

$$\begin{cases} V_t^{p*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \frac{(P_t + c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) \\ \text{or } V_t^{p*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \frac{(P_t + c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t})^2 + 2,_t \frac{\lambda P_t}{\alpha^2\rho^2}\frac{E_{t+1}}{\eta_t}E_t \right) \\ V_t^{g*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - (P_t - c)\cdot(\frac{E_{t+1}}{\eta_t})\beta\rho - \lambda P_t\beta^2\rho^2(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) \\ \text{or } V_t^{g*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - (P_t - c)\cdot(\frac{E_{t+1}}{\eta_t})\beta\rho - \lambda P_t\beta^2\rho^2(\frac{E_{t+1}}{\eta_t})^2 + 2\lambda P_t\beta^2\rho^2\frac{E_{t+1}}{\eta_t}E_t \right) \end{cases} \tag{3.9}$$

We will analyze the optimal results based on Eq. (3.9) in the next Section.

3.4.2. Optimization and Optimal Policy. To obtain the closed-form optimal policy/decision rule in Eq. (3.9), following the previous study (Kim and Powell, 2011; Liu et al, 2021a; Porteus, 2002; Zhou et al., 2016), we know that for any $t \in \{1, 2, \cdots, T\}$, in every stage *t*, the value function $V_t(S(t))$ and $E[V_{t+1}(S(t+1)\,|\,S(t)]$ are concave in $E_t \in [\underline{E}, \bar{E}]$ for each given state $S(t) = S_t(E_t, P_t)$ and $P_t < \infty$ holds. Thus, we will get the following relationship:

$$\frac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}{}^2} = \left(\partial \left(\frac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_t} \cdot \frac{\partial E_t}{\partial E_{t+1}} \right) \Big/ \partial E_t \right) \cdot \frac{\partial E_t}{\partial E_{t+1}} \le 0 \quad (3.10)$$

Therefore, we will get that the second-order derivatives of two sub-optimization problems in Eq. (3.9) are negative (i.e., $\partial V_t^{g*}(S(t)) \big/ \partial E_{t+1}{}^2 < 0$, and $\partial V_t^{p*}(S(t)) \big/ \partial E_{t+1}{}^2 < 0$), so, we can find the unique optimal solutions through the first-order condition of the value functions. Thus, the optimal results of SOC are shown as:

LEMMA 3.1. For the price-maker (PM) electricity merchant, let E^{p*}_{t+1} and E^{g*}_{t+1} are the optimal results in (3.9) for the price-maker scenario, respectively, as shown:

$$\begin{cases} E^{p*}_{t+1} = \arg\max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V^*_{t+1}(S(t+1)\,|\,S(t)] - \frac{P_t + c}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t})^2 + 2\frac{\lambda P_t}{\alpha^2\rho^2}\frac{E_{t+1}}{\eta_t}E_t \right) \\ \text{or} \left(\frac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - \frac{P_t + c}{\alpha\rho\eta_t} - 2\frac{\lambda P_t}{\alpha^2\rho^2\eta_t{}^2}E_{t+1} + 2\frac{\lambda P_t}{\alpha^2\rho^2\eta_t}E_t \right)\Bigg|_{E_{t+1}=E^{p*}_{t+1}} = 0 \\ E^{g*}_{t+1} = \arg\max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V^*_{t+1}(S(t+1)\,|\,S(t)] - (P_t - c)\cdot(\frac{E_{t+1}}{\eta_t})\beta\rho - \lambda P_t(\frac{E_{t+1}}{\eta_t})^2 \cdot \beta^2\rho^2 + 2\lambda P_t\frac{E_{t+1}}{\eta_t}E_t \cdot \beta^2\rho^2 \right) \\ \text{or} \left(\frac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - (P_t - c)\cdot(\frac{\beta\rho}{\eta_t}) - 2\lambda P_t(\frac{E_{t+1}}{\eta_t})\frac{1}{\eta_t}\cdot\beta^2\rho^2 + 2\lambda P_t\frac{E_t}{\eta_t}\cdot\beta^2\rho^2 \right)\Bigg|_{E_{t+1}=E^{g*}_{t+1}} = 0 \end{cases} \quad (3.11)$$

In Eq. (3.11), we can safely draw that there exists two optimal reference points/functions E^{p*}_{t+1} and E^{g*}_{t+1} depend on the current SOC E_t in the storage, the given price P_t, and the intensity of market impact λ. In Eq. (3.11), when market impacts are *not* considered (i.e., $\lambda = 0$), which becomes a special case for the scenario of price-taker, we obtain the following relations:

$$E^{p*}_{t+1(\lambda=0)} = E^{p*}_{t+1(PT)} \quad \text{and} \quad E^{g*}_{t+1(\lambda=0)} = E^{g*}_{t+1(PT)}. \quad (3.12)$$

Assume we ignore the market impact of the merchant's trading decision on price. In that case, the price maker scenario becomes a particular case of the price-taker, then we will get the same optimal decisions. It means that the profit-maximizing electricity merchant, the optimal trading policy at each decision time depends on the forecasted price, the available energy in the storage, and the market impact. Based on the above discussion, the corresponding optimal solutions are given in the following proposition (All proofs are given in Appendix A)

Proposition 3.1: For every stage $t \in \{1, 2, \cdots, T\}$, and positive forecasted electricity price $\hat{P}_t \in P$ (resp. negative forecasted electricity price), when $0 \le \lambda \le \overline{\lambda}_t^{(p,g)}$[1], there exist a unique optimal storage level relationship $\underline{E} \le E_{t+1}^{p*} \le E_{t+1}^{g*} \le \overline{E}$ (resp. $\underline{E} \ge E_{t+1}^{p*} \ge E_{t+1}^{g*} \ge \overline{E}$), which depend on the state of $S(t)$ (i.e., current energy storage E_t and the electricity price P_t), where $\overline{\lambda}_t^{(p,g)} = \left(\frac{P_t + c}{\alpha\rho} - (P_t - c)\beta\rho \right) \Big/ 2P_t \left(\frac{1}{\alpha^2\rho^2} - \beta^2\rho^2 \right) \left(E_t - \frac{\underline{E}}{\eta_t} \right)$.

In this work, we only target the positive electricity prices. Thus, an optimal decision in each state $S(t) = S_t(E_t, P_t) \in \hat{E} \times P$, can be specified as follows:

Case 1: If $\alpha\beta\rho^2 < 1$ (with efficiency loss) or $c \ne 0$ (with operating cost), then there is $E_{t+1}^{p*} \le E_{t+1}^{g*}$, the feasible storage level or SOC can be divide into three regions: buying-and-pumping, generating-and-selling, and do nothing (or idle/offline).

$$\alpha_t^*(E_t, \hat{P}_t) = \begin{cases} \min\{E_{t+1}^{p*} - E_t, \overline{Q}^p\} \text{(buy and pump energy up to } E_{t+1}^{p*}) & E_t \in [\underline{E}, E_{t+1}^{p*}) \\ 0 \quad \text{(keep energy unchanged)} & E_t \in [E_{t+1}^{p*}, E_{t+1}^{g*}] \\ \max\{E_{t+1}^{g*} - E_t, -\overline{Q}^g\} \text{(generate and sell energy down to } E_{t+1}^{g*}) & E_t \in (E_{t+1}^{g*}, \overline{E}] \end{cases} \quad (3.13)$$

Case 2: If $\alpha\beta\rho^2 = 1$ (i.e., without efficiency loss) and $c = 0$ (without considering

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[1]Large-scale storage/PSH capacities are around 1GW to 2GW. Considering a large competitive market (such as MISO which has roughly 190 GW installed capacity) and the limited presence of locational market power due to strong transmission and market monitoring, we focus on analyzing a relatively small market impact.

operating cost), then there is $E^{p*}_{t+1} = E^{g*}_{t+1} = E^{*}_{t+1}$ and the feasible storage level or SOC can be divide into only two regions: buying-and-pumping and generating-and-selling.

$$\alpha^*_t(E_t,P_t)=\begin{cases}\min\{E^*_{t+1}-E_t,\bar{Q}^p\}(\text{buy and pump energy up to } E^*_{t+1}) & E_t\in[\underline{E},E^*_{t+1}]\\ \max\{E^*_{t+1}-E_t,-\bar{Q}^g\}(\text{generate and sell energy down to } E^*_{t+1}) & E_t\in[E^*_{t+1},\bar{E}]\end{cases} \tag{3.14}$$

The first part of proposition 3.1 indicates that the price maker merchant also has three decision choices: generating, pumping, and idle (or offline). By comparing the current storage SOC with the optimal reference points, the merchant can schedule the corresponding optimal actions. If E_t less than E^{p*}_{t+1}, then the electricity merchant (i.e., PSH owner) should buy the power from market, then pump water and bring the SOC level up to E^{p*}_{t+1} as close as possible, and if E_t is larger than E^{g*}_{t+1}, the electricity merchant should release the water from the upper reservoir, generate and sell power to market then result in the SOC level down to E^{g*}_{t+1} as close as possible, however, if the stored energy is within the boundary set forth by the two reference points (i.e., $E^{p*}_{t+1} \le E_t \le E^{g*}_{t+1}$), then keep the SOC unchanged or the electricity merchant should do nothing, respectively.

The second part of proposition 3.1 indicates that without considering both operating cost and efficiency loss, there exists one optimal SOC threshold function E^*_{t+1} depends on the current available energy in storage E_t, the prices P_t, and the market impact λ. The merchant should generate-and-sell the power to market and down the SOC level to E^*_{t+1} or buy-and-pump the energy to the storage and result in the SOC up to E^*_{t+1}, respectively. Based on the two parts of proposition 3.1, we will get the following insight.

Managerial Insight 3.1: *For a price maker electricity merchant, the optimal trading policy at each decision time depends on the given power price, the available energy in the* *storage, and the intensity of market impact. From the respective price-maker to maximize the profit, the SOC range is segmented into three regions by two optimal SOC reference points, which corresponds to one of three distinct actions.*

These results for the electricity merchants bear a critical implication. Because a profit-maximizing merchant only needs to compare the real-time storage SOC with the reference points, she can get the corresponding optimal actions. This insight helps explain the observation and intuition that the merchant should pump (resp. generate) at full capacity when the prices are low (resp. high) if the SOC constraints do not bind, respectively.

3.4.3. Market Impact Analysis. Compared with the traditional study (i.e., without considering the market impact), although we still get similar insights that the feasible SOC can be divided into three regions or two regions, the merchant market impact affects the objective function. When other parameters remain the same except the market impact, based on the above discussion and assumption, the corresponding optimal solution is given in the following proposition (See Appendix A (Proof of proposition 3.2)).

Proposition 3.2: For every stage $t \in \{1, 2, \cdots, T\}$, and forecasted positive price P_t, when both price-taker and price-maker merchants have the same pumping and generating upper limits, the optimal value function and maximum profit have the following relations:

$$\begin{cases} E[V^*_{t+1(\lambda=0)}(S(t+1) \mid S(t)] \geq E[V^*_{t+1(\lambda>0)}(S(t+1) \mid S(t)] \\ \max\limits_{\pi} \sum\limits_{t=1}^{T} E\left[R(q_t, \hat{P}_t)_{(\lambda=0)} \mid S(1) \right] \geq \max\limits_{\pi} \sum\limits_{t=1}^{T} E\left[R(q_t, \hat{P}_t)_{(\lambda\geq 0)} \mid S(1) \right] \end{cases} \tag{3.15}$$

Proposition 3.2 shows that if the electricity merchants' trading decisions are of sufficient magnitude to have a market impact, whose trading decisions will affect the power prices --- when merchants choose to buy electricity, the market load will increase, leading to rising market prices; on the contrary, selling power by a price-marker merchant will increase the supply, resulting in a decrease in market prices--- which influence decision in return. Therefore, with the increases in market impact, the electricity merchant will obtain less profit when price-taker merchants and price-maker merchants have the same generating and pumping limits.

Based on the results in proposition 3.2, we will gain the following insight.

Managerial Insight 3.2: *If the electricity merchant ignores the market impact in power market, she will exaggerate her expectation profit theoretically when the price-taker merchants and price-maker merchants have the same generating and pumping limits offered to ISOs.*

These findings are consistent with the reported results by Felix et al., (2012) and Cruise et al. (2019), which means the merchant will decrease the expected profit with the increasing market impact. We propose our managerial insights and optimal scheduling to employ dynamic programming in this Section based on the forecasted price. Next, we will verify the proposed analytical results through a case study.

3.5. NUMERICAL SIMULATION

In this Section, we first validate the proposed methods and results via two cases to express the analytic findings' characteristics and compared them with the MILP based on synthesis data in Section 3.5.1. Further, Section 3.5.2 uses real data from MISO (2020) electricity prices to indicate optimal insights.

3.5.1. Case Study and Comparison. For simplicity, we use two small cases to show the process detail of the proposed methods. The following cases provide the conditions under which we can get the corresponding optimal analytical results.

Case 1: In this case, we assume there are three time periods ($T = 3$). At each period, the power price takes one of the values in set $P_t = \{p^M, p^L, p^H\} = \{5, 2, 10\}$. For simplify, we assume the storage energy capacity cannot refill it fully in one time period, but fewer than two time periods, it holds that $\underline{E} + \bar{Q}^p \le \bar{E}$ and $\underline{E} + 2\bar{Q}^p \ge \bar{E}$. We also assume the storage can sell it empty in one period (i.e., $\bar{E} - \underline{E} \le \bar{Q}^g$). We assume the storage capacity is 10 (i.e., $\underline{E}=0, \bar{E}=10$), the generating/discharging max capacity is 12 and the pumping/charging max capacity is 7.

For simplicity, let the operating cost be zero, and the pumping/charging, generating/discharging, self-discharging, and transmission efficiencies are one. To verify the effect of market impact, in this case, we assume the merchant's market impact parameter $\lambda = 0.05$. On this basis, we use backward dynamic programming to get the following optimal policy and results:

In Stage 3: The value function is shown as follows:

$$\begin{aligned} V_3 &= -[p^H + \lambda p^H(\underline{E} - E_3)\beta\rho](\underline{E} - E_3)\beta\rho \\ &= -[10 + 0.5(\underline{E} - E_3)](\underline{E} - E_3) = 10E_3 - 0.5{E_3}^2, E_3 \in [0,10] \end{aligned} \tag{3.16}$$

In Stage 2: By using Eq. (3.11), we get the following results:

$$E_3^{g*} = E_3^{p*} = E_3^* = \underset{E_3 \in [\underline{E}, \bar{E}]}{\arg\max} \{V_3^* - p^L E_3 - \lambda P^L [E_3 - E_2]^2\} = (40 + E_2)/6 \tag{3.17}$$

Following proposition 3.1, we will get the following optimal action at stage 2.

$$\alpha_2^* = \begin{cases} \min\{\frac{40+E_2}{6} - E_2, \bar{Q}^p\} = \frac{40-5E_2}{6}, \\ \text{(buying and pumping up to } E_3^*) \text{ if } E_2 \in [0, (40+E_2)/6); \\ \max\{\frac{40+E_2}{6} - E_2, -\bar{Q}^g\} = \frac{40-5E_2}{6}, \\ \text{(generating and selling down to } E_3^*) \text{ if } E_2 \in [(40+E_2)/6, 10]. \end{cases} \tag{3.18}$$

Thus, the optimal value functions in stage 2 are shown as follows:

$$V_2^* = R_2 + V_3^* = \left(320 + 40E_2 - E_2^{\,2}\right)/12 \text{ if } E_2 \in [0,10] \tag{3.19}$$

In Stage 1: By using Eq. (3.11), we get the following results:

$$E_2^{p*} = E_2^{g*} = E_2^* = \underset{E_2 \in [\underline{E}, \bar{E}]}{\arg\max}\{V_2^* - p^M E_2 - \lambda P^M [E_2 - E_1]^2\} = \max\{0, \left(3E_1 - 10\right)/4\} \tag{3.20}$$

Following proposition 3.1, we will get the following optimal action at stage 1.

$$\alpha_1^* = \begin{cases} -E_1 \text{ (generating and seling up to 0)} & \text{if } E_1 \in [0, 10/3); \\ \frac{3E_1 - 10}{4} - E_1 \text{ (generating and seling down to } \frac{3E_1-10}{4}) & \text{if } E_1 \in [10/3, 10]. \end{cases} \tag{3.21}$$

Thus, the optimal value functions at stage 1 are shown as follows:

$$V_1^* = R_1 + V_2^* = \begin{cases} (5E_1 - 0.25E_1^{\,2}) + 80/3 & \text{if } E_1 \in [0, 10/3) \\ \frac{700 + 60E_1 - E_1^{\,2}}{64} + \frac{\left(320 + 40 \cdot \frac{3E_1 - 10}{4} - \left(\frac{3E_1-10}{4}\right)^2\right)}{12} & \text{if } E_1 \in [10/3, 10] \end{cases} \tag{3.22}$$

We will get the following optimal results:

1) If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, (action 1: generate and sell, $\alpha_1^* = -1$), then $E_2 = 0$;

Stage 2: If $E_2 = 0$, (action 2: buy and pump, $\alpha_2^* = 40/6$), then $E_3 = 40 + E_2/6 = 40/6$;

Stage 3: If $E_3 = 40/6$, (action 3: generate and sell, $\alpha_3^* = -40/6$), then $E_4 = 0 = \underline{E}$.

The optimal value at stage 1 is shown as $V_1^* = (5E_1 - 0.25E_1^2) + 80/3 = 31.4167$.

2) If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$,(action 1: generate and sell, $\alpha_1^* = -3.75$), then $E_2 = 1.25$;

Stage 2: If $E_2 = 1.25$, (action 2: buy and pump, $\alpha_2^* = 33.75/6$), then $E_3 = 41.25/6$;

Stage 3: If $E_3 = 41.25/6$, (action 3: generate and sell, $\alpha_3^* = -41.25/6$), then $E_4 = 0$.

The optimal value at stage 1 is shown as

$$V_1^* = \left(700 + 60E_1 - E_1^2\right)/64 + \left(320 + 40 \cdot (3E_1 - 10)/4 - ((3E_1 - 10)/4)^2\right)/12 = 45.9375.$$

Cases 2: In this case, we will show the difference in the optimal policy and corresponding results between both price-taker and price-maker. Following Case 1, let the operating cost be one(i.e., $c = 1$), the pumping and generating efficiencies be 0.9 (i.e., $\alpha = \beta = 0.9$), self-discharging and transmission efficiencies be one (i.e., $\rho = \eta = 1$), while other parameters (i.e., pumping and generating upper limits) remain the same. We assume $\lambda = 0$, $\lambda = 0.01$, and $\lambda = 0.02$ corresponding to different market impact in trading (See Appendix A). The optimal results are shown in Table 3.1.

Table 3.1 Optimal Results with Market Impact

	$\lambda=0, E_1 = 1$	$\lambda=0, E_1 = 5$	$\lambda=0.01, E_1 = 1$	$\lambda=0.01, E_1 = 5$	$\lambda=0.02, E_1 = 1$	$\lambda=0.02, E_1 = 5$
(E_3^{g*}, E_3^{P*})	(10,10)		(10,10)		(10,10)	
(E_2^{g*}, E_2^{P*})	(3,3)		$(\frac{0.2272 + 0.081E_1}{0.13048}, \frac{10E_1/81 + 459/1500}{0.2854})$		$(\frac{0.72 + 0.162E_1}{0.262}, 0)$	
α_3^*	-10	-10	-8.5	-10	-8	-10
α_2^*	7	7	7	5.16	7	5
α_1^*	2	-2	0.5	-0.16	0	0
V_1^*	44.34	64.87	34.1	55.7	28.68	46.9

To verify the proposed method's effectiveness, we also get the optimal results for the above two cases using the traditional MILP model (Chazarra et al., 2018; Zhan et al., 2020) and compare the optimal results that obtained through Markov DP (proposed method this work) and MILP (traditional method). Following the discussion and assumption in Section 3.3, the traditional MILP model for a price-maker (PM) merchant is shown as:

$$\sum_{t=1}^{T}\left((P_t-\lambda P_t q_t^g \beta\rho)\cdot q_t^g\cdot\beta\rho - c(q_t^g\cdot\beta\rho) - (P_t+\lambda P_t\frac{q_t^p}{\alpha\rho})\cdot q_t^p/\alpha\rho - c(q_t^p/\alpha\rho) \right)$$

$$\text{s.t.}\begin{cases} 0\le q_t^g \le \bar{Q}^g\cdot U_t^g \\ q_t^g \le E_t-\underline{E} \\ 0\le q_t^p \le \bar{Q}^p\cdot U_t^p \\ q_t^p\le \bar{E}-E_t \\ U_t^p\in\{0,1\}, U_t^g\in\{0,1\} \\ U_t^p+U_t^g\le 1 \\ E_{t+1}=\eta_t(E_t+q_t^p-q_t^g) \end{cases} \tag{3.23}$$

The parameters and constraints of PSH, market impact, and forecasted prices remain the same as cases 1-2 in Section 3.5.1. The above optimal results (unit commitment and economy dispatch (UCED) and optimal profit) in cases 1-2 are verified in AIMMS, a prescriptive analytics software. We achieve the same optimal results using both MILP methods and dynamic programming for the above two cases.

When the operating cost and efficiency loss are fixed values, the optimal profit and policy are most strongly related to the market impact λ. We confirm these findings by performing additional calculations, as briefly described next. We adjust the value λ from 0 to 0.2 increments of 0.01 and then re-run the cases in the software of AIMMS; the results do not differ materially from those obtained in case 1. It is, therefore, the market impact that will affect the optimal results. In the meanwhile, both cases also verify the relations of $E_{t+1}^{p*} \leq E_{t+1}^{g*}$ for the scenario of price maker.

3.5.2. MISO Case Study. This Section hourly time units as the power prices series $P = \{P_1, P_2, \ldots, P_T\}$ (\$/MW) with 336 stages ($T = 336$) corresponding to two-week periods in MISO for the year 2020 as supplied. The first stage corresponding to the beginning of 06/28/2020 (the power prices are available at https://www.misoenergy.org/). Following the Eq. (3.2), we know that the updated prices at which the electricity merchant buys α_t units power from the market to increase her level (i.e., $\alpha_t > 0$) can be computed by $P_t + \lambda P_t \alpha_t / \alpha\rho$, and at which the electricity merchant sells α_t units power to the market to get revenue (i.e., $\alpha_t < 0$) can be expressed by $P_t + \lambda P_t \alpha_t \beta\rho$.

We assume the minimum and maximum storage capacity (upper reservoir) $\underline{E}$ and $\overline{E}$ are 2 and 20, respectively. Here, $\underline{E} > 0$ means the merchant cannot empty the storage, this is realistic in the power market for a PSH or battery owner. The generating and pumping rate constraint $\overline{Q}^p = 2$ and $\overline{Q}^g = 3$. The unit of measurement of storage can be interpreted as an appropriate MWH. The units of measurement of pumping and generating rate can be expressed as an appropriate MW)—both the pumping and the generating efficiency of $\alpha = \beta = 0.9$. The time $(\overline{E} - \underline{E})/\overline{Q}^g = 6$ hours units for the PSH to empty the upper reservoir, while $(\overline{E} - \underline{E})/\overline{Q}^p = 9$ hours units for the PSH to refill the upper reservoir correspond entirely approximately to the Taum Sauk pumped storage plant in Missouri USA. In this case, the maintenance and operating cost $c = 1$ (\$/MW). We assume $\rho = 1$ and $\eta = 1$.

When the power prices are fixed, the electricity merchant's revenue is determined mostly by the operating cost and efficiency in the trading. Here, we fix both the pumping and generating efficiencies, the operating cost, and focus on the market impact. We also assume the relationship between price and demand throughout the example can be obtained at any point in the time as the demand was varied. Usually, two weeks is an optimization cycle for the Taum Sauk in the power market. The optimal policy obtained from Eq. (3.5) is shown in Figures 3.1 and 3.2 with different initial energy in the storage, respectively.

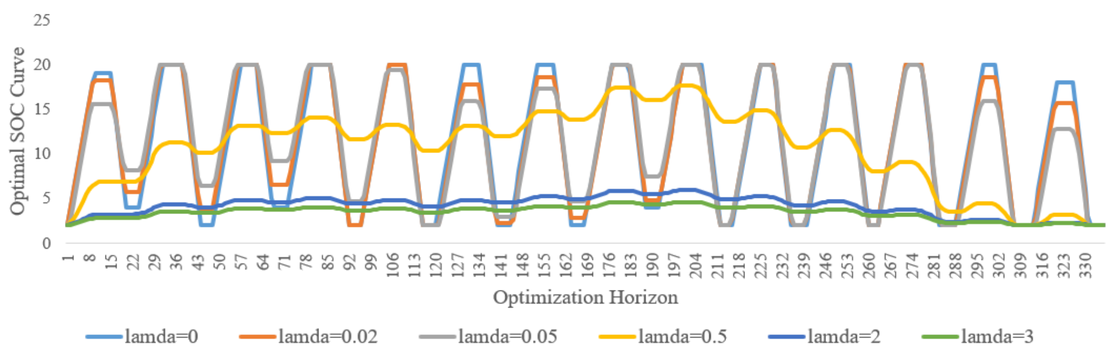

Figure 3.1 Optimal Storage/SOC Change Curve with Market Impact When E1=2MWH

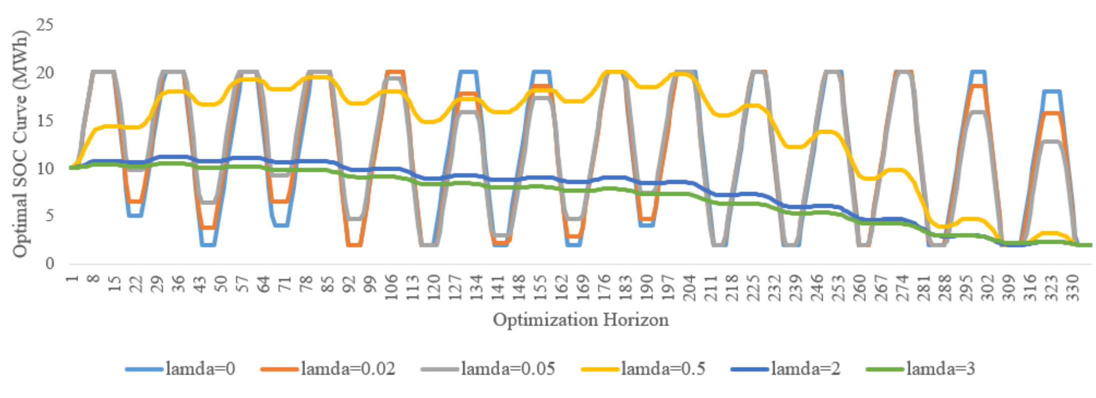

Figure 3.2 Optimal Storage/SOC Change Curve with Market Impact When E1=10MWH

Figures 3.1-3.2 lead us to the following observations and conclusions. When the market impact is small, the merchant will adopt a similar strategy as the traditional policy (i.e., price-taker). We can also find the daily cycle of SOC according to the pattern of price every day. The merchant should buy power as much as possible at lower prices at midnight and sell as much as possible during the day at higher prices. However, when the market impact is large enough, we see that the navy-blue curve ($\lambda = 2$) and green curve ($\lambda = 3$) change very smoothly. To maximize profit, the electricity merchant needs to reduce the energy transition quantity (i.e., amount of energy that buys from the market or sells to the market) each period to lower the negative effect of market impact in trading. With the increase of market impact, the cost of buying will increase; however, the revenue will decrease. To reduce the negative effect of the market impact in trading, the electricity merchant should choose by lowering the power transition quantity at each period. Therefore, a profit-maximizing electricity merchant must perfectly balance the trade-off between the intensity of market impact and the power transition quantity. We will show the relations between the expectation profit the market impact intuitively in Figure 3.3.

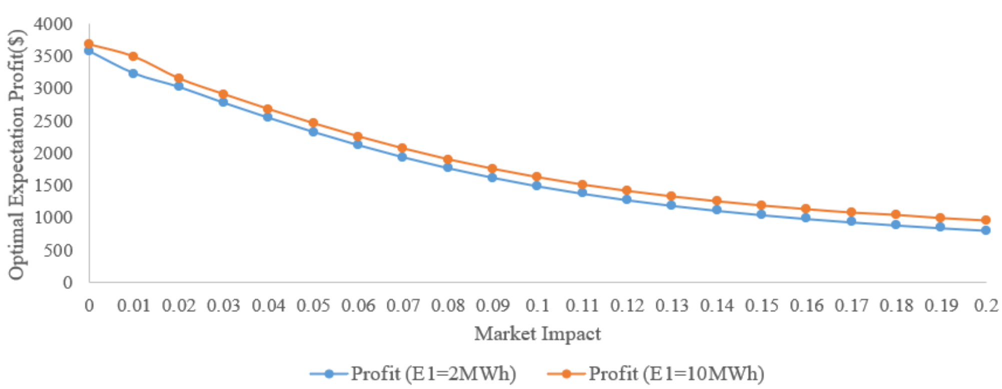

Figure 3.3 The Relations between Optimal Expectation Profit and Market Impact

Figure 3.3 shows that comparing with the traditional study (i.e., price-taker merchant), if both price taker and price maker merchants hold the same pumping and generating upper limits if the market impact is small or the electricity merchant's decisions have a weak market impact on prices, the merchant will get higher profit during trading. In this case, with the decrease in market impact, the cost will go down, and the revenue will rise, which benefits the merchant's profit. As the market impact increases, the cost of buying will increase; however, the revenue will decrease, which results in a lower profit.

The profit-maximizing merchant should reduce her market impact by balancing the intensity of market impact and the power transition quantity. In practice, the ratio of energy storage max capacity and the demand in the power market may be small, which means the market impact is low in trading. So, the merchant will adopt a similar optimal policy as the price taker; however, she may obtain less profit than theoretically maximum profit.

We also re-run the cases in the software of AIMMS by using one day period in MISO for 06/28/2020 as supplied. Once again, our previous conclusions are supported mainly. Therefore, it is reasonable to conclude that our results (i) are robust and representative of the broader set of parameters that we have tested, and (ii) confirm our analytical results and insights. We can draw the numerical simulations are in line with the conclusions made in Section 3.4 from our examination of Figures 3.1–3.3.

From the price-maker situation, results are novel and insightful—giving the electricity merchant an additional set of considerations when her trading decisions impact the market prices.

3.6. SECTION SUMMARY AND ANALYSIS

Although the electricity trading policy has been extensively studied in the wholesale market and inventory management literature, optimal policy research from the respective price-maker whose trading decisions impact *market prices* has only started to gain attention recently. Our study is the first to model the problem as a Markov DP and derive the optimal policy structure for the slow storage to analyze electricity merchant management of managing a storage facility used for arbitrage and whose activities are sufficiently significant to have a market impact. We show that this optimal policy structure generalizes a classic result of Secomandi (2010) and differs significantly from typical threshold policies known to be optimal in the literature without considering the market impact. A focused yet thorough presentation required that we study only the merchant's objectives of maximizing profit.

For a price maker electricity merchant, in the presence of efficiency loss or operating cost, the optimal trading policy corresponding to the reference points/ functions at each decision time not only depend on the current energy availability in the storage and the given prices but also rely on its market price impact. The feasible SOC can be segmented into three regions by two optimal reference points/functions: buying-and-pumping, generating-and-selling, and do nothing (or idle/offline). However, suppose efficiency loss and operating costs are not considered. In that case, the feasible SOC can only be divided into two regions by one unique optimal reference point/function: buying-and-pumping and generating-and-selling. We obtain similar results and insights for electricity merchants after additionally incorporating the value of water/energy at the end of the optimization horizon into our notion of reward functions and value functions.

We identify the merchant's market impact as a critical driver of optimal policy design. If the electricity merchant's market impact is small on the prices, we will get similar results as the scenario price-taker. However, when both price-takers and price-makers are under the same generating and pumping limits offered to ISO, the most surprising finding is that market impact may lead to profit-reducing by increasing the cost of buying and decreasing sales revenue. If, besides, the market impact of electricity is high, then revenue can only partially offset the increased cost. We find that the electricity merchant should lower the adverse effects of the market impact as much as possible by reducing the power transition quantity at each period to maximize the profit. In summary, the scenario of price maker electricity merchants requires different trading strategies than a price taker.

4. ECONOMIC DISPATCH FOR ELECTRICITY MERCHANT WITH ENERGY STORAGE AND WIND PLANT: SOC BASED DECISION MAKING CONSIDERING MARKET IMPACT AND UNCERTAINTIES

4.1. OVERRIEW AND RESEARCH QUESTIONS

In the existing literature, merchants' trading actions are usually assumed not to affect market prices; however, a large-scale energy storage merchant's actions can affect market prices. We approximate the electricity price by a linear function of the quantity of power traded by the merchant in the reward function to achieve decision-making incorporating the market impact and utilize the dynamic programming approach to analyze merchants' optimal multi-period decision-making incorporating market impact, uncertain wind generation, and energy storage constraints.This study (Liu et al., 2022b) was motivated to concentrate on the optimal energy operational decisions scheduling of a merchant who has a co-located storage system and a renewable power plant. In such circumstances, the merchant operates the large-scale energy storage facility to control electricity operation in the wholesale electricity market and incorporate the market impact, the forecasted uncertain wind-generated power, the constraints of energy storage (i.e., PSH capacity, pumping/generating limits, and efficiencies), and the residual value of water in the storage when modeling. This work's analyses are intended to address the following two research questions: (1) How do electricity merchants with co-optimized energy storage and wind farm benefit from considering the market impact of buying and selling power and the uncertain wind generation? (2) What is the difference between the scheduling strategy considering market impact and the traditional scheduling strategy ignoring market impact?

Toward that end, this study relaxes the price-taker assumption and assumes that the impact of the merchant's buy/sell decisions on the market price is approximately linear in the amount of power of buy/sell (Cruise et al., 2019; Liu et al, 2021a; Sioshansi, 2010, 2014). We formulate this problem as a Markov decision process and explore the electricity merchant's optimal joint operational trading strategies by utilizing the dynamic programming approach to maximize profit. To solve this problem and achieve the closed-form analytical results to support multi-period decision-making, this work first split the original problem into three sub-optimization problems corresponding to three available activities of the electricity merchant at each period. Then, the optimal solution for each sub-optimization problem will be addressed based on the Bellman equation. Finally, we combine them and achieve the global conclusions of the original problem to obtain the optimal decision rules in the entire optimization horizon. This is the first work to manage the co-optimized economic dispatch scheduling of the energy storage and wind plants issue, considering the market impact of the merchant's actions and uncertainty of forecasted wind generation through dynamic programming.

This work is organized into six Sections. First, we summarize main contribution in Section 4.2. Section 4.3 models an electricity merchant who has co-located energy storage and wind plants; then, we compare our conclusions with the existing literature in which merchants' market impact is not considered. Section 4.4 demonstrates the proposed results through the synthesis data case study and real data case study of Midcontinent Independent System Operator (MISO), US. Section 4.5 extends our research by examining cases in which market impact is related to generating/pumping limits that offered to ISOs. Finally, Section 4.6 summarizes our study and points out the future research directions.

4.2. THE PRINCIPAL CONTRIBUTIONS

The major contributions of this study are as follows: First, this research overcomes the challenges in achieving analytical results when considering market impact because it will change the traditional linear reward functions that overlook the market impact to nonlinear ones. For a storage-and-renewable energy source electricity merchant, we identify analytically three SOC reference points that rely on the currently available energy inventory in the storage, the forecasted prices, the intensity of the market impact of energy storage in trading, and the predicted available renewable energy source. The storage feasible SOC range (i.e., the energy storage capacity space) will be split into four possible sub-ranges by three SOC reference points corresponding to the previously listed four actions. The merchant can choose the optimal action simply by comparing the current energy inventory in the energy storage with the three optimal SOC reference points. Then, the electricity merchant's unique optimal decisions can be achieved through the sub-range within the current energy inventory level falls.

Second, in contrast to the results from existing studies (i.e., those based on price-taker analyses or ignoring the market impact), our results show that market impact and operating cost can raise the cost of pumping/buying and lower the revenue from generating/selling in each period. As a result, a merchant that ignores her impact on electricity prices will overestimate her expected profit when offering the same generating/discharging and pumping/charging maximum limits of the PSH in each period to ISOs as the price-taker merchant. To decrease the negative effect of the market impact of the merchant in operational decisions, the merchant needs to reduce her energy trading amount at each decision period. Our results find that the market impact influence the merchant’s optimal economic dispatch volume by changing the value of optimal SOC reference point. Although the residual value of energy in the storage does not affect the traditional scheduling policy, it influences the value function to affect the SOC and indirectly changes the scheduling quantity of power. Withholding the offered generating/pumping capacity may be needed to offset the market impact. This paper also confirms the corresponding boundary that wind generation benefits merchants' profit if the wind generation cost is low.

Finally, we extend our research to consider how expected profits are affected by the relation between the intensity of market impact and generating/discharging and pumping/charging maximum limits of the PSH offered to ISOs. Our findings suggest that the profit-maximizing merchant should try to make a trade-off between increasing the power transaction quantity directly and limiting the market impact's detrimental effects by reducing the transaction quantity.

4.3. MODELLING AND OPTIMIZATION

In this Section, we first model the reward and objective functions for electricity merchants with co-located energy storage and renewable power plants. Then, we study the merchant’s optimal joint profit-maximizing strategies and consider the market impact as a function of the forecast price.

4.3.1. Model Setup. Here, we focus on a merchant with energy storage (here, we use PSH to represent large-scale storage in this work) and a renewable power plant (for simplicity, henceforth, we use wind plants to refer to renewable power plants), both of which are co-located and connected to the electricity markets via transmission lines. The merchant adopts a co-optimized storage operation strategy and uses her energy storage plant to manage electricity. In this work, "*we do not study bidding in a forward market, and we assume that any power offered to the wholesale electricity markets is accepted*" (Liu et al., 2022; Sioshansi et al., 2009, Walawalkar et al., 2007; Zhou et al., (2016; 2019).

In this work, we consider discrete time and that the merchant periodically performs operational actions during a finite optimization decision horizon, $t \in \{1, 2, \cdots, T\}$, and assume that the capacity of storage is limited. The PSH has maximum storage capacity $\bar{S}$ (i.e., the total energy/water that could be stored in the upper reservoir) and minimum energy inventory $\underline{S}$, where $\bar{S} > \underline{S} \geq 0$. Following the previous (Harsha and Dahleh, 2015; Jiang and Powell, 2015a, 2015 b; Moarefdoost and Snyder, 2015; Zhou et al., 2016, 2019), we focus on the optimal operating (e.g., charging/pumping, and discharging/generating) policy for a given storage capacity. However, how to optimize the storage capacity, such an approach would be appropriate for solving a different type of problem, thus beyond the scope of this work. The PSH also has generating and pumping limits. Let $\bar{Q}^p$ and $\underline{Q}^p$ represent (respectively) the maximum and minimum limits of pumping that can be stored into the storage in each period, and let $\bar{Q}^g$ and $\underline{Q}^g$ denote (respectively) the upper and lower limits of released energy from the storage in each period. To ensure that the model will remain analytically tractable, this work employs the conventional assumption (as in Kim and Powell, 2011; Liu et al., 2022a, 2022b; Zhou et al., 2016, 2019) that $\underline{Q}^g = \underline{Q}^p = 0$ to build the continuous reward functions. We use w_t to represent the available *wind generation of* the wind plant in period t (in energy units/period). The vector $W = (w_1, w_2, \cdots, w_T)$ represents the sequential levels of available forecasted wind generation.

Following the previous work (Jiang and Powell, 2015a; Kim and Powell, 2011; Qi et al., 2015), wind generation is constrained by the maximum generation capacity $\bar{w}$ of the wind plants to show the uncertainty in modeling. Here, the $w_t \in [0, \bar{W}]$ follows a uniform distribution. In reality, the utility would require the transmission capacity to be sufficiently large for the wind plant, so we do not consider the transmission capacity.

Our research involves three types of efficiency with PSH. The first type of efficiency is a portion $\varphi_t \in (0,1]$, a time-independent efficiency of stored energy that dissipates in one optimization period due to the evaporation, spill rate, and leakage of the PSH. The second type efficiency is denoted by θ and ξ, which represent the efficiency of (respectively) the pumping and generating of the PSH; here, $\theta, \xi \in (0,1]$. The other is $\sigma \in (0,1]$, which represents the efficiency of transmission line, that is, the proportion of electricity that flows out of the transmission line to that which flows into this transmission line. Transmission losses will be happened in two directions of the line (Liu et al., 2022b; Zhou et al., 2019). It follows that the quantities $\xi\sigma \cdot \bar{Q}^g$ and $\bar{Q}^p / \theta\sigma$ are, respectively, the gross generating power capacity and the net pumping power capacity.

We suppose that the merchant's energy storage is large enough, and her generating and pumping decisions have a market impact on electricity prices. As noted previously, there are four possible actions: storing all renewable energy generation and also purchasing electricity to store; storing partial wind generation and selling the rest of renewable energy generation; remaining idle/offline, and generating PSH storage to sell and also selling all wind electricity to the electricity market. Following previous work (e.g., Cruise et al., 2019; Liu et al., 2021a; Sioshansi, 2010, 2014), this work approximates market impact via a linear function of the quantity of power traded by the merchant. Therefore, we get the following updated prices:

$$\hat{P}_t = \begin{cases} \left(P_t + \lambda P_t\left(q_t^p/\theta - w_t\right)/\sigma\right) = P_t\left(1+\lambda\left(q_t^p/\theta - w_t\right)/\sigma\right) & (q_t^p > \theta w_t) \\ \left(P_t - \lambda P_t\left(w_t - q_t^p/\theta\right)\sigma\right) = P_t\left(1-\lambda\left(w_t - q_t^p/\theta\right)\sigma\right) & (0 \le q_t^p \le \theta w_t) \\ \left(P_t - \lambda P_t(q_t^g\xi + w_t)\sigma\right) = P_t\left(1-\lambda(q_t^g\xi + w_t)\sigma\right) & (q_t^g > 0) \end{cases} \tag{4.1}$$

Here, $\hat{P}_t$ is the updated price that results from storing all renewable power generation and purchasing power from the market in energy units of $\left(q_t^p/\theta - w_t\right)/\sigma$, storing partial wind-generated power and selling the rest of to the market in units of $\left(w_t - q_t^p/\theta\right)\sigma$, and generating PSH and selling all wind source in units of $(q_t^g\xi + w_t)\sigma$. Here, the parameter $\lambda \ge 0$ reflects *the market impact factor* of the electricity merchant on electricity prices in trading decisions. The special case of $\lambda = 0$ represents the scenario of a price taker merchant for the traditional study. In the electricity market, time-coupling constraints require that the merchant should decide whether to buy or sell electricity in quantities that reflect the optimal policy based on forecasted prices. The electricity price in period t is denoted by P_t (dollars per unit energy). Both buying and selling prices at time *t* are shown by P_t conveniently for a price taker. The sequential levels of the price by a vector of $P=\left(P_1, P_2, \cdots, P_T\right)$.The P_t is the forecast electricity price, and λP_t is a measurement of the market impact of the energy storage on the price at decision time.

From ISO perspective, power transmission network must be considered explicitly in market clearing. From merchant perspective, power transmission network can be considered in two different approaches, explicitly (through building a quasi-ISO clearing model where power transmission network is often treated as constraints of a lower-level optimization problem) and implicitly (through price forecasting model where historical congestion of power transmission network can be included as an input). Due to concerns with the explicit approach (such as data and model availability, uncertainty and computational challenges), this work uses the latter approach, i.e., implicit consideration of power transmission network which is common in merchant strategy analysis (Li et al. 2007; Radovanovic et al., 2019; Wang et al. 2017).To maximize the profit of the electricity merchant and get the optimal economics dispatch policy of the energy storage, following the previous study (Liu et al., 2021a; Liu et al, 2022a, 2022b; Zhou et al., 2016, 2019), we assume for the merchant that all forecasted prices are known in advance.

By incorporating the market impact in operational decisions and analyzing the co-optimization policy of a merchant who has both co-located energy storage and wind plants, this method produces the model novel and practical and generalizes the current problem (Liu et al., 2021a; Zhou et al., 2019), as it makes the first contribution of this work. Thus, the reward function $R(q_t^g, q_t^p, w_t, P_t)$ from making the decision (q_t^g, q_t^p), which corresponds to the decision time t, the forecast electricity prices P_t, and the forecasted wind power generation w_t, are, when considering the market impact, defined as follows:

$$R(q_t^g, q_t^p, w_t, P_t) = \begin{cases} -P_t\left(1+\lambda\left(q_t^p/\theta - w_t\right)/\sigma\right)\cdot\left(q_t^p/\theta - w_t\right)/\sigma - c^p q_t^p/\theta\sigma - c_w w_t & (q_t^p > \theta w_t) \\ P_t\left(1-\lambda\left(w_t - q_t^p/\theta\right)\sigma\right)\cdot\left(w_t - q_t^p/\theta\right)\sigma - c^p q_t^p/\theta\sigma - c_w w_t & (0 \le q_t^p \le \theta w_t) \\ P_t\left(1-\lambda(q_t^g\xi + w_t)\sigma\right)\cdot(q_t^g\xi + w_t)\cdot\sigma - c^g q_t^g \xi\sigma - c_w w_t & (q_t^g \ge 0) \end{cases} \quad (4.2)$$

The first line in Eq. (4.2) indicates the costs of buying power of electricity merchants from the market. For example, w_t represents available wind generation,

$(q_t^p/\theta - w_t)/\sigma$ indicates the units that the merchant purchases from the market to pump at time *t*, and q_t^p is the increase in storage inventory. This study lets c^g (resp. c^p) (dollar-unit energy) denote the generating (resp. pumping) operating cost for PSH or the discharging (resp. charging) operating cost of the battery (Huang et al., 2018, 2019; Xu et al., 2017). Following Liu et al. (2022b) and Xu et al. (2017), we assume that the generating and pumping operating costs of energy storage are a linear function. The term $c^p \cdot q_t^p/\theta\sigma$ is the pumping operating cost, and $c_w w_t$ is the wind power plant's cost of generation. The second line gives the merchant's rewards from storing part of her wind generation q_t^p while selling the remaining units $(w_t - q_t^p/\theta)\sigma$ to the market. In the third line, $(q_t^g\xi + w_t)$ represents the electricity merchants generated by the PSH and all available wind sources that are sold to the market. The term $c^g q_t^g \xi\sigma$ denotes the generating operating cost of PSH.

This work uses SOC_t to denote as the current available energy inventory in the upper reservoir of PSH at the beginning of decision time *t*. The sequential SOC inventories are represented by $\hat{S} = \left(SOC_1, \cdots, SOC_T\right)$, where $SOC_t \in [\underline{S}, \bar{S}]$ and $\forall t \in \{1, 2, \cdots, T\}$. Feasible actions set based on $SOC_t \in \hat{S}$ is defined as follows:

$$\text{Action}(SOC_t) := \left\{ \begin{matrix} (q_t^g, q_t^p) \in \mathbb{R} : 0 \le q_t^p \le \bar{Q}^p \cdot U_t^p,\ q_t^p \le \bar{S} - SOC_t, \\ 0 \le q_t^g \le \bar{Q}^g \cdot U_t^g,\ q_t^g \le SOC_t - \underline{S} \end{matrix} \right\}. \tag{4.3}$$

This expression gives the upper limit of the quantity of energy that can be charged/pumped and discharged/generated at each optimization period. The first and second constraints define, respectively, the maximum limit of pumping and the space capacity of the upper reservoir. The third and the fourth constraints represent the maximum limit of generating and available energy in the reservoir. Both binary variables U_t^p and U_t^g denote the unit commitment of pumping and generating in decision period $[t, t+1)$ (respectively). Thus, we have $U_t^p + U_t^g \le 1$; here, $U_t^g \in \{0,1\}$ and $U_t^p \in \{0,1\}$, meaning the PSH cannot generate and pump simultaneously. If the PSH is idle, then $U_t^p + U_t^g = 0$.

At decision time $t \in \{1, 2, \cdots, T\}$, the merchant will know the storage inventory SOC_t, the wind generation w_t, the price P_t, and the market impact λ. The decision for each time *t* is denoted by q_t^g or q_t^p, which represents the *SOC change* from time *t* to time *t*+1 prior to considering, respectively, the generating loss and the pumping loss. The "storage self-loss" occurs at the end of decision time *t*, so the energy level at the beginning of decision time *t*+1 is equal to $\varphi_t(SOC_t + q_t^p - q_t^g)$. Hence, the following equation that summarizes the state transition from decision time *t* to decision time *t*+1 for the PSH storage is accurate:

$$SOC_{t+1} = \varphi_t(SOC_t + q_t^p - q_t^g) \tag{4.4}$$

Following Liu et al. (2022a), Secomandi (2010), and Zhou et al. (2019), this study also adopts a single decision (action) variable, and lets q_t (i.e., $q_t = q_t^p - q_t^g$) at as the decision variable of electricity merchant at each decision time $t \in \{1, 2, \cdots, T\}$ to substitute for the original two decision (action) variables q_t^g and q_t^p, which represent *the change of energy inventory or of SOC between two optimization periods t and t+1* (i.e., prior to considering accounting for the efficiency loss). Here, $q_t > 0$ denotes the SOC increase due to the pumping action, $q_t < 0$ means the SOC decrease because of generating, and $q_t = 0$ indicates that the SOC does not change or that the storage remains idle or is offline. The state decision variables at each stage t are SOC_t, w_t, and P_t. Thus, the decision state at stage t can be indicated by $S(t)=S_t(SOC_t, w_t, P_t)$. The merchant aims to achieve the optimal decision policy π to maximize her total expected reward functions overall feasible policies. Her objective function is

$$\max_{\pi}\sum_{t=1}^{T}E\left[R(q_t^g, q_t^p, w_t, P_t)|\ S(1)\right]=\max_{\pi}\sum_{t=1}^{T}E\left[R(q_t, w_t, P_t)|\ S(1)\right] \tag{4.5}$$

subject to the capacity constraints $\max\{-\bar{Q}^g, \underline{S}-SOC_t\}\le q_t \le \min\{\bar{Q}^p, \bar{S}-SOC_t\}$ and to the storage energy balance constraints $SOC_{t+1}=\varphi_t(SOC_t+q_t)$, as well as $w_t \in[0,\bar{W}]$, where $t\in\{1,2,\cdots,T\}$. Both E_1, P_1, and w_1 are the given initial level of the storage and the price in advance. Because the optimization horizon is finite, this work ignores the discount factor in this work. This work uses E to denote the expectations concerning SOC_t, w_t, P_t. In our notation, SOC_1, w_1, and P_1 are, respectively, the given *initial* energy storage inventory, the forecasted wind generation, and advanced electricity price.

Let $V_t(S(t))$ represent the value function of electricity merchant at time *t* and state $S(t)=S_t(SOC_t, w_t, P_t)\in\hat{S}\times W\times P$ .This function of $V_t(S(t))$ satisfies the Bellman equation. Thus, the merchant's value function can be created as

$$V(S(t))=\max_{Action(SOC_t)}[R(q_t, w_t, P_t)+E\left(V_{t+1}(S(t+1)\,|\,S(t)\right)] \tag{4.6}$$

Most on this topic expresses the value of water (VOW) at the last optimization period (residual value of water in the storage) as $V_{T+1}(S(T+1))=0$ (e.g., Secomandi 2010; Zhou et al., 2019). In Eq. (4.6), however, $V_{T+1}(S(T+1))=VOW_{T+1}\cdot SOC_{T+1}$. Here, VOW_{T+1} denotes the VOW in the upper reservoir of PSH at the terminal period (Liu et al. 2022b, Kim and Powell 2011), and SOC_{T+1} denotes the energy inventory level at the beginning of decision time T+1, which also represents the SOC at the end of decision time T.

4.3.2. Model Optimization and Analysis. To obtain the optimal co-optimized decision rules of the electricity merchant, this study first splits the optimization problem in Eq. (4.6) into three sub-problems, as in Eq. (4.7), corresponding to the three different actions described in Eq. (4.2) since only one of these actions is allowed at the same period. Then, we obtain the optimal result to each of these three sub-problems. The corresponding value functions of the electricity merchant on three available actions are shown as follows:

$$V(S(t))=\begin{cases} -P_t\left(1+\lambda\left(\frac{q_t}{\theta}-w_t\right)/\sigma\right)\cdot\left(\frac{q_t}{\theta}-w_t\right)/\sigma-c^p\frac{q_t^p}{\theta\sigma}-c_w w_t+E[V_{t+1}\left(S(t+1)\,|\,S(t)\right)]\ (q_t>\theta w_t) \\ -P_t\left(1+\lambda\left(\frac{q_t}{\theta}-w_t\right)\sigma\right)\cdot\left(\frac{q_t}{\theta}-w_t\right)\cdot\sigma-c^p\frac{q_t^p}{\theta\sigma}-c_w w_t+E[V_{t+1}\left(S(t+1)\,|\,S(t)\right)]\ (0\le q_t\le\theta w_t) \\ -P_t\left(1+\lambda((q_t\xi-w_t)\sigma)\cdot(q_t\xi-w_t)\cdot\sigma+c^g q_t\xi\sigma-c_w w_t+E[V_{t+1}\left(S(t+1)\,|\,S(t)\right)]\ \ (q_t<0) \end{cases} \quad (4.7)$$

Since there is $q_t = SOC_{t+1}/\varphi_t - SOC_t$, to simplify, we use SOC_{t+1} substitute q_t as the decision variable to gain the analytical results, then maximizing Eq. (4.8) enables us to obtain the optimal results by removing the values in the observed current state $S(t)$. The optimal unique action of the electricity merchant at each period will be achieved by comparing the optimal SOC in the next period (i.e., SOC_{t+1}) and the current available SOC (i.e., SOC_t) in the storage. Then, the Bellman equation (Liu et al., 2021a; Liu et al., 2022b; Zhou et al., 2019) can be used to derive the following results:

$$\begin{cases} V_t^{(1)*}(S(t)) = \max\limits_{\underline{E}\le E_{t+1}\le \bar{E}} \left(\begin{aligned} & E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \frac{\lambda P_t}{\theta^2\sigma^2}\frac{SOC_{t+1}}{\varphi_t^2}^2 + \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t} SOC_{t+1}\cdot SOC_t \\ & +(\frac{2\lambda P_t w_t}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma})^2 \frac{SOC_{t+1}}{\varphi_t} \end{aligned} \right) \\ V_t^{(2)*}(S(t)) = \max\limits_{\underline{E}\le E_{t+1}\le \bar{E}} \left(\begin{aligned} & E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \frac{\lambda\sigma^2 P_t}{\theta^2}\frac{SOC_{t+1}}{\varphi_t^2}^2 + \frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t} SOC_{t+1}\cdot SOC_t \\ & +(2\lambda P_t\sigma^2 \frac{w_t}{\theta} - \frac{P_t\sigma^2 + c^p}{\theta\sigma})^2 \frac{SOC_{t+1}}{\varphi_t} \end{aligned} \right) \\ V_t^{(3)*}(S(t)) = \max\limits_{\underline{E}\le E_{t+1}\le \bar{E}} \left(\begin{aligned} & E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \lambda P_t\xi^2\sigma^2 \frac{SOC_{t+1}}{\varphi_t^2}^2 + 2\lambda P_t\xi^2\sigma^2 \frac{SOC_{t+1}}{\varphi_t} SOC_t \\ & +(2\lambda P_t w_t\xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma)^2 \frac{SOC_{t+1}}{\varphi_t} \end{aligned} \right) \end{cases} \tag{4.8}$$

This study next investigates the optimal results based on these expressions. Finally, we get the closed-form optimal co-optimization policy structure of merchant in Eq. (4.9) by following previous research on this topic (i.e., Kim and Powell, 2011; Liu et al., 2022b; Zhou et al., 2019). When incorporating the market impact into the reward function, for the forecast price of electricity P_t, if $P_t < \infty$, then, at each decision state t, the merchant's value function $V_t(S(t))$ and expected total reward $E[V_{t+1}\left(S(t+1)\,|\,S(t)\right)]$ are concave in $SOC_t \in [\underline{S}, \bar{S}]$ for each observed state $S(t) = S_t(SOC_t, w_t, P_t)$. The SOC optimal analytical solution is given by the following lemma (see Appendix B).

LEMMA 4.1. When considering an electricity merchant's market impact in trading decisions, let $SOC_{t+1}^{(1)*}$, $SOC_{t+1}^{(2)*}$, and $SOC_{t+1}^{(3)*}$ be the closed-form optimal SOC results (e.g., SOC reference points in next period) in Eq.(4.9). Then, there are

$$\begin{cases} SOC_{t+1}^{(1)*} = \arg\max_{\underline{S} \le SOC_{t+1} \le \overline{S}} \left(\begin{array}{l} E[V_{t+1}^{*}\left(S(t+1) \mid S(t)\right)] - \frac{\lambda P_t}{\theta^2\sigma^2}\left(\frac{SOC_{t+1}}{\varphi_t} - SOC_t\right)^2 \\ +\left(\frac{2\lambda P_t w_t}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma}\right)\frac{SOC_{t+1}}{\varphi_t} \end{array} \right) \\ SOC_{t+1}^{(2)*} = \arg\max_{\underline{S} \le SOC_{t+1} \le \overline{S}} \left(\begin{array}{l} E[V_{t+1}^{*}\left(S(t+1) \mid S(t)\right)] - \frac{\lambda\sigma^2 P_t}{\theta^2}\left(\frac{SOC_{t+1}}{\varphi_t} - SOC_t\right)^2 \\ +\left(2\lambda P_t\sigma^2 \frac{w_t}{\theta} - \frac{P_t\rho^2 + c^p}{\theta\sigma}\right)\frac{SOC_{t+1}}{\varphi_t} \end{array} \right) \\ SOC_{t+1}^{(3)*} = \arg\max_{\underline{S} \le SOC_{t+1} \le \overline{S}} \left(\begin{array}{l} E[V_{t+1}^{*}\left(S(t+1) \mid S(t)\right)] - \lambda P_t \xi^2\sigma^2\left(\frac{SOC_{t+1}}{\varphi_t} - SOC_t\right)^2 \\ +\left(2\lambda P_t w_t \xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma\right)\frac{SOC_{t+1}}{\varphi_t} \end{array} \right) \end{cases} \tag{4.9}$$

Based on the scenario of an electricity merchant who has co-located PSH and a wind power plant, this lemma has a critical implication. Because the merchant can choose the optimal action simply by comparing the current SOC level in the storage with the above three optimal SOC reference points separately. It now follows from the preceding discussion that our first proposition gives the corresponding optimal results.

Proposition 4.1: For positive forecast electricity prices $\hat{P}_t \in P$ (negative forecast electricity prices) at each stage $t \in \{1, 2, 3, \cdots, T\}$: if $0 \le \lambda \le \min\{\overline{\lambda}_t^{(1,2)}, \overline{\lambda}_t^{(2,3)}\}$, then there exist unique optimal storage inventories $\underline{S} \le SOC_{t+1}^{(1)*} \le SOC_{t+1}^{(2)*} \le SOC_{t+1}^{(3)*} \le \overline{S}$ [2] (resp., $\overline{S} \ge SOC_{t+1}^{(1)*} \ge SOC_{t+1}^{(2)*} \ge SOC_{t+1}^{(3)*} \ge \underline{S}$) that depend on the state $S(t)$, where,

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[2]We discuss large-scale storage with 1–2 gigawatts (GW) capacities. Considering a sizeable competitive wholesale electricity market (such as MISO, which has approximately 50 (off-peak period)–80 GW (on-peak period) demand, roughly 100 GW online capacity) and the limited presence of locational market electricity because of transmission capacity constrained and electricity market monitoring, we only address the case of a relatively small market impact.

$$\begin{cases} \overline{\lambda}^{(1,2)} = P_t \Big/ \dfrac{2P_t}{\theta}\left(\dfrac{1+\sigma^2}{\sigma}\right)\left(SOC_t + w_t\theta - \dfrac{\underline{S}}{\varphi_t}\right); \\ \overline{\lambda}^{(2,3)} = \left(\dfrac{P_t\sigma^2 + c^p}{\theta\sigma} - (P_t\xi\sigma - c^g\xi\sigma)\right) \Big/ 2\sigma^2 P_t\left((SOC_t - \dfrac{\underline{S}}{\varphi_t})(\dfrac{1}{\theta^2} - \xi^2) + (\dfrac{w_t}{\theta} - w_t\xi)\right) \end{cases} \tag{4.10}$$

Therefore, an optimal economic dispatch decision in each state $S(t) = S_t(SOC_t, w_t, P_t) \in \hat{S} \times W \times P$ can be specified as described in the following two cases.

CASE 1: If $\theta w_t < \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$ (less forecasted available wind-generated power), then the feasible SOC range (i.e., the from the lower boundary to the upper boundary of energy storage capacity) can be split into four sub-ranges (i.e., regions or areas): storing all wind power generation and purchasing electricity to store, storing partial wind power generation and selling the rest of it to market, remaining idle or do nothing, and generating PSH and also selling all wind power to the electricity market.

$$q_t^*(S_t) = \begin{cases} \min\{SOC_{t+1}^{(1)*} - SOC_t, \overline{Q}^p\},\ SOC_t \in [\underline{S}, SOC_{t+1}^{(1)*} - \theta w_t], \\ \text{(store renewable and buy electricity, up to } SOC_{t+1}^{(1)*}); \\ \min\{SOC_{t+1}^{(2)*} - SOC_t, \theta w_t\}, SOC_t \in (SOC_{t+1}^{(1)*} - \theta w_t, SOC_{t+1}^{(2)*}], \\ \text{(store renewable without buying up to } SOC_{t+1}^{(2)*}); \\ 0,\ SOC_t \in (SOC_{t+1}^{(2)*}, SOC_{t+1}^{(3)*}],\ \text{(keep SOC unchanged)}; \\ \max\{SOC_{t+1}^{(3)*} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC_{t+1}^{(3)*}, \overline{S}], \\ \text{(generate and sell renewable down to } SOC_{t+1}^{(3)*}). \end{cases} \tag{4.11}$$

CASE 2: If $\theta w_t \geq \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$ (more *available wind-generated power*), then the feasible SOC range of the storage can be segmented into three possible sub-ranges: storing partial wind power generation and selling the rest of it to market, generating PSH and also selling all wind power generation to the electricity market, and idle:

$$q_t^*(S_t)=\begin{cases}\min\{SOC_{t+1}^{(2)*}-SOC_t,\overline{Q}^p\},SOC_t\in[\underline{S},\ SOC_{t+1}^{(2)*}],\\ \text{(store renewable without purchasing, up to } SOC_{t+1}^{(2)*});\\ 0,SOC_t\in[SOC_{t+1}^{(2)*},SOC_{t+1}^{(3)*}]\text{ (keep SOC unchanged)};\\ \max\{SOC_{t+1}^{(3)*}-SOC_t,-\overline{Q}^g\},SOC_t\in(SOC_{t+1}^{(3)*},\ \overline{S}],\\ \text{(generate and sell renewable down to } SOC_{t+1}^{(3)*}).\end{cases}\tag{4.12}$$

Case 1 of Proposition 4.1 shows analytically that, f*or an electricity merchant who has both co-located PSH and wind plant and pursues to maximize her expected profit, if there is less* available forecasted wind power, the SOC of the storage will be segmented into four possible sub-ranges by three analytical SOC reference points ($SOC_{t+1}^{(1)*}, SOC_{t+1}^{(2)*}$, and $SOC_{t+1}^{(3)*}$, *which depend on the price forecast* P_t*, the energy in storage* SOC_t*, the forecast wing generation* w_t*, and the market impact* λ) that correspond to four possible different operational decisions: (1) storing all renewable generation and also purchasing electricity to store, (2) storing partial wind power and selling the rest of it, (3) remaining idle (i.e., offline/do nothing), and (4) releasing PSH and also selling all wind power. If the current available energy in the storage is more than reference point $SOC_{t+1}^{(3)*}$, the merchant will release water from the PSH to generate electricity and also sell all wind-generated electricity to the market, then reduce the SOC level down to $SOC_{t+1}^{(3)*}$. If there is less available energy in the PSH than $SOC_{t+1}^{(1)*}-\theta w_t$ and less available wind power (i.e., $\theta w_t<\min\{SOC_{t+1}^{(1)*},\overline{Q}^p\}$), the merchant should (1) store all the wind power and buy electricity and then (2) increase the SOC inventory up to $SOC_{t+1}^{(1)*}$.

According to Case 2 of the proposition, if there is *more* available wind generation (i.e., $\theta w_t \geq \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$), then the *feasible storage inventory range* will be divided into three sub-ranges by two analytical SOC reference points ($SOC_{t+1}^{(2)*}$ and $SOC_{t+1}^{(3)*}$) that correspond to operational decisions 2–4. In this case, decision one will not happen since the merchant does not need to purchase power from the market to store when there is more available wind power generation. If there is less water in the PSH than $SOC_{t+1}^{(2)*}$, the merchant does not need to buy electricity to increase the SOC level but she can store partially wind power and increase the SOC so that it is to $SOC_{t+1}^{(2)*}$, and then sell the rest of her wind generation. Likewise, if the current available energy inventory in the storage falls within the boundaries established by two analytical reference points (i.e., $SOC_{t+1}^{(2)*} \leq SOC_t \leq SOC_{t+1}^{(3)*}$), then the merchant should do nothing for the PSH storage; and if there is more water in the upper reservoir than the SOC reference point $SOC_{t+1}^{(3)*}$, then the profit-maximizing merchant should (4) release energy from the PSH for generating and also sell all wind power, thereby decreasing the current inventory to $SOC_{t+1}^{(3)*}$.

Further, this study has three special degenerated cases with fewer thresholds, as seen below.

Special Case A: If $\sigma = 1$ (i.e., ignoring the efficiency loss of transmission line), then our results have $SOC_{t+1}^{(1)*} = SOC_{t+1}^{(2)*}$. This means that storing wind power generation or purchasing electricity from the power market to store will yield the merchant the same profit, that is, without considering the energy loss from the power market to storage via the transmission line, as when the merchant purchases electricity to store. Considering the efficiency loss of transmission line, storing merchant's own generated renewable source to storage is better than purchasing power from the market.

Special Case B: If $\theta = \xi = 1$ (i.e., ignoring the pumping and generating efficiency loss) and if $c^p = c^g = 0$ (i.e., ignoring the generating and pumping operating costs), then $SOC_{t+1}^{(2)*} = SOC_{t+1}^{(3)*}$. Moreover, the SOC range can be split into only three (or two) subranges that depend on the forecasted wind generation. In this case, however, no optimal strategy will include the "idle" state.

Special Case C: If $w_t = 0$ (i.e., the available forecasted wind generation equals zero or no wind source), in this case, there will be no storing or selling of wind generation, and our study has only $SOC_{t+1}^{(1)*}$ and $SOC_{t+1}^{(3)*}$ as optimal reference points (See Appendix B). Then this work obtains the optimal policy for the previous study for a merchant with PSH or storage only (Liu and Bo et al., 2021). In our results, the storage state of charge (SOC) is segmented into four possible subranges by three analytical SOC reference points that correspond to four different decisions for the co-optimization merchant, compared to the three decisions in the previous study (Liu et al., 2021a). Obviously, the scenario that electricity merchant only has storage is a particular case for the merchant has storage and wind plant. Proposition 4.1 yields our first insight and application, as follows.

Managemental Insight 4.1: *For an electricity merchant with co-located energy storage and a wind plant, the feasible SOC range of the energy storage is segmented into different sub-ranges by the analytical SOC reference points, which depends mainly upon the current SOC, forecasted electricity price, and available forecasted wind source, and the intensity of market impact. As a result, the merchant will achieve the corresponding optimal operational decision for each subrange.*

To maximize the profit, and if less available renewable source, the SOC of storage will be split into four possible sub-ranges by three analytical reference points $SOC_{t+1}^{(1)*}, SOC_{t+1}^{(2)*}$ and $SOC_{t+1}^{(3)*}$, which correspond to four possible operational actions: storing all wind-generated power and also purchasing electricity to store, storing and selling partial renewable generation, do nothing/idle/offline, and generating PSH to sell and also selling all wind power. By comparing the current SOC level in the storage with the obtained SOC reference points for next period, the merchant can obtain the related optimal operational decisions. However, *if more* available forecasted wind generation, the storage SOC will be segmented into three sub-ranges by two analytical reference points $SOC_{t+1}^{(2)*}$ and $SOC_{t+1}^{(3)*}$, which correspond to three possible different operational decisions: storing and selling partial wind electricity, doing nothing (idle/offline), and generating electricity by PSH to sell and also selling all wind generation. Obviously, the optimal SOC reference points will be adjusted based on the intensity of market impact to support decision-making.

4.3.3. Market Impact and Wind Generation Analysis. This research studies the optimal co-optimized scheduling strategy of a merchant with large-scale energy storage and wind plant, whose trading decisions (i.e., buying or selling) are able to affect electricity prices. In traditional treatments, the electricity merchant is a price taker (Kim and Powell, 2011; Liu et al., 2022b; Zhou et al., 2019) or only addressed the base problem without considering the wind plant (Cruise et al., 2019; Liu et al., 2021a; Secomandi, 2010).

In light of our assumptions and the preceding analysis, the optimal results are described in the next proposition.

Proposition 4.2: (a) If the electricity merchant has large-scale energy storage and wind plant, optimal expected profit is decreasing in the market impact and operating cost of energy storage.

(b) For the electricity merchant with energy storage and wind plant, optimal expected profit increases with the forecasted wind generation $w_t \in [0, \bar{W}], \forall t = \{1, 2, \cdots, T\}$.

(c) Suppose the $q^{*(M)}_{t\ (\lambda \geq 0)}$ (resp. $q^{*(M)}_{t\ (\lambda = 0)}$) represents the optimal actions of electricity merchants *accounting for* the market impact (resp. ignoring the market impact) on power prices, we can draw the following intuitive conclusions for the optimal expected profit of the merchant:

$$\sum_{t=1}^{T} E\left[R(q^{*(M)}_{t\,(\lambda=0)}, w_t, P_t)_{(\lambda \geq 0)} \mid S(1)\right] \leq \sum_{t=1}^{T} E\left[R(q^{*(M)}_{t\,(\lambda \geq 0)}, w_t, P_t)_{(\lambda \geq 0)} \mid S(1)\right] \tag{4.13}$$

Proposition 4.2 is quite intuitive. These conclusions in Part (a) are consistent with the insights stated by Felix et al. (2012) and Liu et al. (2021a). It is straightforward; the merchant will achieve less profit with the increasing of operating cost and market impact. It will increase the cost of buying power from the market and decrease the revenue of selling power to the market by smoothing the difference between the high price at peak hours and low prices at off-peak. Part (b) demonstrates that the electricity merchant should take advantage of renewable wind generation to maximize reward at each period and optimal profit in the optimization horizon. It implies that the merchant with energy storage and wind plant should not curtail wind generation (i.e., generate the wind power based on the max generation capacity of the wind plants installed) to benefit their profit as long as the electricity prices are larger than the generation cost of wind if we do not consider bidding in a forward market. Part (c) shows that if a merchant ignores market impact on the power price and decides from the scenario of price-taker (i.e., the optimal economics dispatch of the storage is optimized on the wrong assumption $\lambda = 0$; however, where the corresponding profit of the merchant is calculated according to the real value of market impact factor λ), she will get less expected profit. Proposition 2 states that the market impact considerably alters the optimal policy structure and optimal expected profit, as detailed by the numerical results presented in Section 4.4.

Managemental Insight 4.2: *The co-optimization merchants will have a lower expected profit with the increase of market impact if price-maker merchant and price-taker merchant submit the same generating and pumping maximum capacity to ISO. However, if the price-maker merchant ignores market impact in trading decisions and follows the price-taker's solutions, she will achieve less optimal expected profit. On the other hand, wind generation benefits merchant's profit if the wind generation cost is low.*

Insight 4.2 has an important implication for the price-maker merchant. The decisions of a merchant naturally affect the market price, so the merchant will have a lower profit when the cost of buying power is increasing, and the revenue from selling power is decreasing. Therefore, to smooth the negative effect of the market impact on buying and selling actions of the merchant, they should reduce the amount of electricity generating or pumping each period. Thus, a merchant with PSH and wind plants must perfectly balance the power transition quantity and market impact intensity and reduce wind power curtailment to maximize profit.

4.4. CASE STUDY AND NUMERICAL SIMULATION

Section 4.4.1 validates the presented approaches and results employing one three-period case to represent the calculation procedure in detail and then compare them with the MILP method through the synthesis data. Additionally, Section 4.4.2 employs real data from MISO to demonstrate the related results and insights.

4.4.1. Synthesis Date Case Study. For simplicity, this Section employs a three-period example to show the detail of the proposed method in Section 4.3. Here, we suppose there are three optimization decision periods ($T=3$). The forecasted price takes set $P_t=\{P_1,P_2,P_3\}=\{5,2,10\}=\{P^M,P^L,P^H\}$ at each period. This work also supposes the merchant cannot fill her energy storage fully in one decision period but less than two (i.e., $\underline{S}+\bar{Q}^p\le\bar{S}$, and $\underline{S}+2\bar{Q}^p\ge\bar{S}$). Meanwhile, the full storage can be emptied in one decision period (i.e., resp. $\bar{S}-\underline{S}\le\bar{Q}^g$). In detail, we suppose the energy storage capacity is 10 (i.e., $\underline{S}=0,\bar{S}=10$), and we suppose the pumping capacity in one period is 7, and the generating capacity is 12 in one period. Suppose the pumping and generating operating costs of the storage $c^p=c^g=0.1$, the pumping and generating efficiencies of the energy storage as well as the transmission efficiency of the line are $\theta=\xi=\sigma=0.9$.

In this case, to illustrate the effect of market impact, this Section supposes the intensity market impact parameter of the merchant is $\lambda=0.01$. In the case study, we focus on the scenario that the electricity merchant has energy storage and a wind plant and also assume the forecasted wind generation is $w_t=\{3,5,0\}=\{w_1,w_2,w_3\}$. Based on Lemma 4.1, our results show that both the generation cost of wind and the self-discharging do not affect the optimal solutions, so we assume the generation cost of wind equals zero (i.e.,

$c_g = 0$). Let the operating cost be 0.1(i.e., $c^p = c^g = 0.1$), and the pumping and generating efficiencies, and efficiency of transmission line be 0.9 (i.e., $\theta = \xi = \sigma = 0.9$). For simplification, in this case study, we assume the residual value of water in storage is zero (i.e., $VOW_4 = 0$). On this basis, we employ the backward dynamic programming approach to achieve the following optimal outcomes:

In decision State 3: Action 3: Since the energy in the storage and the end of the third period is valueless, to maximize the profit, the electricity merchant needs to sell power to the electricity market and bring the SOC $\underline{S} = 0 = E_4^*$ down to the minimum boundary of the storage as long as the electricity prices are positive, thus

$$q_3^*(S_3) = -SOC_3,\ SOC_3 \in (0, \bar{S}] \tag{4.14}$$

Thus, the following value function at stage 3 is achieved:

$$\begin{aligned} V_3^* &= \max\{R_3 + V_4^*\} = -P_t[1 + \lambda(q_t\xi - w_t)\sigma] \cdot (q_t\xi - g_t) \cdot \sigma + c^g q_t \xi\sigma - c_w w_t (q_t < 0) \\ &= 8.019SOC_3 - 0.06561SOC_3^{\,2} \end{aligned} \tag{4.15}$$

In decision state 2: By utilizing the functions (4.9), (4.10), and (4.11) in Section 4.3, we obtain the following outcomes for the optimal SOC reference points at initial of third period or the end of second period:

$$\begin{cases} SOC_3^{(1)*} = \underset{SOC_3 \in [0,10]}{\arg\max} \left(V_3^* - \dfrac{\lambda P_2}{\theta^2\sigma^2}[\dfrac{SOC_3}{\varphi_3} - SOC_2]^2 + (\dfrac{2\lambda P_2 w_2}{\theta\sigma^2} - \dfrac{P_2 + c^p}{\theta\sigma})\dfrac{SOC_3}{\varphi_3} \right) \\ SOC_3^{(2)*} = \underset{SOC_3 \in [0,10]}{\arg\max} \left(V_3^* - \dfrac{\lambda\sigma^2 P_2}{\theta^2}[\dfrac{SOC_3}{\varphi_3} - SOC_2]^2 + (2\lambda P_2\sigma^2 \dfrac{w_2}{\theta} - \dfrac{P_2\sigma^2 + c^p}{\alpha\sigma})\dfrac{SOC_3}{\varphi_3} \right) \\ SOC_3^{(3)*} = \underset{SOC_3 \in [0,10]}{\arg\max} \left(V_3^* - \lambda P_2\xi^2\theta^2[\dfrac{SOC_3}{\varphi_3} - SOC_2]^2 + (2\lambda P_2 w_2 \xi\sigma^2 - P_2\xi\sigma + c^g\xi\sigma)\dfrac{SOC_3}{\varphi_3} \right) \end{cases} \tag{4.16}$$

$$\Rightarrow \begin{cases} SOC_3^{(1)*} = \underset{SOC_3 \in [0,10]}{\arg\max}\left(4.590 \cdot SOC_3 - 0.096 \cdot SOC_3{}^2 + 0.061 \cdot SOC_3 SOC_2 - 0.030 \cdot SOC_2{}^2\right) \\ \quad = (4.590 + 0.061 \cdot SOC_2)/0.192 > 10 = \overline{S} \\ SOC_3^{(2)*} = \underset{SOC_3 \in [0,10]}{\arg\max}\left(4.964 \cdot SOC_3 - 0.086 \cdot SOC_3{}^2 + 0.040 \cdot SOC_3 SOC_2 - 0.020 \cdot SOC_2{}^2\right) \\ \quad = (4.964 + 0.04 \cdot SOC_2)/0.172 > 10 = \overline{S} \\ SOC_3^{(3)*} = \underset{SOC_3 \in [0,10]}{\arg\max}\left(7.355 \cdot SOC_3 - 0.079 \cdot SOC_3{}^2 + 0.026 \cdot SOC_3 SOC_2 - 0.013 \cdot SOC_2{}^2\right) \\ \quad = (7.355 + 0.026 \cdot SOC_2)/0.158 > 10 = \overline{S} \end{cases} \tag{4.17}$$

Here, $\theta w_t = 0.9 \times 5 = 4.5 < 7$, by comparing the current SOC at the initial of second period and the above-obtained reference points, the merchant will obtain the following optimal decision at period 2:

$$q_2^*(S_2) = \begin{cases} 7,\ SOC_2 \in [0,3], \\ \text{(store generation and purchase electricity up to } SOC_3^{1*} = \overline{S}); \\ 10 - SOC_2,\ SOC_2 \in (3, 5.5], \\ \text{(store generation and purchase electricity up to } SOC_3^{1*} = \overline{S}); \\ 10 - SOC_2,\ SOC_2 \in (5.5, 10], \\ \text{(store generation without buying up to } SOC_3^{2*} = \overline{S}). \end{cases} \tag{4.18}$$

Then, the optimal value functions at decision time 3 can be rewritten as

$$V_3^* = \begin{cases} 8.019 \cdot SOC_3 - 0.06561 \cdot SOC_3{}^2 \big|_{SOC_3 = SOC_2 + 7} \\ = 7.1 \cdot SOC_2 + 52.92 - 0.06561 \cdot SOC_2{}^2;\ \text{if } SOC_2 \in [0,3] \\ 8.019 \cdot SOC_3 - 0.06561 \cdot SOC_3{}^2 \big|_{SOC_3 = \overline{S} = 10} \\ = 80.19 - 6.561 = 73.629;\ \text{if } SOC_2 \in (3,10] \end{cases} \tag{4.19}$$

By combining the optimal actions and the corresponding price at period 2, we obtain the following reward functions at period 2:

$$R(q_2, w_2, P_2) = \begin{cases} -7.23 & SOC_2 \in [0,3] \\ -15.74 - 0.6361 \cdot SOC_2{}^2 + 2.93 \cdot SOC_2 & SOC_2 \in (3,5.5] \\ -12.84 + 2.34 \cdot SOC_2 - 0.01 \cdot SOC_2{}^2 & SOC_2 \in (5.5,10] \end{cases} \quad (4.20)$$

Hence, incorporating the Eq. (4.19) and the reward function at period 2, the optimal value functions at the second decision time are obtained:

$$V_2^* = \begin{cases} 7.1 \cdot SOC_2 + 45.69 - 0.06561 \cdot SOC_2{}^2 & SOC_2 \in [0,3] \\ -0.6361 \cdot SOC_2{}^2 + 2.93 \cdot SOC_2 + 57.89 & SOC_2 \in (3,5.5] \\ 2.34 \cdot SOC_2 - 0.01 \cdot SOC_2{}^2 + 60.79 & SOC_2 \in (5.5,10] \end{cases} \quad (4.21)$$

In decision state 1: Similarly, by employing the functions (4.9), (4.10), and (4.11), we will reach optimal SOC reference points at the end of the first period or the initial of the second period as the following solutions:

$$\begin{cases} SOC_2^{(1)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - \frac{\lambda P_1}{\theta^2 \sigma^2} [\frac{SOC_3}{\varphi_3} - SOC_2]^2 + (\frac{2\lambda P_1 w_2}{\theta \sigma^2} - \frac{P_1 + c^p}{\theta \sigma}) \frac{SOC_3}{\varphi_3} \right) \\ SOC_2^{(2)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - \frac{\lambda \sigma^2 P_1}{\theta^2} [\frac{SOC_3}{\varphi_3} - SOC_2]^2 + (2\lambda P_1 \sigma^2 \frac{w_2}{\theta} - \frac{P_1 \sigma^2 + c^p}{\theta \sigma}) \frac{SOC_3}{\varphi_3} \right) \\ SOC_2^{(3)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - \lambda P_1 \xi^2 \sigma^2 [\frac{SOC_3}{\varphi_3} - SOC_2]^2 + (2\lambda P_1 w_2 \xi \sigma^2 - P_1 \xi \sigma + c^g \xi \sigma) \frac{SOC_3}{\varphi_3} \right) \end{cases} \quad (4.22)$$

$$\Rightarrow \begin{cases} SOC_2^{(1)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - \frac{0.05}{0.81 \times 0.81} [SOC_2 - SOC_1]^2 - 5.88 \cdot SOC_2 \right) \\ SOC_2^{(2)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - 0.05 [SOC_2 - SOC_1]^2 - 4.85 \cdot SOC_2 \right) \\ SOC_2^{(3)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - 0.05 \cdot 0.81 \cdot 0.81 [SOC_2 - SOC_1]^2 - 3.75 \cdot SOC_2 \right) \end{cases}$$

Next, we analyze the SOC reference points separately based on Eq. (4.21) and energy storage capacity.

(1) scenario: If $SOC_2 \in [0,\ 3]$

$$\begin{cases} \mathrm{SOC}_2^{(1)*} = \underset{\mathrm{SOC}_2 \in [0,3]}{\arg\max}\left(-0.142\cdot \mathrm{SOC}_2{}^2 + (1.22+0.15\cdot \mathrm{SOC}_1)\mathrm{SOC}_2 + 45.69 - 0.076\cdot \mathrm{SOC}_1{}^2\right) \\ \qquad = \left(1.22+0.15\mathrm{SOC}_1\right)/0.284 > 3 \Rightarrow \mathrm{SOC}_2^{(1)*} = 3 \\ \mathrm{SOC}_2^{(2)*} = \underset{\mathrm{SOC}_2 \in [0,3]}{\arg\max}\left(-0.116\cdot \mathrm{SOC}_2{}^2 + (2.25+0.1\cdot \mathrm{SOC}_1)\mathrm{SOC}_2 + 45.69 - 0.05\cdot \mathrm{SOC}_1{}^2\right) \\ \qquad = \left(2.25+0.1\mathrm{SOC}_1\right)/0.232 > 3 \Rightarrow \mathrm{SOC}_2^{(2)*} = 3 \\ \mathrm{SOC}_2^{(3)*} = \underset{\mathrm{SOC}_2 \in [0,3]}{\arg\max}\left(-0.098\cdot \mathrm{SOC}_2{}^2 + (3.35+0.066\cdot \mathrm{SOC}_1)\mathrm{SOC}_2 + 45.69 - 0.033\cdot \mathrm{SOC}_1{}^2\right) \\ \qquad = \left(3.35+0.066\mathrm{SOC}_1\right)/0.196 > 3 \Rightarrow \mathrm{SOC}_2^{(3)*} = 3 \end{cases} \tag{4.23}$$

(2) Scenario2: If $\mathrm{SOC}_2 \in (3,\ 5.5]$

$$\begin{cases} \mathrm{SOC}_2^{(1)*} = \underset{\mathrm{SOC}_2 \in (3,5.5]}{\arg\max}\left(-0.71\cdot \mathrm{SOC}_2{}^2 - 2.95\cdot \mathrm{SOC}_2 + 0.152\cdot \mathrm{SOC}_1\mathrm{SOC}_2 + 57.89 - 0.076\cdot \mathrm{SOC}_1{}^2\right) \\ \qquad = (-2.95+0.152\cdot \mathrm{SOC}_1)/1.42 < 0 \Rightarrow \mathrm{SOC}_2^{(1)*} = 3 \\ \mathrm{SOC}_2^{(2)*} = \underset{\mathrm{SOC}_2 \in (3,5.5]}{\arg\max}\left(-0.686\cdot \mathrm{SOC}_2{}^2 - 1.92\cdot \mathrm{SOC}_2 + 0.1\cdot \mathrm{SOC}_1\mathrm{SOC}_2 + 57.89 - 0.05\cdot \mathrm{SOC}_1{}^2\right) \\ \qquad = (-1.92+0.1\cdot \mathrm{SOC}_1)/1.372 < 0 \Rightarrow \mathrm{SOC}_2^{(2)*} = 3 \\ \mathrm{SOC}_2^{(3)*} = \underset{\mathrm{SOC}_2 \in (3,5.5]}{\arg\max}\left(-0.669\cdot \mathrm{SOC}_2{}^2 - 0.82\cdot \mathrm{SOC}_2 + 0.066\cdot \mathrm{SOC}_1\mathrm{SOC}_2 + 57.89 - 0.033\cdot \mathrm{SOC}_1{}^2\right) \\ \qquad = \left(-0.82+0.066\cdot \mathrm{SOC}_1\right)/1.338 < 0 \Rightarrow \mathrm{SOC}_2^{(3)*} = 3 \end{cases} \tag{4.24}$$

(3) Scenario 3: If $\mathrm{SOC}_2 \in (5.5,\ 10]$

$$\begin{cases} \mathrm{SOC}_2^{(1)*} = \underset{\mathrm{SOC}_2 \in [5.5,10]}{\arg\max}\left((-3.54+0.152\cdot \mathrm{SOC}_1)\mathrm{SOC}_2 + 60.79 - 0.086\cdot \mathrm{SOC}_2{}^2 - 0.076\cdot \mathrm{SOC}_1{}^2\right) \\ \qquad = (-3.54+0.152\cdot \mathrm{SOC}_1)/0.172 < 0 \Rightarrow \mathrm{SOC}_2^{(1)*} = 5.5 \\ \mathrm{SOC}_2^{(2)*} = \underset{\mathrm{SOC}_2 \in [5.5,10]}{\arg\max}\left((-2.51+0.1\cdot \mathrm{SOC}_1)\mathrm{SOC}_2 + +60.79 - 0.06\cdot \mathrm{SOC}_2{}^2 - 0.05\cdot \mathrm{SOC}_1{}^2\right) \\ \qquad = (-2.51+0.1\cdot \mathrm{SOC}_1)/0.12 < 0 \Rightarrow \mathrm{SOC}_2^{(2)*} = 5.5 \\ \mathrm{SOC}_2^{(3)*} = \underset{\mathrm{SOC}_2 \in [5.5,10]}{\arg\max}\left((-1.41+0.066\cdot \mathrm{SOC}_1)\mathrm{SOC}_2 + 60.79 - 0.043\cdot \mathrm{SOC}_2{}^2 - 0.0328\cdot \mathrm{SOC}_1{}^2\right) \\ \qquad = (-1.41+0.066\cdot \mathrm{SOC}_1)/0.086 < 0 \Rightarrow \mathrm{SOC}_2^{(3)*} = 5.5 \end{cases} \tag{4.25}$$

By comparing the max value, we can find the optimal references among scenario 1, scenario 2, and scenario 3.

Thus, the merchant obtains the following three optimal SOC reference points: $SOC_2^{(1)*} = SOC_2^{(2)*} = SOC_2^{(3)*} = 3$.

Since $\theta w_t < \min\{SOC_{t+1}^{(1)}, \overline{Q}^p\}$ (i.e., $0.9 \cdot 3 = 2.7 < \min\{3, 7\} = 3$), based on proposition 4.1 in Section 4.3, the optimal decisions of the merchant at stage 1 are

$$q_1^*(S_1) = \begin{cases} 3 - SOC_1, \text{if } SOC_1 \in [0, 0.3] (\text{store generation and purchase electricity up to 3}) \\ 3 - SOC_1, \text{if } SOC_1 \in (0.3, 3] (\text{store generation without buying up to 3}) \\ 3 - SOC_1, \text{ if } SOC_1 \in (3, 10] (\text{sell inventory down to 3}) \end{cases} \tag{4.26}$$

When incorporating the market impact of the merchant, based on the forecasted price at period 1 and the optimal action in Eq. (4.26), the reward functions of electricity merchants at stage 1 are shown:

$$R_1(q_1, w_1, P_1) = \begin{cases} -P_1[1 + \lambda(\frac{3 - SOC_1}{0.9} - 3) / 0.9] \cdot (\frac{3 - SOC_1}{0.9} - 3) / 0.9 - 0.1 \frac{3 - SOC_1}{0.81} \text{ if } SOC_1 \in [0, 0.3] \\ -P_1[1 + \lambda(\frac{3 - SOC_1}{0.9} - 3) 0.9] \cdot (\frac{3 - SOC_1}{0.9} - 3) 0.9 - 0.1 \frac{3 - SOC_1}{0.81} \quad \text{if } SOC_1 \in (0.3, 3] \\ -P_1[1 + \lambda(\frac{3 - SOC_1}{0.9} - 3) 0.9] \cdot ((3 - SOC_1) 0.9 - 3) 0.9 - 0.1 (3 - SOC_1) 0.81 \text{ if } SOC_1 \in (3, 10] \end{cases} \tag{4.27}$$

Thus, the optimal value functions of the merchants at first decision state are:

$$V_1^* = \begin{cases} -P_1[1 + \lambda(\frac{3 - SOC_1}{0.9} - 3) / 0.9] \cdot (\frac{3 - SOC_1}{0.9} - 3) / 0.9 - 0.1 \frac{3 - SOC_1}{0.81} \\ +7.1 \cdot SOC_2 + 45.69 - 0.066 \cdot SOC_2^{\ 2} \text{ if } SOC_1 \in [0, 0.3]; \\ -P_1[1 + \lambda(\frac{3 - SOC_1}{0.9} - 3) 0.9] \cdot (\frac{3 - SOC_1}{0.9} - 3) 0.9 - 0.1 \frac{3 - SOC_1}{0.81} \\ +7.1 \cdot SOC_2 + 45.69 - 0.066 \cdot SOC_2^{\ 2} \quad \text{if } SOC_1 \in (0.3, 3]; \\ -P_1[1 + \lambda((3 - SOC_1) 0.9 - 3) 0.9] \cdot ((3 - SOC_1) 0.9 - 3) 0.9 - 0.1 (3 - SOC_1) 0.81 \\ -0.636 \cdot SOC_2^{\ 2} + 2.93 \cdot SOC_2 + 57.89 \text{ if } SOC_1 \in (3, 5.5]; \\ -P_1[1 + \lambda((3 - SOC_1) 0.9 - 3) 0.9] \cdot ((3 - SOC_1) 0.9 - 3) 0.9 - 0.1 (3 - SOC_1) 0.81 \\ +2.34 \cdot SOC_2 - 0.01 \cdot SOC_2^{\ 2} + 60.79 \quad \text{if } SOC_1 \in (5.5, 10]. \end{cases} \tag{4.28}$$

Recall the previous steps, the following optimal trading actions of the merchant at three periods are obtained.

1) If $SOC_1 = 1$ (The SOC in energy storage at the beginning of decision time 1)

State 1: If $SOC_1 = 1$, (store wind generation 2, and make the SOC up to 3, also sell $2/0.9 - 3 = -7/9$ to the market), then the SOC in the storage will approach to $SOC_2 = 3$ (i.e., $q_1^* = 2$, $R_1 = 3.23$);

State 2: If $SOC_2 = 3$, (buying and pumping), then, there is $SOC_3 = 10$ (i.e., $q_2^* = 7, R_2 = -7.23$);

State 3: If $SOC_3 = 10$, (generating and selling), the SOC in the storage will down to $SOC_4 = 0$ (i.e., $q_3^* = -10, R_3 = 73.63$).

By using the predicted electricity prices, the total rewards of the merchant during the optimization horizon are shown as $R = R_1 + R_2 + R_3 = 69.63 = V_1^*$.

2) If $SOC_1 = 5$ (The SOC in energy storage at the beginning of decision time 1)

State 1: If $SOC_1 = 5$,(idle), there is $SOC_2 = 3$ (i.e., $q_1^* = -2$, $R_1 = 20.5$) holding;

State 2: If $SOC_2 = 3$,(buying and pumping), then there exists $SOC_3 = 10$ (i.e., $q_2^* = 7$, $R_2 = -7.23$);

State 3: If $SOC_3 = 10$,(generating and selling), since we have $SOC_4 = 0 = \underline{S}$, so the optimal action in the third period $q_3^* = -10$), so there has $R_3 = 73.63$).

Accordingly, the total rewards of the merchant during the given three optimization periods are $R = R_1 + R_2 + R_3 = 86.91 = V_1^*$.

Compared to the previous study (Liu and Bo et al., 2021) in which the electricity merchant with energy storage only, or the predicted wind power is zero (i.e., *Special Case C*), considering the market impact and $\lambda = 0.01$, the corresponding optimal SOC reference points and profits are shown in Table 4.1 under different two initial SOC in the storage.

Table 4.1 Optimal dispatching strategies and profit of the electricity merchant with energy storage only

	Optimal SOC reference points	Optimal economic dispatch	Total rewards
$SOC_1 = 1$	$SOC_2^{(1)*} = 1.5$; $SOC_3^{(1)*} = 10$	$q_1^* = 0.5$; $q_2^* = 7$; $q_3^* = -8.5$	$R = 34.1$
$SOC_1 = 5$	$SOC_2^{(3)*} = 3$; $SOC_3^{(1)*} = 10$	$q_1^* = -0.16$; $q_2^* = 5.16$; $q_3^* = -10$	$R = 55.7$

This table displays that two optimal SOC reference points, $\mathrm{SOC}_{t+1}^{(1)*}$ and $\mathrm{SOC}_{t+1}^{(3)*}$, were created based on the method proposed in Section 4.3 when ignoring wind power generation. For the scenario, the profit-maximizing merchant has energy storage only and only needed to buy power from the electricity market to store and make the current energy level in the storage up to $\mathrm{SOC}_{t+1}^{(1)*}$ as close as possible when there is less energy in the storage. If the current available energy level in the storage is larger than reference point $\mathrm{SOC}_{t+1}^{(3)*}$, the merchant needs to discharge energy from the storage for selling, then bring the SOC down to $\mathrm{SOC}_{t+1}^{(3)*}$ as close as possible.

To verify our research and the proposed method in this work, we also adopted the classic MILP method (Bo et al., 2021; Liu et al., 2021b; Wang et al., 2021; Wang et al., 2022) to solve the above three-periods case and get the optimal results as well as compare them with the optimal outcomes in Section 4.4.1. It yielded the same optimal results under both the dynamic programming method (i.e., our method in Section 4.4.1) and the MILP (i.e., traditional approach). The above optimal solutions are verified in AIMMS.

4.4.2. Real Data Case Study. This Section will use hourly optimization period units as the electricity prices and wind generation sequence $P=\{P_1, P_2, \dots, P_T\}$ (\$/M.W.) and $W=\{w_1, w_2, \dots, w_T\}$ (MWH) with 336 decision periods (T = 336) corresponding to two-weeks optimization horizons from Dec. 3 to Dec. 18, 2020) in MISO as supplied (the prices data is available at: https://www.misoenergy.org/). The maximum and minimum capacity of the PSH upper reservoir $\overline{S}$ and $\underline{S}$ are 20 and 2 (respectively). Here, $\underline{S}>0$ denotes that the merchant cannot empty the upper reservoir of PSH, which is common in the electricity market for a PSH. The pumping and generating capacity are $\overline{Q}^p=2$ and $\overline{Q}^g=2$. The unit of measurement of PSH can be described as GWH. The units of generating and pumping capacity measurement can be represented as a GW.

Following the previous study, we also assume the pumping and the generating efficiencies of the PSH are $\alpha=\beta=0.9$. The optimization period $(\overline{S}-\underline{S})/\overline{Q}^g=9$ hours units for the PSH to empty the storage, while $(\overline{S}-\underline{S})/\overline{Q}^p=9$ hours units for the PSH to fill the storage fully correspond approximately to the Ludington PSH in Michigan USA (the PSH detail are available at: https://www.consumersenergy.com/company/electric-generation/renewables/hydroelectric/pumped-storage-hydro-electricity). Based on the existing report (Mongird, et al., 2020), we assume the operating cost $c=1$ (\$/MWh). We also ignore the transmission efficiency loss and suppose $\rho=1$ and $\eta=1$. To simplify, we assume that the residual value of the water in the storage is equal to the expected electricity prices during the optimization horizon (i.e., $VOW_{T+1} = \sum_{t=1}^{T} p_t \Big/ T$).

Using the same method proposed by Cruise et al. (2019) to calculate the market impact (e.g., we used the off-peak load and on-peak load and the corresponding prices in the optimization horizon and pumping and generating limits in each period offered to the ISOs to achieve the lambda approximately as a proxy for the market impact[3]).

For a merchant who owns a large storage *(such as the Ludington PSH)* and a wind farm, the results are as follows. The merchants' optimal co-optimized economics dispatch actions are obtained from the value functions (4.6) when the merchant with co-located energy storage and wind plants is displayed in Figure 4.1 and Figure 4.2 under two different initial SOC in the PSH, respectively.

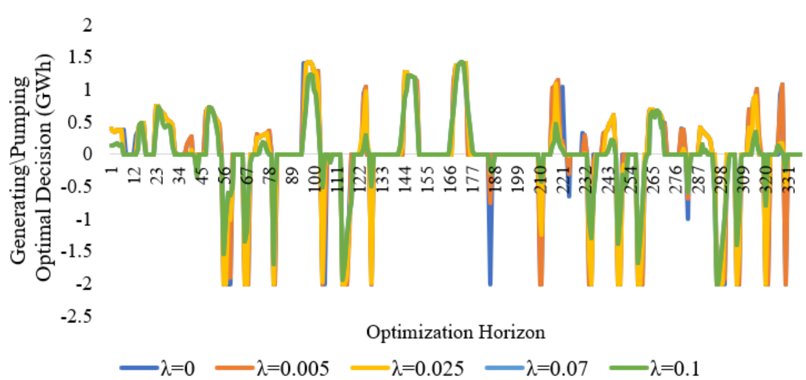


Figure 4.1 The optimal decisions when $SOC_1 = 2$ GWh

[3]Although the merchant has PSH and wind plants, we will ignore the effect of wind generation when we calculate the market impact due to the high uncertainty of renewable generation.

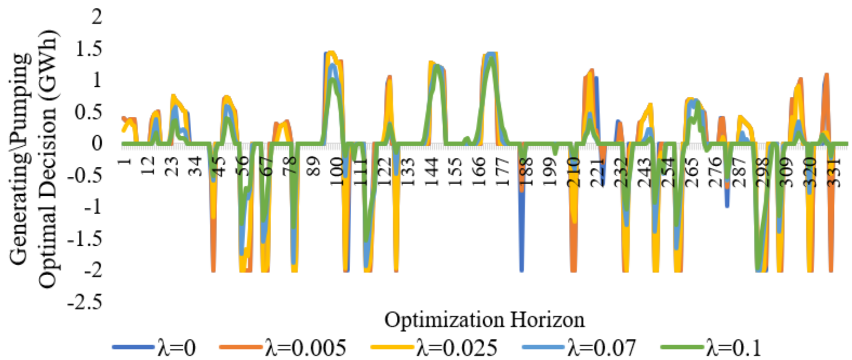


Figure 4.2 The optimal decisions when $SOC_1 = 10$ GWh

Figures 4.1 and 4.2 show that when the market impact factor is small, merchants with a co-located energy storage and wind plant will choose a similar strategy to the traditional strategy (that is, as a price-taker merchant and ignoring the market impact of the energy storage in trading), that is, when the market price of electricity is low, the merchant will buy electricity from market and will resale it later at a high price to maximize the profit. As the intensity of market impact increases (such as $\lambda = 0.1$), the transaction quantity of electricity merchant who has energy storage and wind farm (see green and blue curves) in each period decreases. In this situation, the merchant's profit mainly depends on wind generation and indirectly reduces the energy storage arbitrage function by decreasing the frequent pumping and generating actions.

The optimal actions are obtained from Eq. (4.11) when the merchant with energy storage only (i.e., without wind generation) is shown in Figure 4.3 and Figure 4.4 under different initial SOC in the storage.

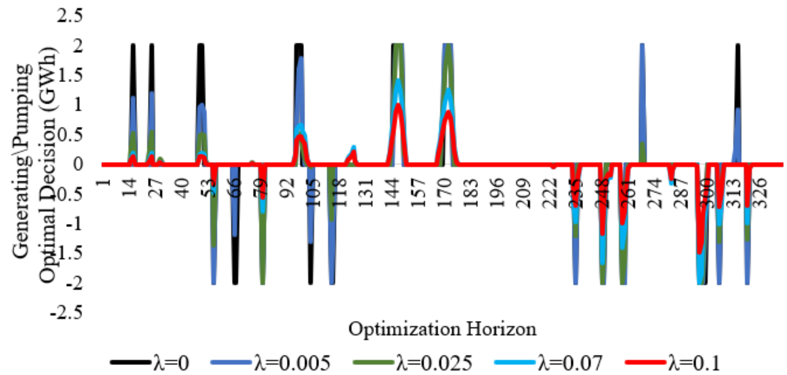


Figure 4.3 The optimal decisions when $SOC_1 = 2$ GWh

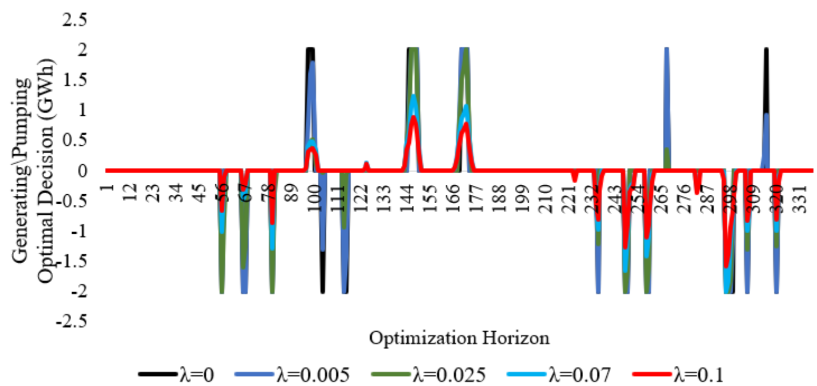
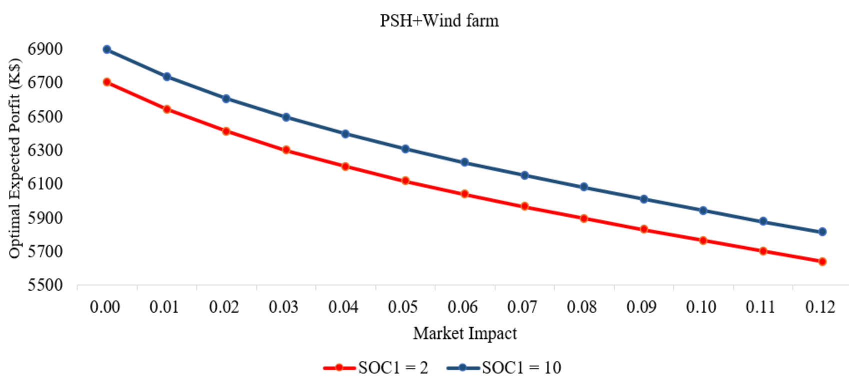


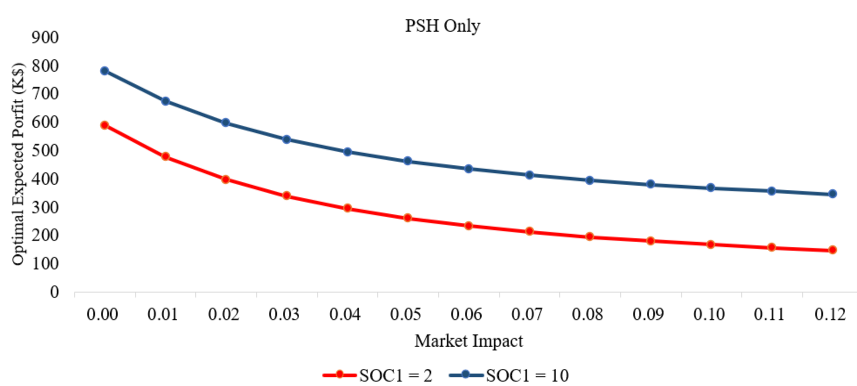
Figure 4.4 The optimal decisions when $SOC_1 = 10$ GWh

Figures 4.3 and 4.4 show that when there is no wind plant, the merchant's dispatching strategy is the same as when there is a wind plant. With the intensity of market influence increasing (such as $\lambda = 0.1$), each period's transaction quantity decreases (see, red curve). Figures 4.3 and 4.4 show the relationship between the optimal action and the intensity of market influence under such a situation, which is the same as that of merchants with only energy storage. With the increasing market impact of the merchant in trading, the cost of purchasing power to pump will rise; however, the revenue will decrease through discharging energy for selling. Therefore, to decrease the negative effect of market impact on operational decisions, the merchant should lower the power transition amount at each decision period to benefit her own profit. Consequently, a profit-maximining merchant with energy storage and wind plants must balance market impact intensity and energy transition quantity.

Figure 4.5 corresponds to the Ludington PSH case for the relationship between the optimal expected profit and the intensity of market impact with wind and without wind plants, respectively.



(a) Merchant with both PSH and wind farm



(b) Merchant with PSH only

Figure 4.5 The relationship between the optimal expected profit and market impact

Figure 4.5 indicates that regardless of whether there is wind power generation, for the large-scale energy storage, the operational trading decisions will *influence the market prices*. However, compared with the existing study, the increases of market impact will lead to decreased maximum expectation profit because purchase costs increase, and sale revenues decrease. It is intuition, considering the market impact of merchants in trading will increase the cost of buying electricity from the market and decrease the revenue of selling electricity to the market.

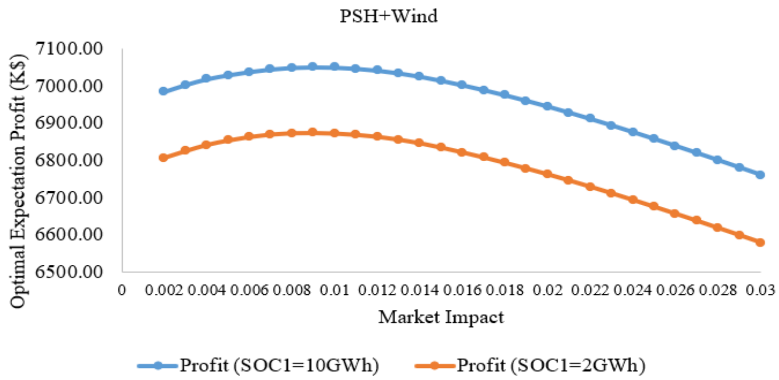
Obviously, if the large-scale energy storage merchant schedules energy in the storage following the scenario of a price taker, she will lose more profit. This part further proves the conclusion of the previous Section through numerical simulation. These results are similar to the reported consequences by and Cruise et al. (2019), Felix et al. (2012), and Liu et al. (2021a). To maximize expected profits, merchants should mitigate the market impact and increase profits by reducing the amount of electricity trading each period to offset the negative effect of market impact.

4.5. EXTENSION RESEARCH: MARKET IMPACT AS A FUNCTION OF OFFERED LIMITS TO ISO

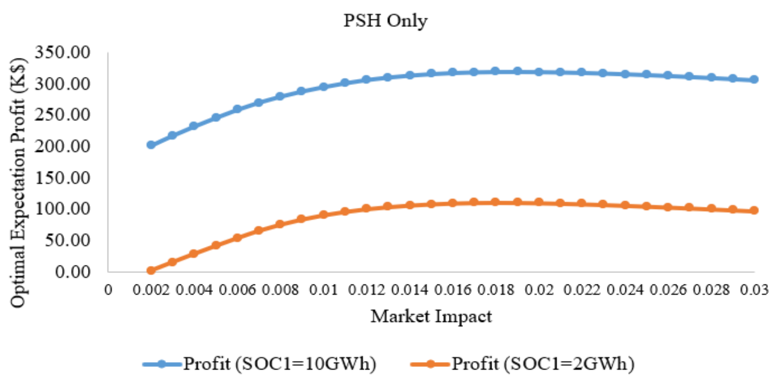
The results presented in Section 4.2 and Section 4.4 show that electricity merchants get less expected profit with growing market impact if both the price-taker electricity merchant and the price-maker electricity merchant offer the exact pumping/generating maximum capacity in one period offered to ISOs. However, in the electricity market, where capacity withholding is allowed, the merchant can adjust their pumping and generating capacity offered to ISOs to change her market impact (Mehdipourpicha and Bo, 2020, 2021). The implications of electricity merchants' market impact change substantially when considering the relationship between that impact and the generating and pumping capacity in each optimization period offered to ISOs. If the market impact is related to offered maximum pumping and generating limits, when the merchant changed her offered pumping (resp. generating) limit from $\bar{Q}_n^p$ (resp. $\bar{Q}_n^g$) to $\bar{Q}_m^p$ (resp. $\bar{Q}_m^g$), and if $\bar{Q}_m^p \le \bar{Q}_n^p$ and $\bar{Q}_m^g \le \bar{Q}_n^g$ hold, we will get $0 \le \lambda_m \le \lambda_n$. Here, the different subscript values show different generating and pumping limits offered to the ISOs.

For the intensity market impact parameter, in this Section, following the previous study (Mehdipourpicha and Bo, 2020, 2021), we shall use the ratio of electricity merchants' offered limits to the total (MISO-wide) online capacity of generators, where the latter is commonly about 100 GW. Thus, different market impacts correspond to different generating and pumping maximum limits in one period that is offered to the ISOs, which may result in different optimal actions and expected profits. For example, a market impact factor of $\lambda = 0.02$ (resp., $\lambda = 0.01$) corresponds to a merchant generating/pumping maximum limit of 2 GW (resp.,1 GW) offered to ISOs. Our results are derived simply by increasing the upper limits of generating and pumping that offered to MISO from 0.1 GW to 3 GW (i.e., $0.001 \le \lambda \le 0.03$); here we also suppose all other parameters are the same as in Section 4.4.

Figure 4.6 illustrates the impact of market impact by adjusting the limits that offered to MISO on the expected profit of the merchant for cases with wind generation (upper panel) and without wind generation (lower panel).



(a) Merchant with both PSH and wind farm



(b) Merchant with PSH only

Figure 4.6 Merchant's optimal expected profit as a function of market impact

Figure 4.6 indicates that, regardless of whether there is wind power generation, the merchant's optimal expected profits first increase and then decrease with their market impact. It follows that a merchant can maximize expected profits by balancing market impact with offered pumping/generating maximum limits to ISOs. From the perspective of profit maximization, the electricity merchant must decide which is more important: the limits of offered transaction to ISOs or the approximately market impact. There is an inherent trade-off between these two factors, since the merchant can—in each period—increase the unit energy/power profit while lowering transaction quantity.

Suppose the market impact is low; in that case, there is a low revenue (due to the limited power transaction) although the unit power profit is high. Hence the merchant should increase the power transmission quantity to enhance her profit by enlarging the max capacity offered to ISOs. The most intriguing result is that raised market impact would result in a reducing unit power profit by raising the cost of purchasing and lowering sales revenue. In that case, we recommend that the merchant should limit their market impact's detrimental effects by—in each period—reducing her generating/pumping limits offered to ISO and increasing profit from unit power.

We affirm these conclusions by conducting additional analyses, as briefly described following. Accordingly, we change the power prices and wind generation corresponding to one day period with 24 stages and seven days period with 168 stages, respectively, (corresponding to one day 12/01/2020 and one week from December 1 to December 7, 2020) in MISO as provided. Once again, our previous findings are mainly supported.

4.6. SECTION SUMMARY AND ANALYSIS

The main objective of this work is to analyze the scenario when the merchants with both co-located large-scale energy storage systems and wind plants and build the co-optimized policy structure of electricity merchants whose actions are sufficiently important to have a market impact on electricity prices. We formulate this problem as a Markov decision process and employ the dynamic programming method to achieve the closed-form analytical results to support multi-period decision-making of merchants. Although there are multiple activities available each period for the electricity merchant, only one of these decisions/actions is allowed at the same time. On this basis, to solve this problem, this work first split the original problem into three sub-optimization problems corresponding to three different actions. Then, the optimal solution for each sub-optimization problem will be addressed based on the Bellman equation. Finally, we combine them and achieve the global conclusions of the original problem. We demonstrate that the obtained optimal strategy policy in this work generalizes the traditional results and differs significantly from usual strategies reported to be optimal in the current published work, neglecting the market impact and the residual value of energy in the storage.

To maximize the profit of electricity merchant who has large-scale energy storage and wind power plant, considering the generating and pumping operating costs and three types of efficiency loss, we find the current optimal economic dispatch strategy of the storage relies on the SOC reference points. These SOC reference points depend on the current SOC inventory in the energy storage, the forecasted electricity prices, available forecasted wind generation currently, and the market impact of energy storage in trading. We show analytically that, for a merchant with both PSH and wind plant, there exist three SOC reference points such that the SOC range is divided into four possible sub-ranges, each of which corresponds to one of four distinct options. The merchant will achieve the unique optimal action by comparing the current SOC in the storage and the SOC reference points. However, suppose operating costs and efficiency loss of the energy storage are not modeled. Then, the feasible SOC range of the storage can also be segmented into two sub-ranges by one unique optimal SOC reference point. In this case, storing renewable generation or buying power for pumping will bring the same cost for a merchant. If we ignore the wind generation or not available renewable source, it equals an electricity merchant with only large-scale energy storage. Our study finds the condition that *wind generation benefits merchants' profit.*

We recognize that the merchant's market impact and the residual value of energy in the storage play essential roles in the optimal strategy design. Although the residual value of energy in the storage affects the value function then influences the optimal decision, this work finds that it does not change the relationship among three optimal SOC reference points, so the residual value cannot revise the traditional policy. Our results also show that the price-maker merchant will obtain similar strategies as the price-taker merchant scenario when the market impact is small. However, considering the market impact and offering the same generating and pumping capacity as the price-taker, we find the market impact would drive profit-reducing by raising the cost of purchasing and lowering sales revenue. In that case, our findings recommend that the merchant needs to mitigate the market impact's negative effect as much as possible by lowering the power transition amount at each decision period to benefit her profit. These new conclusions provide more knowledge of managing differentiated forecasted wind generation, market impact, and co-optimized economic dispatch of energy storage and wind plant. To the extent that a merchant can influence the market impact (e.g., through adjustment of the pumping and generating maximum limits offered to ISOs), we identify conditions under which the trade-off is either beneficial or detrimental to the merchant. These new findings augment our collective knowledge about managing the intensity of market impact and are an essential contribution to research on this topic.

5. OPTIMAL ECONOMIC DISPATCH POLICY FOR PROSUMER WITH ENERGY STORAGE CONSIDERING SELF-CONSUMPTION DEMAND

5.1. OVERVIEW AND RESEARCH QUESTIONS

This study (Liu et al., 2022 d) analyzes how prosumers' power demand or self-consumption (see, Figure 5.1) affects their optimal joint economic dispatch structure. Due to the intermittent and high levels of uncertainty regarding DERs (i.e., solar or wind) generation and the dynamic demand of the prosumer, production and consumption are not always simultaneous, as in a PV system that can only generate during daytime hours and will only produce optimally on long and cloudless days. Thus,there are two possible scenarios at each period where DERs generation can meet prosumers' demand or not. Therefore, a prosumer has grid connected DERs will get involved in two types of exchanges with the grid: energy imports when DER production is insufficient to match self-consumption, and energy flows are from the grid to the home; energy exports when DERs production is greater than or equal self-consumption, and the energy flows are from the home to the grid for use by others.

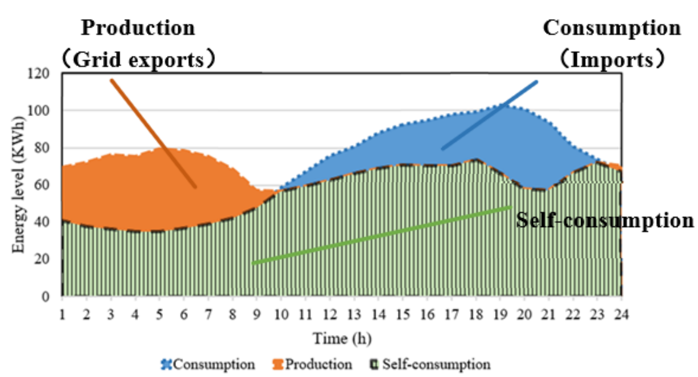


Figure 5.1 Self-consumption for the prosumer with installed wind turbines in 24 hours

Considering these two types of exchanges, a model is carried out under two scenarios to analyze the optimal dispatch decision for prosumers under different load rates. In the first scenario, the DERs generation of the prosumer cannot satisfy her own demand (i.e., production is less than power demand); in such case, when all the DERs generation is consumed, there is still a gap between the demand and supply. Then, the prosumers will have three choices to fulfill that gap: prosumer internal imports (discharging storage), external grid imports (buying electricity), or both. Under this scenario, there are four decisions for prosumers: buying electricity from the market to match the gap and to store, buying electricity and discharging storage to meet the gap, just buying electricity to meet the gap and storage remaining idle, and discharging storage to meet the load and to sell. Second, when the DERs generation is large, the prosumers can satisfy their local demand by DERs (i.e., generation is more than demand). They will carry out energy arbitrage as the traditional electricity merchant (Liu et al., 2022b; Zhou et al., 2019) and have four decisions: storing the remaining DERs production and buying electricity from the electricity market, storing and selling the partial remaining renewable energy, selling all remaining energy and storage remaining idle, and selling all remaining DERs generation and discharging the battery.

In contrast to the previous research, this study analyzes how prosumers' power demand affects their optimal joint economic dispatch structure. Due to the intermittent and high levels of uncertainty regarding DERs generation and the dynamic demand of the prosumer, production and consumption are not always simultaneous; there are two possible scenarios at each period where DERs generation can meet prosumers' demand or not. Therefore, a prosumer who has grid connected DERs will get involved in two types of exchanges with the grid: energy imports when DER production is insufficient to match self-consumption, and energy flows are from the grid to the home; energy exports when DERs production is greater than or equal self-consumption, and the energy flows are from the home to the grid for use by others.

Considering these two types of exchanges, a model is carried out under two scenarios to analyze the optimal dispatch decision for prosumers under different load rates. In the first scenario, the DERs generation of the prosumer cannot satisfy her own demand; in such case, when all the DERs generation is consumed, there is still a gap between the demand and supply. Then, the prosumers will have three potential options to mitigate the power shortage between the self-demand and distributed energy generation: a) buying electricity from the grid; b) discharging the energy storage; c) or both. Under this scenario, there are four decisions for prosumers: buying electricity from the market to match the gap and to store, buying electricity and discharging storage to meet the gap, just buying electricity to meet the gap and storage remaining idle, and discharging storage to meet the load and to sell. Second, when the DERs generation is large, the prosumers can satisfy their local demand by DERs, then the remaining renewable power generation can be sold to the grid or stored in storage. They will carry out energy arbitrage as the traditional electricity merchant (Liu et al., 2022b; Zhou et al., 2019) and have four decisions: storing the remaining DERs production and buying electricity from the electricity market, storing and selling the partial remaining renewable energy, selling all remaining energy and storage remaining idle, and selling all remaining DERs generation and discharging the battery.

Incorporating the self-demand will bring challenges in modeling since these two scenarios cannot happen simultaneously in each period, and different scenarios require different decisions and cost or rewards functions for prosumers. In our study, we first analyze each scenario in isolation to determine the best energy storage scheduling strategy, and then we combine them to determine the best overall solution. Instead of focusing on profit maximization, we consider cost minimization. It should be noted that the model is non-trivial in achieving analytical results by employing a dynamic programming approach when considering the uncertainty of DERs generation and dynamic demand in the problem. Such challenges not fully explored in the research to date are addressed in this paper as original theoretical findings.

The above analyses are intended to answer the following two questions: (1) How do co-optimizing prosumers benefit from considering the dynamic demand and the uncertain DERs generation? (2) What is the difference between the co-optimization scheduling strategy and the traditional co-optimization scheduling strategy ignoring self-consumption demands in dispatching the energy storage? Toward that end, we formulate the prosumer's cost minimization problem as a Markov decision process. We find that results from the perspectives of cost-minimizing and profit-maximizing are equivalent for the prosumer. Then, we employ the prosumer's optimal joint operational trading strategies by approaching the dynamic programming to maximize her own profit. This is conceivably the first paper to manage the co-optimization scheduling of prosumers with energy storage, considering the two scenarios concerned with the uncertainty of DERs generation and self-demand through dynamic programming.

The rest of this work was as follows. Section 5.2 provides the principal contribution of this research. Section 5.3 models a prosumer installing distributed energy source and has energy storage, then explores the optimal storage scheduling to fulfill local demand and arbitrage. Section 5.4 verifies the proposed results based on synthesis data and real data from MST (Missouri University of Science and Technology). Finally, the conclusions and suggestions for future study are discussed in Section 5.5.

5.2. THE PRINCIPLA CONTRIBUTIONS

This research makes two principal contributions. First, for a prosumer with energy storage: by applying the monotony of the cost function and realizing that the above two scenarios (i.e., DER generation is less or more than the user's self-demand) cannot happen simultaneously, the cost functions under each scenario was analyzed separately to find the optimal storage scheduling strategy, which are then combined to get the optimal global solution. This study analytically develops three SOC reference points in each scenario, which depend on the currently available energy in the storage, the forecasted electricity prices, the self-demand of electricity, and the available power of the DERs production. The feasible SOC of energy storage will be segmented into different sub-regions corresponding to the different decisions under two scenarios. The prosumers are able to obtain optimal operations strategies by analyzing the current energy in the storage with the optimal SOC reference points. The self-consumption demand substantially alters the optimal scheduling policy structures by affecting the decisions, cost functions, and SOC reference points and their relationships. We found the analytical results regarding the boundary conditions under which the merchants choose the different optimal economic dispatch policies.

Second, in contrast to the results from traditional studies (i.e., those ignoring the self-demand of co-optimization electricity merchants who have energy storage and renewable power plant), this study shows that the prosumer needs to fulfill her power load priority considering self-demand. If the DERs production can satisfy self-demand, the prosumer will operate as a traditional co-optimization electricity merchant with energy storage and renewable energy sources. We formulate this scenario as a Markov decision process and obtain the analytical solutions for the prosumer's scheduling strategy based on dynamic programming. Based on the analytical results, some conclusions and insights can be obtained. The numerical simulation verified that the method proposed in this paper will reduce the prosumer's electricity bill. Unlike the conventional study, it is required to determine in advance whether the prosumer's DERs generation can satisfy her own power demand and choose different formulas accordingly when implementing our model to achieve the optimal economic dispatch of energy storage for prosumers.

5.3. OPTIMAL DISPATCH FOR PROSUUMER UNDER TWO SCENARIOS

This Section models a prosumer who has distributed energy resource (for simplicity, henceforth, we use a solar panel to refer to DERs), electrical devices, and energy storage. We will propose the analytical optimal energy scheduling strategy of the prosumers who can produce and consume energy based on minimizing electricity bills through dynamic programming (Liu et al., 2022b; Finnah et al., 2022; Zhou et al., 2019).

5.3.1. Model Setup. This research considered the prosumer with the energy storage to balance electricity demand and do electricity trading. When the solar power generation cannot support the prosumer's electricity demand, the prosumer can purchase electricity from the market through the transmission line or release the energy from the storage. The optimal economic dispatch strategy was studied based on the prosumer's electricity bill minimization aim. The prosumer performed the scheduling of the storage periodically

$t \in \{1, 2, \cdots, T\}$ in a finite horizon. The structure of the prosumer transaction is shown in Figure 5.2.

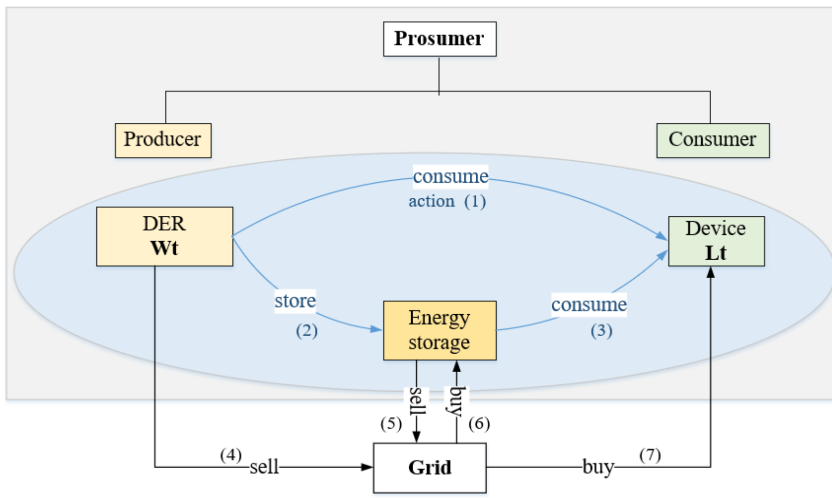


Figure 5.2 The structure of the prosumer transaction.

Figure 5.2 shows that the prosumer owns a solar panel with energy storage (i.e., battery) and power-consuming devices as well as connected to the grid. There are seven activities from the perspective of prosumer, which can be summarized into two classes.

The *first* class of action happens within the prosumer, that is, between the solar panel generator, power-consuming device, and energy storage:

1. The device consumes solar production.
2. Store partial solar generation into energy storage.
3. Discharge storage for device use.

The *second* class of action is the transaction between the prosumer and market:

4. Sell partial solar power production to the market /grid.
5. Discharge storage for sale to the market.

6. Purchase power from the market to charge energy storage.
7. Purchase electricity for device to use.

We focus on the economic dispatch of energy storage for prosumers considering the self-demand during the optimization horizon. Thus, we assume that all the forecasted solar power generation, electricity prices, and power demand based on time series are known in advance in our model (Liu et al., 2022a; Liu et al., 2022b; Zhou et al., 2019).

Assume that the storage capacity is limited, with a maximum capacity of $\overline{E}$ and a minimum capacity of $\underline{E}$, then $\overline{E} > \underline{E} \geq 0$. The storage has charging and discharging capacity limits. The upper bound of charge is defined as $\overline{Q}^{ch}$, and the upper bound of discharge is defined as $\overline{Q}^{dis}$, which represents the maximum capacity that the storage can charge and discharge at a given period. Here, following the conventional study (Jiang and Powell, 2015a; Zhou et al., 2019), the assumed lower bound of charging and discharging is zero. The storage can also choose to be neither charged nor discharged during any period.

Three types of efficiency loss were considered in this model. The first is the charging and discharging efficiency for energy storage, denoted by α and β, and $\alpha, \beta \in (0,1]$. The second type of efficiency is the transmission efficiency, denoted by $\rho \in (0,1]$ and implying the ratio of power flowing out of the transmission line to the power flowing into the other line. The third is $\eta_t \in (0,1]$, an efficiency of storing energy in storage and $1-\eta_t$ means dissipated energy during one period due to the self-discharge of the energy storage.

Following the previous study on energy economic dispatch of the electricity merchant (Zhou et al., 2019, Liu et al., 2021a, Liu et al., 2022b), the prosumer's optimal dispatch decision in period *t* is defined by q_t, which means the energy change in the storage between the period *t* and *t*+1 before considering efficiency loss. Where $q_t > 0$ indicates increased storage level due to charging, $q_t < 0$ represents decreased storage level for the action of discharging, and $q_t = 0$ means the storage level had not changed (i.e., idle or offline). As a result, the above assumption also means charging and discharging cannot occur simultaneously.

Here, E_t denotes as the state of charge (SOC)/storage level at the beginning of the period t, or the storage level at the end of the period t-1, and the sequential levels of the storage is defined by $\hat{E} = (E_1, E_2, \cdots, E_T)$, where $E_t \in [\underline{E}, \overline{E}], \forall t \in \{1, 2, \cdots, T\}$. At the end of the period t, the storage self-discharge occurred, then the energy balance or state transition formulation from the period *t* to *t*+1 is shown as follows:

$$E_{t+1} = \eta_t (E_t + q_t) \tag{5.1}$$

In this study, for simplicity, also it was assumed that the energy storage has linear charging and discharging costs, and $\{c^{ch}, c^{dis}\}$ (USD per unit of energy) is denoted as the operating cost coefficients of the energy storage charge and discharge (Liu et al., 2021a; Liu et al., 2022a). Assuming that solar panel follows a linear production cost c^w, the price of electricity in period *t* is P_t (dollars per unit energy). Solar power generation is denoted by W_t in period t. The power demand/load of the equipment for prosumer in period *t* is defined as L_t. *Due to the intermittent and high levels of uncertainty regarding renewable sources like solar generation and dynamic demand/load of the prosumer in each period,* there are two possible scenarios for the prosumer.

1) The power generation of the solar cannot satisfy her demand/load (i.e., $W_t < L_t$);

2) The power generation of prosumer can meet her demand/load (i.e., $W_t \geq L_t$).

If solar power generation can satisfy the self-demand in time *t*, then the remaining solar generation can be sold to the market or stored in energy storage, depending on the prices in the market. However, if the solar power generation cannot meet the power load at period *t*, there are three methods to fill the power gap between the demand and solar generation: prosumer internal imports (discharging storage), external grid imports (buying electricity), or both.

We first analyze each scenario separately below to find the optimal storage scheduling strategy across scenarios 1) and 2), then we will combine them and get the optimal global solution. Thus, Section 5.3.2 introduces the prosumer's cost function and scheduling decision under scenario 1, corresponding to the periods $t \in \Gamma^-$ (i.e., $W_t < L_t$); and the prosumer's cost function and its optimal dispatch were shown in Section 5.3.3 for scenario 2 during the periods $t \in \Gamma^+$ (i.e., $W_t \geq L_t$). Finally, we will merge them to derive the optimal global solution for $\Gamma = \{1, 2, \cdots, T\} = \Gamma^- \cup \Gamma^+$.

5.3.2. DERS Generation Cannot Satisfy the Self-demand. This Section concerns prosumers' optimal energy scheduling analysis when prosumers' solar power generations are lower than their own electricity demand in some decision periods (i.e., $t \in \Gamma^-$).

(1) Profit function and optimization analysis.

Based on the assumption that when $t \in \Gamma^-$, there are two methods to satisfy the power gap between the production and self-demand: 1) prosumer internal imports: buying electricity from the market; 2) storage external imports: releasing energy from storage to satisfy the gap between the DER generation and self-consumption demand. Therefore, the cost function in *scenario 1* is:

$$C^{-}(q_t, W_t, L_t, P_t) = \begin{cases} P_t\left[q_t/\alpha - \left(W_t - L_t\right)\right]/\rho + C^{ch}\cdot q_t + C^{w}\cdot W_t & \left(q_t \geq 0\right) \\ P_t\left[q_t\beta - \left(W_t - L_t\right)\right]/\rho - C^{dis}\cdot q_t + C^{w}\cdot W_t & \left(-\left(L_t - W_t\right)/\beta \leq q_t < 0\right) \\ P_t\left[q_t\beta - \left(W_t - L_t\right)\right]\rho - C^{dis}\cdot q_t + C^{w}\cdot W_t & \left(q_t \leq -\left(L_t - W_t\right)/\beta\right) \end{cases} \quad (5.2)$$

Prosumer's cost function includes electricity trading cost, battery charge and discharge operating cost and solar power generation cost. The first line in Eq. (5.2) represents the costs when the prosumer buys electricity from the market to meet the gap and store extra energy into the storage. Here, $[q_t/\alpha - (W_t - L_t)]/\rho$ in the first line represents the amount of electricity purchased from the market considering the transmission loss. The second line in Eq. (5.2) represents the net cost when the gap is met by discharging storage and purchasing electricity from the power market, and $[q_t\beta - (W_t - L_t)]\rho$ means the amount of electricity purchased from the market to make up for the shortfall. The last line represents the cost when electricity in the storage is released to fill the gap and be sold to the market, and $[q_t\beta - (W_t - L_t)]\rho$ is equivalent to the portion of the energy storage discharged that is sold to the market.

The prosumer aims to obtain the optimal dispatch decision to minimize her total electricity bills at the beginning (i.e., at the stage 1) during the finite period (i.e., optimization horizon) $t \in \{1, 2, 3, \cdots, T\}$, which is model as Eq. (5.3).

$$\min_{\pi}\sum_{t=1}^{T}\mathrm{E}\left[C(q_t,W_t,L_t,P_t)|\ S(1)\right]$$

$$\text{s.t.}\begin{cases}-\bar{Q}^{dis}\le q_t\le\bar{Q}^{ch}\\ -q_t\le E_t-\underline{E}\\ q_t\le\bar{E}-E_t\\ E_{t+1}=\eta_t(E_t+q_t)\end{cases}\quad\forall t\in\{1,2,3,\cdots,T\}\tag{5.3}$$

Where, E is the expectation concerning (E_t,W_t,L_t,P_t), C represents the cost function for prosumer. Here, E_1, W_1, L_1, and P_1 are the given initial energy level in storage, the forecasted solar production, the load/demand, and the forested electricity price, respectively. The above constraints represent storage capacity constraints and energy balances/state transition. Similarly, the reward function of the prosumer at each period $R^-(q_t,W_t,L_t,P_t)$ under scenario 1 can be given as the Eq. (5.4), which is the electricity trading profit minus battery operating cost and solar power generation cost:

$$R^-(q_t,W_t,L_t,P_t)=\begin{cases}-P_t\left[q_t/\alpha-(W_t-L_t)\right]/\rho-C^{ch}\cdot q_t-C^w\cdot W_t & (q_t\ge 0)\\ -P_t\left[q_t\beta-(W_t-L_t)\right]/\rho+C^{dis}\cdot q_t-C^w\cdot W_t & (-(L_t-W_t)/\beta\le q_t<0)\\ -P_t\left[q_t\beta-(W_t-L_t)\right]\rho+C^{dis}\cdot q_t-C^w\cdot W_t & (q_t\le-(L_t-W_t)/\beta)\end{cases}\tag{5.4}$$

Combing the Eqs. (5.2) and (5.4) forms $-C^-(q_t,W_t,L_t,P_t)=R^-(q_t,W_t,L_t,P_t)$, to arrive at the following relations:

$$\min_{\pi}\sum_{t=1}^{T}\mathrm{E}\left[C(q_t,W_t,L_t,P_t)|\ S(1)\right]\Leftrightarrow\max_{\pi}\sum_{t=1}^{T}\mathrm{E}\left[R^-(q_t,W_t,L_t,P_t)|\ S(1)\right]\tag{5.5}$$

Following the previous study (Zhou et al.,2019; Liu et al., 2022), to establish the Bellman equation, the prosumer's objective function will be built based on profit-maximizing (i.e., the cost-minimizing). This work modeled the prosumer’s optimal scheduling decisions as a Markov decision process during finite horizon $t\in\{1,2,3,\cdots,T\}$.

At the beginning of period t, prosumer knew the storage level E_t, forecasted available solar generation W_t, predicted electricity price P_t, and self-demand/load L_t in advance. Define the state at period *t* as $S(t) = S_t(E_t, W_t, P_t, L_t)$. Based on these states, the prosumer determined the optimal economic scheduling decision q_t, as a decision variable representing the SOC change before considering efficiency loss.

Let $V_t(S(t))$ represents the value function under the state $S(t) = S_t(E_t, W_t, L_t, P_t) \in \hat{E} \times \hat{W} \times \hat{L} \times \hat{P}$ in period *t*, which satisfies the Bellman equation:

$$V(S(t)) = \max_{Action(E_t)} [R^{-}(q_t, W_t, L_t, P_t) + E\left(V_{t+1}(S(t+1) \mid S(t)\right). \tag{5.6}$$

The value function is represented as three sub-optimization problems as shown in Eq. (5.7) when solar generation cannot meet the power load (i.e., *scenario 1)*:

$$\begin{cases} V_t^{(1)*-}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left\{ -P_t \left(\frac{E_{t+1}}{\eta_t} - E_t \right) \Big/ \alpha\rho + P_t \frac{(W_t - L_t)}{\rho} - C^{ch} \cdot \left(\frac{E_{t+1}}{\eta_t} - E_t \right) - C^{w} \cdot W_t + E[V_{t+1}^{*}(S(t+1) \mid S(t))] \right\} \\ V_t^{(2)*-}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left\{ -P_t \left(\frac{E_{t+1}}{\eta_t} - E_t \right) \beta \Big/ \rho + P_t \frac{(W_t - L_t)}{\rho} + C^{dis} \cdot \left(\frac{E_{t+1}}{\eta_t} - E_t \right) - C^{w} \cdot W_t + E[V_{t+1}^{*}(S(t+1) \mid S(t))] \right\} \\ V_t^{(3)*-}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left\{ -P_t \left(\frac{E_{t+1}}{\eta_t} - E_t \right) \beta\rho + P_t (W_t - L_t) \rho + C^{dis} \cdot \left(\frac{E_{t+1}}{\eta_t} - E_t \right) - C^{w} \cdot W_t + E[V_{t+1}^{*}(S(t+1) \mid S(t))] \right\} \end{cases} \tag{5.7}$$

By using the Bellman equation, three optimal energy inventory reference points corresponding to the three value functions in *scenario 1* are shown as Eq. (5.8):

$$\begin{cases} E_{t+1}^{(1)*-} = \arg \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left\{ -\left(P_t / \alpha\rho + C^{ch} \right) E_{t+1} / \eta_t + E[V_{t+1}^{*}(S(t+1) \mid S(t))] \right\} \\ E_{t+1}^{(2)*-} = \arg \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left\{ -\left(P_t \beta / \rho - C^{dis} \right) E_{t+1} / \eta_t + E[V_{t+1}^{*}(S(t+1) \mid S(t))] \right\} \\ E_{t+1}^{(3)*-} = \arg \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left\{ -(P_t \beta\rho - C^{dis}) E_{t+1} / \eta_t + E[V_{t+1}^{*}(S(t+1) \mid S(t))] \right\} \end{cases} \tag{5.8}$$

By investigation and comparison, it was found that the relations between the three reference points are related to forecasted electricity prices, efficiencies of energy storage and transmission line, and charging and discharging operating costs of storage. Thus, we can draw the following results.

Lemma 5.1: The relationship of the optimal reference point is as follows:

$$\begin{cases} 1)\ \text{If } P_t \geq 0, \text{there is } E_{t+1}^{(1)*-} \leq E_{t+1}^{(2)*-} \leq E_{t+1}^{(3)*-}; \\ 2)\ \text{If } -\left(C^{dis}+C^{ch}\right)/\left(1/\alpha\rho-\beta\rho\right) \leq P_t \leq 0, \text{there is } E_{t+1}^{(1)*-} \leq E_{t+1}^{(3)*-} \leq E_{t+1}^{(2)*-}; \\ 3)\ \text{If } -\dfrac{\left(C^{dis}+C^{ch}\right)}{\left(1/\alpha\rho-\beta/\rho\right)} \leq P_t \leq -\dfrac{\left(C^{dis}+C^{ch}\right)}{\left(1/\alpha\rho-\beta\rho\right)}, \text{there is } E_{t+1}^{(3)*-} \leq E_{t+1}^{(1)*-} \leq E_{t+1}^{(2)*-}; \\ 4)\ \text{If } P_t \leq -\left(C^{dis}+C^{ch}\right)/\left(1/\alpha\rho-\beta/\rho\right), \text{there is } E_{t+1}^{(3)*-} \leq E_{t+1}^{(2)*-} \leq E_{t+1}^{(1)*-}. \end{cases} \quad (5.9)$$

Similar to Liu et al. (2021a) and Liu et al. (2022a), this study targeted the positive electricity prices, then from Eq. (5.9), the relationships of three reference points are $E_{t+1}^{(1)*-} \leq E_{t+1}^{(2)*-} \leq E_{t+1}^{(3)*-}$, which leads to the following proposition.

(2) Optimal dispatch for energy storage when $t \in \Gamma^-$.

Proposition 5.1: When the forecasted electricity prices $P_t \geq 0$, there is $E_{t+1}^{(1)*-} \leq E_{t+1}^{(2)*-} \leq E_{t+1}^{(3)*-}$. The optimal dispatch for storage at the period $t \in \Gamma^- \in \{1, 2, \cdots, T\}$ in each state $S(t) = S_t(E_t, W_t, L_t, P_t) \in \hat{E} \times \hat{G} \times \hat{L} \times \hat{P}$ can be shown as follows:

Case1: If the demand gap is small: $(L_t - W_t)/\beta < \min\{\overline{E} - E_{t+1}^{(3)*-}, \overline{Q}^{dis}\}$, the optimal action of the prosumer is obtained as follows:

$$q_t^{*-}(S_t)=\begin{cases}\min\{E_{t+1}^{(1)*-}-E_t,\overline{Q}^{ch}\},\ E_t\in[0,E_{t+1}^{(1)*-}],\\ \text{(buying power for consumption and storing, bringing SOC up to } E_{t+1}^{(1)*-});\\ 0, E_t\in(E_{t+1}^{(1)*-},E_{t+1}^{(2)*-}],\\ \text{(purchasing power for consumption without storing it or changing the SOC)};\\ \max\{E_{t+1}^{(2)*-}-E_t,-(L_t-W_t)/\beta,-\overline{Q}^{dis}\},E_t\in(E_{t+1}^{(2)*-},E_{t+1}^{(3)*-}+(L_t-W_t)/\beta],\\ \text{(discharge and purchasing power for consumption, lowering SOC to } E_{t+1}^{(2)*-});\\ \max\{E_{t+1}^{(3)*-}-E_t,-\overline{Q}^{dis}\},E_t\in(E_{t+1}^{(3)*-}+(L_t-W_t)/\beta,\overline{E}],\\ \text{(discharging for consumption and selling, bringing SOC down to } E_{t+1}^{(3)*-}).\end{cases}\tag{5.10}$$

Case 2: If the demand gap is large: $(L_t-W_t)/\beta\geq\min\{\overline{E}-E_{t+1}^{(3)*-},\overline{Q}^{dis}\}$, the optimal action of prosumer is obtained as follows:

$$q_t^{*-}(S_t)=\begin{cases}\min\{E_{t+1}^{(1)*-}-E_t,\overline{Q}^{ch}\},\ E_t\in[0,E_{t+1}^{(1)*-}],\\ \text{(buying power for consumption and storing, bringing SOC up to } E_{t+1}^{(1)*-});\\ 0, E_t\in(E_{t+1}^{(1)*-},E_{t+1}^{(2)*-}],\\ \text{(buying power for consumption without storing it, keeping SOC unchanged)};\\ \max\{E_{t+1}^{(2)*-}-E_t,-(L_t-W_t)/\beta,-\overline{Q}^{dis}\},E_t\in(E_{t+1}^{(2)*-},\overline{E}],\\ \text{(discharging and buying for consumption, bringing SOC down to } E_{t+1}^{(2)*-}).\end{cases}\tag{5.11}$$

The proposition 5.1 shows the prosumer's optimal scheduling dispatch for energy storage based on three SOC reference points $E_{t+1}^{(1)*-}$, $E_{t+1}^{(2)*-}$,and $E_{t+1}^{(3)*-}$. In scenario 1, if the solar generation cannot meet the power load, then the gap between the supply and demand is $L_t>W_t$. There are four different actions: 1) buying power from the market to meet the gap between production and demand as well as to store; 2) discharging/releasing energy from the storage and buying the power from the market to match the gap; 3) discharging power from storage for satisfying the load gap and sale; 4) remaining idle. The first action's optimal storage level at the end of period t is reference point $E_{t+1}^{(1)*-}$, which means that the decision variable q_t is greater than 0 (i.e., $q_t \geq 0$). Then, the second action corresponds to reference point $E_{t+1}^{(2)*-}$, meaning q_t is between 0 and $-(L_t - W_t)/\beta$ (i.e., $-(L_t - W_t)/\beta < q_t \leq 0$). The third action corresponds to the third reference point $E_{t+1}^{(3)*-}$, and q_t is less than $-(L_t - W_t)/\beta$ (i.e., $q_t \leq -(L_t - W_t)/\beta$). If none of the three reference points can be approached in one period, the prosumer will choose to do nothing (i.e., *action 4*).

The first part (i.e., *Case 1*) shows the optimal dispatch when the load gap is small (i.e., $(L_t - W_t)/\beta < \min\{\bar{E} - E_{t+1}^{(3)*-}, \overline{Q}^{dis}\}$). When energy in the battery/storage is less than $E_{t+1}^{(1)*-}$, the prosumer should buy power to consume and store (*action 1*) then reach the SOC reference point $E_{t+1}^{(1)*-}$. When the energy in the storage/battery falls between $E_{t+1}^{(1)*-}$ and $E_{t+1}^{(2)*-}$, either action takes the SOC away from the optimal reference point, so the prosumer should do nothing. When the current storage level (SOC) is between $E_{t+1}^{(2)*-}$ and $E_{t+1}^{(3)*-} + (L_t - W_t)/\beta$, the prosumer should release energy from the storage and buy power to fulfil the gap between the consumption and production (*action 2*) as well as bring energy inventory in the storage down to $E_{t+1}^{(2)*-}$ as much as possible. When energy in the battery is more than $E_{t+1}^{(3)*-}$, the prosumer should discharge energy from battery, part of which is used to fulfill the gap and part of which is sold to the market(*action 3*) as well as bring SOC down to $E_{t+1}^{(3)*-}$.

The second part of Proposition 5.1 (i.e., *Case 2*) is the optimal scheduling dispatch when the gap is large (i.e., $(L_t - W_t)/\beta \geq \min\{\bar{E} - E_{t+1}^{(3)*-}, \overline{Q}^{dis}\}$). In case 2, the prosumer cannot perform action 3 for a large load gap. This is because when the power in the battery/storage is less than $E_{t+1}^{(1)*-}$, the prosumer should adopt *action 1* and bring SOC of storage up to $E_{t+1}^{(1)*-}$. When the power in the battery is between $E_{t+1}^{(1)*-}$ and $E_{t+1}^{(2)*-}$, the prosumer should do nothing. If power in the battery is more than $E_{t+1}^{(2)*-}$, the prosumer should perform *action 2* and bring SOC of storage down to $E_{t+1}^{(2)*-}$ as close as possible.

On the whole, if the gap between the DERs generation and self-demand is small (i.e., *Case1*), when the energy inventory in storage is low, the prosumer should choose to purchase electricity from the market to meet the demand and keep energy storage idle. If the electricity prices are low, after buying power from the grid to fulfill the gap, the prosumer should also buy electricity to store it into the battery and sell it when the prices are high for energy arbitrage (*actions 1 and 4).* When the energy in storage is in the middle range, the prosumer can meet the gap by electricity discharging the storage or purchasing energy from the power market (*action 2*). When the energy in storage is large, the prosumer should release the power in the battery to match the gap and sell the remaining electricity from the storage to the electricity market (*action3*). If the gap is large (i.e., *Case2*) and the energy in the battery is insufficient, discharging storage cannot meet the gap, she still needs to purchase electricity from the grid to fulfill the gap, then action 3 does not happen.

Special Case: if $\rho = 1$ (i.e., ignoring the transmission loss), the prosumer's cost function (i.e., profit function) corresponding to actions 2 and 3 under *scenario 1* is equivalent. This is because when we ignore the transmission loss, the purchased electricity absolute quantity corresponding to action 2 and the absolute electricity quantity sold to market corresponding to action 3 are the same before considering the transmission loss.

There is $E_{t+1}^{(1)*-} \le E_{t+1}^{(2)*-} = E_{t+1}^{(3)*-}$, which is the special case of Proposition 5.1 and the prosumer's optimal dispatch decision remains unchanged.

5.3.3. DERS Generation Can Satisfy the Self-demand. This Section examines the prosumer's optimal energy scheduling and electricity bill minimization analysis when the prosumer's solar generation can satisfy her power demand (i.e., $t \in \Gamma^+$).

(1) Profit function and optimization analysis.

In stage $t \in \Gamma^+$ (i.e., prosumer's solar power generation can meet her own demand, $W_t \ge L_t$), the following cost functions are obtained (Nascimento and Powell, 2013) in *Scenario 2*, showing as Eq. (5.12):

$$C^+(q_t, W_t, L_t, P_t) = \begin{cases} P_t\left[q_t/\alpha - \left(W_t - L_t\right)\right]/\rho + C^{ch} \cdot q_t + C^w \cdot W_t & \left(q_t \ge \alpha\left(W_t - L_t\right)\right) \\ -P_t\left[-q_t/\alpha + \left(W_t - L_t\right)\right]\rho + C^{ch} q_t + C^w \cdot W_t & \left(0 \le q_t < \alpha\left(W_t - L_t\right)\right) \\ -P_t\left[-q_t\beta + \left(W_t - L_t\right)\right]\rho - C^{dis} \cdot q_t + C^w \cdot W_t & \left(q_t < 0\right) \end{cases} \quad (5.12)$$

The first line of Eq. (5.12) indicates the cost when the prosumer stored the surplus solar power generation after satisfying the power-consuming device demand and also purchased electricity from the market to store into the battery. Additionally, $[q_t/\alpha - (W_t - L_t)]/\rho$ represents the amount of electricity purchased from the market after considering efficiency loss. After meeting the electrical load, the remaining solar generation is $W_t - L_t$. If a partial solar generation is sold to the market and the rest of it is stored in storage, then the cost for the prosumer is shown in the second line of Eq. (5.12), which is the operation cost minus the profit from the electricity sold to the market, and $[-q_t/\alpha + (W_t - L_t)]\rho$ means the electricity quantity that sold to the market. The last line is the cost when the prosumer releases the energy from the storage and discharges the storage as well as sells all remaining solar generation to the market, here $[-q_t\beta+(W_t-L_t)]\rho$ is the amount of electricity sold to the market.

Similarly, the reward functions of the prosumer can be reported as in Eq. (5.13).

$$R^+(q_t,W_t,L_t,P_t)=-C^+=\begin{cases}-P_t\left[q_t/\alpha-\left(W_t-L_t\right)\right]/\rho-C^{ch}\cdot q_t-C^w\cdot W_t & \left(q_t\geq\alpha(W_t-L_t)\right)\\ -P_t\left[q_t/\alpha-\left(W_t-L_t\right)\right]\rho-C^{ch}q_t-C^w\cdot W_t & \left(0\leq q_t<\alpha(W_t-L_t)\right)\\ -P_t\left[q_t\beta-\left(W_t-L_t\right)\right]\rho+C^{dis}\cdot q_t-C^w\cdot W_t & \left(q_t<0\right)\end{cases} \quad (5.13)$$

We also find that $R^+(q_t,W_t,L_t,P_t)=-C^+(q_t,W_t,L_t,P_t)$ hold. Thus, the model can also be built as a profit-maximizing problem. Similar to Section 5.3.2, the objective function in the *Scenario 2* is:

$$\max_{\pi}\sum_{t=1}^{T}E\left[R^+(q_t,W_t,L_t,P_t)\mid S(1)\right]=\min_{\pi}\sum_{t=1}^{T}E\left[-C^+(q_t,W_t,L_t,P_t)\mid S(1)\right] \quad (5.14)$$

Eq. (5.14) indicates that when the renewable energy generation is larger than the demand of prosumer, then the prosumer can store the rest of solar generation or sell it to the market, which is equivalent to the scenario that the electricity merchant with renewable source plants and energy storage as discussed by Zhou et al. (2019) and Liu et al. (2022d). So then, there is the optimal storage level as SOC reference points as follow.

$$\begin{cases}E_{t+1}^{(1)*+}=\arg\max_{\underline{E}\leq E_{t+1}\leq\overline{E}}\left\{-\left(P_t/\alpha\rho+C^{ch}\right)E_{t+1}/\eta_t+E[V_{t+1}^*\left(S(t+1)\mid S(t)\right)]\right\}\\ E_{t+1}^{(2)*+}=\arg\max_{\underline{E}\leq E_{t+1}\leq\overline{E}}\left\{-\left(P_t\,\rho/\alpha+C^{ch}\right)E_{t+1}/\eta_t+E[V_{t+1}^*\left(S(t+1)\mid S(t)\right)]\right\}\\ E_{t+1}^{(3)*+}=\arg\max_{\underline{E}\leq E_{t+1}\leq\overline{E}}\left\{-(P_t\beta\rho-C^{dis})E_{t+1}/\eta_t+E[V_{t+1}^*\left(S(t+1)\mid S(t)\right)]\right\}\end{cases} \quad (5.15)$$

In Eq. (5.15), three SOC reference points correspond to three different actions. By comparing, for positive prices $P_t\geq 0$, we obtained $E_{t+1}^{(1)*+}\leq E_{t+1}^{(2)*+}\leq E_{t+1}^{(3)*+}$ holds. Considering the demand/load of prosumers, the following proposition will be achieved.

(2) Optimal dispatch for energy storage when $W_t - L_t \geq 0$.

When the power generation can meet the load (i.e., $W_t \geq L_t$) after meeting the load by solar generation, the prosumer's dispatch action is similar to Zhou et al. (2019) and Liu et al. (2022b), shown as Proposition 5.2:

Proposition 5.2: For every stage $t \in \Gamma^{+} \in \{1, 2, 3, \cdots, T\}$, and positive forecasted electricity price $P_t \in P$, the optimal storage level as SOC reference points in equation (5.15) has $E_{t+1}^{(1)*+} \leq E_{t+1}^{(2)*+} \leq E_{t+1}^{(3)*+}$. Thus, an optimal decision for profit-maximizing merchant in each state $S(t) = S_t(E_t, W_t, L_t, P_t) \in \hat{E} \times \hat{G} \times \hat{L} \times \hat{P}$ can be shown as follows:

Case 3: If the remaining generation is small: $\alpha(W_t - L_t) < \min\{E_{t+1}^{(1)*+}, \overline{Q}^{ch}\}$, the optimal action of prosumer is obtained as follows:

$$q_t^{*+}(S_t) = \begin{cases} \min\{E_{t+1}^{(1)*+} - E_t, \overline{Q}^{ch}\},\ E_t \in [0, E_{t+1}^{(1)*+} - \alpha(W_t - L_t)], \\ \text{(storing all remaining DERs and purchasing power, bringing SOC up to } E_{t+1}^{(1)*+}\text{);} \\ \min\{E_{t+1}^{(2)*+} - E_t, \alpha(W_t - L_t), \overline{Q}^{ch}\}, E_t \in (E_{t+1}^{(1)*+} - \alpha(W_t - L_t), E_{t+1}^{(2)*+}], \\ \text{(storing remaining DERs without buying power, and increasing SOC to } E_{t+1}^{(2)*+}\text{);} \\ 0, E_t \in (E_{t+1}^{(2)*+}, E_{t+1}^{(3)*+}] \text{ (keeping SOC unchanged);} \\ \max\{E_{t+1}^{(3)*+} - E_t, -\overline{Q}^{dis}\}, E_t \in (E_{t+1}^{(3)*+}, \overline{E}], \\ \text{(selling all remaining DERs and relasing energy, lowering SOC to } E_{t+1}^{(3)*+}\text{).} \end{cases} \quad (5.16)$$

Case 4: If the remaining generation is large: $\alpha(W_t - L_t) \geq \min\{E_{t+1}^{(1)*+}, \overline{Q}^{ch}\}$, the optimal action is shown below:

$$q_t^{*+}(S_t)=\begin{cases}\min\{E_{t+1}^{(2)*+}-E_t,\alpha(W_t-L_t),\overline{Q}^{ch}\},E_t\in[0,E_{t+1}^{(2)*+}],\\ \text{(storing remaining DERs without buying, increasing SOC to } E_{t+1}^{(2)*+});\\ 0,E_t\in[E_{t+1}^{(2)*+},E_{t+1}^{(3)*+}]\text{(keeping SOC unchanged)};\\ \max\{E_{t+1}^{(3)*+}-E_t,-\overline{Q}^{dis}\},E_t\in(E_{t+1}^{(3)*+},\overline{E}],\\ \text{(Selling remaining DERs and discharing energy, lowering SOC to } E_{t+1}^{(3)*+}).\end{cases} \tag{5.17}$$

Special Case: if $\rho=1$ (i.e., ignoring the transmission loss), the prosumer's profit function (cost function) expressions corresponding to actions 1 and 2 under *scenario 2* are the same. Then, there is $E_{t+1}^{(1)*+}=E_{t+1}^{(2)*+}\le E_{t+1}^{(3)*+}$, which is the special case of Proposition 5.2.

5.3.4. Optimal Dispatch for Prosumer with Energy Storage. The optimal scheduling in the two possible scenarios (which correspond to different relationships between the solar generation and self-consumption demand of prosumer), implied the optimal global solution across those scenarios. The prosumer's scheduling strategies were analyzed under two scenarios, respectively.

Propostion 5.1 and Propostion 5.2 show the structure of the optimal dispatch decisions under scenario two. The storage's feasible SOC range can be segmented into several sub-ranges by SOC reference points, and prosumers' dispatch decisions can be selected based on the sub-ranges within which the current SOC falls.

For a given finite optimization horizon $\Gamma=\{1,2,...,T\}$, only scenarios 1 and 2 (solar power generation can satisfy the prosumers' electricity demand or not) exist, that is $\Gamma=\Gamma^-\cup\Gamma^+$. Recall the described optimal scheduling in propositions 5.1 and 5.2, this allows us to get the optimal dispatch of the energy storage is:

Proposition 5.3: For any $t \in \Gamma = \Gamma^- \cup \Gamma^+ = \{1, 2, ..., T\}$, the optimal energy storage scheduling of prosumers for each scenario are summarized below:

$$\begin{cases} 1) \text{ If } t \in \Gamma^-, \text{ there is } q_t^*(S_t) = \{q_t^{*-}(S_t) | S_t\} \\ 2) \text{ If } t \in \Gamma^+, \text{ there is } q_t^*(S_t) = \{q_t^{*+}(S_t) | S_t\} \end{cases} \quad \forall t \in T = \{1, 2, ..., T\} \tag{5.18}$$

This expression in Proposition 5.3 indicates the optimal global results from the perspective of electricity bill minimization or profit maximization. Proposition 5.3 describes the optimal energy scheduling policy structure of the storage and the prosumers' optimal expected minimum electricity bills.

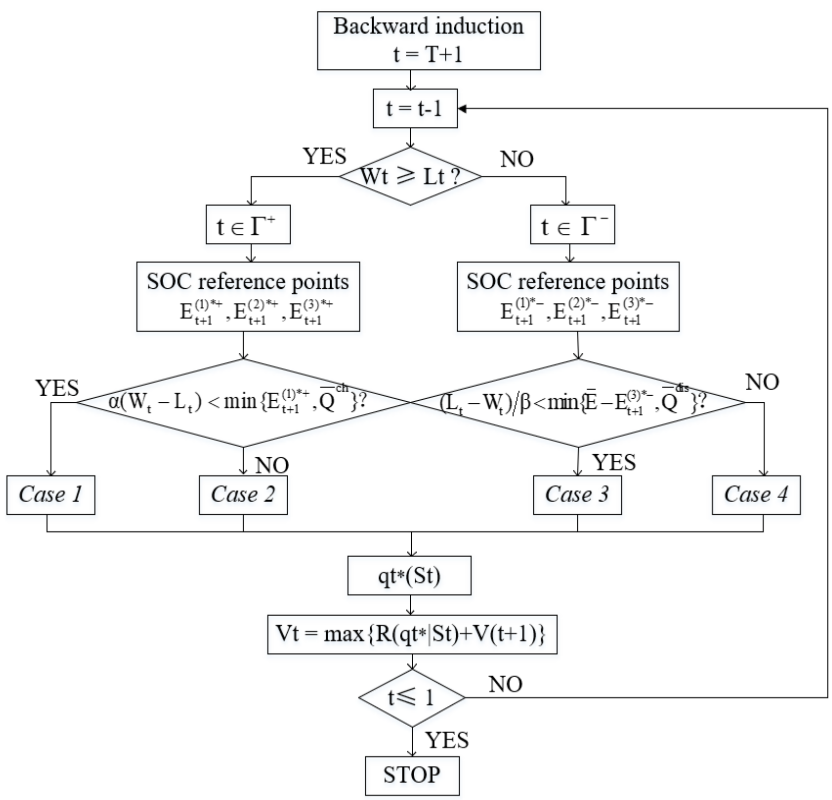


Figure 5.3 The flowchart of obtaining the optimal scheduling decisions for prosumer

Figure 5.3 shows the process of obtaining the optimal scheduling decisions for prosumer in the optimization period $t \in \Gamma = \Gamma^{-} \cup \Gamma^{+} = \{1, 2, ..., T\}$ according to the results proposed above using dynamic programming. First, according to whether the DERs generation can meeting the self-demand or not, we choose the corresponding model and obtain the SOC reference points. Then, by analyzing the relationship of reference point and the demand gap/ the remaining generation, different formula of optimal dispatch decision is given. Last, by the Bellman equation, we perform stepwise iterative solution to obtain the solution of the optimal scheduling problem in the multi-stage optimization period. The result of optimal dispatch in Section 5.3 yields the following insight, as shown below.

Managerial Insight: *Due to the uncertainty of DERs generation and the dynamic demands, an electricity bill-minimization prosumer with storage should decide in advance whether the DERs generation can fulfill her power demand and choose different methods. Our results indicate that the analytical solution based on State-of-Charge (SOC) references points depends on the current energy in storage, the forecast price, available production of solar power, and power demand, which significantly facilitates prosumer decision-making. The feasible state of charge (SOC) range of the storage will be split into different sub-ranges by SOC optimal reference points; the prosumer can conveniently achieve the corresponding optimal decision for each region by comparing the current SOC in the energy storage with the reference points in the next period.*

5.4. CASE STUDY AND NUMERICAL SIMULATION

This Section explains the three-period optimal scheduling of the prosumer using a synthesis data case, which contains two scenarios: generation can satisfy the demand or not. The results will be shown in 5.4.1. Further, Section 5.4.2 uses real electricity price data (from MISO), solar generation, and load from Missouri University of Science and Technology to show relevant results and insights.

5.4.1. Case Study. In this Section, to illustrate the detail of the proposed approaches, a three-period case was shown. For each period, the electricity price was set as $P_t = \{5, 3, 10\}$. Moreover, the residual value of energy in battery at the end of the decision-making cycle was defined as the mean value of the three-period electricity price (i.e., $VOE = 6$).

This study assumed the capacity of storage is ten (i.e., $\underline{E} = 0, \overline{E} = 10$), maximum capacity of a single cycle storage discharge/charge is twelve and seven. It means that the full storage could be emptied in one period, and empty storage energy need more than one period but less than two periods to be filled, which holding that $\underline{E} + \overline{Q}^{ch} \le \overline{E}$ (resp. $\overline{E} - \underline{E} > \overline{Q}^{dis}$) and $\underline{E} + 2\overline{Q}^{ch} \ge \overline{E}$. It's assumed that the operating cost of storage be one (i.e., $C^{ch} = C^{dis} = 1$), and the solar production cost is zero ($C^{W} = 0$) since it does not affect the optimal strategies. Assumed the storage charging, discharging efficiency and transmission efficiency are 0.9 (i.e., $\alpha = \beta = \rho = 0.9$), and do not consider self-discharging loss of storage (i.e., $\eta = 1$), the solar generations are $w_t = \{6, 5, 0\}$, and the local demand is $L_t = \{4, 9, 6\}$.

This case uses the model and approach shown in Section 5.3, and the optimal dispatch result proposed in Proposition 5.3 (optimization from the perspective of prosumer) to get the optimal decisions and SOC reference points with two different initial SOC (i.e., $E_1 = \{1, 5\}$), as shown in Table 5.1 (See Appendix C).

Table 5.1 Optimal results under two perspectives

	$E_1=1$	$E_1=5$	$E_1=1$	$E_1=5$
	Considering self-consumption demand		Ignoring self-consumption demand	
$(E_4^{(1)*},E_4^{(2)*},E_4^{(3)*})$	(0,0,0)		(0,0,0)	
$(E_3^{(1)*},E_3^{(2)*},E_3^{(3)*})$	(10,10,10)		(10,10,10)	
$(E_3^{(1)*},E_3^{(2)*},E_3^{(3)*})$	(0,3,10)		(0,3,10)	
q_3^*	-9.8	-10	-10	-10
q_2^*	7	5	7	5
q_1^*	1.8	0	2	0
V_1^*	-32.479	-10.819	69.741	90.148
Optimal Profit	-32.479	-10.819	-49.139	-28.732
Minimum Bill	32.479	10.819	49.139	28.732

To show the difference between the method proposed in this paper and the traditional study where the electricity merchant with storage and renewable source plants but ignoring the self-demand when making optimization and buys the electricity from the market directly to match her demand (optimization from the perspective of merchant), the optimal results are presented in Table 5.1. Table 5.1 indicates the results under two scenarios: 1) from the perspective of the prosumers who has DERs and energy storage (considering the self-demand during the optimization); and 2) from the perspective of the merchant who has renewable energy resource and energy storage (dispatch the energy storage based on the co-optimization scheduling policy proposed in Section 2; buy the electricity from the market directly to satisfy her power demand).

This study also used the traditional MILP model (Chazarra et al., 2018; Wang et al., 2021) to achieve optimal results, for comparison purpose. The same optimal results for this case were obtained by DP and MILP methods, shown in Table 5.1, which are verified through the implementations in MATLAB. The scheduling decisions, cost functions, SOC reference points, and their relationships are all impacted by the power demand, which significantly influences the analytically optimal scheduling policy structures. Hence, we employ the dynamic programming to obtain the analytical results regarding the boundary conditions under which the merchants choose the different optimal scheduling policies.

We also confirm that the best SOC reference points (i.e., equations (6.8) and (6.9)) for the subsequent period (i.e., period t+1) only relied on the current state and the historical data. As a result, the prosumer only needs to recompute the reference points to make the decision in the first period when the initial SOC in the storage changes. Thus, employing the dynamic programming, the prosumer does not need to completely rerun the model since the initial SOC just modifies the reference points used to make decisions in the first period and has no effect on the economic dispatch policy. However, using the traditional MILP approach, the decision-makers should conduct the entire model again and achieve the best schedule for different initial SOC levels.

5.4.2. MISO Case Study. In this Section, one-month real data from 1 to 31 May 2021 in MISO was used (i.e., T=744), concluding hourly electricity prices series $\mathrm{LMP} = \{\mathrm{LMP}_1, \mathrm{LMP}_2, \ldots, \mathrm{LMP}_T\}$ ($/MW) in the day-ahead market, solar power generation and clear load data with 744 stages (prices information are available at https://www.misoenergy.org/).

The minimum and maximum battery capacities $\underline{E}$ and $\overline{E}$ is denoted by 0 and 300, respectively. The maximum charging and discharging capacities are $\overline{Q}^{ch} = 30$ and $\overline{Q}^{dis} = 50$. Storage quantities used the units of KW hours, and both charging and discharging rates were measure by units of KW. In this case, the charging and discharging efficiencies are $\alpha = \beta = 0.95$, and transmission efficiency is $\rho = 0.95$. Assume it took

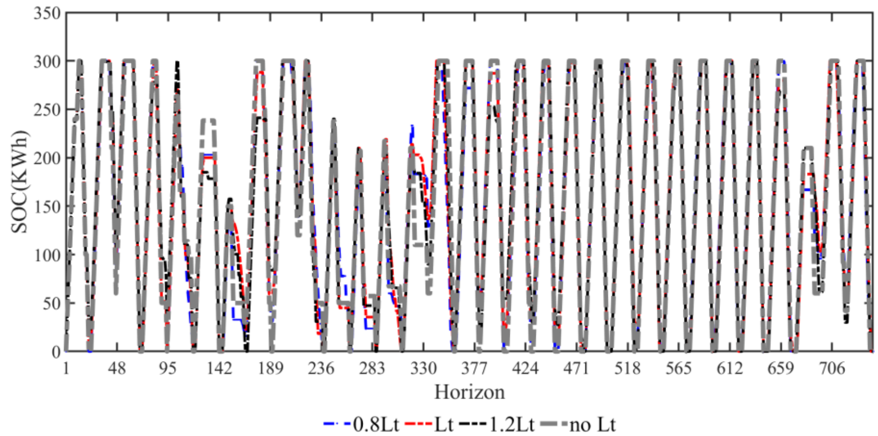
$(\overline{E}-\underline{E})/\overline{Q}^{dis}=6$ hours for energy storage to empty the battery and that refilling the battery needed $(\overline{E}-\underline{E})/\overline{Q}^{ch}=10$ hours. These values above corresponded to the approximate durations shown in Missouri SandT's Eco-Village (see https://cree.mst.edu/laboratories/ecovillage/ for details). And it was assumed that the charging and discharging operating costs were 1 ($C^{ch}=C^{dis}=1$ (\$/ MW)), and the self-discharging were ignored ($\eta=1$).

The solar generation cost was also ignored as it did not affect the optimal strategies ($C^{w}=0$). In this Section, the VOE at the terminal period was not considered (i.e., $VOE_{T+1}=0$). Regularly, one month is an optimization cycle for the community to minimize the monthly electricity bill in the power market.

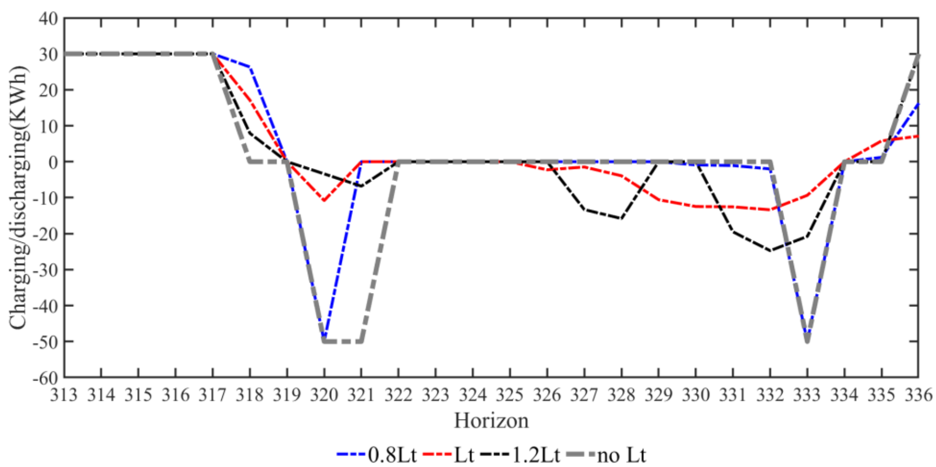
In practice, considering the demand response of self-consumption of the prosumers who have energy storage and distributed energy resources, the original load values were decreased (resp., increased) by 10%, 20% are investigated, respectively. Compared with the existing literature, this is the first work to consider the joint scheduling strategy of prosumers with energy storage.

The plots in Figures. 5.4 and 5.5 show the optimal SOC and scheduling actions of the battery from the perspective of the prosumers under three different demand/load rates when the initial energy is zero in the energy storage. In these figures, No-Lt (grey line) indicates that ignoring the self-consumption demand impacts the energy storage scheduling. Under which the prosumer 1) dispatches the energy storage based on the co-optimization scheduling policy proposed in Section 2; 2) buys the electricity from the market directly to satisfy her own power demand.

To clearly demonstrate the difference between various load levels, we select one-day findings from Figure 5.4 (a) and draw it in Figure 5.4 (b).

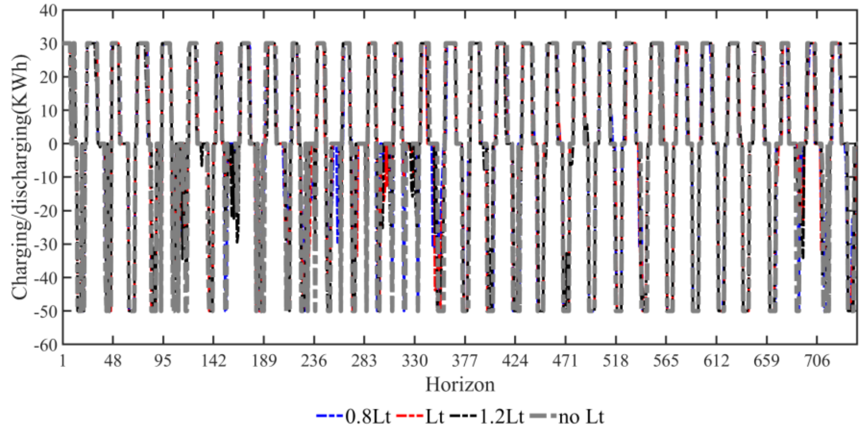


(a) The optimal SOC under different load levels in one month

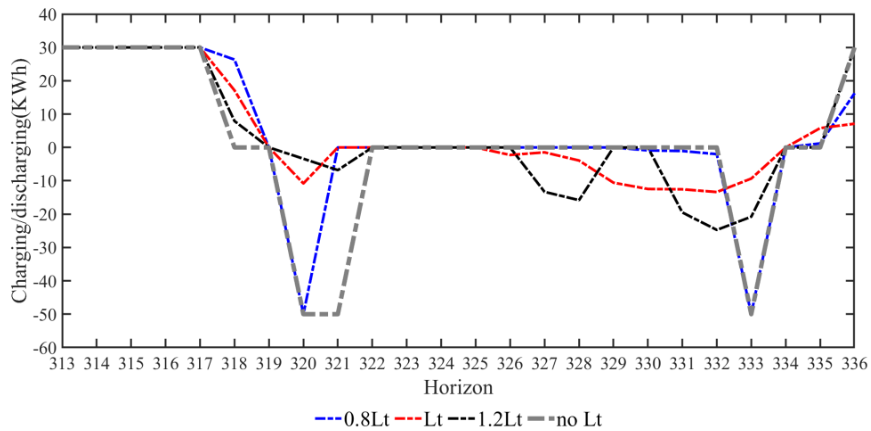


(b) The optimal SOC under different load levels in one day

Figure 5.4 The optimal SOC under different load levels



(a) The optimal actions under different load levels in one month

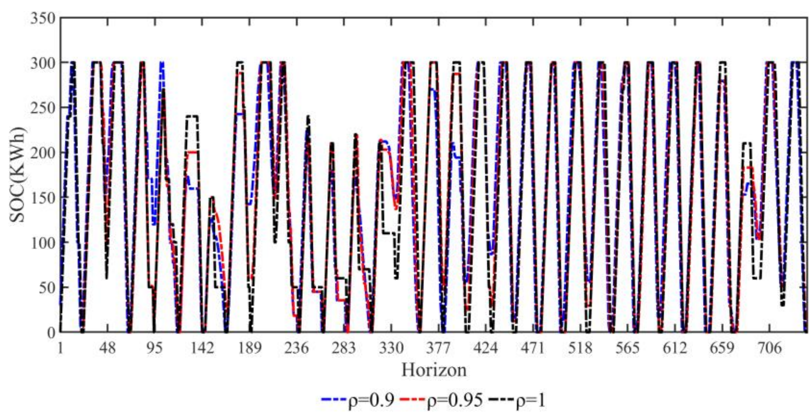


(b) The optimal actions under different load levels in one day

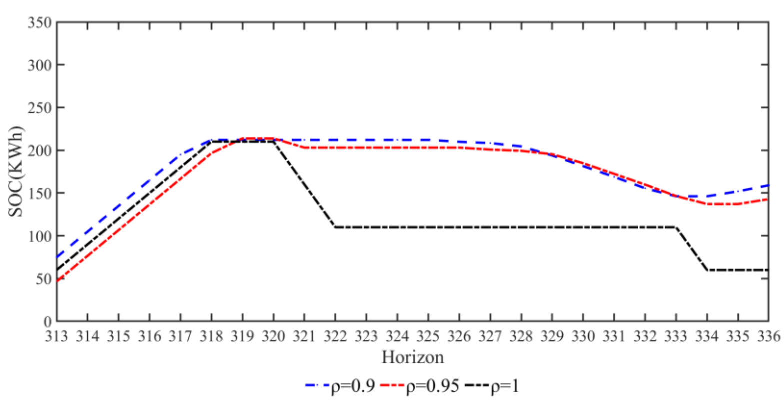
Figure 5.5 The optimal actions under different load levels

Compared with performing scheduling optimization from the traditional merchant ignoring self-demand, considering the self-demand can bring more profit to prosumer and decrease the frequency of idle for energy storage. This attributes to the prosumer can meet her load by discharging power in storage and DER generation rather than buying power for consumption and resulting in transmission loss.

According to Propositions 5.1 and 5.2 in Section 5.3, transmission efficiency played an essential role in decision-making process. Next, there are three different situations: $\rho = \{0.9;\ 0.95;\ 1.0\}$. The prosumer's optimal SOC and dispatch actions curves when considering and ignoring the transmission loss are shown in Figures 5.6 and 5.7 based on the original load demand values. We use one-day results from Figure 5.6 (a) and portray it in Figure 5.6 (b) to highlight the difference between different transmission efficiencies.

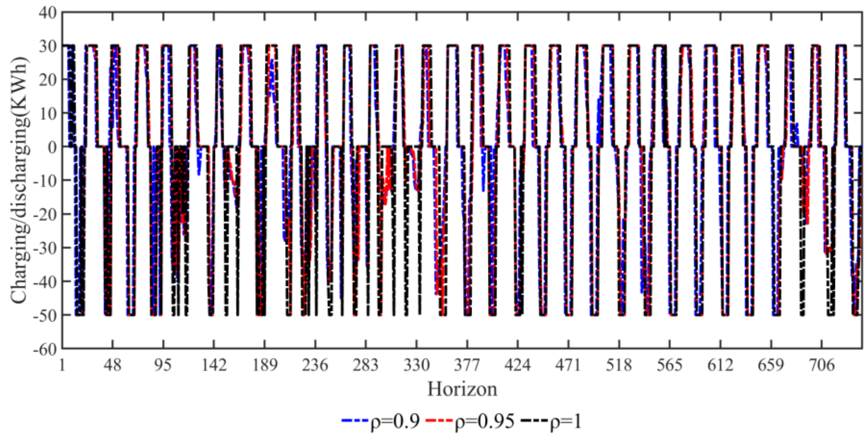


(a) The optimal SOC under different transmission efficiencies in one month

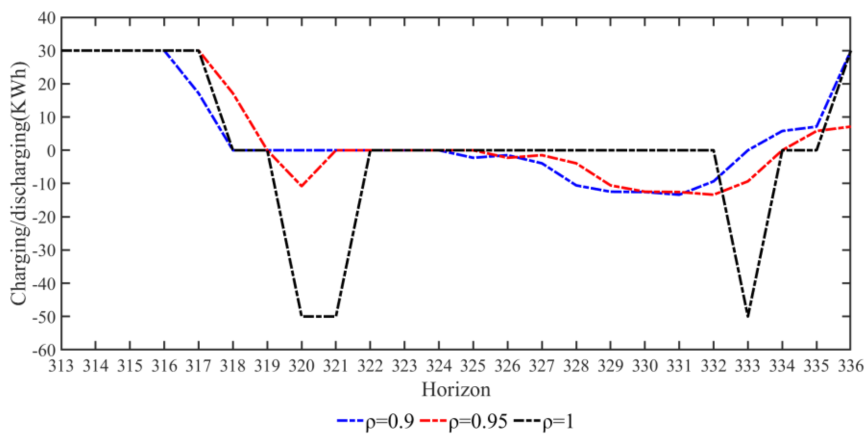


(b) The optimal SOC under different transmission efficiencies in one day

Figure 5.6 The optimal SOC under different transmission efficiencies



(a) The optimal actions under different transmission efficiencies in one month



(b) The optimal actions under different transmission efficiencies in one day

Figure 5.7 The optimal actions under transmission efficiencies

Considering the transmission loss, when the DERs generation is less than load, compared with buying electricity from the market to satisfy the self-consumption, the prosumer is more willing to consume the DERs generation and energy in the battery. On the other hand, when the DERs generation is larger than self-consumption load, the prosumer tends to store the solar power generation and to release it in the future.

Similar to Section 5.4.1, we consider two scenarios from the perspective of prosumer and merchant who do not consider self-demand and show the optimal profit under different transmission losses in Table 5.2.

Table 5.2 Optimal profit for prosumer and traditional merchant

Transmission loss	ρ=0.93	ρ=0.95	ρ=0.97	ρ=0.99	ρ=1
Prosumer considering the self-demand					
Profit	70652.43	79539.34	88872.48	98800.36	104004.97
Traditional merchant ignoring self-demand but buying power for consumption					
Wind-ES profit	1151070.88	1176252.74	1201436.28	1226634.98	1239228.43
Consumption cost	1220680.36	1194981.83	1170343.03	1146699.74	1135232.74
Profit	-69609.48	-18729.09	31093.25	79935.25	103995.69

In Table 5.2, 'Wind-ES profit' represents the profit that the merchant obtains based on the co-optimization scheduling of renewable energy resources and energy storage. The 'Consumption cost' is the cost merchant purchased from the market to meet the self-demand. Then, the profit from the perspective of traditional merchants ignoring self-demand but buying power for consumption is the 'Wind-ES profit' minus the 'Consumption cost,' which is shown in Table 5.2. Results show that, compared with ignoring self-demand during scheduling optimization, considering self-demand makes more profit or less electricity bill because of the transmission loss. Otherwise, with the increase in transmission loss, the cost/bill will increase, and it has a greater impact on merchants under scenario two. This is because merchants ignoring the self-demand will need to buy power from the market for consumption more frequently, producing transmission loss and increasing their bills.

The simulation results show that, for a prosumer who owns DERs and energy storage, taking into account her self-demand during the optimization (fulfilling her self-demand by DER generation, discharging storage, or power purchased from the market) lowers her electricity bill compared to the existing method (dispatching her storage based on the co-optimization scheduling optimization of electricity merchant and buying power directly to satisfy her self-demand). Therefore, when considering the transmission loss, the prosumer should consider her self-demand during the optimization.

5.5. SECTION SUMMARY AND ANALYSIS

This section analyzed the effects of self-consumption demand on the joint economic dispatch of prosumers (energy consumers who are also producers), particularly for prosumers with both energy storage and distributed energy sources (DERs). Studies in the existing literature on the economic dispatch scheduling policy of energy storage, mostly from the perspective of electricity merchants, do not address the impacts of self-consumption demand. We model the optimal scheduling for prosumers with energy storage based on dynamic programming, which achieves the optimal solution via Bellman equations. Furthermore, the optimal dispatch decision structure of this study theorized the classic results known as optimal in the literature, and it did not consider the self-consumption demand. The results led to an optimal strategic structure to support multistage decision-making of prosumers with energy storage.

Because of the uncertainty of distributed energy resource generation and the dynamic demands, it is critical to determine in advance whether the prosumers' DERs generation can meet their own power demand and select different methods accordingly.

Therefore, to obtain the optimal scheduling strategy of the prosumer, different decisions under two scenarios were analyzed and the corresponding three SOC reference points were obtained at each decision stage, which were related to (a) forecasted electricity prices, (b) available DER generation, (c) energy in storage, and (d) electricity self-demand.

Our findings indicate that the analytical solution based on SOC significantly facilitates merchant decision-making. As the feasible SOC range of the storage is split into different sub-regions, the prosumer should make the corresponding scheduling decision to bring the current SOC in storage reach to the corresponding SOC reference point as close as possible considering the storage physical constraints in each sub-region. Compared with traditional profit-maximize electricity merchants with grid-connected storage and renewable energy power plants, but without considering the self-demand. When the DERs generation of the prosumer is larger than her own power demand, the prosumer should adopt similar strategies as the traditional study. However, if the DERs generation is lower than the power demand, the prosumer who prioritizes minimizing her electricity cost will discharge her energy storage or buy electricity from the market to meet her power needs. Finally, our results also show that considering the transmission loss of lines, the prosumer is willing to utilize the power in storage.

6. IMPLICATION OF PTC ON ECONOMICS DISPATCH FOR ELECTRICITY MERCHANTS WITH STORAGE AND WIND FARMS

6.1. OVERVIEW AND RESEARCH QUESTIONS

The production tax credit (PTC) promotes wind energy development, reduces wind power generation costs, and affects merchants' joint economic dispatch, particularly for electricity merchants with both energy storage and wind farms. In this work (Liu et al., 2022a), we focus on the optimal scheduling of a merchant that has both storage (e.g., PSH or battery) and a renewable power plant (wind farms will be used henceforth to mean power plants). In this configuration, the electricity merchant uses energy storage to manage power transactions with the wholesale power market while considering the physical constraints of storage (e.g., the capacity of the energy storage, charging and discharging limitations/efficiencies), and the wind farm receives PTC (production tax credit) by selling the wind-generated electricity to the market in the model. Operational costs and energy value at the end of the optimization horizon were included when calculating the electricity trade on the market.

To capture a wide range of application scenarios, our analyses are carried out in two distinct settings. In the first, a wind farm is receiving PTC by selling the wind generation to the market and has a battery that will be able to buy electricity from the grid to store, but the stored wind generation cannot receive PTC for time-shifting sales (policy one). At this junction, the merchant has four choices: storing wind-generated power and buying power from the market, storing and selling wind-generated power, remaining idle, or discharging storage and selling all wind-generated power to the market. In the second scenario, a wind farm with energy storage that stored wind generation will also qualify for

PTC but storing energy from the grid will disqualify it from receiving PTC. In this scenario, the power stored in the storage must be less than or equal to wind production in each period. Therefore, in this case, an electricity merchant has three choices: storing and selling wind-generated power, remaining idle, or discharging storage and selling all wind-generated power to the market (policy two). We are able to characterize both policies mathematically.

Our analyses aimed to address the following questions: (i) How do joint optimization electricity merchants benefit from considering the PTC? (ii) Under which policy will an individual profit-maximizing merchant achieve more profit? To answer these questions, we first adjust the merchant's traditional reward function by incorporating the PTC subsidy based on the quantity of renewable energy sold to the power market. We then distinguish the optimal decisions of electricity merchant by applying dynamic programming method to maximize his or her profit. We also show how the optimal economic dispatch actions can be modified based on the different PTC credit rates under policies 1 and 2. It should be mentioned that it is nontrivial to achieve analytical results when considering PTC in the problem since it will affect product cost and the traditional structure of reward functions that only consider electricity prices. Such challenges are approached in this work. We believe these are innovative theoretical conclusions that have never been examined before. This was the first work to work on the energy storage scheduling/economics dispatch problem via dynamic programming while considering PTC policies to the best of our knowledge.

The rest of this Section is organized as follows. In Section 6.2, we outline the principal contributions. Section 6.3 modeled a wind farm receiving a PTC or subsidy by selling the wind generation to the market, and it explored battery options to buy electricity from the grid to store. Section 6.4 examines cases in which storing energy from the grid disqualified the wind farm from receiving PTC. We will verify the proposed method based on a synthesis data case and demonstrate the results by a real electricity price data case from MISO in Section 6.5. Finally, the conclusions of this work and research questions for future research are discussed in Section 6.6.

6.2. THE PRINCIPAL CONTRIBUTIONS

This research makes three principal contributions. First, for a wind (or renewable) farm merchant with energy storage, this study analytically showed that the state of charge (SOC) reference points at each decision time depended on the currently available energy, the forecasted price, the PTC credit rates, and the available energy of wind generation. On this basis, the storage SOC was divided into different regions by the SOC reference points corresponding to the four actions: storing renewable generation and buying power from the power market, storing and selling renewable generation, remaining idle, and discharging storage and selling all renewable generation to the market. Then, the optimal merchant's actions could be uniquely determined based on the region within the current SOC falls.

Second, compared to the traditional study in which the PTC was ignored, our results show that the PTC influenced the merchant’s optimal economic dispatch volume by changing the value of the SOC reference point when the PTC was relatively small. However, when the PTC credit rates were relatively large, the traditional optimal storage policy structure was substantially reconstructed by modifying the relationships among the reference points. The analytical results regarding the boundary conditions (i.e., PTC is large or small) under which the merchants choose the different optimal policies were explained. With increased PTC, the frequencies for pumping and generating decreased.

Third, we also investigate whether stored wind generation will qualify the firm to receive PTC but will make it unable to buy electricity from the grid. Although the optimal SOC reference points in this situation are affected, we can derive valuable insights by employing similar analytical procedures. We find that compared to the policy allowing merchants to buy power from the market (i.e., policy 1), which prohibits purchasing electricity from the grid, and all the energy released from the storage qualifies for the PTC (i.e., policy 2) benefits merchants' profit when the PTC subsidy is large according to the current PTC credit rates. We find that the merchant must seek to balance perfectly between policy 1 and policy 2.

6.3. CONSIDERATION OF PTC FOR A MERCHANT WITH BOTH STORAGE AND A WIND FARM UNDER POLICY 1

In this Section, the objective functions for electricity merchants were modeled when the merchant had both energy storage and wind farm power plants (Kim and Powell, 2011; Su et al., 2019; Zhou et al., 2019). A wind farm is receiving PTC by selling wind generation to the market and has energy storage that will be able to buy electricity from the grid to store. The merchant's optimal strategies to maximize the profit and to consider the PTC subsidy based on the forecasted price were studied.

6.3.1. Model Setup. This research focused on the co-optimal scheduling of an owner of storage and wind farm plant (i.e., an electricity merchant) using an inventory policy to manage power in the wholesale market and considering the physical constraints of storage. Considering the production tax credit (PTC), a tax policy and subsidy for renewable electricity production, the merchant adopted a storage strategy and managed electricity, generating electricity by the power plant, and buying and selling electricity on the wholesale electricity market. Using discrete-time parameters, the electricity merchant periodically executed trading decisions during a limited horizon for each period $t \in \{1, 2, 3, \cdots, T\}$.

It was assumed that the storage capacity was finite. The PSH storage had maximum energy capacity $\overline{E}$ and minimum energy level $\underline{E}$, thus $\overline{E} > \underline{E} \geq 0$ .The PSH had generating/discharging and pumping/charging capacity constraints. $\overline{Q}^g$ and $\underline{Q}^g$ are denoted as the upper and lower limits of generating, respectively, that can be sold to the market in each period. $\overline{Q}^P$ and $\underline{Q}^P$ express the pumping maximum and minimum limits that can be bought from the power market in one period. For tractability, the study adopted the conventional assumption (Kim and Powell, 2011; Jiang and Powell, 2015a; Secomandi, 2010; Zhou et al., 2019) that $\underline{Q}^g = \underline{Q}^P = 0$.

Three efficiency types with storage were considered. One type of efficiency is denoted by α and β , describing the efficiency of the pumping/charging and generating/discharging modes, where $\alpha, \beta \in (0,1]$. The other is $\rho \in (0,1]$, which implies the portion of transmission efficiency: the ratio of power flowing out of a transmission line to the power flowing into another. Losses are incurred at the ends of the transmission lines in both directions. A third of them is a portion $\eta_t \in [0,1]$, an efficiency of stored energy dissipates during one period because of the evaporation, leakage, and spill rate of the PSH or self-discharge rate of the battery.

The economic dispatch for each period t is defined by q_t^g or q_t^p to denote the energy change between periods *t* and *t*+1 before considering the efficiency loss. This work uses $\{c_p, c_g\}$ ($/unit energy) denote the operating cost of PSH storage generating and pumping. Following a previous study (Xu et al., 2017), it was assumed that energy storage had a linear operating cost. In this study, c_w indicates the cost for unit wind generation. For simplicity and brevity of exposition, it was assumed that the renewable power plant followed a linear generation cost. The electricity price in period t is P_t (dollars per unit energy), and the sequential levels of the price are denoted by a vector of $P = (P_1, P_2, \cdots, P_T)$. In this work, bidding problems were not considered, and we assumed that all electricity submitted to the market was accepted (Zhou et al., 2016, 2019).

The renewable electricity production tax credit (PTC) is a per-kilowatt-hour (kWh) tax credit for electricity generated using qualified renewable resources. To qualify for this credit, the taxpayer must sell electricity to an unrelated person. In practice, there are two metered technologies for wind farm service (Gautier et al., 2018). An import meter can measure electricity drawn from the grid, and an export meter is used to measure the renewable power supply to the grid to qualify for the PTC subsidy (see https://www.energy.gov/energysaver/grid-connected-renewable-energy-systems for detail). To prevent the merchant from selling all wind-generated electricity to the market to gain high PTC from the government and then buying electricity from the market to store, it was assumed that selling power to the market and purchasing power from the market were not allowed simultaneously.

Based on the assumption above, the reward function $R(q_t^g, q_t^p, w_t, P_t)$ from making decision (q_t^g, q_t^p) corresponding to time *t*, the forecast prices P_t, and the available renewable generation w_t, the PTC subsidy parameter *s* are defined as follows:

$$R^{(s)}(q_t^g, q_t^p, w_t, P_t) = \begin{cases} -P_t \cdot \left(q_t^p/\alpha - w_t\right)/\rho - c_p q_t^p - c_w w_t \;\; (q_t^p > \alpha w_t) \\ P_t \cdot \left(w_t - q_t^p/\alpha\right)\rho - c_p q_t^p + s\left(w_t - q_t^p/\alpha\right) - c_w w_t (0 \le q_t^p \le \alpha w_t) \\ P_t \cdot (q_t^g \beta + w_t) \cdot \rho - c_g q_t^g + s w_t - c_w w_t \;\; (q_t^g \ge 0) \end{cases} \tag{6.1}$$

Here, the parameter *s* in superscript of Eq. (6.1) indicates the situation considering the subsidy under policy 1. A distinguishing feature of this model is that it incorporates the subsidy *s* into the reward function for the power generation sold to the market by a wind turbine. This approach rendered the model both novel and practical, as it made the first contribution to the reward function.

The first line in Eq. (6.1) means the costs when the merchant buys power from the power market, and there is no energy sold to the market in this instance; thus, the merchant does not receive a subsidy. The second line shows the rewards yielded from when the merchant stored part of renewable generation q_t^p to the storage and injected the remaining units $(w_t - q_t^p/\alpha)$ into the transmission line to the market. The additional term $s \cdot (w_t - q_t^p/\alpha)$ represents the PTC from the wind power selling to the market. The third equation, $(q_t^g \beta + w_t)$, shows the reward function when the merchant released the energy from PSH to the market and sold all wind generation. Among all generations sold to the market, additional term $s \cdot w_t$ shows the PTC from the government, which also means that the stored wind generation by time-shifting sales cannot receive the PTC. The terms $c_p q_t^p$,

$c_g q_t^g$, and $c_w w_t$ denote the pumping operating costs paid by the merchant at time *t* for q_t^p units of energy (power), generating operating costs paid by the merchant at time *t* for q_t^g units of energy (power), and the wind generation cost, respectively.

This work did not focus on electricity price forecasting methods. Instead, it was assumed that the merchant observed all power prices in advance, based on the forecasted method, so that controlling the storage was deterministic. The target of the study was the situation when wind farms received PTCs or subsidies by selling wind generation to the market and when they had a battery to be able to buy electricity from the grid to store, as shown in $R^{(s)}(q_t^g, q_t^p, w_t, P_t)$ in Eq. (6.1).

Denote E_t as the SOC level in the PSH storage at the beginning of period *t* and the sequential SOC levels of the PSH as $\hat{E}=\left(E_1, E_2, \cdots, E_T\right)$, where $E_i \in [\underline{E}, \bar{E}], \forall i \in \left\{1, 2, \cdots, T\right\}$. We defined the feasible trading decisions set based on the current SOC level $E_t \in E$ as:

$$\text{Action}(E_t) := \{(q_t^p, q_t^g) \in \mathbb{R} : 0 \le q_t^p \le \bar{Q}^p \cdot U_t^p,\ q_t^p \le \bar{E} - E_t, 0 \le q_t^g \le \bar{Q}^g \cdot U_t^g,\ q_t^g \le E_t - \underline{E}\}. \quad (6.2)$$

Eq. (6.2) represents the maximum quantity of energy that can be pumped and generated. The first two constraints defined the upper pumping boundary because the maximum limit and PSH storage capacity, respectively. The third and fourth constraints showed the upper generating boundary because of the maximum limit and available energy in the PSH storage. Both binary decision variables U_t^p and U_t^g represent the commitment of the unit to pumping and power generation in $[t, t+1)$. Without loss of generality, $U_t^p + U_t^g \leq 1$, where $U_t^p \in \{0,1\}$ and $U_t^g \in \{0,1\}$, means the PSH cannot pump and generate simultaneously, or the battery cannot charge and discharge simultaneously.

At the beginning of period *t*, the electricity merchant knew the SOC of PSH E_t, the wind generation w_t, the forecasted electricity price P_t, and the PTC subsidy s. The action for each period t is defined by q_t^p or q_t^g to represent the water/energy changes between periods *t* and *t*+1 before considering the generating and pumping losses. At the end of period *t*, PSH storage self-discharge occurs, so the SOC at the start of period *t*+1 equals $\eta_t(E_t + q_t^p - q_t^g)$. Thus, the following formulation that outlines the state transition or the energy balance from time *t* to *t*+1 for storage is true:

$$E_{t+1} = \eta_t(E_t + q_t^p - q_t^g)\,. \tag{6.3}$$

6.3.2. Optimization and Analysis. To maximize profit, the merchant periodically produced economic dispatch decisions over a limited horizon in each time $t \in \{1,2,3,\cdots,T\}$. The forecasted electricity price in period t is P_t. Following a previous study (Liu et al., 2021; Zhou *et al.*, 2016; 2019), this work modeled the merchant's economic dispatch strategy as Markov dynamic programming with a finite horizon. The decision variables in each stage t are E_t, w_t, and P_t, and the state at period t is defined by $S(t) = S_t(E_t, w_t, P_t)$. The merchant aimed to find the optimal decision rule π to limited period $t \in \{1,2,3,\cdots,T\}$.

$$\max_{\pi}\sum_{t=1}^{T}E\left[R^{(s)}(q_t^g,q_t^p,w_t,P_t)|\ S(1)\right]$$

$$\text{s.t.}\begin{cases}0\le q_t^g\le \bar{Q}^g\cdot U_t^g\\ q_t^g\le E_t-\underline{E}\\ 0\le q_t^p\le \bar{Q}^p\cdot U_t^p\\ q_t^p\le \bar{E}-E_t \qquad\qquad \forall t\in\{1,2,3,\cdots,T\}\\ U_t^p\in\{0,1\},U_t^g\in\{0,1\}\\ U_t^p+U_t^g\le 1\\ E_{t+1}=\eta_t(E_t+q_t^p-q_t^g)\end{cases} \tag{6.4}$$

This work neglects the discount factor due to the limited optimization horizon, and E is the expectation concerning (E_t,w_t,P_t). Here, E_1, w_1 and P_1 are the given initial storage levels, the available wind generation, and the advance price, respectively. Let $V_t(S(t))$ symbolize the value function under state $S(t)=S_t(E_t,w_t,P_t)\in\hat{E}\times\hat{W}\times\hat{P}$ in period t. This function satisfies the Bellman equation:

$$V(S(t))=\max_{\text{Action}(E_t)}[R^{(s)}(q_t^g,q_t^p,w_t,P_t)+E\left(V_{t+1}(S(t+1)\,|\,S(t))\right)]. \tag{6.5}$$

Prior research in this field typically expressed the VOE at the end optimization horizon or residual value of energy as $V_{T+1}(S(T+1))=0$ (Secomandi, 2010; Zhou et al., 2019). Eq. (6.5), $V_{T+1}(S(T+1))=VOE_{T+1}\cdot E_{T+1}$, indicates the residual value of energy in storage. As a result, the implications for economic dispatch policy design are quite different when PTC is considered in addition to the value of energy in storage at T. Here, E_{T+1} represents the PSH SOC at the beginning of period T+1 and the SOC at the end of period T. Next, the optimal policy decision rule of Eq. (6.5) was established for the slow storage case (Secomandi, 2010).

This research substituted the binary decision variables with an equivalent continuous variable and changed Eq. (6.5) to a traditional Markov Dynamic Programming to obtain the analytical optimal policy rule. Following Porteus (2002), Secomandi (2010), and Zhou et al. (2019), let $E_{t+1} = \eta_t (E_t + q_t^p - q_t^g) = \eta_t (E_t + q_t)$ be the decision variable. The action (decision) q_t was used for each time t to substitute the decision variables q_t^g, and q_t^p representing the SOC changed between t and t+1 before accounting for the energy loss.

Similar to Liu et al. (2021a), Zhou et al. (2016, 2019), $q_t < 0$ represents the SOC decrease due to the action of generating, $q_t > 0$ indicates the SOC increase because of pumping, $q_t = 0$ shows that the SOC did not change, or the electricity merchant did nothing (i.e., idle). Then, there is $q_t = E_{t+1}/\eta_t - E_t$.

The reward function $R^{(s)}(q_t^g, q_t^p, w_t, P_t)$ can be rewritten as $R^{(s)}(q_t, w_t, P_t)$ from decision q_t and is defined as:

$$R^{(s)}(q_t, w_t, P_t) = \begin{cases} -P_t \cdot (q_t/\alpha - w_t)/\rho - c_w w_t - c_p q_t & (q_t > \alpha w_t) \\ -P_t \cdot (q_t/\alpha - w_t) \cdot \rho - s(q_t/\alpha - w_t) - c_w w_t - c_p q_t & (0 \le q_t < \alpha w_t) \\ -P_t \cdot (q_t \beta - w_t) \cdot \rho - c_w w_t + s w_t + c_g q_t & (q_t < 0) \end{cases}. \quad (6.6)$$

If $s = 0$, a traditional reward function is yielded. The PTC subsidy will change the traditional reward functions, which is an innovation of this study. As a merchant who has both energy storage and a wind farm, the objective function is:

$$\max_{\pi} \sum_{t=1}^{T} E\left[R^{(s)}(q_t, w_t, P_t) | S(1) \right]. \quad (6.7)$$

subject to the capacity constraints $\max\{-\bar{Q}^g, \underline{E} - E_t\} \le q_t \le \min\{\bar{Q}^p, \bar{E} - E_t\}$ and the storage energy balance constraints $\eta_t(E_t + q_t) = E_{t+1}$, where $t \in \{1, 2, 3, \cdots, T\}$.

6.3.3. Profit Maximization and Optimal Decision Rule. To obtain the optimal decision rules, the optimization in Eq. (6.7) was first split into three optimizations corresponding to three different actions: *storing renewable generation and buying power from the market, storing and selling renewable generation, and discharging storage and selling all renewable generation to the market.* Then, the optimal solutions for these three optimizations were found:

$$\begin{cases} V_t^{(1\text{-}s)*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left\{ -\left(P_t/\alpha\rho + c_p\right) q_t - w_t(-P_t/\rho + c_w) + E[V_{t+1}^*(S(t+1) \mid S(t)] \right\} \\ V_t^{(2-s)*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left\{ -\left(\left(P_t\rho + s\right)/\alpha + c_p\right) q_t - w_t[-P_t\rho + c_w - s] + E[V_{t+1}^*(S(t+1) \mid S(t)] \right\} \\ V_t^{(3-s)*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left\{ (-\left(P_t\beta\rho - c_g\right) q_t - w_t[-P_t\rho + c_w - s] + E[V_{t+1}^*(S(t+1) \mid S(t)] \right\} \end{cases} . \quad (6.8)$$

The original optimization problem can be subdivided into three subproblems $\max V_t^{(1-s)}(S(t))$, $\max V_t^{(2\text{-}s)}(S(t))$, and $\max V_t^{(3\text{-}s)}(S(t))$ subject to $\max\{-\bar{Q}^g, \underline{E} - E_t\} \le q_t \le \min\{\bar{Q}^p, \bar{E} - E_t\}$, and $\underline{E} \le E_t \le \bar{E}$.

If $V_t(S(t))$ is the value function, it should produce the following conclusions based on the Bellman equation (Puterman, 1994, Zhou et al., 2019). Similarly, maximization Eq. (6.8) can be addressed by removing the given state $S(t)$ (i.e., the given value E_t, w_t, and P_t). The optimal results are:

$$\begin{cases} V_t^{(1\text{-s})*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^*(S(t+1) \,|\, S(t)] - \left(P_t/\alpha\rho + c_p\right) E_{t+1}/\eta_t \right) \\ V_t^{(2\text{-s})*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^*(S(t+1) \,|\, S(t)] - \left(\left(P_t\rho + s\right)/\alpha + c_p\right) E_{t+1}/\eta_t \right) \\ V_t^{(3\text{-s})*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^*(S(t+1) \,|\, S(t) - \left(P_t\beta\rho - c_g\right) E_{t+1}/\eta_t \right) \end{cases} . \tag{6.9}$$

Unlike the previous study (Kim and Powell, 2011; Porteus, 2002; Zhou et al., 2019), we considered the PTC in the reward function, which yielded for every $t \in \{1, 2, 3, \cdots, T\}$, if the price of electricity at time *t*, $P_t < +\infty$ holds, in each stage *t*, the value functions $V_t(S(t))$ and $E[V_{t+1}(S(t+1) \,|\, S(t)]$ are concave in $E_t \in [\underline{E}, \overline{E}]$ for each given state $S(t) = S_t(E_t, w_t, P_t)$. The optimal solutions for these three optimizations were found. Hence, the different SOC solutions, as in Lemma 6.1 (see Appendix D), were obtained.

Lemma 6.1: When considering the PTC subsidy and electricity merchant, let $E_{t+1}^{(1\text{-s})*}$, $E_{t+1}^{(2\text{-s})*}$, and $E_{t+1}^{(3\text{-s})*}$ be the optimal results in Eq. (6.10), respectively:

$$\begin{cases} E_{t+1}^{(1\text{-s})*} = \arg \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^*(S(t+1) \,|\, S(t)] - \left(P_t/\alpha\rho + c_p\right) E_{t+1}/\eta_t \right) \\ E_{t+1}^{(2\text{-s})} = \arg \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^*(S(t+1) \,|\, S(t)] - \left(\left(P_t\rho + s\right)/\alpha + c_p\right) E_{t+1}/\eta_t \right) \\ E_{t+1}^{(3\text{-s})*} = \arg \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^*(S(t+1) \,|\, S(t)] - \left(P_t\beta\rho - c_g\right) E_{t+1}/\eta_t \right) \end{cases} . \tag{6.10}$$

In Eq. (6.10), the first finding for the optimal SOC solution (i.e., SOC reference point) $E_{t+1}^{(1\text{-s})*}$ does not relate to the PTC subsidy because the merchant did not receive the PTC when he or she stored all renewable generation. To be qualified for credit, renewable electricity must be sold by the merchant to the market to an unrelated person.

Second, a wind farm can still receive PTC when the merchant performs the action: *storing and selling renewable generation and discharging storage*. Hence, it was clear that the optimal solution $E_{t+1}^{(2\text{-}s)*}$ is related to the wind generation that was sold to the market and the PTC subsidy that was received.

Third, although the reward function changed when the merchant discharged storage and sold all renewable generation to the market when the PTC was considered, neither the PTC nor the wind generation affected the SOC level. Therefore, the optimal solution $E_{t+1}^{(3\text{-}s)*}$ was obtained and had nothing to do with wind generation.

In Eq. (6.10), the following relations were obtained:

(1) For positive electricity prices $P_t \ge 0$, the following is true:

$$\begin{cases} 1)\text{ If } s \le \left(P_t(1-\rho^2)\right)/\rho, \text{there is } E_{t+1}^{(1\text{-}s)*} \le E_{t+1}^{(2\text{-}s)*} \le E_{t+1}^{(3\text{-}s)*} \\ 2)\text{ If } s \ge \left(P_t(1-\rho^2)\right)/\rho, \text{there is } E_{t+1}^{(2\text{-}s)*} \le E_{t+1}^{(1\text{-}s)*} \le E_{t+1}^{(3\text{-}s)*} \end{cases}. \tag{6.11}$$

(2) For negative electricity prices $P_t < 0$, the study obtained the following:

a) When there is $-(c_g+c_p)/\left(1/\alpha\rho-\beta\rho\right) \le P_t < 0$ holding, for any $s > 0$, we will get the following relationship:

$$E_{t+1}^{(2\text{-}s)*} \le E_{t+1}^{(1\text{-}s)*} \le E_{t+1}^{(3\text{-}s)*}. \tag{6.12}$$

b) When $P_t < -(c_g+c_p)/\left(1/\alpha\rho-\beta\rho\right) < 0$ hold,

$$\begin{cases} 1)\text{ If } 0 < s \le (\alpha\beta-1)\rho P_t - \alpha(c_g+c_p), \text{we will get } E_{t+1}^{(1\text{-}s)*} \ge E_{t+1}^{(2\text{-}s)*} \ge E_{t+1}^{(3\text{-}s)*} \\ 2)\text{ If } s \ge (\alpha\beta-1)\rho P_t - \alpha(c_g+c_p), \text{we will get } E_{t+1}^{(1\text{-}s)*} \ge E_{t+1}^{(3\text{-}s)*} \ge E_{t+1}^{(2\text{-}s)*} \end{cases}. \tag{6.13}$$

Without loss of generality, this work only focused on positive electricity price scenarios. The optimal operation policy was obtained by comparing the current SOC with the SOC reference points. It should be mentioned that when considering PTC and the PTC was large (e.g., $s \ge (P_t(1-\rho^2))/\rho$), the traditional relationship among the optimal SOC reference points was changed. Thus, the following conclusions and insights are proposed that have never been explored before. Based on Eq. (6.11), the corresponding optimal results were delivered in the following proposition (proofs are given in Appendix D):

Proposition 6.1: For every period $t \in \{1,2,3,\cdots,T\}$ and forecasted electricity price $P_t \in P$, unique optimal SOC reference points exist $E_{t+1}^{(1\text{-}s)*}, E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(3\text{-}s)*}$, which depend on the state $S(t)$. When the subsidy per renewable generation sold to the market was small (i.e., $s \le P_t(1-\rho^2)/\rho$), there is $E_{t+1}^{(1\text{-}s)*} \le E_{t+1}^{(2\text{-}s)*} \le E_{t+1}^{(3\text{-}s)*}$, so the optimal action in each state $S(t) = S_t(E_t, w_t, P_t) \in \hat{E} \times \hat{G} \times \hat{P}$ can be specified (see the following).

Case 1) If $\alpha w_t < \min\{E_{t+1}^{(1\text{-}s)*}, \overline{Q}^p\}$ (less wind generation), the SOC can be segmented into four regions: storing renewable generation and buying electricity from the market, storing and selling renewable generation, discharging storage and selling all renewable generation to the market, and doing nothing.

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(1\text{-}s)*} - E_t, \overline{Q}^p\},\ E_t \in [0, E_{t+1}^{(1\text{-}s)*} - \alpha w_t], \\ \text{(store renewable and purchased electricity up to } E_{t+1}^{(1\text{-}s)*}\text{);} \\ \min\{E_{t+1}^{(2\text{-}s)*} - E_t, \alpha w_t\}, E_t \in (E_{t+1}^{(1)*} - \alpha w_t, E_{t+1}^{(2\text{-}s)*}], \\ \text{(store renewable without buying up to } E_{t+1}^{(2\text{-}s)*}\text{);} \\ 0, E_t \in (E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(3\text{-}s)*}] \text{ (keep SOC unchanged);} \\ \max\{E_{t+1}^{(3\text{-}s)*} - E_t, -\overline{Q}^g\}, E_t \in (E_{t+1}^{(3\text{-}s)*}, \overline{E}], \\ \text{(generate and sell renewable down to } E_{t+1}^{(3\text{-}s)*}\text{).} \end{cases} \tag{6.14}$$

Case 2) If $\alpha w_t \ge \min\{E_{t+1}^{(1\text{-}s)*}, \overline{Q}^p\}$ (more wind generation), the SOC will be segmented into three regions: storing and selling renewable generation, discharging storage and selling all renewable generation to the market, and doing nothing (i.e., idle/offline).

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(2\text{-}s)*} - E_t, \alpha w_t, \overline{Q}^p\}, E_t \in [0, E_{t+1}^{(2\text{-}s)*}], \\ \text{(store renewable without buying up to } E_{t+1}^{(2\text{-}s)*}\text{);} \\ 0, E_t \in [E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(3\text{-}s)*}]\text{(keep SOC unchanged);} \\ \max\{E_{t+1}^{(3\text{-}s)*} - E_t, -\overline{Q}^g\}, E_t \in (E_{t+1}^{(3\text{-}s)*}, \overline{E}], \\ \text{(generate and sell renewable down to } E_{t+1}^{(3\text{-}s)*}\text{).} \end{cases} \quad (6.15)$$

The first part of proposition 6.1 (i.e., *Case 1*) analytically shows that *for a PSH and wind farm merchant that seeks to maximize his or her profit when there is less* available wind energy, the storage SOC was divided into four possible regions by three SOC reference points ($E_{t+1}^{(1\text{-}s)*}$, $E_{t+1}^{(2\text{-}s)*}$, and $E_{t+1}^{(3\text{-}s)*}$ *that depend on the currently available energy* E_t*, the forecasted electricity price* P_t*, the PTC* s *, and the wind generation source* w_t) that corresponded to four different actions: 1) storing all wind generation and buying power from the market; 2) storing and selling partial wind generation; 3) remaining idle; and 4) releasing storage and selling all wind generation to the market. If there was less energy in the storage than $E_{t+1}^{(1\text{-}s)*} - \alpha w_t$ and less available renewable energy (i.e., $\alpha w_t < \min\{E_{t+1}^{(1\text{-}s)*}, \overline{Q}^p\}$), the merchant should (a) store all the renewable sources and (b) purchase electricity from the market, then force the SOC up to $E_{t+1}^{(1\text{-}s)*}$ .Otherwise, the optimal action is similar with the second part of Proposition 6.1 as follows.

The second part of Proposition 6.1 (i.e., *Case 2*) indicated that if there were more available renewable sources (i.e., $\alpha w_t \geq \min\{E_{t+1}^{(1\text{-}s)*}, \overline{Q}^p\}$), then the SOC storage was divided into three regions by two optimal reference points ($E_{t+1}^{(2\text{-}s)*}$ and $E_{t+1}^{(3\text{-}s)*}$) that corresponded to actions 2-4. If there was less energy in the storage/reservoir than $E_{t+1}^{(2\text{-}s)*}$, the merchant did not need to purchase power from the market to increase the SOC, but he or she needed to (a) store renewable sources to increase the SOC to $E_{t+1}^{(2\text{-}s)*}$ as close as possible and then (b) sell the reset of the renewable power to the market. Considering the efficiency loss, the income from selling the same power to the market was less than the cost of buying the same energy from the market in the exact time period. Similarly, if the stored energy was within the boundaries of the two optimal reference points (i.e., $E_{t+1}^{(2\text{-}s)*} \le E_t \le E_{t+1}^{(3\text{-}s)*}$), the merchant should do nothing, and if there was more energy in the PSH than in the optimal reference point $E_{t+1}^{(3\text{-}s)*}$, the merchant should release energy from storage and sell all renewable generation to the market, thus reducing the SOC to $E_{t+1}^{(3\text{-}s)*}$ to maximize profit.

Proposition 6.2: If the subsidy per wind generation sold to the market was large (i.e., $s \ge P_t(1-\rho^2)/\rho$), it yielded $E_{t+1}^{(2\text{-}s)*} \le E_{t+1}^{(1\text{-}s)*} \le E_{t+1}^{(3\text{-}s)*}$ for the optimal reference points on the SOC, which depended on the current state $S(t)$ *and* PTC*, and the optimal decisions are specified as follows:*

Case 3) If $\alpha w_t < \min\{E_{t+1}^{(2\text{-}s)*}, \overline{Q}^p\}$ (less wind generation), the feasible SOC can be classified into four regions: storing and selling renewable generation, storing renewable generation and buying electricity from the market, discharging storage and selling all renewable generation to the market, and doing nothing:

1) If $\alpha w_t \ge E_{t+1}^{(1\text{-}s)*} - E_{t+1}^{(2\text{-}s)*}$

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(1\text{-}s)*}-E_t,\overline{Q}^p\}, E_t\in(0,E_{t+1}^{(2\text{-}s)*}-\alpha w_t],\\ \text{(store renewable and purchased electricity up to } E_{t+1}^{(1\text{-}s)*});\\ \min\{E_{t+1}^{(2\text{-}s)*}-E_t,\overline{Q}^p\}, E_t\in(E_{t+1}^{(2\text{-}s)*}-\alpha w_t,E_{t+1}^{(2\text{-}s)*}],\\ \text{(store renewable and without buy up to } E_{t+1}^{(2\text{-}s)*});\\ 0, E_t\in(E_{t+1}^{(2\text{-}s)*},E_{t+1}^{(3\text{-}s)*}] \text{ (keep SOC unchanged)};\\ \max\{E_{t+1}^{(3\text{-}s)*}-E_t,-\overline{Q}^g\}, E_t\in(E_{t+1}^{(3\text{-}s)*},\overline{E}],\\ \text{(generate and sell renewable down to } E_{t+1}^{(3\text{-}s)*}).\end{cases} \quad (6.16)$$

2) If $\alpha w_t < E_{t+1}^{(1\text{-}s)*}-E_{t+1}^{(2\text{-}s)*}$

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(1\text{-}s)*}-E_t,\overline{Q}^p\}, E_t\in(0,E_{t+1}^{(2\text{-}s)*}-\alpha w_t],\\ \text{(store renewable and purchased electricity up to } E_{t+1}^{(1\text{-}s)*});\\ \min\{E_{t+1}^{(2\text{-}s)*}-E_t,\overline{Q}^p\}, E_t\in(E_{t+1}^{(2\text{-}s)*}-\alpha w_t,E_{t+1}^{(2\text{-}s)*}],\\ \text{(store renewable and without buy up to } E_{t+1}^{(2\text{-}s)*});\\ \min\{E_{t+1}^{(1\text{-}s)*}-E_t,\overline{Q}^p\}, E_t\in(E_{t+1}^{(2\text{-}s)*},E_{t+1}^{(1\text{-}s)*}-\alpha w_t],\\ \text{(store renewable and purchased electricity up to } E_{t+1}^{(1\text{-}s)*});\\ 0, E_t\in(E_{t+1}^{(1\text{-}s)*}-\alpha w_t,E_{t+1}^{(3\text{-}s)*}] \text{ (keep SOC unchanged)};\\ \max\{E_{t+1}^{(3\text{-}s)*}-E_t,-\overline{Q}^g\}, E_t\in(E_{t+1}^{(3\text{-}s)*},\overline{E}],\\ \text{(generate and sell renewable down to } E_{t+1}^{(3\text{-}s)*}).\end{cases} \quad (6.17)$$

Case 4) If $\min\{E_{t+1}^{(1\text{-}s)*},\overline{Q}^p\}\geq\alpha w_t\geq\min\{E_{t+1}^{(2\text{-}s)*},\overline{Q}^p\}$ (middle-level wind generation), the feasible SOC can be segmented into three or four regions:

1) If $E_{t+1}^{(1)*}-\alpha w_t\leq E_{t+1}^{(2\text{-}s)*}\leq E_{t+1}^{(1)*}$, the SOC is divided into three regions: storing renewable generation and buying power from the market, discharging/generating storage and selling all renewable generation to the market, and doing nothing (i.e., idle/offline):

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(2\text{-}s)*}-E_t,\alpha w_t,\overline{Q}^p\}, E_t\in(0,E_{t+1}^{(2)*}],\\ \text{(store renewable and without buy up to } E_{t+1}^{(2\text{-}s)*});\\ 0, E_t\in(E_{t+1}^{(2\text{-}s)*},E_{t+1}^{(3\text{-}s)*}] \text{ (keep SOC unchanged)};\\ \max\{E_{t+1}^{(3\text{-}s)*}-E_t,-\overline{Q}^g\}, E_t\in(E_{t+1}^{(3\text{-}s)*},\overline{E}],\\ \text{(generate and sell renewable down to } E_{t+1}^{(3\text{-}s)*}).\end{cases} \quad (6.18)$$

2) If $E_{t+1}^{(1)*}\ge E_{t+1}^{(1)*}-\alpha w_t\ge E_{t+1}^{(2\text{-}s)*}$, the SOC has four regions: storing renewable generation and buying power from the market, storing and selling renewable generation, discharging/generating storage and selling all renewable generation to the market, and doing nothing (i.e., idle/offline):

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(2\text{-}s)*}-E_t,\overline{Q}^p\}, E_t\in(0,E_{t+1}^{(2\text{-}s)*}],\\ \text{(store renewable and without buy up to } E_{t+1}^{(2\text{-}s)*});\\ \min\{E_{t+1}^{(1\text{-}s)*}-E_t,\overline{Q}^p\}, E_t\in(E_{t+1}^{(2\text{-}s)*},E_{t+1}^{(1\text{-}s)*}-\alpha w_t],\\ \text{(store renewable and purchased electricity up to } E_{t+1}^{(1\text{-}s)*});\\ 0, E_t\in(E_{t+1}^{(1\text{-}s)*}-\alpha w_t,E_{t+1}^{(3\text{-}s)*}] \text{ (keep SOC unchanged)};\\ \max\{E_{t+1}^{(3\text{-}s)*}-E_t,-\overline{Q}^g\}, E_t\in(E_{t+1}^{(3\text{-}s)*},\overline{E}],\\ \text{(generate and sell renewable down to } E_{t+1}^{(3\text{-}s)*}).\end{cases} \quad (6.19)$$

Case 5) If $\alpha w_t\ge\min\{E_{t+1}^{(1\text{-}s)*},\overline{Q}^p\}$ (more wind generation), the feasible SOC can also be divided into three regions: storing and purchasing electricity, generating PSH storage and selling all renewable generation to the market, and doing nothing. Thus, an optimal action in each state $S(t)=S_t(E_t,g_t,P_t)\in\hat{E}\times G\times P$ can be shown as follows:

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(2\text{-}s)*}-E_t,\alpha w_t,\overline{Q}^p\},\ E_t\in[0,E_{t+1}^{(2\text{-}s)*}]\text{(store renewable up to }E_{t+1}^{(2\text{-}s)*})\\ 0,E_t\in(E_{t+1}^{(2)*},E_{t+1}^{(3\text{-}s)*}]\text{ (keep SOC unchanged)}\\ \max\{E_{t+1}^{(3\text{-}s)*}-E_t,-\overline{Q}^g\},E_t\in(E_{t+1}^{(3\text{-}s)*},\overline{E}]\text{(generate and sell renewable down to }E_{t+1}^{(3\text{-}s)*})\end{cases}. \quad (6.20)$$

Compared to Proposition 6.1, the first part of Proposition 6.2 (i.e., *Case 3*) analytically shows that *for a PSH and wind farm merchant seeking to maximize his or her profit and with relatively less* available wind energy, the storage SOC was divided into four or five regions by three or four SOC reference points ($E_{t+1}^{(2\text{-}s)*}$, $E_{t+1}^{(1\text{-}s)*}$ *and* $E_{t+1}^{(3\text{-}s)*}$*, which depend on the current state* S(t) *and PTC, and the relationship between them with* αw_t) that corresponded to actions 1-4. I) If the current SOC in the storage/reservoir was more than $E_{t+1}^{(3\text{-}s)*}$, the merchant should release energy from storage and generate energy for the market and then decrease the SOC to $E_{t+1}^{(3\text{-}s)*}$. II) If there was less energy in the storage/reservoir than $E_{t+1}^{(1\text{-}s)*}$ and more energy in the storage/reservoir than $E_{t+1}^{(2\text{-}s)*}$, the optimal actions would be as follows: if there were fewer available renewable sources (i.e., $\alpha w_t < E_{t+1}^{(1\text{-}s)*}-E_{t+1}^{(2\text{-}s)*}$), then the merchant should (a) store all the renewable sources and (b) purchase power from the power market, then increase the SOC up to $E_{t+1}^{(1\text{-}s)*}$. If there were more available renewable sources (i.e., $\alpha w_t \ge E_{t+1}^{(1\text{-}s)*}-E_{t+1}^{(2\text{-}s)*}$), the merchant should do nothing. III) If there was less energy in the storage/reservoir than $E_{t+1}^{(2\text{-}s)*}$ but the SOC could approach $E_{t+1}^{(2\text{-}s)*}$ by storing power without buying, the merchant should store renewable sources and increase the SOC as close as possible to $E_{t+1}^{(2\text{-}s)*}$. Otherwise, if there was less energy in the storage/reservoir than $E_{t+1}^{(2\text{-}s)*}-\alpha w_t$, they should increase SOC up to $E_{t+1}^{(1\text{-}s)*}$ by storing all the renewable sources and purchasing power from the market. When the merchant faces two optimal choices: storing renewable generation and buying power from the power market to increase SOC up to $E_{t+1}^{(1\text{-}s)*}$ or storing and selling renewable energy to decrease SOC as close as possible to $E_{t+1}^{(2\text{-}s)*}$, the second choice was more profitable and should be preferred.

The second part of Proposition 6.2 (i.e., *Case 4*) indicated that if there was a middle available wind source (i.e., $\min\{E_{t+1}^{(2\text{-}s)*}, \overline{Q}^{p}\} \le \alpha w_t \le \min\{E_{t+1}^{(1\text{-}s)*}, \overline{Q}^{p}\}$), then the SOC storage was divided into three or four regions by two or three SOC reference points ($E_{t+1}^{(2\text{-}s)*}$, $E_{t+1}^{(1\text{-}s)*} - \alpha w_t$, and $E_{t+1}^{(3\text{-}s)*}$) that corresponded to actions 1-4 or 2-4. If there was less energy in the storage/reservoir than $E_{t+1}^{(2\text{-}s)*}$, it was recommended that the merchant store renewable sources and increase SOC as close as possible to $E_{t+1}^{(2\text{-}s)*}$. If there was less energy in the storage than $E_{t+1}^{(1\text{-}s)*}$ and more energy in the storage than $E_{t+1}^{(2\text{-}s)*}$, similarly, the optimal actions will be as follows: If there was less available wind source (i.e., $E_{t+1}^{(1)*} \ge E_{t+1}^{(1)*} - \alpha w_t \ge E_{t+1}^{(2\text{-}s)*}$), then the merchant should (a) store all the renewable source and (b) purchase electricity from the market, then increase SOC to $E_{t+1}^{(1\text{-}s)*}$. If there were more available renewable sources (i.e., $E_{t+1}^{(1)*} - \alpha w_t \le E_{t+1}^{(2\text{-}s)*} \le E_{t+1}^{(1)*}$), it is recommended that the merchant do nothing. If there was more energy in the PSH than the reference point $E_{t+1}^{(3\text{-}s)*}$, the merchant should release energy from storage and sell all renewable generation, then reduce the SOC to $E_{t+1}^{(3\text{-}s)*}$.

The last part of Proposition 6.2 (i.e., *Case 5*) indicated that if there was more available renewable source (i.e., $\alpha w_t \ge \min\{E_{t+1}^{(1\text{-}s)*}, \overline{Q}^{p}\}$), then the storage SOC should be divided into three regions by two SOC reference points $E_{t+1}^{(2\text{-}s)*}$ and $E_{t+1}^{(3\text{-}s)*}$ that corresponded to actions 2-4. The merchant only needed to store renewable sources, increase the SOC up to $E_{t+1}^{(2\text{-}s)*}$, and then sell partial renewable power to the market.

Special Case A): If $\rho=1$ (i.e., without transmission efficiency loss), $E_{t+1}^{(2\text{-}s)*} \le E_{t+1}^{(1\text{-}s)*} \le E_{t+1}^{(3\text{-}s)*}$ for any $s \ge 0$ is yielded, which means that the traditional results of proposition 6.1 do not hold.

Special Case B): If $w_t=0$ (i.e., no wind generation or the forecasted wind generation equals zero), the merchant had storage/PSH only, and the optimal reference point $E_{t+1}^{(2\text{-}s)*}$ did not exist.

For wind power, the merchant has two choices: store it in PSH/ storage (and sell it at a later time) or sell it to the market at the time the wind power is generated. There are two opportunity costs for the two choices respectively. The opportunity cost of the first choice is the lost PTC subsidies because the wind power is not sold to the market at the time of generation. The opportunity cost of the second choice is the lost arbitrage profit from selling the power at a more preferable price point at a later time. When the opportunity cost of storing, the first opporunity cost, is less than the opporunity cost of selling, the second opportunity cost, (i.e., $s \le P_t(1-\rho^2)/\rho$), the relationship between the reference points remain the same as those in the traditional study which do not consider PTC, but the PTC subsidy will affect the transaction quantity. However, sometimes the opportunity cost of storing is more than selling (i.e., $s > P_t(1-\rho^2)/\rho$). In this case, the PTC subsidy affects the optimal dispatch policy structure by changing the relationships among the reference points. Only when electricity price is low and wind generation is small, the merchant will buy electricity from the market to store for energy arbitrage. Propositions 6.1 and 6.2 yielded the insight and application, as follows.

Managemental Insight: *For a profit-maximization merchant with both storage and a wind farm, three states of charge (SOC) reference points depend on the current energy level, the forecast price, the available energy of wind source, and the PTC subsidy. The SOC range was split into different regions by SOC reference points. The electricity merchant will obtain identical optimal operations by examining the current storage SOC with the optimal SOC reference points.*

6.3.4. Production Tax Credit (PTC) Analysis. In traditional treatments, electricity merchants do not consider subsidies. The optimal scheduling strategy for merchants owning a wind farm with energy storage when considering PTC was studied. As described in propositions 6.1 and 6.2, the subsidy policy can influence trading decisions: when the subsidy was large, electricity merchants tended to sell electricity instead of storing and buying electricity. When subsidies rose, merchants sold more electricity, which is a dynamic that affected merchants' decisions.

Proposition 6.3: For any given level of current energy storage E_t, available wind generation w_t, and forecast price P_t ($t \in \{1, 2, 3, \cdots, T\}$), the production tax credit (PTC) was considered, and then the optimal value functions exhibited the following relations:

$$\begin{cases} E[V^*_{t+1(s>0)}\left(S(t+1) \mid S(t)\right)] \geq E[V^*_{t+1(s=0)}\left(S(t+1) \mid S(t)\right)] \\ \max_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t)_{(s>0)} \mid S(1)\right] \geq \max_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t)_{(s=0)} \mid S(1)\right] \end{cases} . \quad (6.21)$$

Proposition 6.3 stated that the PTC considerably altered the optimal economic dispatch policy structure and optimal expected profits of electricity merchants under policy one. An electricity merchant who ignores PTC on the power market will overestimate his or her power generation costs. The decisions of an electricity merchant were naturally affected, so when the subsidy was relatively large, the electricity merchant made trading decisions that deviated from optimal scheduling.

To avoid selling all wind-generated electricity to the market to gain high PTC from the government rather than storing electricity for smoothing loads to improve system operation stability. Next, a merchant with a wind farm and an energy storage and could also obtain PTC for stored wind generation, but he or she could not buy electricity from the grid to store. That is, storing energy from the grid disqualified one from receiving PTC.

6.4. RECEIVING PTC PROHIBITS BUYING ELECTRICITY FROM THE GRID TO STORE UNDER POLICY 2

The findings presented in Section 6.3 depended on a wind farm receiving PTC by selling the wind generation to market and that the merchant also had a battery to buy electricity from the grid to store. The implications for the optimal decision changed when the power stored in the energy storage had to be less than or equal to the wind production, and the stored wind generation was also qualifying for PTC. Therefore, in Section 6.4.1, this work proposed a model for policy 2. Then, the optimal scheduling solutions in Section 6.4.2 were derived.

6.4.1. Model Setup. It was also assumed that selling power to the market and purchasing power from the market were not allowed simultaneously. Compared to Section 6.3, wind farm merchants receiving PTC cannot buy electricity from the grid to store; thus, there are only three possible actions: a) storing and selling renewable generation, b) idling, and c) discharging storage and selling all renewable generation to the market.

Thus, we can generate the following reward functions:

$$R^{(PTC)}(q_t, w_t, P_t) = \begin{cases} -P_t(q_t/\alpha - w_t)\cdot\rho - s(q_t/\alpha - w_t) - c_w w_t - c_p q_t \ \ (0 \le q_t \le \alpha w_t) \\ -P_t\cdot(q_t\beta - w_t)\cdot\rho - c_w w_t - s(q_t\beta - w_t) + c_g q_t \ \ (q_t < 0) \end{cases} . \quad (6.22)$$

Unlike Eq. (6.1), if the wind farm merchants receiving PTC cannot buy electricity from the grid to store, then all energy sold to the market is wind generation, so all the energy released from the storage was qualifying for the PTC.

6.4.2. Optimization and Analysis. Similar to Section 6.3, with the subsidy, the objective function is shown:

$$\max_{\pi} \sum_{t=1}^{T} E\left[R^{(PTC)}(q_t, w_t, P_t) | S(1)\right]. \quad (6.23)$$

To obtain the optimal decision rule, we first decompose the optimization in Eq. (6.23) into two optimizations that corresponded to two different actions: a) and c). The capacity constraints and the balance constraints on stored energy remained unchanged. Then, by optimizing the value function $V_t(E_t, w_t, \hat{P}_t))$, subject to $\underline{E} \le E_{t+1} \le \overline{E}$, the following equations were yielded based on the Bellman equation:

$$\begin{cases} V_t^{(2\text{-}PTC)*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left\{ -\left((P_t\rho + s)/\alpha + c_p\right)q_t - w_t[-P_t\rho + c_g - s] + E[V_{t+1}^*(S(t+1) | S(t)] \right\} \\ V_t^{(3\text{-}PTC)*}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left\{ -\left((P_t\rho + s)\beta - c_g\right)q_t - w_t[-P_t\rho + c_g - s] + E[V_{t+1}^*(S(t+1) | S(t)] \right\} \end{cases} . \quad (6.24)$$

Using the same method in Section 6.3, the optimal solutions of these two suboptimizations were found.

Lemma 6.2: When considering the PTC subsidy and the electricity merchant, let $E_{t+1}^{(2\text{-}PTC)*}$ and $E_{t+1}^{(3\text{-}PTC)*}$ be the optimal results in Eq. (6.24), respectively:

$$\begin{cases} E_{t+1}^{(2-PTC)*} = \arg\max_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^{*}(S(t+1) \mid S(t)] - \left((P_t\rho + s)/\alpha + c_p \right) E_{t+1}/\eta_t \right) \\ E_{t+1}^{(3-PTC)*} = \arg\max_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^{*}(S(t+1) \mid S(t)] - \left((P_t\rho + s)\beta - c_g \right) E_{t+1}/\eta_t \right) \end{cases} . \quad (6.25)$$

In Eq. (6.25), we obtain the following relations:

(1) For electricity prices $P_t \ge -\left(\alpha(c_g + c_p) + s(1-\beta\alpha)\right) \big/ (\rho - \beta\alpha\rho)$, there is

$$E_{t+1}^{(2-PTC)*} \le E_{t+1}^{(3-PTC)*} . \quad (6.26)$$

(2) For electricity prices $P_t < -\left(\alpha(c_g + c_p) + s(1-\beta\alpha)\right) \big/ (\rho - \beta\alpha\rho) < 0$, there is

$$E_{t+1}^{(2-PTC)*} \ge E_{t+1}^{(3-PTC)*} . \quad (6.27)$$

The optimal analytical results were obtained from the perspective of profit maximization. Similarly, positive electricity prices were still targeted. The conclusions and insights that follow were based on these analytical results (see Appendix B).

Proposition 6.4: For every stage $t \in \{1, 2, 3, \cdots, T\}$ and positive forecasted electricity prices $\hat{P}_t \in P$, unique optimal storage levels $E_{t+1}^{(2\text{-PTC})*} \le E_{t+1}^{(3\text{-PTC})*}$ were generated, and they depended on the state $S(t)$ and PTC. The optimal decisions of electricity merchant were specified as follows:

The feasible SOC of storage can be divided into three regions: storing and selling renewable generation, discharging/generating storage and selling all renewable generation to the market, and doing nothing (i.e., idle/offline). Thus, an optimal action in each state $S(t) = S_t(E_t, w_t, P_t) \in \hat{E} \times \hat{W} \times \hat{P}$ was specified:

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(2-PTC)*} - E_t, \overline{Q}^p, \alpha w_t\}, E_t \in [0, E_{t+1}^{(2-PTC)*}], \\ \text{(store renewable bring SOC up to } E_{t+1}^{(2-PTC)*}); \\ 0, E_t \in (E_{t+1}^{(2-PTC)*}, E_{t+1}^{(3)*}] \text{ (keep SOC unchanged)}; \\ \max\{E_{t+1}^{(3-PTC)*} - E_t, -\overline{Q}^g\}, E_t \in (E_{t+1}^{(3-PTC)*}, \overline{E}]; \\ \text{(discharge make SOC down to } E_{t+1}^{(3-PTC)*}) \end{cases} . \quad (6.28)$$

This proposition indicated that two SOC reference points $E_{t+1}^{(2\text{-}PTC)*}$ and $E_{t+1}^{(3-PTC)*}$ were included, and the merchant only needed to store a portion of the renewable source, increase the SOC as close as possible to $E_{t+1}^{(2\text{-}PTC)*}$, and then sell the rest of the renewable power to the market. If the current storage level in the storage/reservoir was more than $E_{t+1}^{(3-PTC)*}$, the merchant should generate or discharge energy from PSH and sell all renewable generation to the market, then decrease the SOC to $E_{t+1}^{(3-PTC)*}$. Proposition 6.4 displays the optimal economic dispatch decision for the merchant when considering PTC under policy two.

Overall, the findings confirmed that when a PTC-subsidized wind farm with energy storage cannot buy electricity from the grid for storage, it affects the optimal SOC reference points, but these insights were qualitatively unchanged by such adjustments.

6.5. NUMERICAL SIMULATION AND CASE STUDY

We first validated the stated approaches and outcomes using a synthesis data case to show the analysis process under the two policies and compared them with MILP in Section 6.5.1. Furthermore, Section 6.5.2 used a real data case from MISO electricity prices, wind generation, and PTC to show the related insights.

6.5.1. Case Study and Comparison. For simplicity, a three-period case was used to explain the details of the proposed approaches. In this case, it was assumed that there were three periods ($T=3$). At each period, the electricity price took one of the values in set $P_t=\{p^M, p^L, p^H\}=\{6,3,10\}$. This study assumed that the storage capacity was 10 (i.e., $\underline{E}=0, \overline{E}=10$), the maximum generating/discharging capacity was 12 and the maximum pumping/charging capacity was 7. This means that full storage could be emptied in one period, and empty storage could not be filled in one period, but it could be filled in fewer than two periods. It holds that $\underline{E}+\overline{Q}^p \le \overline{E}$ (resp. $\overline{E}-\underline{E} > \overline{Q}^g$) and $\underline{E}+2\overline{Q}^p \ge \overline{E}$. Assume the operating cost is one (i.e., $c_p=c_g=1$), the wind generation cost is zero (i.e., $c_w=0$), the charging, discharging and transmission efficiencies are 0.9 (i.e., $\alpha=\beta=0.9=\rho$), the self-discharging efficiency is one (i.e., $\eta=1$), and the wind generation is $w_t=\{3,5,0\}$. This research employed the method proposed in Section 6.3 and the optimal actions proposed in Propositions 6.1 and 6.2 to obtain the optimal actions and SOC reference points under three different PTC credit rates (i.e., $s=\{0,1,3\}$) with two different initial SOCs, as shown in Table 6.1 (the proof method is given in Appendix D).

Table 6.1 Optimal Results with PTC Credit Rates Under Policy 1

	$s=3, E_1=1$	$s=3, E_1=5$	$s=1, E_1=1$	$s=1, E_1=5$	$s=0, E_1=1$	$s=0, E_1=5$
$(E_4^{(1)*}, E_4^{(2)*}, E_4^{(3)*})$	(0,0,0)		(0,0,0)		(0,0,0)	
$(E_3^{(1)*}, E_3^{(2)*}, E_3^{(3)*})$	(10,0,10)		(10,0,10)		(10,10,10)	
$(E_2^{(1)*}, E_2^{(2)*}, E_2^{(3)*})$	(0,0,10)		(0,0,10)		(0,3,10)	
q_3^*	-8	-10	-8	-10	-10	-10
q_2^*	7	5	7	5	7	5
q_1^*	0	0	0	0	2	0
V_1^*	65.7407	89.3481	59.7407	83.3478	56.9	80.3478

Table 6.1 indicates that PTC affects the optimal actions. Compared with the current study (i.e., an electricity merchant ignoring the PTC), increasing PTC leads to an increased maximum expectation of profits. When wind farm merchants cannot buy power from the grid to store (i.e., policy 2) to receive PTC subsidies, then, the method proposed in Section 6.4 is used to obtain the SOC reference points and the optimal actions under three different PTC credit rates (i.e., $s=\{0,1,3\}$) with two initial SOCs, as shown in Table 6.2.

Table 6.2 Optimal Results with PTC Credit Rates Under Policy2

	$s=3, E_1=1$	$s=3, E_1=5$	$s=1, E_1=1$	$s=1, E_1=5$	$s=0, E_1=1$	$s=0, E_1=5$
$(E_4^{(2)*}, E_4^{(3)*})$	(0,0)		(0,0)		(0,0)	
$(E_3^{(2)*}, E_3^{(3)*})$	(10,10)		(10,10)		(10,10)	
$(E_2^{(2)*}, E_2^{(3)*})$	(0,10)		(0,10)		(5.5,10)	
q_3^*	-5.5	-9.5	-5.5	-9.5	-8.2	-10
q_2^*	4.5	4.5	4.5	4.5	4.5	4.5
q_1^*	0	0	0	0	2.7	0.5
V_1^*	74.6	113.8	58.7	90.7	51.02	79.2

Table 6.2 shows the optimal actions and profits when the wind farm merchant receives a PTC subsidy, but he or she cannot buy electricity from the grid to store (i.e., policy 2). Compared with Table 6.1, we find that when the subsidy is large, the profit under policy 2 may be higher, indicating that electricity merchants are willing to give up the opportunity to buy electricity and qualify for the subsidy for all wind generation.

We also employed the traditional MILP model (Chazarra et al., 2018; Wang et al., 2021) to obtain the optimal solutions and compared them with the optimal results in Table 6.1 and Table 6.2. The same optimal results were obtained under both dynamic programming and MILP methods for Case 1. The above optimal results and optimal profits are verified in AIMMS.

6.5.2. MISO Case Study. In this Section, 1-hour time units were used for the electricity price series in the day-ahead market $LMP = \{LMP_1, LMP_2, \ldots, LMP_T\}$ (\$/MW). Wind generation with T = 336 stages corresponding to a 2-week period from 1 to 14 May 2021 in MISO (all electricity prices and wind generation data are available at https://www.misoenergy.org/) was also used. This work adopted three different PTC credit rates 0to show the optimal results.

Values 2000 and 20000 were assigned the minimum and maximum SOC of storage (upper reservoir of PSH) $\underline{E}$ and $\overline{E}$, respectively. Here, $\underline{E} > 0$ indicates that the merchant cannot empty the upper reservoir, which is a realistic statement for a PSH in the power market. The maximum generation rate and pumping rate are $\overline{Q}^p = 2000$ and $\overline{Q}^g = 2000$, respectively. Units of MW hours were used for storage quantities. Units of GW were used to measure both pumping and generating rates, where the pumping and generating efficiencies in this case are $\alpha = \beta = 0.9$. It was assumed that it took $(\overline{E} - \underline{E})/\overline{Q}^g = 9$ hours for the PSH to empty the upper reservoir and that it took $(\overline{E} - \underline{E})/\overline{Q}^p = 9$ hours to refill the upper reservoir; these values corresponded to the approximate durations exhibited by a large-scale pumped storage hydropower plant in Ludington, Michigan (see https://www.consumersenergy.com/for details).

In this case, the pumping and generating operating costs were calculated as $c_p = c_g = 1$ (\$/MW). Self-discharging was ignored, and it was assumed that $\eta = 1$. The wind generation cost did not affect the optimal results; therefore, the wind generation cost was ignored, and the study only focused on the PTC subsidy. In this Section, the VOE at time T. (i.e., $VOE_{T+1} = 0$) was not considered. Regularly, two weeks is an optimization cycle for Ludington in the power market.

Based on Propositions 6.1 and 6.2 in Sections 6.3 and 6.4, the transmission efficiency and PTC played a crucial role in determining the optimal actions. Thus, this study considered two different situations: $\rho = 0.9$ and $\rho = 1$.

The renewable electricity PTC was a per kWh credit for electricity generated using qualified energy resources[4]. Under the current law, facilities for which construction began before January 1, 2021, may be qualified for the PTC. However, the credit rates are various for wind facilities that depend on the year when one began construction. Following Sherlock's report (2020) and the Renewable Electricity Production Tax Credit (see https://www.epa.gov/lmop/renewable-electricity-production-tax-credit-information/), five different PTC Credit Rates (i.e.,0,1.0,1.5,2.3,2.5¢/kWh; that are 0,10,15,23,25$/MWh) were considered to yield results.

For a merchant owning both energy storage (such as the Ludington PSH) and a wind farm, the plots in Figures 6.1 and 6.2 show the optimal scheduling under policy 1 and policy 2 with the initial energy in the storage of 2000 MW hours (i.e., 2 GWh) when the transmission efficiency was $\rho = 0.9$. The optimal decisions are made from the perspectives of the PSH owner, considering the four different PTC credit rates and the traditional approach of ignoring the PTC in each figure.

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[4] The renewable electricity production credit can be found in §45 of the Internal Revenue Code (IRC).

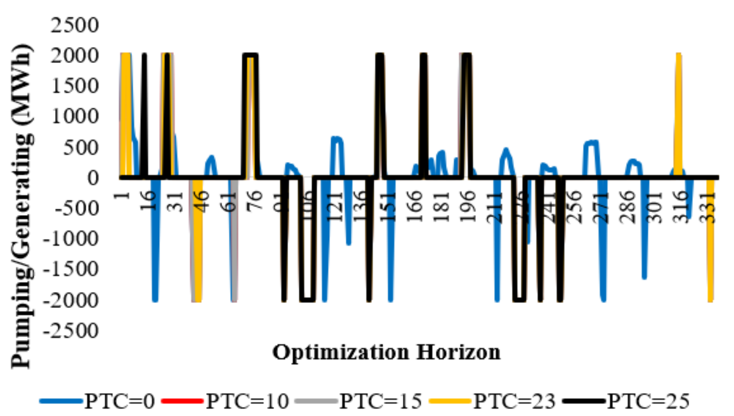

Figure 6.1 The optimal actions under policy 1

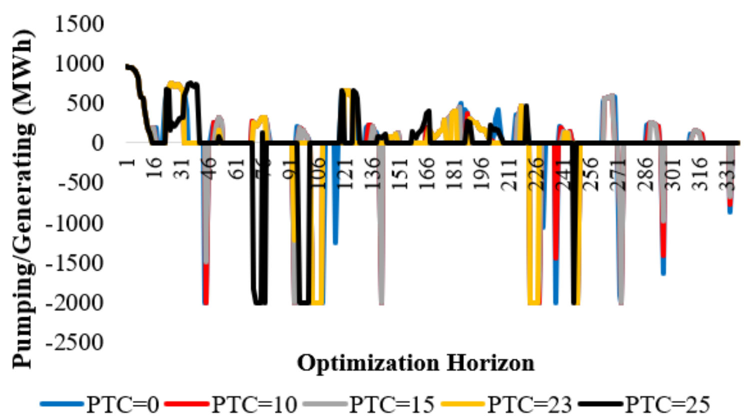

Figure 6.2 The optimal actions under policy 2

The plots in Figure 6.3 and Figure 6.4 illustrate the optimal SOC curves under policy 1 and policy 2 that relate to the optimal operations in Figure 6.1 and Figure 6.2 with the initial energy in the storage of 2 GWh from the perspectives of the PSH owners to maximize their profit considering the four different PTC credit rates and the traditional approach of ignoring the PTC in each figure.

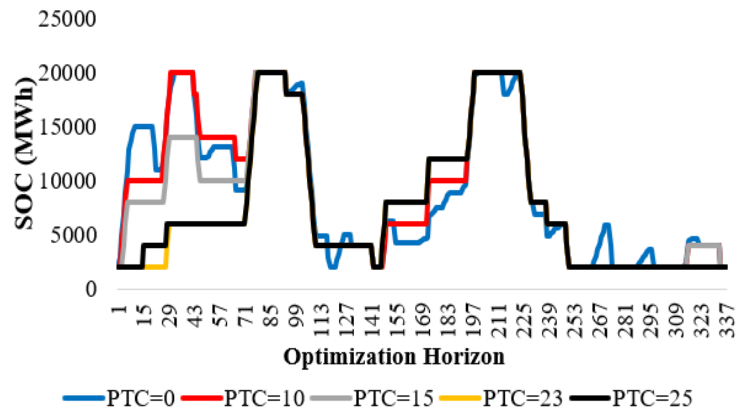

Figure 6.3 The optimal SOC curves under policy 1

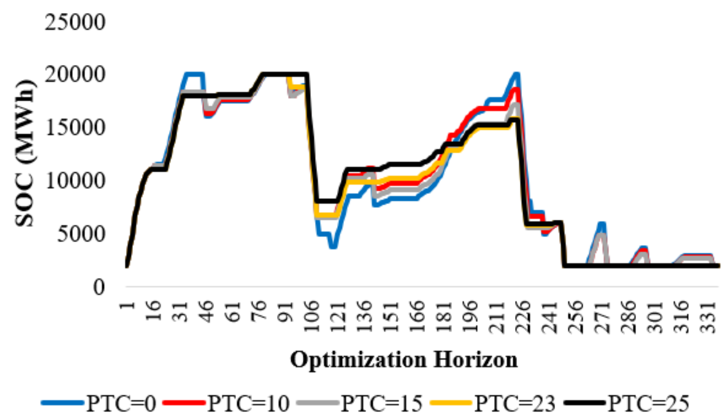

Figure 6.4 The optimal SOC curves under policy 2

Figures 6.1-6.4 led to the following observations and conclusions: the electricity merchant required different trading strategies because the PTC subsidy changed the objective function. Hence, it affects trading decisions under two different policies. Obviously, under policy 2, the charging electricity amount in each period is less than the optimal decision under policy 1 since the power stored in the battery must be less than or equal to the wind production under policy 2. Therefore, a merchant must strike a perfect trade-off between the PTC credit rates and the power transition quantity.

The optimal actions were obtained from Proposition 6.2 when the merchant ignored the transmission loss (i.e., $\rho = 1$), which is shown in Figure. 6.5 and Figure. 6.6 with different PTC policies when the initial energy (i.e., E1) in the storage was 2 GWh.

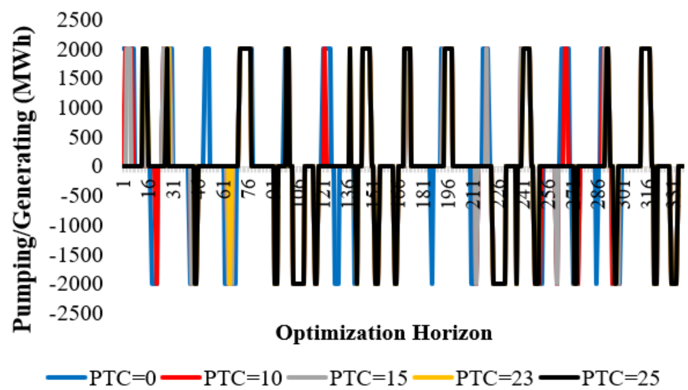

Figure 6.5 The optimal actions under policy 1

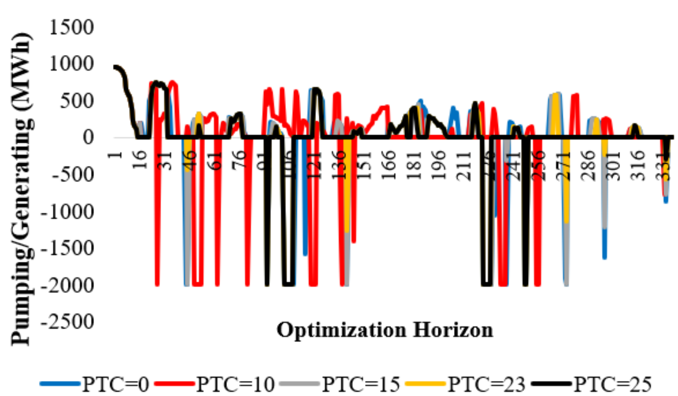

Figure 6.6 The optimal actions under policy 2

Compared with Figures 6.1 and 6.2, ignoring the loss of transmission efficiency will increase the opportunity to buy and sell electricity, and then the frequency of pumping and generating will be increased. The plots in Figure 6.7 and Figure 6.8 illustrate the optimal SOC curves with initial energy (i.e., E_1) in the storage of 2000 MW hours under two PTC policies that corresponded to the optimal actions in Figure 6.5 and Figure 6.6 when the merchant ignored the transmission loss.

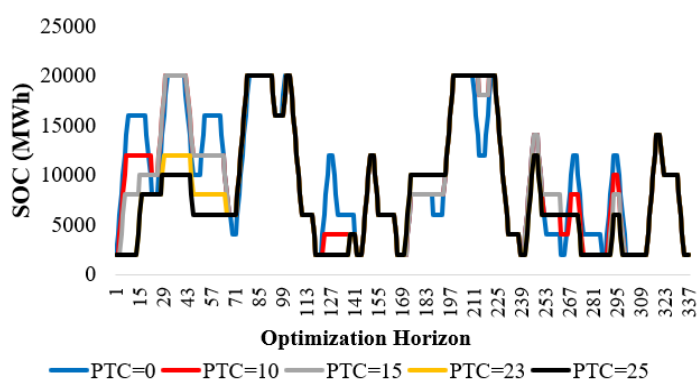

Figure 6.7 The optimal SOC curves under Policy 1

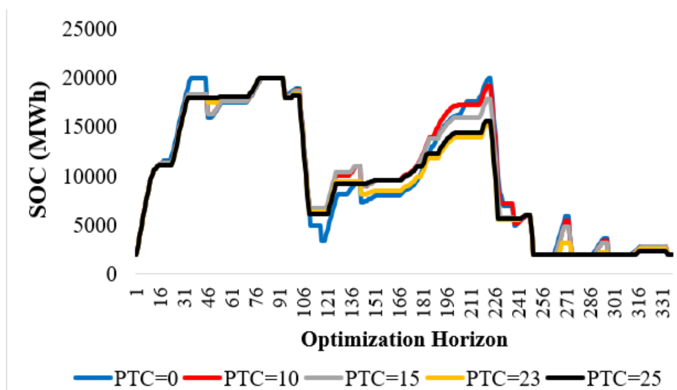

Figure 6.8 The optimal SOC curves under policy 2

Figures 6.5-6.8 show that regardless of whether there was transmission loss, the PTC credit rates impacted the optimal decisions for profit maximization for the merchant with energy storage and wind farms. This part proved the conclusion outlined in Section 6.3 and Section 6.4 through numerical simulation.

Figures 6.1-6.8 show that the frequency of pumping and generating (i.e., the number of trades during the optimization horizon) was reduced with the increased PTC. Without considering the PTC, or when the PTC was small, the electricity merchants only made optimal economic dispatching decisions based on the electricity prices. Under policy 1, when the price was very low, the electricity merchant took actions: 1) storing all wind generation and buying power from the market or 2) storing and selling partial wind generation. When the price was fairly high, the electricity merchant chose action 4), releasing storage and selling all wind generation to the market. Otherwise, the merchant took action 3) doing nothing (i.e., selling all the wind generation to market for subsidies). Under policy 2, the merchants have the three actions (2, 3 and 4) to choose from.

Table 6.3 reports simulation results for the number of idle actions—based on 336 decision periods in two weeks—of an electricity merchant with both storage and a wind farm under different PTC credit rates.

Table 6.3 Number of Idle Actions with PTC Credit Rates Under Two Policies

Transmission efficiency $\rho = 0.9$					
Policies/PTC	PTC=0	PTC=10$/MWh	PTC=15$/MWh	PTC=23$/MWh	PTC=25$/MWh
Policy 1	196	291	291	295	301
Policy 2	161	192	207	253	259
Transmission efficiency $\rho = 1$					
Policies/PTC	PTC=0	PTC=10$/MWh	PTC=15$/MWh	PTC=23$/MWh	PTC=25$/MWh
Policy 1	195	231	239	253	255
Policy 2	160	185	199	231	246

This table reports the number of instances in which the merchant chose to do nothing during the optimization horizon to maximize his or her profit. In the second row, where PTC = 0, we can see that there are 191 (resp., 161) periods when the merchant remains idle in two weeks of optimization when PTC is not considered under policy 1 (resp., policy 2). With improvements in the PTC credits, idle frequencies will increase monotonically. For different PTC credits, the number of idle actions (i.e., the storage had done nothing, and all the wind power generation was sold to the market) under policy 1 is greater than that under policy 2.

According to the current PTC credit rates, when the PTC was large, the merchant was willing to sell wind power generation to obtain large subsidies instead of storing the wind generation, even if the market prices were low. Generally speaking, the increase of PTC and efficiency loss will lead to reduced frequency of the action of storing wind generation for electricity merchants. Therefore, the frequencies of pumping and generation are decreased. Furthermore, compared with policy 1, the number of idle actions for merchants under policy 2 is smaller. This is because under policy 2, merchants chose to store wind generation and sell it when prices were high and could also receive subsidies. Next, we compare the optimal results for the preceding two PTC policies and then analyze which PTC policy merchants should adopt to maximize profit. Table 6.4 demonstrates the Ludington PSH optimal profit results with initial energy in the storage of 2000 MW hours under two policies when the merchant considered and ignored the transmission loss.

Table 6.4 Optimal Profit with PTC Credit Rates Under Two Policies

Transmission efficiency $\rho = 0.9$					
Profit	PTC=0	PTC=10$/MWh	PTC=15$/MWh	PTC=23$/MWh	PTC=25$/MWh
Profit on Policy 1($)	2,588,335	3,442,529	3,914,204	4,694,023	4,891,781
Profit on Policy 2($)	2,504,431	3,445,301	3,922,480	4,697,404	4,893,548
Transmission efficiency $\rho = 1$					
Policies	PTC=0	PTC=10$/MWh	PTC=15$/MWh	PTC=23$/MWh	PTC=25$/MWh
Profit on Policy 1($)	3,229,613	4,017,197	4,452,313	5,195,041	5,387,410
Profit on Policy 2($)	2,794,687	3,733,006	4,208,833	4,977,473	5,172,282

Table 6.4 exhibits the optimal profits under two PTC policies when the wind generation and power price correspond to 2 weeks from 1 to 14 May 2021 in the MISO for a merchant having both storage and a wind farm. According to the current PTC credit rates, the electricity merchant will achieve more profit under policy 2 than policy 1 when the PTC subsidy is large. Compared to allowing merchants to buy power from the market at a low price for time-shifting resales, a large PTC subsidy will bring more profit. If the PTC subsidy is small for electricity merchants, they should aim to buy power from the market at a low price and sell power at a high price to increase their profit by energy arbitrage. Therefore, policy 1 benefits an electricity merchant's profit. However, if the PTC subsidy is large, then the increased revenue from energy arbitrage cannot offset any loss because stored wind generation for time-shifting sales cannot receive PTC under policy 1. To obtain subsidies, under policy 1, the merchant can only sell electricity in the current period, while under policy 2, the merchant can store wind generation in storage when the prices are low and sell it when the electricity prices are high and receive subsidies at the same time.

Tables 6.3 and 6.4 are consistent with our intuition: the electricity merchant derives more profit from the wholesale power market with the improvement of transmission efficiency in the power market. It is straightforward that the PTC led to high maximum expectations of profit because the PTC policy reduced the wind generation costs and increased the profit. When the transmission efficiency is equal to 1, the optimal profit under policy 1 is greater than that under policy 2. This is because policy 2 relies on storage for arbitrage, while policy 1 relies more on directly selling wind power to obtain the PTC, so no transmission loss is more beneficial to policy 1.

To verify the robustness of our conclusions, we conducted a numerical simulation of the cases where uses hourly time units as the power prices and the wind generation with 336 stages corresponding to two weeks from December 3 to 18, 2020 in MISO as supplied. The results are qualitatively unchanged from our previous simulations. (See numerical simulation in Appendix D)

6.6. SECTION SUMMARY AND ANALYSIS

This research is the first to model the joint economic dispatch problem via dynamic programming for an electricity merchant with energy storage and a wind farm receiving PTC by selling wind generation to market. We derived the optimal policy structure from supporting multistage decision-making. The optimal decision structure theorized the classic results, and it differed from conventional policies, known as optimal in the literature, without considering the PTC. Toward the end of establishing a reasonably tractable framework and deriving valuable insights, we have made some simplifying assumptions about the linear market impact and the zero lower limits of generation and pumping. In this section,we also assume that the merchant's charging and discharging decisions do not affect electricity prices.

For an electricity merchant with energy storage and a wind farm, the optimal scheduling policy that corresponds to the optimal SOC reference points at each decision period depends on (a) the current SOC in the storage, (b) the forecast electricity prices, (c) the availability of wind generation, and (d) the PTC. The study analytically showed that when the wind farm can receive PTC by selling the wind generation to market and be able to buy electricity from the grid to store, there were three SOC reference points. The SOC range was segmented into four regions, each of which corresponded to one of four distinct actions. On the other hand, we further find that if a wind farm is receiving PTC but cannot buy electricity from the grid to store, there are two SOC reference points, such that the feasible SOC range was split into three regions, each of which corresponded to one of three distinct actions.

The optimal energy storage decision of the electricity merchant was achieved by simply comparing the current SOC of storage with the optimal SOC reference points. However, the PTC credit rates changed the traditional relationships between optimal SOC reference points. Under the two policies, PTC played a significant driver of optimal policy reform. Under policy one and allowing the wind farm to buy power from the market when a wind farm receives PTC, the merchant should perfectly balance the opportunity cost of wind generation of electricity stored in the storage and the cost of buying electricity from the market for storage to arbitrage. When the PTC subsidy was small, although similar results as the traditional optimal policy structure were found, the optimal economic dispatch volume was changed. On the other hand, based on the current PTC credit rates, if the PTC was large, the merchants should adopt the corresponding optimal actions that all wind-generated electricity sells to the market for high PTC subsidies. Only when the electricity price is low and wind generation is small should the merchant buy electricity from the market to store to arbitrage, which will affect the merchant's decision.

This study showed that with an increase in PTC, the frequency trading decision (i.e., charging and discharging) was reduced. Our results also confirmed that an electricity merchant should adopt policy two when the PTC subsidy is large, prohibiting the purchase of electricity from the grid and all the energy released from storage qualified merchants for

PTC subsidy and will produce more profit to the merchants. Otherwise, the electricity merchant should take advantage of policy one to receive the PTC subsidy if possible. These new findings add to the collective knowledge of managing differentiated wind generation, PTC subsidy policies and joint economic dispatch of energy storage and wind farm, which constitutes a significant contribution to this research topic.

7. OPTIMAL ECONOMIC DISPATCH BETWEEN PROFIT-MAXIMIIZNG ELECTRICITY MERCHANTS AND SOCIAL WELFARE MAXIMZING INDEPENDENT SYSTEM OPERATORS

7.1. OVERVIEW AND RESEARCH QUESTIONS

The ISO can generate more accurate price forecasts than can electricity merchants, as the ISO has the most comprehensive information about market operations. From the perspective of the ISO, modeling and optimizing these energy storage resources and generators across multiple market-clearing processes and planning studies with uncertainties and incomplete information raise new challenges. Using energy storage flexibility to deal with realized uncertainties in the multi-stage clearing process of electricity markets, in contrast, can induce deviation in the multi-stage scheduling procedures. The financial risks associated with the schedule variation may be unacceptable to energy storage owners. Unlike previous research on the economic dispatch policies of electricity merchants (Liu et al., 2022b; Zhou et al., 2016, 2019), the goal of this study problems (Liu et al., 2022e) is to address these challenges and investigate whether a decentralized profit-maximizing merchant with energy storage or traditional generators should comply with the centralized ISO's dispatch decisions if the ISO sends price information to the merchant.

We are thus motivated to focus on the following three dispatching. The first problem concerns ISO's goal to minimize the system cost, which considers electricity supply and demand balance constraints and other physical characteristics. The second problem is the optimal scheduling of the energy storage merchant's profit maximization through buying power from the market at low prices and selling power to the market at high prices. The third problem is the economic dispatch problem of traditional thermal generator merchants. We begin by determining the outcome for traditional generators (i.e., thermal, nuclear, etc.). Then, in terms of energy storage electricity merchants, this study focuses on two scenarios (Kim and Powell 2011; Liu et al., 2022b; Qi et al., 2015; Secomandi, 2010; Zhou et al., 2016, 2019): one with only energy storage and another with energy storage and renewable power plants. In the first scenario, the merchant has three options: (a) purchasing power to store, (b) remaining idle, and (c) discharging storage and selling electricity to the grid. If the merchant has energy storage and renewable power plants, there are four options: (i) storing all renewable power production and buying electricity from the market, (ii) storing and selling partial renewable production, (iii) remaining idle, and (iv) discharging storage and selling all renewable production to the grid. In an extension, we explore our research through the account of a wind farm-and-storage merchant qualified for production tax credits and prohibited from purchasing electricity from the grid to store as well as investigate a co-optimizing electricity merchant with a wind farm and a PHS facility with two linked upper and lower reservoirs.

Our analysis addresses the following question: (1) Under which conditions will an individual profit-maximizing merchant and a welfare-maximizing ISO arrive at the same optimal economic dispatch? (2) To what extent can an ISO's scheduling decisions be exploited by the merchant to increase her profit and improve the social welfare of the system? To answer these two questions, we first assume that the electricity generation cost of traditional generators is linear. To solve this problem and facilitate our modeling through strong duality theory, we select to relax the nonconvex constraints of charging/pumping and discharging/generating that cannot occur simultaneously and develop the corresponding Lagrangian function. The Karush-Kuhn-Tucker (KKT) condition is utilized to analyze sufficient conditions for such an exact relaxation. Later, we evaluate the associated dual problems of the ISO and energy storage merchant scheduling models, which ignore the nonconvex constraints, and identify the condition under which the ISO and merchant arrive at the identical optimal economic dispatch results. Finally, we extend our investigation numerically by examining quadratic electricity generation costs. To the best of our knowledge, this is the first study to investigate the relationship of optimal scheduling solutions between ISOs and merchants while taking energy storage characteristics and the uncertainty of renewable energy generation into account.

The remainder of this work is organized as follows. Section 7.2 provides the principal contributions of this work. Section 7.3 presents the development of three scheduling models and the associated dual models of the ISO, the merchant with only energy storage, and traditional generators. Here, we study the optimal decision-making relationship between the ISO and merchants. Section 7.4 focuses on the electricity merchant who owns both energy storage and renewable energy sources and generators. Section 7.5, we present the results when we use realistic MISO data to perform a numerical simulation to validate the stated results. Section 7.6 examines a renewable-power-plant-and-storage merchant who receives PTCs but is not permitted to purchase electricity from the grid. Section 7.7 contains an investigation of a co-optimizing electricity merchant with a wind farm and a PHS facility with upper and lower reservoirs. We analyze cases in which the quadratic generation cost is considered and numerically demonstrate the robustness of our conclusions in Section 7.8. Finally, we conclude in Section 7.9 with a summary of our findings and some suggestions for future research.

7.2. THE PRINCIPAL CONTRIBUTIONS

This study provides three significant contributions. First, our results illustrate that, in optimal economic dispatch solutions, generating/discharging and pumping/charging cannot occur simultaneously for any positive forecasted electricity prices. We found that, if the ISO sends the cleared electricity prices to the merchant based on social welfare maximization, and the merchant dispatches the energy storage based on that price, the ISO and the energy storage merchant (resp., generators) have the same optimal charging/pumping and discharging/generating (resp., optimal electricity generation) decisions if such profit-maximizing merchant has a unique optimal solution. When a merchant's scheduling problem has multiple optimal solutions, the merchant can still achieve maximum profit by seeking her own profit optimization or following the ISO's social welfare optimization scheduling. As a result, the merchant has an incentive to let the ISO schedule her energy storage or generators; in this instance, the merchant's profit is maximized while the social welfare of the system is improved.

Second, following the same approach, we use a duality model to demonstrate that, if conditions are "ideal" (namely, forecast prices align with LMPs or the ISO sends the cleared prices to the merchants), the profit-maximizing electricity merchant and the social welfare-maximizing ISO will arrive at the same optimal dispatch on energy storage and renewable energy source or traditional generators when the merchant's scheduling problem has a unique solution. The merchant can still achieve the maximum profit by following the ISO's dispatch even if she has multiple options to schedule her energy storage and renewable power plants or traditional generators. Thus, a wind-and-storage merchant also has an incentive to let the ISO take over her operations because the ISO may be better equipped with information to obtain more accurate price predictions to guide the dispatch decision and, as a result, achieve higher social welfare for the market as well as higher profit for each merchant.

Finally, we extend our research to consider a wind farm that (a) receives production tax credits (PTCs), (b) has energy storage, and (c) is prohibited from buying electricity from the grid. Thus, the energy stored in the storage can be no greater than the power produced from the wind farm. Although the PTCs affect the optimal state of charge (SOC) reference points, we can derive valuable insights by employing similar analytical procedures. In addition, we investigate a co-optimizing electricity merchant with a wind farm and a PHS facility with upper and lower reservoirs, and the results are consistent. We also tested our research by numerical simulation, using realistic MISO data to verify our findings, taking into consideration the quadratic generation cost function, which is untraceable, and considering, in particular, the energy balancing constraint. Our results show that the electricity merchant can achieve the maximum optimal profit by following ISO's scheduling, which demonstrates that our conclusions are robust and hold to quadratic generating costs. If we can forecast pricing accurately, parallel computing to solve the profit maximization problems of many decentralized individual merchants simultaneously will be more efficient than handling the large-scale centralized economic dispatch problem from the perspective of ISO.

7.3. OPTIMAL DECISIONS BETWEEN MERCHANT AND ISO

The ISO schedules the traditional generators and energy storage in the power system. We investigate the relationship of the optimal dispatch decisions for conventional generators and energy storage between the perspective of the ISO and merchant, the ISO scheduling problem, and two merchants' scheduling problems, in which only one has energy storage and the other has a traditional generator modeled in this Section. The parameter setup is presented in Section 7.3.1. In Sections 7.3.2 and 7.3.3, we build primal and dual models from two perspectives: (a) a centralized ISO's scheduling model to maximize the social welfare of the system, and (b) a decentralized generator merchant's and a storage merchant's scheduling model based on the profit-maximizing perspective. Finally, in Section 7.3.4, we examine the relationships of best scheduling decisions for energy storage and an individual traditional generator between the ISO and merchants.

7.3.1. Model Setup. In this Section, we assume that energy storage (for simplicity, henceforth, we use PSH to refer to energy storage) and other generators (e.g., thermal, coal, nuclear) are connected to the electricity market through transmission lines. To maintain tractability and to model the dual problem, it is assumed that the generation and operation cost for generators and storage is a linear function (El-Meligy et al., 2022; Soofi and Manshadi, 2022; Xu et al., 2017) and the transmission efficiency losses are ignored, as in Ostrowski et al. (2012) and Xu et al. 2017. In practice, a merchant who buys electricity from or sells electricity to the MISO market to optimize her profit based on the power that she injects into and withdraws from the bus does not have to account for transmission loss. As a result, this assumption is realistic and reasonable.

We work in discrete time, during which the ISO makes a set of economic dispatch decisions that minimize the system's operating cost while satisfying the operational market requirement and physical constraints. Both the ISO and merchant make a set of scheduling decisions in discrete time periodically over a finite horizon, $t \in \{1, 2, \cdots, T\}$. In the ISO's scheduling model, it is assumed that an ISO operates energy storage and other generators g_i^h, where $i \in \{1, 2, 3, \cdots, M\}$, here, and M denotes the number of generators.

For simplicity, it is assumed that each generator has one unit. Assuming that the unit electricity generation cost for generator *i* in period t is C_{it}^h, then the dispatched generation cost for generator *i* is $C_{it}^h \cdot g_{it}^h$. It is also assumed that the generators have the generation capacity constraints and denote $\overline{G}_i^h$ and $\underline{G}_i^h$ as the upper and lower limits of power generation that can be allocated to the grid for each period, respectively. In the scheduling model for a generator merchant, it is assumed that the merchant operates only a traditional generator-*I*. Then, to be consistent with that in the ISO's model, the unit-energy generation cost is C_{It}^h, and it has upper and lower limits of power generation, denoted as $\overline{G}_I^h$ and $\underline{G}_I^h$, respectively. The generator merchant decides the generation amount g_{It}^h for each period *t* based on forecasted electricity price to maximize her expected profit.

In the scheduling model for an energy storage merchant, it is assumed that energy storage has the maximum energy capacity $\overline{E}$ (i.e., the total energy that can be stored) and the minimum energy level $\underline{E}$, where $\overline{E} > \underline{E} \geq 0$; hence, storage capacity is finite. It's also assumed that storage has charging/pumping and discharging/generating capacity limitations in each period. Let $\underline{Q}^p$ and $\underline{Q}^p$ represent the upper and lower limits, respectively, of the charging/pumping electricity amount (i.e., the amount of electricity that can be stored to the storage) in each period. Here, $\overline{Q}^g$ and $\underline{Q}^g$ denote the upper and lower limits, respectively, of the discharging/generating electricity amount in each period (i.e., the amount of electricity that can be released from the storage). When the lower limits of charging and discharging (i.e., $\underline{Q}^{p}$ and $\underline{Q}^{g}$) are greater than 0, binary variables of unit commitments are required to ensure that charging/pumping and discharging/generating cannot happen simultaneously, making it challenging to build the corresponding dual model. To keep the model analytically tractable, we follow the conventional studies and assume $\underline{Q}^{g} = \underline{Q}^{P} = 0$ (see Jiang and Powell, 2015b; Kim and Powell, 2011; Secomandi, 2010; Zhou et al., 2016, 2019). Charging/pumping and discharging/generating efficiency are considered and denoted by α and β; here, $\alpha, \beta \in (0,1]$. This study lets c^{p} (resp., c^{g}) (dollars unit energy) denote the pumping/charging (resp., generating/discharging) unit energy operating cost for energy storage (Mongird et al., 2020; Xu et al., 2017).

We assume q_t^p and q_t^g denote the dispatch decisions for each period *t*, which represent the energy change in storage between period *t* and period *t* + 1 before accounting for the charging/pumping and discharging/generating energy loss. Let E_t represent the current SOC or energy inventory in storage or the upper reservoir at the beginning of *t*. Therefore, the storage energy balance (or state transition) from period t to period *t* + 1 is: $E_{t+1} = E_t + q_t^p - q_t^g$.

7.3.2. Primal-Dual Scheduling Model for the ISO. The system cost from the ISO's perspective includes energy storage operating costs and other generators' electricity generation costs, that is, the objective function; then, the dispatch problem from the prespective of ISO is as follows:

$$\min\Big[\underbrace{\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h}\cdot g_{it}^{h}}_{\text{electricity generation cost}} + \underbrace{\sum_{t=1}^{T}\left(c^{p}\cdot q_{t}^{p} + c^{g}\cdot q_{t}^{g}\right)}_{\text{storage operating cost}}\Big]$$

$$\text{s.t.}\begin{cases} 0 \le q_t^p \le \bar{Q}^p, & (\underline{\chi}^p_t, \bar{\chi}^p_t) \\ 0 \le q_t^g \le \bar{Q}^g, & (\underline{\chi}^g_t, \bar{\chi}^g_t) \\ \underline{E} \le E_t \le \bar{E}, & (\underline{\theta}_t, \bar{\theta}_t) \\ \underline{G}_i^h \le g_{it}^h \le \bar{G}_i^h, & (\underline{\beta}_{it}, \bar{\beta}_{it}) \\ \sum_{i=1}^{M} g_{it}^h + q_t^g\beta - q_t^p/\alpha = D_t, & (\mu_t) \\ E_t - q_t^g + q_t^p = E_{t+1}, & (\gamma_{t+1}) \\ q_t^g \cdot q_t^p = 0 \end{cases} \tag{7.1}$$

(Huang et al., 2020; Xu et al. 2017).

The first three constraints in Eq. (7.1) represent the storage charging/discharging (pumping/generating) capacity constraints and the capacity constraints of energy in storage. The fourth constraint indicates the generation constraints of generators. The fifth line is the energy balance constraint for the electricity supply that satisfies the demand. Here, D_t is the electricity demand at period *t*, which is determined based on customers' bidding quantity in the electricity market, and we assume that it is known in advance. The energy storage state transition constraint from the current period *t* to the next period *t* + 1 is represented by the sixth constraint. Here, $q_t^g \cdot q_t^p = 0$ represents charging/pumping and discharging/generating, which cannot happen simultaneously (i.e., energy storage can only charge or discharge (pump or generate) during a single period).

The unit commitment decision includes binary decision variables in the model, making the dual model difficult to develop, especially when energy balance constraints are considered. To maintain traceable and analytical results, we analyze dispatch decisions only in Eq. (7.1), not unit commitment decisions. If $0 < \underline{G}_i^h$, all generators must run in this system (generating units). If $\underline{G}_i^h = 0$, this means the ISO has a unit commitment and economic dispatch problem for $t \in \{1, 2, 3, \cdots, T\}$. Here, g_{it}^h represents the decision variables/generation for other generators except storage.

Because the problem of Eq. (7.1) is not convex, to satisfy strong duality and keep the ISO's scheduling problem convex, we first relax the non-convex constraint $q_t^g \cdot q_t^p = 0$ and derive the Lagrange function. Then, from the perspective of the ISO, we identify the sufficient condition for avoiding simultaneous pumping/charging and generating/discharging using the KKT condition.

First, we get the following Lagrange function after relaxing this non-convex constraint $q_t^g \cdot q_t^p = 0$.

$$
\begin{aligned}
L &= \sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^h \cdot g_{it}^h + \sum_{t=1}^{T}\left(c^g q_t^g + c^p q_t^p\right) + \mu 1_t \left(D_t - \left(\sum_{i=1}^{M} g_{it}^h + q_t^g \beta - q_t^p / \alpha \right)\right) \\
&+ \gamma 1_{t+1}\left(E_{t+1} - E_t + q_t^g - q_t^p\right) + \underline{\chi} 1_t^p \cdot \left(-q_t^p\right) + \bar{\chi} 1_t^p \cdot \left(q_t^p - \bar{Q}^p\right) + \underline{\chi} 1_t^g \cdot \left(-q_t^g\right) + \bar{\chi} 1_t^g \cdot \left(q_t^g - \bar{Q}^g\right) \\
&+ \underline{\theta} 1_t \cdot \left(-E_t + \underline{E}\right) + \bar{\theta} 1_t \cdot \left(E_t - \bar{E}\right) + \underline{\beta} 1_{it} \cdot \left(-g_{it}^h + \underline{G}_i^h\right) + \bar{\beta} 1_{it}\left(g_{it}^h - \bar{G}_i^h\right)
\end{aligned}
\tag{7.2}
$$

where $\{\underline{\chi} 1_t^p, \underline{\chi} 1_t^g, \underline{\theta} 1_t, \underline{\beta} 1_{it}, \bar{\chi} 1_t^p, \bar{\chi} 1_t^g, \bar{\theta} 1_t, \bar{\beta} 1_{it}\} \geq 0$ are Lagrange multipliers.

Second, the KKT condition is utilized to determine whether there are sufficient conditions for such relaxation. The following proposition demonstrates the sufficient condition for $q_t^g \cdot q_t^p = 0$ holding. (All proofs are provided in Appendix A)

Proposition 7.1: We demonstrate the following sufficient condition for avoiding simultaneous pumping/charging and generating/discharging of energy storage from the perspectives of the ISO, and we get the same following condition:

$$c^g + c^p > -u1_t^*(1/\alpha - \beta) \tag{7.3}$$

where $u1_t^*$ is the Lagrange multiplier/shade price of the energy balance constraint (i.e., optimal LMP at period t), c^p and c^g is the energy storage's charging/discharging operating cost, and α and β are the charging and discharging efficiency, respectively.

The constraint $q_t^g \cdot q_t^p = 0$ is obviously always true for the positive electricity prices (i.e., $u1_t^* = LMP_t > 0$) by Proposition 7.1. This sufficient condition in Eq. (7.3) illustrates that, taking into account the efficiency loss and operating cost of the energy storage, for all positive electricity prices, generating and pumping cannot occur simultaneously in optimal economic dispatch solutions from the perspective of the ISO. When electricity prices are positive, the cost of conducting simultaneous dispatches is higher than the cost of conducting separate dispatches. Negative electricity prices are uncommon and rarely occur in most locational electricity markets (Zhou et al., 2016) due to transmission capacity constraints and electricity market monitoring, which is not the focus of this work. There, it can be shown that energy storage can be enforced to charge or discharge only in a single period for any positive electricity prices.

Thus, the non-convex constraint of $q_t^g \cdot q_t^p = 0$ can thus be relaxed because, for the ISO, strong duality theorem establishes that the duality problem can be written as (All proofs are provided in Appendix E).

$$\max\left(\begin{array}{l}\sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_{t}^{p}+\bar{Q}^{g}\cdot\bar{\chi}_{t}^{g}+\underline{E}\cdot\underline{\theta}_{t}+\bar{E}\cdot\bar{\theta}_{t}\right)+\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}+\bar{G}_{i}^{h}\cdot\bar{\beta}_{it}\right)\\+\sum_{t=1}^{T}\mu_{t}\cdot D_{t}-E_{1}\cdot\gamma_{2}+E_{T+1}\cdot\gamma_{T+1}\end{array}\right)$$

$$\text{s.t.}\begin{cases}\text{for } q_{t}^{p}:\ \underline{\chi}_{t}^{p}+\bar{\chi}_{t}^{p}-\mu_{t}/\alpha-\gamma_{t}=c^{p}/\alpha,\\ \text{for } q_{t}^{g}:\ \underline{\chi}_{t}^{g}+\bar{\chi}_{t}^{g}+\mu_{t}\beta+\gamma_{t}=c^{g}\beta,\\ \text{for } g_{it}^{h}:\ \underline{\beta}_{it}+\bar{\beta}_{it}+\mu_{t}=C_{it}^{h}, \qquad \forall t\in\{1,2,\cdots,T\}\\ \text{for } E_{t}:\ -\gamma_{t}+\gamma_{t+1}+\underline{\theta}_{t}+\bar{\theta}_{t}=0,\\ \text{Where}, \underline{\chi}_{t}^{p},\underline{\chi}_{t}^{g},\underline{\theta}_{t},\underline{\beta}_{it}\geq 0;\ \bar{\chi}_{t}^{p},\bar{\chi}_{t}^{g},\bar{\theta}_{t},\bar{\beta}_{it}\leq 0.\end{cases} \tag{7.4}$$

Here, $\{\underline{\chi}^{g}_{t},\bar{\chi}^{g}_{t},\underline{\chi}^{p}_{t},\bar{\chi}^{p}_{t},\underline{\theta}_{t},\bar{\theta}_{t},\underline{\beta}_{it},\bar{\beta}_{it},\mu_{t},\gamma_{2},\gamma_{t+1},\gamma_{T+1}\}$ represent the corresponding dual variables based on Eq. (7.1) constraints without considering the non-convex constraint of $q_t^g \cdot q_t^p = 0$. To achieve our research goal of analyzing the relationship between ISO and electricity merchants' optimal results, we create the primal and dual models from the perspective of merchants.

7.3.3. Primal-Dual Scheduling Models for Merchant and Generators. We examine the case when an electricity merchant has only a PSH (Secomandi, 2010; Zhou et al., 2016), and a traditional generator-*I* (Xu et al., 2017*)*. Both are connected to the markets via transmission lines. In the scheduling model, the merchant has to decide the amount of electricity to buy or sell in each period. In this work, we consider discrete time and that the two merchants periodically execute scheduling decisions during a limited horizon to maximize their expected profit based on the forecasted electricity price P_t; here, $t\in\{1,2,\cdots,T\}$.

Unlike Gianfreda and Bunn (2018) and Cruise et al. (2019), this work models the merchant as a price taker (Liu et al., 2022b; Zhou et al., 2016, 2019), which means the merchant's own dispatch decisions have no effect on market electricity prices. A price-taker PSH merchant's profit maximization objective includes storage scheduling profit (profit from selling energy to the market minus the cost of buying power to pump in PSH from the market) minus operating cost. To facilitate comparison and maintain the same optimization goal as the ISO's perspective, the modified merchant's optimal dispatching model is shown.

$$\max \sum_{t=1}^{T} [\underbrace{P_t\left(q_t^g\beta - q_t^p/\alpha\right)}_{\text{storage trading revenue}} - \underbrace{\left(c^p \cdot q_t^p + c^g \cdot q_t^g\right)}_{\text{storage operating cost}}] = \min\left(\sum_{t=1}^{T} -\left(P_t\left(q_t^g\beta - q_t^p/\alpha\right) + \left(c^p \cdot q_t^p + c^g \cdot q_t^g\right)\right)\right)$$

$$\text{s.t.}\begin{cases} 0 \le q_t^p \le \bar{Q}^p, & (\underline{\chi}^p_t, \bar{\chi}^p_t) \\ 0 \le q_t^g \le \bar{Q}^g, & (\underline{\chi}^g_t, \bar{\chi}^g_t) \\ \underline{E} \le E_t \le \bar{E}, & (\underline{\theta}_t, \bar{\theta}_t) \\ E_t - q_t^g + q_t^p = E_{t+1}, & (\gamma_{t+1}) \\ q_t^g \cdot q_t^p = 0 \end{cases} \tag{7.5}$$

The first three constraints indicate the PSH pumping capacity, generating capacity, and energy storage capacity. The fourth constraint is the storage energy balance constraint, and the last line denotes that pumping and generating cannot occur simultaneously, where $t \in \{1, 2, 3, \cdots, T\}$.

To maintain the convexity and satisfy a strong duality in the scheduling model of the merchant, similar to that of the ISO, first, we relax the non-convex constraint $q_t^g \cdot q_t^p = 0$ and build the corresponding Lagrange function of Eq. (7.5).

$$L=\sum_{t\in T}\Big(\big(c^{g}-P_t\beta\big)q_t^{g}+\big(c^{p}+P_t/\alpha\big)q_t^{p}\Big)+\gamma 2_{t+1}\big(E_{t+1}-E_t+q_t^{g}-q_t^{p}\big)$$
$$+\underline{\chi}2_t^{p}\cdot\big(-q_t^{p}\big)+\overline{\chi}2_t^{p}\cdot\big(q_t^{p}-\overline{Q}^{p}\big)+\underline{\chi}2_t^{g}\cdot\big(-q_t^{g}\big)+\overline{\chi}2_t^{g}\cdot\big(q_t^{g}-\overline{Q}^{g}\big) \tag{7.6}$$
$$+\underline{\theta}2_t\cdot\big(-E_t+\underline{E}\big)+\overline{\theta}2_t\cdot\big(E_t-\overline{E}\big)$$

Second, employing the KKT condition, we get the following sufficient condition for this relaxation, where P_t represents the forecasted price at period t.

$$c^{g}+c^{p}>-P_t(1/\alpha-\beta) \tag{7.7}$$

Thus, as the first contribution of this study, Eq. (7.7) always holds for any positive price $P_t\geq 0,\ \forall t=\{1,2,\cdots,T\}$. It also demonstrates that optimal pumping/charging and generating/discharging decisions cannot occur simultaneously for any positive electricity prices for the profit-maximizing merchant. If the ISO sends the cleared electricity prices to the merchant, there is $P_t=u1_t^{*}$, which is the same as that in Proposition 7.1. Therefore, in this case, inEq.s (7.7) and (7.3) are identical (all proofs are provided in Appendix A).

Similar to the ISO's model, after relaxing the non-convex constraint of $q_t^{g}\cdot q_t^{p}=0$, for the merchant, the strong duality theorem establishes that the duality problem can be written as:

$$\max\sum_{t=1}^{T}\Big(\overline{Q}^{p}\cdot\overline{\chi}_t^{p}+\overline{Q}^{g}\cdot\overline{\chi}_t^{g}+\underline{E}\cdot\underline{\theta}_t+\overline{E}\cdot\overline{\theta}_t\Big)-E_1\cdot\gamma_2+E_{T+1}\cdot\gamma_{T+1}$$
$$\text{s.t.}\begin{cases}\text{for } q_t^{p}\text{: } \underline{\chi}_t^{p}+\overline{\chi}_t^{p}-\gamma_t=\big(c^{p}+P_t\big)/\alpha, & \\ \text{for } q_t^{g}\text{: } \underline{\chi}_t^{g}+\overline{\chi}_t^{g}+\gamma_t=\big(c^{g}-P_t\big)\beta, & \forall t\in\{1,2,\cdots,T\}\\ \text{for } E_t\text{: } -\gamma_t+\gamma_{t+1}+\underline{\theta}_t+\overline{\theta}_t=0, & \\ \text{Where, } \underline{\chi}_t^{p},\underline{\chi}_t^{g},\underline{\theta}_t\geq 0;\overline{\chi}_t^{p},\overline{\chi}_t^{g},\overline{\theta}_t\leq 0 & \end{cases} \tag{7.8}$$

Here, $\{\underline{\chi}^{g}_{t}, \overline{\chi}^{g}_{t}, \underline{\chi}^{p}_{t}, \overline{\chi}^{p}_{t}, \underline{\theta}_{t}, \overline{\theta}_{t}, \mu_{t}, \gamma_{2}, \gamma_{t+1}, \gamma_{T+1}\}$ refer to the dual variables that correspond to the constraints in Eq. (7.5) without considering the non-convex constraint of $q^{g}_{t} \cdot q^{p}_{t} = 0$. For simplification and comparison, we use the same dual variables for the same constraints as that of the ISO's dual model.

The economics dispatch model for a traditional merchant operates a generator-*I* that takes into account the generator's constraints to maximize her expected profit, shown as follows (Hua and Baldick, 2017):

$$\max \sum_{t=1}^{T} \left(P_{t} - C^{h}_{It}\right) \cdot g^{h}_{It} \Leftrightarrow \min \sum_{t=1}^{T} \left(C^{h}_{It} - P_{t}\right) \cdot g^{h}_{It}$$

$$\text{s.t.} \begin{cases} g^{h}_{It} \geq \underline{G}^{h}_{I}, & (\underline{\beta}_{It}) \\ g^{h}_{It} \leq \overline{G}^{h}_{I}. & (\overline{\beta}_{It}) \end{cases} \tag{7.9}$$

Here, $\left\{\underline{\beta}_{It}, \overline{\beta}_{It}\right\}$ are the corresponding dual variables of Eq. (7.9). Therefore, the duality model of the primal problem in Eq. (7.9) is obtained below (Hua and Baldick, 2017; Schiro et al., 2016):

$$\max \sum_{t=1}^{T} \left(\underline{G}^{h}_{I} \cdot \underline{\beta}_{It} + \overline{G}^{h}_{I} \cdot \overline{\beta}_{It}\right)$$

$$\text{s.t.} \begin{cases} \underline{\beta}_{It} + \overline{\beta}_{It} = C^{h}_{It} - P_{t} \\ \underline{\beta}_{It} \geq 0, \overline{\beta}_{It} \leq 0. \end{cases} \tag{7.10}$$

In Eq. (7.1), the principal goal of an ISO in the electricity market is to maximize social welfare by dispatching the generators as well as supplying energy to customers at the lowest prices. For Eqs. (7.5) and (7.8), in contrast, the goal of an electricity merchant with energy storage or a traditional generator is to maximize her profit. According to Schiro et al. (2016) and Hua and Baldick (2017), if the ISO's objective function is convex and satisfies a strong duality, a perfect electricity price forecast provides incentives that enable generators' profit-maximizing actions to align with the ISO welfare-maximizing solutions. These authors, however, did not take into account energy storage. Thus, we investigate the relationship between the optimal dispatch decisions for energy storage and generators under these models when the ISO sends the cleared electricity prices to the merchant.

7.3.4. Optimal Scheduling Results Analysis Between ISO and Merchant. We identify the relationships between optimal solutions by Propositions 7.2 and 7.3, which allows us to optimize the objective functions in duality problems (Eqs. (7.4), (7.8), and (7.10)). Suppose that the merchant maximizes her profit by using the electricity prices cleared by the ISO or the merchant's forecasted prices perfectly (i.e., $P_t = \mu_t^* = LMP_t, \forall t \in \{1, 2, \cdots, T\}$). Using this assumption, we obtain the next proposition (see Appendix E).

Proposition 7.2A: Suppose we ignore the transmission efficiency loss and the transmission capacity as well as suppose that electricity prices are predicted perfectly (i.e., $P_t = \mu_t^* = LMP_t, \forall t \in \{1, 2, \cdots, T\}$). If the primal problem of merchant profit maximizing (i.e., Eq. (7.5)) has a unique optimal solution, then we can draw the following conclusion for the energy storage optimal actions $(q_t^{p*(S)}, q_t^{g*(S)})$ of the ISO and $(q_t^{p*(M)}, q_t^{g*(M)})$ of the electricity merchant:

$$\begin{cases} \text{1) If } C_{it}^h < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^h,\ q_t^{p*(S)} = q_t^{p*(M)}, \text{and } q_t^{g*(S)} = q_t^{g*(M)}; \\ \text{2) If } C_{it}^h = P_t = LMP_t \text{ and } \underline{G}_i^h \le g_{it}^{h*} \le \bar{G}_i^h,\ q_t^{p*(S)} = q_t^{p*(M)}, \text{and } q_t^{g*(S)} = q_t^{g*(M)}; \\ \text{3) If } C_{it}^h > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^h,\ q_t^{p*(S)} = q_t^{p*(M)}, \text{and } q_t^{g*(S)} = q_t^{g*(M)}. \end{cases} \quad (7.11)$$

If the lower bound of power generation of thermal generators is 0, that is, $\underline{G}_i^h = 0, \forall i \in \{1,2,\cdots,M\}$, then we can rewrite Eq. (7.11) as

$$\begin{cases} 1) \text{ If } C_{it}^h < P_t = LMP_t \text{ and } g_{it}^{h*} = \overline{G}_i^h,\ q_t^{p*(S)} = q_t^{p*(M)}, \text{and } q_t^{g*(S)} = q_t^{g*(M)}; \\ 2) \text{ If } C_{it}^h = P_t = LMP_t \text{ and } 0 \le g_{it}^{h*} \le \overline{G}_i^h,\ q_t^{p*(S)} = q_t^{p*(M)}, \text{and } q_t^{g*(S)} = q_t^{g*(M)}; \\ 3) \text{ If } C_{it}^h > P_t = LMP_t \text{ and } g_{it}^{h*} = 0,\ q_t^{p*(S)} = q_t^{p*(M)}, \text{and } q_t^{g*(S)} = q_t^{g*(M)}. \end{cases} \quad (7.12)$$

Proposition 7.2A shows the equivalent conditions for storage scheduling decisions from the perspective of the ISO and merchant, when the merchant's scheduling problem has a unique optimal solution. In period t, for *M* generating units, each generator *i* has a generation cost C_{it}^h. The following three equivalence conditions apply to the ISO and merchant while making optimal storage scheduling decisions:

(1) If the generating cost of the generator *i* is less than the cleared electricity price (i.e., $C_{it}^h < LMP_t$), the generation of the generator *i* reaches the upper bound of the generating $\overline{G}_i^h$ to maximize the social welfare.

(2) If the generating cost of the unit is equal to the cleared electricity price (i.e., $C_{it}^h = LMP_t$), then there is no additional generation constraint for the "marginal" generator; that is, the generation falls between the upper and lower limits of the unit. The optimal profit of the marginal generator is zero because cost equals income. In this situation, there are several optimal options from the generator's perspective; however, given the load balance constraint, there is only one optimal solution from the ISO's perspective.

(3) If the generating cost of the unit is greater than the cleared price (i.e., $C_{it}^h > LMP_t$), then the generating generator *i* reaches its lower boundary $\underline{G}_i^h$ to minimize the generation cost; in this case, the ISO and merchant will make identical optimal decisions. The

conditions and assumptions (Bo et al., 2021; Hua and Baldick, 2017) in these three scenarios have been discovered to always be true to minimize the generation cost, implying that, when the merchant has a unique optimal solution and the ISO can provide price information to the merchant, the ISO and merchant will arrive at the same optimal storage scheduling decision.

Proposition 7.2B: If the primal problem of the energy storage merchant (i.e., Eq. (7.5)) has multiple optimal solutions, then we can draw the following conclusion for the optimal actions $(q_t^{p*(S)}, q_t^{g*(S)})$ of the ISO and $(q_t^{p*(M)}, q_t^{g*(M)})$ of the merchant:

$$
\begin{aligned}
&\max\left(\sum_{t=1}^{T}\left(\mathrm{LMP}_t\left(q_t^g\beta - q_t^p/\alpha\right) - \left(c^p \cdot q_t^p + c^g \cdot q_t^p\right)\right)\right) \\
&= \sum_{t=1}^{T}\left(\mathrm{LMP}_t\left(q_t^{g*(S)}\beta - q_t^{p*(S)}/\alpha\right) - \left(c^p \cdot q_t^{p*(S)} + c^g \cdot q_t^{g*(S)}\right)\right) \qquad (7.13) \\
&= \sum_{t=1}^{T}\left(\mathrm{LMP}_t\left(q_t^{g*(M)}\beta - q_t^{p*(M)}/\alpha\right) - \left(c^p \cdot q_t^{p*(M)} + c^g \cdot q_t^{g*(M)}\right)\right)
\end{aligned}
$$

When the profit maximization problem for merchants has multiple optimal solutions, Proposition 7.2B shows the relationship between storage scheduling strategies from two perspectives. Assuming that the electricity merchant can get a cleared electricity price from the ISO based on social welfare maximization, the optimal profit earned by the merchant based on her profit maximization is equal to the profit obtained by following the ISO's scheduling.

Similar to the equivalent relationship for the optimal dispatches of energy storage between the perspective of the merchant and the ISO, the relations between the optimal actions $g_{it}^{h*(S)}$ of ISO and $g_{it}^{h*(M)}$ of generator $\forall i \in \{1,2,\cdots,M\}, \forall t \in \{1,2,\cdots,T\}$ can be drawn:

Proposition 7.3A: For the generator, if the forecasted price matches the actual LMPs, when the primal problem in Eq. (7.9) from generator profit-maximizing has a unique optimal solution:

$$\begin{cases} 1) \text{ If } C_{it}^{h} < P_t = LMP_t \text{ and } g_{it}^{h*} = \overline{G}_i^{h},\ g_{it}^{h*(S)} = g_{it}^{h*(M)}; \\ 2) \text{ If } C_{it}^{h} = P_t = LMP_t \text{ and } \underline{G}_i^{h} \le g_{it}^{h*} \le \overline{G}_i^{h},\ g_{it}^{h*(S)} = g_{it}^{h*(M)}\}; \\ 3) \text{ If } C_{it}^{h} > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^{h},\ g_{it}^{h*(S)} = g_{it}^{h*(M)}. \end{cases} \quad (7.14)$$

Proposition 7.3B: For the generator, if the forecasted price aligns with the actual LMPs and when the primal problem in Eq. (7.9) from generator profit-maximizing has multiple optimal solutions:

$$\max \sum_{t=1}^{T} \left(LMP_t - C_{It}^{h} \right) \cdot g_{It}^{h} = \sum_{t=1}^{T} \left(LMP_t - C_{It}^{h} \right) \cdot g_{It}^{h*(S)} = \sum_{t=1}^{T} \left(LMP_t - C_{It}^{h} \right) \cdot g_{It}^{h*(M)} \quad (7.15)$$

Proposition 7.3 shows the equivalent condition for the traditional generator i between the perspective of the ISO and generator by the strong duality theorem, which is consistent with the conclusions reported by Hua and Baldick (2017) and Baldick (2018), who focus solely on traditional generating firms and do not consider the energy storage scenario.

Propositions 7.2A and 7.3A demonstrate that, if an electricity merchant (resp. a generator) accurately predicts electricity prices or the ISO sends the cleared electricity prices to the merchant (resp. generator) based on a welfare-maximizing solution, the merchant with energy storage (resp. the traditional generator) and the ISO will arrive at the same optimal economic dispatch. Propositions 7.2B and 7.3B imply that merchants with storage or a generator have the incentive to follow the ISO schedule when ISOs are able to produce more accurate price forecasts and incorporate the operating characteristics of energy storage or a traditional generator directly, which is another contribution of this study. Propositions 7.2 yields Managerial Insight 7.1:

Managemental Insight 7.1: *An ISO dispatch decision that incorporates storage operating characteristics will be able to maximize both the social welfare of the system and the merchant's profit. This statement holds not only when there is a unique optimal solution (from the merchant's perspective) but also when there are multiple optimal solutions.*

This insight reflects that, for a strictly convex profit maximization problem of an electricity merchant and generator, a perfect price forecast would incentivize the merchant and generator—in response to the prices—to self-dispatch in a way that is consistent with the ISO's dispatch. The issue signifies the convexity of the electricity merchant's problem and satisfies the strong duality after relaxing the constraint that pumping and generating cannot happen simultaneously. Our findings not only strengthen those of previous studies but also restructure the results to accommodate the energy storage scenario.

7.4. OPTIMAL DECISIONS BETWEEN CO-OPTIMIZED MERCHANT AND ISO

In this Section, we addresses the case of an electricity merchant who owns and operates a renewable plant (we use wind farms to refer to renewable power plants for convenience) as well as PSH, both of which are connected to the electricity markets via transmission lines (Kim and Powell, 2011; Qi et al., 2015; Zhou et al., 2019). We investigate the relationship between joint optimal scheduling decisions between the merchant who has PSH and wind farms or generator owner to maximize her profit and the ISO who schedules the energy storage and wind farms directly in the electricity market to maximize the social welfare.

7.4.1. Model Setup. Unlike the merchants with storage only, as seen in Section 7.3, merchants with wind farms and storage must decide on the best energy storage scheduling and the best wind power generating decisions. In this Section, we use the same physical characteristics to define the energy storage/PSH constraints as was done in Section 7.3. We use w_t to represent the available *wind generation of* the wind plant in period t (in energy units/period). The vector $W = (w_1, w_2, \cdots, w_T)$ represents the sequential levels of available forecasted wind generation. Following previous work (Jiang and Powell, 2015a; Kim and Powell, 2011; Liu et al., 2022b; Qi et al., 2015; Zhou et al., 2019), wind generation is constrained by the maximum generation capacity $\bar{W}$ of the wind plant to show the uncertainty in modeling (i.e., $0 = \underline{W} \le w_t \le \bar{W}$).

For simplicity, the merchant would require the transmission capacity to be sufficiently large for her wind plant; thus, we do not consider the transmission capacity. The unit generation cost for wind is defined as c^w, and the wind power generation cost in period *t* is $c^w \cdot w_t$. Then, there are three decision variables: q_t^g, q_t^p, and w_t. Compared to what is presented in Section 7.3, from the perspective of the ISO, the objective function should include the cost of wind power generation. The scheduling model is shown below (Bo et al., 2021; Huang et al., 2020):

$$\min[\underbrace{\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h}\cdot g_{it}^{h}}_{\text{generating cost}}+\underbrace{\sum_{t=1}^{T}\left(c^{g}q_{t}^{g}+c^{p}q_{t}^{p}\right)}_{\text{ES operating cost}}+\underbrace{\sum_{t=1}^{T}c^{w}\cdot w_{t}}_{\text{wind genearation cost}}]$$

$$\text{s.t.}\begin{cases}0\le q_{t}^{p}\le \bar{Q}^{p},\\ 0\le q_{t}^{g}\le \bar{Q}^{g},\\ \underline{E}\le E_{t}\le \bar{E},\\ g_{it}^{h}\le g_{it}^{h}\le \bar{G}_{i}^{h},\\ E_{t}-q_{t}^{g}+q_{t}^{p}=E_{t+1},\\ q_{t}^{g}\cdot q_{t}^{p}=0,\\ \sum_{i=1}^{M} g_{it}^{h}+w_{t}+q_{t}^{g}\beta-q_{t}^{p}/\alpha=D_{t},\\ \underline{W}\le w_{t}\le \bar{W}.\end{cases} \tag{7.16}$$

The first six lines of constraints in the ISO scheduling Eq. (7.16) are the same as the problem in Section 7.3 as Eq. (7.4), as we employ the same energy storage psychical properties. The difference in Eq. (7.16) from Eq. (7.4) is that the electricity supply matches the demand balance constraint in the seventh line, on the energy supply side, which includes wind power generation w_t. In addition, we add a new constraint for wind power generation (i.e., $\underline{W}\le w_t \le \bar{W}$).

Similar to what is presented in Section 7.3, we first relax the non-convex constraint of $q_t^g\cdot q_t^p=0$ in Eq. (7.16) and get the Lagrange functions. Then, the KKT condition is used to analyze the sufficient conditions for $q_t^g\cdot q_t^p=0$. The same results are obtained as in Eq. (7.17), implying that pumping and generating cannot occur simultaneously. The non-convex constraint of $q_t^g\cdot q_t^p=0$ is confirmed to always hold for all positive electricity prices from the perspective of the ISO (all proofs are provided in Appendix E).

$$c^g + c^p > -u1_t^*(1/\alpha - \beta) \tag{7.17}$$

After relaxing the non-convex constraint, we maintain the relaxed ISO's scheduling problem convexity and conduct the corresponding dual problem as well as satisfy the strong duality. As previously stated, an electricity merchant with wind farms and storage has four options to maximize her profit: (a) storing all wind power production and purchasing electricity from the grid, (b) storing and selling partial wind power production, (c) remaining idle, and (d) discharging storage and selling all wind power production to the market. Thus, following previous studies (Liu et al., 2022b; Zhou et al., 2019), based on the above four actions, the reward function $R(q_t^g, q_t^p, w_t, P_t)$ from making the decision (q_t^g, q_t^p, w_t), which corresponds to time t, the forecast prices P_t, is defined as follows:

$$R(q_t^p, q_t^g, w_t, P_t) = \begin{cases} -P_t \cdot \left(q_t^p/\alpha - w_t\right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p + c^w \cdot w_t\right) & (q_t^p > w_t) \\ -P_t \cdot \left(q_t^p/\alpha - w_t\right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p + c^w \cdot w_t\right) & (0 \le q_t^p < w_t) \\ P_t \cdot \left(q_t^g \beta + w_t\right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p + c^w \cdot w_t\right) & (q_t^g \ge 0) \end{cases} \tag{7.18}$$

Similar to what is presented in Section 7.3, the transmission efficiency loss is ignored. The profit of the merchant obtained from trading energy from storage (revenue from selling electricity minus the cost of buying electricity) and the profit from wind power generation are included in the merchant's profit maximization objective. To facilitate comparison, the merchant's profit maximization objective is reformulated as a cost minimization problem, and the scheduling model is illustrated below:

$$\max \sum_{t=1}^{T} [\underbrace{(P_t\beta - c^g)q_t^g - (c^p + P_t/\alpha)q_t^p}_{\text{energy trading profit}} + \underbrace{(P_t - c^w)\cdot w_t}_{\text{wind generation profit}}]$$

$$= \min \sum_{t=1}^{T} \left((c^g - P_t\beta)q_t^g + (c^p + P_t/\alpha)q_t^p + (c^w - P_t)\cdot w_t \right)$$

$$\text{s.t.} \begin{cases} 0 \le q_t^p \le \bar{Q}^p, \\ 0 \le q_t^g \le \bar{Q}^g, \\ \underline{E} \le E_t \le \bar{E}, \\ E_t - q_t^g + q_t^p = E_{t+1}, \\ q_t^g \cdot q_t^p = 0, \\ \underline{W} \le w_t \le \bar{W}. \end{cases} \tag{7.19}$$

The constraint is assigned to wind power generation, represented as $\underline{W} \le w_t \le \bar{W}$. In the same way, we first relax the non-convex constraint $q_t^g \cdot q_t^p = 0$ and analyze the sufficient condition from the perspective of the merchant. We get the same sufficient condition shown as in (7.20) by developing the Lagrange function and KKT condition.

$$c^g + c^p > -P_t(1/\alpha - \beta) \tag{7.20}$$

Eqs. (7.17) and (7.20) show that the optimal charging/pumping and discharging/generating decisions cannot occur at the same time from the view of a profit-maximizing merchant who operates both wind farms and storage and the ISO.

7.4.2. Optimal Scheduling Results Analysis Between ISO and Merchant. We build the corresponding dual problem without considering this non-convex constraint to derive the relationship of optimal solutions between the co-optimization profit-maximizing profit merchant and social welfare-maximizing ISO. If the ISO sends the cleared electricity prices to the electricity merchant or the merchant can predict the electricity prices perfectly (i.e., $P_t = \mu_t^* = LMP_t, \forall t \in \{1, 2, \cdots, T\}$), according to strong duality theory, the following two statements are demonstrated (See Appendix B).

Proposition 7.4A: Assume that the transmission efficiency loss is neglected, that electricity prices are accurately predicted (i.e., $P_t = \mu_t^* = LMP_t, \forall t \in \{1, 2, \cdots, T\}$), or that the ISO provides the cleared prices to the merchant, and that the primal profit maximization problem of the merchant in Eq. (7.19) has a unique optimal solution. In such a situation, the following conclusion applies to the ISO's optimal actions $(q_t^{p*(S)}, q_t^{g*(S)}, w_t^{*(S)})$ and the joint wind and storage merchant's optimal actions $(q_t^{p*(M)}, q_t^{g*(M)}, w_t^{*(M)})$:

$$\begin{cases} \text{1) If } C_{it}^h < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^h, \text{ then } q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ \text{2) If } C_{it}^h = P_t = LMP_t \text{ and } \underline{G}_i^h \le g_{it}^{h*} \le \bar{G}_i^h, \text{ then } q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ \text{3) If } C_{it}^h > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^h, \text{ then } q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}. \end{cases} \quad (7.21)$$

If the lower bound of power generation of thermal generators is 0, that is $\underline{G}_i^h = 0, \forall i \in \{1, 2, \cdots, M\}$, then (7.21) can be rewritten as

$$\begin{cases} \text{1) If } C_{it}^h < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^h,\ q_t^{p*(S)} = q_t^{p*(M)}, q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ \text{2) If } C_{it}^h = P_t = LMP_t \text{ and } 0 \le g_{it}^{h*} \le \bar{G}_i^h,\ q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ \text{3) If } C_{it}^h > P_t = LMP_t \text{ and } g_{it}^{h*} = 0,\ q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}. \end{cases} \quad (7.22)$$

Proposition 7.4A shows the equivalent condition for optimal energy storage scheduling and wind generation decisions from the perspectives of the ISO and the electricity merchant. When the profit-maximization electricity merchant with wind farms and storage has a unique optimal solution, the equivalent condition for optimal wind generation and storage scheduling decision under the two perspectives is the same as that in Proposition 7.2A. This means that, if the merchant predicts prices accurately or the ISO distributes the cleared electricity prices based on social welfare maximization to the merchant, and the merchant utilizes those prices to construct a schedule, the ISO and merchant will arrive at the same optimal economic dispatch for storage and wind farms.

Proposition 7.4B: If the primal problem of the merchant's operating both storage and a wind farm has multiple optimal solutions (i.e., Eq. (7.19)), then the following is the conclusion for the optimal actions $(q_t^{p*(S)}, q_t^{g*(S)}, w_t^{*(S)})$ of the ISO and $(q_t^{p*(M)}, q_t^{g*(M)}, w_t^{*(M)})$ of the merchant:

$$\begin{aligned} &\max\left(\sum_{t=1}^{T}\left(LMP_t\left(q_t^g\beta - q_t^p/\alpha\right) + \left(LMP_t - c^w\right)\cdot w_t - \left(c^p \cdot q_t^p + c^g \cdot q_t^p\right)\right)\right) \\ &= \sum_{t=1}^{T}\left(LMP_t\left(q_t^{g*(S)}\beta - q_t^{p*(S)}/\alpha\right) + \left(LMP_t - c^w\right)\cdot w_t^{*(S)} - \left(c^p \cdot q_t^{p*(S)} + c^g \cdot q_t^{g*(S)}\right)\right) \\ &= \sum_{t=1}^{T}\left(LMP_t\left(q_t^{g*(M)}\beta - q_t^{p*(M)}/\alpha\right) + \left(LMP_t - c^w\right)\cdot w_t^{*(M)} - \left(c^p \cdot q_t^{p*(M)} + c^g \cdot q_t^{g*(M)}\right)\right) \end{aligned} \quad (7.23)$$

When the electricity merchant's optimal scheduling problem has multiple optimal solutions, Proposition 7.4B shows that the profit obtained by the merchant based on her profit maximization is always the same as the profit obtained by that of the merchant who follows the ISO's scheduling, implying that merchants with wind farms and storage have a motivation to follow the ISO's schedule when ISOs are able to produce more accurate price forecasts because the ISO has the most comprehensive information about the market operation. This finding shows that a perfect price forecast would motivate a merchant to self-dispatch energy storage and wind farms in a way that is consistent with the ISO's economics dispatch.

Similar to the traditional generator merchant scenario proposed in Section 7.4 (see Proposition 7.3), Hua and Baldick (2017) and Baldick (2018) observed that, when the ISO sends the cleared price to generators, the generating companies' individual profit-maximizing decisions align with the ISO's welfare-maximizing solution. Propositions 7.4 results in Managerial Insight 7.2:

Managemental Insight 7.2: *An ISO dispatch decision that incorporates storage operating characteristics and uncertain wind generation will be able to maximize the social welfare and the merchant's profit. This statement holds not only when there is a unique optimal solution (from the profit-maximizing view of the power merchant, the social welfare perspective of the ISO, or both) but also when there are multiple optimal solutions. It implies that letting the ISO schedule PSH, wind farms, and generators will improve the social welfare and maximize the profit of the electricity merchant.*

This insight indicates that, for a strictly convex profit maximization problem, a perfect price forecast would incentivize the merchant, in response to the prices, to self-dispatch the energy storage and wind farms or generators in a way that is consistent with the ISO's dispatch. If the merchant optimizes only her expected profit based on the forecasted prices, the accurate price forecast would help her to profit. Obviously, if the merchant's forecasted prices match the actual LMP, she will make the greatest profit. As the ISO may be better equipped with information to obtain more accurate price predictions to guide the dispatch decision and, therefore, achieve higher social welfare for the market as well as higher profit for each participant, the wind-and-storage merchant or generator owner has the incentive to let the ISO control her operations. The merchant-ISO equivalent connection for optimal decisions is novel and instructive, giving the ISO a new set of considerations when scheduling energy storage, renewable power plants, and generators.

7.5. NUMERICAL SIMULATION AND CASE STUDY

In Sections 7.3 and 7.4, we recommend the best scheduling strategies for electricity merchants who own energy storage and those who have both storage and wind farms. This research yields some significant managerial insights by utilizing strong duality theory under the assumption that the profit maximization merchants can forecast prices perfectly or that the electricity generation cost minimization ISO can send the cleared electricity prices (or share all the LMP created data) to the merchant and generators. In this section, we utilize a real-world example to verify our findings.

Considering the state transition time of PSH, in this study, we concentrate only on the day-ahead market and on hourly-period optimization for numerical simulation. Thus, the state transition behaviors of PSH can be completed within an hour (Wang et al., 2021). We use 1-hour time units for the power prices series in the day-ahead market $\mathrm{LMP} = \{\mathrm{LMP}_1, \mathrm{LMP}_2, \ldots, \mathrm{LMP}_T\}$ (\$/MW). To be consistent with the model in Sections 7.3 and 7.4, we consider the linear generation cost and ignore the transmission efficiency and line capacity. Then, the analytical results from the ISO and merchant perspectives are tested in both scenarios, taking into consideration the storage merchant or joint wind-storage merchant, respectively. The minimum and maximum storage levels (lower and upper reservoir) $\underline{E}$ and $\overline{E}$ are given the values 2 and 20, respectively. In this case, $\underline{E} > 0$ means that the storage cannot be emptied, which is a realistic statement for a PSH. The maximum discharging and charging (generating and pumping) capacity are $\overline{Q}^p = 2$ and $\overline{Q}^g = 2$, respectively. Units of GW hours are used for energy quantities. Both charging/pumping and discharging/generating rates are measured in GW units. In this case, the charging and discharging efficiency is $\alpha = \beta = 0.9$. We assume that the PSH takes $(\overline{E} - \underline{E})/\overline{Q}^g = 9$ hours to empty the upper reservoir and $(\overline{E} - \underline{E})/\overline{Q}^p = 9$ hours to refill it; these values correspond to the approximate times exhibited by the pumped storage plant in Ludington, Michigan (

seehttps://www.consumersenergy.com/company/electricgeneration/renewables/hydroelectric/pumped-storage-hydro-electricity for details).

In this section, we consider the linear generation costs in the ISO and generator model as well as ignore the value of water in the storage at the end of the optimization horizon. First, we consider storage merchants and verify the results delivered by Proposition 7.2 in Section 7.3. We choose the actual model of MISO—about 5,000 generators (e.g., thermal, nuclear) and only one PSH plant (e.g., Ludington)—and neglect the transmission line's capacity in the system. Then, we randomly select a thermal generator-*I* as the traditional generator merchant. We use the day-ahead hourly load as our measure of demand, with 24 periods that correspond to the 1-day period of January 1, 2022 (data on the load and wind generation are available at https://www.misoenergy.org/).

We developed an analytical expression for the relationship between the electricity merchant's optimal actions and the ISO's optimal scheduling decision under two scenarios, separately. Therefore, in Section 7.5.1, we verify the results for an electricity merchant who has only a PSH merchant and a traditional generator. Then, we compare the optimal scheduling for the ISO and merchant with both storage and a wind farm.

7.5.1. Numerical Simulation for Scenario 1. The plots in Figures 7.1 and 7.2 correspond to the Ludington PSH optimal scheduling—with initial energy in the storage of 2 GW hours (Figure 7.1) and 10 GW hours (Figure 7.2). The optimal decisions from the perspectives of the ISO and the PSH owner are shown in each figure. The parenthetical values of 0.9 and 0.8 represent the generating/pumping (discharging/charging) efficiency of storage in the optimization.

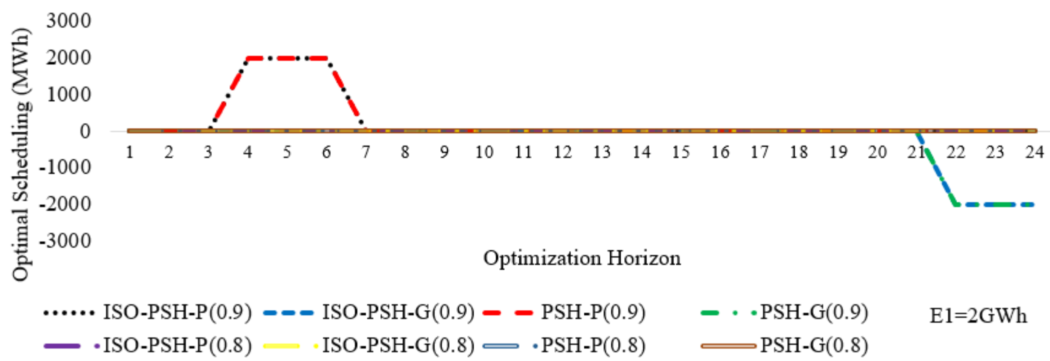

Figure 7.1 Optimal policy that considers a PSH merchant when $E_1 = 2$ GWh

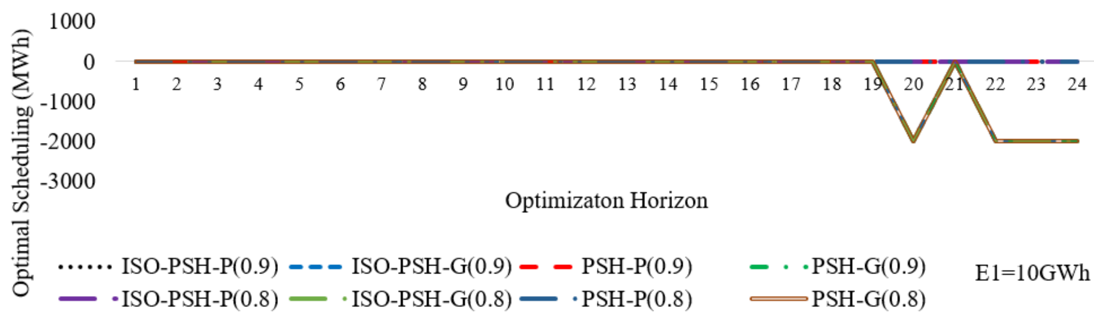

Figure 7.2 Optimal policy that considers a PSH merchant when $E_1 = 10$ GWh

In these figures, ISO-PSH-P (resp., ISO-PSH-G) indicates the optimal pumping (resp., generating) scheduling of PSH from the ISO's social welfare-maximizing perspective, and PSH-P (resp., PSH-G) indicates the optimal pumping (resp., generating) scheduling of PSH from the electricity merchant's profit-maximizing perspective.

Figure 7.3 shows the optimal generation of a traditional generator from the two perspectives. ISO-G indicates the ISO's social welfare-maximizing perspective, and Mer-G indicates the optimal generation from the generator's profit-maximizing perspective.

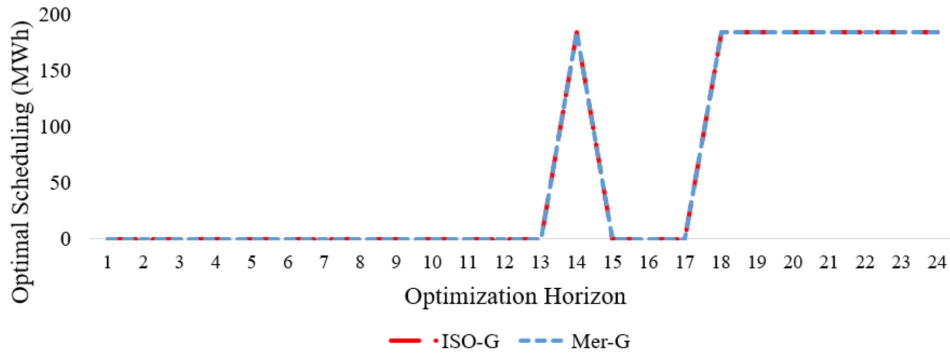

Figure 7.3 Optimal policy for a traditional generator

Each figure confirms that the profit-maximizing generator complies with the ISO's dispatch decisions if that merchant can accurately predict the actual electricity price that the ISO sends to each generating unit while considering the efficiency loss of storage.

7.5.2. Numerical Simulation for Scenario2. We consider the scenario in which the merchant operates storage and a wind farm to verify the results delivered by Proposition 7.4 in Section 7.4. On the basis of the above numerical simulation, we add a new decision variable of wind power generation, w_t, with $T = 24$ periods that correspond to a 1-day period of January 1, 2022, in MISO (power prices and wind production values are available at https://www.misoenergy.org/). The wind power generation has generation capacity constraints, and the lower and upper limits ($\underline{W}$ and $\overline{W}$) are given the values 0 and 2 GW hours, respectively.

Figures 7.4 and 7.5 show the optimal scheduling with the initial 2 GW hours (Figure 4) and 10 GW hours (Figure 7.5). Different from the figures above, the ISO and electricity merchant decide the optimal pumping and generation scheduling and determine the amount of wind power generation. The plots in Figures 7.4 and 7.5 also consider the optimal results under the generating/pumping (discharging/charging) efficiency of storage as 0.9 and 0.8.

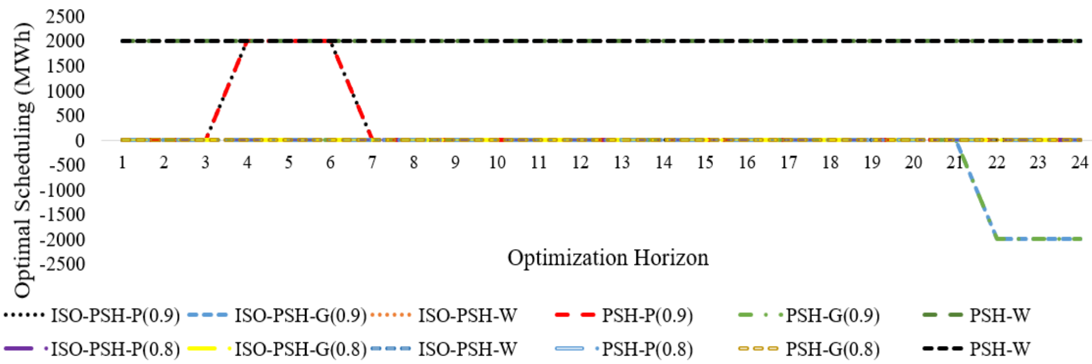

Figure 7.4 Optimal policy that considers a co-optimization merchant when $E_1 = 2\text{GWh}$

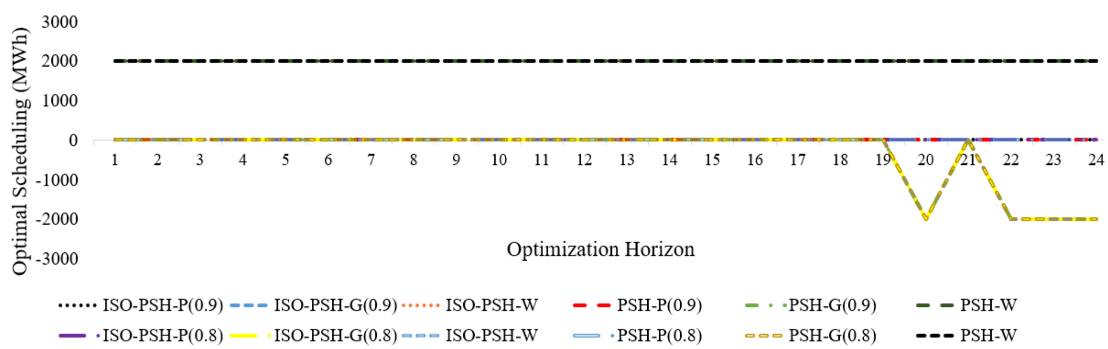

Figure 7.5 Optimal policy that considers a co-optimization merchant when $E_1 = 10\text{GWh}$

Figures 7.4 and 7.5 add the optimal solution for wind generation from the perspective of the ISO and merchant. ISO-PSH-W indicates the optimal wind power generation from the ISO's social welfare-maximizing perspective, while PSH-W represents the optimal wind generation from the perspective of the PSH merchant based on her profit maximization. Again, these figures confirm the conclusion in Section 7.4 that the electricity merchant who utilizes the wind farms and PSH also arrives at the same optimal solutions as the ISO.

The ISO's social welfare maximizing goal aligns with the merchant's profit maximization goal based on accurate price information. If we can get an exact pricing estimate, we won't have to deal with ISO's large-scale economic dispatch problem, which is taking a long time to solve. Rather, we may solve the profit maximization problem for each merchant by sending price information to them. Then, because each problem is extremely small and readily addressed, parallel computing can be utilized to handle many such merchant problems at the same time.

7.6. CONSIDERING THE PRODUCTION TAX CREDIT(PTC)

The optimal decisions derived in Section 7.4 consider the energy storage capacity and discharging/generating and charging/pumping capacity constraints of PSH. We also consider two types of efficiency related to PSH storage: the operating cost of storage and the generation cost of wind farms. In practice, however, generous economic subsidies, such as PTCs, are offered by the federal to state government to promote the development of renewable sources of power; these subsidies have reduced U.S. wind power costs by 70% (Cullen, 2013; DOE, 2016; Siler-Evans et al., 2013). Moreover, PTCs have spurred more than $143 billion worth of private investment in U.S. wind farms over the past ten years (https://www.awea.org/policy-and-issues/tax-policy).

In this Section, we address the situation of a wind farm merchant with storage (PSH or battery) who receives PTCs and so is prohibited from purchasing electricity from the market to store (the policy 2 in Section 6). In other words, storing energy from the market will disqualify her from receiving PTCs; hence, in each period, the power stored by the merchant cannot exceed her wind production (Liu et al., 2022b). PTCs will significantly affect the scheduling decision of merchants who operate a wind farm and storage. We propose a model that captures the scenario just described. We then derive the conclusion that the optimal scheduling decisions between profit-maximizing merchant and social welfare-maximizing ISO still hold.

We continue to assume that the merchant cannot simultaneously sell electricity to the grid and purchase power from the market. Because merchants with PTC-subsidized wind farms cannot buy electricity from the grid for purposes of storage, there are three possible actions: (a) storing and selling partial wind generation, (b) remaining idle, or (c) discharging storage and selling all wind generation to the grid. We follow the model developed in Section 7.4 and incorporate the subsidy(s) into the profit function for the wind-generated electricity sold to the grid, where the updated reward functions are:

$$R^{(PTC)}(q_t^p, q_t^g, w_t, P_t) = \begin{cases} -P_t(\frac{q_t^p}{\alpha} - w_t) - s(\frac{q_t^p}{\alpha} - w_t) - c_w w_t - c_p q_t^p & (0 \le q_t^p \le \alpha w_t) \\ P_t \cdot (q_t^g \beta + w_t) - c_w w_t - s(-q_t^g \beta - w_t) - c_g q_t^g & (q_t^g < 0) \end{cases} \quad (7.24)$$

The PTC in the (·) subscript indicates the situation in which the merchant receives PTCs. Following Sherlock (2020), this work also assumes that the PTC is a per-kilowatt-hour (kWh) tax credit for electricity generated using qualified energy resources. The first equation gives the reward when the merchant commits some of her wind generation q_t^p to storage and sells the remaining units $(g_t - q_t/\alpha)$ to the market. The additional term $s \cdot (g_t - q_t/\alpha)$ represents the federally subsidized wind power that is being sold to the market. In the second equation, $(q_t\beta - g_t)$ corresponds to the electricity merchant's releasing power from storage to the grid and selling all her wind-generated energy. Among all energy sold to the market, the term $s \cdot g_t$ captures the federal subsidy for wind energy.

From the perspective of the ISO, as compared with what was presented in Section 7.4, the wind-generation cost will be decreased due to the subsidy when the wind production is sold to the electricity market. Wind production that is stored, in contrast, is not currently eligible for the government's PTC subsidy, but it was when it was sold to the market. With the PTC in consideration, the scheduling problem of ISOs can be reformulated as Eq. (7.25).

$$\begin{aligned}&\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}-s\beta\right)\cdot q_{t}^{g}+\sum_{t=1}^{T}\left(c^{p}+s/\alpha\right)\cdot q_{t}^{p}+\sum_{t=1}^{T}\left(c^{w}-s\right)w_{t}\right)\\&\text{s.t.}\begin{cases}q_{t}^{p}-\alpha w_{t}\leq 0,\\ \text{Other constraints are the same as those in Section 7.4.}\end{cases}\quad \forall t\in\{1,2,\cdots,T\}\end{aligned}\tag{7.25}$$

Considering the PTC from the perspective of an electricity merchant who has a wind farm and storage, to facilitate comparison and maintain the same optimization goal as an ISO's perspective, Eq. (7.26) is modified, and the updated scheduling problem of merchant is shown below:

$$\begin{aligned}&\min\sum_{t\in T}\left(\left(c^{g}-P_{t}\beta-s\beta\right)q_{t}^{g}+\left(c^{p}+P_{t}/\alpha+s/\alpha\right)q_{t}^{p}+\left(c^{w}-P_{t}-s\right)\cdot w_{t}\right)\\&\text{s.t.}\begin{cases}q_{t}^{p}-\alpha w_{t}\leq 0,\\ \text{Other constraints are the same as those in Section 7.4.}\end{cases}\quad \forall t\in\{1,2,\cdots,T\}\end{aligned}\tag{7.26}$$

Similar to what is seen in Sections 7.3 and 7.4, we relax the non-convex constraint by the KKT condition and find the sufficient condition from the perspectives of both the ISO and the merchant:

$$c^{g}+c^{p}+s\cdot\left(1/\alpha-\beta\right)>-LMP_{t}(1/\alpha-\beta)\tag{7.27}$$

Eq. (7.27) also guarantees that the optimal charging/pumping and discharging/ generating actions cannot occur simultaneously for any positive prices from the perspectives of the social welfare-maximizing ISO and the profit-maximizing electricity merchant. The relationships of the optimal scheduling decisions between the ISO and merchant who have energy storage and wind farms are then established using the strong duality theorem, which is the same as Proposition 7.4 (all proofs are provided in Appendix E). In addition, based on the centralized ISO electricity generation cost-minimizing model and the expected profit-maximizing problem of traditional decentralized generators (e.g., thermal, nuclear, natural gas), we reach the same conclusions as Hua and Baldick (2017) and Baldick (2018), who discovered that traditional generators have the same optimal economic dispatch decisions.

7.7. A WIND FARM MERCHANT WITH TWO CONNECTED PSH RESERVOIRS

A typical PSH system includes two reservoirs at different elevations, with the upper reservoir's storing hydraulic potential energy. During off-peak periods, the merchant can store energy by pumping water from the lower reservoir to the upper reservoir and then generate energy to sell during peak periods by releasing water from the upper reservoir to the lower reservoir (Al-Swaiti et al., 2017; Avci et al., 2021; Ding et al., 2014). In this Section, we address the co-optimization of a wind farm and a PSH facility in a power system with two connected reservoirs.

For the PSH facility, energy can be stored by pumping water from the lower reservoir to the upper reservoir and be generated by releasing water from the upper reservoir to the lower reservoir. We assume that q_t^p and q_t^r represent the dispatch decisions, which indicate the energy change in the upper and lower reservoirs (i.e., the electricity quantity of pumping and releasing). In each period, let $\bar{Q}^p$ denotes the maximum amount of water that can be pumped from the lower reservoir to the upper reservoir, and $\bar{Q}^r$ denotes the maximum amount of water that can be released from the upper reservoir to the lower reservoir. Both the upper and lower reservoirs have capacity constraints. Upper boundaries are denoted by $\bar{E}^U$ and $\bar{E}^L$, whereas lower limits are 0 (i.e., $\underline{E}^U = \underline{E}^L = 0$) for the upper and lower reservoirs, respectively, and water overflow is not permitted. In period t, the amount of water/energy in the upper and lower reservoirs is given as E_t^U and E_t^L.

To simplify and model this scenario, we assume that the PHS is a closed-loop facility with no natural inflow or spill into either reservoir (Lu et al., 2018). The ISO scheduling model, which includes a PSH, a wind farm, and other traditional generators, is presented below:

$$\min[\underbrace{\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^h \cdot g_{it}^h}_{\text{electricity generation cost}} + \underbrace{\sum_{t=1}^{T}\left(c^r q_t^r + c^p q_t^p\right)}_{\text{PSH operating cost}} + \underbrace{\sum_{t=1}^{T} c^w \cdot w_t}_{\text{wind generation cost}}]$$

$$\text{s.t.}\begin{cases} 0 \le q_t^p \le \bar{Q}^p, q_t^p \le E_t^L, \\ 0 \le q_t^r \le \bar{Q}^r, q_t^r \le E_t^U, \\ 0 \le E_t^U \le \bar{E}^U, 0 \le E_t^L \le \bar{E}^L, \\ E_t^U - q_t^r + q_t^p = E_{t+1}^U, \\ E_t^L + q_t^r - q_t^p = E_{t+1}^L, \\ \underline{G}_i^h \le g_{it}^h \le \bar{G}_i^h, \\ \sum_{i=1}^{M} g_{it}^h + q_t^r \beta - q_t^p / \alpha = D_t, \\ \underline{W} \le w_t \le \bar{W}, \\ q_t^r \cdot q_t^p = 0. \end{cases} \tag{7.28}$$

The model differs from the traditional energy storage model if both the lower and upper reservoirs are included. First, we evaluate not only the upper reservoir's energy balance but also that of the lower reservoir. Second, in contrast to the analyses in Sections 7.3 and 7.4, pumping dispatch decisions are limited by pumping capacity, upper reservoir available space, and amount of energy available in the lower reservoir. Third, generating activities are limited by the quantity of energy available in the upper reservoir, generating capacity, and the amount of available space in the lower reservoir that can be stored.

Similarly, the merchant who operates a wind farm and a PSH can still take four actions: (a) pumping water from the lower reservoir to store energy in the upper reservoir using both wind power generation and electricity purchased from the market, (b) pumping water from the lower reservoir to store energy in the upper reservoir using partial wind power generation and selling the remaining wind generation to the market, (c) releasing the water from the upper reservoir and selling all the wind generation to the market, and (d) keeping unchanged or idle. The merchant's scheduling model is obtained as Eq. (7.29):

$$\max \sum_{t=1}^{T} \left(P_t \cdot \left(q_t^r \beta - q_t^p / \alpha \right) - \left(c^r \cdot q_t^r + c^p \cdot q_t^p \right) + \sum_{t=1}^{T} \left(P_t - c^w \right) \cdot w_t \right)$$

$$\Leftrightarrow \min \left(\sum_{t \in T} \left(\left(c^r - P_t \beta \right) q_t^r + \left(c^p + P_t / \alpha \right) q_t^p \right) + \sum_{t=1}^{T} \left(c^w - P_t \right) \cdot w_t \right)$$

$$\text{s.t.} \begin{cases} 0 \le q_t^p \le \bar{Q}^p, q_t^p \le E_t^L, \\ 0 \le q_t^r \le \bar{Q}^r, q_t^r \le E_t^U, \\ 0 \le E_t^U \le \overline{E}^U, 0 \le E_t^L \le \overline{E}^L, \\ E_t^U - q_t^r + q_t^p = E_{t+1}^U, E_t^L + q_t^r - q_t^p = E_{t+1}^L, \\ \underline{W} \le w_t \le \bar{W}, \\ q_t^r \cdot q_t^p = 0. \end{cases} \tag{7.29}$$

Using the same approach as in Sections 7.3 and 7.4, first, the non-convex constraint of $q_t^r \cdot q_t^p = 0$ that the pumping and releasing cannot occur concurrently can be demonstrated by holding on to all positive electricity prices from the perspectives of the ISO and merchant. The results for the equivalent condition for the merchant's and generator owner's optimal dispatch decisions are, thus, the same as in Section 7.4 (all proofs are included in Appendix E).

7.8. QUADRATIC ELECTRICITY GENERATION COST

The results presented in Sections 7.3 and 7.4 are based on the assumption that generators have linear electricity generating costs, enabling us to build a dual problem and derive the findings and insight. Many studies, such as Sioshansi (2014), Hua and Baldick (2017), and Yu et al. (2020), have modeled the electricity generation cost as a quadratic function, which is more precise and realistic as compared to the linear electricity generation cost. In this Section, we extend our model presented in Sections 7.3 and 7.4 by adopting quadratic electricity generation costs and investigating the optimal decisions in regard to the relationship between the ISO and electricity merchant.

7.8.1. Model Setup. When the generating cost function for generators is characterized as a quadratic function, the optimization model can be reformulated as follows from the perspective of the ISO, who schedules energy storage and other generators:

$$\min[\underbrace{\sum_{t=1}^{T}\sum_{i=1}^{M}\left(a_{it}\cdot\left(g_{it}^{h}\right)^{2}+b_{it}\cdot g_{it}^{h}\right)}_{\text{electricity generation cost}}+\underbrace{\sum_{t=1}^{T}\left(c^{p}\cdot q_{t}^{p}+c^{g}\cdot q_{t}^{g}\right)}_{\text{storage operating cost}}] \tag{7.30}$$

s.t., The constraints are the same as (7.1) in Section 7.3.

The electricity-generation cost for other generators and the storage operation cost are included in the objective function of the ISO scheduling model. Following the previous studies (Hua and Baldick, 2017; Sioshansi, 2014; Wang et al., 2021; Yu et al., 2020), the electricity generation cost of the generators is modeled as a quadratic function, as shown in Eq. (7.30), and the constraints in Eq. (7.30) are the same as Eq. (7.1) in Section 7.3. The challenge of the problem in Eq. (7.30) is represented in quadratic programming, so obtaining the dual problem based on strong duality theory is challenge. As a result, the analytical conclusions about the optimal solution for the relationship between profit-maximizing merchants and social welfare-maximizing ISOs cannot be achieved using the approaches utilized in Section 7.3.

Further, when a merchant operates storage and a wind farm, modeling the electricity cost function of generating generators is a quadratic function. The ISO's model is given in Eq. (7.31):

$$\min[\underbrace{\sum_{t=1}^{T}\sum_{i=1}^{M}\left(a_{it}\cdot\left(g_{it}^{h}\right)^{2}+b_{it}\cdot g_{it}^{h}\right)}_{\text{electricity generation cost}}+\underbrace{\sum_{t=1}^{T}\left(c^{g}q_{t}^{g}+c^{p}q_{t}^{p}\right)}_{\text{storage operating cost}}+\underbrace{\sum_{t=1}^{T}c^{w}w_{t}}_{\text{wind generation cost}}] \tag{7.31}$$

$$\text{s.t., The constraints are the same those in Section 7.4.}$$

The objective function in Eq. (7.31) includes the (a) electricity generation cost of traditional generators, (b) operating cost of energy storage, and (c) cost of wind-power generation. It is too complex to develop the dual model when the ISO's problems are characterized as a quadratic function. Further, it is difficult to obtain analytical findings using the Lagrange function when the energy balance constraint is considered. This Section presents data simulation to test the aforementioned results to see whether the conclusions made using the linear cost function hold in the two scenarios below.

7.8.2. Numerical Simulation and Analysis. We test two scenarios: a merchant who owns only energy storage and another who operates both storage and wind farms. Except for the data for the coefficients of power-generating cost, we use the same data as in Section 7.5 when doing numerical simulation.

Figure 7.6 shows the optimal decisions from the perspectives of the ISO and merchant, with initial storage energy of 2 GW hours under the two scenarios. The parenthetical value of 0.9 represents the PSH pumping or generating efficiency, and N represents the quadratic electricity generation cost. The other labels in the figures have the same meaning as those in Section 7.5.

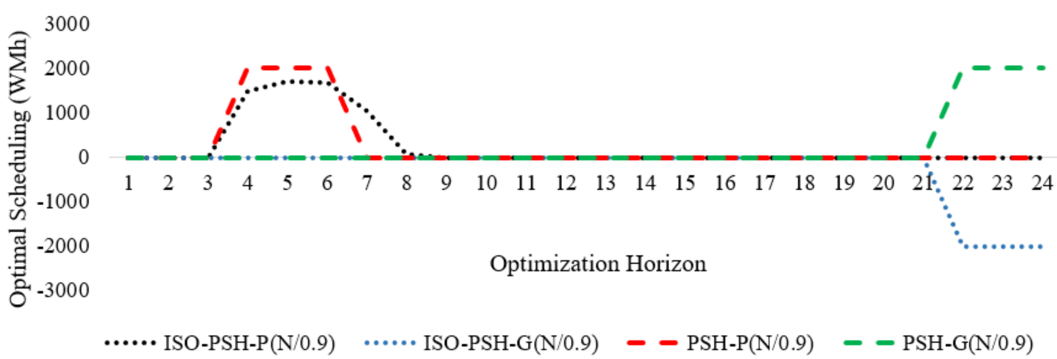

Figure 7.6 Optimal policy, considering a PSH merchant and quadratic generation cost

When the initial SOC in the storage E1 = 2GMh and the pumping/generating efficiency is 0.9, Figure 7.6 illustrates the optimal policy from the ISO and PSH merchant perspectives, taking into account the quadratic electricity generating cost. The simulation result reveals that the optimal decisions for energy storage differ from the perspectives of the ISO and merchant; nonetheless, the merchant can achieve the same profits by following the ISO schedule or pursuing her own profit maximization (which is $168,019.559). This implies that the merchant, ISO, or both have various optimal options, which is consistent with Proposition 7.2B's implications.

Figure 7.7 depicts the best policy from the ISO's and electricity merchant's viewpoints, taking into account both wind farms and energy storage as well as the quadratic generating cost of other generators in the system. The optimal scheduling decision for energy storage from the social welfare maximizing ISO differs from that of the profit-maximization merchant, similar to the results seen in Figure 7.5, whereas optimal wind power generation always reaches the upper limit. Both the merchant and ISO benefit from lower wind-generation costs. In addition, the merchant can still make the same optimal profit (which is $5,711,217.335) by following the ISO's schedule or maximizing her own profit. The results indicate that the optimization problem of the merchant, ISO, or both have numerous optimal solutions and are consistent with Proposition 7.3B's finding.

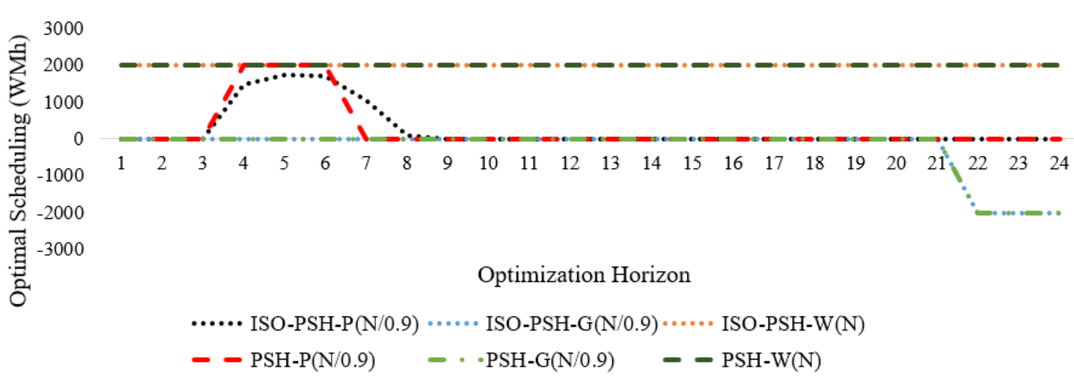

Figure 7.7 Optimal policy, considering a co-optimization merchant and quadratic electricity generation cost

Figures 7.6 and 7.7 show that the conclusions obtained based on linear electricity-generation costs are still valid under two scenarios when the generating cost of units is characterized as a quadratic function. Although the dispatch options may be different from the two optimization goals when the ISO sends the cleared prices to the merchant or the merchant can accurately predict the electricity prices, the merchant can earn the same optimal profit by seeking her own profit or following the ISO's schedule. Regardless of the expected effect of this alternative definition of the optimal actions, the qualitative insights acquired before are unchanged.

7.9. SECTION SUMMARY AND ANALYSIS

Independent system operators, such as MISO, are improving their approaches to integrating energy storage and renewable energy resources into electricity markets so that they can benefit from the resources' fast-ramping capabilities and bring significant value to the grid. Following the current market rules, whereby the electricity merchant optimizes charging and discharging hours to maximize her expected profit, this study investigates whether allowing the ISO to optimize energy storage and generators while taking into account energy storage and generators' capacity constraints, the multi-stage clearing process of electricity markets, uncertainty of renewable generation, and other factors can benefit the system and energy storage owners as well as generators.

This is the first study that utilizes duality theory to investigate the interaction between profit-maximizing electricity merchants and social welfare-maximizing ISOs while considering energy storage and renewable power plants. For energy storage merchants, we concentrated on two scenarios: those with only storage and those with a wind farm and storage. Unlike the classical studies of Secomandi (2010) and Zhou et al. (2016, 2019), which investigated how to obtain analytically optimal scheduling solutions for energy storage, we investigated the relationship between optimal economic dispatch decisions from two perspectives. As a result, we were able to identify equivalent conditions and achieve managerial insight. Our results indicate that we don't need to address the large-scale UCED problem if we have accurate price forecasts (because it takes a lot of time to solve). Rather, we can address the problem of profit maximization for each merchant or generator (although there are many such problems, one for each merchant or generator, but each problem is very small and can be solved easily, and we can use parallel computing to solve many such problems simultaneously).

For an energy storage merchant or generator, under the assumption that the centralized ISO sends the cleared electricity prices to the decentralized merchant or generator and the electricity generation cost function of traditional generators is linear, we show that, if the forecast prices align with the actual LMPs or the ISO sends the cleared prices to the electricity merchant, our results further indicate that a merchant with a wind farm and storage will arrive at the same optimal economic dispatch decisions (charging, discharging, and wind power generating) as those from the perspective of the ISO (when there exists a unique optimal solution for the merchant). Further, even if there were multiple optimal solutions to the merchant scheduling problem, the merchant could still achieve the maximum profit if she follows the ISO schedule. Using the KKT condition, language function, and complementary slackness, we also discovered the sufficient condition that the optimal generating and pumping decisions cannot occur simultaneously for any positive electricity prices for the ISO and the energy storage merchant’s non-convex problem. Our findings show that the prior report about the optimal economic dispatch decisions between traditional generators and ISO is still valid.

In Section 7.6, we find that, if a wind farm with storage is receiving production tax credits and, thus, is not allowed to purchase electricity from the market for storage, although both the optimal scheduling decisions—and, hence, the optimal profit—are affected by those credits, our statements about the relationship between the ISO and merchants are still valid. The same results apply to the co-optimization of a wind farm and a PHS with upper and lower reservoirs in Section 7.7. Finally, we consider the quadratic electricity generation cost in Section 7.8, and the numerical results show that the merchant still obtains the maximum profit if she follows the ISO's dispatch. We conclude that, if an ISO includes the storage and renewable power plant as well as traditional generators in his social welfare-maximizing problem with a reasonably good electricity price forecast, the solution will enhance the system's social welfare and the merchant's profit, which incentivizes the merchant and generator to follow the ISO's economic dispatch. These new findings augment our collective knowledge about managing the scheduling of energy storage, generators, and renewable energy sources and are an essential contribution to the research on this topic.

8. CONCLUSIONS

8.1. THE MAIN WORK AND CONTRIBUTIONS

To investigate optimal economic dispatch strategies for energy storage in electricity markets, we apply dynamic programming, mixed integer linear programming, nonlinear optimization, duality theory, and Lagrange relaxation. This work studies the optimal economic dispatch strategy of energy storage in electricity markets from four perspectives. First, we investigate the scheduling policy of an electricity merchant with only energy storage. Second, we explore the co-optimization economic dispatch strategy for a merchant with energy storage and renewable energy sources. Third, we examine the impact of the self-consumption demand rate on economic dispatch for prosumers with energy storage. Fourth, we analyze the effects of PTC on economic dispatch for electricity merchants with storage and wind farms. Finally, we identify the relationship of the optimal economic dispatch above scenarios between profit-maximizing merchants and social-welfare maximizing ISOs.

This work employs dynamic programming theory to investigate merchants' optimal economic dispatch considering the market impact and physical characteristics of storage systems. Our findings showed that the State-of-Charge (SOC) based analytical solution significantly facilitates energy storage merchants' decision-making. The SOC range is segmented into three regions by two optimal SOC reference points, which depend on the available energy in storage, forecasted electricity prices, and market impact. By comparing the current storage SOC with the reference points, the merchant can get the corresponding optimal actions. We analytically show that if the merchant neglects the market impact power market, she will exaggerate her expectation profit when the price-taker and price-maker merchants have the same generating and pumping upper limits. The profit-maximizing merchant must, therefore, assay to balance the trade-off correctly between the intensity of market impact and the dispatched power. Our findings are verified by numerical simulation, and the results demonstrate the ramifications for electricity merchants in energy arbitrage decisions.

This study investigated how the market impact of energy storage and uncertainty of wind generation affect co-optimized scheduling policy, specifically for merchants who have both energy storage and wind plants. Our results first demonstrate that for a merchant with co-located energy storage facilities and wind power plants, the energy storage's feasible state of charge (SOC) range can be segmented into four possible sub-ranges by three analytical SOC reference points. The unique optimal trading decision can be achieved by comparing the current energy inventory and the SOC reference points in the next period. Second, our results show that market impact and uncertainties substantially change the optimal storage scheduling policy by impacting the values of reference points. To smooth the negative effect of the merchant’s market impact on buying and selling actions, the merchant should reduce the amount of electricity generating or pumping each period to maximize profit. Moreover, we identify and investigate the trade-off between increasing the unit power profit and lowering the transaction quantity. Our findings provide co-optimized scheduling guidance for electricity merchants with co-located energy storage and renewable power plants systems.

This work analyzed the effects of self-consumption demand on the joint economic dispatch of prosumers (energy consumers who are also producers), particularly for prosumers with both energy storage and distributed renewable energy sources. Self-consumption demand can affect operational decisions; if the renewable power generation can satisfy self-consumption demand, then the remaining renewable power generation can be sold to the grid or stored in storage. On the other hand, if renewable power generation cannot meet the self-consumption demand, there are three potential options to fulfill the power shortage between the self-demand and distributed energy generation: a) buying electricity from the grid; b) discharging the energy storage; c) or both. In this paper, the above two situations were analyzed separately to find the optimal storage scheduling strategy, and the results were combined to get the optimal global solution. This study focused on prosumers' economic decision-making while considering self-consumption demand and the physical constraints of a battery based on dynamic programming. Our study showed that feasible state of charge (SOC) range of a storage can be segmented into several sub-ranges by SOC reference points under the above two scenarios. As a result, a prosumers' optimal scheduling can be uniquely and conveniently selected based on the sub-ranges within which the current SOC falls. The results therefore provided multistage decision-making guidance for prosumers with energy storage.

Two common PTC policies are studied – in the first, a wind farm is receiving PTC by selling the wind generation to the market and has storage to be able to buy electricity from the grid to store but the stored wind generation cannot receive PTC; in the second, the stored wind generation can also qualify for PTC but purchasing energy from the grid will not be allowed. We then employ dynamic programming to study merchants' optimal decision-making while considering PTC and the physical characteristics of storage systems. We analytically show that the state of charge (SOC) range can be segmented into different regions by SOC reference points under two PTC policies. The merchant's optimal action can be conveniently and uniquely determined based on the region within which the current SOC falls. Moreover, this study illustrates that PTC could substantially alter the optimal scheduling policy structures by affecting reference points and their relationships. The results showed that the frequencies for charging and discharging storage decisions decreased with an increase in PTC subsidy. Last, we confirm that although the first policy allows merchants to buy electricity from the market, the second policy can bring more profits when the PTC is large at current PTC rates. The findings provided multistage decision-making guidance for electricity merchants in the wholesale power market.

This study analyzes whether allowing the ISO to schedule the generators and energy storage (pumping/charging and generating/discharging as well as electricity generation schedules are optimized by ISO), taking into consideration multiple operating modes, energy limitation constraints, and multi-stage clearing process of electricity markets, can bring economic benefits to the social-welfare maximizing (i.e., electricity generation cost-minimizing) centralized ISO and the decentralized profit-maximizing electricity merchant and generators. To that purpose, we construct the primal and dual dispatching problems from three perspectives: the electricity merchant, the traditional generators, and the ISO. We analytically identify that when the ISO sends the cleared electricity prices, based on the social welfare-maximizing solutions, to the merchant, under which the merchants benefit from letting ISOs dispatch their energy storage (and wind farms) or generators directly, considering the linear electricity generation cost. This implies that an electricity merchant has the incentive to let the ISO take over the merchant’s operations, or that instead of analyzing the large-scale centralized economic dispatch problem from the perspective of ISO, we can utilize parallel computing to handle the profit maximization problems of many individual merchants simultaneously and more efficiently. Further, we investigate two scenarios: (a) a wind farm merchant who receives production tax credits and has storage but is not permitted to store power purchased from the grid, and (b) a wind farm merchant who has a pumped hydro energy storage facility with upper and lower reservoirs. We also analyze the quadratic power generation cost from the perspective of the ISO. Our findings are supported by numerical simulation and provide merchants and ISO with decision-making guidance.

8.2. FUTURE WORK

For analytical tractability, we assume in this work that the market impact of electricity merchants follows a simple linear relation. There are usually two approaches to model market impact---an equilibrium model or a conjecture variation model. This work's approach is a conjecture variation. Another connected concern for future research is confirming how to model the market impact in an equilibrium model and construct the corresponding reward functions. Although we expect our work's results to hold for other relationships, confirming this expectation is a worthy goal. It seems likely that our work's main structural results will also hold for other types of relation—when merchants choose to buy electricity, the market load will increase, leading to rising market prices; on the contrary, selling power by a price-marker merchant increases the supply and lowers price. Therefore, exploring this topic is a promising avenue for future research.

To establish a reasonable and tractable framework and derive insightful results, we have followed the conventional assumptions about the generating and pumping minimum limitations to get continuous reward functions. Further research could be undertaken that relaxes these assumptions and extends our research on this problem. It would also be worthwhile to investigate generating and pumping lower limitations in positive values other than zero. The results and optimal optimization scheduling proposed in this work are developed via dynamic programming based on the static price forecast for the entire horizon. Another related consideration for future work arises: Should the merchant's decision be adjusted to account for this changing price uncertainty?

This study led to some simplifying assumptions about DERs generation and self-demand and assumed both are accurately predictable and regarded as known before the decision at each stage. The managerial insights and optimal scheduling proposed in this work are developed via dynamic programming based on the single-point forecasts for the entire horizon. Future research could relax these assumptions, and approximate dynamic programming is a valuable tool to address the problem, incorporating uncertainties of their predictions and thereby extending our understanding. In addition, this paper only studied the optimal scheduling policy of prosumers with storage. In the future, the optimal scheduling between prosumers in the community considering the P2P (peer to peer) energy trade will be explored. Moreover, considering the investment cost of energy storage, by employing parametric analysis and sensitivity analysis to identify the right energy storage to upgrade or add capacity is another direction of future work.

To construct a tractable framework and obtain valuable insights, we developed simplified assumptions about the electricity prices that cleared the ISO and were sent to the merchant aligned with the actual LMP. It's difficult to get an accurate price forecast; future studies could tighten these assumptions and then extend our understanding of this problem, including how we can get an accurate price forecast and what inaccuracy level of the price forecast can be tolerated. We also assume that the transmission line capacity is sufficiently large. It is likely that the main structural results of this work will hold after relaxing the sampling assumptions. Another avenue worth exploring is the impact of the transmission line capacity on optimal scheduling. We tested the quadratic electricity generation cost numerically and suggest research on how a quadratic duality model can be built. In addition, an examination of how negative electricity prices affect the relationship between the ISO and merchant optimal decisions is needed.

APPENDIX A.

PROOF OF SECTION 3

Proof of Proposition 3.1:

(1) The uniqueness of the optimal results:

The current payoff rewards are shown as follows for the price maker:

$$R(\alpha_t,\hat{P}_t)=\begin{cases}-(P_t+\lambda P_t\alpha_t\beta\rho)\cdot\alpha_t\cdot\beta\rho+c\cdot\alpha_t\cdot\beta\rho & (\alpha_t\le 0)\text{ (generating-and-selling)}\\ -(P_t+\dfrac{\lambda P_t\alpha_t}{\alpha\rho})\cdot\dfrac{\alpha_t}{\alpha\rho}-c\cdot\dfrac{\alpha_t}{\alpha\rho} & (\alpha_t\ge 0)\text{ (buying-and-pumping)}\end{cases} \tag{A1}$$

Where, α_t is the energy/inventory change from period t to period t+1 before accounting for energy loss. By using the same method $E_{t+1}=\eta_t\cdot(E_t+\alpha_t)$, we will get the following rewards function at time t.

$$R(\alpha_t,\hat{P}_t)=\begin{cases}-\dfrac{(P_t+c)}{\alpha\rho}(\dfrac{E_{t+1}}{\eta_t}-E_t)-\dfrac{\lambda P_t}{\alpha^2\rho^2}(\dfrac{E_{t+1}}{\eta_t}-E_t)^2\\ -(P_t-c)\beta\rho\cdot(\dfrac{E_{t+1}}{\eta_t}-E_t)-\lambda P_t\beta^2\rho^2(\dfrac{E_{t+1}}{\eta_t}-E_t)^2\end{cases}$$

$$\Leftrightarrow\begin{cases}-\dfrac{(P_t+c)}{\alpha\rho}(\dfrac{E_{t+1}}{\eta_t})+\dfrac{(P_t+c)}{\alpha\rho}E_t-\dfrac{\lambda P_t}{\alpha^2\rho^2}(\dfrac{E_{t+1}}{\eta_t})^2+2\dfrac{\lambda P_t}{\alpha^2\rho^2}\dfrac{E_{t+1}}{\eta_t}E_t-\dfrac{\lambda P_t}{\alpha^2\rho^2}(E_t)^2\\ -(P_t-c)\cdot(\dfrac{E_{t+1}}{\eta_t})\beta\rho+(P_t-c)E_t\cdot\beta\rho-\lambda P_t\beta^2\rho^2(\dfrac{E_{t+1}}{\eta_t})^2+2\lambda P_t\beta^2\rho^2\dfrac{E_{t+1}}{\eta_t}E_t-\lambda P_t\beta^2\rho^2\cdot(E_t)^2\end{cases} \tag{A2}$$

We also have the following value function:

$$V_t(E_t,\hat{P}_t))=[R(\alpha_t,\hat{P}_t)+E(V(S(t+1)\,|\,S(t)]$$

Optimization of the value function $V_t(E_t,\hat{P}_t))$, subject to $\underline{E}\le E_{t+1}\le\overline{E}$, we will get the following equations based on the Bellman equation.

$$\begin{cases} V_t^{P*}(S(t)) = \max_{\underline{E} \le E_{t+1} \le \bar{E}} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - \frac{(P_t + c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t} - E_t) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) \\ \text{or } V_t^{P*}(S(t)) = \max_{E_{t+1} \in [\underline{E}, \bar{E}]} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - \frac{P_t + c}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t})^2 + 2\frac{\lambda P_t}{\alpha^2\rho^2}\frac{E_{t+1}}{\eta_t}E_t \right) \quad \text{(A3-1)} \\ V_t^{g*}(S(t)) = \max_{\underline{E} \le E_{t+1} \le \bar{E}} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - (P_t - c)\beta\rho \cdot (\frac{E_{t+1}}{\eta_t} - E_t) - \lambda P_t \beta^2\rho^2(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) \\ \text{or } V_t^{g*}(S(t)) = \max_{E_{t+1} \in [\underline{E}, \bar{E}]} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - (P_t - c) \cdot (\frac{E_{t+1}}{\eta_t})\beta\rho - \lambda P_t(\frac{E_{t+1}}{\eta_t})^2 \cdot \beta^2\rho^2 + 2\lambda P_t \frac{E_{t+1}}{\eta_t} E_t \cdot \beta^2\rho^2 \right) \quad \text{(A3-2)} \end{cases}$$

We know that any $t \in \{1,2,\cdots,T\}$, in every stage t, the value function $V_t(S(t))$ and $E[V_{t+1}^*(S(t+1) \mid S(t)]$ are concave in $E_t \in [\underline{E}, \bar{E}]$ for each given state $S(t) = S_t(E_t, g_t, P_t)$. Clearly, $E[V_{t+1}^*(S(t+1) \mid S(t)]$ and functions (A3-1) and (A3-2) are concave in $E_{t+1} \in [\underline{E}, \bar{E}]$ for each given state $S(t) = S_t(E_t, P_t)$ by through the following equivalence relations:

$$\frac{\partial E[V_{t+1}^*(S(t+1) \mid S(t)]}{\partial E_{t+1}{}^2} = \frac{\partial\left(\frac{\partial E[V_{t+1}^*(S(t+1) \mid S(t)]}{\partial E_{t+1}}\right)}{\partial E_{t+1}} = \frac{\partial\left(\frac{\partial E[V_{t+1}^*(S(t+1) \mid S(t)]}{\partial E_t} \cdot \frac{\partial E_t}{\partial E_{t+1}}\right)}{\partial E_t} \cdot \frac{\partial E_t}{\partial E_{t+1}}$$

$$= \left(\frac{\partial E[V_{t+1}^*(S(t+1) \mid S(t)]}{\partial E_t{}^2} \cdot \frac{\partial E_t}{\partial E_{t+1}} + \frac{\partial E[V_{t+1}^*(S(t+1) \mid S(t)]}{\partial E_t} \cdot \frac{\partial E_t}{\partial E_{t+1}\partial E_t}\right) \cdot \frac{\partial E_t}{\partial E_{t+1}} .$$

$$= \left(\frac{\partial E[V_{t+1}^*(S(t+1) \mid S(t)]}{\partial E_t{}^2} \cdot \left(\frac{\partial E_t}{\partial E_{t+1}}\right)^2\right) \le 0$$

1) By optimizing the function (A3-1), subject to $E_{t+1} \in [\underline{E}, \bar{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\frac{\partial V_t^{P*}(S(t))}{\partial E_{t+1}} = \frac{\partial E[V_{t+1}^*(S(t+1) \mid S(t)]}{\partial E_{t+1}} - \frac{P_t + c}{\alpha\rho\eta_t} - 2\frac{\lambda P_t}{\alpha^2\rho^2\eta_t{}^2}E_{t+1} + 2\frac{\lambda P_t}{\alpha^2\rho^2\eta_t}E_t$$

Furthermore, the second-order derivative is as follows:

$$\frac{\partial V_t^{p*}(S(t))}{\partial E_{t+1}{}^2} = \frac{\partial\left(E[V_{t+1}^*(S(t+1)\,|\,S(t)]\right)}{\partial E_{t+1}{}^2} - 2\frac{\lambda P_t}{\alpha^2\rho^2\eta_t{}^2} < 0 .$$

Since the second-order derivative is negative, we can find the unique optimal solutions through the first-order condition. We also will get the following optimal results:

$$\begin{cases} E_{t+1}^{p*} = \arg\max_{E_{t+1}\in[\underline{E},\overline{E}]}\left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \frac{P_t + c}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t})^2 + 2\frac{\lambda P_t}{\alpha^2\rho^2}\frac{E_{t+1}}{\eta_t}E_t \right) \\ \text{or } E_{t+1}^{p*} = \arg\max_{E_{t+1}\in[\underline{E},\overline{E}]}\left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \frac{(P_t + c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t} - E_t) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) \\ \text{or } \left.\left(\frac{\partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - \frac{P_t + c}{\alpha\rho\eta_t} - 2\frac{\lambda P_t}{\alpha^2\rho^2\eta_t{}^2}E_{t+1} + 2\frac{\lambda P_t}{\alpha^2\rho^2\eta_t}E_t \right)\right|_{E_{t+1}=E_{t+1}^{p*}} = 0 \end{cases} \quad (A4)$$

2) By optimizing the function (A3-2), subject to $E_{t+1}\in[\underline{E},\overline{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\frac{\partial V_t^{g*}(S(t))}{\partial E_{t+1}} = \frac{\partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - (P_t - c)\cdot(\frac{\beta\rho}{\eta_t}) - 2\lambda P_t(\frac{E_{t+1}}{\eta_t})\frac{1}{\eta_t}\cdot\beta^2\rho^2 + 2\lambda P_t\frac{E_t}{\eta_t}\cdot\beta^2\rho^2$$

Furthermore, the second-order derivative is as follows:

$$\frac{\partial V_t^{g*}(S(t))}{\partial E_{t+1}{}^2} = \frac{\partial\left(E[V_{t+1}^*(S(t+1)\,|\,S(t)]\right)}{\partial E_{t+1}{}^2} - 2\lambda P_t(\frac{\beta\rho}{\eta_t})^2 < 0 .$$

Since the second-order derivative is negative, we can find the unique optimal solutions through the first-order condition. We also will get the following optimal results:

$$\begin{cases} E_{t+1}^{g*} = \arg\max_{E_{t+1}\in[\underline{E},\overline{E}]}\left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - (P_t - c)\cdot(\frac{E_{t+1}}{\eta_t})\beta\rho - \lambda P_t(\frac{E_{t+1}}{\eta_t})^2\cdot\beta^2\rho^2 + 2\lambda P_t\frac{E_{t+1}}{\eta_t}E_t\cdot\beta^2\rho^2 \right) \\ \text{or } E_{t+1}^{g*} = \arg\max_{E_{t+1}\in[\underline{E},\overline{E}]}\left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - (P_t - c)\beta\rho\cdot(\frac{E_{t+1}}{\eta_t} - E_t) - \lambda P_t\beta^2\rho^2(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) \\ \text{or } \left.\left(\frac{\partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - (P_t - c)\cdot(\frac{\beta\rho}{\eta_t}) - 2\lambda P_t(\frac{E_{t+1}}{\eta_t})\frac{1}{\eta_t}\cdot\beta^2\rho^2 + 2\lambda P_t\frac{E_t}{\eta_t}\cdot\beta^2\rho^2 \right)\right|_{E_{t+1}=E_{t+1}^{g*}} = 0 \end{cases} \quad (A5)$$

(2) The relations among two reference points:

To simplify exposition, we define two auxiliary functions by using (A4) and (A5).

$$\begin{cases} F(E_{t+1})^{(p)} = \dfrac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - \dfrac{P_t + c}{\alpha\rho\eta_t} - 2\dfrac{\lambda P_t}{\alpha^2\rho^2\eta_t^{\,2}} E_{t+1} + 2\dfrac{\lambda P_t}{\alpha^2\rho^2\eta_t} E_t \\ F(E_{t+1})^{(g)} = \dfrac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - (P_t - c)\cdot(\dfrac{\beta\rho}{\eta_t}) - 2\lambda P_t(\dfrac{E_{t+1}}{\eta_t})\dfrac{1}{\eta_t}\cdot\beta^2\rho^2 + 2\lambda P_t\dfrac{E_t}{\eta_t}\cdot\beta^2\rho^2 \end{cases} \tag{A6}$$

Obviously, the corresponding first-order functions of (A6) is the second-order derivative of (A3-1) and (A3-2) as follows:

$$\begin{cases} \dfrac{\partial F(E_{t+1})^{(p)}}{\partial E_{t+1}} = \dfrac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E^2_{t+1}} - \dfrac{2\lambda P_t}{\alpha^2\rho^2\eta_t^{\,2}} = \dfrac{\partial V_t^{(p)*}(S(t))}{\partial E^2_{t+1}} < 0 \\ \dfrac{\partial F(E_{t+1})^{(g)}}{\partial E_{t+1}} = \dfrac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E^2_{t+1}} - \dfrac{2\lambda P_t\beta^2\rho^2}{\eta_t^{\,2}} = \dfrac{\partial V_t^{(g)*}(S(t))}{\partial E^2_{t+1}} < 0 \end{cases}$$

Thus, we know that two auxiliary functions are all decreasing with $E_{t+1} \in [\underline{E}, \overline{E}]$. We also find the following relationships for first-order functions of (A6), that is

$$\left|\frac{\partial F(E_{t+1})^{(p)}}{\partial E_{t+1}}\right| \geq \left|\frac{\partial F(E_{t+1})^{(g)}}{\partial E_{t+1}}\right|.$$

1) For all $E_{t+1} \in \left[\underline{E}, \overline{E}\right]$, if $\max F(E_{t+1})^{(p)} \leq \max F(E_{t+1})^{(g)}$, then we will obtain $E^{(p)*}_{t+1} \leq E^{(g)*}_{t+1}$

$$\max F(E_{t+1})^{(p)} = \left.\frac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}}\right|_{E_{t+1}=\underline{E}} - \frac{2\lambda P_t}{\alpha^2\rho^2\eta_t^{\,2}}\underline{E} + \frac{2\lambda P_t}{\alpha^2\rho^2\eta_t}E_t - \frac{P_t + c}{\alpha\rho\eta_t}$$

$$\leq \max F(E_{t+1})^{(g)} = \left.\frac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}}\right|_{E_{t+1}=\underline{E}} - \frac{2\lambda P_t\beta^2\rho^2}{\eta_t^{\,2}}\underline{E} + \frac{2\lambda P_t\beta^2\rho^2}{\eta_t}E_t - \frac{P_t\beta\rho - c\beta\rho}{\eta_t}$$

$$\Leftrightarrow \left(-\frac{2\lambda P_t}{\alpha^2\rho^2\eta_t^{\,2}}\underline{E} + \frac{2\lambda P_t}{\alpha^2\rho^2\eta_t}E_t - \frac{P_t + c}{\alpha\rho\eta_t}\right) \leq \left(-\frac{2\lambda P_t\beta^2\rho^2}{\eta_t^{\,2}}\underline{E} + \frac{2\lambda P_t\beta^2\rho^2}{\eta_t}E_t - \frac{(P_t\beta\rho - c\beta\rho)}{\eta_t}\right) \tag{A7}$$

$$\Leftrightarrow \left(2\lambda P_t(\frac{1}{\alpha^2\rho^2} - \beta^2\rho^2)\left(E_t - \frac{\underline{E}}{\eta_t}\right)\right) \leq \left(\frac{P_t + c}{\alpha\rho} - (P_t - c)\beta\rho\right) \Rightarrow \lambda \leq \frac{\left(\dfrac{P_t + c}{\alpha\rho} - (P_t - c)\beta\rho\right)}{2P_t(\dfrac{1}{\alpha^2\rho^2} - \beta^2\rho^2)\left(E_t - \dfrac{\underline{E}}{\eta_t}\right)} = \overline{\lambda}_t^{(p,g)}$$

If the current SOC reaches the lower boundary of storage (i.e., $E_t - \underline{E}/\eta_t = 0$) at period t, for any positive electricity price $P_t \geq 0$ and $\lambda \geq 0$, we will get $E_{t+1}^{(p)*} \leq E_{t+1}^{(g)*}$. In this situation, the merchant only has two actions: do nothing, and buy power from the market.

To sum up, for every stage $t \in \{1,2,\cdots,T\}$ and positive prices $P_t \geq 0$, when the market impact satisfies $0 \leq \lambda \leq \overline{\lambda}_t^{(p,g)}$, we can draw the relationship for the optimal SOC:

$$E_{t+1}^{(p)*} \leq E_{t+1}^{(g)*}$$

Obviously, if $P_t \leq 0$, there is $E_{t+1}^{(p)*} \geq E_{t+1}^{(g)*}$ when $0 \leq \lambda \leq \overline{\lambda}_t^{(p,g)}$.

Therefore, for positive prices $P_t \geq 0$, we have the following results:

1) If $\alpha\beta\rho^2 < 1$, or $c > 0$ holds, there is

$$\underline{E} \leq E_{t+1}^{p*} \leq E_{t+1}^{g*} \leq \overline{E} \tag{A8}$$

2) If there are $\alpha\beta\rho^2 = 1$ and $c = 0$ holding, we will get

$$V_t^{p*}(S(t)) = V_t^{g*}(S(t)) = \max_{E_{t+1} \in [\underline{E},\overline{E}]} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - P_t \cdot \frac{E_{t+1}}{\eta_t} - \lambda P_t (\frac{E_{t+1}}{\eta_t})^2 + 2\lambda P_t \frac{E_{t+1}}{\eta_t} E_t \right)$$

Thus, there is

$$E_{t+1}^{g*} = E_{t+1}^{p*} = E_{t+1}^{*} \tag{A9}$$

In summary, we get the following results:

1) If $\alpha\beta\rho^2 < 1$ (considering efficiency loss) or $c \neq 0$ (considering the operating cost),

$$\alpha_t^*(E_t, \hat{P}_t) = \begin{cases} \min\{E_{t+1}^{p*} - E_t, \overline{Q}^p\} (\text{buy and pump energy up to } E_{t+1}^{p*}) & E_t \in [\underline{E}, E_{t+1}^{p*}) \\ 0 \quad (\text{keep energy unchanged}) & E_t \in [E_{t+1}^{p*}, E_{t+1}^{g*}] \\ \max\{E_{t+1}^{g*} - E_t, -\overline{Q}^g\} (\text{generate and sell energy down to } E_{t+1}^{g*}) & E_t \in (E_{t+1}^{g*}, \overline{E}] \end{cases} \tag{A10}$$

2) If $\alpha\beta\rho^2 = 1$ (no efficiency loss) and $c = 0$ (without considering the operating cost),

$$\alpha_t^*(E_t, P_t) = \begin{cases} \min\{E_{t+1}^* - E_t, \bar{Q}^p\}(\text{buy and pump energy up to } E_{t+1}^*) & E_t \in [\underline{E}, E_{t+1}^*] \\ \max\{E_{t+1}^* - E_t, -\bar{Q}^g\}(\text{generate and sell energy down to } E_{t+1}^*) & E_t \in [E_{t+1}^*, \bar{E}] \end{cases} \quad \text{(A11)}$$

<u>Proof of Proposition 3.2:</u>

Recall the proof the proposition 3.1, for any given state $S(t)$ and we can get the following results:

$$\begin{cases} E_{t+1}^{P*} = \arg\max_{E_{t+1}\in[\underline{E},\bar{E}]} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \frac{\lambda P_t}{\alpha^2\rho^2}[\frac{E_{t+1}}{\eta_t} - E_t]^2 - \frac{(P_t + c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) \right) \\ E_{t+1}^{g*} = \arg\max_{E_{t+1}\in[\underline{E},\bar{E}]} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \lambda P_t\beta^2\rho^2[\frac{E_{t+1}}{\eta_t} - E_t]^2 - (P_t - c)\beta\rho\cdot(\frac{E_{t+1}}{\eta_t}) \right) \end{cases}.$$

Recall the Proof of Proposition 3.1, and we also have the following results:

$$\begin{cases} E_{t+1(\lambda=0)}^{P*} = \arg\max \left(E[V_{t+1(PT)}^*(S(t+1)\,|\,S(t)] - \frac{(P_t + c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) \right) \\ E_{t+1(\lambda=0)}^{g*} = \arg\max \left(E[V_{t+1(PT)}^*(S(t+1)\,|\,S(t)] - (P_t - c)\cdot(\frac{E_{t+1}}{\eta_t})\beta\rho \right) \end{cases}$$

By using the payoff rewards function (A1), we have the following relations:

$$\frac{\partial R(\alpha_t, \hat{P}_t)}{\partial\lambda} = \begin{cases} -P_t(\alpha_t\beta\rho)^2 \le 0 \quad (\alpha_t \le 0) \\ -P_t(\frac{\alpha_t}{\alpha\rho})^2 \le 0 \quad (\alpha_t \ge 0) \end{cases} \text{ and } \frac{\partial R(\alpha_t, \hat{P}_t)}{\partial\lambda} = -(P_t\alpha_t\beta\rho)\cdot\alpha_t\cdot\beta\rho - (\frac{P_t\alpha_t}{\alpha\rho})\cdot\frac{\alpha_t}{\alpha\rho} \le 0.$$

By using $V(S(t)) = \max_{Action(E_t)}[R(\alpha_t, \hat{P}_t) + \sum_{i=t+1}^{T} R(\alpha_t, \hat{P}_t)]$, then, for every stage $t \in \{1, 2, \cdots, T\}$ and positive prices $P_t \ge 0$, the value function of $V_{t+1}^*(S(t+1)\,|\,S(t)$ decreases with the market impact parameter λ. Thus, we will get the following relations:

$$E[V_{t+1(\lambda=0)}^*(S(t+1)\,|\,S(t)] \ge E[V_{t+1(\lambda\ge 0)}^*(S(t+1)\,|\,S(t)] \quad \text{(A12)}$$

In this way, we also get the following:

$$E[V^*_{t+1(\lambda=0)}(S(t+1)\,|\,S(1)] \geq E[V^*_{t+1(\lambda\geq 0)}(S(t+1)\,|\,S(1)]$$

Therefore, we obtain the following optimal results:

$$\max_{\pi}\sum_{t=1}^{T} E\left[R(q_t,\hat{P}_t)_{(\lambda=0)}\,|\,S(1)\right] \geq \max_{\pi}\sum_{t=1}^{T} E\left[R(q_t,\hat{P}_t)_{(\lambda\geq 0)}\,|\,S(1)\right] \tag{A13}$$

Case1 in Section 3. Without Considering the Efficiency and Operation Cost

Three stages: Prices: $P=\{5,2,10\}=\{P_1,P_2,P_3\}$, initial SOC: E_1, $\underline{E}=0$, and $\bar{E}=10$.

Assumption and Constraints:

Let the operating cost be zero (i.e., $c=0$), the efficiencies of pumping, generating, self-discharging, and transmission are one (i.e., $\alpha=\beta=\rho=\eta=1$).

$\bar{Q}^p=7$ (i.e., cannot fill up in one period)

$\bar{Q}^g=12$ (i.e., can be emptied in one period)

We assume the merchant's market impact parameter $\lambda=0.05$.

Similarly, by using backward DP to obtain the following optimal value functions:

In Stage 3:

Action 3: the merchant should sell power to the market and make the storage level down to $\underline{E}$.

$$V_3=-[p^H+\lambda p^H(\underline{E}-E_3)\beta\rho](\underline{E}-E_3)\beta\rho=-[10+0.5(\underline{E}-E_3)](\underline{E}-E_3)=10E_3-0.5E_3{}^2, E_3\in[0,10]$$

In Stage 2:

By using the equations (A4) and (A5), we will get the following results for price maker merchant:

$$E_3^{g*} = E_3^{p*} = E_3^* = \arg\max_{E_3 \in [\underline{E}, \bar{E}]} \{V_3^* - p^L E_3 - \lambda P^L [E_3 - E_2]^2\}$$

$$= \arg\max_{E_3 \in [0,10]} \{10E_3 - 0.5{E_3}^2 - 2E_3 - 0.1[E_3 - E_2]^2\} \quad \text{(A14)}$$

$$= \arg\max_{E_3 \in [0,10]} \{(8 + 0.2E_2)E_3 - 0.6{E_3}^2 - 0.1{E_2}^2\} = (40 + E_2)/6$$

Following proposition 3.2, we will get the following results:

① If $E_2 < (40 + E_2)/6$ (i.e., $E_2 < 8$), the merchant should adopt buying-and-pumping up to $E_3^* = (40 + E_2)/6$ as close as possible.

② If $E_2 \geq (40 + E_2)/6$ (i.e., $E_2 \geq 8$), the merchant should adopt generating and selling down to $E_3^* = (40 + E_2)/6$ as close as possible.

Thus, we will get the following optimal action at stage 2.

$$\alpha_2^* = \begin{cases} \min\{\frac{40 + E_2}{6} - E_2, \bar{Q}^p\} = \frac{40 - 5E_2}{6} \geq 0 \\ \text{(buying and pumping up to } E_3^*) \text{ if } E_2 \in [0, \frac{40 + E_2}{6}) \\ \max\{\frac{40 + E_2}{6} - E_2, -\bar{Q}^g\} = \frac{40 - 5E_2}{6} < 0 \\ \text{(generating and selling down to } E_3^*) \text{if } E_2 \in [\frac{40 + E_2}{6}, 10] \end{cases} \quad \text{(A15)}$$

The reward payoff functions at stage 2 are shown as follows:

$$R_2 = \begin{cases} -(P^L + \lambda P^L \alpha_2) \cdot \alpha_2 \quad \text{if } E_2 \in [0, \frac{40 + E_2}{6}) \\ -(P^L + \lambda P^L \alpha_2) \cdot \alpha_2 \quad \text{if } E_2 \in [\frac{40 + E_2}{6}, 10] \end{cases} = \begin{cases} -(2 + 0.1\frac{40 - 5E_2}{6}) \cdot \frac{40 - 5E_2}{6} \quad \text{if } E_2 \in [0, \frac{40 + E_2}{6}) \\ -(2 + 0.1\frac{40 - 5E_2}{6}) \cdot \frac{40 - 5E_2}{6} \quad \text{if } E_2 \in [\frac{40 + E_2}{6}, 10] \end{cases}$$

$$= -\left((16 - 0.5E_2)/6\right) \cdot 16 - 0.5E_2 \left((40 - 5E_2)/6\right) \quad \text{if } E_2 \in [0, 10]$$

The optimal value function at stage 3 can be rewritten as

$$V_3^* = (10 - 0.5E_3)E_3 = 10\frac{40 + E_2}{6} - 0.5(\frac{40 + E_2}{6})^2 = \frac{3200 + 40E_2 - {E_2}^2}{72}$$

Thus, the optimal value function at stage 2 is shown as follows:

$$V_2^* = \max(R_2 + V_3^*) = \left(320 + 40E_2 - E_2{}^2\right)/12 \;\text{ if }\; E_2 \in [0,10]$$

In Stage 1:

By using the equation (A4) and (A5), we will get the following results:

$$\begin{aligned} E_2^{p*} = E_2^{g*} = E_2^* &= \underset{E_2 \in [\underline{E},\bar{E}]}{\arg\max}\{V_2^* - p^M E_2 - \lambda P^M [E_2 - E_1]^2\} \\ &= \underset{E_2 \in [0,10]}{\arg\max}\{\left(320 + 40E_2 - E_2{}^2\right)/12 - 5E_2 - 0.25[E_2 - E_1]^2\} \qquad \text{(A16)} \\ &= \underset{E_2 \in [0,10]}{\arg\max}\{\left(320 - (20 - 6E_1)E_2 - 4E_2{}^2 - 3E_1{}^2]\right)/12\} = \left(3E_1 - 10\right)/4 \end{aligned}$$

For any given $E_1 \in [0,10]$, there is $E_1 \geq \left(3E_1 - 10\right)/4$, and we will get $E_2^* = \left(3E_1 - 10\right)/4 \in [-2.5,5]$. We also get $E_1 \geq 10/3$ by using $\left(3E_1 - 10\right)/4 \geq 0$. That is if $0 < E_1 < 10/3$, $E_2^* = 0$, $E_2^* - E_1 < 0$, and if $10/3 \leq E_1 < 10$, $E_2^* = \left(3E_1 - 10\right)/4$, and $E_2^* - E_1 < 0$. In this way, the electricity merchant should adopt the following policy:

① if $E_1 \in [0,10/3)$, the merchant should adopt generating and selling down to $E_2^* = 0$.

② If if $E_1 \in [10/3,10]$, the merchant should adopt generating and selling down to $E_2^* = \left(3E_1 - 10\right)/4$.

Thus, we will get the following optimal action at stage 1.

$$\alpha_1^* = \begin{cases} -E_1 < 0 \text{ (generating and selling down to 0)} & \text{if } E_1 \in [0,10/3) \\ \left(3E_1 - 10\right)/4 - E_1 < 0 \text{ (generating and selling down to } \left(3E_1 - 10\right)/4\text{)} & \text{if } E_1 \in [10/3,10] \end{cases} \qquad \text{(A17)}$$

The reward payoff functions at stage 1 are shown as follows:

$$R_1 = \begin{cases} -(P^M + \lambda P^M \alpha_1)\cdot \alpha_1 \text{ if } E_1 \in [0,10/3) \\ -(P^M + \lambda P^M \alpha_1)\cdot \alpha_1 \text{ if } E_1 \in [10/3,10] \end{cases} = \begin{cases} -(5+0.25(-E_1))\cdot(-E_1), & \text{if } E_1 \in [0,10/3) \\ (5-0.25(10+E_1)/4)\cdot(10+E_1)/4, & \text{if } E_1 \in [10/3,10] \end{cases}$$

$$= \begin{cases} 5E_1 - 0.25E_1{}^2, & \text{if } E_1 \in [0,10/3) \\ (700+60E_1 - E_1{}^2)/64, & \text{if } E_1 \in [10/3,10] \end{cases}$$

Thus, the optimal value functions at stage 1 are shown as follows:

$$V_1^* = \max(R_1 + V_2^*) = \begin{cases} (5E_1 - 0.25E_1{}^2) + \frac{320+40E_2 - E_2{}^2}{12}\Big|_{E_2^*=0} & \text{if } E_1 \in [0,\frac{10}{3}) \\ (\frac{700+60E_1 - E_1{}^2}{64}) + \frac{320+40E_2 - E_2{}^2}{12}\Big|_{E_2^*=\frac{3E_1-10}{4}} & \text{if } E_1 \in [\frac{10}{3},10] \end{cases}$$

$$= \begin{cases} (5E_1 - 0.25E_1{}^2) + 320/12 & \text{if } E_1 \in [0,10/3) \\ (\frac{700+60E_1 - E_1{}^2}{64}) + \frac{320+40\frac{3E_1-10}{4} - (\frac{3E_1-10}{4})^2}{12} & \text{if } E_1 \in [\frac{10}{3},10] \end{cases}$$

We will get the following optimal results:

1) If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$,(action 1: generating and selling), there has

$$E_2 = 0 \text{ (i.e., } \alpha_1^* = -1,\ R_1 = 5E_1 - 0.25E_1{}^2 = 4.75\text{)}.$$

Stage 2: If $E_2 = 0$,(action 2: buying and pumping), there has

$$E_3 = \frac{40+E_2}{6} = \frac{40}{6} \text{ (i.e., } \alpha_2^* = \frac{40}{6},\ R_2 = -(\frac{16-0.5E_2}{6})\cdot\frac{40-5E_2}{6} = -\frac{160}{9}\text{)}.$$

Stage 3: If $E_3 = \frac{40}{6}$,(action 3: generating and selling), there is

$$E_4 = 0 = \underline{E} \text{ (i.e., } \alpha_3^* = -\frac{40}{6}, R_3 = 10E_3 - 0.5E_3{}^2 = 10\times\frac{40}{6} - 0.5(\frac{40}{6})^2 = \frac{400}{9}\text{)}.$$

Total rewards $R = R_1 + R_2 + R_3 = 4.75 - \frac{160}{9} + \frac{400}{9} = 31.4167$.

The optimal value at stage 1 is shown as

$V_1^* = (5\times 1 - 0.25\times 1^2) + \frac{320}{12} = 4.75 + \frac{80}{3} = 31.4167$.

2) If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$, (action 1: generating and selling), there is

$$E_2 = \frac{3E_1 - 10}{4} = \frac{3\times 5 - 10}{4} = 1.25 \quad \text{(i.e., } \alpha_1^* = -3.75,\ R_1 = \frac{700 + 60E_1 - E_1^{\,2}}{64} = \frac{975}{64}\text{);}$$

Stage 2: If $E_2 = 1.25$, (action 2: buying and pumping), there has

$$E_3 = \frac{40 + E_2}{6} = \frac{41.25}{6} \quad \text{(i.e., } \alpha_2^* = \frac{33.75}{6},\ R_2 = -(\frac{16 - 0.5E_2}{6})\cdot\frac{40 - 5E_2}{6} = -\frac{123}{48}\times\frac{135}{24}\text{);}$$

Stage 3: If $E_3 = \frac{41.25}{6}$, (action 3: generating and selling), there is

$$E_4 = 0 = \underline{E} \text{ (i.e., } \alpha_3^* = -\frac{41.25}{6},\ R_3 = 10E_3 - 0.5E_3^{\,2} = 10\times\frac{40}{6} - 0.5(\frac{40}{6})^2 = \frac{412.5}{6} - \frac{(41.25)^2}{72}\text{).}$$

Total rewards $R = R_1 + R_2 + R_3 = \frac{975}{64} - \frac{123}{48}\times\frac{135}{24} + \frac{412.5}{6} - \frac{(41.25)^2}{72} = 45.9375$.

The optimal value at stage 1 is shown as

$$V_1^* = (\frac{975}{64}) + \frac{320 + 40\frac{5}{4} - (\frac{5}{4})^2}{12} = (\frac{975}{64}) + \frac{320}{12} + \frac{775}{16\times 12} = \frac{735}{16} = 45.9375.$$

<u>Case2. Market Impact $\lambda = 0$</u>

Three stages (Price: $P_t = \{5, 2, 10\}$ and E_1 (initial SOC/given value), $\underline{E} = 0$, and $\overline{E} = 10$.

Assumption and Constraints:

Let the operating cost be one (i.e., $c = 1$), the pumping and generating efficiencies be

0.9 (i.e., $\alpha = \beta = 0.9$), self-discharging, and transmission efficiencies are one (i.e., $\rho = \eta = 1$).

$\bar{Q}^p = 7$ (i.e., cannot fill up in one period)

$\bar{Q}^g = 12$ (i.e., can be emptied in one period)

In Stage 3:

Action 3: the merchant should sell power and make the storage level down to $\underline{E}$ due to $V_4 = 0$.

$$V_3 = -(p^H - c)(\underline{E} - E_3)\beta\rho = 8.1E_3, E_3 \in [0,10]$$

In Stage 2:

By using the equation (A4) and (A5), we will get the following results:

$$\begin{cases} E_3^{g*} = \arg\max\limits_{E_3 \in [\underline{E}, \bar{E}]} \{V_3^* - (p^L - c)E_3\beta\rho\} = \arg\max\limits_{E_3 \in [0,10]} \{8.1E_3 - (2-1)\cdot 0.9 \cdot E_3\} = \bar{E} = 10 \\ E_3^{p*} = \arg\max\limits_{E_3 \in [\underline{E}, \bar{E}]} \{V_3^* - (p^L + c)E_3/\alpha\rho\} = \arg\max\limits_{E_3 \in [0,10]} \{8.1E_3 - 3E_3/0.9\} = \bar{E} = 10 \end{cases} \tag{A18}$$

If $E_2 < E_3^* = 10 = \bar{E}$, buying-and-pumping up to 10 (i.e., $\bar{E}$) as much as possible.

① If $E_2 \in [0,3)$, $\alpha_2^* = \min\{E_3^* - E_2, \bar{Q}^p\} = \min\{10 - E_2, 7\} = 7$, we will get $E_3 = E_2 + \bar{Q}^p = E_2 + 7 < E_3^*$.

② If $E_2 \in [3,10]$, $\alpha_2^* = \min\{E_3^* - E_2, \bar{Q}^p\} = \min\{10 - E_2, 7\} = 10 - E_2$, we will get $E_3 = \bar{E} = E_3^*$.

Following proposition 3.1, we will get the following optimal actions:

$$\alpha_2^* = \begin{cases} \bar{Q}^p = 7 > 0 \text{ (buying and pumping up to } E_3^{p*} = E_2 + \bar{Q}^p = E_2 + 7 < \bar{E}) & \text{if } E_2 \in [0,3) \\ 10 - E_2 > 0 \text{ (buying and pumping down to } E_3^{p*} = \bar{E}) & \text{if } E_2 \in [3,10] \end{cases} \tag{A19}$$

Thus, the reward functions at stage 2 are shown as follows:

$$R_2 = \begin{cases} -P_t \cdot \dfrac{\bar{Q}^p}{\alpha\rho} - c \cdot \dfrac{\bar{Q}^p}{\alpha\rho} & \text{if } E_2 \in [0,3) \\ -P_t \cdot \dfrac{10-E_2}{\alpha\rho} - c \cdot \dfrac{10-E_2}{\alpha\rho} & \text{if } E_2 \in [3,10] \end{cases} = \begin{cases} -23.33 & \text{if } E_2 \in [0,3) \\ \dfrac{10}{9}(3E_2 - 30) & \text{if } E_2 \in [3,10] \end{cases}$$

So, the optimal value function at stage 3 can be rewritten to

$$V_3^* = \begin{cases} (p^H - c)(E_2 + \bar{Q}^p - \underline{E})\beta\rho & \text{if } E_2 \in [\underline{E}, \bar{E} - \bar{Q}^p); \\ (p^H - c)(\bar{E} - \underline{E})\beta\rho & \text{if } E_2 \in [\bar{E} - \bar{Q}^p, \bar{E}]. \end{cases} \text{, that is } V_3^* = \begin{cases} 8.1(E_2 + 7) & \text{if } E_2 \in [0,3) \\ 81 & \text{if } E_2 \in [3,10] \end{cases}$$

The optimal value functions at stage 2 are shown as:

$$V_2^* = \max(R_2 + V_3^*) = \begin{cases} -(P^L + c) \cdot \left(\bar{Q}^p / \alpha\rho\right) + (p^H - c)(E_2 + \bar{Q}^p - \underline{E})\beta\rho & \text{if } E_2 \in [\underline{E}, \bar{E} - \bar{Q}^p) \\ -(P^L + c)\left((10 - E_2)/\alpha\rho\right) + (p^H - c)(\bar{E} - \underline{E})\beta\rho & \text{if } E_2 \in [\bar{E} - \bar{Q}^p, \bar{E}] \end{cases}$$

$$= \begin{cases} 8.1E_2 + 33.37 & \text{if } E_2 \in [0, 3) \\ (10/3)E_2 + (143/3) & \text{if } E_2 \in [3, 10] \end{cases}$$

In Stage 1:

Similarly, by using the Eqs. (A4) and (A5), we get the following results:

$$\begin{cases} E_2^{g*} = \arg\max_{E_2 \in [\underline{E}, \bar{E}]} \{V_2^* - (p^M - c)E_2\beta\rho\} = \arg\max_{E_2 \in [0,10]} \begin{cases} 4.5E_2 + 33.37 & \text{if } E_2 \in [0,3) \\ -\dfrac{4}{15}E_2 + \dfrac{143}{3} & \text{if } E_2 \in [3,10] \end{cases} = 3 = \bar{E} - \bar{Q}^p \\ E_2^{p*} = \arg\max_{E_2 \in [\underline{E}, \bar{E}]} \left(V_2^* - \dfrac{(P^M + c)}{\alpha\rho} E_2 \right) = \arg\max_{E_2 \in [0,10]} \begin{cases} \dfrac{43}{30}E_2 + 33.37 & \text{if } E_2 \in [0,3) \\ -\dfrac{10}{3}E_2 + \dfrac{143}{3} & \text{if } E_2 \in [3,10] \end{cases} = 3 = \bar{E} - \bar{Q}^p \end{cases} \quad \text{(A20)}$$

Following proposition 3.1, we will get the following actions:

$$\alpha_1^* = \begin{cases} 3 - E_1 > 0 \text{ (buying-and-pumping up to } \bar{E} - \bar{Q}^p = 3) & \text{if } E_1 \in [0,3) \\ 3 - E_1 < 0 \text{ (generating-and-selling down to } \bar{E} - \bar{Q}^p = 3) & \text{if } E_1 \in [3,10] \end{cases} \quad \text{(A21)}$$

Thus, the reward functions at stage 1 are shown as follows:

$$R_1 = \begin{cases} -P^M \cdot (\bar{E} - \bar{Q}^p - E_1)/\alpha\rho - c \cdot (\bar{E} - \bar{Q}^p - E_1)/\alpha\rho \text{ (buy) if } E_1 \in [0, 3) \\ -P^M (\bar{E} - \bar{Q}^p - E_1) \cdot \beta\rho + c \cdot (\bar{E} - \bar{Q}^p - E_1) \cdot \beta\rho \text{ (sell) if } E_1 \in [3, 10] \end{cases}$$

$$= \begin{cases} -6 \cdot [(3 - E_1)/0.9] \text{ if } E_1 \in [0,3) \\ -3.6 \cdot (3 - E_1) \text{ if } E_1 \in [3,10] \end{cases}$$

Therefore, the optimal value functions at stage 1 are shown as

$$V_1^* = \max(R_1 + V_2^*) = R_1 + \begin{cases} -(P^L + c) \cdot \dfrac{\bar{Q}^p}{\alpha\rho} + (p^H - c)(E_2 + \bar{Q}^p - \underline{E})\beta\rho \Big|_{E_2^* = \bar{E} - \bar{Q}^p} \text{ if } E_2 \in [\underline{E}, \bar{E} - \bar{Q}^p) \\ -(P^L + c) \dfrac{10 - E_2}{\alpha\rho} + (p^H - c)(\bar{E} - \underline{E})\beta\rho \Big|_{E_2^* = \bar{E} - \bar{Q}^p} \text{ if } E_2 \in [\bar{E} - \bar{Q}^p, \bar{E}] \end{cases}$$

$$= \begin{cases} (20/3)E_1 + 37.67 \text{ if } E_1 \in [0,3) \\ 3.6E_1 + 46.87 \text{ if } E_1 \in [3,10] \end{cases}$$

To sum up, we will get the following optimal results:

1) If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, (action 1: buying and pumping), we will get

$E_2 = 3$ (i.e., $\alpha_1^* = 2$, $R_1 = -5 \times 2 / 0.9 - 1 \times 2 / 0.9 = -13.33$).

Stage 2: If $E_2 = 3$,(action 2: buying and pumping), we will get

$E_3 = 10$ (i.e., $\alpha_2^* = 7$, $R_2 = -2 \times 7 / 0.9 - 1 \times 7 / 0.9 = -23.33$).

Stage 3: If $E_3 = 10$,(action 3: generating and selling), there has

$E_4 = 0$ (i.e., $\alpha_3^* = -10$, $R_3 = -10 \times (-10) \times 0.9 + 1 \times (-10) \times 0.9 = 81$).

Total rewards $R = 81 - 13.33 - 23.33 = 44.34$.

The optimal value at stage 1 is shown as $V_1^* = (20/3)E_1 + 37.67 = 44.34$.

2) If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$,(action 1: generating and selling), we have

$$E_2 = 3 \text{ (i.e., } \alpha_1^* = -2, R_1 = -5\times(-2)\times 0.9 + 1\times(-2)\times 0.9 = 7.2\text{)}.$$

Stage 2: If $E_2 = 3$,(action 2: buying and pumping), we will get

$$E_3 = 10 \text{ (i.e., } \alpha_2^* = 7, R_2 = -2\times 7/0.9 - 1\times 7/0.9 = -23.33\text{)}.$$

Stage 3: If $E_3 = 10$,(action 3: generating and selling), there has

$$E_4 = 0 \text{ (i.e., } \alpha_3^* = -10, R_3 = -10\times(-10)\times 0.9 + 1\times(-10)\times 0.9 = 81\text{)}.$$

Thus, the total rewards $R = 81 + 7.2 - 23.33 = 64.87$.

The optimal value at stage 1 is shown as $V_1^* = 3.6E_1 + 46.87 = 64.87$.

Case 3. (Market Impact $\lambda = 0.01$)

The efficiencies $\alpha = \beta = 0.9; \rho = \eta = 1$; the operating cost $c = 1$.

Unlike Case 2, we assume the merchant's market impact parameter $\lambda = 0.01$, and we use the same method to obtain the following optimal value functions:

In Stage 3:

Action 3: Sell power to market and make the storage level down to $\underline{E} = 0$.

$$V_3 = -[p^H + \lambda p^H(\underline{E} - E_3)\beta\rho - c](\underline{E} - E_3)\beta\rho = 8.1E_3 - 0.081E_3^{\ 2}, E_3 \in [0,10]$$

In Stage 2:

$$\begin{cases} E_3^{g*} = \arg\max_{E_3\in[\underline{E},\bar{E}]}\{V_3^* - (p^L - c)E_3\beta\rho - \lambda P^L(\beta\rho)^2[E_3 - E_2]^2\} \\ \quad = \arg\max_{E_3\in[0,10]}\{8.1E_3 - 0.081E_3{}^2 - (2-1)\cdot 0.9\cdot E_3 - 0.02(0.81)[E_3 - E_2]^2\} \\ \quad = \arg\max_{E_3\in[0,10]}\{(7.2+0.0324E_2)E_3 - \frac{243}{1000}E_3{}^2 - 0.0162E_2{}^2\} \\ \quad = \min_{E_3\in[0,10]}\{\frac{500}{243}(7.2+0.0324E_2),10\} = 10 = \bar{E} \\ E_3^{p*} = \arg\max_{E_3\in[\underline{E},\bar{E}]}\{V_3^* - (p^L + c)\frac{E_3}{\alpha\rho} - \frac{\lambda P^L}{(\alpha\rho)^2}[E_3 - E_2]^2\} \\ \quad = \arg\max_{E_3\in[0,10]}\{8.1E_3 - 0.081E_3{}^2 - \frac{3}{0.9}E_3 - \frac{0.02}{0.81}[E_3 - E_2]^2\} \\ \quad = \arg\max_{E_3\in[0,10]}\{(\frac{143}{30} + \frac{4}{81}E_2)E_3 - \frac{8561}{81000}E_3{}^2 - \frac{4}{81}E_2{}^2\} \\ \quad = \min_{E_3\in[0,10]}\{\frac{50(3861+40E_2)}{8561},10\} = 10 = \bar{E} \end{cases} \tag{A22}$$

Thus, we will get the following optimal actions at stage 2.

$$\alpha_2^* = \begin{cases} \bar{Q}^p = 7 > 0 \text{ (buying and pumping up to } E_3^{p*} = E_2 + \bar{Q}^p = E_2 + 7 < \bar{E}) & \text{if } E_2 \in [0,3) \\ 10 - E_2 > 0 \text{ (buying and pumping up to } E_3^{p*} = \bar{E}) & \text{if } E_2 \in [3,10] \end{cases} \tag{A23}$$

The reward payoff functions at stage 2 are shown as follows:

$$R_2 = \begin{cases} -(2+0.02\alpha_2/0.9)\cdot\alpha_2/0.9 - 1\cdot\alpha_t/0.9 = -25.5432 & \text{if } E_2 \in [0,3) \\ -(2+0.02\alpha_2/0.9)\cdot\alpha_2/0.9 - 1\cdot\alpha_t/0.9 = \frac{-2900+310E_2-2E_2{}^2}{81} & \text{if } E_2 \in [3,10] \end{cases}$$

The optimal value functions at stage 3 can be rewritten as

$$V_3^* = \begin{cases} 8.1E_3 - 0.081E_3{}^2 |_{E_3^*=E_2+7} = -0.081E_2{}^2 + 6.966E_2 + 52.731 & \text{if } E_2 \in [0,3) \\ 8.1E_3 - 0.081E_3{}^2 |_{E_3^*=10} = 72.9 & \text{if } E_2 \in [3,10] \end{cases}$$

Thus, the optimal value functions at stage 2 are shown as follows:

$$V_2^* = \max(R_2 + V_3^*) = \begin{cases} -0.081E_2{}^2 + 6.966E_2 + 27.1878 & \text{if } E_2 \in [0,3) \\ \frac{310E_2 - 2E_2{}^2}{81} + \frac{30049}{810} & \text{if } E_2 \in [3,10] \end{cases}$$

In Stage 1:

$$E_2^{g*} = \underset{E_2 \in [\underline{E}, \bar{E}]}{\arg\max}\{V_2^* - (p^M - c)E_2\beta\rho - \lambda P^M\beta^2\rho^2[E_2 - E_1]^2\}$$

$$= \underset{E_2 \in [0,10]}{\arg\max}\begin{cases} -\frac{243}{2000}E_2{}^2 + (3.366 + \frac{162}{2000}E_1)E_2 + 27.1878 - \frac{81}{2000}E_1{}^2 & \text{if } E_2 \in [0,3) \\ -0.0652E_2{}^2 + (\frac{92}{405} + \frac{162}{2000}E_1)E_2 - \frac{81}{2000}E_1{}^2 + \frac{30049}{810} & \text{if } E_2 \in [3,10] \end{cases} \quad \text{(A24)}$$

If $E_2 \in [0,3)$, in the first equation of (A24), we will get $E_2^{g*} = 3$.

If $E_2 \in [3,10]$, in the second equation of (A25), we get

$E_2^{g*} = \frac{0.2272 + 0.081E_1}{0.13048}$ if $E_1 \in (4.5992,10]$. We also find there is

$$[-0.0652E_2{}^2 + (\frac{92}{405} + \frac{162}{2000}E_1)E_2 - \frac{81}{2000}E_1{}^2 + \frac{30049}{810}\,|_{E_2 = \frac{0.2272+0.081E_1}{0.1304}}]$$

$$-[-\frac{243}{2000}E_2{}^2 + (3.366 + \frac{162}{2000}E_1)E_2 + 27.1878 - \frac{81}{2000}E_1\,|_{E_2=3}] > 0$$

Therefore, if $E_1 \in (4.5992,10]$, we will get $E_2^{g*} = \frac{0.2272 + 0.081E_1}{0.13048}$

$$E_2^{P*} = \underset{E_2 \in [\underline{E}, \bar{E}]}{\arg\max}\left(V_2^* - \frac{\lambda P^M}{\alpha^2\rho^2}[E_2 - E_1]^2 - \frac{(P^M + c)}{\alpha\rho}E_2\right)$$

$$= \underset{E_2 \in [0,10]}{\arg\max}\begin{cases} -0.1427E_2{}^2 + (\frac{10}{81}E_1 + \frac{449}{1500})E_2 - \frac{5}{81}E_1{}^2 + 27.1878 & \text{if } E_2 \in [0,3) \\ -\frac{7}{81}E_2{}^2 + (\frac{10}{81}E_1 - \frac{230}{81})E_2 - \frac{5}{81}E_1{}^2 + \frac{30049}{810} & \text{if } E_2 \in [3,10] \end{cases} \quad \text{(A25)}$$

If $E_2 \in [0,3)$, in the first equation of (A25), if $E_1 \in [0,1.89]$, there is $E_2^{p*} = \frac{(\frac{10}{81}E_1 + \frac{459}{1500})}{0.2854}$.

If $E_2 \in [3,10]$, in the second equation of (A25), if $E_1 \in [0,1.89]$, there is $E_2^{P*}=3$.

$$\left(-0.1427E_2^{\ 2}+(\frac{10}{81}E_1+\frac{449}{1500})E_2-\frac{5}{81}E_1^{\ 2}+27.1878|_{E_2^{p*}=3}\right)$$

$$-\left(-0.1427E_2^{\ 2}+(\frac{10}{81}E_1+\frac{449}{1500})E_2-\frac{5}{81}E_1^{\ 2}+27.1878|_{E_2^{p*}=\frac{(\frac{10}{81}E_1+\frac{459}{1500})}{0.2854}}\right)<0$$

Therefore, if $E_1 \in [0,1.89)$, $E_2^{p*}=\dfrac{(\frac{10}{81}E_1+\frac{459}{1500})}{0.2854}=0.4326E_1+1.072$.

The optimal value functions at stage 2 can be rewritten as follows:

$$V_2^*=\begin{cases} -0.081E_2^{\ 2}+6.966E_2+27.1878|_{E_2^{p*}=\frac{(\frac{10}{81}E_1+\frac{459}{1500})}{0.2854}} = 34.56+2.94E_1-0.015E_1^{\ 2} & \text{if } E_1\in[0,1.89) \\ 3.83E_2-0.024E_2^{\ 2}+37.10|_{E_2^{p*}=E_1}=3.83E_1-0.024E_1^{\ 2}+37.10 & \text{if } E_1\in[1.89,4.6] \\ 3.83E_2-0.024E_2^{\ 2}+37.10|_{E_2^{g*}=\frac{0.2272+0.081E_1}{0.13048}}=43.695+2.327E_1-0.009E_1^{\ 2} & \text{if } E_1\in(4.6,10] \end{cases}$$

The optimal actions at stage 1 are shown as

$$\alpha_1^*=\begin{cases} 0.4326E_1+1.0722-E_1=-0.5674E_1+1.0722>0 \\ \text{(buying and pumping up to } E_2^{p*}=0.4326E_1+1.0722) \quad \text{if } E_1\in[0,1.89); \\ 0 \quad \text{(do nothing)} \qquad \text{if } E_1\in[1.89,4.5992]; \\ 1.741+0.621E_1-E_1=1.741-0.379E_1<0 \\ \text{(generating and selling down to } E_2^{g*}=1.741+0.621E_1) \quad \text{if } E_1\in(4.5992,10]. \end{cases} \qquad \text{(B26)}$$

The reward payoff functions at stage 1 are shown as follows:

$$R_1=\begin{cases}-(P^M+\lambda P^M\alpha_1/\alpha\rho+c)\cdot\alpha_1/\alpha\rho & \text{if } E_1\in[0,1.89)\\ 0 & \text{if } E_1\in[1.89,4.6]\\ -(P^M+\lambda P^M\alpha_1\beta\rho-c)\cdot\alpha_1\beta\rho & \text{if } E_1\in(4.6,10]\end{cases}$$

$$=\begin{cases}-(6+0.01\cdot 5(-0.5674E_1+1.0722)/0.9)(-0.5674E_1+1.0722)/0.9 & \text{if } E_1\in[0,1.89)\\ 0 & \text{if } E_1\in[1.89,4.6]\\ -(4+0.01\cdot 5\cdot 0.9(1.741-0.379E_1))\cdot(1.741-0.379E_1)\cdot 0.9 & \text{if } E_1\in(4.6,10]\end{cases}$$

$$=\begin{cases}-7.22+3.86E_1-0.02E_1^2 & \text{if } E_1\in[0,1.89)\\ 0 & \text{if } E_1\in[1.89,4.6]\\ -6.39+1.429E_1-0.006E_1^2 & \text{if } E_1\in(4.6,10]\end{cases}$$

Thus, the optimal value functions at stage 1 are shown as follows:

$$V_1^*=\max(R_1+V_2^*)=\begin{cases}-7.22+3.86E_1-0.02E_1^2+34.56+2.94E_1-0.015E_1^2 & \text{if } E_1\in[0,1.89)\\ 0+3.83E_1-0.024E_1^2+37.10 & \text{if } E_1\in[1.89,4.6]\\ -6.39+1.429E_1-0.006E_1^2+43.695+2.327E_1-0.009E_1^2 & \text{if } E_1\in(4.6,10]\end{cases}$$

$$=\begin{cases}27.34+6.8E_1-0.035E_1^2 & \text{if } E_1\in[0,1.89)\\ 3.83E_1-0.024E_1^2+37.10 & \text{if } E_1\in[1.89,4.6]\\ 37.31+3.76E_1-0.015E_1^2 & \text{if } E_1\in(4.6,10]\end{cases}$$

In summary, we will get the following optimal results:

1) If $E_1=1$ (The initial SOC in the storage)

Stage 1: If $E_1=1$, (action 1: buying and pumping), there is

$E_2=1.5$ (i.e., $\alpha_1^*=0.5$, $R_1=-(5+5\times0.01\times0.5/0.9)\times0.5/0.9-1\times0.5/0.9=-3.35$).

Stage 2: If $E_2=1.5$, (action 2: buying and pumping), there is

$E_3=8.5$ (i.e., $\alpha_2^*=7$, $R_2=-(2+2\times0.01\times7/0.9)\times7/0.9-1\times7/0.9=-24.54$).

Stage 3: If $E_3=8.5$,(action 3: generating and selling), there is

$E_4 = 0$ (i.e., $\alpha_3^* = -8.5$, $R_3 = -(10 - 0.01 \times 10 \times 8.5 \times 0.9) \times (-8.5) \times 0.9 + 1 \times (-8.5) \times 0.9 = 63.00$).

Total rewards $R = R_1 + R_2 + R_3 = -3.4 - 24.5 + 63.0 = 34.1$.

The optimal value at stage 1 is $V_1^* = 27.34 + 6.8 - 0.035 = 34.1$, if $E_1 = 1$.

2) If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$,(action 1: generating and selling), we will get

$E_2 = 4.84$ (i.e., $\alpha_1^* = -0.16$, $R = R_1 + R_2 + R_3 = -3.4 - 24.5 + 63.0 = 34.1$).

Stage 2: If $E_2 = 4.84$,(action 2: buying and pumping), there is

$E_3 = 10$ (i.e., $\alpha_2^* = 5.16$, $R_2 = -(2 + 2 \times 0.01 \times 5.16 / 0.9) \times 5.16 / 0.9 - 1 \times 5.16 / 0.9 = -17.86$).

Stage 3: If $E_3 = 10$,(action 3: generating and selling), there is

$E_4 = 0 = \underline{E}$ (i.e., $\alpha_3^* = -10$, $R_3 = -(10 - 0.01 \times 100 \times 0.9) \times (-10) \times 0.9 + 1 \times (-10) \times 0.9 = 72.9$).

Total rewards $R = R_1 + R_2 + R_3 = 0.57 - 17.86 + 72.9 = 55.7$.

The optimal value at stage 1 is shown as

$V_1^* = 37.31 + 3.76 \times 5 - 0.015 \times 25 = 55.7$, if $E_1 = 5$.

Case 4. Market Impact $\lambda = 0.02$

In this case, we assume the merchant's market impact parameter $\lambda = 0.02$.

Similarly, by using backward DP to obtain the following optimal value functions:

In Stage 3:

Action 3: the merchant should sell power to the market and lower the storage level down to $\underline{E} = 0$ due to $V_4 = 0$. The optimal value functions in stage 3 are:

$$V_3 = -[p^H + \lambda p^H(\underline{E} - E_3)\beta\rho - c](\underline{E} - E_3)\beta\rho$$
$$= [9 + 0.2(-E_3)0.9](-E_3)0.9 = 8.1E_3 - 0.162{E_3}^2, E_3 \in [0,10]$$

In Stage 2:

By using the equation (A4) and (A5) for price maker merchant, we will get the following results:

$$\begin{cases} E_3^{g*} = \underset{E_3 \in [\underline{E},\bar{E}]}{\arg\max}\{V_3^* - (p^L - c)E_3\beta\rho - \lambda P^L(\beta\rho)^2[E_3 - E_2]^2\} \\ \quad = \underset{E_3 \in [0,10]}{\arg\max}\{8.1E_3 - 0.162{E_3}^2 - 1\cdot 0.9\cdot E_3 - 0.04(0.81)[E_3 - E_2]^2\} \\ \quad = \underset{E_3 \in [0,10]}{\arg\max}\{(7.2 + 0.0648E_2)E_3 - 0.1944{E_3}^2 - 0.0324{E_2}^2\} = \underset{E_3 \in [0,10]}{\min}\{\frac{7.2 + 0.0648E_2}{0.3888}, 10\} = 10 = \bar{E} \\ E_3^{p*} = \underset{E_3 \in [\underline{E},\bar{E}]}{\arg\max}\{V_3^* - (p^L + c)\frac{E_3}{\alpha\rho} - \frac{\lambda P^L}{(\alpha\rho)^2}[E_3 - E_2]^2\} \\ \quad = \underset{E_3 \in [0,10]}{\arg\max}\{8.1E_3 - 0.162{E_3}^2 - \frac{3}{0.9}E_3 - \frac{0.04}{0.81}[E_3 - E_2]^2\} \\ \quad = \underset{E_3 \in [0,10]}{\arg\max}\{(4.67 + 0.09E_2)E_3 - 0.211{E_3}^2 - 0.05{E_2}^2\} = \underset{E_3 \in [0,10]}{\min}\{\frac{4.767 + 0.09E_2}{0.422}, 10\} = 10 = \bar{E} \end{cases} \quad \text{(A27)}$$

Following proposition 3.1, we will get the following results:

② If $E_2 < E_3^{p*} - \bar{Q}^p = 3$, buying-and-pumping up to $E_3^* = E_2 + \bar{Q}^p = E_2 + 7$, $\alpha_2^* = \bar{Q}^p = 7 > 0$.

③ If $E_2 \geq E_3^{p*} - \bar{Q}^p = 3$, buying-and-pumping up to $E_3^* = \bar{E} = 10$, $\alpha_2^* = E_3^{p*} - E_2 = 10 - E_2 > 0$.

Thus, we get the following optimal actions at stage 2 based on proposition 3.2.

$$\alpha_2^* = \begin{cases} \bar{Q}^p = 7 > 0 \text{ (buying and pumping up to } E_3^{p*} = E_2 + \bar{Q}^p < \bar{E}) & \text{if } E_2 \in [0,3) \\ 10 - E_2 > 0 \text{ (buying and pumping up to } E_3^{p*} = \bar{E}) & \text{if } E_2 \in [3,10] \end{cases} \quad \text{(A28)}$$

The reward payoff functions at stage 2 are shown as follows:

$$R_2 = \begin{cases} -(P^L + \lambda P^L \frac{\alpha_2}{\alpha\rho}) \cdot \frac{\alpha_2}{\alpha\rho} - c \cdot \frac{\alpha_2}{\alpha\rho} & \text{if } E_2 \in [0,\ E_3^{p*} - \bar{Q}^p) \\ -(P^L + \lambda P^L \frac{\alpha_2}{\alpha\rho}) \cdot \frac{\alpha_2}{\alpha\rho} - c \cdot \frac{\alpha_2}{\alpha\rho} & \text{if } E_2 \in [E_3^{p*} - \bar{Q}^p, 10] \end{cases}$$

$$= \begin{cases} -25.7531 & \text{if } E_2 \in [0,3) \\ \frac{10}{9}(-\frac{2}{45}E_2{}^2 + \frac{35}{9}E_2 - \frac{310}{9}) & \text{if } E_2 \in [3,10] \end{cases}$$

The optimal value functions are stage 3 can be rewritten as

$$V_3^* = \begin{cases} 8.1E_3 - 0.162E_3{}^2 |_{E_3^* = E_2 + 7} = -0.162E_2{}^2 + 5.832E_2 + 48.762 & \text{if } E_2 \in [0,3) \\ 8.1E_3 - 0.162E_3{}^2 |_{E_3^* = 10} = 64.8 & \text{if } E_2 \in [3,10] \end{cases}$$

Thus, the optimal value functions in stage 2 are shown as follows:

$$V_2^* = \max(R_2 + V_3^*) = \begin{cases} -0.162E_2{}^2 + 5.832E_2 + 23.01 & \text{if } E_2 \in [0,3) \\ -0.05E_2{}^2 + 4.32E_2 + 26.53 & \text{if } E_2 \in [3,10] \end{cases}$$

In Stage 1:

Similarly, by using the equations (A4) and (A5) for price maker merchant, we will get the following results:

$$E_2^{g*} = \underset{E_2 \in [\underline{E}, \bar{E}]}{\arg\max}\{V_2^* - (p^M - c)E_2\beta\rho - \lambda P^M \beta^2 \rho^2 [E_2 - E_1]^2\}$$
$$= \underset{E_2 \in [0,10]}{\arg\max} \begin{cases} -0.243E_2{}^2 + (2.232 + 0.162E_1)E_2 - 0.081E_1{}^2 + 23.0089 & \text{if } E_2 \in [0,3) \\ -0.131E_2{}^2 + (0.72 + 0.162E_1)E_2 - 0.081E_1{}^2 + 26.5284 & \text{if } E_2 \in [3,10] \end{cases} \quad \text{(A29)}$$

① If $E_2 \in [0,3)$, in the first equation of (B16), for any given $E_1 \in [0,10]$, there is

$(2.232 + 0.162E_1)/0.486 > 2.232/0.486 > 3$, that is $E_2^{g*} = 3$ if $E_2 \in [0,3)$.

② If $E_2 \in [3,10]$, in the second equation of (B16), for any given $E_1 \in [0,10]$, we get the relation of $2.748 \le (0.72 + 0.162E_1)/0.262 \le 8.931$.

By using $(0.72 + 0.162E_1)/0.262 \le E_1 \Rightarrow 0.72 + 0.162E_1 \le 0.262E_1 \Rightarrow E_1 \ge 7.2$

Thus, if $E_1 \in [7.2,10], E_2^{g*} = (0.72+0.162E_1)/0.262$, we also find that

$$[-0.131E_2^{\,2} + (0.72+0.162E_1)E_2 - 0.081E_1^{\,2} + 26.5284 |_{E_2=\frac{0.72+0.162E_1}{0.262}}]$$

$$-[-0.243E_2^{\,2} + (2.232+0.162E_1)E_2 - 0.081E_1^{\,2} + 23.0089 |_{E_2=3}]$$

$$=[-0.131(\frac{1}{0.262})^2 + \frac{1}{0.262}](0.72+0.162E_1)^2 - 0.081E_1^{\,2} + 26.5284$$

$$-[-0.243\cdot 9 + (2.232+0.162E_1)\cdot 3 - 0.081E_1^{\,2} + 23.0089]$$

$$= -0.0309E_1^{\,2} + 27.5177 + 0.4452E_1 - [0.486E_1 - 0.081E_1^{\,2} + 27.5179]$$

$$= 0.0501E_1^{\,2} - 0.0408E_1 - 0.0002$$

So, for any $E_1 \in [7.2,10]$, the relations of $0.0501E_1^{\,2} - 0.0408E_1 - 0.0002 > 0$ holds.

Therefore, if $E_1 \in (7.2976,10], E_2^{g*} = (0.72+0.162E_1)/0.262$ in (B16)

$$E_2^{P*} = \arg\max_{E_2 \in [\underline{E},\bar{E}]} \left(V_2^* - \frac{\lambda P^M}{\alpha^2 \rho^2}[E_2 - E_1]^2 - \frac{(P^M + c)}{\alpha\rho} E_2 \right)$$
$$= \arg\max_{E_2 \in [0,10]} \begin{cases} -0.285E_2^{\,2} + (0.246E_1 - 0.835)E_2 - 0.123E_1^{\,2} + 23.0089 & \text{if } E_2 \in [0,3) \\ -0.173E_2^{\,2} + (0.246E_1 - 2.347)E_2 - 0.123E_1^{\,2} + 26.5284 & \text{if } E_2 \in [3,10] \end{cases} \quad \text{(A30)}$$

① If $E_2 \in [0,3)$, in the first equation of (A30), for any given $E_1 \in [0,10]$, there $-1.4649 \le (0.246E_1 - 0.835)/0.57 \le 2.851$ holding. So, $E_2^{p*} \in [0, 2.851]$.

However, by using $(0.246E_1 - 0.835)/0.57 \ge E_1 \Leftrightarrow 0.246E_1 - 0.835 \ge 0.57E_1 \Rightarrow E_1 \le -2.577$, so, there is an available $E_2^{p*}=0$ if $E_2 \in [0,3)$.

② If $E_2 \in [3,10]$, in the second equation of (A30), for any given $E_1 \in [0,10]$, there is $(0.246E_1 - 2.347)/0.346 \le 0.327$.

So, the function of $-0.173E_2^{\,2} + (0.246E_1 - 2.347)E_2 - 0.123E_1^{\,2} + 26.5284$ decreases with E_2 on [3, 10]. Thus, we will get $E_2^{p*}=3$, if $E_2 \in [3,10]$. We also find that

$$-0.285E_2{}^2 + (0.246E_1 - 0.835)E_2 - 0.123E_1{}^2 + 23.0089\Big|_{E_2^{P*}=0}$$

$$-[-0.285E_2{}^2 + (0.246E_1 - 0.835)E_2 - 0.123E_1{}^2 + 23.0089\Big|_{E_2^{P*}=0}]$$

$$= -0.123E_1{}^2 + 23.0089 - [-0.123E_1{}^2 + 17.9304 + 0.738E_1\ ]$$

$$= 5.0785 - 0.738E_1 > 0 \text{ if } E_1 = 0 \text{ or } E_1 \le 3$$

In this way, the electricity merchant should adopt the following policy

① If $E_1 \in [0,7.2)$, the merchant should do nothing.

② If $E_1 \in (7.2,10]$, the merchant should adopt generating and selling and lower the storage down to $E_2^{g*} = \left(0.72 + 0.162E_1\right)/0.262$.

Thus, we will get the following optimal action at stage 1.

$$\alpha_1^* = \begin{cases} 0 \text{ (do nothing)} & \text{if } E_1 \in [0,7.2] \\ E_1 - \dfrac{0.72+0.162E_1}{0.262} \text{(generating and selling down to } \dfrac{0.72+0.162E_1}{0.262}\text{)} & \text{if } E_1 \in (7.2,10] \end{cases} \quad \text{(A31)}$$

Therefore, the optimal value functions at stage 1 are shown as follows:

$$V_2^* = \max(R_2 + V_3^*) = \begin{cases} -0.162E_2{}^2 + 5.832E_2 + 23.01 & \text{if } E_2 \in [0,3) \\ -0.05E_2{}^2 + 4.32E_2 + 26.53 & \text{if } E_2 \in [3,10] \end{cases}$$

The reward payoff functions at stage 1 are shown as follows:

$$R_1 = \begin{cases} 0 & \text{if } E_1 \in [0,7.2] \\ -(P^M + \lambda P^M \alpha_1 \beta\rho - c)\cdot \alpha_1 \beta\rho & \text{if } E_1 \in (7.2,10] \end{cases}$$

$$= \begin{cases} 0 & \text{if } E_1 \in [0,7.2] \\ 0.93 - 0.12E_1 - 0.0012E_1{}^2 & \text{if } E_1 \in (7.2,10] \end{cases}$$

Thus, the optimal value functions in stage 1 are shown as follows:

$$V_1^* = \max(R_1 + V_2^*) = \begin{cases} -0.05E_1{}^2 + 4.32E_1 + 26.5284 & \text{if } E_1 \in [3,7.2] \\ 38.8 + 2.38E_1 - 0.0202E_1{}^2 & \text{if } E_1 \in (7.2,10] \end{cases}$$

To sum up, we will get the following optimal results:

1) If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, (action 1: do nothing), we get $E_2 = 1$ (i.e., $\alpha_1^* = 0$, $R_1 = 0$);

Stage 2: If $E_2 = 1$, (action 2: buying and pumping), then, we will get

$E_3 = 8$ (i.e., $\alpha_2^* = 7$, $R_2 = -(2 + 2 \times 0.02 \times 7 / 0.9) \times 7 / 0.9 - 1 \times 7 / 0.9 = -25.7531$);

Stage 3: If $E_3 = 8$, (action 3: generating and selling), then, we have

$E_4 = 0 = \underline{E}$ (i.e., $\alpha_3^* = -8$, $R_3 = -(10 - 0.02 \times 80 \times 0.9) \times (-8) \times 0.9 + 1 \times (-8) \times 0.9 = 54.432$).

Total rewards is shown as $R = R_1 + R_2 + R_3 = -25.753 + 54.432 = 28.679$.

The optimal value at stage 1 is $V_1^* = -0.162E_1^2 + 5.832E_1 + 23.0089 = 28.679$ if $E_1 = 1$

2) If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$, (action 1: do nothing), we get $E_2 = 5$ (i.e., $\alpha_1^* = 0$, $R_1 = 0$) holds;

Stage 2: If $E_2 = 5$, (action 2: buying and pumping), then there exists

$E_3 = 10$ (i.e., $\alpha_2^* = 5$, $R_2 = -(2 + 2 \times 0.02 \times 5 / 0.9) \times 5 / 0.9 - 1 \times 5 / 0.9 = -17.9012$);

Stage 3: If $E_3 = 10$, (action 3: generating and selling), then we have

$E_4 = 0 = \underline{E}$ (i.e., $\alpha_3^* = -10$); $R_3 = 8.1 \times 10 - 0.162 \times 100 = 64.8$).

Therefore, total rewards in three periods are shown as $R = R_1 + R_2 + R_3 = 46.9$.

The optimal value in stage 1 is $V_1^* = -0.05 \times 25 + 4.32 \times 5 + 26.5284 = 46.9$ if $E_1 = 5$

APPENDIX B.

PROOF OF SECTION 4

<u>Proof of Lemma 4.1:</u>

(1) The uniqueness of the SOC reference points:

Based on the equation (4.5), by replacing q_t with SOC_{t+1} as the decision variable via $SOC_{t+1}/\varphi_t - SOC_t = q_t$, we get the following rewards functions.

$$R(q_t, w_t, P_t) = \begin{cases} -P_t(\frac{q_t}{\theta} - w_t)/\sigma - \lambda P_t/\sigma^2[(\frac{q_t}{\theta})^2 - 2\frac{q_t}{\theta}w_t + w_t{}^2] - c^p\frac{q_t}{\theta\sigma} - c_w w_t \ \ (q_t \geq \theta w_t) \\ -P_t(\frac{q_t}{\theta} - w_t)\cdot\sigma - \lambda P_t\sigma^2[(\frac{q_t}{\theta})^2 - 2\frac{q_t}{\theta}w_t + w_t{}^2] - c^p\frac{q_t}{\theta\sigma} - c_w w_t \ \ (0 \leq q_t \leq \theta w_t) \\ -P_t(q_t\theta - w_t)\cdot\sigma - \lambda P_t\sigma^2[(q_t\xi)^2 - 2(q_t\xi)w_t + w_t{}^2] + c^g q_t\xi\sigma - c_w w_t \ \ (q_t \leq 0) \end{cases} \quad \text{(B1)}$$

In the end of period T, the value function is shown:

$$V_T(S(T)) = [R(q_T, w_T, p_T) + E[V_{T+1}(S(T+1)\,|\,S(T))]] = [R(q_T, w_T, p_T) + VOW_{T+1}\cdot SOC_{T+1}]$$

Thus, we get the following three sub-optimization value functions:

$$\begin{cases} V_T^{(1)*}(S(T)) = \max\limits_{\underline{S}\leq SOC_{T+1}\leq\bar{S}} \begin{Bmatrix} -\frac{\lambda P_T}{\theta^2\sigma^2}q_T{}^2 + (\frac{2\lambda P_T w_T}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma})q_T - w_T(\frac{\lambda P_T}{\sigma^2}w_T - \frac{P_T}{\sigma} + c_w) \\ +E[V_{T+1}^*(S(T+1)\,|\,S(T))] \end{Bmatrix} \\ V_T^{(2)*}(S(T)) = \max\limits_{\underline{S}\leq SOC_{T+1}\leq\bar{S}} \begin{Bmatrix} -\frac{\lambda\rho^2 P_T}{\sigma^2}q_T{}^2 + (2\lambda P_T\sigma^2\frac{w_T}{\theta} - \frac{P_T\sigma^2 + c^p}{\theta\sigma})q_T - w_T[P_T\sigma(\lambda w_T\sigma - 1) + c_w] \\ +E[V_{T+1}^*(S(T+1)\,|\,S(T))] \end{Bmatrix} \\ V_T^{(3)*}(S(T)) = \max\limits_{\underline{S}\leq SOC_{T+1}\leq\bar{S}} \begin{Bmatrix} -\lambda P_T\xi^2\sigma^2 q_T{}^2 + (2\lambda P_T w_T\xi\sigma^2 - P_T\xi\sigma - c^g\xi\sigma)q_T - w_T[P_T\sigma(\lambda w_T\sigma - 1) + c_w] \\ +E[V_{T+1}^*(S(T+1)\,|\,S(T))] \end{Bmatrix} \end{cases} \quad \text{(B2)}$$

We can get the optimal results to the equation (B3) by removing the given state $S(T)$ (i.e., the given values SOC_T, w_T, and P_T) when maximizing the (B2) . So, we get the following equivalent equations:

$$\begin{cases} V_T^{(1)*}(S(T)) = \max\limits_{\underline{S} \le SOC_{T+1} \le \bar{S}} \begin{Bmatrix} -\frac{\lambda P_T}{\theta^2\sigma^2}\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)^2 + \left(\frac{2\lambda P_T w_T}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma}\right)\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right) \\ + VOW_{T+1} \cdot SOC_{T+1} \end{Bmatrix} \\ V_T^{(2)*}(S(T)) = \max\limits_{\underline{S} \le SOC_{T+1} \le \bar{S}} \begin{Bmatrix} -\frac{\lambda \rho^2 P_T}{\theta^2}\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)^2 + \left(2\lambda P_T \sigma^2 \frac{w_T}{\theta} - \frac{P_T\sigma^2 + c^p}{\theta\sigma}\right)\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right) \\ + VOW_{T+1} \cdot SOC_{T+1} \end{Bmatrix} \\ V_T^{(3)*}(S(T)) = \max\limits_{\underline{S} \le SOC_{T+1} \le \bar{S}} \begin{Bmatrix} -\lambda P_T \xi^2 \sigma^2 \left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)^2 + \left(2\lambda P_T w_T \xi\sigma^2 - P_T\xi\sigma - c^g\xi\sigma\right)\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right) \\ + VOW_{T+1} \cdot SOC_{T+1} \end{Bmatrix} \end{cases} \tag{B3}$$

The first-order derivative of $V_T^*(S(T))$ (i.e., best response functions) on SOC_{T+1} are:

$$\begin{cases} \frac{\partial V_T^{(1)*}(S(T))}{\partial SOC_{T+1}} = -2\frac{\lambda P_T}{\theta^2\sigma^2}\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)\frac{1}{\varphi_T} + \left(\frac{2\lambda P_T w_T}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma}\right)\frac{1}{\varphi_T} + VOW_{T+1} \\ \frac{\partial V_T^{(2)*}(S(T))}{\partial SOC_{T+1}} = -2\frac{\lambda\sigma^2 P_T}{\theta^2}\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)\frac{1}{\varphi_T} + \left(2\lambda P_T\sigma^2\frac{w_T}{\theta} - \frac{P_T\sigma^2 + c^p}{\theta\sigma}\right)\frac{1}{\varphi_T} + VOW_{T+1} \\ \frac{\partial V_T^{(3)*}(S(T))}{\partial SOC_{T+1}} = -2\lambda P_T\xi^2\sigma^2\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)\frac{1}{\varphi_T} + \left(2\lambda P_T w_T\xi\sigma^2 - P_T\xi\sigma - c^g\xi\sigma\right)\frac{1}{\varphi_T} + VOW_{T+1} \end{cases} \tag{B4}$$

We have the following second-order derivative functions of $V_T^*(S(T))$ on SOC_{T+1}.

$$\begin{cases} \frac{\partial^2 V_T^{(1)*}(S(T))}{\partial {SOC_{T+1}}^2} = \left(-2\frac{\lambda P_T}{\theta^2\sigma^2}\frac{1}{{\varphi_T}^2}\right) < 0; \\ \frac{\partial^2 V_T^{(2)*}(S(T))}{\partial {SOC_{T+1}}^2} = -2\frac{\lambda\sigma^2 P_T}{\theta^2}\frac{1}{{\varphi_T}^2} < 0; \\ \frac{\partial^2 V_T^{(3)*}(S(T))}{\partial {SOC_{T+1}}^2} = -2\lambda P_T\xi^2\sigma^2\frac{1}{{\varphi_T}^2} < 0. \end{cases} \tag{B5}$$

Because the second-order derivative function is negative, we can achieve the unique optimal results by using the first-order function. Therefore, $V_T^{(1)*}(S(t))$, $V_T^{(2)*}(S(t))$, and $V_T^{(3)*}(S(t))$ have a unique optimal solution on $SOC_{T+1} \in [\underline{S}, \bar{S}]$.

Then the Bellman equation can be used to derive the following results:

$$\begin{cases} SOC_{T+1}^{(1)*} = \arg \max_{\underline{S} \le SOC_{T+1} \le \bar{S}} \left(\begin{array}{l} E[V_{T+1}^{*}\left(S(T+1)\,|\,S(T)\right)] - \dfrac{\lambda P_t}{\theta^2\sigma^2}(\dfrac{SOC_{T+1}}{\varphi_T} - SOC_T)^2 \\ +(\dfrac{2\lambda P_T w_T}{\theta\sigma^2} - \dfrac{P_T + c^p}{\theta\sigma})\dfrac{SOC_{T+1}}{\varphi_T} \end{array} \right) \\ SOC_{T+1}^{(2)*} = \arg \max_{\underline{S} \le SOC_{T+1} \le \bar{S}} \left(\begin{array}{l} E[V_{T+1}^{*}\left(S(T+1)\,|\,S(T)\right)] - \dfrac{\lambda\sigma^2 P_T}{\theta^2}(\dfrac{SOC_{T+1}}{\varphi_T} - SOC_T)^2 \\ +(2\lambda P_T \rho^2 \dfrac{w_T}{\theta} - \dfrac{P_T\sigma^2 + c^p}{\theta\sigma})\dfrac{SOC_{T+1}}{\varphi_T} \end{array} \right) \\ SOC_{T+1}^{(3)*} = \arg \max_{\underline{S} \le SOC_{T+1} \le \bar{S}} \left(\begin{array}{l} E[V_{T+1}^{*}\left(S(T+1)\,|\,S(T)\right)] - \lambda P_T \xi^2 \sigma^2 (\dfrac{SOC_{T+1}}{\varphi_T} - SOC_T)^2 \\ +(2\lambda P_T w_T \xi\sigma^2 - P_T \xi\sigma + c^g \xi\sigma)\dfrac{SOC_{T+1}}{\varphi_T} \end{array} \right) \end{cases} \qquad (B6)$$

Similarly, for the any state at $t \in \{1,2,\cdots,T\}$, by maximizing of the value function $V_t(SOC_t, w_t, P_t)$, subject to $SOC_{t+1} \in [\underline{S},\ \bar{S}]$, we will obtain the following optimal functions based on the Bellman equation.

$$\begin{cases} V_t^{(1)*}(S(t)) = \max_{\underline{S} \le SOC_{t+1} \le \bar{S}} \left\{ -\dfrac{\lambda P_t}{\theta^2\sigma^2} q_t^{\ 2} + (\dfrac{2\lambda P_t w_t}{\theta\sigma^2} - \dfrac{P_t + c^p}{\theta\sigma})q_t - w_t(\dfrac{\lambda P_t}{\sigma^2} w_t - \dfrac{P_t}{\sigma} + c_w) + E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] \right\} \\ \Leftrightarrow \max_{\underline{S} \le SOC_{t+1} \le \bar{S}} \left(E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \dfrac{\lambda P_t}{\theta^2\sigma^2} \dfrac{SOC_{t+1}}{\varphi_t^{\ 2}}^{2} + \dfrac{2\lambda P_t}{\theta^2\sigma^2\varphi_t} SOC_{t+1} \cdot SOC_t + (\dfrac{2\lambda P_t w_t}{\theta\sigma^2} - \dfrac{P_t + c^p}{\theta\sigma})\dfrac{SOC_{t+1}}{\varphi_t} \right) \qquad (B7\text{-}1) \\ V_t^{(2)*}(S(t)) = \max_{\underline{S} \le SOC_{t+1} \le \bar{S}} \left\{ -\dfrac{\lambda\sigma^2 P_t}{\theta^2} q_t^{\ 2} + (2\lambda P_t \sigma^2 \dfrac{w_t}{\theta} - \dfrac{P_t\sigma^2 + c^p}{\theta\sigma})q_t - w_t[P_t\sigma(\lambda w_t \sigma - 1) + c_w] + E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] \right\} \\ \Leftrightarrow \max_{\underline{S} \le SOC_{t+1} \le \bar{S}} \left(E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \dfrac{\lambda\rho^2 P_t}{\theta^2} \dfrac{SOC_{t+1}}{\varphi_t^{\ 2}}^{2} + \dfrac{2\lambda\sigma^2 P_t}{\theta^2} \dfrac{SOC_{t+1}}{\varphi_t} SOC_t + (2\lambda P_t \sigma^2 \dfrac{w_t}{\theta} - \dfrac{P_t\sigma^2 + c^p}{\theta\sigma})\dfrac{SOC_{t+1}}{\eta_t} \right) \qquad (B7\text{-}2) \\ V_t^{(3)*}(S(t)) = \max_{\underline{S} \le SOC_{t+1} \le \bar{S}} \left\{ -\lambda P_t \xi^2 \sigma^2 q_t^{\ 2} + (2\lambda P_t w_t \xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma)q_t - w_t[P_t\sigma(\lambda w_t\sigma - 1) + c_w] + E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] \right\} \\ \Leftrightarrow \max_{\underline{S} \le SOC_{t+1} \le \bar{S}} \left(E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \lambda P_t \xi^2 \sigma^2 \dfrac{SOC_{t+1}}{\varphi_t^{\ 2}}^{2} + 2\lambda P_t \xi^2 \sigma^2 \dfrac{SOC_{t+1}}{\varphi_t} SOC_t + (2\lambda P_t w_t \xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma)\dfrac{SOC_{t+1}}{\varphi_t} \right) (B7\text{-}3) \end{cases}$$

Based on the proof at last decision period T, we know that for every optimization period $t \in \{1, 2, \cdots, T\}$, and in every state *t*, both $V_t(S(t))$ and $E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]$ are concave functions on $SOC_t \in [\underline{S},\ \bar{S}]$ for any given decision state

$S(t)=S_t(SOC_t,w_t,P_t)$. Clearly, $E[V_{t+1}^*(S(t+1)|S(t))]$ and functions (B7-1)-(B7-3) are concave in $SOC_{t+1}\in[\underline{S},\ \overline{S}]$ for each given state $S(t)=S_t(SOC_t,w_t,P_t)$ by using

$$\frac{\partial E[V_{t+1}^*(S(t+1)|S(t))]}{\partial SOC_{t+1}{}^2}=\frac{\partial\left(\frac{\partial E[V_{t+1}^*(S(t+1)|S(t))]}{\partial SOC_{t+1}}\right)}{\partial SOC_{t+1}}=\frac{\partial\left(\frac{\partial E[V_{t+1}^*(S(t+1)|S(t))]}{\partial SOC_t}\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}}\right)}{\partial SOC_t}\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}}$$

$$=\left(\frac{\partial E[V_{t+1}^*(S(t+1)|S(t))]}{\partial SOC_t{}^2}\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}}+\frac{\partial E[V_{t+1}^*(S(t+1)|S(t))]}{\partial SOC_t}\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}\partial SOC_t}\right)\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}}$$

$$=\left(\frac{\partial E[V_{t+1}^*(S(t+1)|S(t))]}{\partial SOC_t{}^2}\cdot\left(\frac{\partial SOC_t}{\partial SOC_{t+1}}\right)^2\right)\le 0$$

1) When $q_t>\theta w_t$, by maximizing the equation (B7-1), subject to $SOC_{t+1}\in[\underline{S},\ \overline{S}]$, we get the following best response function (i.e., first-order derivative):

$$\frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}}=\frac{\partial E[V_{t+1}^*(S(t+1)|S(t))]}{\partial SOC_{t+1}}-\frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t{}^2}SOC_{t+1}+\frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t}SOC_t+\frac{2\lambda P_t w_t}{\theta\sigma^2\varphi_t}-\frac{P_t+c^p}{\theta\sigma\varphi_t}.$$

The second-order derivative function: $\frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}^2}=\frac{\partial E[V_{t+1}^*(S(t+1)|S(t))]}{\partial SOC_{t+1}^2}-\frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t{}^2}<0$.

Thus, we can achieve the optimal references points solutions using the first-order function because the second-order derivative is negative. Therefore, we will obtain the subsequent optimal consequences:

$$\begin{cases} SOC_{t+1}^{(1)*}=\arg\max\limits_{\underline{S}\le SOC_{t+1}\le\overline{S}}\left(\begin{array}{l} E[V_{t+1}^*(S(t+1)|S(t))]-\frac{\lambda P_t}{\theta^2\sigma^2}\frac{SOC_{t+1}{}^2}{\varphi_t{}^2}+\frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t}SOC_{t+1}\cdot SOC_t \\ +(\frac{2\lambda P_t w_t}{\theta\sigma^2}-\frac{P_t+c^p}{\theta\sigma})\frac{SOC_{t+1}}{\varphi_t} \end{array}\right) \\ \text{or}\ \frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}}=\frac{\partial E[V_{t+1}^*(S(t+1)|S(t))]}{\partial SOC_{t+1}}-\left(\frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t{}^2}-\frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t}\right)SOC_t+\frac{2\lambda P_t w_t}{\theta\sigma^2\varphi_t}-\frac{P_t+c^p}{\theta\sigma\varphi_t}\Bigg|_{SOC_{t+1}=SOC_{t+1}^{(1)*}}=0 \end{cases}\quad\text{(B8)}$$

2) Similarly, when $0\le q_t\le\theta w_t$, by optimizing the function (B7-2), we can obtain the unique optimal reference points using the first-order function, and the optimal

solutions are:

$$\begin{cases} SOC_{t+1}^{(2)*} = \arg\max_{\underline{S} \le SOC_{t+1} \le \bar{S}} \left(\begin{array}{l} E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \frac{\lambda\sigma^2 P_t}{\theta^2} \frac{SOC_{t+1}^{\ 2}}{\varphi_t^{\ 2}} + \frac{2\lambda\sigma^2 P_t}{\theta^2 \varphi_t} SOC_{t+1} \cdot SOC_t \\ +(2\lambda P_t \sigma^2 \frac{w_t}{\theta} - \frac{P_t\sigma^2 + c^p}{\theta\sigma}) \frac{E_{t+1}}{\varphi_t} \end{array} \right) \\ \text{or} \frac{\partial V_t^{(2)*}(S(t))}{\partial SOC_{t+1}} = \frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_{t+1}} - \left(\frac{2\lambda\sigma^2 P_t}{\theta^2 \varphi_t^{\ 2}} - \frac{2\lambda\sigma^2 P_t}{\theta^2 \varphi_t} \right) SOC_t - (2\lambda P_t \sigma^2 \frac{w_t}{\theta} - \frac{P_t\sigma^2 + c^p}{\theta\sigma}) \frac{1}{\varphi_t} \Big|_{SOC_{t+1} = SOC_{t+1}^{(2)*}} = 0 \end{cases} \quad \text{(B9)}$$

3) When $q_t < 0$, by optimizing the function (B7-3), we will obtain the optimal SOC results as follows:

$$\begin{cases} SOC_{t+1}^{(3)*} = \arg\max_{\underline{S} \le SOC_{t+1} \le \bar{S}} \left(E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \lambda P_t \xi^2 \sigma^2 \frac{E_{t+1}^{\ 2}}{\varphi_t^{\ 2}} + 2\lambda P_t \xi^2 \sigma^2 \frac{E_{t+1}}{\varphi_t} E_t + (2\lambda P_t w_t \xi \sigma^2 - P_t \xi \sigma + c^g \xi \sigma) \frac{SOC_{t+1}}{\varphi_t} \right) \\ \text{or} \frac{\partial V_t^{(3)*}(S(t))}{\partial SOC_{t+1}} = \frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_{t+1}} - \left(\frac{2\lambda P_t \xi^2 \sigma^2}{\varphi_t^{\ 2}} - \frac{2\lambda P_t \xi^2 \sigma^2}{\varphi_t} \right) SOC_t + \frac{2\lambda P_t w_t \xi \sigma^2 - P_t \xi \sigma + c^g \xi \sigma}{\eta_t} \Big|_{SOC_{t+1} = SOC_{t+1}^{(3)*}} = 0 \end{cases}$$

(B10)

(2) The relations among three SOC optimal results/SOC reference points:

We define three auxiliary functions based on (B8) – (B10) to simplify illumination.

$$\left\{ F(SOC_{t+1})^{(1)} = \frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}};\ F(SOC_{t+1})^{(2)} = \frac{\partial V_t^{(2)*}(S(t))}{\partial SOC_{t+1}};\ F(SOC_{t+1})^{(3)} = \frac{\partial V_t^{(3)*}(S(t))}{\partial SOC_{t+1}} \right. \quad \text{(B11)}$$

The related first-order functions of (B11) correspond to the second-order derivative functions of (B7-1), (B7-2), and (B7-3) are shown:

$$\begin{cases} \frac{\partial F(SOC_{t+1})^{(1)}}{\partial SOC_{t+1}} = \frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}^{\ 2}} = \frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_{t+1}^{2}} - \frac{2\lambda P_t}{\theta^2 \sigma^2 \varphi_t^{\ 2}} < 0 \\ \frac{\partial F(SOC_{t+1})^{(2)}}{\partial SOC_{t+1}} = \frac{\partial V_t^{(2)*}(S(t))}{\partial SOC_{t+1}^{\ 2}} = \frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_{t+1}^{\ 2}} - \frac{2\lambda\sigma^2 P_t}{\theta^2 \varphi_t^{\ 2}} < 0 \\ \frac{\partial F(SOC_{t+1})^{(3)}}{\partial SOC_{t+1}} = \frac{\partial V_t^{(3)*}(S(t))}{\partial SOC_{t+1}^{\ 2}} = \frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_{t+1}^{\ 2}} - \frac{2\lambda P_t \xi^2 \sigma^2}{\varphi_t^{\ 2}} < 0 \end{cases} \quad \text{(B12)}$$

Based on (B12), we find that the above three defined auxiliary functions are all decreasing with $SOC_{t+1} \in [\underline{S}, \overline{S}]$. We also get the following relations among the first-order functions of (B11).

$$\left|\partial F(SOC_{t+1})^{(1)} / \partial SOC_{t+1}\right| \geq \left|\partial F(SOC_{t+1})^{(2)} / \partial SOC_{t+1}\right| \geq \left|\partial F(SOC_{t+1})^{(3)} / \partial SOC_{t+1}\right|.$$

1) For all $SOC_{t+1} \in [\underline{S}, \overline{S}]$, if $\max F(SOC_{t+1})^{(1)} \leq \max F(SOC_{t+1})^{(2)}$, then we will obtain $SOC_{t+1}^{(1)*} \leq SOC_{t+1}^{(2)*}$.

$$\max F(SOC_{t+1})^{(1)} = F(SOC_{t+1} = \underline{S})^{(1)} \leq \max F(SOC_{t+1})^{(2)} = F(SOC_{t+1} = \underline{S})^{(2)}$$

$$\Leftrightarrow \left(-\frac{2\lambda P_t}{\theta^2 \sigma^2 \varphi_t{}^2} \underline{S} + \frac{2\lambda P_t}{\theta^2 \sigma^2 \varphi_t} SOC_t + \frac{2\lambda P_t w_t}{\theta \sigma^2 \varphi_t} - \frac{P_t + c^p}{\theta \sigma \varphi_t} \right)$$

$$\leq \left(-\frac{2\lambda \sigma^2 P_t}{\theta^2 \varphi_t{}^2} \underline{S} + \frac{2\lambda \sigma^2 P_t}{\theta^2 \varphi_t} SOC_t + 2\lambda P_t \sigma^2 \frac{w_t}{\theta} \frac{1}{\varphi_t} - \frac{P_t \sigma^2 + c^p}{\theta \sigma} \frac{1}{\varphi_t} \right)$$

$$\Rightarrow \lambda \leq \frac{P_t(1-\sigma^2)}{\theta \sigma} \Bigg/ \frac{2P_t}{\theta^2}\left(\frac{1}{\sigma^2} - \sigma^2\right)\left(SOC_t + w_t \theta - \frac{\underline{S}}{\varphi_t}\right) = \overline{\lambda}_t^{(1,2)} = P_t \Bigg/ \frac{2P_t}{\theta}\left(\frac{1+\sigma^2}{\sigma}\right)\left(SOC_t + w_t \theta - \frac{\underline{S}}{\varphi_t}\right)$$

2) For all $SOC_{t+1} \in [\underline{S}, \overline{S}]$, if $\max F(SOC_{t+1})^{(2)} \leq \max F(SOC_{t+1})^{(3)}$, then we will obtain $SOC_{t+1}^{(2)*} \leq SOC_{t+1}^{(3)*}$.

$$\max F(SOC_{t+1})^{(2)} = F(SOC_{t+1} = \underline{S})^{(2)} \leq \max F(SOC_{t+1})^{(3)} = F(SOC_{t+1} = \underline{S})^{(3)}$$

$$\Leftrightarrow \left(-\frac{2\lambda \sigma^2 P_t}{\theta^2 \varphi_t{}^2} \underline{S} + \frac{2\lambda \sigma^2 P_t}{\theta^2 \varphi_t} E_t + 2\lambda P_t \sigma^2 \frac{w_t}{\theta} \frac{1}{\varphi_t} - \frac{P_t \sigma^2 + c^p}{\theta \sigma} \frac{1}{\varphi_t} \right)$$

$$\leq \left(-\frac{2\lambda P_t \xi^2 \sigma^2}{\varphi_t{}^2} \underline{S} + \frac{2\lambda P_t \xi^2 \sigma^2}{\varphi_t} SOC_t + \frac{2\lambda P_t w_t \xi \sigma^2}{\varphi_t} - \frac{(P_t \xi \sigma - c^g \xi \sigma)}{\varphi_t} \right)$$

$$\Leftrightarrow \lambda 2\sigma^2 P_t \left((SOC_t - \frac{\underline{S}}{\varphi_t})(\frac{1}{\theta^2} - \xi^2) + (\frac{w_t}{\theta} - w_t \xi) \right) \leq \left(\frac{P_t \sigma^2 + c^p}{\theta \rho} - (P_t \xi \sigma - c^g \xi \sigma) \right)$$

$$\Rightarrow \lambda \leq \left(\frac{P_t \sigma^2 + c^p}{\theta \sigma} - (P_t \xi \sigma - c^g \xi \sigma) \right) \Bigg/ 2\rho^2 P_t \left((SOC_t - \frac{\underline{S}}{\varphi_t})(\frac{1}{\theta^2} - \xi^2) + w_t (\frac{1}{\theta} - \xi) \right) = \overline{\lambda}_t^{(2,3)}$$

If both the available wind generation equals zero (i.e., $w_t = 0$), and the current energy inventory reaches the minimum limit of storage (i.e., $SOC_t - \underline{S}/\varphi_t = 0$) at optimization period t, for any forecasted price $P_t \geq 0$ and market impact of energy storage $\lambda \geq 0$, there exists $SOC_{t+1}^{(1)*} \leq SOC_{t+1}^{(2)*} \leq SOC_{t+1}^{(3)*}$.

To sum up, for positive prices $P_t \geq 0$, when the market impact of energy storage meets condition $0 \leq \lambda \leq \min\{\overline{\lambda}_t^{(1,2)}, \overline{\lambda}_t^{(2,3)}\}$, thus, we can get the following relations among three optimal SOC reference points:

$$SOC_{t+1}^{(1)*} \leq SOC_{t+1}^{(2)*} \leq SOC_{t+1}^{(3)*}$$

Obviously, if $P_t \leq 0$, we get $SOC_{t+1}^{(1)*} \geq SOC_{t+1}^{(2)*} \geq SOC_{t+1}^{(3)*}$ when there is $0 \leq \lambda \leq \min\left\{\overline{\lambda}_t^{(1,2)}, \overline{\lambda}_t^{(2,3)}\right\}$.

<u>Proof of Proposition 4.1:</u>

(1) Optimal Solutions (without consider the capacity of transmission line):

1) $\theta w_t < \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$

$$q_t^*(S_t) = \begin{cases} \min\{SOC_{t+1}^{(1)*} - SOC_t, \overline{Q}^p\},\ SOC_t \in [\underline{S}, SOC_{t+1}^{(1)*} - \theta w_t], \\ \text{(store generation and buy electricity up to } SOC_{t+1}^{(1)*}\text{);} \\ \min\{SOC_{t+1}^{(2)*} - SOC_t, \theta w_t\}, SOC_t \in (SOC_{t+1}^{(1)*} - \theta w_t, SOC_{t+1}^{(2)*}], \\ \text{(store generation without buying up to } SOC_{t+1}^{(2)*}\text{);} \\ 0,\ SOC_t \in (SOC_{t+1}^{(2)*}, SOC_{t+1}^{(3)*}] \text{ (keep inventory unchanged);} \\ \max\{SOC_{t+1}^{(3)*} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC_{t+1}^{(3)*}, \overline{S}] \text{(sell inventory down to } SOC_{t+1}^{(3)*}\text{).} \end{cases} \quad \text{(B13)}$$

2) $\theta w_t \geq \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$

$$q_t^*(S_t)=\begin{cases}\min\{SOC_{t+1}^{(2)*}-SOC_t,\overline{Q}^p,\theta w_t\},SOC_t\in[\underline{S},SOC_{t+1}^{(2)*}]\text{(store generation up to }SOC_{t+1}^{(2)*})\\ 0,\ SOC_t\in[SOC_{t+1}^{(2)*},SOC_{t+1}^{(3)*}]\text{ (keep inventory unchanged)}\\ \max\{SOC_{t+1}^{(3)*}-SOC_t,-\overline{Q}^g\},SOC_t\in(SOC_{t+1}^{(3)*},\overline{S}]\text{(sell inventory down to }SOC_{t+1}^{(3)*})\end{cases}\quad \text{(B14)}$$

Special case:

a) If $(\theta=\xi=\sigma=1,c^p=c^g=0)$, then we will get $SOC_{t+1}^{(1)*}=SOC_{t+1}^{(2)*}=SOC_{t+1}^{(3)*}=SOC_{t+1}^*$.

1) $\theta w_t<\min\{SOC_{t+1}^*,\overline{Q}^p\}$

$$q_t^*(S_t)=\begin{cases}\min\{SOC_{t+1}^*-SOC_t,\overline{Q}^p\},\ SOC_t\in[\underline{S},SOC_{t+1}^*-\theta w_t],\\ \text{(store generation and purchased electricity up to }SOC_{t+1}^*);\\ \max\{SOC_{t+1}^*-SOC_t,-\overline{Q}^g\},SOC_t\in(SOC_{t+1}^*,\overline{S}],\\ \text{(sell inventory down to }SOC_{t+1}^*).\end{cases}\quad \text{(B15)}$$

2) $\theta w_t\geq\min\{SOC_{t+1}^*,\overline{Q}^p\}$

$$q_t^*(S_t)=\begin{cases}\min\{SOC_{t+1}^*-SOC_t,\overline{Q}^p,\theta w_t\},SOC_t\in[\underline{S},SOC_{t+1}^*]\text{(store generation up to }SOC_{t+1}^*)\\ \max\{SOC_{t+1}^*-SOC_t,-\overline{Q}^g\},SOC_t\in(SOC_{t+1}^*,\overline{S}]\text{(sell inventory down to }SOC_{t+1}^*)\end{cases}\quad \text{(B16)}$$

b) If $\sigma=1$ (transmission efficiency), then we will get $SOC_{t+1}^{(1)*}=SOC_{t+1}^{(2)*}$.

1) $\theta w_t<\min\{SOC_{t+1}^{(1)*},\overline{Q}^p\}$

$$q_t^*(S_t)=\begin{cases}\min\{SOC_{t+1}^{(1)*}-SOC_t,\overline{Q}^p\},\ SOC_t\in[\underline{S},SOC_{t+1}^{(1)*}-\theta w_t],\\ \text{(store generation and buy power to }SOC_{t+1}^{(1)*});\\ 0,\ SOC_t\in(SOC_{t+1}^{(2)*},SOC_{t+1}^{(3)*}]\text{ (keep inventory unchanged);}\\ \max\{SOC_{t+1}^{(3)*}-SOC_t,-\overline{Q}^g\},SOC_t\in(SOC_{t+1}^{(3)*},\overline{S}]\text{(sell power to }SOC_{t+1}^{(3)*}).\end{cases}\quad \text{(B17)}$$

2) $\theta w_t\geq\min\{SOC_{t+1}^*,\overline{Q}^p\}$

$$q_t^*(S_t)=\begin{cases}\min\{SOC_{t+1}^{(2)*}-SOC_t,\overline{Q}^p,\theta w_t\}, SOC_t\in[\underline{S},SOC_{t+1}^{(2)*}] (\text{store generation up to } SOC_{t+1}^{(2)*})\\ 0, SOC_t\in[SOC_{t+1}^{(2)*},SOC_{t+1}^{(3)*}] (\text{keep inventory unchanged})\\ \max\{SOC_{t+1}^{(3)*}-SOC_t,-\overline{Q}^g\}, SOC_t\in(SOC_{t+1}^{(3)*},\overline{S}] (\text{sell inventory down to } SOC_{t+1}^{(3)*})\end{cases} \tag{B18}$$

c) If $(\theta=\xi=1, c^p=c^g=0)$ (we have $SOC_{t+1}^{(2)*}=SOC_{t+1}^{(3)*}$)

1) $\theta w_t<\min\{SOC_{t+1}^{(1)*},\overline{Q}^p\}$

$$q_t^*(S_t)=\begin{cases}\min\{SOC_{t+1}^{(1)*}-SOC_t,\overline{Q}^p\},\ SOC_t\in[\underline{S},SOC_{t+1}^{(1)*}-\theta w_t],\\ (\text{store renewable and buy power up to } SOC_{t+1}^{(1)*});\\ \min\{SOC_{t+1}^{(2)*}-SOC_t,\theta w_t\}, SOC_t\in(SOC_{t+1}^{(1)*}-\theta w_t,SOC_{t+1}^{(2)*}],\\ (\text{store renewable without buying up to } SOC_{t+1}^{(2)*});\\ \max\{SOC_{t+1}^{(3)*}-SOC_t,-\overline{Q}^g\}, SOC_t\in(SOC_{t+1}^{(3)*},\overline{S}] (\text{sell energy down to } SOC_{t+1}^{(3)*}).\end{cases} \tag{B19}$$

2) $\theta w_t\geq\min\{SOC_{t+1}^{*},\overline{Q}^p\}$

$$q_t^*(S_t)=\begin{cases}\min\{SOC_{t+1}^{(2)*}-SOC_t,\overline{Q}^p,\theta w_t\}, SOC_t\in[\underline{S},SOC_{t+1}^{(2)*}],\\ (\text{store renewable generation up to } SOC_{t+1}^{(2)*});\\ \max\{SOC_{t+1}^{(3)*}-SOC_t,-\overline{Q}^g\}, SOC_t\in(SOC_{t+1}^{(3)*},\overline{S}] (\text{sell energy down to } SOC_{t+1}^{(3)*}).\end{cases} \tag{B20}$$

<u>Proof of Proposition 4.2:</u>

Recall the proof the proposition 4.1, when the merchant who has PSH and wind plants, for any given state $S(t)$, we can also obtain the following outcomes:

$$\begin{cases} SOC^{(1)*}_{t+1\ (\lambda>0)} = \arg \max\limits_{\underline{S}\le SOC_{t+1}\le\bar{S}} \left(\begin{array}{l} E[V^*_{t+1}\left(S(t+1)\,|\,S(t)\right)] - \dfrac{\lambda P_t}{\theta^2\sigma^2}(\dfrac{SOC_{t+1}}{\varphi_t} - SOC_t)^2 \\ +(\dfrac{2\lambda P_t w_t}{\theta\sigma^2} - \dfrac{P_t + c^p}{\theta\sigma})\dfrac{SOC_{t+1}}{\varphi_t} \end{array} \right) \\ SOC^{(2)*}_{t+1\ (\lambda>0)} = \arg \max\limits_{\underline{S}\le SOC_{t+1}\le\bar{S}} \left(\begin{array}{l} E[V^*_{t+1}\left(S(t+1)\,|\,S(t)\right)] - \dfrac{\lambda\sigma^2 P_t}{\theta^2}(\dfrac{SOC_{t+1}}{\varphi_t} - SOC_t)^2 \\ +(2\lambda P_t\rho^2\dfrac{w_t}{\theta} - \dfrac{P_t\sigma^2 + c^p}{\theta\sigma})\dfrac{SOC_{t+1}}{\varphi_t} \end{array} \right) \\ SOC^{(3)*}_{t+1\ (\lambda>0)} = \arg \max\limits_{\underline{S}\le SOC_{t+1}\le\bar{S}} \left(\begin{array}{l} E[V^*_{t+1}\left(S(t+1)\,|\,S(t)\right)] - \lambda P_t\xi^2\sigma^2(\dfrac{SOC_{t+1}}{\varphi_t} - SOC_t)^2 \\ +(2\lambda P_t w_t\xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma)\dfrac{SOC_{t+1}}{\varphi_t} \end{array} \right) \end{cases} \tag{B21}$$

Through the rewards function of (B1), for any positive forecasted prices and decision state $t \in \{1, 2, \cdots, T\}$, there exist the following relationships:

$$\frac{\partial R(q_t, w_t, P_t)}{\partial\lambda} = \begin{cases} -P_t/\sigma^2 \cdot (q_t/\theta - w_t)^2 \le 0 & (q_t > \theta w_t) \\ -P_t\sigma^2 \cdot (q_t/\theta - w_t)^2 \le 0 & (0 \le q_t < \theta w_t) \\ -P_t\sigma^2 \cdot (q_t\xi - w_t)^2 \le 0 & (q_t < 0) \end{cases} \Rightarrow \frac{\partial R(q_t, w_t, P_t)}{\partial\lambda} \le 0 \tag{B22}$$

Suppose the $q^{*(M)}_{t\ (\lambda\ge0)}$ (resp. $q^{*(M)}_{t\ (\lambda=0)}$) represents the optimal actions of electricity merchants considering the market impact (resp. without considering market impact) in trading decisions. Thus, $\sum_{t=1}^{T} R(q^{*(M)}_{t\ (\lambda\ge0)}, w_t, P_t) \ge \sum_{t=1}^{T} R(q^{*(M)}_{t\ (\lambda=0)}, w_t, P_t)$ holds, which means the value function of the merchant $V^*_{t+1\ (\lambda>0)}(S(t+1)\,|\,S(t))$ decreases with the increasing of market impact, then there are:

$$\begin{cases} E[V^*_{t+1\ (\lambda\ge0)}\left(S(t+1)\,|\,S(t)\right)] \le E[V^*_{t+1\ (\lambda=0)}\left(S(t+1)\,|\,S(t)\right)] \\ \max\limits_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t)_{(\lambda\ge0)}\,|\,S(1)\right] \le \max\limits_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t)_{(\lambda=0)}\,|\,S(1)\right] \end{cases} \tag{B23}$$

Obviously, if a price-maker merchant ignores her market impact and follows the price-taker's optimal economic dispatch, we can draw the following relationship:

$$\begin{aligned}\max_{\pi}\sum_{t=1}^{T}E\left[R(q_t,w_t,P_t)_{(\lambda\geq 0)}\,|\,S(1)\right]&=\sum_{t=1}^{T}E\left[R(q^{*(M)}_{t\,(\lambda\geq 0)},w_t,P_t)_{(\lambda\geq 0)}\,|\,S(1)\right]\\&\geq\sum_{t=1}^{T}E\left[R(q^{*(M)}_{t\,(\lambda=0)},w_t,P_t)_{(\lambda\geq 0)}\,|\,S(1)\right]\end{aligned}\tag{B24}$$

Using the rewards function (B1), we get the following first-order response function:

$$\frac{\partial R(q_t,w_t,P_t)}{\partial w_t}=\begin{cases}\frac{P_t}{\sigma}-\lambda P_t/\sigma^2[-2(\frac{q_t}{\theta})+2w_t]-c_w=\frac{P_t}{\sigma}-c_w+2\lambda P_t/\sigma^2\left(\frac{q_t}{\theta}-w_t\right)(q_t\geq\theta w_t)\\P_t\sigma-\lambda P_t\sigma^2[-2(\frac{q_t}{\theta})+2w_t]-c_w=P_t\sigma-c_w+2\lambda P_t\sigma^2\left(\frac{q_t}{\theta}-w_t\right)(0\leq q_t\leq\theta w_t)\\P_t\sigma-\lambda P_t\sigma^2[-2(q_t\xi)+2w_t]-c_w=P_t\sigma-c_w+2\lambda P_t\sigma^2\left(q_t\xi-w_t\right)(q_t\leq 0)\end{cases}\tag{B25}$$

We have the following relationship for $\partial R(q_t,w_t,P_t)/\partial w_t$ based on equation (B25).

$$\begin{cases}\partial R_1(q_t,w_t,P_t)/\partial w_t=P_t/\sigma-c_w+2\lambda P_t/\sigma^2\left(q_t/\theta-w_t\right)\geq 0\\\Rightarrow P_t/\sigma\geq c_w\ \ (q_t\geq\theta w_t);\\\partial R_2(q_t,w_t,P_t)/\partial w_t=P_t\sigma-c_w+2\lambda P_t\sigma^2\left(q_t/\theta-w_t\right)\geq 0\\\Rightarrow\lambda\leq(P_t\sigma-c_w)/2P_t\sigma^2\left(w_t-q_t/\theta\right)<\bar{\lambda}^{(1,2)}_t\ \ (0\leq q_t\leq\theta w_t);\\\partial R_3(q_t,w_t,P_t)/\partial w_t=P_t\sigma-c_w+2\lambda P_t\sigma^2\left(q_t\xi-2w_t\right)\geq 0\\\Rightarrow\lambda\leq(P_t\sigma-c_w)/2P_t\sigma^2\left(w_t-q_t\xi\right)<\bar{\lambda}^{(2,3)}_t\ \ (q_t\leq 0).\end{cases}\tag{B26}$$

It implies that the merchant with PSH and wind plants needs to generate the wind power based on the max capacity of the wind turbines installed to benefit her profit.

Next, we will analyze how the operation cost influences the optimal scheduling policy of the energy storage and the revenue of the electricity merchant. Then, based on the rewards function of (B1), we will get the following first-order response function:

$$\frac{\partial R(q_t, w_t, P_t)}{\partial c^p} = \begin{cases} -(q_t/\theta\sigma)\ (q_t \geq \theta w_t) \\ -(q_t/\theta\sigma)\ (0 \leq q_t \leq \theta w_t) \end{cases}; \frac{\partial R(q_t, w_t, P_t)}{\partial c^g} = q_t \xi \sigma\ (q_t \leq 0) \tag{B27}$$

Based on the equation (B27), We get the following relationship for the reward functions on the generating and pumping cost.

$$\begin{cases} \dfrac{\partial R(q_t, w_t, P_t)}{\partial c^p} \leq 0, \dfrac{\partial R(q_t, w_t, P_t)}{\partial c^g} \leq 0 \end{cases} \tag{B28}$$

It is straightforward; the merchant will achieve less profit by increasing the operating cost. It plays a similar role as the market impact.

APPENDIX C.

PROOF OF SECTION 5

Proof of Lemma 5.1:

1) The uniqueness of the optimal results:

The current payoff rewards in the *scenario1*: Electricity generation cannot meet the power load (i.e., $(W_t - L_t) < 0$) are shown as follows for the prosumer:

$$R^{-}(q_t, W_t, L_t, P_t) = \begin{cases} -P_t\left[q_t/\alpha - (W_t - L_t)\right]/\rho - C^{ch} \cdot q_t - C^{w} \cdot W_t & (q_t \geq 0) \\ -P_t\left[q_t\beta - (W_t - L_t)\right]/\rho + C^{dis} \cdot q_t - C^{w} \cdot W_t & (-(L_t - W_t)/\beta < q_t \leq 0) \\ -P_t\left[q_t\beta - (W_t - L_t)\right]\rho + C^{dis} \cdot q_t - C^{w} \cdot W_t & (q_t \leq -(L_t - W_t)/\beta) \end{cases} \quad \text{(C1)}$$

where, q_t is the energy change from period t to period t+1 before accounting for energy loss. For the state at $t \in \{1, 2, \cdots, T\}$, by optimizaing the value function $V_t(E_t, W_t, L_t, P_t)$, subject to $\underline{E} \leq E_{t+1} \leq \bar{E}$, we will get the following equations:

$V_t(S_t) = [R(q_t, W_t, L_t, P_t) + E[V_{t+1}(S(t+1) \mid S(t))]$, where $E[V_{T+1}(S(T+1) \mid S(T)) = VOE_{T+1} \cdot E_{T+1}$.

Thus, we get the following three sub-optimization value functions:

$$V_t^{*}(S(t)) = \begin{cases} V_t^{(1)*-}(S(t)) = \max\limits_{\underline{E} \leq E_{t+1} \leq \bar{E}} \left\{-P_t q_t/\alpha\rho + P_t(W_t - L_t)/\rho - C^{ch} \cdot q_t - C^{w} \cdot W_t + E[V_{t+1}^{*}(S(t+1) \mid S(t))]\right\} \\ V_t^{(2)*-}(S(t)) = \max\limits_{\underline{E} \leq E_{t+1} \leq \bar{E}} \left\{-P_t q_t\beta/\rho + P_t(W_t - L_t)/\rho + C^{dis} \cdot q_t - C^{w} \cdot W_t + E[V_{t+1}^{*}(S(t+1) \mid S(t))]\right\} \\ V_t^{(3)*-}(S(t)) = \max\limits_{\underline{E} \leq E_{t+1} \leq \bar{E}} \left\{-P_t q_t\beta\rho + P_t(W_t - L_t)\rho + C^{dis} \cdot q_t - C^{w} \cdot W_t + E[V_{t+1}^{*}(S(t+1) \mid S(t))]\right\} \end{cases} \quad \text{(C2)}$$

By replacing q_t with E_{t+1} as the decision variable through $E_{t+1}/\eta_t - E_t = q_t$, we will get the following rewards function at time t. Maximizing (C2) can be approached by obtaining the optimal results to the equation (C3) by removing the given state $S(t)$ (i.e., the given E_t, W_t, L_t, and P_t). By doing so, we get the following equivalent equations:

$$\left\{\begin{array}{l} V_t^{(1)*-}(S(t)) = \max\limits_{\underline{E}\le E_{t+1}\le\bar{E}} \left\{-P_t\left(\frac{E_{t+1}}{\eta_t} - E_t\right)\Big/\alpha\rho + P_t\left(W_t - L_t\right)/\rho - C^{ch}\cdot\left(\frac{E_{t+1}}{\eta_t} - E_t\right) - C^w\cdot W_t + E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]\right\} \\ \Leftrightarrow V_t^{(1)*-}(S(t)) = \max\limits_{\underline{E}\le E_{t+1}\le\bar{E}} \left\{-P_t\left(E_{t+1}/\eta_t\right)/\alpha\rho - C^{ch}\cdot\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]\right\} \qquad (C3-1) \\ V_t^{(2)*-}(S(t)) = \max\limits_{\underline{E}\le E_{t+1}\le\bar{E}} \left\{-P_t\left(\frac{E_{t+1}}{\eta_t} - E_t\right)\beta\Big/\rho + P_t\left(W_t - L_t\right)/\rho + C^{dis}\cdot\left(\frac{E_{t+1}}{\eta_t} - E_t\right) - C^w\cdot W_t + E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]\right\} \\ \Leftrightarrow V_t^{(2)*-}(S(t)) = \max\limits_{\underline{E}\le E_{t+1}\le\bar{E}} \left\{-P_t\left(E_{t+1}/\eta_t\right)\beta/\rho + C^{dis}\cdot\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]\right\} \qquad (C3-2) \\ V_t^{(3)*-}(S(t)) = \max\limits_{\underline{E}\le E_{t+1}\le\bar{E}} \left\{-P_t\left(\frac{E_{t+1}}{\eta_t} - E_t\right)\beta\rho + P_t\left(W_t - L_T\right)\rho + C^{dis}\cdot\left(\frac{E_{t+1}}{\eta_t} - E_t\right) - C^w\cdot W_t + E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]\right\} \\ \Leftrightarrow V_t^{(3)*-}(S(t)) = \max\limits_{\underline{E}\le E_{t+1}\le\bar{E}} \left\{-P_t\left(E_{t+1}/\eta_t\right)\beta\rho + C^{dis}\cdot\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]\right\} \qquad (C3-3) \end{array}\right. \qquad (C3)$$

For every $t \in \{1,2,\cdots,T\}$, in every stage t, the value function $V_t(S(t))$ and $E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]$ are concave in $E_t \in [\underline{E},\bar{E}]$ for each given state $S(t) = S_t(E_t, W_t, L_t, P_t)$. Clearly, $E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]$ and functions (C1-1)-(C3-3) are concave in $E_{t+1} \in [\underline{E},\bar{E}]$ for each given state $S(t) = S_t(E_t, W_t, L_t, P_t)$ by using

$$\begin{aligned} \frac{\partial E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]}{\partial E_{t+1}{}^2} &= \frac{\partial\left(\partial E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]/\partial E_{t+1}\right)}{\partial E_{t+1}} \\ &= \frac{\partial\left(\partial E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]/\partial E_t\cdot\left(\partial E_t/\partial E_{t+1}\right)\right)}{\partial E_t}\cdot\frac{\partial E_t}{\partial E_{t+1}} \\ &= \left(\frac{\partial E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]}{\partial E_t{}^2}\cdot\frac{\partial E_t}{\partial E_{t+1}} + \frac{\partial E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]}{\partial E_t}\cdot\frac{\partial E_t}{\partial E_{t+1}\partial E_t}\right)\cdot\frac{\partial E_t}{\partial E_{t+1}} \\ &= \left(\frac{\partial E[V_{t+1}^*\left(S(t+1)\,|\,S(t)\right)]}{\partial E_t{}^2}\cdot\left(\frac{\partial E_t}{\partial E_{t+1}}\right)^2\right) \le 0 \end{aligned}$$

Therefore, $V_t^{(1)*-}(S(t))$, $V_t^{(2)*-}(S(t))$,and $V_t^{(3)*-}(S(t))$ have a unique optimal solution on $E_{t+1} \in [\underline{E},\bar{E}]$.

(1) When $q_t \ge 0$, by optimizing the function (C3-1), subject to $E_{t+1} \in [\underline{E},\bar{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\partial V_t^{(1)*-}(S(t))/\partial E_{t+1} = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t))]/\partial E_{t+1} - \left(P_t/\alpha\rho + C^{ch}\right)/\eta_t$$

The second-order derivative: $\partial V_t^{(1)*-}(S(t))/\partial E_{t+1}^2 = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t))]/\partial E_{t+1}^2 \le 0$.

Since the second-order derivative is non-positive, we can get the following unique optimal solution through the first-order condition.

$$\begin{cases} E_{t+1}^{(1)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{ -\left(\dfrac{P_t}{\alpha\rho} + C^{ch}\right)\left(\dfrac{E_{t+1}}{\eta_t} - E_t\right) + P_t\left(W_t - L_t\right)/\rho - C^w \cdot W_t + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \\ E_{t+1}^{(1)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{ -P_t\left(E_{t+1}/\eta_t\right)/\alpha\rho - C^{ch}\cdot\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \\ \text{or } \partial V_t^{(1)*-}(S(t))/\partial E_{t+1} = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t))]/\partial E_{t+1} - \left(P_t/\alpha\rho + C^{ch}\right)/\eta_t \Big|_{E_{t+1}=E_{t+1}^{(1)*-}} = 0 \end{cases} \quad \text{(C4)}$$

(2) When $-\left(L_t - W_t\right)/\beta < q_t \le 0$, by optimizing the function (C3-2), subject to $E_{t+1} \in [\underline{E}, \bar{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\partial V_t^{(2)*-}(S(t))/\partial E_{t+1} = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t))]/\partial E_{t+1} - \left(P_t\,\beta/\rho - C^{dis}\right)/\eta_t$$

The second-order derivative: $\partial V_t^{(2)*-}(S(t))/\partial E_{t+1}^2 = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t))]/\partial E_{t+1}^2 \le 0$.

Since the second-order derivative is non-positive, we can find the unique optimal solutions through the first-order condition, and the optimal SOC results (SOC reference points) are shown:

$$\begin{cases} E_{t+1}^{(2)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{ -\left(\dfrac{P_t\beta}{\rho} - C^{dis}\right)\left(\dfrac{E_{t+1}}{\eta_t} - E_t\right) + P_t\left(W_t - L_t\right)/\rho - C^w \cdot W_t + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \\ E_{t+1}^{(2)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{ -P_t\left(E_{t+1}/\eta_t\right)\beta/\rho + C^{dis}\cdot\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \\ \text{or } \partial V_t^{(2)*-}(S(t))/\partial E_{t+1} = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t))]/\partial E_{t+1} - \left(P_t\,\beta/\rho - C^{dis}\right)/\eta_t \Big|_{E_{t+1}=E_{t+1}^{(2)*-}} = 0 \end{cases} \quad \text{(C5)}$$

(3) When $q_t \le -\left(L_t - W_t\right)/\beta$, by optimizing the function (C3-3), subject to $E_{t+1} \in [\underline{E}, \bar{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\partial V_t^{(3)*}(S(t))/\partial E_{t+1} = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t))]/\partial E_{t+1} - (P_t\beta\rho - C^{dis})/\eta_t$$

The second-order derivative: $\partial V_t^{(3)*-}(S(t))/\partial E_{t+1}^2 = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t))]/\partial E_{t+1}^2 \le 0$.

Similarly, we can find the unique optimal solutions through the first-order condition. We will get the following optimal SOC results:

$$\begin{cases} E_{t+1}^{(3)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \overline{E}} \left\{ -(P_t\beta\rho - C^{dis})\left(\dfrac{E_{t+1}}{\eta_t} - E_t\right) + P_t\left(W_t - L_t\right)\rho - C^w \cdot W_t + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \\ E_{t+1}^{(3)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \overline{E}} \left\{ -P_t\left(E_{t+1}/\eta_t\right)\beta\rho + C^{dis}\cdot\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \\ \text{or } \partial V_t^{(3)*}(S(t))/\partial E_{t+1} = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t))]/\partial E_{t+1} - (P_t\beta\rho - C^{dis})/\eta_t \Big|_{E_{t+1}=E_{t+1}^{(3)*-}} = 0 \end{cases} \quad \text{(C6)}$$

To sum up, the Bellman equation can be used to derive the following results:

$$\begin{cases} E_{t+1}^{(1)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \overline{E}} \left\{ -\left(P_t/\alpha\rho + C^{ch}\right)\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \\ E_{t+1}^{(2)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \overline{E}} \left\{ -\left(P_t\,\beta/\rho - C^{dis}\right)\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \\ E_{t+1}^{(3)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \overline{E}} \left\{ -(P_t\beta\rho - C^{dis})\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \end{cases} \quad \text{(C7)}$$

2) The relations among three SOC optimal results/SOC reference points:

(1) Recall the proof 1), for the state at $t \in \{1,2,\cdots,T\}$,there have the following two equations:

$$\begin{cases} E_{t+1}^{(1)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \overline{E}} \left\{ -\left(P_t/\alpha\rho + C^{ch}\right)\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \\ E_{t+1}^{(2)*-} = \arg\max\limits_{\underline{E}\le E_{t+1}\le \overline{E}} \left\{ -\left(P_t\,\beta/\rho - C^{dis}\right)\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*(S(t+1)\,|\,S(t))] \right\} \end{cases}$$

Then, we can get the following inequations:

$$\left.\begin{aligned}&(a)\ \left\{-\left(P_t/\alpha\rho+C^{ch}\right)\left(E_{t+1}^{(1)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(1)*-},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\\&(b)\ \geq\left\{-\left(P_t/\alpha\rho+C^{ch}\right)\left(E_{t+1}^{(2)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(2)*-},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\\&(c)\ \left\{-\left(P_t\,\beta/\rho-C^{dis}\right)\left(E_{t+1}^{(2)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(2)*-},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\\&(d)\ \geq\left\{-\left(P_t\,\beta/\rho-C^{dis}\right)\left(E_{t+1}^{(1)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(1)*-},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\end{aligned}\right\}\qquad (C8)$$

Based on the above inequations, we can get the relationship of $(a)-(d)\geq(b)-(c)$.

That is, for any given current state $S(t)=S_t(E_t,W_t,W_t,P_t)\in\hat{E}\times W\times L\times P$, we will get:

$$\left(P_t/\alpha\rho+C^{ch}-P_t\,\beta/\rho+C^{dis}\right)\left(E_{t+1}^{(1)*-}/\eta_t-E_{t+1}^{(2)*-}/\eta_t\right)\leq 0 \qquad (C9)$$

Since there is $0<\rho\leq 1$. Then, we can get the following inequations:

1) If $\frac{P_t}{\alpha\rho}+C^{ch}-\frac{P_t\beta}{\rho}P+C^{dis}\geq 0\Leftrightarrow P_t\geq-\frac{\left(C^{dis}+C^{ch}\right)\rho}{(1/\alpha-\beta)}$ hold, we will get $E_{t+1}^{(1)*-}\leq E_{t+1}^{(2)*-}$;

2) If $\frac{P_t}{\alpha\rho}+C^{ch}-\frac{P_t\beta}{\rho}P+C^{dis}\leq 0\Leftrightarrow P_t\leq-\frac{\left(C^{dis}+C^{ch}\right)\rho}{(1/\alpha-\beta)}$, there is $E_{t+1}^{(1)*-}\geq E_{t+1}^{(2)*-}$.

(2) Recall the proof 1), we also have the following two equations:

$$\begin{cases}E_{t+1}^{(2)*-}=\arg\max\limits_{\underline{E}\leq E_{t+1}\leq\bar{E}}\left\{-\left(P_t\,\beta/\rho-C^{dis}\right)\left(E_{t+1}/\eta_t\right)+E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]\right\}\\E_{t+1}^{(3)*-}=\arg\max\limits_{\underline{E}\leq E_{t+1}\leq\bar{E}}\left\{-(P_t\beta\rho-C^{dis})\left(E_{t+1}/\eta_t\right)+E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]\right\}\end{cases}$$

Then, we can get the following inequations:

$$\left.\begin{aligned}&(e)\ \left\{-\left(P_t\,\beta/\rho-C^{dis}\right)\left(E_{t+1}^{(2)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(2)*-},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\geq\\&(f)\ \left\{-\left(P_t\,\beta/\rho-C^{dis}\right)\left(E_{t+1}^{(3)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(3)*-},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\\&(g)\ \left\{-(P_t\beta\rho-C^{dis})\left(E_{t+1}^{(3)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(3)*-},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\geq\\&(h)\ \left\{-(P_t\beta\rho-C^{dis})\left(E_{t+1}^{(2)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(2)*-},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\end{aligned}\right\}\qquad (C10)$$

Obviously, there is $(e)-(h)\ge(f)-(g)$, that is

$$\left(P_t\beta/\rho-P_t\beta\rho\right)\left(E_{t+1}^{(3)*-}/\eta_t-E_{t+1}^{(2)*-}/\eta_t\right)\ge 0 \tag{C11}$$

1) For any given price $P_t\ge 0$, we will get the relationship of $E_{t+1}^{(2)*-}\le E_{t+1}^{(3)*-}$.

2) For any given price $P_t<0$, there is $E_{t+1}^{(2)*-}\ge E_{t+1}^{(3)*-}$.

(3) Recall the proof 1), we also have the following two equations:

$$\begin{cases} E_{t+1}^{(1)*}=\arg\max\limits_{\underline{E}\le E_{t+1}\le\overline{E}}\left\{-\left(P_t/\alpha\rho+C^{ch}\right)\left(E_{t+1}/\eta_t\right)+E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]\right\} \\ E_{t+1}^{(3)*}=\arg\max\limits_{\underline{E}\le E_{t+1}\le\overline{E}}\left\{-(P_t\beta\rho-C^{dis})\left(E_{t+1}/\eta_t\right)+E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]\right\} \end{cases}$$

Then, we can get the following inequations:

$$\left.\begin{array}{l} \text{(i) } \left\{-\left(P_t/\alpha\rho+C^{ch}\right)\left(E_{t+1}^{(1)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(1)*},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\ge \\ \text{(j) } \left\{-\left(P_t/\alpha\rho+C^{ch}\right)\left(E_{t+1}^{(3)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(3)*},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\} \\ \text{(k) } \left\{-(P_t\beta\rho-C^{dis})\left(E_{t+1}^{(3)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(3)*},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\}\ge \\ \text{(l) } \left\{-(P_t\beta\rho-C^{dis})\left(E_{t+1}^{(1)*-}/\eta_t\right)+E\left[V_{t+1}^{*}\left(S_{t+1}(E_{t+1}^{(1)*},W_{t+1},L_{t+1},P_{t+1})\,|\,S(t)\right)\right]\right\} \end{array}\right\} \tag{C12}$$

Obviously, there is $(i)-(l)\ge(j)-(k)$, that is

$$\left(P_t/\alpha\rho+C^{ch}-P_t\beta\rho+C^{dis}\right)\left(E_{t+1}^{(3)*-}/\eta_t-E_{t+1}^{(1)*-}/\eta_t\right)\ge 0 \tag{C13}$$

Since there is $0<\rho\le 1$,

1) If $P_t/\alpha\rho+C^{ch}-P_t\beta\rho+C^{dis}\ge 0\Leftrightarrow P_t\ge-(C^{dis}+C^{ch})/\left(1/\alpha\rho-\beta\rho\right)$ hold, we will get $E_{t+1}^{(1)-*}\le E_{t+1}^{(3)-*}$.

2) If $P_t/\alpha\rho+C^{ch}-P_t\beta\rho+C^{dis}\le 0\Leftrightarrow P_t\le-(C^{dis}+C^{ch})/\left(1/\alpha\rho-\beta\rho\right)$, there is $E_{t+1}^{(1)-*}\ge E_{t+1}^{(3)-*}$.

We have the following relationship:

$$(1/\alpha\rho-\beta\rho)>(1/\alpha\rho-\beta/\rho)>0\Rightarrow 0<\frac{(C^{dis}+C^{ch})}{(1/\alpha\rho-\beta\rho)}<\frac{(C^{dis}+C^{ch})}{(1/\alpha\rho-\beta/\rho)}\Rightarrow 0>-\frac{(C^{dis}+C^{ch})}{(1/\alpha\rho-\beta\rho)}>-\frac{(C^{dis}+C^{ch})}{(1/\alpha\rho-\beta/\rho)}$$

Thus, we can get the following results:

$$\begin{cases} \text{1) If } P_t\geq 0, \text{there are } E_{t+1}^{(2)*-}\leq E_{t+1}^{(3)*-}, E_{t+1}^{(1)*-}\leq E_{t+1}^{(3)*-}, \text{and} E_{t+1}^{(1)*-}\leq E_{t+1}^{(2)*-}; \\ \text{2) If } -(C^{dis}+C^{ch})/(1/\alpha\rho-\beta\rho)\leq P_t\leq 0, \\ \text{there are} E_{t+1}^{(1)*-}\leq E_{t+1}^{(3)*-},\ E_{t+1}^{(2)*-}\geq E_{t+1}^{(3)*-}, \text{and} E_{t+1}^{(1)*-}\leq E_{t+1}^{(2)*-}; \\ \text{3) If } -\frac{(C^{dis}+C^{ch})}{(1/\alpha\rho-\beta/\rho)}\leq P_t\leq -\frac{(C^{dis}+C^{ch})}{(1/\alpha\rho-\beta\rho)}, \\ \text{there are} E_{t+1}^{(1)*-}\geq E_{t+1}^{(3)*-},\ E_{t+1}^{(2)*-}\geq E_{t+1}^{(3)*-}, \text{and} E_{t+1}^{(1)*-}\leq E_{t+1}^{(2)*-}; \\ \text{4) If } P_t\leq -(C^{dis}+C^{ch})/(1/\alpha\rho-\beta/\rho\rho), \\ \text{there are} E_{t+1}^{(1)*-}\geq E_{t+1}^{(3)*-},\ E_{t+1}^{(2)*-}\geq E_{t+1}^{(3)*-}, \text{and} E_{t+1}^{(1)*-}\geq E_{t+1}^{(2)*-}. \end{cases} \quad \text{(C14)}$$

Therefore,

$$\begin{cases} \text{1) If } P_t\geq 0, \text{there is } E_{t+1}^{(1)*-}\leq E_{t+1}^{(2)*-}\leq E_{t+1}^{(3)*-}; \\ \text{2) If } -(C^{dis}+C^{ch})/(1/\alpha\rho-\beta\rho)\leq P_t\leq 0, \text{there is} E_{t+1}^{(1)*-}\leq E_{t+1}^{(3)*-}\leq E_{t+1}^{(2)*-}; \\ \text{3) If } -\frac{(C^{dis}+C^{ch})}{(1/\alpha\rho-\beta/\rho)}\leq P_t\leq -\frac{(C^{dis}+C^{ch})}{(1/\alpha\rho-\beta\rho)}, \text{there is} E_{t+1}^{(2)*-}\geq E_{t+1}^{(1)*-}\geq E_{t+1}^{(3)*-}; \\ \text{4) If } P_t\leq -(C^{dis}+C^{ch})/(1/\alpha\rho-\beta/\rho), \text{there is } E_{t+1}^{(1)*-}\geq\ E_{t+1}^{(2)*-}\geq E_{t+1}^{(3)*-}. \end{cases} \quad \text{(C15)}$$

<u>Proof of Proposition 5.1:</u>

For Positive price, we will get the following results about optimal solutions:

If $P_t\geq 0$, there is $E_{t+1}^{(1)*-}\leq E_{t+1}^{(2)*-}\leq E_{t+1}^{(3)*-}$

1) *Case 1:* If $(L_t-W_t)/\beta<\min\{\bar{E}-E_{t+1}^{(3)*-},\overline{Q}^{dis}\}\Leftrightarrow\alpha(W_t-L_t)>\max\{E_{t+1}^{(3)*-}-\bar{E},-\overline{Q}^{dis}\}$

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(1)*-}-E_t,\overline{Q}^{ch}\},\ E_t\in[0,E_{t+1}^{(1)*-}],\\ \text{(buying power for consuming and storing, bring SOC up to } E_{t+1}^{(1)*-}\text{);}\\ 0, E_t\in(E_{t+1}^{(1)*-},E_{t+1}^{(2)*-}],\\ \text{(buying power for consuming without storing, keep SOC unchanged);}\\ \max\{E_{t+1}^{(2)*-}-E_t,-\overline{Q}^{dis}\}, E_t\in(E_{t+1}^{(2)*-},E_{t+1}^{(3)*-}],\\ \text{(discharging and buying partial energy for consuming, bring SOC down to } E_{t+1}^{(2)*-}\text{);}\\ \max\{E_{t+1}^{(3)*-}-E_t,-\overline{Q}^{dis}\}, E_t\in(E_{t+1}^{(3)*-},\overline{E}],\\ \text{(discharging energy for consuming and selling, bring SOC down to } E_{t+1}^{(3)*-}\text{).}\end{cases} \quad \text{(C16)}$$

2) *Case 2:* If $(L_t-W_t)/\beta>\min\{\overline{E}-E_{t+1}^{(3)*-},\overline{Q}^{dis}\}\Leftrightarrow\alpha(W_t-L_t)<\max\{E_{t+1}^{(3)*-}-\overline{E},-\overline{Q}^{dis}\}$

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(1)*-}-E_t,\overline{Q}^{ch}\},\ E_t\in[0,E_{t+1}^{(1)*-}],\\ \text{(buying power for consuming and storing, bring SOC up to } E_{t+1}^{(1)*-}\text{);}\\ 0, E_t\in(E_{t+1}^{(1)*-},E_{t+1}^{(2)*}],\\ \text{(buying power for consuming without storing, keep SOC unchanged);}\\ \max\{E_{t+1}^{(2)*-}-E_t,-\overline{Q}^{dis}\}, E_t\in(E_{t+1}^{(2)*-},\overline{E}],\\ \text{(discharging and buying partial energy for consuming, bring SOC down to } E_{t+1}^{(2)*-}\text{).}\end{cases} \quad \text{(C17)}$$

<u>Proof of Lemma 5.2</u>

1)The uniqueness of the optimal results:

The current payoff rewards in the *scenario2*: Electricity generation can meet the power load (i.e., $(W_t-L_t)\geq 0$) are shown as follows for the prosumer:

$$R^+(q_t,W_t,L_t,P_t)=\begin{cases}-P_t\left[q_t/\alpha-(W_t-L_t)\right]/\rho-C^{ch}\cdot q_t-C^w\cdot W_t & \left(q_t\geq\alpha(W_t-L_t)\right)\\ -P_t\cdot\left[q_t/\alpha-(W_t-L_t)\right]\cdot\rho-C^w\cdot W_t-C^{ch}q_t & (0\leq q_t<\alpha(W_t-L_t))\\ -P_t\left[q_t\beta-(W_t-L_t)\right]\rho+C^{dis}\cdot q_t-C^w\cdot W_t & (q_t<0)\end{cases} \quad \text{(C18)}$$

where, q_t is the energy change from period t to period t+1 before accounting for energy loss. For the state at $t \in \{1,2,\cdots,T\}$, optimization the value function $V_t(E_t, W_t, L_t, P_t)$, subject to $\underline{E} \le E_{t+1} \le \overline{E}$, we will get the following equations:

$V_t(S_t) = [R(q_t, W_t, L_t, P_t) + E[V_{t+1}(S(t+1) \mid S(t))]$, where $E[V_{T+1}(S(T+1) \mid S(T)) = VOE_{T+1} \cdot E_{T+1}$.

Thus, we get the following three sub-optimization value functions:

$$V_t^{*+}(S(t)) = \begin{cases} V_t^{(1)*+}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left\{ -P_t\left(q_t/\alpha\rho\right) + P_t\left(W_t - L_t\right)/\rho - C^{ch} \cdot q_t - C^w \cdot W_t + E[V_{t+1}^*\left(S(t+1) \mid S(t)\right)] \right\} \\ V_t^{(2)*+}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left\{ -P_t q_t \rho/\alpha + P_t\left(W_t - L_t\right)\rho - C^{ch} \cdot q_t - C^w \cdot W_t + E[V_{t+1}^*\left(S(t+1) \mid S(t)\right)] \right\} \\ V_t^{(3)*+}(S(t)) = \max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left\{ -P_t q_t \beta\rho + P_t\left(W_t - L_t\right)\rho + C^{dis} \cdot q_t - C^w \cdot W_t + E[V_{t+1}^*\left(S(t+1) \mid S(t)\right)] \right\} \end{cases} \quad \text{(C19)}$$

Following the previous study (Zhou et al., 2019), we derive the following results:

$$\begin{cases} E_{t+1}^{(1)*+} = \arg\max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left\{ -\left(P_t/\alpha\rho + C^{ch}\right)\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*\left(S(t+1) \mid S(t)\right)] \right\} \\ E_{t+1}^{(2)*+} = \arg\max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left\{ -\left(P_t\rho/\alpha + C^{ch}\right)\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*\left(S(t+1) \mid S(t)\right)] \right\} \\ E_{t+1}^{(3)*+} = \arg\max\limits_{\underline{E} \le E_{t+1} \le \overline{E}} \left\{ -(P_t\beta\rho - C^{dis})\left(E_{t+1}/\eta_t\right) + E[V_{t+1}^*\left(S(t+1) \mid S(t)\right)] \right\} \end{cases} \quad \text{(C20)}$$

Similar, we can get the following results:

$$\begin{cases} \text{1) If } P_t \ge 0, \text{there is } E_{t+1}^{(1)*+} \le E_{t+1}^{(2)*+} \le E_{t+1}^{(3)*+}; \\ \text{2) If } -\left(C^{dis} + C^{ch}\right)/\left(1/\alpha\rho - \beta\rho\right) \le P_t \le 0, \text{there is } E_{t+1}^{(2)*+} \le E_{t+1}^{(1)*+} \le E_{t+1}^{(3)*+}; \\ \text{3) If } -\dfrac{\left(C^{ch} + C^{dis}\right)}{\left(\rho/\alpha - \beta\rho\right)} \le P_t \le -\dfrac{\left(C^{ch} + C^{dis}\right)}{\left(1/\alpha\rho - \beta\rho\right)}, \text{there is } E_{t+1}^{(1)*+} \ge E_{t+1}^{(3)*+} \ge E_{t+1}^{(2)*+}; \\ \text{4) If } P_t \le -\left(C^{ch} + C^{dis}\right)/\left(\rho/\alpha - \beta\rho\right), \text{there is } E_{t+1}^{(1)*+} \ge E_{t+1}^{(2)*+} \ge E_{t+1}^{(3)*+}. \end{cases} \quad \text{(C21)}$$

<u>Proof of Proposition 5.2:</u>

For Positive price, we will get the following results about optimal solutions:

1) *Case 3:* If $\alpha(W_t - L_t) < \min\{E_{t+1}^{(1)*+}, \overline{Q}^{ch}\}$, the optimal action of prosumer is obtained as follows:

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(1)*+}-E_t,\overline{Q}^{ch}\},\ E_t\in[0,E_{t+1}^{(1)*+}-\alpha(W_t-L_t)],\\ \text{(store residual power and purchase electricity up to } E_{t+1}^{(1)*+});\\ \min\{E_{t+1}^{(2)*+}-E_t,\alpha(W_t-L_t),\overline{Q}^{ch}\},E_t\in(E_{t+1}^{(1)*+}-\alpha(W_t-L_t),E_{t+1}^{(2)*+}],\\ \text{(store residual power without buying up to } E_{t+1}^{(2)*+});\\ 0,E_t\in(E_{t+1}^{(2)*+},E_{t+1}^{(3)*+}]\text{ (keep inventory unchanged)};\\ \max\{E_{t+1}^{(3)*+}-E_t,-\overline{Q}^{dis}\},E_t\in(E_{t+1}^{(3)*+},\overline{E}],\\ \text{(Sell redisual power and relase energy down to } E_{t+1}^{(3)*+}).\end{cases}\quad (C22)$$

2) *Case 4:* If $\alpha(W_t-L_t)\geq\min\{E_{t+1}^{(1)*+},\overline{Q}^{ch}\}$, the optimal action of prosumer is obtained as follows:

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(2)*+}-E_t,\alpha(W_t-L_t)\overline{Q}^{ch},\},E_t\in[0,E_{t+1}^{(2)*+}],\\ \text{(store residual power without buying up to } E_{t+1}^{(2)*+});\\ 0,E_t\in[E_{t+1}^{(2)*+},E_{t+1}^{(3)*+}]\text{(keep inventory unchanged)};\\ \max\{E_{t+1}^{(3)*+}-E_t,-\overline{Q}^{dis}\},E_t\in(E_{t+1}^{(3)*+},\overline{E}],\\ \text{(Sell redisual power and relase energy down to } E_{t+1}^{(3)*+}).\end{cases}\quad (C23)$$

Cases Study

In this case, we assume there are three time periods ($T=3$). At each period, the power price takes one of the values in set $P_t=\{5,3,10\}$. We also assume the storage energy capacity cannot refill it fully or sell it empty in one time period, but fewer than two time periods. In detail, when the full (resp. empty) storage can be emptied (resp. filled up) in more than one period but fewer than two periods, it holds that $\underline{E}+\overline{Q}^p\leq\overline{E}$ (resp., $\overline{E}-\underline{E}>\overline{Q}^g$) and $\underline{E}+2\overline{Q}^p\geq\overline{E}$ (resp., $\overline{E}-\underline{E}\leq2\overline{Q}^g$). We assume the storage capacity is 10 (i.e.,

$\underline{E} = 0, \overline{E} = 10$), the generating/discharging max capacity is 12 and the pumping/charging max capacity is 7. Let the operating cost be one(i.e., $C^{ch} = C^{dis} = 1$),wind generation cost (i.e., $c_w = 0$), the pumping/ generating efficiencies and transmission efficiencies be 0.9 (i.e., $\alpha = \beta = 0.9 = \rho$), self-discharging be one (i.e., $\eta = 1$), the wind generations are $w_t = \{6, 5, 0\}$, and the local load/demand is $L_t = \{4, 9, 6\}$.

Case 1

In stage 4:

$$VOE_4 = (5 + 3 + 10)/3 = 6 \Rightarrow V_4 = VOE_4 \cdot E_4 = 6E_4$$

We will get the following optimal references points:

$$\begin{cases} E_4^{(1)*} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -\left(P_3/\alpha\rho + C^{ch} \right)\left(E_4/\eta_3 - E_3 \right) + E[V_4^*(S(4) \mid S(3))] \right\} \\ E_4^{(2)*} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -\left(P_3\beta/\rho - C^{dis} \right)\left(E_4/\eta_3 - E_3 \right) + E[V_4^*(S(4) \mid S(3))] \right\} \\ E_4^{(3)*} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -(P_3\beta\rho - C^{dis})\left(E_4/\eta_3 - E_3 \right) + E[V_4^*(S(4) \mid S(3))] \right\} \end{cases}$$

Then, plug in the data, we will get the following equations:

$$\begin{cases} E_4^{(1)*} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -(10/0.81 + 1)E_4 + 6E_4 \right\} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -7.3457E_4 \right\} \\ E_4^{(2)*} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -(10 \cdot 0.9/0.9 - 1)E_4 + 6E_4 \right\} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -3E_4 \right\} \\ E_4^{(3)*} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -(10 \cdot 0.81 - 1)E_4 + 6E_4 \right\} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -1.1E_4 \right\} \end{cases} \qquad (C24)$$

$$\Rightarrow E_4^{(1)*} = E_4^{(2)*} = E_4^{(3)*} = 0$$

In Stage 3: There is $W_3 - L_3 = 0 - 6 < 0$.

Action 3: Release power and make the storage level down to $\underline{E} = 0 = E_4^*$, thus,

$V_4^* = 0$. Therefore, the optimal action is

$$q_3^*(S_3) = -E_3, E_3 \in (0, \overline{E}], \tag{C25}$$
(sell energy and make SOC down to 0 as close as possible)

The reward function at stage3 is shown as

$$R_3 = \begin{cases} -P_3\left[q_3\beta - \left(W_3 - L_3\right)\right]/\rho + C^{dis} \cdot q_3 - C^w \cdot W_3 & (-\left(6-0\right)/0.9 < -E_3 \le 0) \\ -P_3\left[q_3\beta - \left(W_3 - L_3\right)\right]\rho + C^{dis} \cdot q_3 - C^w \cdot W_3 & \left(-E_3 \le -\left(6-0\right)/0.9\right) \end{cases}$$
$$= \begin{cases} -10\left[6 - 0.9E_3\right]/0.9 - E_3 = \left[-60 + 9E_3\right]/0.9 - E_3 = 9E_3 - 60/0.9 & (E_3 < 6/0.9) \\ -10\left[6 - 0.9E_3\right]0.9 - E_3 = \left[-60 + 9E_3\right]0.9 - E_3 = 7.1E_3 - 54 & \left(E_3 \ge 6/0.9\right) \end{cases} \tag{C26}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_3^* = \max\{R_3 + V_4^*\} = \begin{cases} 9E_3 - 600/9 & (E_3 < 6/0.9) \\ 7.1E_3 - 54 & (E_3 \ge 6/0.9) \end{cases} \tag{C27}$$

In stage 2: There has $W_2 - L_2 = 5 - 9 = -4 < 0$.

By using the Eqs. (C4), (C5), and (C6), we get the following results for merchants:

$$\begin{cases} E_3^{(1)*-} = \arg \max\limits_{\underline{E} \le E_3 \le \overline{E}} \left\{V_3^* - \left(P_2/\alpha\rho + C^{ch}\right)\left(E_3/\eta_3 - E_2\right)\right\} \\ \quad = \arg \max\limits_{\underline{E} \le E_3 \le \overline{E}} \left\{V_3^* - \left(3/0.81 + 1\right)\left(E_3 - E_2\right)\right\} = \arg \max\limits_{\underline{E} \le E_3 \le \overline{E}} \left\{V_3^* - 4.7037E_3 + 4.7037E_2\right\} \\ E_3^{(2)*-} = \arg \max\limits_{\underline{E} \le E_3 \le \overline{E}} \left\{V_3^* - \left(P_2\,\beta/\rho - C^{dis}\right)\left(E_3/\eta_3 - E_2\right)\right\} \\ \quad = \arg \max\limits_{\underline{E} \le E_3 \le \overline{E}} \left\{V_3^* - \left(3 - 1\right)\left(E_3 - E_2\right)\right\} = \arg \max\limits_{\underline{E} \le E_3 \le \overline{E}} \left\{V_3^* - 2E_3 + 2E_2\right\} \\ E_3^{(3)*-} = \arg \max\limits_{\underline{E} \le E_3 \le \overline{E}} \left\{V_3^* - (P_2\beta\rho - C^{dis})\left(E_3/\eta_3 - E_2\right)\right\} \\ \quad = \arg \max\limits_{\underline{E} \le E_3 \le \overline{E}} \left\{V_3^* - \left(3 \cdot 0.81 - 1\right)\left(E_3 - E_2\right)\right\} = \arg \max\limits_{\underline{E} \le E_3 \le \overline{E}} \left\{V_3^* - 1.43E_3 + 1.43E_2\right\} \end{cases} \tag{C28}$$

(1) Scenario1: If $E_3 \in [0,\ 6/0.9]$, there is $V_3^* = 9E_3 - 600/9$.

$$\begin{cases} E_3^{(1)*-} = \arg\max_{E_3 \in [0,\ 6/0.9]} \{9E_3 - 600/9 - 4.7037E_3 - 4.7037E_2\} \\ \qquad = \arg\max_{E_3 \in [0,\ 6/0.9]} \{4.2963E_3 - 600/9 + 4.7037E_2\} \Rightarrow E_3^{(1)*} = 6/0.9; \\ E_3^{(2)*-} = \arg \arg\max_{E_3 \in [0,\ 6/0.9]} \{9E_3 - 600/9 - 2E_3 - 2E_2\} \\ \qquad = \arg\max_{E_3 \in [0,\ 6/0.9]} \{7E_3 - 600/9 + 2E_2\} \Rightarrow E_3^{(2)*} = 6/0.9; \\ E_3^{(3)*-} = \arg\max_{E_3 \in [0,\ 6/0.9]} \{9E_3 - 600/9 - 1.43E_3 - 1.43E_2\} \\ \qquad = \arg\max_{E_3 \in [0,\ 6/0.9]} \{7.57E_3 - 600/9 + 1.43E_2\} \Rightarrow E_3^{(3)*} = 6/0.9. \end{cases} \tag{C29}$$

(2) Scenario2: If $E_3 \in [6/0.9, 10]$, there is $V_3^* = 7.1E_3 - 54$.

$$\begin{cases} E_3^{(1)*-} = \arg\max_{E_3 \in [6/0.9,\ 10]} \{7.1E_3 - 54 - 4.7037E_3 - 4.7037E_2\} \\ \qquad = \arg\max_{E_3 \in [6/0.9,\ 10]} \{2.3963E_3 - 54 + 4.7037E_2\} \Rightarrow E_3^{(1)*} = \bar{E} = 10; \\ E_3^{(2)*-} = \arg\max_{E_3 \in [6/0.9,\ 10]} \{7.1E_3 - 54 - 2E_3 - 2E_2\} \\ \qquad = \arg\max_{E_3 \in [6/0.9,\ 10]} \{5.1E_3 - 54 + 2E_2\} \Rightarrow E_3^{(2)*} = \bar{E} = 10; \\ E_3^{(3)*-} = \arg\max_{E_3 \in [6/0.9,\ 10]} \{7.1E_3 - 54 - 1.43E_3 - 1.43E_2\} \\ \qquad = \arg\max_{E_3 \in [6/0.9,\ 10]} \{5.67E_3 - 54 + 1.43E_2\} \Rightarrow E_3^{(3)*} = \bar{E} = 10. \end{cases} \tag{C30}$$

Next, we will compare the max value and pick up the optimal references point between the above two scenarios.

(1) Compare $E_3^{(1)*}$

$$\left\{\begin{array}{l} 1)\begin{cases} \text{If}\quad E_3 \in [0,\ 6/0.9] \Rightarrow E_3^{(1)*-} = 6/0.9 \\ E_3^{(1)*-} = \arg\max_{E_3\in[0,\ 6/0.9]}\left\{4.2963E_3 - 600/9 + 4.7037E_2\right\} \\ \Rightarrow 4.2963E_3 - 600/9 + 4.7037E_2\ |_{E_3^{(1)*-}=6/0.9} = -38.0437 + 4.7037E_2\ \ \left(E_2 < 6/0.9\right) \end{cases} \\ 2)\begin{cases} \text{If}\quad E_3 \in [6/0.9,\ 10] \Rightarrow E_3^{(1)*-} = 10 \\ E_3^{(1)*-} = \arg\max_{E_3\in[6/0.9,\ 10]}\left\{2.3963E_3 - 54 + 4.7037E_2\right\} \\ \Rightarrow 2.3963E_3 - 54 + 4.7037E_2\ |_{E_3^{(1)*-}=10} = -30.037 + 4.7037E_2 \end{cases} \end{array}\right\} \tag{C31}$$

$$\Rightarrow E_3^{(1)*-} = 10$$

(2) Compare $E_3^{(2)*}$

$$\left\{\begin{array}{l} 1)\begin{cases} \text{If}\quad E_3 \in [0,\ 6/0.9] \Rightarrow E_3^{(2)*-} = 6/0.9 \\ E_3^{(2)*-} = \arg\max_{E_3\in[0,\ 6/0.9]}\left\{7E_3 - 600/9 + 2E_2\right\} \\ \Rightarrow 7E_3 - 600/9 + 2E_2\ |_{E_3^{(2)*-}=6/0.9} = -20.1 + 2E_2\ \ \left(E_2 > 6/0.9\right) \end{cases} \\ 2)\begin{cases} \text{If}\quad E_3 \in [6/0.9,\ 10] \Rightarrow E_3^{(2)*-} = 10 \\ E_3^{(2)*-} = \arg\max_{E_3\in[6/0.9,\ 10]}\left\{5.1E_3 - 54 + 2E_2\right\} \\ \Rightarrow 5.1E_3 - 54 + 2E_2\ |_{E_3^{(2)*-}=10} = -3 + 2E_2 \end{cases} \end{array}\right\} \Rightarrow E_3^{(2)*-} = 10 \tag{C32}$$

(3) Compare $\mathbf{E}_3^{(3)*}$

$$\left.\begin{array}{l} 1)\begin{cases} \text{If } \ E_3 \in [0,\ 6/0.9] \Rightarrow E_3^{(3)*-} = 6/0.9 \\ E_3^{(3)*-} = \arg \max\limits_{E_3 \in [0,\ 6/0.9]} \{7.57E_3 - 600/9 + 1.43E_2\} \\ \Rightarrow 7.57E_3 - 600/9 + 1.43E_2 \,|_{E_3^{(3)*-}=6/0.9} = -16.201 + 1.43E_2 \end{cases} \\ 2)\begin{cases} \text{If } \ E_3 \in [6/0.9,\ 10] \Rightarrow E_3^{(3)*-} = 10 \\ E_3^{(3)*-} = \arg \max\limits_{E_3 \in [6/0.9,\ 10]} \{5.67E_3 - 54 + 1.43E_2\} \\ \Rightarrow 5.67E_3 - 54 + 1.43E_2 \,|_{E_3^{(3)*-}=10} = 2.7 + 1.43E_2 \end{cases} \end{array}\right\} \Rightarrow E_3^{(3)*-} = 10 \qquad \text{(C33)}$$

Thus, we will get the optimal reference points at stage 3 that are shown as:

$$E_3^{(1)*} = E_3^{(2)*} = E_3^{(3)*} = 10 \qquad \text{(C34)}$$

Since there are $(L_t - W_t)/\beta \geq \min\{\overline{E} - E_{t+1}^{(3)*-}, \overline{Q}^g\}$. The optimal actions at stage 2 are shown as

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(1)*-} - E_t, \overline{Q}^{ch}\},\ E_t \in [0, E_{t+1}^{(1)*-}], \\ \text{(buying power for consuming and storing, bring SOC up to } E_{t+1}^{(1)*-}\text{);} \\ 0, E_t \in (E_{t+1}^{(1)*-}, E_{t+1}^{(2)*}], \\ \text{(buying power for consuming without storing, keep SOC unchanged);} \\ \max\{E_{t+1}^{(2)*-} - E_t, -\overline{Q}^{dis}\}, E_t \in (E_{t+1}^{(2)*-}, \overline{E}], \\ \text{(discharging and buying partial energy for consuming, bring SOC down to } E_{t+1}^{(2)*-}\text{).} \end{cases} \qquad \text{(C35)}$$

$$\Rightarrow q_2^*(S_2) = \begin{cases} 7, & E_2 \in [0,3] \\ 10 - E_2, & E_2 \in [3,\ 10] \end{cases}$$

The reward payoff functions at stage 2 are shown as follows:

$$\begin{aligned} R_2 &= -P_2\left[q_2/\alpha - (W_2 - L_2)\right]/\rho - C^{ch}\cdot q_2 - C^w \cdot W_2 \quad (q_2 \geq 0) \\ &= \begin{cases} -3\left[7/0.9 - (5-9)\right]/0.9 - 7 \\ -3\left[(10 - E_2)/0.9 - (5-9)\right]/0.9 - (10 - E_2) \end{cases} = \begin{cases} -46.2593 & (E_2 < 3) \\ 4.7037E_2 - 60.337 & (E_2 \geq 3) \end{cases} \end{aligned} \qquad \text{(C36)}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_2^* = \max\{R_2 + V_3^*\} = \begin{cases} 7.1E_3 - 54 - 46.2593_{|E_3=E_2+7} & (E_2 < 3) \\ 7.1E_3 - 54 + 4.7037E_2 - 60.337_{|E_3=10} & (E_2 \geq 3) \end{cases}$$
$$= \begin{cases} 7.1E_2 - 50.5593 & (E_2 < 3) \\ 4.7037E_2 - 43.337 & (E_2 \geq 3) \end{cases} \quad \text{(C37)}$$

In Stage 1: There exists $W_1 - L_1 = 6 - 4 > 0$.

By using the equations (C12), (C13), and (C14), we will get the following results for merchants:

$$\begin{cases} E_2^{(1)*+} = \arg\max_{\underline{E} \leq E_2 \leq \overline{E}} \left\{ V_2^* - \left(P_1/\alpha\rho + C^{ch} \right)\left(E_2/\eta_1 - E_1 \right) \right\} \\ \quad = \arg\max_{\underline{E} \leq E_2 \leq \overline{E}} \left\{ V_2^* - \left(5/0.81 + 1 \right)\left(E_2 - E_1 \right) \right\} = \arg\max_{\underline{E} \leq E_2 \leq \overline{E}} \left\{ V_2^* - 7.1728\left(E_2 - E_1 \right) \right\} \\ E_2^{(2)*+} = \arg\max_{\underline{E} \leq E_2 \leq \overline{E}} \left\{ V_2^* - \left(P_1\rho/\alpha + C^{ch} \right)\left(E_2/\eta_1 - E_1 \right) \right\} \\ \quad = \arg\max_{\underline{E} \leq E_2 \leq \overline{E}} \left\{ V_2^* - \left(5 + 1 \right)\left(E_2 - E_1 \right) \right\} = \arg\max_{\underline{E} \leq E_2 \leq \overline{E}} \left\{ V_2^* - 6\left(E_2 - E_1 \right) \right\} \\ E_2^{(3)*+} = \arg\max_{\underline{E} \leq E_2 \leq \overline{E}} \left\{ V_2^* - (P_1\beta\rho - C^{dis})\left(E_2/\eta_1 - E_1 \right) \right\} \\ \quad = \arg\max_{\underline{E} \leq E_2 \leq \overline{E}} \left\{ V_2^* - (5 * 0.81 - 1)\left(E_2 - E_1 \right) \right\} = \arg\max_{\underline{E} \leq E_2 \leq \overline{E}} \left\{ V_2^* - 3.05\left(E_2 - E_1 \right) \right\} \end{cases} \quad \text{(C38)}$$

(1) Scenario1: If $E_2 \in (0,3]$, there is $V_2^* = 7.1E_2 - 50.5593$.

$$\begin{cases} E_2^{(1)*+} = \arg\max_{E_2 \in (0,3]} \left\{ 7.1E_2 - 50.5593 - 7.1728\left(E_2 - E_1 \right) \right\} \\ \quad = \arg\max_{E_2 \in (0,3]} \left\{ -0.0728E_2 - 50.5593 + 7.1728E_1 \right\} \Rightarrow E_2^{(1)*+} = 0; \\ E_2^{(2)*+} = \arg\max_{E_2 \in (0,3]} \left\{ 7.1E_2 - 50.5593 - 6\left(E_2 - E_1 \right) \right\} \\ \quad = \arg\max_{E_2 \in (0,3]} \left\{ 1.1E_2 - 50.5593 + 6E_1 \right\} \Rightarrow E_2^{(2)*+} = 3; \\ E_2^{(3)*+} = \arg\max_{E_2 \in (0,3]} \left\{ 7.1E_2 - 50.5593 - 3.05\left(E_2 - E_1 \right) \right\} \\ \quad = \arg\max_{E_2 \in (0,3]} \left\{ 4.05E_2 - 50.5593 + 3.05E_1 \right\} \Rightarrow E_2^{(3)*+} = 3. \end{cases} \quad \text{(C39)}$$

(2) Scenario2: If $E_2 \in [3,\ 10]$, there is $V_3^* = 4.7037E_2 - 43.337$.

$$\begin{cases} E_2^{(1)*+} = \arg\max_{E_2 \in [3,\ 10]} \left\{ 4.7037E_2 - 43.337 - 7.1728\left(E_2 - E_1\right) \right\} \\ \qquad = \arg\max_{E_2 \in [3,\ 10]} \left\{ -2.4691E_2 - 43.337 + 7.1728E_1 \right\} \Rightarrow E_2^{(1)*+} = 3; \\ E_2^{(2)*+} = \arg\max_{E_2 \in [3,\ 10]} \left\{ 4.7037E_2 - 43.337 - 6\left(E_2 - E_1\right) \right\} \\ \qquad = \arg\max_{E_2 \in [3,\ 10]} \left\{ -1.2963E_2 - 43.337 + 6E_1 \right\} \Rightarrow E_2^{(2)*+} = 3; \\ E_2^{(3)*+} = \arg\max_{E_2 \in [3,\ 10]} \left\{ 4.7037E_2 - 43.337 - 3.05\left(E_2 - E_1\right) \right\} \\ \qquad = \arg\max_{E_2 \in [3,\ 10]} \left\{ 1.6537E_2 - 43.337 + 3.05E_1 \right\} \Rightarrow E_2^{(3)*+} = 10. \end{cases} \tag{C40}$$

Next, we will compare the max value and pick up the optimal references point between the above two scenarios.

(1) Compare $E_2^{(1)*+}$

$$\left| \begin{array}{l} 1) \begin{cases} \text{If } E_2 \in [0,\ 3] \Rightarrow E_2^{(1)*+} = 0 \\ E_2^{(1)*-} = \arg\max_{E_3 \in [0,\ 3]} \left\{ -0.0728E_2 - 50.5593 + 7.1728E_1 \right\} \\ \Rightarrow -0.0728E_2 - 50.5593 + 7.1728E_1 \,|_{E_2^{(1)*+}=0} = -50.5593 + 7.1728E_1 \end{cases} \\ 2) \begin{cases} \text{If } E_2 \in [3,\ 10] \Rightarrow E_2^{(1)*+} = 3 \\ E_2^{(1)*-} = \arg\max_{E_3 \in [6/0.9,\ 10]} \left\{ -2.4691E_2 - 43.337 + 7.1728E_1 \right\} \\ \Rightarrow -2.4691E_2 - 43.337 + 7.1728E_1 \,|_{E_2^{(1)*+}=3} = -50.7443 + 7.1728E_1, \left(E_1 < 3\right) \end{cases} \end{array} \right. \tag{C41}$$

(2) Compare $E_2^{(3)*+}$

$$\left\{\begin{array}{l} 1)\begin{cases} \text{If} \quad E_2 \in [0,\ 3] \Rightarrow E_2^{(3)*+} = 3 \\ E_2^{(3)*-} = \arg \max\limits_{E_2 \in [0,\ 3]} \{4.05E_2 - 50.5593 + 3.05E_1\} \\ \Rightarrow \{4.05E_2 - 50.5593 + 3.05E_1\} |_{E_2^{(3)*+}=3} = -38.4093 + 3.05E_1 \end{cases} \\ 2)\begin{cases} \text{If} \quad E_2 \in [3,\ 10] \Rightarrow E_2^{(3)*+} = 10 \\ E_2^{(3)*+} = \arg \max\limits_{E_2 \in [3,\ 10]} \{1.6537E_2 - 43.337 + 3.05E_1\} \\ \Rightarrow 1.6537E_2 - 43.337 + 3.05E_1 |_{E_2^{(3)*+}=10} = -26.8 + 3.05E_1 \end{cases} \end{array}\right\} \Rightarrow E_2^{(3)*+} = 10 \quad \text{(C42)}$$

Thus, we will get the optimal reference points at stage 2 that are shown as:

$$E_2^{(1)*}=0,\ E_2^{(2)*}=3,\ E_2^{(3)*}=10 \quad \text{(C43)}$$

Similarly, because there are $W_1 - L_1 = 6 - 4 > 0$, and $\alpha(W_t - L_t) \geq \min\{E_{t+1}^{(1)*+}, \overline{Q}^p\}$, the optimal actions at stage 1 are shown as

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(2)*+} - E_t, \overline{Q}^p, \alpha(W_t - L_t)\}, E_t \in [0, E_{t+1}^{(2)*+}], \\ \text{(store residual power without buying up to } E_{t+1}^{(2)*+}\text{);} \\ 0, E_t \in [E_{t+1}^{(2)*+}, E_{t+1}^{(3)*+}] \text{(keep inventory unchanged);} \\ \max\{E_{t+1}^{(3)*+} - E_t, -\overline{Q}^g\}, E_t \in (E_{t+1}^{(3)*+}, \overline{E}], \\ \text{(Sell redisual power and relase energy down to } E_{t+1}^{(3)*+}\text{).} \end{cases} = \begin{cases} 1.8, & E_1 \in [0,1.2] \\ 3 - E_1, & E_1 \in (1.2,\ 3] \\ 0, & E_1 \in (3,\ 10] \end{cases} \quad \text{(C44)}$$

The reward payoff functions at stage 1 are shown as follows:

$$R_1 = -P_1 \cdot (\frac{q_1}{\alpha} - (W_1 - L_1)) \cdot \rho - C^W W_1 - C^{ch} q_1 = \begin{cases} -1.8, & E_1 \in [0,1.2] \\ 6E_1 - 9, & E_1 \in (1.2,\ 3] \\ 9, & E_1 \in (3,\ 10] \end{cases} \quad \text{(C45)}$$

Therefore, we will get the following optimal value functions at stage 1/initial stage.

$$V_1^* = \max\{R_1 + V_2^*\} = \begin{cases} 7.1E_2 - 50.5593 - 1.8 \,|_{E_2 = E_1 + 1.8} \\ 7.1E_2 - 50.5593 + 6E_1 - 9 \,|_{E_2 = 3} \\ 4.7037E_2 - 43.337 + 9 \,|_{E_2 = E_1} \end{cases} = \begin{cases} 7.1E_1 - 39.5793 \quad (E_1 < 1.2) \\ 6E_1 - 38.2593 \quad (1.2 < E_1 < 3) \\ 4.7037E_1 - 34.337 \ (E_1 > 3) \end{cases} \quad \text{(C46)}$$

The corresponding optimal actions are shown:

In stage 1, $q_1^*(S_1) = \{1.8, E_1 \in [0,1.2];\ 3 - E_1, E_1 \in (1.2,\ 3];\ 0, E_1 \in (3,\ 10]$

In stage 2, $q_2^*(S_2) = \{7,\ E_2 \in [0,3];\ 10 - E_2,\ E_2 \in [3,\ 10]$

In stage 3,

$q_3^*(S_3) = -E_3, E_3 \in (0, \overline{E}]$(sell energy and make SOC down to 0 as close as possible)

To sum up, we get the following results:

If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, (action 1: store the remaining wind generation), then we will get $E_2 = 2.8$ (i.e., $q_1^* = 1.8$, $R_1 = -1.8$);

Stage 2: If $E_2 = 2.8$, (action 2: store renewable and purchased electricity up to $\overline{Q}^p$), then, we will get $E_3 = 9.8$ (i.e., $q_2^* = 7, R_2 = -46.2593$);

Stage 3: If $E_3 = 9.8$,(action 3: generating and selling), then, we have $E_4 = 0$ (i.e., $q_3^* = -9.8, R_3 = 15.58$).

Based on the forecasted price, total rewards are shown as $R = R_1 + R_2 + R_3 = -32.4793 = V_1^*$.

If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$,(action 1: store the remaining wind generation), then, the relation of $E_2 = 5$ (i.e., $q_1^* = 0$, $R_1 = 9$) holds;

Stage 2: If $E_2 = 5$,(action 2: store renewable and purchased electricity up to $E_{t+1}^{(1\text{-}s)*}$), then there exists $E_3 = 10$ (i.e., $q_2^* = 5$, $R_2 = -36.8185$);

Stage 3: If $E_3 = 10$,(action 3: generating and selling), then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -10$, $R_3 = 17$).

Therefore, total rewards are shown as $R = R_1 + R_2 + R_3 = -10.8185 = V_1^*$ if $E_1 = 5$.

<u>Case 2:</u>

In stage 4:

$$VOE_4 = (5+3+10)/3 = 6 \Rightarrow V_4 = VOE_4 \cdot E_4 = 6E_4$$

We will get the following optimal references points:

$$\begin{cases} E_4^{(1)*} = \arg\max\limits_{\underline{E} \le E_4 \le \overline{E}} \left\{ -\left(P_3/\alpha\rho + C^{ch} \right)\left(E_4/\eta_3 - E_3 \right) + E[V_4^*\left(S(4) \mid S(3) \right)] \right\} \\ E_4^{(2)*} = \arg\max\limits_{\underline{E} \le E_4 \le \overline{E}} \left\{ -\left(P_3\beta/\rho - C^{dis} \right)\left(E_4/\eta_3 - E_3 \right) + E[V_4^*\left(S(4) \mid S(3) \right)] \right\} \\ E_4^{(3)*} = \arg\max\limits_{\underline{E} \le E_4 \le \overline{E}} \left\{ -(P_3\beta\rho - C^{dis})\left(E_4/\eta_3 - E_3 \right) + E[V_4^*\left(S(4) \mid S(3) \right)] \right\} \end{cases}$$

Then, plug in the data, we will get the following equations:

$$\begin{cases} E_4^{(1)*} = \arg\max\limits_{\underline{E} \le E_4 \le \overline{E}} \left\{ -\left(10/0.81 + 1 \right) E_4 + 6E_4 \right\} = \arg\max\limits_{\underline{E} \le E_4 \le \overline{E}} \left\{ -7.3457E_4 \right\} \\ E_4^{(2)*} = \arg\max\limits_{\underline{E} \le E_4 \le \overline{E}} \left\{ -\left(10 \cdot 0.9/0.9 - 1 \right) E_4 + 6E_4 \right\} = \arg\max\limits_{\underline{E} \le E_4 \le \overline{E}} \left\{ -3E_4 \right\} \\ E_4^{(3)*} = \arg\max\limits_{\underline{E} \le E_4 \le \overline{E}} \left\{ -(10 \cdot 0.81 - 1) E_4 + 6E_4 \right\} = \arg\max\limits_{\underline{E} \le E_4 \le \overline{E}} \left\{ -1.1E_4 \right\} \end{cases} \tag{C47}$$

$$\Rightarrow E_4^{(1)*} = E_4^{(2)*} = E_4^{(3)*} = 0$$

In Stage 3: There is $W_3 - L_3 = 0 - 0 = 0$.

Action 3: Release power and make the storage level down to $\underline{E} = 0 = E_4^*$, thus,

$V_4^* = 0$. Therefore, the optimal action is

$$q_3^*(S_3) = -E_3, E_3 \in (0, \overline{E}] \text{(sell energy and make SOC down to 0 as close as possible)} \quad \text{(C48)}$$

The reward function at stage3 is shown as

$$\begin{aligned} R_3 &= \left\{ -P_3\left[q_3\beta - \left(W_3 - L_3\right)\right]\rho + C^{dis}\cdot q_3 - C^w \cdot W_3 \quad \left(-E_3 \le 0\right) \right. \\ &= \left\{ -10\left[0 - 0.9E_3\right]0.9 - E_3 = 7.1E_3 \quad \left(E_3 \ge 0\right) \right. \end{aligned} \quad \text{(C49)}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_3^* = \max\{R_3 + V_4^*\} = \left\{ 7.1E_3 \quad (E_3 \ge 0) \right. \quad \text{(C50)}$$

In stage 2: There is $W_2 - L_2 = 5 - 0 = 5 \ge 0$.

By using the equations (C4), (C5), and (C6), we get the following results:

$$\begin{cases} E_3^{(1)*+} = \arg\max\limits_{\underline{E}\le E_3\le \overline{E}} \left\{ V_3^* - \left(P_2/\alpha\rho + C^{ch}\right)\left(E_3/\eta_2 - E_2\right) \right\} \\ \qquad = \arg\max\limits_{\underline{E}\le E_3\le \overline{E}} \left\{ V_3^* - \left(3/0.81 + 1\right)\left(E_3 - E_2\right) \right\} = \arg\max\limits_{\underline{E}\le E_3\le \overline{E}} \left\{ V_3^* - 4.7037E_3 + 4.7037E_2 \right\} \\ E_3^{(2)*+} = \arg\max\limits_{\underline{E}\le E_3\le \overline{E}} \left\{ V_3^* - \left(P_2\rho/\alpha + C^{ch}\right)\left(E_3/\eta_3 - E_2\right) \right\} \\ \qquad = \arg\max\limits_{\underline{E}\le E_3\le \overline{E}} \left\{ V_3^* - \left(3 + 1\right)\left(E_3 - E_2\right) \right\} = \arg\max\limits_{\underline{E}\le E_3\le \overline{E}} \left\{ V_3^* - 4E_3 + 4E_2 \right\} \\ E_3^{(3)*+} = \arg\max\limits_{\underline{E}\le E_3\le \overline{E}} \left\{ V_3^* - (P_2\beta\rho - C^{dis})\left(E_3/\eta_3 - E_2\right) \right\} \\ \qquad = \arg\max\limits_{\underline{E}\le E_3\le \overline{E}} \left\{ V_3^* - \left(3\cdot 0.81 - 1\right)\left(E_3 - E_2\right) \right\} = \arg\max\limits_{\underline{E}\le E_3\le \overline{E}} \left\{ V_3^* - 1.43E_3 + 1.43E_2 \right\} \end{cases} \quad \text{(C51)}$$

$E_3 \in [0,\ 10]$, there is $V_3^* = 7.1E_3$.

$$\begin{cases} E_3^{(1)*+} = \arg\max\limits_{E_3\in[0,\ 10]} \left\{ 7.1E_3 - 4.7037E_3 + 4.7037E_2 \right\} = \arg\max\limits_{E_3\in[0,\ 6/0.9]} \left\{ 2.2963E_3 + 4.7037E_2 \right\} \\ E_3^{(2)*+} = \arg\arg\max\limits_{E_3\in[0,\ 10]} \left\{ 7.1E_3 - 4E_3 + 4E_2 \right\} = \arg\max\limits_{E_3\in[0,\ 6/0.9]} \left\{ 3.1E_3 + 4E_2 \right\} \\ E_3^{(3)*+} = \arg\max\limits_{E_3\in[0,\ 10]} \left\{ 7.1E_3 - 1.43E_3 + 1.43E_2 \right\} = \arg\max\limits_{E_3\in[0,\ 6/0.9]} \left\{ 5.67E_3 + 1.43E_2 \right\} \end{cases} \quad \text{(C52)}$$

$$\Rightarrow E_3^{(1)*} = E_3^{(2)*} = E_3^{(3)*} = 10$$

Thus, we will get the optimal reference points at stage 3 that are shown as:

$$E_3^{(1)*} = E_3^{(2)*} = E_3^{(3)*} = 10 \tag{C53}$$

Since there are $\alpha(W_t - L_t) < \min\{E_{t+1}^{(1)*+}, \overline{Q}^{ch}\}$. The optimal actions at stage 2 are

$$q_t^{*+}(S_t) = \begin{cases} \min\{E_{t+1}^{(1)*+} - E_t, \overline{Q}^{ch}\},\ E_t \in [0, E_{t+1}^{(1)*+} - \alpha(W_t - L_t)], \\ \text{(store residual power and purchase electricity up to } E_{t+1}^{(1)*+}); \\ \min\{E_{t+1}^{(2)*+} - E_t, \alpha(W_t - L_t), \overline{Q}^{ch}\}, E_t \in (E_{t+1}^{(1)*+} - \alpha(W_t - L_t), E_{t+1}^{(2)*+}], \\ \text{(store residual power without buying up to } E_{t+1}^{(2)*+}); \\ 0, E_t \in (E_{t+1}^{(2)*+}, E_{t+1}^{(3)*+}] \text{ (keep inventory unchanged)}; \\ \max\{E_{t+1}^{(3)*+} - E_t, -\overline{Q}^{dis}\}, E_t \in (E_{t+1}^{(3)*+}, \overline{E}], \\ \text{(Sell redisual power and relase energy down to } E_{t+1}^{(3)*+}). \end{cases} \tag{C54}$$

$$\Rightarrow q_2^{*}(S_2) = \begin{cases} 7, & E_2 \in [0,3] \\ 10 - E_2, & E_2 \in [3,\ 5.5] \\ 10 - E_2, & E_2 \in [5.5,\ 10] \end{cases}$$

The reward payoff functions at stage 2 are shown as follows:

$$R_2 = \begin{cases} -P_2\left[q_2/\alpha - (W_2 - L_2)\right]/\rho - C^{ch} \cdot q_2 - C^{w} \cdot W_2 & (q_2 \geq 4.5) \\ -P_2\left[q_2/\alpha - (W_2 - L_2)\right]\rho - C^{ch} q_2 - C^{w} \cdot W_2 & (0 \leq q_2 < 4.5) \end{cases}$$

$$= \begin{cases} -3\left[7/0.9 - (5-0)\right]/0.9 - 7 \\ -3\left[(10 - E_2)/0.9 - (5-0)\right]/0.9 - (10 - E_2) \\ -3\left[(10 - E_2)/0.9 - (5-0)\right]0.9 - (10 - E_2) \end{cases} = \begin{cases} -16.2593 & (E_2 < 3) \\ 4.7037E_2 - 30.3704 & (3 \leq E_2 < 5.5) \\ 4E_2 - 26.5 & (E_2 \geq 5.5) \end{cases} \tag{C55}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_2^{*} = \max\{R_2 + V_3^{*}\} = \begin{cases} 7.1E_3 - 16.2593_{|E_3 = E_2 + 7} \\ 7.1E_3 + 4.7037E_2 - 30.3704_{|E_3 = 10} \\ 7.1E_3 + 4E_2 - 26.5_{|E_3 = 10} \end{cases} = \begin{cases} 7.1E_2 + 33.4407 & (E_2 < 3) \\ 4.7037E_2 + 40.6296 & (3 \leq E_2 < 5.5) \\ 4E_2 + 44.5 & (E_2 \geq 5.5) \end{cases} \tag{C56}$$

In Stage 1: There has $W_1 - L_1 = 6 - 0 > 0$.

By using the equations (A4), (A5), and (A6), we get the following results:

$$\begin{cases} E_2^{(1)*+} = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left\{ V_2^* - \left(P_1/\alpha\rho + C^{ch} \right)\left(E_2/\eta_1 - E_1 \right) \right\} \\ \quad = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left\{ V_2^* - \left(5/0.81 + 1 \right)\left(E_2 - E_1 \right) \right\} = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left\{ V_2^* - 7.1728\left(E_2 - E_1 \right) \right\} \\ E_2^{(2)*+} = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left\{ V_2^* - \left(P_1\rho/\alpha + C^{ch} \right)\left(E_2/\eta_1 - E_1 \right) \right\} \\ \quad = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left\{ V_2^* - \left(5 + 1 \right)\left(E_2 - E_1 \right) \right\} = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left\{ V_2^* - 6\left(E_2 - E_1 \right) \right\} \\ E_2^{(3)*+} = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left\{ V_2^* - (P_1\beta\rho - C^{dis})\left(E_2/\eta_1 - E_1 \right) \right\} \\ \quad = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left\{ V_2^* - (5*0.81 - 1)\left(E_2 - E_1 \right) \right\} = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left\{ V_2^* - 3.05\left(E_2 - E_1 \right) \right\} \end{cases} \quad \text{(C57)}$$

(1) Scenario1: If $E_2 \in (0,3]$, there is $V_2^* = 7.1E_2 + 33.4407$.

$$\begin{cases} E_2^{(1)*+} = \arg\max_{E_2 \in (0,3]} \left\{ 7.1E_2 + 33.4407 - 7.1728\left(E_2 - E_1 \right) \right\} \\ \quad = \arg\max_{E_2 \in (0,3]} \left\{ -0.0728E_2 + 33.4407 + 7.1728E_1 \right\} \Rightarrow E_2^{(1)*+} = 0; \\ E_2^{(2)*+} = \arg\max_{E_2 \in (0,3]} \left\{ 7.1E_2 + 33.4407 - 6\left(E_2 - E_1 \right) \right\} \\ \quad = \arg\max_{E_2 \in (0,3]} \left\{ 1.1E_2 + 33.4407 + 6E_1 \right\} \Rightarrow E_2^{(2)*+} = 3; \\ E_2^{(3)*+} = \arg\max_{E_2 \in (0,3]} \left\{ 7.1E_2 + 33.4407 - 3.05\left(E_2 - E_1 \right) \right\} \\ \quad = \arg\max_{E_2 \in (0,3]} \left\{ 4.05E_2 + 33.4407 + 3.05E_1 \right\} \Rightarrow E_2^{(3)*+} = 3. \end{cases} \quad \text{(C58)}$$

(2) Scenario2: If $E_2 \in [3,\ 10]$, there is $V_2^* = 4.7037E_2 + 40.6296$.

$$\begin{cases} E_2^{(1)*+} = \arg\max_{E_2 \in [3,\ 10]} \left\{4.7037E_2 + 40.6296 - 7.1728\left(E_2 - E_1\right)\right\} \\ \qquad = \arg\max_{E_2 \in [3,\ 10]} \left\{-2.4691E_2 + 40.6296 + 7.1728E_1\right\} \Rightarrow E_2^{(1)*+} = 3; \\ E_2^{(2)*+} = \arg\max_{E_2 \in [3,\ 10]} \left\{4.7037E_2 + 40.6296 - 6\left(E_2 - E_1\right)\right\} \\ \qquad = \arg\max_{E_2 \in [3,\ 10]} \left\{-1.2963E_2 + 40.6296 + 6E_1\right\} \Rightarrow E_2^{(2)*+} = 3; \\ E_2^{(3)*+} = \arg\max_{E_2 \in [3,\ 10]} \left\{4.7037E_2 + 40.6296 - 3.05\left(E_2 - E_1\right)\right\} \\ \qquad = \arg\max_{E_2 \in [3,\ 10]} \left\{1.6537E_2 + 40.6296 + 3.05E_1\right\} \Rightarrow E_2^{(3)*+} = 10. \end{cases} \tag{C59}$$

Next, we will compare the max value and pick up the optimal references point between the above two scenarios.

(3) Compare $E_2^{(1)*+}$

$$\left| \begin{array}{l} 1)\begin{cases} \text{If} \quad E_2 \in [0,\ 3] \Rightarrow E_2^{(1)*+} = 0 \\ E_2^{(1)*-} = \arg\max_{E_3 \in [0,\ 3]} \left\{-0.0728E_2 + 33.4407 + 7.1728E_1\right\} \\ \Rightarrow -0.0728E_2 + 33.4407 + 7.1728E_1 \,|_{E_2^{(1)*+}=0} = 33.4407 + 7.1728E_1 \end{cases} \\ 2)\begin{cases} \text{If} \quad E_2 \in [3,\ 10] \Rightarrow E_2^{(1)*+} = 3 \\ E_2^{(1)*-} = \arg\max_{E_3 \in [6/0.9,\ 10]} \left\{-2.4691E_2 + 40.6296 + 7.1728E_1\right\} \\ \Rightarrow -2.4691E_2 + 40.6296 + 7.1728E_1 \,|_{E_2^{(1)*+}=3} = 30.2223 + 7.1728E_1, \quad \left(E_1 < 3\right) \end{cases} \end{array} \right\} \tag{C60}$$

$$\Rightarrow E_2^{(1)*+} = 0$$

(4) Compare $E_2^{(3)*+}$

$$\left.\begin{cases} 1)\begin{cases} \text{If} \quad E_2 \in [0,\ 3] \Rightarrow E_2^{(3)*+} = 3 \\ E_2^{(3)*-} = \arg\max\limits_{E_2 \in [0,\ 3]} \{4.05E_2 + 33.4407 + 3.05E_1\} \\ \Rightarrow \{4.05E_2 + 33.4407 + 3.05E_1\} |_{E_2^{(3)*+}=3} = 45.5907 + 3.05E_1 \end{cases} \\ 2)\begin{cases} \text{If} \quad E_2 \in [3,\ 10] \Rightarrow E_2^{(3)*+} = 10 \\ E_2^{(3)*+} = \arg\max\limits_{E_2 \in [3,\ 10]} \{1.6537E_2 + 40.6296 + 3.05E_1\} \\ \Rightarrow 1.6537E_2 + 40.6296 + 3.05E_1 |_{E_2^{(3)*+}=10} = 57.1666 + 3.05E_1 \end{cases} \end{cases}\right\} \Rightarrow E_2^{(3)*+} = 10 \qquad \text{(C61)}$$

Thus, we will get the optimal reference points at stage 2 that are shown as:

$$E_2^{(1)*} = 0, \quad E_2^{(2)*} = 3, \quad E_2^{(3)*} = 10 \qquad \text{(C62)}$$

Similarly, because there are $W_1 - L_1 = 6 - 0 > 0$, and $\alpha(W_t - L_t) \geq \min\{E_{t+1}^{(1)*+}, \overline{Q}^p\}$, the optimal actions at stage 1 are shown as

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(2)*+} - E_t, \overline{Q}^p, \alpha(W_t - L_t)\}, E_t \in [0, E_{t+1}^{(2)*+}], \\ \text{(store residual power without buying up to } E_{t+1}^{(2)*+}\text{);} \\ 0, E_t \in [E_{t+1}^{(2)*+}, E_{t+1}^{(3)*+}] \text{(keep inventory unchanged);} \\ \max\{E_{t+1}^{(3)*+} - E_t, -\overline{Q}^g\}, E_t \in (E_{t+1}^{(3)*+}, \overline{E}], \\ \text{(Sell redisual power and relase energy down to } E_{t+1}^{(3)*+}\text{).} \end{cases} \qquad \text{(C63)}$$

$$\Rightarrow q_1^*(S_1) = \begin{cases} 3 - E_1, & E_1 \in [0,\ 3] \\ 0, & E_1 \in (3,\ 10] \end{cases}$$

The reward payoff functions at stage 1 are shown as follows:

$$R_1 = -P_1\left[q_1/\alpha - \left(W_1 - L_1\right)\right]\rho - C^{ch} \cdot q_1 - C^w \cdot W_1 \qquad \left(q_1 \geq 0\right)$$

$$= \begin{cases} -5\left[(3 - E_1)/0.9 - \left(6 - 0\right)\right]0.9 - (3 - E_1) \ \left(E_1 < 3\right) \\ -5\left[-\left(6 - 0\right)\right]0.9 - 0 \ \left(E_1 \geq 3\right) \end{cases} = \begin{cases} 6E_1 + 9 \ \left(E_1 < 3\right) \\ 27 \ \left(E_1 \geq 3\right) \end{cases} \qquad \text{(C64)}$$

Therefore, we will get the following optimal value functions at stage 1/initial stage.

$$V_1^* = \max\{R_1 + V_2^* = \begin{cases} 6E_1 + 7.1E_2 + 42.4407|_{E_2=3} & (E_1 < 3) \\ 27 + 4.7037E_2 + 40.6296|_{E_2=E_1} & (3 \le E_1 < 5.5) \\ 27 + 4E_2 + 44.5|_{E_2=E_1} & (E_1 \ge 5.5) \end{cases}$$

$$= \begin{cases} 6E_1 + 63.7407 & (E_1 < 3) \\ 4.7037E_1 + 67.6296 & (3 \le E_1 < 5.5) \\ 4E_1 + 71.5 & (E_1 \ge 5.5) \end{cases} \tag{C65}$$

The corresponding optimal actions are shown:

In stage 1, $q_1^*(S_1) = \begin{cases} 3 - E_1, & E_1 \in [0,\ 3] \\ 0, & E_1 \in (3,\ 10] \end{cases}$; In stage 2, $q_2^*(S_2) = \begin{cases} 7, & E_2 \in [0,3] \\ 10 - E_2, & E_2 \in [3,\ 10] \end{cases}$

In stage 3,

$q_3^*(S_3) = -E_3, E_3 \in (0, \overline{E}]$(sell energy and make SOC down to 0 as close as possible)

To sum up, we get the following results:

If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, (action 1: store the remaining wind generation), then we will get $E_2 = 3$ (i.e., $q_1^* = 2$, $R_1 = 15$);

Stage 2: If $E_2 = 3$, (action 2: store renewable and purchased electricity up to $\overline{Q}^P$), then, we will get $E_3 = 10$ (i.e., $q_2^* = 7, R_2 = -16.2593$);

Stage 3: If $E_3 = 10$,(action 3: generating and selling), then, we have $E_4 = 0$ (i.e., $q_3^* = -10, R_3 = 71$).

The total rewards are shown as $R = R_1 + R_2 + R_3 = 69.7407 = V_1^*$.

If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$,(action 1: store the remaining wind generation), then, the relation of $E_2 = 5$ (i.e., $q_1^* = 0$, $R_1 = 27$) holds;

Stage 2: If $E_2 = 5$,(action 2: store renewable and purchased electricity up to $E_{t+1}^{(1\text{-}s)^*}$), then there exists $E_3 = 10$ (i.e., $q_2^* = 5$, $R_2 = -6.8519$);

Stage 3: If $E_3 = 10$,(action 3: generating and selling), then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -10$, $R_3 = 71$).

Therefore, total rewards in three periods are shown as $R = R_1 + R_2 + R_3 = 90.1481 = V_1^*$ if $E_1 = 5$.

APPENDIX D.

PROOF OF SECTION 6

Proof of Lemma 6.1

1) The uniqueness of the optimal results:

We show the current payoff rewards as follows for the merchant with the renewable power plant and storage and when the merchant can receive PTC *s* per renewable generation sold to the market:

$$R^{(s)}(q_t,w_t,P_t)=\begin{cases}-P_t\cdot(q_t/\alpha-w_t)/\rho-c_w w_t-c_p q_t & (q_t>\alpha w_t)\\ -P_t\cdot(q_t/\alpha-w_t)\cdot\rho-s(q_t/\alpha-w_t)-c_w w_t-c_p q_t & (0\le q_t<\alpha w_t)\\ -P_t\cdot(q_t\beta-w_t)\cdot\rho-c_w w_t+sw_t+c_g q_t & (q_t<0)\end{cases} \quad \text{(D1)}$$

Where, q_t is the energy/inventory change from period t to period t+1 before accounting for energy loss. By using the method $E_{t+1}=\eta_t\cdot(E_t+q_t)$, we will get the following rewards function at time t.

$$R^{(s)}(q_t,w_t,P_t)=\begin{cases}-(P_t/\alpha\rho)q_t-c_p q_t-w_t(-P_t/\rho+c_w) & (q_t>\alpha w_t)\\ -((P_t\rho+s)/\alpha)q_t-c_p q_t-w_t[-P_t\rho+c_w-s] & (0\le q_t<\alpha w_t)\\ -P_t\beta\rho q_t+c_g q_t-w_t[-P_t\rho+c_w-s] & (q_t<0)\end{cases}$$

$$\Leftrightarrow\begin{cases}-(P_t/\alpha\rho)(E_{t+1}/\eta_t-E_t)-c_p(E_{t+1}/\eta_t-E_t)-w_t(-P_t/\rho+c_w) & (q_t>\alpha w_t)\\ -((P_t\rho+s)/\alpha)(E_{t+1}/\eta_t-E_t)-c_p(E_{t+1}/\eta_t-E_t)-w_t[-P_t\rho+c_w-s] & (0\le q_t<\alpha w_t)\\ -P_t\beta\rho(E_{t+1}/\eta_t-E_t)+c_g(E_{t+1}/\eta_t-E_t)-w_t[-P_t\rho+c_w-s] & (q_t<0)\end{cases} \quad \text{(D2)}$$

In the end of the period T or in the beginning of the period T+1:

$$V_T(E_T,P_T)=[R(q_T,w_T,P_T)+VOE_{T+1}\cdot E_{T+1}]$$

Where,

$$\begin{cases} V_T^{(1\text{-}s)*}(S(T)) = \max\limits_{\underline{E}\le E_{T+1}\le\bar{E}}\left\{-\left(P_t/\alpha\rho\right)\cdot q_T - c_p q_T - w_T(-P_T/\rho + c_w) + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\right\} \\ V_T^{(2-s)*}(S(T)) = \max\limits_{\underline{E}\le E_{T+1}\le\bar{E}}\left\{-(P_T\rho + s)/\alpha\cdot q_T - c_p q_T - w_T[-P_T\rho + c_w - s] + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\right\} \\ V_T^{(3-s)*}(S(T)) = \max\limits_{\underline{E}\le E_{T+1}\le\bar{E}}\left\{-P_t\beta\rho q_T + c_g q_T - w_T[-P_T\rho + c_w - s] + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\right\} \end{cases} \quad (D3)$$

Here, we can also get the following relation of equivalence:

$$\begin{cases} V_T^{(1\text{-}s)*}(S(T)) = \max\limits_{\underline{E}\le E_{T+1}\le\bar{E}}\left\{-\left(P_t/\alpha\rho + c_p\right)\left(E_{T+1}/\eta_T - E_T\right) + VOE_{T+1}\cdot E_{T+1}\right\} \\ V_T^{(2-s)*}(S(T)) = \max\limits_{\underline{E}\le E_{T+1}\le\bar{E}}\left\{-\left((P_T\rho + s)/\alpha + c_p\right)\left(E_{T+1}/\eta_T - E_T\right) + VOE_{T+1}\cdot E_{T+1}\right\} \\ V_T^{(3-s)*}(S(T)) = \max\limits_{\underline{E}\le E_{T+1}\le\bar{E}}\left\{-\left(P_t\beta\rho - c_g\right)\left(E_{T+1}/\eta_T - E_T\right) + VOE_{T+1}\cdot E_{T+1}\right\} \end{cases} \quad (D4)$$

The best response functions (first-order derivative) of $V_T^*(S(T))$ on E_{T+1} are:

$$\begin{cases} \partial V_T^{(1\text{-}s)*}(S(T))\big/\partial E_{T+1} = -\left(P_t/\alpha\rho + c_p\right)\big/\eta_T + VOE_{T+1} \\ \partial V_T^{(2\text{-}s)*}(S(T))\big/\partial E_{T+1} = -\left((P_T\rho + s)/\alpha + c_p\right)\big/\eta_T + VOE_{T+1} \\ \partial V_T^{(3\text{-}s)*}(S(T))\big/\partial E_{T+1} = -\left(P_t\beta\rho - c_g\right)\big/\eta_T + VOE_{T+1} \end{cases} \quad (D5)$$

We also get the following second-order derivative functions of $V_T^*(S(T))$ on E_{T+1}.

$$\partial^2 V_T^{(1\text{-}s)*}(S(T))\big/\partial {E_{T+1}}^2 = 0;\ \partial^2 V_T^{(2\text{-}s)*}(S(T))\big/\partial {E_{T+1}}^2 = 0;\ \partial^2 V_T^{(3\text{-}s)*}(S(T))\big/\partial {E_{T+1}}^2 = 0 \quad (D6)$$

Since there is $P_T/\alpha\rho + c_p \ge (P_T\rho + s)/\alpha + c_p \Leftrightarrow P_T \ge (P_T\rho + s)\rho \Leftrightarrow s \le P_T(1-\rho^2)/\rho$, thus

(1) when $s \le P_T(1-\rho^2)/\rho$, there are $\left(P_T\beta\rho - c_g\right)\big/\eta_T < \left((P_T\rho + s)/\alpha + c_p\right)\big/\eta_T \le \left(P_t/\alpha\rho + c_p\right)\big/\eta_T$

(a) If $VOE_{T+1} < \left(P_T\beta\rho - c_g\right)\big/\eta_T$,there are $\partial V_T^{(1\text{-}s)*}(S(T))\big/\partial E_{T+1} < 0$, $\partial V_T^{(2\text{-}s)*}(S(T))\big/\partial E_{T+1} < 0$, and $\partial V_T^{(3\text{-}s)*}(S(T))\big/\partial E_{T+1} < 0$. We get the following optimal results:

$$\begin{cases} E_{T+1}^{(1\text{-}s)*} = \arg\max\limits_{\underline{E} \le E_{T+1} \le \overline{E}} \left\{ -\left(P_t/\alpha\rho + c_p\right)\left(E_{T+1}/\eta_T - E_T\right) + E[V_{T+1}^{*}(S(T+1) \mid S(T)]\right\} = \underline{E} \\ E_{T+1}^{(2\text{-}s)*} = \arg\max\limits_{\underline{E} \le E_{T+1} \le \overline{E}} \left\{ -\left((P_T\rho + s)/\alpha + c_p\right)\left(E_{T+1}/\eta_T - E_T\right) + E[V_{T+1}^{*}(S(T+1) \mid S(T)]\right\} = \underline{E} \\ E_{T+1}^{(3\text{-}s)*} = \arg\max\limits_{\underline{E} \le E_{T+1} \le \overline{E}} \left\{ -\left(P_t\beta\rho - c_g\right)\left(E_{T+1}/\eta_T - E_T\right) + E[V_{T+1}^{*}(S(T+1) \mid S(T)]\right\} = \underline{E} \end{cases} \quad \text{(D7)}$$

(b) If $\left(P_T\beta\rho - c_g\right)/\eta_T < V_{T+1} < \left((P_T\rho + s)/\alpha + c_p\right)/\eta_T$, there are $\partial V_T^{(1\text{-}s)*}(S(T))/\partial E_{T+1} < 0$, $\partial V_T^{(2\text{-}s)*}(S(T))/\partial E_{T+1} < 0$, and $\partial V_T^{(3\text{-}s)*}(S(T))/\partial E_{T+1} > 0$. We get the following optimal results:

$$\left\{ E_{T+1}^{(1\text{-}s)*} = \underline{E}; \quad E_{T+1}^{(2\text{-}s)*} = \underline{E}; \quad E_{T+1}^{(3\text{-}s)*} = \overline{E} \right. \quad \text{(D8)}$$

(c) If $\left((P_T\rho + s)/\alpha + c_p\right)/\eta_T < VOE_{T+1} < \left(P_T/\alpha\rho + c_p\right)/\eta_T$, there are $\partial V_T^{(1\text{-}s)*}(S(T))/\partial E_{T+1} < 0$, $\partial V_T^{(2\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, and $\partial V_T^{(3\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, then we will get the following optimal results:

$$\left\{ E_{T+1}^{(1\text{-}s)*} = \underline{E}; \quad E_{T+1}^{(2\text{-}s)*} = \overline{E}; \quad E_{T+1}^{(3\text{-}s)*} = \overline{E} \right. \quad \text{(D9)}$$

(d) If $VOE_{T+1} > \left(P_T/\alpha\rho + c_p\right)/\eta_T$,there are $\partial V_T^{(1\text{-}s)*}(S(T))/\partial E_{T+1} > 0$ $\partial V_T^{(2\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, and $\partial V_T^{(3\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, we have the following optimal results:

$$\left\{ E_{T+1}^{(1\text{-}s)*} = \overline{E}; \quad E_{T+1}^{(2\text{-}s)*} = \overline{E}; \quad E_{T+1}^{(3\text{-}s)*} = \overline{E} \right. \quad \text{(D10)}$$

(2) When $s > P_T(1-\rho^2)/\rho$, there are $(P_T\beta\rho - c_g)/\eta_T < (P_T/\alpha\rho + c_p)/\eta_T < \left((P_T\rho + s)/\alpha + c_p\right)/\eta_T$

(e) If $VOE_{T+1} < \left(P_T\beta\rho - c_g\right)/\eta_T$,there are $\partial V_T^{(1\text{-}s)*}(S(T))/\partial E_{T+1} < 0$, $\partial V_T^{(2\text{-}s)*}(S(T))/\partial E_{T+1} < 0$, and $\partial V_T^{(3\text{-}s)*}(S(T))/\partial E_{T+1} < 0$, we get the following optimal results:

$$\begin{cases} E_{T+1}^{(1\text{-}s)*} = \arg\max_{\underline{E}\le E_{T+1}\le \overline{E}} \left\{-\left(P_t/(\alpha\rho)+c_p\right)\left(E_{T+1}/\eta_T - E_T\right) + E[V_{T+1}^{*}(S(T+1)\,|\,S(T)]\right\} = \underline{E} \\ E_{T+1}^{(2\text{-}s)*} = \arg\max_{\underline{E}\le E_{T+1}\le \overline{E}} \left\{-\left((P_T\rho+s)/\alpha+c_p\right)\left(E_{T+1}/\eta_T - E_T\right) + E[V_{T+1}^{*}(S(T+1)\,|\,S(T)]\right\} = \underline{E} \\ E_{T+1}^{(3\text{-}s)*} = \arg\max_{\underline{E}\le E_{T+1}\le \overline{E}} \left\{-\left(P_T\beta\rho-c_g\right)\left(E_{T+1}/\eta_T - E_T\right) + E[V_{T+1}^{*}(S(T+1)\,|\,S(T)]\right\} = \underline{E} \end{cases} \quad \text{(D11)}$$

(f) If $(P_T\beta\rho-c_g)/\eta_T < VOE_{T+1} < (P_T/\alpha\rho+c_p)/\eta_T$,there are $\partial V_T^{(1\text{-}s)*}(S(T))/\partial E_{T+1} < 0$, $\partial V_T^{(2\text{-}s)*}(S(T))/\partial E_{T+1} < 0$, and $\partial V_T^{(3\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, we get the following optimal results:

$$\left\{E_{T+1}^{(1\text{-}s)*} = \underline{E};\quad E_{T+1}^{(2\text{-}s)*} = \underline{E};\quad E_{T+1}^{(3\text{-}s)*} = \overline{E}\right. \quad \text{(D12)}$$

(g) If $(P_T/\alpha\rho+c_p)/\eta_T < VOE_{T+1} < \left((P_T\rho+s)/\alpha+c_p\right)/\eta_T$, there are $\partial V_T^{(1\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, $\partial V_T^{(2\text{-}s)*}(S(T))/\partial E_{T+1} < 0$, and $\partial V_T^{(3\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, we get the following optimal results:

$$\left\{E_{T+1}^{(1\text{-}s)*} = \overline{E};\quad E_{T+1}^{(2\text{-}s)*} = \underline{E};\quad E_{T+1}^{(3\text{-}s)*} = \overline{E}\right. \quad \text{(D13)}$$

(h) If $VOE_{T+1} > \left((P_T\rho+s)/\alpha+c_p\right)/\eta_T$, there are $\partial V_T^{(1\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, $\partial V_T^{(2\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, and $\partial V_T^{(3\text{-}s)*}(S(T))/\partial E_{T+1} > 0$, we have the following optimal results:

$$\left\{E_{T+1}^{(1\text{-}s)*} = \overline{E};\quad E_{T+1}^{(2\text{-}s)*} = \overline{E};\quad E_{T+1}^{(3\text{-}s)*} = \overline{E}\right. \quad \text{(D14)}$$

For the state at $t \in \{1,2,\cdots,T\}$

By optimizing of the value function $V_t(E_t, w_t, P_t)$, subject to $\underline{E} \le E_{t+1} \le \overline{E}$, we will get the following equations based on the Bellman equation.

$$\begin{cases} V_t^{(1-s)*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{-\left(P_t/\alpha\rho + c_p\right)q_t - w_t(-P_t/\rho + c_w) + E[V_{t+1}^*(S(t+1)\,|\,S(t)]\right\} \\ \qquad \Leftrightarrow \max_{\underline{E}\le E_{t+1}\le \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - (P_t/\alpha\rho + c_p)\cdot(E_{t+1}/\eta_t)\right) \\ V_t^{(2-s)*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{-\left((P_t\rho + s)/\alpha + c_p\right)q_t - w_t[-P_t\rho + c_w - s] + E[V_{t+1}^*(S(t+1)\,|\,S(t)]\right\} \\ \qquad \Leftrightarrow \max_{\underline{E}\le E_{t+1}\le \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \left((P_t\rho + s)/\alpha + c_p\right)\cdot(E_{t+1}/\eta_t)\right) \\ V_t^{(3-s)*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{(-\left(P_t\beta\rho - c_g\right)q_t - w_t[-P_t\rho + c_w - s] + E[V_{t+1}^*(S(t+1)\,|\,S(t)]\right\} \\ \qquad \Leftrightarrow \max_{\underline{E}\le E_{t+1}\le \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t) - (P_t\beta\rho - c_g)\cdot(E_{t+1}/\eta_t)\right) \end{cases} \tag{D15}$$

Recall the previous proof at state T, we know that every $t \in \{1,2,3,\cdots,T\}$, in every stage *t*, the value function $V_t(S(t))$ and $E[V_{t+1}^*(S(t+1)\,|\,S(t)]$ are concave in $E_t \in [\underline{E}, \bar{E}]$ for each given state $S(t) = S_t(E_t, w_t, P_t)$ .Clearly, $E[V_{t+1}^*(S(t+1)\,|\,S(t)]$ and functions (D15) are concave in $E_t \in [\underline{E}, \bar{E}]$ for each given state $S(t) = S_t(E_t, g_t, P_t)$ by using

$$\partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]/\partial E_{t+1}{}^2 = \left(\partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]/\partial E_t{}^2 \cdot \left(\partial E_t/\partial E_{t+1}\right)^2\right) \le 0$$

(1) When $q_t > \alpha w_t$, by optimizing the function (D15), subject to $E_{t+1} \in [\underline{E}, \bar{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\frac{\partial V_t^{(1\text{-}s)*}(S(t))}{\partial E_{t+1}} = \frac{\partial\left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \left(P_t/\alpha\rho\right)\cdot\left(E_{t+1}/\eta_t\right)\right)}{\partial E_{t+1}} = \frac{\partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - \left(\frac{P_t}{\alpha\rho} + c_p\right)\frac{1}{\eta_t}$$

The second-order derivative： $\partial V_t^{(1\text{-}s)*}(S(t))/\partial E_{t+1}^2 = \partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]/\partial E_{t+1}^2 < 0 < 0$

Since the second-order derivative is negative, we can find the unique optimal solutions through the first-order condition. We also will get the following optimal results:

$$E_{t+1}^{(1\text{-}s)*} = \arg\max_{\underline{E}\le E_{t+1}\le \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \left(P_t/\alpha\rho + c_p\right)\left(E_{t+1}/\eta_t\right)\right) \tag{D16}$$

(2) When $0 \le q_t < \alpha w_t$, by optimizing the function (D15), subject to $E_{t+1} \in [\underline{E}, \bar{E}]$, we can

derive the response function (i.e., first-order derivative) as follows:

$$\frac{\partial\left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]-\left((P_t\rho+s)/\alpha\right)\cdot\left(E_{t+1}/\eta_t\right)\right)}{\partial E_{t+1}}=\frac{\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}}-\left(\frac{(P_t\rho+s)}{\alpha}+c_p\right)\frac{1}{\eta_t}]$$

The second-order derivative: $\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]\big/\partial E_{t+1}^{2}<0$.

Since the second-order derivative is negative, we can find the unique optimal solutions through the first-order condition. We also will get the following optimal results:

$$E_{t+1}^{(2\text{-}s)*}=\arg\max_{\underline{E}\le E_{t+1}\le\bar{E}}\left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]-\left((P_t\rho+s)/\alpha+c_p\right)\left(E_{t+1}/\eta_t\right)\right)\qquad\text{(D17)}$$

(3) When $q_t<0$, by optimizing the function (D15), subject to $E_{t+1}\in[\underline{E},\bar{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\frac{\partial\left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]-P_t\beta\rho\left(E_{t+1}/\eta_t\right)\right)}{\partial E_{t+1}}=\frac{\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}}-\frac{P_t\beta\rho-c_g}{\eta_t}$$

The second-order derivative: $\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]\big/\partial E_{t+1}^{2}<0$.

Similarly, we can find the unique optimal solutions through the first-order condition. We also will get the following optimal results:

$$E_{t+1}^{(3\text{-}s)*}=\arg\max_{\underline{E}\le E_{t+1}\le\bar{E}}\left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]-\left(P_t\beta\rho-c_g\right)\left(E_{t+1}/\eta_t\right)\right)\qquad\text{(D18)}$$

2)The relations among three reference points:

(1) Recall the proof 1), for the state at $t\in\{1,2,\cdots,T\}$,there have the following two equations:

$$\begin{cases}E_{t+1}^{(1\text{-}s)*}=\arg\max_{\underline{E}\le E_{t+1}\le\bar{E}}\left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]-\left(P_t/\alpha\rho+c_p\right)\left(E_{t+1}/\eta_t\right)\right)\\ E_{t+1}^{(2\text{-}s)}=\arg\max_{\underline{E}\le E_{t+1}\le\bar{E}}\left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)]-\left((P_t\rho+s)/\alpha+c_p\right)\left(E_{t+1}/\eta_t\right)\right)\end{cases}$$

Then, we can get the following inequations:

$$\left.\begin{array}{l}\text{(a) } \left(E[V_{t+1}(E_{t+1}^{(1\text{-}s)*}, P_{t+1}) \mid (E_t, P_t)] - \left(P_t/\alpha\rho + c_p\right) E_{t+1}^{(1\text{-}s)*}/\eta_t\right) \geq \\ \text{(b) } \left(E[V_{t+1}(E_{t+1}^{(2\text{-}s)*}, P_{t+1}) \mid (E_t, P_t)] - \left(P_t/\alpha\rho + c_p\right) E_{t+1}^{(2\text{-}s)*}/\eta_t\right) \\ \text{(c) } \left(E[V_{t+1}(E_{t+1}^{(2\text{-}s)*}, P_{t+1}) \mid (E_t, P_t)] - \left((P_t\rho + s)/\alpha + c_p\right) E_{t+1}^{(2\text{-}s)*}/\eta_t\right) \geq \\ \text{(d) } \left(E[V_{t+1}(E_{t+1}^{(1\text{-}s)*}, P_{t+1}) \mid (E_t, P_t)] - \left((P_t\rho + s)/\alpha + c_p\right) E_{t+1}^{(1\text{-}s)*}/\eta_t\right)\end{array}\right\} \quad \text{(D19)}$$

Based on the above inequations, we can get the relationship of $(a)-(d) \geq (b)-(c)$.

That is, for any given current state $S(t) = S_t(E_t, w_t, P_t) \in \hat{E} \times \hat{W} \times \hat{P}$, we will get:

$$\left((P_t/\alpha\rho + c_p) - \left((P_t\rho + s)/\alpha + c_p\right)\right)[E_{t+1}^{(2\text{-}s)*}/\eta_t - E_{t+1}^{(1\text{-}s)*}/\eta_t] \geq 0$$

Here, we have the following relation:

a) $P_t/\alpha\rho \geq (P_t\rho + s)/\alpha \geq 0 \Leftrightarrow P_t \geq (P_t\rho + s)\rho \Leftrightarrow P_t - P_t\rho^2 \geq s\rho \Leftrightarrow s \leq P_t(1-\rho^2)/\rho$;

b) $0 \leq P_t/\alpha\rho \leq (P_t\rho + s)/\alpha \Leftrightarrow 0 \leq P_t \leq (P_t + s)\rho^2 \Leftrightarrow P_t - P_t\rho^2 \leq s\rho \Leftrightarrow s \geq P_t(1-\rho^2)/\rho$

Therefore, we will obtain the following relationship:

1) For positive electricity price $P_t \geq 0$, if $s \leq P_t(1-\rho^2)/\rho$, there is $E_{t+1}^{(1\text{-}s)*} \leq E_{t+1}^{(2\text{-}s)*}$.

2) For positive electricity price $P_t \geq 0$, if $s \geq P_t(1-\rho^2)/\rho$, we will get $E_{t+1}^{(1\text{-}s)*} \geq E_{t+1}^{(2\text{-}s)*}$.

Obviously, if $P_t < 0$, we will get $s \geq 0 \geq P_t(1-\rho^2)/\rho$.

So, for any $s \geq 0$, we will get $E_{t+1}^{(1\text{-}s)*} \geq E_{t+1}^{(2\text{-}s)*}$ for all the negative electricity price.

(2) Recall the proof 1), we also have the following two equations:

$$\begin{cases} E_{t+1}^{(2\text{-}s)*} = \arg\max\limits_{\underline{E} \leq E_{t+1} \leq \bar{E}} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - \left((P_t\rho + s)/\alpha + c_p\right) \cdot (E_{t+1}/\eta_t)\right) \\ E_{t+1}^{(3\text{-}s)*} = \arg\max\limits_{\underline{E} \leq E_{t+1} \leq \bar{E}} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - \left(P_t\beta\rho - c_g\right) \cdot (E_{t+1}/\eta_t)\right) \end{cases}$$

Then, we can get the following inequations:

$$\left.\begin{array}{l}\text{(e) } \left(E[V^*_{t+1}(E^{(2\text{-}s)*}_{t+1},P_{t+1})\,|\,(E_t,P_t)] - \left((P_t\rho+s)/\alpha + c_p\right)\cdot\left(E^{(2\text{-}s)*}_{t+1}/\eta_t\right)\right) \geq \\ \text{(f) } \left(E[V^*_{t+1}(E^{(3\text{-}s)*}_{t+1},P_{t+1})\,|\,(E_t,P_t)] - \left((P_t\rho+s)/\alpha + c_p\right)\cdot\left(E^{(3\text{-}s)*}_{t+1}/\eta_t\right)\right) \\ \text{(g) } E[V^*_{t+1}(E^{(3\text{-}s)*}_{t+1},P_{t+1})\,|\,(E_t,P_t)] - \left(P_t\beta\rho - c_g\right)\cdot\left(E^{(3\text{-}s)*}_{t+1}/\eta_t\right) \geq \\ \text{(h) } E[V^*_{t+1}(E^{(2\text{-}s)*}_{t+1},P_{t+1})\,|\,(E_t,P_t)] - \left(P_t\beta\rho - c_g\right)\cdot\left(E^{(2\text{-}s)*}_{t+1}/\eta_t\right)\end{array}\right\} \quad \text{(D20)}$$

Obviously, there is $(e)-(h) \geq (f)-(g)$, that is

$$0 \geq \left((P_t\beta\rho - c_g) - \left((P_t\rho+s)/\alpha + c_p\right)\right)\cdot\left(E^{(3\text{-}s)*}_{t+1}/\eta_t - E^{(2\text{-}s)*}_{t+1}/\eta_t\right)$$

For any $P_t \geq 0$, Since there is $0<\rho\leq 1, 0<\alpha\leq 1, 0<\beta\leq 1$, $c_g > 0$, and $c_p > 0$, so, $\left(P_t\beta\rho - c_g\right) < \left((P_t\rho+s)/\alpha + c_p\right)$ hold.

Therefore, we will get the relationship $E^{(2\text{-}s)*}_{t+1} \leq E^{(3\text{-}s)*}_{t+1}$.

Thus, we draw the relationship for positive prices $P_t \geq 0$ and $s \leq P_t(1-\rho^2)/\rho$:

$$E^{(1\text{-}s)*}_{t+1} \leq E^{(2\text{-}s)*}_{t+1} \leq E^{(3\text{-}s)*}_{t+1} \quad \text{(D21)}$$

If $P_t \leq 0$, we have $\left((P_t\beta\rho - c_g) - \left((P_t\rho+s)/\alpha + c_p\right)\right)\cdot\left(E^{(3\text{-}s)*}_{t+1}/\eta_t - E^{(2\text{-}s)*}_{t+1}/\eta_t\right) \leq 0$.

Here, we have the following relation:

a) $\left(P_t\beta\rho - c_g\right) - \left(\dfrac{(P_t\rho+s)}{\alpha} + c_p\right) \geq 0 \Leftrightarrow \left(P_t\beta\rho - c_g\right)\alpha \geq (P_t\rho+s) + \alpha c_p \Leftrightarrow s \leq (\alpha\beta-1)\rho P_t - \alpha\left(c_g + c_p\right)$;

b) $\left(P_t\beta\rho - c_g\right) - \left(\dfrac{(P_t\rho+s)}{\alpha} + c_p\right) \leq 0 \Leftrightarrow \left(P_t\beta\rho - c_g\right)\alpha \leq (P_t\rho+s) + \alpha c_p \Leftrightarrow s \geq (\alpha\beta-1)\rho P_t - \alpha\left(c_g + c_p\right)$.

For the negative price if $s \geq 0$ and $s \leq \left(\alpha\beta-1\right)\rho P_t - \alpha\left(c_g + c_p\right)$ the following relation: hold, we can draw

$$(\alpha\beta-1)\rho P_t - \alpha\left(c_g + c_p\right) \geq 0 \Rightarrow (\alpha\beta-1)\rho P_t \geq \alpha\left(c_g + c_p\right) \Rightarrow P_t \leq -\alpha\left(c_g + c_p\right)\Big/\left((1-\alpha\beta)\rho\right)$$

Therefore, we will obtain the following relationship:

1) For $P_t < -\alpha(c_g + c_p)/((1-\alpha\beta)\rho)$, if $0 < s \le (\alpha\beta - 1)\rho P_t - \alpha(c_g + c_p)$, there is $E_{t+1}^{(2\text{-}s)*} \ge E_{t+1}^{(3\text{-}s)*}$.

2) For $P_t < -\alpha(c_g + c_p)/((1-\alpha\beta)\rho)$, if $s \ge (\alpha\beta - 1)\rho P_t - \alpha(c_g + c_p)$, we get $E_{t+1}^{(2\text{-}s)*} \le E_{t+1}^{(3\text{-}s)*}$.

To sum up, for negative prices $P_t < -\alpha(c_g + c_p)/(1-\alpha\beta)\rho$ and $0 < s \le (\alpha\beta - 1)\rho P_t - \alpha(c_g + c_p)$:

$$E_{t+1}^{(1\text{-}s)*} \ge E_{t+1}^{(2\text{-}s)*} \ge E_{t+1}^{(3\text{-}s)*} \tag{D22}$$

In this way, if there have $-\alpha(c_g + c_p)/((1-\alpha\beta)\rho) \le P_t < 0$ and $s \ge 0$, we will get $E_{t+1}^{(2\text{-}s)*} \le E_{t+1}^{(3\text{-}s)*}$.

（3）Recall the proof 1), we also have the following two equations:

$$\begin{cases} E_{t+1}^{(1\text{-}s)*} = \arg\max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - \left(P_t/\alpha\rho + c_p\right)\left(E_{t+1}/\eta_t\right)\right) \\ E_{t+1}^{(3\text{-}s)*} = \arg\max\limits_{\underline{E} \le E_{t+1} \le \bar{E}} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - \left(P_t\beta\rho - c_g\right)\left(E_{t+1}/\eta_t\right)\right) \end{cases}$$

Then, we can get the following inequations:

$$\left.\begin{array}{l} \text{(i)} \left(E[V_{t+1}(E_{t+1}^{(1\text{-}s)*}, P_{t+1}) \mid (E_t, P_t)] - \left(P_t/\alpha\rho + c_p\right)\left(E_{t+1}^{(1\text{-}s)*}/\eta_t\right)\right) \ge \\ \text{(j)} \left(E[V_{t+1}(E_{t+1}^{(3\text{-}s)*}, P_{t+1}) \mid (E_t, P_t)] - \left(P_t/\alpha\rho + c_p\right)\left(E_{t+1}^{(3\text{-}s)*}/\eta_t\right)\right) \\ \text{(k)} \ E[V_{t+1}^*(E_{t+1}^{(3\text{-}s)*}, P_{t+1}) \mid (E_t, P_t)] - \left(P_t\beta\rho - c_g\right)\left(E_{t+1}^{(3\text{-}s)*}/\eta_t\right) \ge \\ \text{(l)} \ E[V_{t+1}^*(E_{t+1}^{(1\text{-}s)*}, P_{t+1}) \mid (E_t, P_t)] - \left(P_t\beta\rho - c_g\right)\left(E_{t+1}^{(1\text{-}s)*}/\eta_t\right) \end{array}\right\} \tag{D23}$$

Obviously, there is $(i) - (l) \ge (j) - (k)$, that is

$$0 \ge \left((P_t\beta\rho - c_g) - (P_t/\alpha\rho + c_p + c_p)\right)\left(E_{t+1}^{(3\text{-}s)*}/\eta_t - E_{t+1}^{(1\text{-}s)*}/\eta_t\right)$$

For $P_t \geq 0$, $\left(P_t/\alpha\rho + c_p\right) \geq \left(P_t\beta\rho - c_g\right)$ holds due to $0<\rho\leq 1, 0<\alpha\leq 1, 0<\beta\leq 1$, $c_g > 0$, and $c_p > 0$. Therefore, we will get the relationship $E_{t+1}^{(1\text{-}s)*} \leq E_{t+1}^{(3\text{-}s)*}$.

To sum up, we can draw the relationship for positive prices $P_t \geq 0$ and $s \geq P_t(1-\rho^2)/\rho$:

$$E_{t+1}^{(2\text{-}s)*} \leq E_{t+1}^{(1\text{-}s)*} \leq E_{t+1}^{(3\text{-}s)* } \tag{D24}$$

Obviously, if $\left((P_t\beta\rho - c_g) - (\frac{P_t}{\alpha\rho} + c_p)\right) \geq 0 \Leftrightarrow (\beta\rho - \frac{1}{\alpha\rho})P_t \geq c_g + c_p \Rightarrow P_t \leq -\frac{(c_g + c_p)}{(1/\alpha\rho - \beta\rho)}$ holds, there is $E_{t+1}^{(1-s)*} \geq E_{t+1}^{(3-s)*}$. And if $-\frac{(c_g + c_p)}{(1/\alpha\rho - \beta\rho)} \leq P_t \leq 0$, we still have $E_{t+1}^{(1\text{-}s)*} \leq E_{t+1}^{(3\text{-}s)*}$.

Therefore, we can draw the following results:

For positive electricity prices $P_t \geq 0$

$$\begin{cases} \text{If } s \leq P_t(1-\rho^2)/\rho, \text{there is } E_{t+1}^{(1\text{-}s)*} \leq E_{t+1}^{(2\text{-}s)*} \leq E_{t+1}^{(3\text{-}s)*} \\ \text{If } s \geq P_t(1-\rho^2)/\rho, \text{there is } E_{t+1}^{(2\text{-}s)*} \leq E_{t+1}^{(1\text{-}s)*} \leq E_{t+1}^{(3\text{-}s)*} \end{cases} \tag{D25}$$

For negative electricity prices $-(c_g + c_p)/(1/\alpha\rho - \beta\rho) \leq P_t < 0$.

$$\left\{ \text{If } s > 0, \text{we will get } E_{t+1}^{(2\text{-}s)*} \leq E_{t+1}^{(1\text{-}s)*} \leq E_{t+1}^{(3\text{-}s)*} \right. \tag{D26}$$

For negative electricity prices $P_t < -(c_g + c_p)/(1/\alpha\rho - \beta\rho)$.

$$\begin{cases} \text{1) If } 0 < s \leq (\alpha\beta - 1)\rho P_t - \alpha\left(c_g + c_p\right), \text{we will get } E_{t+1}^{(1\text{-}s)*} \geq E_{t+1}^{(2\text{-}s)*} \geq E_{t+1}^{(3\text{-}s)*} \\ \text{2) If } s \geq (\alpha\beta - 1)\rho P_t - \alpha\left(c_g + c_p\right), \text{we will get } E_{t+1}^{(1\text{-}s)*} \geq E_{t+1}^{(3\text{-}s)*} \geq E_{t+1}^{(2\text{-}s)*} \end{cases} \tag{D27}$$

<u>Proof of Proposition 6.3: Production Tax Credit (PTC) analysis</u>

For the electricity merchant with storage and wind farm considering production tax credit (PTC), recall the proof the Lemma 6.1, for any given state $S(t)$, and we can get the following results:

$$\begin{cases} E^{(1\text{-}s)*}_{t+1(s>0)} = \arg\max_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{-\left(P_t/\alpha\rho + c_p\right)\left(E_{t+1}/\eta_t - E_t\right) + E[V^*_{t+1}(S(t+1)\,|\,S(t)]\right\} \\ E^{(2\text{-}s)*}_{t+1(s>0)} = \arg\max_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{-\left((P_t\rho + s)/\alpha + c_p\right)\left(E_{t+1}/\eta_t - E_t\right) + sw_t + E[V^*_{t+1}(S(t+1)\,|\,S(t)]\right\} \\ E^{(3\text{-}s)*}_{t+1(s>0)} = \arg\max_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{-\left(P_t\beta\rho - c_g\right)\left(E_{t+1}/\eta_t - E_t\right) + sw_t + E[V^*_{t+1}(S(t+1)\,|\,S(t)]\right\} \end{cases}$$

If the electricity merchants ignore the production tax credit (PTC) in trading decisions (i.e., traditional study), we have the following results:

$$\begin{cases} E^{(1\text{-}s)*}_{t+1(s=0)} = \arg\max_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{-\left(P_t/\alpha\rho + c_p\right)\left(E_{t+1}/\eta_t - E_t\right) + E[V^*_{t+1}(S(t+1)\,|\,S(t)]\right\} \\ E^{(2\text{-}s)*}_{t+1(s=0)} = \arg\max_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{-\left(P_t\rho/\alpha + c_p\right)\left(E_{t+1}/\eta_t - E_t\right) + E[V^*_{t+1}(S(t+1)\,|\,S(t)]\right\} \\ E^{(3\text{-}s)*}_{t+1(s=0)} = \arg\max_{\underline{E}\le E_{t+1}\le \bar{E}} \left\{-\left(P_t\beta\rho - c_g\right)\left(E_{t+1}/\eta_t - E_t\right) + E[V^*_{t+1}(S(t+1)\,|\,S(t)]\right\} \end{cases} \qquad \text{(D28)}$$

By using the payoff rewards function (D1), there have the following relations:

$$\frac{R^{(s)}(q_t, w_t, P_t)}{\partial s} = \begin{cases} 0 & (q_t > \alpha g_t) \\ -(q_t/\alpha - w_t) \ge 0 & (0 \le q_t < \alpha g_t) \\ w_t \ge 0 & (q_t < 0) \end{cases}$$

Then, for positive electricity prices and state $t \in \{1, 2, 3, \cdots, T\}$, we will get

$$\partial R^{(s)}(q_t, w_t, P_t)/\partial s \ge 0 \qquad \text{(D29)}$$

Thus, we will get the following relations: $\sum_{t=1}^{T} R^{(s)}(q^*_t, w_t, P_t)\Big|_{s\ge 0} \ge \sum_{t=1}^{T} R^{(s)}(q^*_t, w_t, P_t)\Big|_{s=0}$,

and the value function of $V^*_{t+1(s>0)}(S(t+1)\,|\,S(t))$ increases with the PTC (s), then we will get the following relations:

$$E[V^*_{t+1(s\ge 0)}\left(S(t+1)\,|\,S(t)\right)] \ge E[V^*_{t+1(s=0)}\left(S(t+1)\,|\,S(t)\right)] \qquad \text{(D30)}$$

In this way, we will get

$$\max_{\pi}\sum_{t=1}^{T} E\left[R^{(s)}(q_t, w_t, P_t)_{(s\geq 0)} \mid S(1)\right] \geq \max_{\pi}\sum_{t=1}^{T} E\left[R^{(s)}(q_t, w_t, P_t)_{(s=0)} \mid S(1)\right] \qquad \text{(D31)}$$

<u>Proof of Lemma 6.2</u>

1) The uniqueness of the optimal results:

The current payoff rewards are shown as follows for the merchant when the merchant can receive PTC s per wind generation sold to the market:

$$R^{(PTC)}(q_t, w_t, P_t) = \begin{cases} -P_t(q_t/\alpha - w_t)\cdot\rho - s(q_t/\alpha - w_t) - c_w w_t - c_p q_t & (0 \leq q_t \leq \alpha w_t) \\ -P_t\cdot(q_t\beta - w_t)\cdot\rho - c_w w_t - s(q_t\beta - w_t) + c_g q_t & (q_t < 0) \end{cases} \qquad \text{(D32)}$$

Where, q_t is the energy/inventory change from period t to period t+1 before accounting for energy loss. Recall the proof for Lemma 6.1 in Appendix A, we will get the following rewards function at time t.

$$\begin{aligned} R^{(PTC)}(q_t, w_t, P_t) &= \begin{cases} -P_t(q_t/\alpha - w_t)\cdot\rho - s(q_t/\alpha - w_t) - c_w w_t - c_p q_t & (0 \leq q_t \leq \alpha w_t) \\ -P_t\cdot(q_t\beta - w_t)\cdot\rho - c_w w_t - s(q_t\beta - w_t) + c_g q_t & (q_t < 0) \end{cases} \\ &= \begin{cases} -(P_t\rho + s)\, q_t/\alpha + P_t w_t\cdot\rho + s w_t - c_w w_t - c_p q_t & (0 \leq q_t \leq \alpha w_t) \\ -(P_t\rho + s) q_t\beta + (P_t\rho + s) w_t - c_w w_t + c_g q_t & (q_t < 0) \end{cases} \end{aligned} \qquad \text{(D33)}$$

Similarly, recall the proof for Lemma 6.1, we can achieve the following optimal SOC results:

$$\begin{cases} E_{t+1}^{(2\text{-}PTC)*} = \arg\max_{\underline{E}\leq E_{t+1}\leq \bar{E}} \left(E[V_{t+1}^{*}(S(t+1) \mid S(t)] - (P_t\rho + s)/\alpha\cdot E_{t+1}/\eta_t - c_p\cdot E_{t+1}/\eta_t \right) \\ \text{or } \partial E[V_{t+1}^{*}(S(t+1) \mid S(t)]/\partial E_{t+1} - \left((P_t\rho + s)/\alpha + c_p\right)/\eta_t \Big|_{E_{t+1}=E_{t+1}^{(2\text{-}PTC)*}} = 0 \end{cases} \qquad \text{(D34)}$$

and

$$\begin{cases} E_{t+1}^{(3\text{-PTC})*} = \arg\max_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - (P_t\rho + s)\beta \cdot \left(E_{t+1}/\eta_t\right) + c_g \cdot E_{t+1}/\eta_t \right) \\ \text{or } \partial E[V_{t+1}^*(S(t+1) \mid S(t)]/\partial E_{t+1} - \left((P_t\rho + s)\beta - c_g\right)/\eta_t \Big|_{E_{t+1}=E_{t+1}^{(3\text{-PTC})*}} = 0 \end{cases} \quad \text{(D35)}$$

The relations among two reference points:

（1）Recall the proof 1), we can have the following two equations:

$$\begin{cases} E_{t+1}^{(2-\text{PTC})*} = \arg\max_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - \left((P_t\rho + s)/\alpha + c_p\right) \cdot \left(E_{t+1}/\eta_t\right)\right) \\ E_{t+1}^{(3-\text{PTC})*} = \arg\max_{\underline{E} \le E_{t+1} \le \overline{E}} \left(E[V_{t+1}^*(S(t+1) \mid S(t)] - \left((P_t\rho + s)\beta - c_g\right) \cdot \left(E_{t+1}/\eta_t\right)\right) \end{cases}$$

Then, we can get the following inequations:

$$\left.\begin{array}{l} (a')\ \left(E[V_{t+1}^*(E_{t+1}^{(2-\text{PTC})*}, P_{t+1}) \mid (E_t, P_t)] - \left((P_t\rho + s)/\alpha + c_p\right)\left(E_{t+1}^{(2-\text{PTC})*}/\eta_t\right)\right) \ge \\ (b')\ \left(E[V_{t+1}^*(E_{t+1}^{(3-\text{PTC})*}, P_{t+1}) \mid (E_t, P_t)] - \left((P_t\rho + s)/\alpha + c_p\right) \cdot \left(E_{t+1}^{(3-\text{PTC})*}/\eta_t\right)\right) \\ (c')\ \left(E[V_{t+1}^*(E_{t+1}^{(3-\text{PTC})*}, P_{t+1}) \mid (E_t, P_t)] - \left((P_t\rho + s)\beta - c_g\right) \cdot \left(E_{t+1}^{(3-\text{PTC})*}/\eta_t\right)\right) \ge \\ (d')\ \left(E[V_{t+1}^*(E_{t+1}^{(2\text{-PTC})*}, P_{t+1}) \mid (E_t, P_t)] - \left((P_t\rho + s)\beta - c_g\right)\left(E_{t+1}^{(2-\text{PTC})*}/\eta_t\right)\right) \end{array}\right\}$$

Obviously, there is $(a') - (d') \ge (b') - (c')$, that is

$$\left(((P_t\rho + s)\beta - c_g) - ((P_t\rho + s)/\alpha + c_p)\right)(E_{t+1}^{(2-\text{PTC})*}/\eta_t - E_{t+1}^{(3-\text{PTC})*}/\eta_t) \ge 0$$

For positive prices $P_t \ge 0$, there is $((P_t\rho + s)\beta - c_g) - ((P_t\rho + s)/\alpha + c_p) < 0$, then we will get

$$E_{t+1}^{(2-\text{PTC})*} \le E_{t+1}^{(3-\text{PTC})*}.$$

For $P_t \le 0$, if $\left(((P_t\rho + s)\beta - c_g) - ((P_t\rho + s)/\alpha + c_p)\right)(E_{t+1}^{(2-\text{PTC})*}/\eta_t - E_{t+1}^{(3-\text{PTC})*}/\eta_t) \ge 0$ holds.

$$\begin{cases} \left((P_t\rho + s)\beta - c_g\right) - \left(\dfrac{(P_t\rho + s)}{\alpha} + c_p\right) \ge 0 \Leftrightarrow s \le \dfrac{P_t\rho(\alpha\beta - 1) - \alpha\left(c_g + c_p\right)}{(1-\alpha\beta)} \Leftrightarrow 0 > P_t \ge -\dfrac{s(1-\alpha\beta) + \alpha\left(c_g + c_p\right)}{\rho(1-\alpha\beta)} \\ \left((P_t\rho + s)\alpha\beta - (P_t\rho + s)\right) - \alpha\left(c_g + c_p\right) \le 0 \Leftrightarrow s \ge \dfrac{P_t\rho(\alpha\beta - 1) - \alpha\left(c_g + c_p\right)}{(1-\alpha\beta)} \Leftrightarrow P_t \le -\dfrac{s(1-\alpha\beta) + \alpha\left(c_g + c_p\right)}{\rho(1-\alpha\beta)} < 0 \end{cases}$$

Therefore, we will get the following relationship:

If $P_t \geq -\left(\alpha(c_g + c_p) + s(1-\beta\alpha)\right) / \left(\rho(1-\beta\alpha)\right)$,there is

$$E_{t+1}^{(2-PTC)*} \leq E_{t+1}^{(3-PTC)*} \quad \text{(D36)}$$

If $P_t \leq -\left(\alpha\left(c_g + c_p\right) + s(1-\beta\alpha)\right) / \left(\rho(1-\beta\alpha)\right) < 0$ there is

$$E_{t+1}^{(2-PTC)*} \geq E_{t+1}^{(3-PTC)*} \quad \text{(D37)}$$

For Positive price, we will get the following results:

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(2-PTC)*} - E_t, \overline{Q}^p\},\ E_t \in [0, E_{t+1}^{(2-PTC)*}], \\ \text{(store renewable without buying up to } E_{t+1}^{(2-PTC)*}\text{);} \\ 0, E_t \in (E_{t+1}^{(2-PTC)*}, E_{t+1}^{(3)*}] \text{ (keep SOC unchanged);} \\ \max\{E_{t+1}^{(3-PTC)*} - E_t, -\overline{Q}^g\}, E_t \in (E_{t+1}^{(3-PTC)*}, \overline{E}]; \\ \text{(discharge and sell renewable down to } E_{t+1}^{(3-PTC)*}\text{).} \end{cases} \quad \text{(D38)}$$

Cases Study

In this case, we assume there are three time periods (T=3). At each period, the power price takes one of the values in set $P_t = \{p^M, p^L, p^H\} = \{6,3,10\}$. We also assume the storage energy capacity cannot refill it fully or sell it empty in one time period, but fewer than two time periods. In detail, when the full (resp. empty) storage can be emptied (resp. filled up) in more than one period but fewer than two periods, it holds that $\underline{E} + \overline{Q}^p \leq \overline{E}$ (resp. $\overline{E} - \underline{E} > \overline{Q}^g$) and $\underline{E} + 2\overline{Q}^p \geq \overline{E}$ (resp. $\overline{E} - \underline{E} \leq 2\overline{Q}^g$). We assume the storage capacity is 10 (i.e., $\underline{E} = 0, \overline{E} = 10$), the generating/discharging max capacity is 12 and the pumping/charging max capacity is 7. Let the operating cost be one(i.e., $c_p = c_g = 1$),wind generation cost (i.e., $c_w = 0$), the pumping and generating efficiencies be 0.9 (i.e.,

$\alpha=\beta=0.9=\rho$), self-discharging and transmission efficiencies be one (i.e., $\eta=1$), the wind generation are $w_t=\{3,5,0\}$.

Using the above related data, we will show the method to get the optimal solution based on the optimal actions proposed in propositions 6.1 and 6.2. In order to compare with the traditional study that without considering the PTC, the optimal results under three different PTC credit rates (i.e., $s=\{0,1,3\}$) under two initial SOC (i.e., $E_1=\{1,5\}$).

Case 1(Policy 1): When the PTC s=3.

In stage 4:

$$VOE_4=(6+3+10)/3=6.33 \Rightarrow V_4=VOE_4\cdot E_4=6.33E_4$$

Plug in the data, we will get the following optimal references points:

$$\begin{cases} E_4^{(1\text{-}s)*}=\arg\max\limits_{\underline{E}\le E_4\le\overline{E}}\left\{-\left(P_3/\alpha\rho+c_p\right)\left(E_4/\eta_3\right)+E[V_4^*(S(4)\,|\,S(3))]\right\} \\ \qquad =\arg\max\limits_{\underline{E}\le E_4\le\overline{E}}\left\{-\left(10/0.81+1\right)E_4+6.33E_4\right\}=\arg\max\limits_{\underline{E}\le E_4\le\overline{E}}\left\{-7.0157E_4\right\}\Rightarrow E_4^{(1\text{-}s)*}=0 \\ E_4^{(2\text{-}s)*}=\arg\max\limits_{\underline{E}\le E_4\le\overline{E}}\left\{-\left((P_3\rho+s)/\alpha+c_p\right)\left(E_4/\eta_3\right)+E[V_4^*(S(4)\,|\,S(3))]\right\} \\ \qquad =\arg\max\limits_{\underline{E}\le E_4\le\overline{E}}\left\{-\left((10\times0.9+3)/0.9+1\right)E_4+6.33E_4\right\}=\arg\max\limits_{\underline{E}\le E_4\le\overline{E}}\left\{-8.0033E_4\right\}\Rightarrow E_4^{(2\text{-}s)*}=0 \\ E_4^{(3\text{-}s)*}=\arg\max\limits_{\underline{E}\le E_3\le\overline{E}}\left\{-\left(P_3\beta\rho-c_g\right)\left(E_4/\eta_3\right)+E[V_4^*(S(4)\,|\,S(3))]\right\} \\ \qquad =\arg\max\limits_{\underline{E}\le E_4\le\overline{E}}\left\{-\left(10\times0.81-1\right)E_4+6.33E_4\right\}=\arg\max\limits_{\underline{E}\le E_4\le\overline{E}}\left\{-0.77E_4\right\}\Rightarrow E_4^{(3\text{-}s)*}=0 \end{cases} \tag{D39}$$

In Stage 3:

Action 3: Release power and make the storage level down to $\underline{E}=0=E_4^*$, thus, $V_4^*=0$

$$q_3^*(S_3)=-E_3,\ E_3\in(0,\overline{E}], \tag{D40}$$

(sell energy and make SOC down to 0 as close as possible)

The reward function at stage3 is shown as

$$R_3^{(s)} = -P_3 \cdot (q_3\beta - w_3) \cdot \rho - c_w w_3 + s w_3 + c_g q_3$$
$$= -10 \cdot ((-E_3)0.9 - 0) \cdot 0.9 + (-E_3) = 7.1E_3 \ \ (q_3 < 0) \quad \text{(D41)}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_3^* = \max\{R_3^{(s)} + V_4^*\} = 7.1E_3 \quad \text{(D42)}$$

In stage 2:

By using the equations (D16), (D17), and (D18), we will get the following results for merchants:

$$\begin{cases} E_3^{(1\text{-}s)*} = \arg\max\limits_{\underline{E} \le E_3 \le \overline{E}} \left(V_3^* - \left(\dfrac{P_2}{\alpha\rho} + c_p \right) \dfrac{E_3}{\eta_2} \right) = \arg\max\limits_{\underline{E} \le E_3 \le \overline{E}} \left(7.1E_3 - \left(\dfrac{3}{0.9 \times 0.9} + 1 \right) \dfrac{E_3}{\eta_2} \right) \\ = \arg\max\limits_{\underline{E} \le E_3 \le \overline{E}} \left(2.3963E_3 \right) \Rightarrow E_3^{(1\text{-}s)*} = 10; \\ E_3^{(2\text{-}s)} = \arg\max\limits_{\underline{E} \le E_3 \le \overline{E}} \left(V_3^* - \left(\dfrac{P_2\rho + s}{\alpha} + c_p \right) \dfrac{E_3}{\eta_2} \right) = \arg\max\limits_{\underline{E} \le E_3 \le \overline{E}} \left(7.1E_3 - \left(\dfrac{3 \times 0.9 + 3}{0.9} + 1 \right) \dfrac{E_3}{\eta_2} \right) \\ = \arg\max\limits_{\underline{E} \le E_3 \le \overline{E}} \left(-0.23E_3 \right) \Rightarrow E_3^{(2\text{-}s)*} = 0; \\ E_3^{(3\text{-}s)*} = \arg\max\limits_{\underline{E} \le E_3 \le \overline{E}} \left(V_3^* - \left(P_2\beta\rho - c_g \right) \dfrac{E_3}{\eta_2} \right) = \arg\max\limits_{\underline{E} \le E_3 \le \overline{E}} \left(7.1E_3 - \left(3 \times 0.9 \times 0.9 - 1 \right) \dfrac{E_3}{\eta_2} \right) \\ = \arg\max\limits_{\underline{E} \le E_3 \le \overline{E}} \left(5.67E_3 \right) \Rightarrow E_3^{(3\text{-}s)*} = 10. \end{cases} \quad \text{(D43)}$$

Thus, we will get the optimal reference points at stage 3 that are:

$$E_3^{(1\text{-}s)*} = 10, E_3^{(2\text{-}s)*} = 0, E_3^{(3\text{-}s)*} = 10\,. \quad \text{(D44)}$$

Since there are $s \ge P_2(1-\rho^2)/\rho$, $\alpha w_2 \ge \min\{E_{t+1}^{(2\text{-}s)*}, \overline{Q}^p\}$, and $E_{t+1}^{(1)*} \ge E_{t+1}^{(1)*} - \alpha w_t \ge E_{t+1}^{(2\text{-}s)*}$. The optimal actions at stage 2 are shown as

$$q_t^*(S_t)=\begin{cases}\min\{E_{t+1}^{(2\text{-}s)*}-E_t,\overline{Q}^p\}, E_t\in(0,E_{t+1}^{(2\text{-}s)*}],\\ \text{(store renewable and without buy up to } E_{t+1}^{(2\text{-}s)*});\\ \min\{E_{t+1}^{(1\text{-}s)*}-E_t,\overline{Q}^p\}, E_t\in(E_{t+1}^{(2\text{-}s)*},E_{t+1}^{(1\text{-}s)*}-\alpha w_t],\\ \text{(store renewable and purchased electricity up to } E_{t+1}^{(1\text{-}s)*});\\ 0, E_t\in(E_{t+1}^{(1\text{-}s)*}-\alpha w_t, E_{t+1}^{(3\text{-}s)*}] \text{ (keep SOC unchanged)};\\ \max\{E_{t+1}^{(3\text{-}s)*}-E_t,-\overline{Q}^g\}, E_t\in(E_{t+1}^{(3\text{-}s)*},\overline{E}],\\ \text{(generate and sell renewable down to } E_{t+1}^{(3\text{-}s)*}).\end{cases} \quad \text{(D45)}$$

$$\Rightarrow q_2^*(S_2)=\begin{cases}7, & E_2\in(0,3]\\ 10-E_2, & E_2\in(3,5.5]\\ 0, & E_2\in(5.5,10]\end{cases}$$

The reward payoff functions at stage 2 are shown as follows:

$$R_2^{(s)}=\begin{cases}-P_2\cdot(q_2/\alpha-w_2)/\rho-c_w w_2-c_p q_2\\ -P_2\cdot(q_2/\alpha-w_2)\cdot\rho-s(q_2/\alpha-w_2)-c_w w_2-c_p q_2\end{cases}$$

$$=\begin{cases}-3\cdot(7/0.9-5)/0.9-7, & E_2\in(3,5.5]\\ -3\cdot((10-E_2)/0.9-5)/0.9-(10-E_2), & E_2\in(3,5.5]\\ -3\cdot(0/0.9-5)\cdot0.9-3(0/0.9-5)-0, & E_2\in(5.5,10]\end{cases} \quad \text{(D46)}$$

$$=\begin{cases}-16.2593, & E_2\in(0,3]\\ 4.7037E_2-30.3704, & E_2\in(3,5.5]\\ 28.5, & E_2\in(5.5,10]\end{cases}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_2^*=\max\{R_2^{(s)}+V_3^*\}=\begin{cases}7.1E_3-16.2593_{|E_3=E_2+7}\\ 7.1E_3+4.7037E_2-30.3704_{|E_3=10}\\ 7.1E_3+28.5_{|E_3=E_2}\end{cases}=\begin{cases}7.1E_2+33.4407, & E_2\in(0,3]\\ 4.7037E_2+40.6296, & E_2\in(0,5.5]\\ 7.1E_2+28.5, & E_2\in(5.5,10]\end{cases}$$

(D47)

In Stage 1:

By using the equations (D16), (D17), and (D18), we get the following results:

$$\begin{cases} E_2^{(1\text{-s})*} = \arg\max\limits_{\underline{E}\le E_2\le\overline{E}}\left(V_2^* - \left(\dfrac{P_1}{\alpha\rho}+c_p\right)\dfrac{E_2}{\eta_1}\right) = \arg\max\limits_{\underline{E}\le E_2\le\overline{E}}\left(V_2^* - \left(\dfrac{6}{0.9\times 0.9}+1\right)\dfrac{E_2}{\eta_1}\right) \\ \quad = \arg\max\limits_{\underline{E}\le E_2\le\overline{E}}\left(V_2^* - 8.4074E_2 + 28.5\right); \\ E_2^{(2\text{-s})} = \arg\max\limits_{\underline{E}\le E_2\le\overline{E}}\left(V_2^* - \left(\dfrac{P_1\rho+s}{\alpha}+c_p\right)\dfrac{E_2}{\eta_1}\right) = \arg\max\limits_{\underline{E}\le E_2\le\overline{E}}\left(V_2^* - \left(\dfrac{6\times 0.9+3}{0.9}+1\right)\dfrac{E_2}{\eta_1}\right) \\ \quad = \arg\max\limits_{\underline{E}\le E_2\le\overline{E}}\left(V_2^* - 10.3233E_2 + 28.5\right); \\ E_2^{(3\text{-s})*} = \arg\max\limits_{\underline{E}\le E_2\le\overline{E}}\left(V_2^* - \left(P_1\beta\rho - c_g\right)\dfrac{E_2}{\eta_1}\right) = \arg\max\limits_{\underline{E}\le E_2\le\overline{E}}\left(V_2^* - \left(6\times 0.9\times 0.9-1\right)\dfrac{E_2}{\eta_1}\right) \\ \quad = \arg\max\limits_{\underline{E}\le E_2\le\overline{E}}\left(V_2^* - 3.86E_2 + 28.5\right). \end{cases} \tag{D48}$$

(1) Scenario1: If $E_2 \in (0,3]$, there is $V_2^* = 7.1E_2 + 33.4407$.

$$\begin{cases} E_2^{(1\text{-s})*} = \arg\max\limits_{E_2\in[0,\ 3]}\left(7.1E_2 + 33.4407 - \left(6/0.81+1\right)E_2/\eta_1\right) \\ \quad = \arg\max\limits_{E_2\in[0,\ 3]}\left(-1.3074E_2 + 33.4407\right) \Rightarrow E_2^{(1\text{-s})*} = 0; \\ E_2^{(2\text{-s})} = \arg\max\limits_{E_2\in[0,\ 3]}\left(7.1E_2 + 33.4407 - \left(\left(6\times 0.9+3\right)/0.9+1\right)E_2/\eta_1\right) \\ \quad = \arg\max\limits_{E_2\in[0,\ 3]}\left(-3.2233E_2 + 33.4407\right) \Rightarrow E_2^{(2\text{-s})*} = 0; \\ E_2^{(3\text{-s})*} = \arg\max\limits_{E_2\in[0,\ 3]}\left(7.1E_2 + 33.4407 - \left(6\times 0.9\times 0.9-1\right)E_2/\eta_1\right) \\ \quad = \arg\max\limits_{E_2\in[0,\ 3]}\left(3.24E_2 + 33.4407\right) \Rightarrow E_2^{(3\text{-s})*} = 3. \end{cases} \tag{D49}$$

(2) Scenario2: If $E_2 \in (3,5.5]$, there is $V_2^* = 4.7037E_2 + 40.6296$.

$$\begin{cases} E_2^{(1\text{-s})*} = \arg\max_{E_2\in[3,5.5]}\left(4.7037E_2 + 40.6296 - (6/0.81+1)E_2/\eta_1\right) \\ \quad = \arg\max_{E_2\in[3,5.5]}\left(-3.7037E_2 + 40.6296\right) \Rightarrow E_2^{(1\text{-s})*} = 3; \\ E_2^{(2\text{-s})} = \arg\max_{E_2\in[3,5.5]}\left(4.7037E_2 + 40.6296 - \left((6\times0.9+3)/0.9+1\right)E_2/\eta_1\right) \\ \quad = \arg\max_{E_2\in[3,5.5]}\left(-5.6296E_2 + 40.6296\right) \Rightarrow E_2^{(2\text{-s})*} = 3; \\ E_2^{(3\text{-s})*} = \arg\max_{E_2\in[3,5.5]}\left(4.7037E_2 + 40.6296 - (6\times0.9\times0.9-1)E_2/\eta_1\right) \\ \quad = \arg\max_{E_2\in[3,5.5]}\left(0.8437E_2 + 40.6296\right) \Rightarrow E_2^{(3\text{-s})*} = 5.5. \end{cases} \tag{D50}$$

(3) Scenario3: If $E_2 \in (5.5,10]$, there is $V_2^* = 7.1E_2 + 28.5$.

$$\begin{cases} E_2^{(1\text{-s})*} = \arg\max_{E_2\in[5.5,10]}\left(7.1E_2 + 28.5 - (6/0.81+1)E_2/\eta_1\right) \\ \quad = \arg\max_{E_2\in[5.5,10]}\left(-1.3074E_2 + 28.5\right) \Rightarrow E_2^{(1\text{-s})*} = 5.5; \\ E_2^{(2\text{-s})} = \arg\max_{E_2\in[5.5,10]}\left(7.1E_2 + 28.5 - \left((6\times0.9+3)/0.9+1\right)E_2/\eta_1\right) \\ \quad = \arg\max_{E_2\in[5.5,10]}\left(-3.2233E_2 + 28.5\right) \Rightarrow E_2^{(2\text{-s})*} = 5.5; \\ E_2^{(3\text{-s})*} = \arg\max_{E_2\in[5.5,10]}\left(7.1E_2 + 28.5 - (6\times0.9\times0.9-1)E_2/\eta_1\right) \\ \quad = \arg\max_{E_2\in[5.5,10]}\left(3.24E_2 + 28.5\right) \Rightarrow E_2^{(3\text{-s})*} = 10. \end{cases} \tag{D51}$$

Next, we will choose the optimal references point between the above three scenarios.

(4) Compare $E_2^{(1\text{-s})*}$

$$\left\{\begin{array}{l} 1)\begin{cases} \text{If } \ E_2 \in [0,\ 3] \Rightarrow E_2^{(1\text{-s})*} = 0 \\ \max\limits_{E_2 \in [0,\ 3]} \left(-1.3074E_2 + 33.4407\right) \Rightarrow -1.3074E_2 + 33.4407 \,|_{E_2^{(1\text{-s})*}=0} = 33.4407 \end{cases} \\ 2)\begin{cases} \text{If } \ E_2 \in [3,\ 5.5] \Rightarrow E_2^{(1\text{-s})*} = 3 \\ \max\limits_{E_2 \in [3,5.5]} \left(-3.7037E_2 + 40.6296\right) \Rightarrow -3.7037E_2 + 40.6296 \,|_{E_2^{(1\text{-s})*}=3} = 29.5185 \end{cases} \\ 3)\begin{cases} \text{If } \ E_2 \in [5.5,\ 10] \Rightarrow E_2^{(1\text{-s})*} = 5.5 \\ \max\limits_{E_2 \in [5.5,10]} \left(-1.3074E_2 + 28.5\right) \Rightarrow -1.3074E_2 + 28.5 \,|_{E_2^{(1\text{-s})*}=5.5} = 21.3093 \end{cases} \end{array}\right\} \qquad \text{(D52)}$$

$$\Rightarrow E_2^{(1\text{-s})*} = 0$$

(5) Compare $E_2^{(2\text{-s})*}$

$$\left\{\begin{array}{l} 1)\begin{cases} \text{If } \ E_2 \in [0,\ 3] \Rightarrow E_2^{(2\text{-s})*} = 0 \\ E_2^{(2\text{-s})*} = \arg\max\limits_{E_2 \in [0,\ 3]} \left(-3.2233E_2 + 33.4407\right) \Rightarrow -3.2233E_2 + 33.4407 \,|_{E_2^{(2\text{-s})*}=0} = 33.4407 \end{cases} \\ 2)\begin{cases} \text{If } \ E_2 \in [3,\ 5.5] \Rightarrow E_2^{(2\text{-s})*} = 3 \\ E_2^{(2\text{-s})*} = \arg\max\limits_{E_2 \in [3,5.5]} \left(-5.6296E_2 + 40.6296\right) \Rightarrow -5.6296E_2 + 40.6296 \,|_{E_2^{(2\text{-s})*}=3} = 23.7408 \end{cases} \\ 3)\begin{cases} \text{If } \ E_2 \in [5.5,\ 10] \Rightarrow E_2^{(2\text{-s})*} = 5.5 \\ E_2^{(2\text{-s})*} = \arg\max\limits_{E_2 \in [5.5,10]} \left(-3.2233E_2 + 28.5\right) \Rightarrow -3.2233E_2 + 28.5 \,|_{E_2^{(2\text{-s})*}=5.5} = 10.77185 \end{cases} \end{array}\right\} \qquad \text{(D53)}$$

$$\Rightarrow E_2^{(2\text{-s})*} = 0$$

(6) Compare $E_2^{(3\text{-s})*}$

$$\left|\begin{array}{l} 1)\begin{cases} \text{If } \ E_2 \in [0,\ 3] \Rightarrow E_2^{(3\text{-s})*} = 3 \\ E_2^{(3\text{-s})*} = \arg\max\limits_{E_2 \in [0,\ 3]} \left(3.24E_2 + 33.4407\right) \Rightarrow 3.24E_2 + 33.4407 \,|_{E_2^{(3\text{-s})*}=3} = 43.1607 \end{cases} \\ 3)\begin{cases} \text{If } \ E_2 \in [3,\ 5.5] \Rightarrow E_2^{(3\text{-s})*} = 5.5 \\ E_2^{(3\text{-s})*} = \arg\max\limits_{E_2 \in [3,5.5]} \left(0.8437E_2 + 40.6296\right) \Rightarrow 0.8437E_2 + 40.6296 \,|_{E_2^{(3\text{-s})*}=5.5} = 45.27 \end{cases} \\ 3)\begin{cases} \text{If } \ E_2 \in [5.5,\ 10] \Rightarrow E_2^{(3\text{-s})*} = 10 \\ E_2^{(3\text{-s})*} = \arg\max\limits_{E_2 \in [5.5,10]} \left(3.24E_2 + 28.5\right) \Rightarrow 3.24E_2 + 28.5 \,|_{E_2^{(3\text{-s})*}=10} = 60.9 \end{cases} \end{array}\right\} \qquad \text{(D54)}$$

$$\Rightarrow E_2^{(3\text{-s})*} = 10$$

Thus, we will get the optimal reference points at stage 2 that are shown as:

$$E_2^{(1\text{-}s)*} = E_2^{(2\text{-}s)*} = 0, E_2^{(3\text{-}s)*} = 10 \tag{D55}$$

Similarly, because there are $s \geq P_1(1-\rho^2)/\rho$, $\alpha w_1 \geq \min\{E_{t+1}^{(2\text{-}s)*}, \overline{Q}^p\}$, and $E_{t+1}^{(1)*} - \alpha w_t \leq E_{t+1}^{(2\text{-}s)*} \leq E_{t+1}^{(1)*}$, the optimal actions at stage 1 are shown as

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(2\text{-}s)*} - E_t, \alpha w_t, \overline{Q}^p\}, E_t \in (0, E_{t+1}^{(2)*}], \\ \text{(store renewable and without buy up to } E_{t+1}^{(2\text{-}s)*}); \\ 0, E_t \in (E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(3\text{-}s)*}] \text{ (keep SOC unchanged)}; \\ \max\{E_{t+1}^{(3\text{-}s)*} - E_t, -\overline{Q}^g\}, E_t \in (E_{t+1}^{(3\text{-}s)*}, \overline{E}], \\ \text{(generate and sell renewable down to } E_{t+1}^{(3\text{-}s)*}). \end{cases} \tag{D56}$$

$$\Rightarrow q_1^*(S_1) = 0, E_t \in (E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(3\text{-}s)*}] \Leftrightarrow q_1^*(S_1) = 0, E_1 \in (0, \overline{E}] \text{(keep SOC unchanged)}$$

The reward payoff functions at stage 1 are shown as follows:

$$\begin{aligned} R_1^{(s)} &= -P_1 \cdot (q_1/\alpha - w_1) \cdot \rho - s(q_1/\alpha - w_1) - c_w w_1 - c_p q_1 \\ &= -6 \cdot (0/0.9 - 3) \cdot 0.9 - 3(0/0.9 - 3) = 25.2, \qquad E_1 \in (0, \overline{E}] \end{aligned} \tag{D57}$$

Therefore, we will get the following optimal value functions at stage 1/initial stage.

$$V_1^* = \max\{R_1^{(s)} + V_2^*\} = \begin{cases} 7.1E_1 + 58.6407, & E_1 \in (0,3] \\ 4.7037E_1 + 65.8296, & E_1 \in (0,5.5] \\ 7.1E_1 + 53.7, & E_1 \in (5.5,10] \end{cases} \tag{D58}$$

The corresponding optimal actions are shown:

In stage 1, $q_1^*(S_1) = 0, \quad E_1 \in (0, \overline{E}]$ (keep SOC unchanged)

In stage 2, $q_2^*(S_2) = \{7, \ E_2 \in (0,3]; \ 10 - E_2, \ E_2 \in (3,5.5]; \ 0, \ E_2 \in (5.5,10]$

In stage 3,

$q_3^*(S_3) = -E_3, E_3 \in (0, \overline{E}]$(sell energy and make SOC down to 0 as close as possible)

To sum up, we get the following results:

If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, (action: sell generation 3 to the market, and keep SOC unchanged), then we will get $E_2 = 1$ (i.e., $q_1^* = 0, R_1 = 25.2$);

Stage 2: If $E_2 = 1$, (action: store renewable and purchased electricity up to $\overline{Q}^p$), then, we will get $E_3 = 8$ (i.e., $q_2^* = 7, R_2 = -16.2593$);

Stage 3: If $E_3 = 8$,(action: generating and selling), then, we have $E_4 = 0$ (i.e., $q_3^* = -8, R_3 = 56.8$).

Based on the forecasted price, total rewards are $R = R_1 + R_2 + R_3 = 65.7407 = V_1^*$.

If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$,(action 1: do nothing), then, the relation of $E_2 = 5$ (i.e., $q_1^* = 0, R_1 = 25.2$) holds;

Stage 2: If $E_2 = 5$,(action 2: store renewable and purchased electricity up to $E_{t+1}^{(1-s)*}$), then there exists $E_3 = 10$ (i.e., $q_2^* = 5, R_2 = -6.8519$);

Stage 3: If $E_3 = 10$,(action 3: generating and selling), then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -10, R_3 = 71$).

Therefore, total rewards in three periods are $R = R_1 + R_2 + R_3 = 89.3481 = V_1^*$ if $E_1 = 5$.

Case 2 (Policy 1): When PTC s=1

Recall the proof of Case 1, we will get the following results:

$$\begin{cases} E_4^{(1\text{-}s)*} = E_4^{(2\text{-}s)*} = E_4^{(3\text{-}s)*} = 0 \\ E_3^{(1\text{-}s)*} = E_3^{(2\text{-}s)*} = E_3^{(3\text{-}s)*} = 10 \\ E_2^{(1\text{-}s)*} = E_2^{(2\text{-}s)*} = 0, E_2^{(3\text{-}s)*} = 10 \end{cases} \tag{D59}$$

The corresponding optimal value function at stage 1 are shown:

$$V_1^* = \max\{R_1^{(s)} + V_2^*\} = \begin{cases} 7.1E_1 + 52.6407, & E_1 \in [0,3] \\ 4.7037E_1 + 59.8293, & E_1 \in [3,5.5] \\ 5.11E_1 + 57.589, & E_1 \in [5.5,3] \end{cases} \tag{D60}$$

In stage 1, $q_1^*(S_1) = 0$, if $E_1 \in (0,\overline{E}]$(keep SOC unchanged)

In stage 2, $q_2^*(S_2) = \{7, \text{ if } E_2 \in [0,3]; \; 10 - E_2, \text{ if } E_2 \in [3,10]\}$

In stage 3,

$q_3^*(S_3) = -E_3$, if $E_3 \in (0,\overline{E}]$(sell energy and make SOC down to 0 as close as possible)

If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, then we will get $E_2 = 1$ (i.e., $q_1^* = 0$, $R_1 = 19.2$);

Stage 2: If $E_2 = 1$, then, we will get $E_3 = 8$ (i.e., $q_2^* = 7, R_2 = -16.2593$);

Stage 3: If $E_3 = 8$, then, we have $E_4 = 0$ (i.e., $q_3^* = -8, R_3 = 56.8$).

Based on the forecasted price, total rewards are $R = R_1 + R_2 + R_3 = 59.7407 = V_1^*$.

If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$, then, the relation of $E_2 = 5$ (i.e., $q_1^* = 0, R_1 = 19.2$) holds;

Stage 2: If $E_2 = 5$, then there exists $E_3 = 10$ (i.e., $q_2^* = 5$, $R_2 = -6.8522$);

Stage 3: If $E_3 = 10$, then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -10, R_3 = 71$).

Therefore, total rewards in three periods are $R = R_1 + R_2 + R_3 = 83.3478 = V_1^*$ if $E_1 = 5$.

Case 3 (Policy 1): When PTC s=0

Recall the proof of Case 1, we will get the following results:

$$\begin{cases} E_4^{(1\text{-s})*} = E_4^{(2\text{-s})*} = E_4^{(3\text{-s})*} = 0 \\ E_3^{(1\text{-s})*} = E_3^{(2\text{-s})*} = E_3^{(3\text{-s})*} = 10 \\ E_2^{(1\text{-s})*} = 0, E_2^{(2\text{-s})*} = 3, E_2^{(3\text{-s})*} = 10 \end{cases} \tag{D61}$$

The corresponding optimal value function at stage 1 are shown:

$$V_1^* = \begin{cases} 7.1E_1 + 49.9107, & E_1 \in [0, 0.3] \\ 7E_1 + 49.9407, & E_1 \in [0.3, 3] \\ 4.7037E_1 + 56.8293, & E_1 \in [3, 5.5] \\ 4E_1 + 60.7, & E_1 \in [5.5, 10] \end{cases} \tag{D62}$$

The optimal decisions are shown as follows:

In stage 1, $q_1^*(S_1) = \{2.7, \text{ if } E_1 \in [0, 0.3);\ \ 3 - E_1, \text{ if } E_1 \in (0, 0.3];\ \ 0, \text{ if } E_1 \in (3, 10]\}$;

In stage 2, $q_2^*(S_2) = \{7, \quad \text{if } E_2 \in [0, 3]; \quad 10 - E_2, \quad \text{if } E_2 \in [3, 10]\}$

In stage 3,

$q_3^*(S_3) = -E_3, \text{ if } E_3 \in (0, \overline{E}]$(sell energy and make SOC down to 0 as close as possible)

If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, ($q_1^* = 2$, $R_1 = 2.2$); Stage 2: If $E_2 = 3$, ($q_2^* = 7, R_2 = -16.2593$);

Stage 3: If $E_3 = 10$, ($q_3^* = -10, R_3 = 71$).

Based on the forecasted price, total rewards are $R = R_1 + R_2 + R_3 = 56.9407 = V_1^*$.

If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$, ($q_1^* = 0, R_1 = 16.2$); Stage 2: If $E_2 = 5$, ($q_2^* = 5$, $R_2 = -6.8522$);

Stage 3: If $E_3 = 10$, then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -10$, $R_3 = 71$).

Therefore, total rewards in three periods are $R = R_1 + R_2 + R_3 = 80.3478 = V_1^*$ if $E_1 = 5$.

Case 4(Policy 2): When the PTC s=3.

In stage 4:

$$VOE_4 = (6+3+10)/3 = 6.33 \Rightarrow V_4 = VOE_4 \cdot E_4 = 6.33E_4$$

Plug in the data, we will get the following optimal references points:

$$\begin{cases} E_4^{(2\text{-PTC})*} = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -\left((P_3\rho + s)/\alpha + c_p \right)\left(E_4/\eta_3 \right) + E[V_4^*(S(4) \mid S(3))] \right\} \\ \quad = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -\left((10*0.9+3)/0.9+1 \right) E_4 + 6.33E_4 \right\} \\ \quad = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -8.0033E_4 \right\} \Rightarrow E_4^{(2\text{-PTC})*} = 0 \\ E_4^{(3\text{-PTC})*} = \arg\max_{\underline{E} \le E_3 \le \overline{E}} \left\{ -\left((P_3\rho + s)\beta - c_g \right)\left(E_4/\eta_3 \right) + E[V_4^*(S(4) \mid S(3))] \right\} \\ \quad = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -\left((10*0.9+3)*0.9-1 \right) E_4 + 6.33E_4 \right\} \\ \quad = \arg\max_{\underline{E} \le E_4 \le \overline{E}} \left\{ -3.47E_4 \right\} \Rightarrow E_4^{(3\text{-PTC})*} = 0 \end{cases} \tag{D63}$$

In Stage 3:

Action 3: Release power and make the storage level down to $\underline{E} = 0 = E_4^*$, thus, $V_4^* = 0$.

$$q_3^*(S_3) = -E_3, E_3 \in (0, \overline{E}], \tag{D64}$$

(sell energy and make SOC down to 0 as close as possible)

The reward function at stage3 is shown as

$$\begin{aligned} R_3^{(PTC)} &= -P_3 \cdot (q_3\beta - w_3) \cdot \rho - c_w w_3 - s(q_3\beta - w_3) + c_g q_3 \qquad (q_3 < 0) \\ &= -10 \cdot \left((-E_3)0.9 - 0 \right) \cdot 0.9 - 3 \times \left((-E_3)0.9 - 0 \right) + (-E_3) = 9.8E_3 \end{aligned} \tag{D65}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_3^* = \max\{R_3^{(PTC)} + V_4^*\} = 9.8E_3 \tag{D66}$$

In stage 2:

By using the equations (D3) and (D4), we have the following results for merchants:

$$\begin{cases} E_3^{(2\text{-PTC})} = \arg\max_{\underline{E}\le E_3\le \overline{E}}\left(V_3^* - \left((P_2\rho+s)/\alpha + c_p\right)\left(E_3/\eta_2\right)\right) \\ \quad = \arg\max_{\underline{E}\le E_3\le \overline{E}}\left(9.8E_3 - \left(\frac{3\times 0.9+3}{0.9}+1\right)\frac{E_3}{\eta_2}\right) = \arg\max_{\underline{E}\le E_3\le \overline{E}}\left(2.47E_3\right) \\ E_3^{(3\text{-PTC})*} = \arg\max_{\underline{E}\le E_3\le \overline{E}}\left(V_3^* - \left((P_2\rho+s)\beta - c_g\right)\frac{E_3}{\eta_2}\right) \\ \quad = \arg\max_{\underline{E}\le E_3\le \overline{E}}\left(9.8E_3 - \left((3*0.9+3)*0.9-1\right)\frac{E_3}{\eta_2}\right) = \arg\max_{\underline{E}\le E_3\le \overline{E}}\left(5.67E_3\right) \end{cases} \tag{D66}$$

Thus, we will get the optimal reference points at stage 3 that are:

$$E_3^{(2\text{-PTC})*} = E_3^{(3\text{-PTC})*} = 10\,. \tag{D67}$$

The optimal actions at stage 2 are shown as

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(2-\text{PTC})*} - E_t, \overline{Q}^p, \alpha w_t\},\ E_t \in [0, E_{t+1}^{(2-\text{PTC})*}], \\ \text{(store renewable bring SOC up to } E_{t+1}^{(2-\text{PTC})*}\text{);} \\ 0, E_t \in (E_{t+1}^{(2-\text{PTC})*}, E_{t+1}^{(3)*}] \text{ (keep SOC unchanged);} \\ \max\{E_{t+1}^{(3-\text{PTC})*} - E_t, -\overline{Q}^g\}, E_t \in (E_{t+1}^{(3-\text{PTC})*}, \overline{E}], \\ \text{(discharge make SOC down to } E_{t+1}^{(3-\text{PTC})*}\text{).} \end{cases} \tag{D68}$$

$$\Rightarrow q_2^*(S_2) = \min\{10 - E_2, 4.5\} \text{ if } E_2 \in (0,10] \Leftrightarrow q_2^*(S_2) = \begin{cases} 4.5, & E_2 \in (0,5.5] \\ 10 - E_2, & E_2 \in (3,5.5] \end{cases}$$

The reward payoff functions at stage 2 are shown as follows:

$$\begin{aligned} R_2^{(\text{PTC})} &= -P_2\cdot(q_2/\alpha - w_2)\cdot\rho - s(q_2/\alpha - w_2) - c_w w_2 - c_p q_2 \\ &= \begin{cases} -3\cdot(4.5/0.9-5)\cdot 0.9 - 3\cdot(4.5/0.9-5) - 4.5, & E_2 \in (0,5.5] \\ -3\cdot\left(\frac{10-E_2}{0.9} - 5\right)\cdot 0.9 - 3\cdot\left(\left(\frac{10-E_2}{0.9} - 5\right)\right) - \left(10 - E_2\right), & E_2 \in (5.5,10] \end{cases} \\ &= \begin{cases} -4.5, & E_2 \in (0,5.5] \\ 7.33E_2 - 44.8, & E_2 \in (5.5,10] \end{cases} \end{aligned} \tag{D69}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_2^* = \max\{R_2^{(PTC)} + V_3^*\} = \begin{cases} 9.8E_2 + 39.6, & E_2 \in (0,5.5] \\ 7.33E_2 + 53.2, & E_2 \in (5.5,10] \end{cases} \tag{D70}$$

In Stage 1:

By using the equations (D3) and (D4), we have the following results for merchants:

$$\begin{cases} E_2^{(2\text{-PTC})} = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left(V_2^* - \left(\frac{P_1\rho + s}{\alpha} + c_p \right) \frac{E_2}{\eta_1} \right) = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left(V_2^* - \left(\frac{6*0.9+3}{0.9} + 1 \right) \frac{E_2}{\eta_1} \right) \\ \quad = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left(V_2^* - 10.33E_2 \right) \\ E_2^{(3\text{-PTC})*} = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left(V_2^* - \left((P_1\rho + s)\beta - c_g \right) \frac{E_2}{\eta_1} \right) = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left(V_2^* - \left((6 \times 0.9 + 3) \cdot 0.9 - 1 \right) \frac{E_2}{\eta_1} \right) \\ \quad = \arg\max_{\underline{E} \le E_2 \le \overline{E}} \left(V_2^* - 6.56E_2 \right) \end{cases} \tag{D71}$$

(1) scenario1: If $E_2 \in (0,5.5]$, $V_2^* = 9.8E_2 + 39.6$,

$$\begin{cases} E_2^{(2\text{-PTC})} = \arg\max_{E_2 \in [0,5.5]} \left(V_2^* - 10.33E_2 \right) = \arg\max_{E_2 \in [0,5.5]} \left(9.8E_2 + 39.6 - 10.33E_2 \right) \\ \quad = \arg\max_{E_2 \in [0,5.5]} \left(-0.53E_2 + 39.6 \right) \Rightarrow E_2^{(2\text{-PTC})*} = 0 \\ E_2^{(3\text{-PTC})*} = \arg\max_{E_2 \in [0,5.5]} \left(V_2^* - 6.56E_2 \right) = \arg\max_{E_2 \in [0,5.5]} \left(9.8E_2 + 39.6 - 6.56E_2 \right) \\ \quad = \arg\max_{E_2 \in [0,5.5]} \left(3.24E_2 + 39.6 \right) \Rightarrow E_2^{(3\text{-PTC})*} = 5.5 \end{cases} \tag{D72}$$

(2) scenario2: If $E_2 \in (5.5,10]$, $V_2^* = 7.33E_2 + 53.2$.

$$\begin{cases} E_2^{(2\text{-PTC})} = \arg\max_{E_2 \in [5.5,10]} \left(V_2^* - 10.33E_2 \right) = \arg\max_{E_2 \in [5.5,10]} \left(7.33E_2 + 53.2 - 10.33E_2 \right) \\ \quad = \arg\max_{E_2 \in [5.5,10]} \left(-3E_2 + 53.2 \right) \Rightarrow E_2^{(2\text{-PTC})*} = 5.5 \\ E_2^{(3\text{-PTC})*} = \arg\max_{E_2 \in [5.5,10]} \left(V_2^* - 6.56E_2 \right) = \arg\max_{E_2 \in [5.5,10]} \left(7.33E_2 + 53.2 - 6.56E_2 \right) \\ \quad = \arg\max_{E_2 \in [5.5,10]} \left(0.77E_2 + 53.2 \right) \Rightarrow E_2^{(3\text{-PTC})*} = 10 \end{cases} \tag{D72}$$

Next, pick up the optimal references point between the above two scenarios.

(3) Compare $E_2^{(2\text{-PTC})*}$

$$\left\{\begin{array}{l} 1)\begin{cases} \text{If} \quad E_2 \in [0,\ 5.5] \Rightarrow E_2^{(2\text{-PTC})*} = 0 \\ E_2^{(2\text{-PTC})*} = \arg\max_{E_2 \in [0,5.5]} \left(-0.53E_2 + 39.6\right) \Rightarrow -0.53E_2 + 39.6\,|_{E_2^{(2\text{-PTC})*}=0} = 39.6 \end{cases} \\ 2)\begin{cases} \text{If} \quad E_2 \in [5.5,\ 10] \Rightarrow E_2^{(2\text{-PTC})*} = 5.5 \\ E_2^{(2\text{-PTC})*} = \arg\max_{E_2 \in [5.5,10]} \left(-3E_2 + 53.2\right) \Rightarrow -3E_2 + 53.2\,|_{E_2^{(2\text{-PTC})*}=5.5} = 36.2 \end{cases} \end{array}\right\} \quad \text{(D73)}$$

$$\Rightarrow E_2^{(2\text{-PTC})*} = 0$$

(4) Compare $E_2^{(3\text{-PTC})*}$

$$\left\{\begin{array}{l} 1)\begin{cases} \text{If} \quad E_2 \in [0,\ 5.5] \Rightarrow E_2^{(3\text{-PTC})*} = 5.5 \\ E_2^{(3\text{-PTC})*} = \arg\max_{E_2 \in [0,5.5]} \left(3.24E_2 + 39.6\right) \Rightarrow 3.24E_2 + 39.6\,|_{E_2^{(3\text{-PTC})*}=5.5} = 57.42 \end{cases} \\ 2)\begin{cases} \text{If} \quad E_2 \in [5.5,\ 10] \Rightarrow E_2^{(3\text{-PTC})*} = 10 \\ E_2^{(3\text{-PTC})*} = \arg\max_{E_2 \in [5.5,10]} \left(0.77E_2 + 53.2\right) \Rightarrow 0.77E_2 + 53.2\,|_{E_2^{(3\text{-PTC})*}=10} = 60.9 \end{cases} \end{array}\right\} \quad \text{(D74)}$$

$$\Rightarrow E_2^{(3\text{-PTC})*} = 10$$

Thus, we will get the optimal reference points at stage 2 that are shown as:

$$E_2^{(2\text{-PTC})*} = 0,\ E_2^{(3\text{-PTC})*} = 10 \quad \text{(D75)}$$

The optimal actions at stage 1 are shown as

$$q_t^*(S_t) = \begin{cases} \min\{E_{t+1}^{(2-\text{PTC})*} - E_t, \overline{Q}^p\},\ E_t \in [0, E_{t+1}^{(2-\text{PTC})*}], \\ \text{(store renewable bring SOC up to } E_{t+1}^{(2-\text{PTC})*}); \\ 0, E_t \in (E_{t+1}^{(2-\text{PTC})*}, E_{t+1}^{(3)*}] \text{ (keep SOC unchanged)}; \Rightarrow q_1^*(S_1) = 0, E_1 \in (0,10] \\ \max\{E_{t+1}^{(3-\text{PTC})*} - E_t, -\overline{Q}^g\}, E_t \in (E_{t+1}^{(3-\text{PTC})*}, \overline{E}], \\ \text{(discharge make SOC down to } E_{t+1}^{(3-\text{PTC})*}), \end{cases} \quad \text{(C40)}$$

The reward payoff functions at stage 1 are shown as follows:

$$\begin{aligned} R_1^{(\text{PTC})} &= -P_1 \cdot (q_1/\alpha - w_1) \cdot \rho - s(q_1/\alpha - w_1) - c_w w_1 - c_p q_1 \\ &= -6 \cdot (0/0.9 - 3) \cdot 0.9 - 3(0/0.9 - 3) = 25.2,\ \ E_1 \in (0, \overline{E}] \end{aligned} \quad \text{(D76)}$$

Therefore, we will get the following optimal value functions at stage 1/initial stage.

$$V_1^* = \max\{R_1^{(PTC)} + V_2^*\} = \begin{cases} 9.8E_1 + 64.8, & E_1 \in (0,5.5] \\ 7.33E_1 + 78.4, & E_1 \in (5.5,10] \end{cases} \quad \text{(D77)}$$

The corresponding optimal actions are shown:

In stage 1, $q_1^*(S_1) = 0$, if $E_1 \in (0, \overline{E}]$(keep SOC unchanged)

In stage 2, $q_2^*(S_2) = \{4.5, \text{ if } E_2 \in (0,5.5]; \ 10 - E_2, \text{ if } E_2 \in (5.5,10]\}$

In stage 3, $q_3^*(S_3) = -E_3$, if $E_3 \in (0, \overline{E}]$(sell energy and make SOC down to 0)

To sum up, we get the following results:

If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, (sell wind generation 3 to the market, and keep SOC unchanged), then we will get $E_2 = 1$ (i.e., $q_1^* = 0$, $R_1 = 25.2$);

Stage 2: If $E_2 = 1$, (store all the wind generation 4.5), then, we will get $E_3 = 5.5$ (i.e., $q_2^* = 4.5, R_2 = -4.5$);

Stage 3: If $E_3 = 5.5$,(generating and selling), then, we have $E_4 = 0$ (i.e., $q_3^* = -5.5, R_3 = 53.9$).

Based on the forecasted price, total rewards are shown as $R = R_1 + R_2 + R_3 = 74.6 = V_1^*$.

If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$,(do nothing), then, $E_2 = 5$ (i.e., $q_1^* = 0$, $R_1 = 25.2$) holds;

Stage 2: If $E_2 = 5$,(store all the wind generation 4.5), then there exists $E_3 = 9.5$ (i.e., $q_2^* = 4.5$, $R_2 = -4.5$);

Stage 3: If $E_3 = 9.5$,(generating and selling), then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -9.5, R_3 = 93.1$).

Therefore, total rewards in three periods are $R = R_1 + R_2 + R_3 = 113.8 = V_1^*$ if $E_1 = 5$.

<u>Case 5 (Policy 2): When PTC s=1</u>

Recall the proof of Case 4, we will get the following results:

$$\begin{cases} E_4^{(2\text{-PTC})*} = E_4^{(3\text{-PTC})*} = 0; \\ E_3^{(2\text{-PTC})*} = E_3^{(3\text{-PTC})*} = 10; \\ E_2^{(2\text{-PTC})*} = 0, E_2^{(3\text{-PTC})*} = 10 \end{cases} \tag{D78}$$

The corresponding optimal value function at stage 1 are shown:

$$V_1^* = \max\{R_1^{(PTC)} + V_2^*\} = \begin{cases} 8E_2 + 31.5 + 19.2_{|E_2 = E_1}, \\ 5.11E_2 + 47.389 + 19.2_{|E_2 = E_1} \end{cases} = \begin{cases} 8E_1 + 50.7, \quad E_1 \in (0, 5.5] \\ 5.11E_1 + 66.589, E_1 \in (5.5, 10] \end{cases} \tag{D79}$$

In stage 1, $q_1^*(S_1) = 0$, if $E_1 \in (0, \overline{E}]$(keep SOC unchanged)

In stage 2, $q_2^*(S_2) = \{4.5, \text{ if } E_2 \in [0, 5.5]; \;\; 10 - E_2, \text{ if } E_2 \in [5.5, 10]\}$

In stage 3, $q_3^*(S_3) = -E_3$, if $E_3 \in (0, \overline{E}]$(sell energy and make SOC down to 0)

If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, $q_1^* = 0$, $R_1 = 19.2$; Stage 2: If $E_2 = 1$, $q_2^* = 4.5, R_2 = -4.5$;

Stage 3: If $E_3 = 5.5$, $q_3^* = -5.5, R_3 = 44$.

Based on the forecasted prices, total rewards are shown as $R = R_1 + R_2 + R_3 = 58.7 = V_1^*$.

If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$, ($q_1^* = 0$, $R_1 = 19.2$); Stage 2: If $E_2 = 5$, ($q_2^* = 4.5$, $R_2 = -4.5$);

Stage 3: If $E_3 = 9.5$, ($q_3^* = -9.5$, $R_3 = 76$).

Therefore, total rewards in three periods are $R = R_1 + R_2 + R_3 = 90.7 = V_1^*$ if $E_1 = 5$.

Case 6 (Policy 2): When PTC s=0

Recall the proof of Case 4, we will get the following results:

$$\begin{cases} E_4^{(2\text{-PTC})*} = E_4^{(3\text{-PTC})*} = 0; \\ E_3^{(2\text{-PTC})*} = E_3^{(3\text{-PTC})*} = 10; \\ E_2^{(2\text{-PTC})*} = 5.5, E_2^{(3\text{-PTC})*} = 10 \end{cases} \tag{D80}$$

The corresponding optimal value function at stage 1 are shown:

$$V_1^* = \{7.1E_1 + 43.92, \text{ if } E_1 \in (0, 2.8]; 7E_1 + 44.2, \text{if } E_1 \in (2.8, 5.5]; 4E_1 + 60.7, \text{if } E_1 \in (5.5, 10]\} \quad \text{(C46)}$$

In stage 1, $q_1^*(S_1) = \{2.7, \text{if } E_1 \in (0, 2.8];\ 5.5 - E_t, \text{if } E_1 \in (2.8, 5.5];\ 0, \text{if } E_1 \in (5.5, 10]\ \}$;

In stage 2, $q_2^*(S_2) = \{4.5,\ \text{ if } E_2 \in [0, 5.5];\ \ 10 - E_2,\ \text{if } E_2 \in [5.5, 10]\}$

In stage 3, $q_3^*(S_3) = -E_3$, if $E_3 \in (0, \overline{E}]$(keep SOC unchanged)

If $E_1 = 1$ (The initial SOC in the storage)

Stage 1: If $E_1 = 1$, $q_1^* = 2.7$, $R_1 = -2.7$; Stage 2: If $E_2 = 3.7$, $q_2^* = 4.5, R_2 = -4.5$;

Stage 3: If $E_3 = 8.2$, $q_3^* = -8.2, R_3 = 58.22$).

Total rewards are $R = R_1 + R_2 + R_3 = 51.02 = V_1^*$.

If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$, $q_1^* = 0.5$, $R_1 = 12.7$; Stage 2: If $E_2 = 5.5$, $q_2^* = 4.5$, $R_2 = -4.5$;

Stage 3: If $E_3 = 10$, $q_3^* = -10$, $R_3 = 71$.

Therefore, total rewards in three periods are $R = R_1 + R_2 + R_3 = 79.2 = V_1^*$ if $E_1 = 5$.

APPENDIX E.

PROOF OF SECTION 7

Proof of Proposition 7.1: Discharging and Charging cannot happen Simultaneously.

The primal problem from the perspective of ISO is shown as follows:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{p}\cdot q_{t}^{p}+c^{g}\cdot q_{t}^{g}\right)\right)$$

$$\text{s.t.}\begin{cases}0\le q_{t}^{p}\le \bar{Q}^{p},\\ 0\le q_{t}^{g}\le \bar{Q}^{g},\\ \underline{E}\le E_{t}\le \bar{E},\\ \underline{G}_{i}^{h}\le g_{it}^{h}\le \bar{G}_{i}^{h},\\ \sum_{i=1}^{M}g_{it}^{h}+q_{t}^{g}\beta-q_{t}^{p}/\alpha=D_{t},\\ E_{t}-q_{t}^{g}+q_{t}^{p}=E_{t+1},\\ q_{t}^{g}\cdot q_{t}^{p}=0\end{cases} \tag{E1}$$

The primal scheduling problem from the perspective of the merchant is shown:

$$\max\left(\sum_{t=1}^{T}\left(P_{t}\left(q_{t}^{g}\beta-q_{t}^{p}/\alpha\right)-\left(c^{p}\cdot q_{t}^{p}+c^{g}\cdot q_{t}^{p}\right)\right)\right)$$

$$\text{s.t.}\begin{cases}0\le q_{t}^{p}\le \bar{Q}^{p},\\ 0\le q_{t}^{g}\le \bar{Q}^{g},\\ q_{t}^{g}\cdot q_{t}^{p}=0\\ \underline{E}\le E_{t}\le \bar{E},\\ E_{t}-q_{t}^{g}+q_{t}^{p}=E_{t+1}.\end{cases} \tag{E2}$$

Discharging/generating and charging/pumping cannot happen simultaneously for energy storage. Hence, we have the following non-convex complementary constraints from the ISO/Battery perspective:

$$q_{t}^{p}\cdot q_{t}^{g}=0 \tag{E3}$$

After relaxing the non-convex constraints, we can rewrite the ISO model as follows:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}q_{t}^{g}+c^{p}q_{t}^{p}\right)\right)$$

$$\text{s.t.}\begin{cases}-q_t^p\le 0, q_t^p-\bar{Q}^p\le 0 & (\underline{\chi}1_t^p,\bar{\chi}1_t^p)\\ -q_t^g\le 0, q_t^g-\bar{Q}^g\le 0 & (\underline{\chi}1_t^g,\bar{\chi}1_t^g)\\ -E_t+\underline{E}\le 0, E_t-\bar{E}\le 0 & (\underline{\theta}1_t,\bar{\theta}1_t)\\ -g_{it}^h+\underline{G}_i^h\le 0, g_{it}^h-\bar{G}_i^h\le 0 & (\underline{\beta}1_{it},\bar{\beta}1_{it})\\ D_t-\left(\sum_{i=1}^{M}g_{it}^h+q_t^g\beta-q_t^p/\alpha\right)=0, & (\mu 1_t)\\ E_t-q_t^g+q_t^p=E_{t+1}, & (\gamma 1_{t+1})\\ \text{Where}, \underline{\chi}1_t^p,\underline{\chi}1_t^g,\underline{\theta}1_t,\underline{\beta}1_{it},\bar{\chi}1_t^p,\bar{\chi}1_t^g,\bar{\theta}1_t,\bar{\beta}1_{it}\ge 0;\end{cases} \tag{E4}$$

First, utilizing the primal model in (E4), we obtain the following Lagrange function after relaxing the non-convex constraint from the perspective of ISO.

$$\begin{aligned}L=&\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}q_{t}^{g}+c^{p}q_{t}^{p}\right)+\mu 1_t\left(D_t-\left(\sum_{i=1}^{M}g_{it}^h+q_t^g\beta-q_t^p/\alpha\right)\right)\\ &+\gamma 1_{t+1}\left(E_{t+1}-E_t+q_t^g-q_t^p\right)+\underline{\chi}1_t^p\cdot\left(-q_t^p\right)+\bar{\chi}1_t^p\cdot\left(q_t^p-\bar{Q}^p\right)+\underline{\chi}1_t^g\cdot\left(-q_t^g\right)+\bar{\chi}1_t^g\cdot\left(q_t^g-\bar{Q}^g\right)\\ &+\underline{\theta}1_t\cdot\left(-E_t+\underline{E}\right)+\bar{\theta}1_t\cdot\left(E_t-\bar{E}\right)+\underline{\beta}1_{it}\cdot\left(-g_{it}^h+\underline{G}_i^h\right)+\bar{\beta}1_{it}\left(g_{it}^h-\bar{G}_i^h\right)\end{aligned} \tag{E5}$$

Second, the KKT condition is used to determine whether there are sufficient conditions for such an exact relaxation. In the KKT condition for the primal problem from the perspective of ISO, the derivative of the Lagrange function concerning energy storage discharging/generating variables q_t^g must equal zero; hence the following equation holds ($\forall t\in\{1,2,\cdots,T\}$)

$$\partial L/\partial q_t^g\Big|_{q_t^g=q_t^{g*(S)}}=c^g-u1_t^*\beta+\gamma 1_{t+1}^*+\bar{\chi}1_t^{g*}-\underline{\chi}1_t^{g*}=0 \tag{E6}$$

Similarly, the optimal response function of (E5) on energy storage

charging/pumping variables $q_t^p,\forall t\in\{1,2,\cdots,T\}$ is

$$\partial L/\partial q_t^p\big|_{q_t^p=q_t^{p*(S)}} = c^p + u1_t^*/\alpha - \gamma 1_{t+1}^* + \overline{\chi}1_t^{p*} - \underline{\chi}1_t^{p*} = 0 \quad \text{(E7)}$$

Assume there exist $q_t^{g*(S)} > 0$ and $q_t^{g*(S)} > 0$ as the optimal solution of (E4) from the perspective of ISO at time t. Because of the complementing slackness conditions, $\underline{\chi}1_t^{g*} = 0$ and $\underline{\chi}1_t^{p*} = 0$ hold. Merging (E6) and (E7), we get the following equation

$$c^g + c^p + \overline{\chi}1_t^{g*} + \overline{\chi}1_t^{p*} - u1_t^*\beta + u1_t^*/\alpha = 0 \quad \text{(E8)}$$

When there are $\overline{\chi}1_t^{p*} \ge 0$ and $\overline{\chi}1_t^{g*} \ge 0$, the equation (A8) can be rewritten as

$$c^g + c^p < u1_t^*\beta - u1_t^*/\alpha = -u1_t^*(1/\alpha - \beta) \quad \text{(E9)}$$

The necessary condition for $q_t^{g*(S)} > 0$ and $q_t^{p*(S)} > 0$ is described in Eq. (E8). As a result, the sufficient condition for such exact relaxation of the complementary constraint of Eq.(E3) is

$$c^g + c^p > -u1_t^*(1/\alpha - \beta) \quad \text{(E10)}$$

Obviously, this is true when $u1_t^* > 0, \forall t \in \{1, 2, \cdots, T\}$ is holding.

As a result, for all positive electricity prices, discharging/generating and charging/pumping cannot occur at the same time for the social welfare-maximizing ISO.

Similarly, after relaxing the non-convex restriction, the primal problem from the perspective of the PSH owner of equation (E2) is rebuilt below:

$$\max \sum_{t=1}^{T} \left(P_t \cdot \left(q_t^g \beta - q_t^p / \alpha \right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p \right) \right)$$

$$\Leftrightarrow \min \left(\sum_{t \in T} \left(\left(c^g - P_t \beta \right) q_t^g + \left(c^p + P_t / \alpha \right) q_t^p \right) \right)$$

$$\text{s.t.} \begin{cases} -q_t^p \le 0, q_t^p - \bar{Q}^p \le 0 & (\underline{\chi}2_t^p, \bar{\chi}2_t^p) \\ -q_t^g \le 0, q_t^g - \bar{Q}^g \le 0 & (\underline{\chi}2_t^g, \bar{\chi}2_t^g) \\ -E_t + \underline{E} \le 0, E_t - \bar{E} \le 0 & (\underline{\theta}2_t, \bar{\theta}2_t) \\ E_t - q_t^g + q_t^p = E_{t+1}, & (\gamma 2_{t+1}) \\ \text{Where,} \left\{ \underline{\chi}2_t^p, \underline{\chi}2_t^g, \underline{\theta}2_t, \bar{\chi}2_t^p, \bar{\chi}2_t^g, \bar{\theta}2_t \right\} \ge 0. \end{cases} \tag{E11}$$

Then, based on (E11), we derive the following Lagrange functions:

$$\begin{aligned} L = \sum_{t \in T} & \left(\left(c^g - P_t \beta \right) q_t^g + \left(c^p + P_t / \alpha \right) q_t^p \right) + \gamma 2_{t+1} \left(E_{t+1} - E_t + q_t^g - q_t^p \right) \\ & + \underline{\chi}2_t^p \cdot \left(-q_t^p \right) + \bar{\chi}2_t^p \cdot \left(q_t^p - \bar{Q}^p \right) + \underline{\chi}2_t^g \cdot \left(-q_t^g \right) + \bar{\chi}2_t^g \cdot \left(q_t^g - \bar{Q}^g \right) \\ & + \underline{\theta}2_t \cdot \left(-E_t + \underline{E} \right) + \bar{\theta}2_t \cdot \left(E_t - \bar{E} \right) \end{aligned} \tag{E12}$$

The first-order derivative function of the Lagrange function (E12) on the energy storage discharging/generating variable $q_t^g, \forall t \in \{1, 2, \cdots, T\}$ must equal zero according to the KKT condition. As a result, the equation below is correct.

$$\partial L / \partial q_t^g \Big|_{q_t^g = q_t^{g*(M)}} = c^g - P_t \beta + \gamma 2_{t+1}^* + \bar{\chi}2_t^{g*} - \underline{\chi}2_t^{g*} = 0 \tag{E13}$$

Similarly, the following equation holds for energy storage charging/pumping variable $q_t^p, \forall t \in \{1, 2, \cdots, T\}$.

$$\partial L / \partial q_t^p \Big|_{q_t^p = q_t^{p*(M)}} = c^p + P_t / \alpha - \gamma 2_{t+1}^* + \bar{\chi}2_t^{p*} - \underline{\chi}2_t^{p*} = 0 \tag{E14}$$

Assume that in the optimal solutions of the primal problem from the perspective of the PSH owner, $q_t^{g*(M)} > 0$ and $q_t^{p*(M)} > 0$ exist. $\underline{\chi}2_t^{g*} = 0$ and $\underline{\chi}2_t^{p*} = 0$, on the other hand, are based on slackness conditions that are complementary. The following equation holds when Eqs. (E13) and (E14) are combined.

$$c^g - P_t\beta + c^p + P_t/\alpha + \bar{\chi}2_t^{g*} + \bar{\chi}2_t^{p*} = 0 \qquad \text{(E15)}$$

Because both $\underline{\chi}2_t^{g*} \geq 0$ and $\underline{\chi}2_t^{p*} \geq 0$ hold true, the Eq. (E15) will be rewritten as

$$c^g + c^p < -P_t/\alpha + P_t\beta = -P_t(1/\alpha - \beta) \qquad \text{(E16)}$$

The equation of (e16) describes the necessary condition for $q_t^{g*(M)} > 0$ and $q_t^{p*(M)} > 0$. Hence, the sufficient condition for the exact relaxation of the complementary constraint of (E3) is

$$c^g + c^p > -P_t(1/\alpha - \beta) \qquad \text{(E17)}$$

Obviously, the equation (E17) is always holding when the forecasted prices $P_t > 0, \forall t \in \{1, 2, \cdots, T\}$.

Therefore, for all positive electricity prices, discharging and charging cannot happen simultaneously to maximize the profit of electricity merchants.

Proof of Proposition 7.2:

(1) Primal and duality problems for ISO, storage merchant, and traditional generator from the respective of ISO:

The primal scheduling problem from perspectives of ISO who operates energy storage and traditional generators is shown as follow:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}q_{t}^{g}+c^{p}q_{t}^{p}\right)\right)$$

$$\text{s.t.}\begin{cases} q_t^p \ge 0,\;\; q_t^p \le \bar{Q}^p, & (\underline{\chi}^p{}_t, \bar{\chi}^p{}_t) \\ q_t^g \ge 0,\; q_t^g \le \bar{Q}^g, & (\underline{\chi}^g{}_t, \bar{\chi}^g{}_t) \\ E_t \ge \underline{E},\; E_t \le \bar{E}, & (\underline{\theta}_t, \bar{\theta}_t) \\ g_{it}^h \ge \underline{G}_i^h,\;\; g_{it}^h \le \bar{G}_i^h, & (\underline{\beta}_{it}, \bar{\beta}_{it}) \\ \sum_{i=1}^{M} g_{it}^h + q_t^g\beta - q_t^p/\alpha = D_t, & (\mu_t) \\ \text{start state}: E_1 - q_1^g + q_1^p = E_2, & (\gamma_2) \\ \text{middle state}: E_t - q_t^g + q_t^p = E_{t+1}, & (\gamma_{t+1}) \\ \text{end state}: E_T - q_T^g + q_T^p = E_{T+1}, & (\gamma_{T+1}) \end{cases} \tag{E18}$$

Here, $\left\{\underline{\chi}^g{}_t, \bar{\chi}^g{}_t, \underline{\chi}^p{}_t, \bar{\chi}^p{}_t, \underline{\theta}_t, \bar{\theta}_t, \underline{\beta}_{it}, \bar{\beta}_{it}, \mu_t, \gamma_2, \gamma_{t+1}, \gamma_{T+1}\right\}$ are the corresponding dual variables based on the constraints in (E18). The duality model of the fundamental problem in (E18) is as follows as a result of the application of duality theory:

$$\max\sum_{t=1}^{T}\left(\bar{Q}^p\cdot\bar{\chi}_t^p+\bar{Q}^g\cdot\bar{\chi}_t^g+\underline{E}\cdot\underline{\theta}_t+\bar{E}\cdot\bar{\theta}_t\right)+\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h\cdot\underline{\beta}_{it}+\bar{G}_i^h\cdot\bar{\beta}_{it}\right)+\sum_{t=1}^{T}\mu_t\cdot D_t-E_1\cdot\gamma_2+E_{T+1}\cdot\gamma_{T+1}$$

$$\text{s.t.}\begin{cases} \text{for } q_t^p:\; \underline{\chi}_t^p + \bar{\chi}_t^p - \mu_t/\alpha - \gamma_t = c^p, & \\ \text{for } q_t^g:\; \underline{\chi}_t^g + \bar{\chi}_t^g + \mu_t\beta + \gamma_t = c^g, & \\ \text{for } g_{it}^h:\; \underline{\beta}_{it} + \bar{\beta}_{it} + \mu_t = C_{it}^h, & \forall t \in \{1,2,\cdots,T\} \\ \text{for } E_t:\; -\gamma_t + \gamma_{t+1} + \underline{\theta}_t + \bar{\theta}_t = 0, & \\ \text{Where}, \left\{\underline{\chi}_t^p, \underline{\chi}_t^g, \underline{\theta}_t, \underline{\beta}_{it}\right\} \ge 0;\; \left\{\bar{\chi}_t^p, \bar{\chi}_t^g, \bar{\theta}_t, \bar{\beta}_{it}\right\} \le 0. & \end{cases} \tag{E19}$$

From the respective of energy storage merchant: The primal problem of electricity merchants with PSH or energy storage only is shown as follows:

$$\max\sum_{t=1}^{T}\left(P_t\cdot\left(q_t^g\beta-q_t^p/\alpha\right)-\left(c^g\cdot q_t^g+c^p\cdot q_t^p\right)\right)\Leftrightarrow\min\left(\sum_{t\in T}\left(\left(c^g-P_t\beta\right)q_t^g+\left(c^p+P_t/\alpha\right)q_t^p\right)\right)$$

$$\text{s.t.}\begin{cases} q_t^p\ge 0,\ q_t^p\le\bar{Q}^p, & (\underline{\chi}^p{}_t,\bar{\chi}^p{}_t)\\ q_t^g\ge 0,\ q_t^g\le\bar{Q}^g, & (\underline{\chi}^g{}_t,\bar{\chi}^g{}_t)\\ E_t\ge\underline{E},\ E_t\le\bar{E}, & (\underline{\theta}_t,\bar{\theta}_t)\\ \text{start state}: E_1-q_1^g+q_1^p=E_2, & (\gamma_2)\\ \text{middle state}: E_t-q_t^g+q_t^p=E_{t+1}, & (\gamma_{t+1})\\ \text{end state}: E_T-q_T^g+q_T^p=E_{T+1}, & (\gamma_{T+1})\end{cases} \tag{E20}$$

Similarly, $\left\{\underline{\chi}^g{}_t,\bar{\chi}^g{}_t,\underline{\chi}^p{}_t,\bar{\chi}^p{}_t,\underline{\theta}_t,\bar{\theta}_t,\mu_t,\gamma_2,\gamma_{t+1},\gamma_{T+1}\right\}$ are the corresponding dual variables of (E20). Therefore, the duality model of the primal problem in (E20) is obtained below:

$$\max\sum_{t=1}^{T}\left(\bar{Q}^p\cdot\bar{\chi}_t^p+\bar{Q}^g\cdot\bar{\chi}_t^g+\underline{E}\cdot\underline{\theta}_t+\bar{E}\cdot\bar{\theta}_t\right)-E_1\cdot\gamma_2+E_{T+1}\cdot\gamma_{T+1}$$

$$\text{s.t.}\begin{cases}\text{for } q_t^p\text{: } \underline{\chi}_t^p+\bar{\chi}_t^p-\gamma_t=c^p+P_t/\alpha,\\ \text{for } q_t^g\text{: } \underline{\chi}_t^g+\bar{\chi}_t^g+\gamma_t=c^g-P_t\beta, & \forall t\in\{1,2,\cdots,T\}\\ \text{for } E_t:\ -\gamma_t+\gamma_{t+1}+\underline{\theta}_t+\bar{\theta}_t=0,\\ \text{Where}, \underline{\chi}_t^p,\underline{\chi}_t^g,\underline{\theta}_t\ge 0;\bar{\chi}_t^p,\bar{\chi}_t^g,\bar{\theta}_t\le 0.\end{cases} \tag{E21}$$

From the respective of traditional generator *I* : The primal problem of merchant who operates a traditional generator is shown as follows:

$$\max\sum_{t=1}^{T}\left(P_t-C_{It}^h\right)\cdot g_{It}^h\Leftrightarrow\min\sum_{t=1}^{T}\left(C_{It}^h-P_t\right)\cdot g_{It}^h$$

$$\text{s.t.}\begin{cases} g_{It}^h\ge\underline{G}_I^h, & (\underline{\beta}_{It})\\ g_{It}^h\le\bar{G}_I^h, & (\bar{\beta}_{It})\end{cases} \tag{E22}$$

Similarly, $\left\{\underline{\beta}_{It},\bar{\beta}_{It}\right\}$ are the corresponding dual variables of (E22). Therefore, the duality model of the primal problem in (E22) is obtained below:

$$\max \sum_{t=1}^{T}\left(\underline{G}_{I}^{h}\cdot\underline{\beta}_{It}+\overline{G}_{I}^{h}\cdot\overline{\beta}_{It}\right)$$
$$\text{s.t.}\begin{cases}\underline{\beta}_{It}+\overline{\beta}_{It}=C_{It}^{h}-P_{t}\\ \underline{\beta}_{It}\geq 0,\overline{\beta}_{It}\leq 0;\end{cases} \tag{E23}$$

(2) Compare these duality problems

1) We suppose that $\left\{\underline{\chi}_{t}^{g*(1)},\overline{\chi}_{t}^{g*(1)},\underline{\chi}_{t}^{p*(1)},\overline{\chi}_{t}^{p*(1)},\gamma_{t}^{*(1)},\underline{\theta}_{t}^{*(1)},\overline{\theta}_{t}^{*(1)},\underline{\beta}_{it}^{*(1)},\overline{\beta}_{it}^{*(1)},\mu_{t}^{*}\right\}$ represents the best solutions to the duality problem from ISO's perspective (i.e., (E19)). Then, plugging these optimal results into the duality problem (E19), we get

$$\max\left(\begin{array}{l}\sum_{t=1}^{T}\left(\overline{Q}^{p}\cdot\overline{\chi}_{t}^{p*(1)}+\overline{Q}^{g}\cdot\overline{\chi}_{t}^{g*(1)}+\underline{E}\cdot\underline{\theta}_{t}^{*(1)}+\overline{E}\cdot\overline{\theta}_{t}^{*(1)}\right)+\sum_{t=1}^{T}\mu_{t}^{*}\cdot D_{t}-E_{1}\cdot\gamma_{2}^{*(1)}+E_{T+1}\cdot\gamma_{T+1}^{*(1)}\\ +\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}^{*(1)}+\overline{G}_{i}^{h}\cdot\overline{\beta}_{it}^{*(1)}\right)\end{array}\right)$$
$$\text{s.t.}\begin{cases}\text{for } q_{t}^{p}\text{: }\underline{\chi}_{t}^{p*(1)}+\overline{\chi}_{t}^{p*(1)}-\mu_{t}^{*}/\alpha-\gamma_{t}^{*(1)}=c^{p},\\ \text{for } q_{t}^{g}\text{: }\underline{\chi}_{t}^{g*(1)}+\overline{\chi}_{t}^{g*(1)}+\mu_{t}^{*}\beta+\gamma_{t}^{*(1)}=c^{g},\\ \text{for } g_{it}^{h}\text{: }\underline{\beta}_{it}^{*(1)}+\overline{\beta}_{it}^{*(1)}+\mu_{t}^{*}=C_{it}^{h}, \qquad \forall t\in\{1,2,\cdots,T\}\\ \text{for } E_{t}:-\gamma_{t}^{*(1)}+\gamma_{t+1}^{*(1)}+\underline{\theta}_{t}^{*(1)}+\overline{\theta}_{t}^{*(1)}=0,\\ \text{Where,}\left\{\underline{\chi}_{t}^{p},\underline{\chi}_{t}^{g},\underline{\theta}_{t},\underline{\beta}_{it}\right\}\geq 0;\ \left\{\overline{\chi}_{t}^{p},\overline{\chi}_{t}^{g},\overline{\theta}_{t},\overline{\beta}_{it}\right\}\leq 0.\end{cases} \tag{E24}$$

Obviously, $\left\{\underline{\chi}_{t}^{g*(1)},\overline{\chi}_{t}^{g*(1)},\underline{\chi}_{t}^{p*(1)},\overline{\chi}_{t}^{p*(1)},\gamma_{t}^{*(1)},\underline{\theta}_{t}^{*(1)},\overline{\theta}_{t}^{*(1)},\underline{\beta}_{it}^{*(1)},\overline{\beta}_{it}^{*(1)}\right\}$ are still the optimal solutions in the following equation (E24) when we assign $\mu_{t}=\mu_{t}^{*}$:

$$\max \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_t^{p}+\bar{Q}^{g}\cdot\bar{\chi}_t^{g}+\underline{E}\cdot\underline{\theta}_t+\bar{E}\cdot\bar{\theta}_t\right)+\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^{h}\cdot\underline{\beta}_{it}+\bar{G}_i^{h}\cdot\bar{\beta}_{it}\right)+\sum_{t=1}^{T}\mu_t^{*}\cdot D_t-E_1\cdot\gamma_2+E_{T+1}\cdot\gamma_{T+1}$$

$$\text{s.t.}\begin{cases}\text{for } q_t^p:\ \underline{\chi}_t^{p}+\bar{\chi}_t^{p}-\gamma_t=c^p+\mu_t^{*}/\alpha,\\ \text{for } q_t^g:\ \underline{\chi}_t^{g}+\bar{\chi}_t^{g}+\gamma_t=c^g-\mu_t^{*}\beta,\\ \text{for } g_{it}^h:\ \underline{\beta}_{it}+\bar{\beta}_{it}+\mu_t=C_{it}^h, \qquad \forall t\in\{1,2,\cdots,T\}\\ \text{for } E_t:\ -\gamma_t+\gamma_{t+1}+\underline{\theta}_t+\bar{\theta}_t=0,\\ \text{Where, } \underline{\chi}_t^{p},\underline{\chi}_t^{g},\underline{\theta}_t,\underline{\beta}_{it}\geq 0;\ \bar{\chi}_t^{p},\bar{\chi}_t^{g},\bar{\theta}_t,\bar{\beta}_{it}\leq 0.\end{cases} \tag{E25}$$

The Eq.(E25) can be divided into two subproblems, each with the following optimal solutions:

$$\max \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_t^{p}+\bar{Q}^{g}\cdot\bar{\chi}_t^{g}+\underline{E}\cdot\underline{\theta}_t+\bar{E}\cdot\bar{\theta}_t\right)+\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^{h}\cdot\underline{\beta}_{it}+\bar{G}_i^{h}\cdot\bar{\beta}_{it}\right)+\sum_{t=1}^{T}\mu_t^{*}\cdot D_t-E_1\cdot\gamma_2+E_{T+1}\cdot\gamma_{T+1}$$

$$\Leftrightarrow\begin{cases}(1)\max \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_t^{p}+\bar{Q}^{g}\cdot\bar{\chi}_t^{g}+\underline{E}\cdot\underline{\theta}_t+\bar{E}\cdot\bar{\theta}_t\right)+\sum_{t=1}^{T}\mu_t^{*}\cdot D_t-E_1\cdot\gamma_2+E_{T+1}\cdot\gamma_{T+1}\\ (2)\max \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^{h}\cdot\underline{\beta}_{it}+\bar{G}_i^{h}\cdot\bar{\beta}_{it}\right)\end{cases} \tag{E26}$$

Based on the constraints in equation (E25) and $\forall t\in\{1,2,\cdots,T\}$, we can get the following relations:

$$\begin{cases}\underline{\chi}_t^{p}+\bar{\chi}_t^{p}-\gamma_t=c^p+\mu_t^{*}/\alpha,\\ \underline{\chi}_t^{g}+\bar{\chi}_t^{g}+\gamma_t=c^g-\mu_t^{*}\beta,\\ \underline{\beta}_{it}+\bar{\beta}_{it}+\mu_t=C_{it}^h,\\ -\gamma_t+\gamma_{t+1}+\underline{\theta}_t+\bar{\theta}_t=0,\\ \text{Where, } \underline{\chi}_t^{p},\underline{\chi}_t^{g},\underline{\theta}_t,\underline{\beta}_{it}\geq 0;\\ \qquad \bar{\chi}_t^{p},\bar{\chi}_t^{g},\bar{\theta}_t,\bar{\beta}_{it}\leq 0,\gamma_t\in\mathbb{R}.\end{cases}\Leftrightarrow(1)\begin{cases}\underline{\chi}_t^{p}+\bar{\chi}_t^{p}-\gamma_t=c^p+\mu_t^{*}/\alpha,\\ \underline{\chi}_t^{g}+\bar{\chi}_t^{g}+\gamma_t=c^g-\mu_t^{*}\beta,\\ -\gamma_t+\gamma_{t+1}+\underline{\theta}_t+\bar{\theta}_t=0,\\ \text{Where, } \underline{\chi}_t^{p},\underline{\chi}_t^{g},\underline{\theta}_t\geq 0;\\ \qquad \bar{\chi}_t^{p},\bar{\chi}_t^{g},\bar{\theta}_t\leq 0,\gamma_t\in\mathbb{R}.\end{cases}+(2)\begin{cases}\underline{\beta}_{it}+\bar{\beta}_{it}+\mu_t^{*}=C_{it}^h\\ \underline{\beta}_{it}\geq 0,\bar{\beta}_{it}\leq 0.\end{cases} \tag{E27}$$

The equation (E27) states that the dual variables set of $\{\underline{\chi}_t^{g*(1)},\bar{\chi}_t^{g*(1)},\underline{\chi}_t^{p*(1)},\bar{\chi}_t^{p*(1)},\gamma_t^{*(1)},\underline{\theta}_t^{*(1)},\bar{\theta}_t^{*(1)}\}$ is independent of $\{\underline{\beta}_{it}^{*(1)},\bar{\beta}_{it}^{*(1)}\}$ for any given $\mu_t=\mu_t^{*}$. As a result, the optimal solutions to *subproblem one* and *subproblem two* should produce the same optimal results as Eq. (E28); then, the new duality problem can be divided into two subproblems:

Subproblem one:

$$\max \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_{t}^{p}+\bar{Q}^{g}\cdot\bar{\chi}_{t}^{g}+\underline{E}\cdot\underline{\theta}_{t}+\bar{E}\cdot\bar{\theta}_{t}\right)+\sum_{t=1}^{T}\mu_{t}^{*}\cdot D_{t}-E_{1}\cdot\gamma_{2}+E_{T+1}\cdot\gamma_{T+1}$$

$$\Leftrightarrow \max \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_{t}^{p}+\bar{Q}^{g}\cdot\bar{\chi}_{t}^{g}+\underline{E}\cdot\underline{\theta}_{t}+\bar{E}\cdot\bar{\theta}_{t}\right)-E_{1}\cdot\gamma_{2}+E_{T+1}\cdot\gamma_{T+1} \tag{E28}$$

$$\text{s.t.}\begin{cases}\underline{\chi}_{t}^{p}+\bar{\chi}_{t}^{p}-\gamma_{t}=c^{p}+\mu_{t}^{*}/\alpha, \\ \underline{\chi}_{t}^{g}+\bar{\chi}_{t}^{g}+\gamma_{t}=c^{g}-\mu_{t}^{*}\beta, \\ -\gamma_{t}+\gamma_{t+1}+\underline{\theta}_{t}+\bar{\theta}_{t}=0, \\ \text{Where},\left\{\underline{\chi}_{t}^{p},\underline{\chi}_{t}^{g},\underline{\theta}_{t}\right\}\geq 0;\left(\bar{\chi}_{t}^{p},\bar{\chi}_{t}^{g},\bar{\theta}_{t}\right)\leq 0,\gamma_{t}\in\mathbb{R}.\end{cases} \qquad \forall t\in\{1,2,3,\cdots,T\}$$

Subproblem two:

$$\max \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}+\bar{G}_{i}^{h}\cdot\bar{\beta}_{it}\right)$$

$$\text{s.t.}\begin{cases}\underline{\beta}_{it}+\bar{\beta}_{it}+\mu_{t}^{*}=C_{it}^{h} \\ \underline{\beta}_{t}\geq 0,\bar{\beta}_{t}\leq 0;\end{cases} \tag{E29}$$

Let $P_t = \mu^*_t$, which indicates the electricity merchant can make a perfect price prediction, or the ISO sends the cleared LMP to the merchant, which is obtained based on the shadow price of energy balance constraint. Then, comparing the *subproblem one* from the ISO's perspective and the duality problem from the merchant's perspective.

The subproblem one from ISO:

$$\max \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_t^{p}+\bar{Q}^{g}\cdot\bar{\chi}_t^{g}+\underline{E}\cdot\underline{\theta}_t+\bar{E}\cdot\bar{\theta}_t\right)-E_1\cdot\gamma_2+E_{T+1}\cdot\gamma_{T+1}$$

$$\Rightarrow\left\{\underline{\chi}_t^{\,g*(1)},\bar{\chi}_t^{\,g*(1)},\underline{\chi}_t^{\,p*(1)},\bar{\chi}_t^{\,p*(1)},\gamma_t^{*(1)},\underline{\theta}_t^{*(1)},\bar{\theta}_t^{*(1)}\right\}$$

$$\text{s.t.}\begin{cases}\underline{\chi}_t^{p}+\bar{\chi}_t^{p}-\gamma_t=c^{p}+\mu_t^{*}/\alpha,\\ \underline{\chi}_t^{g}+\bar{\chi}_t^{g}+\gamma_t=c^{g}-\mu_t^{*}\beta,\\ -\gamma_t+\gamma_{t+1}+\underline{\theta}_t+\bar{\theta}_t=0,\\ \text{Where},\left\{\underline{\chi}_t^{p},\underline{\chi}_t^{g},\underline{\theta}_t\right\}\ge 0;\ \left\{\bar{\chi}_t^{p},\bar{\chi}_t^{g},\bar{\theta}_t\right\}\le 0,\gamma_t\in\mathbb{R}.\end{cases} \tag{E30}$$

Duality problem from storage merchant:

$$\max \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_t^{p}+\bar{Q}^{g}\cdot\bar{\chi}_t^{g}+\underline{E}\cdot\underline{\theta}_t+\bar{E}\cdot\bar{\theta}_t\right)-E_1\cdot\gamma_2+E_{T+1}\cdot\gamma_{T+1}$$

$$\Rightarrow\left\{\underline{\chi}_t^{\,g*(2)},\bar{\chi}_t^{\,g*(2)},\underline{\chi}_t^{\,p*(2)},\bar{\chi}_t^{\,p*(2)},\gamma_t^{*(2)},\underline{\theta}_t^{*(2)},\bar{\theta}_t^{*(2)}\right\}$$

$$\text{s.t.}\begin{cases}\underline{\chi}_t^{p}+\bar{\chi}_t^{p}-\gamma_t=c^{p}+P_t/\alpha,\\ \underline{\chi}_t^{g}+\bar{\chi}_t^{g}+\gamma_t=c^{g}-P_t\beta,\\ -\gamma_t+\gamma_{t+1}+\underline{\theta}_t+\bar{\theta}_t=0,\\ \underline{\chi}_t^{p},\underline{\chi}_t^{g},\underline{\theta}_t\ge 0;\\ \bar{\chi}_t^{p},\bar{\chi}_t^{g},\bar{\theta}_t\le 0;\\ \gamma_t\in\mathbb{R}.\end{cases}\Leftrightarrow\begin{cases}\underline{\chi}_t^{p}+\bar{\chi}_t^{p}-\gamma_t=c^{p}+\mu_t^{*}/\alpha,\\ \underline{\chi}_t^{g}+\bar{\chi}_t^{g}+\gamma_t=c^{g}-\mu_t^{*}\beta,\\ -\gamma_t+\gamma_{t+1}+\underline{\theta}_t+\bar{\theta}_t=0,\\ \underline{\chi}_t^{p},\underline{\chi}_t^{g},\underline{\theta}_t\ge 0;\\ \bar{\chi}_t^{p},\bar{\chi}_t^{g},\bar{\theta}_t\le 0;\\ \gamma_t\in\mathbb{R}.\end{cases} \tag{E31}$$

Due to the identical objective function and constraints, subproblem one from the ISO perspective is equivalent to the duality problem from the perspective of the storage merchant. Thus

$$\left\{\underline{\chi}_t^{\,g*(1)},\bar{\chi}_t^{\,g*(1)},\underline{\chi}_t^{\,p*(1)},\bar{\chi}_t^{\,p*(1)},\gamma_t^{*(1)},\underline{\theta}_t^{*(1)},\bar{\theta}_t^{*(1)}\right\}=\left\{\underline{\chi}_t^{\,g*(2)},\bar{\chi}_t^{\,g*(2)},\underline{\chi}_t^{\,p*(2)},\bar{\chi}_t^{\,p*(2)},\gamma_t^{*(2)},\underline{\theta}_t^{*(2)},\bar{\theta}_t^{*(2)}\right\}$$

$$=\left\{\underline{\chi}_t^{\,g*},\bar{\chi}_t^{\,g*},\underline{\chi}_t^{\,p*},\bar{\chi}_t^{\,p*},\gamma_t^{*},\underline{\theta}_t^{*},\bar{\theta}_t^{*}\right\}$$

Then, comparing the *subproblem two* from the ISO's perspective and the duality problem from the traditional generator's perspective.

Subproblem two from ISO:

$$\max \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}_{it} + \overline{G}_i^h \cdot \overline{\beta}_{it}\right) \Leftrightarrow \max \sum_{t=1}^{T}\sum_{i=1}^{M\backslash I}\left[\left(\underline{G}_i^h \cdot \underline{\beta}_{it} + \overline{G}_i^h \cdot \overline{\beta}_{it}\right) + \left(\underline{G}_I^h \cdot \underline{\beta}_{It} + \overline{G}_I^h \cdot \overline{\beta}_{It}\right)\right]$$

$$\text{s.t.}\begin{cases} \underline{\beta}_{it} + \overline{\beta}_{it} + \mu_t^{*} = C_{it}^h \\ \underline{\beta}_{it} \geq 0, \overline{\beta}_{it} \leq 0. \end{cases} \tag{E32}$$

Dual problem from the generator I:

$$\max \sum_{t=1}^{T}\left(\underline{G}_I^h \cdot \underline{\beta}_{It} + \overline{G}_I^h \cdot \overline{\beta}_{It}\right)$$

$$\text{s.t.}\begin{cases} \underline{\beta}_{It} + \overline{\beta}_{It} = C_{It}^h - P_t \\ \underline{\beta}_{It} \geq 0, \overline{\beta}_{It} \leq 0. \end{cases} \tag{E33}$$

Due to the identical objective function and constraints, the optimal generation for generator *I* from the ISO perspective $\left\{\underline{\beta}_{It}^{*(1)}, \overline{\beta}_{It}^{*(1)}\right\}$ is equivalent to the duality problem from the perspective of generator *I* $\left\{\underline{\beta}_{It}^{*(3)}, \overline{\beta}_{It}^{*(3)}\right\}$. Thus

$$\left\{\underline{\beta}_{It}^{*(1)}, \overline{\beta}_{It}^{*(1)}\right\} = \left\{\underline{\beta}_{It}^{*(3)}, \overline{\beta}_{It}^{*(3)}\right\} = \left\{\underline{\beta}_{It}^{*}, \overline{\beta}_{It}^{*}\right\} \tag{E34}$$

(3) Use the duality theorem to compare the optimal solution $\left\{q_t^{p*(M)}, q_t^{g*(M)},\ g_{It}^{h*(M)}\right\}$ for electricity merchants and $\left\{q_t^{p*(S)}, q_t^{g*(S)},\ g_{It}^{h*(S)}\right\}$ from ISOs.

From ISO perspective, the objective functions of primal and dual problems are:

$$\begin{cases} \text{Primal: } \min\left(\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}q_{t}^{g(S)}+c^{p}q_{t}^{p(S)}\right)\right) \\ \text{Dual: } \max\left(\begin{array}{l}\sum_{t=1}^{T}\left(\overline{Q}^{p}\cdot\overline{\chi}_{t}^{p}+\overline{Q}^{g}\cdot\overline{\chi}_{t}^{g}+\underline{E}\cdot\underline{\theta}_{t}+\overline{E}\cdot\overline{\theta}_{t}\right)+\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}+\overline{G}_{i}^{h}\cdot\overline{\beta}_{it}\right) \\ +\sum_{t=1}^{T}\mu_{t}\cdot D_{t}-E_{1}\cdot\gamma_{2}+E_{T+1}\cdot\gamma_{T+1}\end{array}\right)\end{cases}$$

From storage merchant perspective, the objective functions of primal and dual problems are:

$$\begin{cases} \text{Primal}:\min\left(\sum_{t\in T}\left(\left(c^{g}-P_{t}\beta\right)q_{t}^{g(M)}+\left(c^{p}+P_{t}/\alpha\right)q_{t}^{p(M)}\right)\right) \\ \text{Dual}:\max\sum_{t=1}^{T}\left(\overline{Q}^{p}\cdot\overline{\chi}_{t}^{p}+\overline{Q}^{g}\cdot\overline{\chi}_{t}^{g}+\underline{E}\cdot\underline{\theta}_{t}+\overline{E}\cdot\overline{\theta}_{t}\right)-E_{1}\cdot\gamma_{2}+E_{T+1}\cdot\gamma_{T+1}\end{cases}$$

From the perspective of generator-*I*, the objective functions of primal and dual problems are:

$$\begin{cases} \text{Primal}:\min\sum_{t=1}^{T}\left(C_{It}^{h}-P_{t}\right)\cdot g_{It}^{h} \\ \text{Dual}:\max\sum_{t=1}^{T}\left(\underline{G}_{I}^{h}\cdot\underline{\beta}_{It}+\overline{G}_{I}^{h}\cdot\overline{\beta}_{It}\right)\end{cases}$$

(4) Relation between the ISO and storage merchant scheduling models

We can derive the following from the strong duality theorem, which states that the optimal objective function value of the primal problem and the duality problem are equal:

$$\begin{cases} \sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h}\cdot g_{it}^{h*}+\sum_{t=1}^{T}\left(c^{g}q_{t}^{g*(S)}+c^{p}q_{t}^{p*(S)}\right)=\left\{\begin{array}{l}\sum_{t=1}^{T}\left(\overline{Q}^{p}\cdot\overline{\chi}_{t}^{p*}+\overline{Q}^{g}\cdot\overline{\chi}_{t}^{g*}+\underline{E}\cdot\underline{\theta}_{t}^{*}+\overline{E}\cdot\overline{\theta}_{t}^{*}\right)+\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}^{*}+\overline{G}_{i}^{h}\cdot\overline{\beta}_{it}^{*}\right) \\ +\sum_{t=1}^{T}\mu_{t}^{*}\cdot D_{t}-E_{1}\cdot\gamma_{2}^{*}+E_{T+1}\cdot\gamma_{T+1}^{*}\end{array}\right\} \\ \sum_{t\in T}\left(\left(c^{g}-P_{t}\beta\right)q_{t}^{g*(M)}+\left(c^{p}+P_{t}/\alpha\right)q_{t}^{p*(M)}\right)=\sum_{t=1}^{T}\left(\overline{Q}^{p}\cdot\overline{\chi}_{t}^{p*}+\overline{Q}^{g}\cdot\overline{\chi}_{t}^{g*}+\underline{E}\cdot\underline{\theta}_{t}^{*}+\overline{E}\cdot\overline{\theta}_{t}^{*}\right)-E_{1}\cdot\gamma_{2}^{*}+E_{T+1}\cdot\gamma_{T+1}^{*}\end{cases}$$

The energy balance constraint always holds. Let's consider the following equation:

$$\sum_{i=1}^{M} g_{it}^{h*} + q_t^{g*(S)}\beta - q_t^{p*(S)} / \alpha = D_t .$$

Then, we can derive the following equivalent equations:

$$\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h} \cdot g_{it}^{h*} + \sum_{t=1}^{T}\left(c^g q_t^{g*(S)} + c^p q_t^{p*(S)}\right)$$
$$= \sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h} \cdot g_{it}^{h*} + \sum_{t=1}^{T}\left(c^g q_t^{g*(S)} + c^p q_t^{p*(S)}\right) + \sum_{t=1}^{T} P_t \cdot \left(D_t + q_t^{p*(S)} / \alpha - q_t^{g*(S)}\beta - \sum_{i=1}^{M} g_{it}^{h*} \right)$$
$$= \sum_{t=1}^{T}\sum_{i=1}^{M} \left(C_{it}^{h} - P_t\right) \cdot g_{it}^{h*} + \sum_{t=1}^{T}\left(\left(c^g - P_t\beta\right) q_t^{g*(S)} + \left(c^p + P_t / \alpha\right) q_t^{p*(S)}\right) + \sum_{t=1}^{T} P_t \cdot D_t$$
$$= \sum_{t=1}^{T}\left(\bar{Q}^p \cdot \bar{\chi}_t^{p*} + \bar{Q}^g \cdot \bar{\chi}_t^{g*} + \underline{E} \cdot \underline{\theta}_t^{\,*} + \bar{E} \cdot \bar{\theta}_t^{\,*}\right) + \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}_{it}^{*} + \bar{G}_i^h \cdot \bar{\beta}_{it}^{*}\right) + \sum_{t=1}^{T} \mu_t^{\,*} \cdot D_t - E_1 \cdot \gamma_2^{\,*} + E_{T+1} \cdot \gamma_{T+1}^{\,*}$$
$$= \sum_{t=1}^{T}\left(\left(c^g - P_t\beta\right) q_t^{g*(M)} + \left(c^p + P_t / \alpha\right) q_t^{p*(M)}\right) + \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}^{*}_{\ it} + \bar{G}_i^h \cdot \bar{\beta}^{*}_{\ it}\right) + \sum_{t=1}^{T} P_t \cdot D_t (\text{if } P_t = \mu_t^*)$$

That is,

$$\sum_{t=1}^{T}\sum_{i=1}^{M} \left(C_{it}^{h} - P_t\right) \cdot g_{it}^{h*} + \sum_{t=1}^{T}\left(\left(c^g - P_t\beta\right) q_t^{g*(S)} + \left(c^p + P_t / \alpha\right) q_t^{p*(S)}\right)$$
$$= \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}^{*}_{\ it} + \bar{G}_i^h \cdot \bar{\beta}^{*}_{\ it}\right) + \sum_{t=1}^{T}\left(\left(c^g - P_t\beta\right) q_t^{g*(M)} + \left(c^p + P_t / \alpha\right) q_t^{p*(M)}\right)$$

To find the relation for the optimal decisions between these two problems, we first confirm the following equations:

$$\sum_{t=1}^{T}\sum_{i=1}^{M} \left(C_{it}^{h} - P_t\right) \cdot g_{it}^{h*} = \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}^{*}_{\ it} + \bar{G}_i^h \cdot \bar{\beta}^{*}_{\ it}\right) \qquad \text{(E35)}$$

We can also find there is $\sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(C_{it}^{h} - \mu_t^{\,*}\right) g_{it}^{h*}\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}^{*}_{\ it} + \bar{G}_i^h \cdot \bar{\beta}^{*}_{\ it}\right)$.

In Eq. (D32), we have $\underline{\beta}_{it} + \bar{\beta}_{it} + \mu_t^{\,*} = C_{it}^{h}$ (i.e., $\underline{\beta}_{it}^{*} + \bar{\beta}_{it}^{*} = C_{it}^{h} - \mu_t^{\,*}$). Thus, we only need to find that there is $\sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(\underline{\beta}^{*}_{\ it} + \bar{\beta}^{*}_{\ it}\right) g_{it}^{h*}\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}^{*}_{\ it} + \bar{G}_i^h \cdot \bar{\beta}^{*}_{\ it}\right)$ holding.

Obviously, there is

$$\sum_{t=1}^{T}\sum_{i=1}^{M}\left(g_{it}^{h*}\cdot\underline{\beta}_{it}^{*}+g_{it}^{h*}\cdot\overline{\beta}_{it}^{*}\right)=\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}^{*}+\overline{G}_{i}^{h}\cdot\overline{\beta}_{it}^{*}\right)\Leftrightarrow\sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(g_{it}^{h*}-\underline{G}_{i}^{h}\right)\underline{\beta}_{it}^{*}\right)=\sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(\overline{G}_{i}^{h}-g_{it}^{h*}\right)\overline{\beta}_{it}^{*}\right).$$

Since there are $\underline{\beta}_{it}\geq 0,\overline{\beta}_{it}\leq 0$, and $\underline{G}_{i}^{h}\leq g_{it}^{h}\leq\overline{G}_{i}^{h}$, we can get $\left(g_{it}^{h*}-\underline{G}_{i}^{h}\right)\underline{\beta}_{it}^{*}\geq 0$ and $\left(\overline{G}_{i}^{h}-g_{it}^{h*}\right)\overline{\beta}_{it}^{*}\leq 0$. Thus, we have the following relation:

$$\left\{\sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(g_{it}^{h*}-\underline{G}_{i}^{h}\right)\underline{\beta}_{it}^{*}\right)\geq 0;\ \sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(\overline{G}_{i}^{h}-g_{it}^{h*}\right)\overline{\beta}_{it}^{*}\right)\leq 0\right.$$

If $\sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(g_{it}^{h*}-\underline{G}_{i}^{h}\right)\underline{\beta}_{it}^{*}\right)=\sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(\overline{G}_{i}^{h}-g_{it}^{h*}\right)\overline{\beta}_{it}^{*}\right)$ holding, then there is $\sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(g_{it}^{h*}-\underline{G}_{i}^{h}\right)\underline{\beta}_{it}^{*}\right)=\sum_{t=1}^{T}\left(\sum_{i=1}^{M}\left(\overline{G}_{i}^{h}-g_{it}^{h*}\right)\overline{\beta}_{it}^{*}\right)=0$.

That is, for $\forall i\in\{1,2,\cdots,M\}$, and $\forall t\in\{1,2,\cdots,T\}$, there is

$$\left(g_{it}^{h*}-\underline{G}_{i}^{h}\right)\underline{\beta}_{it}^{*}=\left(\overline{G}_{i}^{h}-g_{it}^{h*}\right)\overline{\beta}_{it}^{*}=0 \tag{E36}$$

Next, recall the proof of *subproblem two* from the ISO's perspective:

$$\max\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}+\overline{G}_{i}^{h}\cdot\overline{\beta}_{it}\right)\Rightarrow\left\{\underline{\beta}_{it}^{*},\overline{\beta}_{it}^{*},\mu_{t}^{*}=LMP_{t}\right\}$$

$$\text{s.t.}\begin{cases}\underline{\beta}_{it}+\overline{\beta}_{it}+\mu_{t}^{*}=C_{it}^{h}\\ \underline{\beta}_{it}\geq 0,\overline{\beta}_{it}\leq 0;\\ \mu_{t}^{*}=LMP_{t}\end{cases}$$

Here, we assume $\mu_{t}=\mu_{t}^{*}=LMP_{t}=P_{t}$. Let $\underline{\beta}_{it}=C_{it}^{h}-(\overline{\beta}_{it}+P_{t})=C_{it}^{h}-P_{t}-\overline{\beta}_{it}$, we have the following:

$$\begin{cases}\underline{\beta}_{it}=C_{it}^{h}-P_{t}-\overline{\beta}_{it}\geq 0\\ \overline{\beta}_{it}\leq 0\end{cases}\Rightarrow\begin{cases}\overline{\beta}_{it}\leq C_{it}^{h}-P_{t}\\ \overline{\beta}_{it}\leq 0\end{cases}\Rightarrow\overline{\beta}_{it}\leq\min\left\{0,C_{it}^{h}-P_{t}\right\}$$

As a result, the objection function of subproblem two can be rewritten as:

$$\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}^{h}\cdot\underline{\beta}_{it}+\overline{G}^{h}\cdot\overline{\beta}_{it}\right)=\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}^{h}\cdot\left(C_{it}^{h}-\left(\overline{\beta}_{it}+P_{t}\right)\right)+\overline{G}^{h}\cdot\overline{\beta}_{it}\right)=\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}^{h}\cdot\left(C_{it}^{h}-P_{t}\right)+\left(\overline{G}^{h}-\underline{G}^{h}\right)\cdot\overline{\beta}_{it}\right)$$

Since $(\overline{G}^{h}-\underline{G}^{h})\geq 0$ is holding, the objection function of subproblem two increases with $\overline{\beta}_{it}$. In this case, when $\overline{\beta}_{it}=\min\{0,C_{it}^{h}-P_{t}\}$, the objection function in subproblem two reaches its maximum value. Then, we can derive $\underline{\beta}_{it}$ as follows:

$$\underline{\beta}_{it}=C_{it}^{h}-P_{t}-\overline{\beta}_{it}=\begin{cases}C_{it}^{h}-P_{t}-\left(C_{it}^{h}-P_{t}\right)=0, & \text{if } C_{it}^{h}-P_{t}<0\\ C_{it}^{h}-P_{t}-0=C_{it}^{h}-P_{t}, & \text{if } C_{it}^{h}-P_{t}>0\end{cases}=\max\left\{0,C_{it}^{h}-P_{t}\right\}$$

That is,

$$\left\{\overline{\beta}_{it}^{\ *}=\min\left\{0,C_{it}^{h}-P_{t}\right\};\ \underline{\beta}_{it}^{\ *}=\max\left\{0,C_{it}^{h}-P_{t}\right\},\ \forall i\in\left\{1,2,\cdots,M\right\}\right. \qquad \text{(E37)}$$

1) When there has $C_{it}^{h}>P_{t}=LMP_{t},\forall i\in\left\{1,2,\cdots,M\right\}$, we get

$$\begin{cases}\overline{\beta}_{it}^{\ *}=\min\left\{0,C_{it}^{h}-P_{t}\right\}=0\\ \underline{\beta}_{it}^{\ *}=\max\left\{0,C_{it}^{h}-P_{t}\right\}=C_{it}^{h}-P_{t}\end{cases}$$

, then (D36) can be shown as

$(g_{it}^{h*}-\underline{G}_{i}^{h})\cdot(C_{it}^{h}-P_{t})=(\overline{G}_{i}^{h}-g_{it}^{h*})\cdot 0=0$, thus, we will get $g_{it}^{h*}=\underline{G}_{i}^{h}$.

2) When there has $C_{it}^{h}=P_{t}=LMP_{t},\forall i\in\left\{1,2,\cdots,M\right\}$, we can get $\begin{cases}\overline{\beta}_{it}^{\ *}=0\\ \underline{\beta}_{it}^{\ *}=0\end{cases}$, then the equation (A36) is satisfied. Thus, there is $\underline{G}_{i}^{h}\leq g_{it}^{h*}\leq\overline{G}_{i}^{h}$.

3) When $C_{it}^{h}<P_{t}=LMP_{t},\forall i\in\left\{1,2,\cdots,M\right\}$ is holding, we can get

$$\begin{cases}\overline{\beta}_{it}^{\ *}=C_{it}^{h}-P_{t}\\ \underline{\beta}_{it}^{\ *}=0\end{cases}$$

, then equation (A36) can be shown as

$(g_{it}^{h*}-\underline{G}_{i}^{h})\cdot 0=(\overline{G}_{i}^{h}-g_{it}^{h*})(C_{it}^{h}-P_{t})=0$, thus, we can obtain $g_{it}^{h*}=\overline{G}_{i}^{h}$.

Thus, for $\forall i \in \{1, 2, \cdots, M\}$ the following conclusion can be drawn:

$$\begin{cases} 1)\text{ If } C_{it}^{h} < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^{h}; \\ 2)\text{ or If } C_{it}^{h} = P_t = LMP_t \text{ and } \underline{G}_i^{h} \le g_{it}^{h*} \le \bar{G}_i^{h}; \\ 3)\text{ or If } C_{it}^{h} > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^{h}; \end{cases} \tag{E38}$$

$$\text{there are } \sum_{t=1}^{T}\left(\sum_{i=1}^{M} C_{it}^{h} g_{it}^{h*} - \sum_{i=1}^{M} P_t g_{it}^{h*}\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^{h} \cdot \underline{\beta}_{it}^{*} + \bar{G}_i^{h} \cdot \bar{\beta}_{it}^{*}\right).$$

When there is $\sum_{t=1}^{T}\left(\sum_{i=1}^{M} C_{it}^{h} g_{it}^{h*} - \sum_{i=1}^{M} P_t g_{it}^{h*}\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^{h} \cdot \underline{\beta}_{it}^{*} + \bar{G}_i^{h} \cdot \bar{\beta}_{it}^{*}\right)$ holding, we can achieve the following equivalence relationship

$\sum_{t=1}^{T}\left((c^g - P_t\beta)q_t^{g*(S)} + (c^p + P_t/\alpha)q_t^{p*(S)}\right) = \sum_{t \in T}\left((c^g - P_t\beta)q_t^{g*(M)} + (c^p + P_t/\alpha)q_t^{p*(M)}\right)$, which is also the objection function of primal problem from the merchant's perspective. In this case, the merchant will get the optimal profit if she follows ISO's dispatch.

(5) The relation between the ISO's and the generator *I's* scheduling models

We can derive the following from the strong duality theorem, which states that the optimal objective function value of the primal problem and the duality problem are equal:

$$\begin{cases} \sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h} \cdot g_{it}^{h*(S)} + \sum_{t=1}^{T}\left(c^g q_t^{g*(S)} + c^p q_t^{p*(S)}\right) = \left\{ \begin{aligned} &\sum_{t=1}^{T}\left(\bar{Q}^p \cdot \bar{\chi}_t^{p*} + \bar{Q}^g \cdot \bar{\chi}_t^{g*} + \underline{E} \cdot \underline{\theta}_t^{*} + \bar{E} \cdot \bar{\theta}_t^{*}\right) + \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^{h} \cdot \underline{\beta}_{it}^{*} + \bar{G}_i^{h} \cdot \bar{\beta}_{it}^{*}\right) \\ &+ \sum_{t=1}^{T} \mu_t^{*} \cdot D_t - E_1 \cdot \gamma_2^{*} + E_{T+1} \cdot \gamma_{T+1}^{*} \end{aligned} \right\} \\ \sum_{t=1}^{T}\left(C_{It}^{h} - P_t\right) \cdot g_{It}^{h*(M)} = \sum_{t=1}^{T}\left(\underline{G}_I^{h} \cdot \underline{\beta}_{It}^{*} + \bar{G}_I^{h} \cdot \bar{\beta}_{It}^{*}\right) \end{cases}$$

Then, we can derive the following equivalent equations:

$$\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h}\cdot g_{it}^{h*}+\sum_{t=1}^{T}\left(c^{g}q_{t}^{g*(S)}+c^{p}q_{t}^{p*(S)}\right)$$
$$=\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h}\cdot g_{it}^{h*}+\sum_{t=1}^{T}\left(c^{g}q_{t}^{g*(S)}+c^{p}q_{t}^{p*(S)}\right)+\sum_{t=1}^{T}P_{t}\cdot\left(D_{t}+q_{t}^{p*(S)}/\alpha-q_{t}^{g*(S)}\beta-\sum_{i=1}^{M}g_{it}^{h*}\right)$$
$$=\sum_{t=1}^{T}\sum_{i=1}^{M}\left(C_{it}^{h}-P_{t}\right)\cdot g_{it}^{h*}+\sum_{t=1}^{T}\left(\left(c^{g}-P_{t}\beta\right)q_{t}^{g*(S)}+\left(c^{p}+P_{t}/\alpha\right)q_{t}^{p*(S)}\right)+\sum_{t=1}^{T}P_{t}\cdot D_{t}$$
$$=\sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_{t}^{p*}+\bar{Q}^{g}\cdot\bar{\chi}_{t}^{g*}+\underline{E}\cdot\underline{\theta}_{t}^{*}+\bar{E}\cdot\bar{\theta}_{t}^{*}\right)+\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}^{*}+\bar{G}_{i}^{h}\cdot\bar{\beta}_{it}^{*}\right)+\sum_{t=1}^{T}\mu_{t}^{*}\cdot D_{t}-E_{1}\cdot\gamma_{2}^{*}+E_{T+1}\cdot\gamma_{T+1}^{*}$$
$$=\sum_{t=1}^{T}\left(\left(c^{g}-P_{t}\beta\right)q_{t}^{g*(M)}+\left(c^{p}+P_{t}/\alpha\right)q_{t}^{p*(M)}\right)+\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}^{*}+\bar{G}_{i}^{h}\cdot\bar{\beta}_{it}^{*}\right)+\sum_{t=1}^{T}P_{t}\cdot D_{t}\,(\text{if } P_{t}=\mu_{t}^{*})$$

That is,

$$\sum_{t=1}^{T}\sum_{i=1}^{M\backslash I}\left(C_{it}^{h}-P_{t}\right)\cdot g_{it}^{h*}+\sum_{t=1}^{T}\left(C_{It}^{h}-P_{t}\right)\cdot g_{It}^{h*(S)}+\sum_{t=1}^{T}\left(\left(c^{g}-P_{t}\beta\right)q_{t}^{g*(S)}+\left(c^{p}+P_{t}/\alpha\right)q_{t}^{p*(S)}\right)$$
$$=\sum_{t=1}^{T}\sum_{i=1}^{M\backslash I}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}^{*}+\bar{G}_{i}^{h}\cdot\bar{\beta}_{it}^{*}\right)+\sum_{t=1}^{T}\left(C_{It}^{h}-P_{t}\right)\cdot g_{It}^{h*(M)}+\sum_{t=1}^{T}\left(\left(c^{g}-P_{t}\beta\right)q_{t}^{g*(M)}+\left(c^{p}+P_{t}/\alpha\right)q_{t}^{p*(M)}\right)$$

When the conditions (E38) are holding on, there has

$$\sum_{t=1}^{T}\sum_{i=1}^{M\backslash I}\left(C_{it}^{h}-P_{t}\right)\cdot g_{it}^{h*}+\sum_{t=1}^{T}\left(\left(c^{g}-P_{t}\beta\right)q_{t}^{g*(S)}+\left(c^{p}+P_{t}/\alpha\right)q_{t}^{p*(S)}\right)$$
$$=\sum_{t=1}^{T}\sum_{i=1}^{M\backslash I}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}^{*}+\bar{G}_{i}^{h}\cdot\bar{\beta}_{it}^{*}\right)+\sum_{t=1}^{T}\left(\left(c^{g}-P_{t}\beta\right)q_{t}^{g*(M)}+\left(c^{p}+P_{t}/\alpha\right)q_{t}^{p*(M)}\right).$$

Therefore, there exist $\sum_{t=1}^{T}\left(C_{It}^{h}-P_{t}\right)\cdot g_{It}^{h*(S)}=\sum_{t=1}^{T}\left(C_{It}^{h}-P_{t}\right)\cdot g_{It}^{h*(M)}$, which is also the objective function of the primal problem for the generator *I* to maximize her own profit. In this case, the generator will get the optimal profit if she follows ISO's dispatch. For $\forall i\in\{1,2,\cdots,M\},\forall t\in\{1,2,\cdots,T\}$,the relations between the *optimal actions* $q_{t}^{*(S)}=\{q_{t}^{p*(S)},q_{t}^{g*(S)}\}$ *of ISO and* $q_{t}^{*(M)}=\{q_{t}^{p*(M)},q_{t}^{g*(M)}\}$ *of storage merchant can be drawn:*

(1) For the PSH merchant, if the forecasted price matches the actual LMPs, *when the primal problem from profit-maximizing has a unique optimal solution:*

$$\begin{cases} 1) \text{ If } C_{it}^{h} < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^{h},\ \left\{q_t^{p*(S)}, q_t^{g*(S)}\right\} = \left\{q_t^{p*(M)}, q_t^{g*(M)}\right\}; \\ 2) \text{ If } C_{it}^{h} = P_t = LMP_t \text{ and } \underline{G}_i^{h} \le g_{it}^{h*} \le \bar{G}_i^{h},\ \left\{q_t^{p*(S)}, q_t^{g*(S)}\right\} = \left\{q_t^{p*(M)}, q_t^{g*(M)}\right\}; \\ 3) \text{ If } C_{it}^{h} > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^{h},\ \left\{q_t^{p*(S)}, q_t^{g*(S)}\right\} = \left\{q_t^{p*(M)}, q_t^{g*(M)}\right\}. \end{cases} \quad \text{(E39)}$$

(2) *If the lower bound of power generation of thermal generators is 0, that is* $\underline{G}_i^{h} = 0, \forall i \in \{1, 2, \cdots, M\}$, *then we can rewrite* (E39) *as*

$$\begin{cases} 1) \text{ If } C_{it}^{h} < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^{h},\ q_t^{p*(S)} = q_t^{p*(M)}, \text{and } q_t^{g*(S)} = q_t^{g*(M)}; \\ 2) \text{ If } C_{it}^{h} = P_t = LMP_t \text{ and } 0 \le g_{it}^{h*} \le \bar{G}_i^{h},\ q_t^{p*(S)} = q_t^{p*(M)}, \text{and } q_t^{g*(S)} = q_t^{g*(M)}; \\ 3) \text{ If } C_{it}^{h} > P_t = LMP_t \text{ and } g_{it}^{h*} = 0,\ q_t^{p*(S)} = q_t^{p*(M)}, \text{and } q_t^{g*(S)} = q_t^{g*(M)}. \end{cases} \quad \text{(E40)}$$

(3) For the storage merchant, if the forecasted price aligns with the actual LMPs, *when the primal problem from profit-maximizing has multiple optimal solutions:*

$$\begin{aligned} & \max \sum_{t=1}^{T} \left(LMP_t \cdot \left(q_t^{g}\beta - q_t^{p} / \alpha \right) - \left(c^{g} \cdot q_t^{g} + c^{p} \cdot q_t^{p} \right) \right) \\ & = \sum_{t=1}^{T} \left(LMP_t \cdot \left(q_t^{g*(S)}\beta - q_t^{p*(S)} / \alpha \right) - \left(c^{g} \cdot q_t^{g*(S)} + c^{p} \cdot q_t^{p*(S)} \right) \right) \\ & = \sum_{t=1}^{T} \left(LMP_t \cdot \left(q_t^{g*(M)}\beta - q_t^{p*(M)} / \alpha \right) - \left(c^{g} \cdot q_t^{g*(M)} + c^{p} \cdot q_t^{p*(M)} \right) \right) \end{aligned} \quad \text{(E41)}$$

For $\forall i \in \{1, 2, \cdots, M\}, \forall t \in \{1, 2, \cdots, T\}$, the relations between the *optimal actions* $g_{it}^{h*(S)}$ *of ISO and* $g_{It}^{h*(M)}$ *of generator-I can be drawn:*

(1) For the generator, if the forecasted price matches the actual LMPs, *when the primal problem from individual profit-maximizing has a unique optimal solution:*

$$\begin{cases} 1) \text{ If } C_{it}^{h} < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^{h},\ g_{it}^{h*(S)} = g_{it}^{h*(M)}; \\ 2) \text{ If } C_{it}^{h} = P_t = LMP_t \text{ and } \underline{G}_i^{h} \le g_{it}^{h*} \le \bar{G}_i^{h},\ g_{it}^{h*(S)} = g_{it}^{h*(M)}\}; \\ 3) \text{ If } C_{it}^{h} > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^{h},\ g_{it}^{h*(S)} = g_{it}^{h*(M)}. \end{cases} \quad \text{(E42)}$$

(2) For the generator, if the forecasted price aligns with the actual LMPs, *when the primal problem from individual profit-maximizing has multiple optimal solutions:*

$$\max \sum_{t=1}^{T}\left(LMP_t - C_{It}^{h}\right)\cdot g_{It}^{h} = \sum_{t=1}^{T}\left(LMP_t - C_{It}^{h}\right)\cdot g_{It}^{h*(S)} = \sum_{t=1}^{T}\left(LMP_t - C_{It}^{h}\right)\cdot g_{It}^{h*(M)} \qquad \text{(E43)}$$

<u>Proof of Scenario for Merchant with Energy Storage and Wind Farms</u>

Discharging/Generating and Charging/Pumping cannot happen at the same period.

Considering the merchant who manages both wind farms and energy storage, the scheduling problem from the perspective of the ISO is shown as follows:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h}\cdot g_{it}^{h} + \sum_{t=1}^{T}\left(c^{g}q_{t}^{g} + c^{p}q_{t}^{p}\right) + \sum_{t=1}^{T} c^{w}\cdot w_t\right)$$

$$\text{s.t.}\begin{cases} 0 \le q_t^p \le \bar{Q}^p, \\ 0 \le q_t^g \le \bar{Q}^g, \\ \underline{E} \le E_t \le \bar{E}, \\ \underline{g}_{it}^{h} \le g_{it}^{h} \le \bar{G}_i^h, \\ E_t - q_t^g + q_t^p = E_{t+1}, \\ \sum_{i=1}^{M} g_{it}^{h} + w_t + q_t^g\beta - q_t^p/\alpha = D_t, \\ \underline{W} \le w_t \le \bar{W}, \\ q_t^g \cdot q_t^p = 0. \end{cases} \qquad \text{(E44)}$$

If ignoring the transmission efficiency loss and startup cost of PSH or energy storage, the reward function of $R(q_t^p, q_t^g, w_t, P_t)$ can be rewritten as

$$R(q_t^p, q_t^g, w_t, P_t) = \begin{cases} -P_t \cdot \left(q_t^p / \alpha - w_t\right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p + c^w \cdot w_t\right) & (q_t^p > w_t) \\ -P_t \cdot \left(q_t^p / \alpha - w_t\right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p + c^w \cdot w_t\right) & (0 \le q_t^p < w_t) \\ P_t \cdot \left(q_t^g \beta + w_t\right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p + c^w \cdot w_t\right) & (q_t^g > 0) \end{cases} \tag{E45}$$

$$= \begin{cases} -P_t \cdot \left(q_t^p / \alpha\right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p\right) + \left(P_t - c^w\right) \cdot w_t & (q_t^p \ge 0) \\ P_t \cdot \left(q_t^g \beta\right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p\right) + \left(P_t - c^w\right) \cdot w_t & (q_t^g \ge 0) \end{cases}$$

Then, the objective function of electricity merchants is shown as follows:

$$\max \sum_{t=1}^{T} \left(P_t \cdot \left(q_t^g \beta - q_t^p / \alpha\right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p\right) + \left(P_t - c^w\right) \cdot w_t\right)$$

$$\text{s.t.} \begin{cases} 0 \le q_t^p \le \bar{Q}^p, \\ 0 \le q_t^g \le \bar{Q}^g, \\ \underline{W} \le w_t \le \bar{W}, \\ E_t - q_t^g + q_t^p = E_{t+1}, \\ \underline{E} \le E_t \le \bar{E}, \\ q_t^g \cdot q_t^p = 0. \end{cases} \quad \forall t \in \{1, 2, \cdots, T\} \tag{E46}$$

Suppose the charging/generating and discharging/generating cannot happen in one period. We have the following non-convex complementary constraint from the perspective of ISO or electricity merchant:

$$q_t^p \cdot q_t^g = 0 \tag{E47}$$

Similar to Appendix A, we use the KKT condition to analyze sufficient conditions for such an exact relaxation of Eq. (E4). The primal problem from perspective of ISO after only relaxing the non-convex constraint is shown:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}q_{t}^{g}+c^{p}q_{t}^{p}\right)+\sum_{t=1}^{T}c^{w}\cdot w_{t}\right)$$

$$\text{s.t.}\begin{cases}-q_{t}^{p}\le 0, q_{t}^{p}-\bar{Q}^{p}\le 0 & (\underline{\chi}1_{t}^{p},\bar{\chi}1_{t}^{p})\\ -q_{t}^{g}\le 0, q_{t}^{g}-\bar{Q}^{g}\le 0 & (\underline{\chi}1_{t}^{g},\bar{\chi}1_{t}^{g})\\ -E_{t}+\underline{E}\le 0, E_{t}-\bar{E}\le 0 & (\underline{\theta}1_{t},\bar{\theta}1_{t})\\ -g_{it}^{h}+\underline{G}_{i}^{h}\le 0, g_{it}^{h}-\bar{G}_{i}^{h}\le 0 & (\underline{\beta}1_{it},\bar{\beta}1_{it})\\ D_{t}-\left(\sum_{i=1}^{M}g_{it}^{h}+q_{t}^{g}\beta+w_{t}-q_{t}^{p}/\alpha\right)=0, & (\mu 1_{t})\\ E_{t}-q_{t}^{g}+q_{t}^{p}=E_{t+1}, & (\gamma 1_{t+1})\\ -w_{t}\le -\underline{W}, w_{t}\le \bar{W} & (\underline{\omega}1_{it},\bar{\omega}1_{it})\\ \text{Where,}\left\{\underline{\chi}1_{t}^{p},\underline{\chi}1_{t}^{g},\underline{\theta}1_{t},\underline{\beta}1_{it},\bar{\chi}1_{t}^{p},\bar{\chi}1_{t}^{g},\bar{\theta}1_{t},\bar{\beta}1_{it}\right\}\ge 0. \end{cases} \tag{E48}$$

The Lagrange function of equation (E49) is obtained:

$$\begin{aligned}L=&\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}q_{t}^{g}+c^{p}q_{t}^{p}\right)+\sum_{t=1}^{T}c^{w}\cdot w_{t}+\mu 1_{t}\left(D_{t}-\left(\sum_{i=1}^{M}g_{it}^{h}+w_{t}+q_{t}^{g}\beta-q_{t}^{p}/\alpha\right)\right)\\ &+\gamma 1_{t+1}\left(E_{t+1}-E_{t}+q_{t}^{g}-q_{t}^{p}\right)+\underline{\chi}1_{t}^{p}\cdot\left(-q_{t}^{p}\right)+\bar{\chi}1_{t}^{p}\cdot\left(q_{t}^{p}-\bar{Q}^{p}\right)+\underline{\chi}1_{t}^{g}\cdot\left(-q_{t}^{g}\right)+\bar{\chi}1_{t}^{g}\cdot\left(q_{t}^{g}-\bar{Q}^{g}\right)\\ &+\underline{\theta}1_{t}\cdot\left(-E_{t}+\underline{E}\right)+\bar{\theta}1_{t}\cdot\left(E_{t}-\bar{E}\right)+\underline{\beta}1_{it}\cdot\left(-g_{it}^{h}+\underline{G}_{i}^{h}\right)+\bar{\beta}1_{it}\left(g_{it}^{h}-\bar{G}_{i}^{h}\right)\\ &+\underline{\omega}1_{it}\cdot\left(-w_{t}+\underline{W}\right)+\bar{\omega}1_{it}\left(w_{t}-\bar{W}\right)\end{aligned} \tag{E49}$$

In the KKT condition for the primal problem from the perspective of ISO, the first-order derivative of the Lagrangian function concerning energy storage discharging variables q_t^g and charging variables q_t^p must equal to zero; hence the following equation holds (i.e., $\forall t\in\{1,2,\cdots,T\}$)

$$\begin{cases}\partial L/\partial q_{t}^{g}\Big|_{q_{t}^{g}=q_{t}^{g*(S)}}=c^{g}-u1_{t}^{*}\beta+\gamma 1_{t+1}^{*}+\bar{\chi}1_{t}^{g*}-\underline{\chi}1_{t}^{g*}=0\\ \partial L/\partial q_{t}^{p}\Big|_{q_{t}^{p}=q_{t}^{p*(S)}}=c^{p}+u1_{t}^{*}/\alpha-\gamma 1_{t+1}^{*}+\bar{\chi}1_{t}^{p*}-\underline{\chi}1_{t}^{p*}=0\end{cases} \tag{E50}$$

Assume $q_t^{g*(S)} > 0$ and $q_t^{p*(S)} > 0$ are the optimal solutions of the primal problem from perspectives of ISO at time t. We also know there have $\underline{\chi}1_t^{g*} = 0$ and $\underline{\chi}1_t^{p*} = 0$ because of the complementary slackness conditions. When you combine two sub-equations in (E50), you get the following equation.

$$c^g + c^p + \overline{\chi}1_t^{g*} + \overline{\chi}1_t^{p*} - u1_t^*\beta + u1_t^*/\alpha = 0 \tag{E51}$$

Because there are $\overline{\chi}1_t^{p*} \geq 0$ and $\overline{\chi}1_t^{g*} \geq 0$, the equation (E52) can be rewritten as

$$c^g + c^p < u1_t^*\beta - u1_t^*/\alpha = -u1_t^*(1/\alpha - \beta) \tag{E52}$$

The Eq. (B9) describes the necessary condition for $q_t^{g*(S)} > 0$ and $q_t^{p*(S)} > 0$ .Hence, the sufficient condition for exact relaxation of the complementary constraint of (E53) is

$$c^g + c^p > -u1_t^*(1/\alpha - \beta) \tag{E53}$$

Similarly, the equation (E53) is true for any $u1_t^* > 0$,where $\forall t \in \{1, 2, \cdots, T\}$.

The primal problem from the perspective of an electricity merchant who has energy storage and wind farms after relaxing the non-convex constraint is shown as follows:

$$\min\left(\sum_{t\in T}\left(\left(c^g - P_t\beta\right)q_t^g + \left(c^p + P_t/\alpha\right)q_t^p + \left(c^w - P_t\right)\cdot w_t\right)\right)$$
$$\text{s.t.}\begin{cases} -q_t^p \leq 0, q_t^p - \overline{Q}^p \leq 0 & (\underline{\chi}2_t^p, \overline{\chi}2_t^p) \\ -q_t^g \leq 0, q_t^g - \overline{Q}^g \leq 0 & (\underline{\chi}2_t^g, \overline{\chi}2_t^g) \\ -E_t + \underline{E} \leq 0, E_t - \overline{E} \leq 0 & (\underline{\theta}2_t, \overline{\theta}2_t) \\ E_t - q_t^g + q_t^p = E_{t+1}, & (\gamma 2_{t+1}) \\ -w_t \leq -\underline{W}, w_t \leq \overline{W} & (\underline{\omega}2_{it}, \overline{\omega}2_{it}) \\ \text{Where,}\left\{\underline{\chi}2_t^p, \underline{\chi}2_t^g, \underline{\theta}2_t, \underline{\omega}2_{it}, \overline{\chi}2_t^p, \overline{\chi}2_t^g, \overline{\theta}2_t, \overline{\omega}2_{it}\right\} \geq 0. \end{cases} \tag{E54}$$

We have the following Lagrange function based on the Eq. (E54):

$$L=\sum_{t\in T}\Big(\big(c^g-P_t\beta\big)q_t^g+\big(c^p+P_t/\alpha\big)q_t^p+\big(c^w-P_t\big)\cdot w_t\Big)+\gamma 2_{t+1}\big(E_{t+1}-E_t+q_t^g-q_t^p\big)$$
$$+\underline{\chi}2_t^p\cdot\big(-q_t^p\big)+\overline{\chi}2_t^p\cdot\big(q_t^p-\overline{Q}^p\big)+\underline{\chi}2_t^g\cdot\big(-q_t^g\big)+\overline{\chi}2_t^g\cdot\big(q_t^g-\overline{Q}^g\big)+\underline{\theta}2_t\cdot\big(-E_t+\underline{E}\big) \quad \text{(E55)}$$
$$+\overline{\theta}2_t\cdot\big(E_t-\overline{E}\big)+\underline{\omega}2_{it}\cdot\big(-w_t+\underline{W}\big)+\overline{\omega}2_{it}\big(w_t-\overline{W}\big)$$

Based on the KKT condition for the primal problem from the perspective of the merchant, the first-order derivative of the Lagrangian function with respect to energy storage discharging variable q_t^g and charging variable q_t^p must equal to zero. Hence, the following equation holds (∀t∈T)

$$\begin{cases}\partial L/\partial q_t^g\big|_{q_t^g=q_t^{g*(M)}}=c^g-P_t\beta+\gamma 2_{t+1}^*+\overline{\chi}2_t^{g*}-\underline{\chi}2_t^{g*}=0\\ \partial L/\partial q_t^p\big|_{q_t^p=q_t^{p*(M)}}=c^p+P_t/\alpha-\gamma 2_{t+1}^*+\overline{\chi}2_t^{p*}-\underline{\chi}2_t^{p*}=0\end{cases} \quad \text{(E56)}$$

Assume $q_t^{g*(M)}>0$ and $q_t^{p*(M)}>0$ represent the optimal solutions of the primal problem from perspectives of merchant. Then, we will get $\underline{\chi}2_t^{g*}=0$ and $\underline{\chi}2_t^{p*}=0$ because of the complementary slackness conditions. In Eq.(E56), there are $\underline{\chi}2_t^{g*}\geq 0$ and $\underline{\chi}2_t^{p*}\geq 0$. Summing Eq. (E56) and the following inequation holds

$$c^g+c^p<-P_t/\alpha+P_t\beta=-P_t(1/\alpha-\beta) \quad \text{(E57)}$$

The Eq.(E57) shows the necessary condition for $q_t^{g*(M)}>0$ and $q_t^{p*(M)}>0$. Hence, the sufficient condition for the exact relaxation of the complementary constraint of (E49) is shown as follows:

$$c^g+c^p>-P_t(1/\alpha-\beta) \quad \text{(E58)}$$

Therefore, for all positive electricity prices, discharging/generating and charging/pumping cannot happen simultaneously for profit-maximizing merchants.

Proof of Proposition 7.4:

(1) *For ISO*: incorporating the wind generation, we get the following primal problem after relaxing the non-convex constraint $q_t^p \cdot q_t^g = 0$:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^h \cdot g_{it}^h + \sum_{t=1}^{T}\left(c^g q_t^g + c^p q_t^p\right) + \sum_{t=1}^{T} c^w w_t\right)$$

$$\text{s.t.}\begin{cases} 0 \le q_t^p \le \overline{Q}^p, & (\underline{\chi}^p{}_t, \overline{\chi}^p{}_t) \\ 0 \le q_t^g \le \overline{Q}^g, & (\underline{\chi}^g{}_t, \overline{\chi}^g{}_t) \\ \underline{W} \le w_t \le \overline{W}, & (\underline{\nu}_t, \overline{\nu}_t) \\ E_t - q_t^g + q_t^p = E_{t+1}, & (\gamma_{t+1}) \qquad \forall t \in \{1, 2, \cdots, T\} \\ \sum_{i=1}^{M} g_{it}^h + w_t + q_t^g \beta - q_t^p / \alpha = D_t, & (\mu_t) \\ \underline{E} \le E_t \le \overline{E}, & (\underline{\theta}_t, \overline{\theta}_t) \\ \underline{G}_i{}^h \le g_{it}^h \le \overline{G}_i{}^h, & (\underline{\beta}_{it}, \overline{\beta}_{it}) \end{cases} \tag{E59}$$

Here, $\{\underline{\chi}^g{}_t, \overline{\chi}^g{}_t, \underline{\chi}^p{}_t, \overline{\chi}^p{}_t, \underline{\theta}_t, \overline{\theta}_t, \underline{\nu}_t, \overline{\nu}_t, \underline{\beta}_{it}, \overline{\beta}_{it}, \mu_t, \gamma_2, \gamma_{t+1}, \gamma_{T+1}\}$ represent the corresponding dual variables based on the constraints in (E59). The duality model of the primal problem of (E59) is shown as:

$$\max\left(\begin{array}{l} \sum_{t=1}^{T}\left(\overline{Q}^p \cdot \overline{\chi}_t^p + \overline{Q}^g \cdot \overline{\chi}_t^g + \underline{E} \cdot \underline{\theta}_t + \overline{E} \cdot \overline{\theta}_t + \underline{W} \cdot \underline{\nu}_t + \overline{W} \cdot \overline{\nu}_t\right) \\ + \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}_{it} + \overline{G}_i^h \cdot \overline{\beta}_{it}\right) + \sum_{t=1}^{T} \mu_t \cdot D_t - E_1 \cdot \gamma_2 + E_{T+1} \cdot \gamma_{T+1} \end{array}\right)$$

$$\text{s.t.}\begin{cases} \text{for } q_t^p\text{: } \underline{\chi}_t^p + \overline{\chi}_t^p - \mu_t / \alpha - \gamma_t = c^p, \\ \text{for } q_t^g\text{: } \underline{\chi}_t^g + \overline{\chi}_t^g + \mu_t \beta + \gamma_t = c^g, \\ \text{for } g_{it}^h\text{: } \underline{\beta}_{it} + \overline{\beta}_{it} + \mu_t = C_{it}^h, & \forall t \in \{1, 2, \cdots, T\} \\ \text{for } w_t\text{: } \underline{\nu}_t + \overline{\nu}_t + \mu_t = c^w, \\ \text{for } E_t\text{: } -\gamma_t + \gamma_{t+1} + \underline{\theta}_t + \overline{\theta}_t = 0, \\ \text{Where}, \left\{\underline{\chi}_t^p, \underline{\chi}_t^g, \underline{\theta}_t, \underline{\nu}_t, \underline{\beta}_{it}\right\} \ge 0; \ \left\{\overline{\chi}_t^p, \overline{\chi}_t^g, \overline{\theta}_t, \overline{\nu}_t, \overline{\beta}_{it}\right\} \le 0. \end{cases} \tag{E60}$$

(2) From merchant with storage and wind farm: After relaxing the non-convex constraint, the primal scheduling problem of the electricity merchant who manages both wind farms and energy storage is shown as follows:

$$\max \sum_{t=1}^{T} \left(P_t \cdot \left(q_t^g \beta - q_t^p / \alpha \right) - \left(c^g \cdot q_t^g + c^p \cdot q_t^p \right) + \left(P_t - c^w \right) \cdot w_t \right)$$

$$\Leftrightarrow \min \sum_{t \in T} \left(\left(c^g - P_t \beta \right) q_t^g + \left(c^p + P_t / \alpha \right) q_t^p + \left(c^w - P_t \right) \cdot w_t \right)$$

$$\text{s.t.} \begin{cases} 0 \le q_t^p \le \bar{Q}^p, & (\underline{\chi}^p{}_t, \bar{\chi}^p{}_t) \\ 0 \le q_t^g \le \bar{Q}^g, & (\underline{\chi}^g{}_t, \bar{\chi}^g{}_t) \\ \underline{W} \le w_t \le \bar{W}, & (\underline{\nu}_t, \bar{\nu}_t) \quad \forall t \in \{1, 2, \cdots, T\} \\ E_t - q_t^g + q_t^p = E_{t+1}, & (\gamma_{t+1}) \\ \underline{E} \le E_t \le \bar{E}, & (\underline{\theta}_t, \bar{\theta}_t) \end{cases} \tag{E61}$$

Similarly, $\{\underline{\chi}^g{}_t, \bar{\chi}^g{}_t, \underline{\chi}^p{}_t, \bar{\chi}^p{}_t, \underline{\theta}_t, \bar{\theta}_t, \underline{\nu}_t, \bar{\nu}_t, \mu_t, \gamma_2, \gamma_{t+1}, \gamma_{T+1}\}$ are the corresponding dual variables in model (E61). Therefore, the corresponding duality problem of the primal problem in (E61) is obtained below:

$$\max \sum_{t=1}^{T} \left(\bar{Q}^p \cdot \bar{\chi}_t^p + \bar{Q}^g \cdot \bar{\chi}_t^g + \underline{E} \cdot \underline{\theta}_t + \bar{E} \cdot \bar{\theta}_t + \underline{W} \cdot \underline{\nu}_t + \bar{W} \cdot \bar{\nu}_t \right) - E_1 \cdot \gamma_2 + E_{T+1} \cdot \gamma_{T+1}$$

$$\text{s.t.} \begin{cases} \text{for } q_t^p\text{: } \underline{\chi}_t^p + \bar{\chi}_t^p - \gamma_t = c^p + P_t / \alpha, \\ \text{for } q_t^g\text{: } \underline{\chi}_t^g + \bar{\chi}_t^g + \gamma_t = c^g - P_t \beta, \\ \text{for } w_t\text{: } \underline{\nu}_t + \bar{\nu}_t = c^w - P_t, \qquad \forall t \in \{1, 2, \cdots, T\} \\ \text{for } E_t\text{: } -\gamma_t + \gamma_{t+1} + \underline{\theta}_t + \bar{\theta}_t = 0, \\ \text{Where}, \underline{\chi}_t^p, \underline{\chi}_t^g, \underline{\theta}_t, \underline{\nu}_t \ge 0; \bar{\chi}_t^p, \bar{\chi}_t^g, \bar{\theta}_t, \bar{\nu}_t \le . \end{cases} \tag{E62}$$

We can split the new duality problem (E62) into two subproblems when the electricity merchant can make a perfect price forecast, or the ISO sends the cleared LMP to the merchant and assume $P_t = \mu^*_t = LMP_t$. Then, using the duality theorem, compare the best solution between $q_t^{*(S)} = \left\{ q_t^{p*(S)}, q_t^{g*(S)}, w_t^{*(S)} \right\}$ of ISO and $q_t^{*(M)} = \left\{ q_t^{p*(M)}, q_t^{g*(M)}, w_t^{*(M)} \right\}$ of merchant.

Thus, for $\forall i \in \{1, 2, \cdots, M\}$ the following conclusion can be drawn:

$$\begin{cases} 1) \text{ If } C_{it}^h < P_t = LMP_t \text{ and } g_{it}^{h*} = \overline{G}_i^h; \\ 2) \text{ or If } C_{it}^h = P_t = LMP_t \text{ and } \underline{G}_i^h \le g_{it}^{h*} \le \overline{G}_i^h; \\ 3) \text{ or If } C_{it}^h > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^h; \end{cases} \tag{E63}$$

$$\text{there is } \sum_{t=1}^{T} \left(\sum_{i=1}^{M} C_{it}^h g_{it}^{h*} - \sum_{i=1}^{M} P_t g_{it}^{h*} \right) = \sum_{t=1}^{T} \sum_{i=1}^{M} \left(\underline{G}_i^h \cdot \underline{\beta}^*_{it} + \overline{G}_i^h \cdot \overline{\beta}^*_{it} \right).$$

When $\sum_{t=1}^{T} \left(\sum_{i=1}^{M} C_{it}^h g_{it}^{h*} - \sum_{i=1}^{M} P_t g_{it}^{h*} \right) = \sum_{t=1}^{T} \sum_{i=1}^{M} \left(\underline{G}_i^h \cdot \underline{\beta}^*_{it} + \overline{G}_i^h \cdot \overline{\beta}^*_{it} \right)$ is holding, we achieve

$$\sum_{t=1}^{T} \left(\left(c^g - P_t \beta \right) q_t^{g*(S)} + \left(c^p + P_t / \alpha \right) q_t^{p*(S)} + \left(c^w - P_t \right) \cdot w_t^{*(S)} \right) = \sum_{t \in T} \left(\left(c^g - P_t \beta \right) q_t^{g*(M)} + \left(c^p + P_t / \alpha \right) q_t^{p*(M)} + \left(c^w - P_t \right) \cdot w_t^{*(M)} \right)$$

, which is the objection function of the primal problem from merchant perspective. In this situation, the merchant will also get the maximum profit. The relations between the *optimal actions* $q_t^{*(S)} = \left\{ q_t^{p*(S)}, q_t^{g*(S)}, w_t^{*(S)} \right\}$ *of ISO and* $q_t^{*(M)} = \left\{ q_t^{p*(M)}, q_t^{g*(M)}, w_t^{*(M)} \right\}$ *of electricity merchant can be drawn:*

(1) For the merchant who operates both wind farms and energy storage, *if the forecasted price aligns with the actual LMPs (i.e.,* $P_t = LMP_t, \forall t \in \{1, 2, \cdots, T\}$*) and when the primal problem from individual profit-maximizing has a unique optimal solution, we have the following results*

$$\begin{cases} \text{1) If } C_{it}^{h} < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^h, \text{ then } q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ \text{2) If } C_{it}^{h} = P_t = LMP_t \text{ and } \underline{G}_i^h \le g_{it}^{h*} \le \bar{G}_i^h, \text{ then } q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ \text{3) If } C_{it}^{h} > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^h, \text{ then } q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}. \end{cases} \quad \text{(E64)}$$

(2) For the merchant, *if the forecasted price aligns with the actual LMPs and when the primal problem from individual profit-maximizing in equation (E61) has multiple optimal solutions, we have the following relations*

$$\begin{aligned} &\max\left(\sum_{t=1}^{T} \left(LMP_t \left(q_t^g \beta - q_t^p / \alpha \right) + \left(LMP_t - c^w \right) \cdot w_t - \left(c^p \cdot q_t^p + c^g \cdot q_t^p \right) \right) \right) \\ &= \sum_{t=1}^{T} \left(LMP_t \left(q_t^{g*(S)} \beta - q_t^{p*(S)} / \alpha \right) + \left(LMP_t - c^w \right) \cdot w_t^{*(S)} - \left(c^p \cdot q_t^{p*(S)} + c^g \cdot q_t^{g*(S)} \right) \right) \qquad \text{(E65)} \\ &= \sum_{t=1}^{T} \left(LMP_t \left(q_t^{g*(M)} \beta - q_t^{p*(M)} / \alpha \right) + \left(LMP_t - c^w \right) \cdot w_t^{*(M)} - \left(c^p \cdot q_t^{p*(M)} + c^g \cdot q_t^{g*(M)} \right) \right) \end{aligned}$$

<u>Proof of Considering the Production Tax Credit (PTC)</u>

Discharging/Generating and Charging/Pumping cannot happen at the same period.

Considering the production tax credit (PTC) offered by the federal government in the US, the scheduling problem from the perspective of the ISO is shown as follows:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}-s\beta\right)\cdot q_{t}^{g}+\sum_{t=1}^{T}\left(c^{p}+s/\alpha\right)\cdot q_{t}^{p}+\sum_{t=1}^{T}\left(c^{w}-s\right)w_{t}\right)$$

$$\text{s.t.}\begin{cases} q_t^p-\alpha w_t\le 0, \\ 0\le q_t^p\le \bar{Q}^p, \\ 0\le q_t^g\le \bar{Q}^g, \\ \underline{W}\le w_t\le \bar{W}, \\ E_t-q_t^g+q_t^p=E_{t+1}, \qquad\qquad \forall t\in\{1,2,\cdots,T\} \\ \sum_{i=1}^{M}g_{it}^{h}+w_t+q_t^g\beta-q_t^p/\alpha=D_t, \\ \underline{E}\le E_t\le \bar{E}, \\ \underline{G}_i^{\,h}\le g_{it}^{h}\le \bar{G}_i^{\,h}, \\ q_t^g\cdot q_t^p=0. \end{cases} \tag{E66}$$

To qualify for PTC, wind farm merchant with storage will not be allowed to store energy in storage from the grid. The reward function of $R(q_t^p,q_t^g,w_t,P_t)$ can be shown as

$$R^{(PTC)}(q_t^p,q_t^g,w_t,P_t)=\begin{cases} -P_t(\frac{q_t^p}{\alpha}-w_t)-s(\frac{q_t^p}{\alpha}-w_t)-c_w w_t-c_p q_t^p \quad (0\le q_t^p\le \alpha w_t) \\ P_t\cdot(q_t^g\beta+w_t)-c_w w_t-s(-q_t^g\beta-w_t)-c_g q_t^g \ \ (q_t^g<0) \end{cases} \tag{E67}$$

$$\Leftrightarrow R^{(PTC)}(q_t^p,q_t^g,w_t,P_t)=\left(-P_t/\alpha-c_p-s/\alpha\right)\cdot q_t^p+\left(P_t\beta-c_g+s\beta\right)\cdot q_t^g+\left(P_t+s-c_w\right)\cdot w_t$$

Then, the objective function of electricity merchants is shown as follows:

$$
\max\sum_{t=1}^{T}\left(\left(P_t\beta-c_g+s\beta\right)\cdot q_t^g+\left(-P_t/\alpha-c_p-s/\alpha\right)\cdot q_t^p+\left(P_t+s-c_w\right)\cdot w_t\right)
$$

$$
\Leftrightarrow \min\sum_{t\in T}\left(\left(c^g-P_t\beta-s\beta\right)q_t^g+\left(c^p+P_t/\alpha+s/\alpha\right)q_t^p+\left(c^w-P_t-s\right)\cdot w_t\right)
$$

$$
\text{s.t.}\begin{cases} q_t^p-\alpha w_t\le 0, \\ 0\le q_t^p\le \bar{Q}^p, \\ 0\le q_t^g\le \bar{Q}^g, \\ \underline{W}\le w_t\le \bar{W}, \qquad \forall t\in\{1,2,\cdots,T\} \\ E_t-q_t^g+q_t^p=E_{t+1}, \\ \underline{E}\le E_t\le \bar{E}, \\ q_t^g\cdot q_t^p=0. \end{cases} \tag{E68}
$$

Suppose the charging/pumping and discharging/generating cannot happen in one period. Then, we have the following non-convex complementary constraint from the perspective of ISO or merchant:

$$
q_t^p\cdot q_t^g=0 \tag{E69}
$$

Recall the proof of Appendix A, we use the KKT condition to analyze sufficient conditions for such an exact relaxation of Eq. (E69). The primal problem from perspective of ISO after only relaxing the non-convex constraint is shown:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}-s\beta\right)\cdot q_{t}^{g}+\sum_{t=1}^{T}\left(c^{p}+s/\alpha\right)\cdot q_{t}^{p}+\sum_{t=1}^{T}\left(c^{w}-s\right)w_{t}\right)$$

$$\text{s.t.}\begin{cases}-q_{t}^{p}\le 0, q_{t}^{p}-\bar{Q}^{p}\le 0, q_{t}^{p}-\alpha w_{t}\le 0, & (\underline{\chi}1_{t}^{p},\bar{\chi}1_{t}^{p},\chi 1_{t}^{P})\\ -q_{t}^{g}\le 0, q_{t}^{g}-\bar{Q}^{g}\ \le 0 & (\underline{\chi}1_{t}^{g},\bar{\chi}1_{t}^{g})\\ -E_{t}+\underline{E}\le 0, E_{t}-\bar{E}\le 0 & (\underline{\theta}1_{t},\bar{\theta}1_{t})\\ -g_{it}^{h}+\underline{G}_{i}^{h}\le 0, g_{it}^{h}-\bar{G}_{i}^{h}\le 0 & (\underline{\beta}1_{it},\bar{\beta}1_{it})\\ D_{t}-\left(\sum_{i=1}^{M}g_{it}^{h}+q_{t}^{g}\beta+w_{t}-q_{t}^{p}/\alpha\right)=0, & (\mu 1_{t})\\ E_{t}-q_{t}^{g}+q_{t}^{p}=E_{t+1}, & (\gamma 1_{t+1})\\ -w_{t}\le -\underline{W}, w_{t}\le \bar{W} & (\underline{\omega}1_{it},\bar{\omega}1_{it})\\ \text{Where}, \chi 1_{t}^{P},\underline{\chi}1_{t}^{p},\underline{\chi}1_{t}^{g},\underline{\theta}1_{t},\underline{\beta}1_{it},\bar{\chi}1_{t}^{p},\bar{\chi}1_{t}^{g},\bar{\theta}1_{t},\bar{\beta}1_{it}\ge 0. & \end{cases} \tag{E70}$$

The Lagrange function of Eq. (E70) is obtained:

$$\begin{aligned}L=&\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{g}-s\beta\right)\cdot q_{t}^{g}+\sum_{t=1}^{T}\left(c^{p}+s/\alpha\right)\cdot q_{t}^{p}+\sum_{t=1}^{T}\left(c^{w}-s\right)w_{t}\\ &+\mu 1_{t}\left(D_{t}-\left(\sum_{i=1}^{M}g_{it}^{h}+w_{t}+q_{t}^{g}\beta-q_{t}^{p}/\alpha\right)\right)+\gamma 1_{t+1}\left(E_{t+1}-E_{t}+q_{t}^{g}-q_{t}^{p}\right)\\ &+\underline{\chi}1_{t}^{p}\cdot\left(-q_{t}^{p}\right)+\bar{\chi}1_{t}^{p}\cdot\left(q_{t}^{p}-\bar{Q}^{p}\right)+\chi 1_{t}^{P}\cdot\left(q_{t}^{p}-\alpha w_{t}\right)+\underline{\chi}1_{t}^{g}\cdot\left(-q_{t}^{g}\right)+\bar{\chi}1_{t}^{g}\cdot\left(q_{t}^{g}-\bar{Q}^{g}\right)\\ &+\underline{\theta}1_{t}\cdot\left(-E_{t}+\underline{E}\right)+\bar{\theta}1_{t}\cdot\left(E_{t}-\bar{E}\right)+\underline{\beta}1_{it}\cdot\left(-g_{it}^{h}+\underline{G}_{i}^{h}\right)+\bar{\beta}1_{it}\left(g_{it}^{h}-\bar{G}_{i}^{h}\right)\\ &+\underline{\omega}1_{it}\cdot\left(-w_{t}+\underline{W}\right)+\bar{\omega}1_{it}\left(w_{t}-\bar{W}\right)\end{aligned} \tag{E71}$$

In the KKT condition for the primal problem from the perspective of ISO, the first-order derivative of the Lagrangian function concerning PSH generating variables q_{t}^{g} and pumping variables q_{t}^{p} must equal to zero; hence the following equation holds, where, $\forall t\in\{1,2,\cdots,T\}$.

$$\begin{cases}\partial L/\partial q_{t}^{g}\Big|_{q_{t}^{g}=q_{t}^{g*(S)}}=c^{g}-s\beta-u1_{t}^{*}\beta+\gamma 1_{t+1}^{*}+\bar{\chi}1_{t}^{g*}-\underline{\chi}1_{t}^{g*}=0\\ \partial L/\partial q_{t}^{p}\Big|_{q_{t}^{p}=q_{t}^{p*(S)}}=c^{p}+s/\alpha+u1_{t}^{*}/\alpha-\gamma 1_{t+1}^{*}+\bar{\chi}1_{t}^{p*}-\underline{\chi}1_{t}^{p*}+\chi 1_{t}^{P*}=0\end{cases} \tag{E72}$$

Assume $q_t^{g*(S)} > 0$ and $q_t^{p*(S)} > 0$ are the optimal solutions of the primal problem from perspectives of ISO at time t. We also know there have $\underline{\chi}1_t^{g*} = 0$ and $\underline{\chi}1_t^{p*} = 0$ because of the complementary slackness conditions. When we combine two functions in Eq. (E72), we get the following equation.

$$c^g - s\beta + c^p + s/\alpha + \overline{\chi}1_t^{g*} + \overline{\chi}1_t^{p*} + \chi1_t^{P*} - u1_t^*\beta + u1_t^*/\alpha = 0 \quad \text{(E73)}$$

Because there are $\overline{\chi}1_t^{p*} \geq 0, \chi1_t^{P*} \geq 0$, and $\overline{\chi}1_t^{g*} \geq 0$, the Eq. (E73) can be rewritten as follows:

$$c^g - s\beta + c^p + s/\alpha < u1_t^*\beta - u1_t^*/\alpha = -u1_t^*(1/\alpha - \beta) \quad \text{(E74)}$$

The Eq.(E74) describes the necessary condition for $q_t^{g*(S)} > 0$ and $q_t^{p*(S)} > 0$. Hence, the sufficient condition for the exact relaxation of the complementary constraint of equation (E71) is

$$c^g + c^p + s/\alpha - s\beta > -u1_t^*(1/\alpha - \beta) \quad \text{(E75)}$$

Similarly, the Eq.(E75) is always true for $u1_t^* > 0$, where, $\forall t \in \{1, 2, \cdots, T\}$.

Similarly, the primal problem from the perspective of an electricity merchant who has energy storage and wind farms after relaxing the non-convex constraint is:

$$\min\left(\sum_{t\in T}\left(\left(c^g - P_t\beta - s\beta\right)q_t^g + \left(c^p + P_t/\alpha + s/\alpha\right)q_t^p + \left(c^w - P_t - s\right)\cdot w_t\right)\right)$$

$$\text{s.t.}\begin{cases} -q_t^p \leq 0, q_t^p - \overline{Q}^p \leq 0, q_t^p - \alpha w_t \leq 0, & (\underline{\chi}2_t^p, \overline{\chi}2_t^p, \chi2_t^P) \\ -q_t^g \leq 0, q_t^g - \overline{Q}^g \leq 0 & (\underline{\chi}2_t^g, \overline{\chi}2_t^g) \\ -E_t + \underline{E} \leq 0, E_t - \overline{E} \leq 0 & (\underline{\theta}2_t, \overline{\theta}2_t) \\ E_t - q_t^g + q_t^p = E_{t+1}, & (\gamma2_{t+1}) \\ -w_t \leq -\underline{W}, w_t \leq \overline{W} & (\underline{\omega}2_{it}, \overline{\omega}2_{it}) \\ \text{Where}, \chi2_t^P, \underline{\chi}2_t^p, \underline{\chi}2_t^g, \underline{\theta}2_t, \underline{\omega}2_{it}, \overline{\chi}2_t^p, \overline{\chi}2_t^g, \overline{\theta}2_t, \overline{\omega}2_{it} \geq 0. \end{cases} \quad \text{(E76)}$$

We have the following Lagrange function based on Eq. (E76):

$$\begin{aligned} L = \sum_{t\in T}\Big(&\left(c^g - P_t\beta - s\beta\right)q_t^g + \left(c^p + P_t/\alpha + s/\alpha\right)q_t^p + \left(c^w - P_t - s\right)\cdot w_t\Big) + \gamma 2_{t+1}\left(E_{t+1} - E_t + q_t^g - q_t^p\right) \\ &+ \underline{\chi}2_t^p\cdot\left(-q_t^p\right) + \bar{\chi}2_t^p\cdot\left(q_t^p - \bar{Q}^p\right) + \chi 2_t^p\cdot\left(q_t^p - \alpha w_t\right) + \underline{\chi}2_t^g\cdot\left(-q_t^g\right) + \bar{\chi}2_t^g\cdot\left(q_t^g - \bar{Q}^g\right) \\ &+ \underline{\theta}2_t\cdot\left(-E_t + \underline{E}\right) + \bar{\theta}2_t\cdot\left(E_t - \bar{E}\right) + \underline{\omega}2_{it}\cdot\left(-w_t + \underline{W}\right) + \bar{\omega}2_{it}\left(w_t - \bar{W}\right) \end{aligned} \quad (E77)$$

Based on the KKT condition for the primal problem from the perspective of the merchant, the first-order derivative of the Lagrangian function with respect to energy storage discharging variables q_t^g and charging variable q_t^p must equal to zero, where, $\forall t \in \{1, 2, \cdots, T\}$. Hence, the following equation holds.

$$\begin{cases} \partial L/\partial q_t^g\big|_{q_t^g = q_t^{g*(M)}} = c^g - P_t\beta - s\beta + \gamma 2_{t+1}^* + \bar{\chi}2_t^{g*} - \underline{\chi}2_t^{g*} = 0 \\ \partial L/\partial q_t^p\big|_{q_t^p = q_t^{p*(M)}} = c^p + P_t/\alpha + s/\alpha - \gamma 2_{t+1}^* + \bar{\chi}2_t^{p*} + \chi 2_t^{P*} - \underline{\chi}2_t^{p*} = 0 \end{cases} \quad (E78)$$

Assume $q_t^{g*(M)} > 0$ and $q_t^{p*(M)} > 0$ represent the optimal solutions of the primal problem from perspectives of merchant. Then, we will get $\underline{\chi}2_t^{g*} = 0$, $\underline{\chi}2_t^{p*} = 0$ based on the complementary slackness conditions.

In Eq.(E78), there are $\underline{\chi}2_t^{g*} \geq 0, \chi 2_t^{P*} \geq 0$ and $\underline{\chi}2_t^{p*} \geq 0$.

Merging two sub-equations in (E78) and the following inequation holds

$$c^g + c^p + s\cdot\left(1/\alpha - \beta\right) < -P_t/\alpha + P_t\beta = -P_t(1/\alpha - \beta) \quad (E79)$$

The Eq.(E79) describes the necessary condition for $q_t^{g*(M)} > 0$ and $q_t^{p*(M)} > 0$. Hence, the sufficient condition for the exact relaxation of the complementary constraint of (E71) is shown as follows:

$$c^g + c^p + s\cdot\left(1/\alpha - \beta\right) > -P_t(1/\alpha - \beta) \quad (E80)$$

Therefore, for all positive electricity prices, discharging/generating and charging/pumping cannot happen simultaneously for profit-maximizing merchants.

Proof of Proposition 7.5:

For ISO: considering the production tax credit (PTC), we get the following primal problem after relaxing the non-convex constraint $q_t^p \cdot q_t^g = 0$:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^h \cdot g_{it}^h + \sum_{t=1}^{T}\left(c^g - s\beta\right)\cdot q_t^g + \sum_{t=1}^{T}\left(c^p + s/\alpha\right)\cdot q_t^p + \sum_{t=1}^{T}\left(c^w - s\right) w_t\right)$$

$$\text{s.t.}\begin{cases} q_t^p - \alpha w_t \le 0, & (\chi^p{}_t) \\ 0 \le q_t^p \le \bar{Q}^p, & (\underline{\chi}^p{}_t, \bar{\chi}^p{}_t) \\ 0 \le q_t^g \le \bar{Q}^g, & (\underline{\chi}^g{}_t, \bar{\chi}^g{}_t) \\ \underline{W} \le w_t \le \bar{W}, & (\underline{\nu}_t, \bar{\nu}_t) \\ E_t - q_t^g + q_t^p = E_{t+1}, & (\gamma_{t+1}) \\ \sum_{i=1}^{M} g_{it}^h + w_t + q_t^g\beta - q_t^p/\alpha = D_t, & (\mu_t) \\ \underline{E} \le E_t \le \bar{E}, & (\underline{\theta}_t, \bar{\theta}_t) \\ \underline{G}_i{}^h \le g_{it}^h \le \bar{G}_i{}^h, & (\underline{\beta}_{it}, \bar{\beta}_{it}) \end{cases} \quad \forall t \in \{1, 2, \cdots, T\} \tag{E81}$$

The duality model of Eq. (E81) is shown as:

$$\max \begin{pmatrix} \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot\bar{\chi}_{t}^{p}+\bar{Q}^{g}\cdot\bar{\chi}_{t}^{g}+\underline{E}\cdot\underline{\theta}_{t}+\bar{E}\cdot\bar{\theta}_{t}+\underline{W}\cdot\underline{\nu}_{t}+\bar{W}\cdot\bar{\nu}_{t}\right) \\ +\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}+\bar{G}_{i}^{h}\cdot\bar{\beta}_{it}\right)+\sum_{t=1}^{T}\mu_{t}\cdot D_{t}-E_{1}\cdot\gamma_{2}+E_{T+1}\cdot\gamma_{T+1} \end{pmatrix}$$

$$\text{s.t.}\begin{cases} \text{for } q_t^p\text{: } \chi^p{}_t+\underline{\chi}_t^p+\bar{\chi}_t^p-\mu_t/\alpha-\gamma_t=c^p+s/\alpha, \\ \text{for } q_t^g\text{: } \underline{\chi}_t^g+\bar{\chi}_t^g+\mu_t\beta+\gamma_t=c^g-s\beta, \\ \text{for } g_{it}^h\text{: } \underline{\beta}_{it}+\bar{\beta}_{it}+\mu_t=C_{it}^h, \\ \text{for } w_t\text{: } -\alpha\chi^p{}_t+\underline{\nu}_t+\bar{\nu}_t+\mu_t=c^w-s, \\ \text{for } E_t\text{: } -\gamma_t+\gamma_{t+1}+\underline{\theta}_t+\bar{\theta}_t=0, \\ \text{Where,}\left\{\underline{\chi}_t^p,\underline{\chi}_t^g,\underline{\theta}_t,\underline{\nu}_t,\underline{\beta}_{it}\right\}\geq 0;\ \left\{\chi^p{}_t,\bar{\chi}_t^p,\bar{\chi}_t^g,\bar{\theta}_t,\bar{\nu}_t,\bar{\beta}_{it}\right\}\leq 0. \end{cases} \quad \forall t\in\{1,2,\cdots,T\} \tag{E82}$$

From merchant with storage and wind farm: After relaxing the non-convex constraint, the primal scheduling problem of the electricity merchant who manages both wind farms and energy storage when considering the PTC is shown as follows:

$$\max\sum_{t=1}^{T}\left((P_t\beta-c_g+s\beta)\cdot q_t^g+(-P_t/\alpha-c_p-s/\alpha)\cdot q_t^p+(P_t+s-c_w)\cdot w_t\right)$$

$$\Leftrightarrow \min\sum_{t\in T}\left((c^g-P_t\beta-s\beta)q_t^g+(c^p+P_t/\alpha+s/\alpha)q_t^p+(c^w-P_t-s)\cdot w_t\right)$$

$$\text{s.t.}\begin{cases} q_t^p-\alpha w_t\leq 0, & (\chi^p{}_t) \\ 0\leq q_t^p\leq\bar{Q}^p, & (\underline{\chi}^p{}_t,\bar{\chi}^p{}_t) \\ 0\leq q_t^g\leq\bar{Q}^g, & (\underline{\chi}^g{}_t,\bar{\chi}^g{}_t) \\ \underline{W}\leq w_t\leq\bar{W}, & (\underline{\nu}_t,\bar{\nu}_t) \\ E_t-q_t^g+q_t^p=E_{t+1}, & (\gamma_{t+1}) \\ \underline{E}\leq E_t\leq\bar{E}, & (\underline{\theta}_t,\bar{\theta}_t) \end{cases} \quad \forall t\in\{1,2,\cdots,T\} \tag{E83}$$

Similarly, $\{\chi^p{}_t,\underline{\chi}^g{}_t,\bar{\chi}^g{}_t,\underline{\chi}^p{}_t,\bar{\chi}^p{}_t,\underline{\theta}_t,\bar{\theta}_t,\underline{\nu}_t,\bar{\nu}_t,\mu_t,\gamma_2,\gamma_{t+1},\gamma_{T+1}\}$ are the corresponding dual variables in Eq.(E83). Therefore, the duality problem of the primal problem in Eq.(E83) is obtained below:

$$\max \sum_{t=1}^{T} (\bar{Q}^p \cdot \bar{\chi}_t^p + \bar{Q}^g \cdot \bar{\chi}_t^g + \underline{E} \cdot \underline{\theta}_t + \bar{E} \cdot \bar{\theta}_t + \underline{W} \cdot \underline{\nu}_t + \bar{W} \cdot \bar{\nu}_t) - E_1 \cdot \gamma_2 + E_{T+1} \cdot \gamma_{T+1}$$

$$\text{s.t.} \begin{cases} \text{for } q_t^p: \ \chi^p{}_t + \underline{\chi}_t^p + \bar{\chi}_t^p - \gamma_t = c^p + P_t / \alpha + s / \alpha, \\ \text{for } q_t^g: \ \underline{\chi}_t^g + \bar{\chi}_t^g + \gamma_t = c^g - P_t\beta - s\beta, \\ \text{for } w_t: \ -\alpha\chi^p{}_t + \underline{\nu}_t + \bar{\nu}_t = c^w - P_t - s, \qquad \forall t \\ \text{for } E_t: \ -\gamma_t + \gamma_{t+1} + \underline{\theta}_t + \bar{\theta}_t = 0, \\ \text{Where}, \underline{\chi}_t^p, \underline{\chi}_t^g, \underline{\theta}_t, \underline{\nu}_t \geq 0; \chi^p{}_t, \bar{\chi}_t^p, \bar{\chi}_t^g, \bar{\theta}_t, \bar{\nu}_t \leq 0 \end{cases} \tag{E84}$$

Similar to Appendix A, we can split the new duality problem of Eq.(C17) into two subproblems when the electricity merchant makes a perfect price forecast, or the ISO sends the cleared LMP to the merchant and assume $P_t = \mu^*{}_t = LMP_t, \forall t \in \{1, 2, \cdots, T\}$. Then, using the duality theorem, compare the best solution between $q_t{}^{*(S)} = \{q_t{}^{p*(S)}, q_t{}^{g*(S)}, w_t{}^{*(S)}\}$ of ISO and $q_t{}^{*(M)} = \{q_t{}^{p*(M)}, q_t{}^{g*(M)}, w_t{}^{*(M)}\}$ of merchant.

Thus, for $\forall i \in \{1, 2, \cdots, M\}$ the following conclusion can be drawn:

$$\begin{cases} \text{1) If } C_{it}^h < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^h; \\ \text{2) or If } C_{it}^h = P_t = LMP_t \text{ and } \underline{G}_i^h \leq g_{it}^{h*} \leq \bar{G}_i^h; \\ \text{3) or If } C_{it}^h > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^h; \end{cases} \tag{E85}$$

there are $\sum_{t=1}^{T}\left(\sum_{i=1}^{M} C_{it}^h g_{it}^{h*} - \sum_{i=1}^{M} P_t g_{it}^{h*}\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}^*{}_{it} + \bar{G}_i^h \cdot \bar{\beta}^*{}_{it}\right)$.

When $\sum_{t=1}^{T}\left(\sum_{i=1}^{M} C_{it}^h g_{it}^{h*} - \sum_{i=1}^{M} P_t g_{it}^{h*}\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^h \cdot \underline{\beta}^*{}_{it} + \bar{G}_i^h \cdot \bar{\beta}^*{}_{it}\right)$ is holding, we can achieve

$$\sum_{t=1}^{T}\left(\left(c^g - P_t\beta - s\beta\right) q_t^{g*(S)} + \left(c^p + P_t / \alpha + s / \alpha\right) q_t^{p*(S)} + \left(c^w - P_t - s\right) \cdot w_t^{*(S)}\right)$$
$$= \sum_{t \in T}\left(\left(c^g - P_t\beta - s\beta\right) q_t^{g*(M)} + \left(c^p + P_t / \alpha + s / \alpha\right) q_t^{p*(M)} + \left(c^w - P_t - s\right) \cdot w_t^{*(M)}\right)$$

The relations between the *optimal actions* $q_t^{*(S)} = \left\{ q_t^{p*(S)}, q_t^{g*(S)}, w_t^{*(S)} \right\}$ *of ISO and* $q_t^{*(M)} = \left\{ q_t^{p*(M)}, q_t^{g*(M)}, w_t^{*(M)} \right\}$ *of electricity merchant can be drawn:*

(1) For the merchant, *if the forecasted price aligns with the actual LMPs (i.e.,* $P_t = LMP_t, \forall t \in \{1, 2, \cdots, T\}$ *) and when the primal problem from individual profit-maximizing has a unique optimal solution, we will get*

$$\begin{cases} 1) \text{ If } C_{it}^h < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^h, \text{ then } q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ 2) \text{ If } C_{it}^h = P_t = LMP_t \text{ and } \underline{G}_i^h \le g_{it}^{h*} \le \bar{G}_i^h, \text{ then } q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ 3) \text{ If } C_{it}^h > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^h, \text{ then } q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}. \end{cases} \quad \text{(E86)}$$

(2) *If the lower bound of power generation of thermal generators is 0, that is* $\underline{G}_i^h = 0, \forall i \in \left\{1, 2, \cdots, M\right\}$, *then equation* (E86) *can be rewritten as*

$$\begin{cases} 1) \text{ If } C_{it}^h < P_t = LMP_t \text{ and } g_{it}^{h*} = \bar{G}_i^h,\ q_t^{p*(S)} = q_t^{p*(M)}, q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ 2) \text{ If } C_{it}^h = P_t = LMP_t \text{ and } 0 \le g_{it}^{h*} \le \bar{G}_i^h,\ q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}; \\ 3) \text{ If } C_{it}^h > P_t = LMP_t \text{ and } g_{it}^{h*} = 0,\ q_t^{p*(S)} = q_t^{p*(M)},\ q_t^{g*(S)} = q_t^{g*(M)}, w_t^{*(S)} = w_t^{*(M)}. \end{cases} \quad \text{(E87)}$$

(3) For the merchant, *if the forecasted price perfectly and when the primal problem from profit-maximizing in Eq.(E83) has multiple optimal solutions, we have*

$$\begin{aligned} &\max \sum_{t=1}^{T} \Big(\big(LMP_t\beta - c_g + s\beta \big) \cdot q_t^g + \big(-LMP_t / \alpha - c_p - s / \alpha \big) \cdot q_t^p + \big(LMP_t + s - c_w \big) \cdot w_t \Big) \\ &= \sum_{t=1}^{T} \Big(\big(LMP_t\beta - c_g + s\beta \big) \cdot q_t^{g*(S)} + \big(-LMP_t / \alpha - c_p - s / \alpha \big) \cdot q_t^{p*(S)} + \big(LMP_t + s - c_w \big) \cdot w_t^{*(S)} \Big) \\ &= \sum_{t=1}^{T} \Big(\big(LMP_t\beta - c_g + s\beta \big) \cdot q_t^{g*(M)} + \big(-LMP_t / \alpha - c_p - s / \alpha \big) \cdot q_t^{p*(M)} + \big(LMP_t + s - c_w \big) \cdot w_t^{*(M)} \Big) \end{aligned} \quad \text{(E88)}$$

Recall the proof of Appendix D, for $\forall i \in \left\{1, 2, \cdots, M\right\}, \forall t \in \left\{1, 2, \cdots, T\right\}$, the relations between the *optimal actions* $g_{it}^{h*(S)}$ *of ISO and* $g_{it}^{h*(M)}$ *of generator-I are still holding.*

(4) For the generator, if the forecasted price matches the actual LMPs, *when the primal problem from individual profit-maximizing has a unique optimal solution:*

$$\begin{cases} 1)\ \text{If } C_{it}^{h} < P_t = LMP_t \text{ and } g_{it}^{h*} = \overline{G}_i^{h},\ g_{it}^{h*(S)} = g_{it}^{h*(M)}; \\ 2)\ \text{If } C_{it}^{h} = P_t = LMP_t \text{ and } \underline{G}_i^{h} \le g_{it}^{h*} \le \overline{G}_i^{h},\ g_{it}^{h*(S)} = g_{it}^{h*(M)}\}; \\ 3)\ \text{If } C_{it}^{h} > P_t = LMP_t \text{ and } g_{it}^{h*} = \underline{G}_i^{h},\ g_{it}^{h*(S)} = g_{it}^{h*(M)}. \end{cases} \quad \text{(E89)}$$

(5) For the generator, if the forecasted price aligns with the actual LMPs, *when the primal problem from individual profit-maximizing has multiple optimal solutions:*

$$\max \sum_{t=1}^{T} \left(LMP_t - C_{It}^{h}\right) \cdot g_{It}^{h} = \sum_{t=1}^{T} \left(LMP_t - C_{It}^{h}\right) \cdot g_{It}^{h*(S)} = \sum_{t=1}^{T} \left(LMP_t - C_{It}^{h}\right) \cdot g_{It}^{h*(G)} \quad \text{(E90)}$$

<u>Proof of Scenaro for a Wind Farm and PSH Merchant With Two Linked Reservoirs</u>

The scheduling model for merchant operating a wind farm and PSH facility with two connected reservoirs is obtained:

$$\max \sum_{t=1}^{T} \left(P_t \cdot \left(q_t^r \beta - q_t^p / \alpha\right) - \left(c^r \cdot q_t^r + c^p \cdot q_t^p\right) + \sum_{t=1}^{T} \left(P_t - c^w\right) \cdot w_t \right)$$

$$\Leftrightarrow \min \left(\sum_{t \in T} \left(\left(c^r - P_t \beta\right) q_t^r + \left(c^p + P_t / \alpha\right) q_t^p \right) + \sum_{t=1}^{T} \left(c^w - P_t\right) \cdot w_t \right)$$

$$\text{s.t.} \begin{cases} q_t^p \ge 0,\ \ q_t^p \le \overline{Q}^p, q_t^p \le E_t^L, \\ q_t^r \ge 0,\ q_t^r \le \overline{Q}^r, q_t^r \le E_t^U, \\ E_t^U \ge 0, E_t^U \le \overline{E}^U, \\ E_t^L \ge 0, E_t^L \le \overline{E}^U, \\ E_t^U - q_t^r + q_t^p = E_{t+1}^U, \\ E_t^L + q_t^r - q_t^p = E_{t+1}^L, \\ \underline{W} \le w_t \le \overline{W}, \\ q_t^r \cdot q_t^p = 0. \end{cases} \quad \text{(E91)}$$

The scheduling model from ISO's perspective when considering a wind farm and the PSH facility with two connected reservoirs (i.e., both the upper reservoir and the lower reservoir) is shown below:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{r}q_{t}^{r}+c^{p}q_{t}^{p}\right)+\sum_{t=1}^{T}c^{w}\cdot w_{t}\right)$$

$$\text{s.t.}\begin{cases} q_t^p \ge 0,\ \ q_t^p \le \bar{Q}^p, q_t^p \le E_t^L, \\ q_t^r \ge 0,\ q_t^r \le \bar{Q}^r, q_t^r \le E_t^U, \\ E_t^U \ge 0, E_t^U \le \bar{E}^U \\ E_t^L \ge 0, E_t^L \le \bar{E}^U \\ g_{it}^h \ge \underline{G}_i^h,\ \ g_{it}^h \le \bar{G}_i^h, \\ \sum_{i=1}^{M} g_{it}^h + q_t^r\beta - q_t^p/\alpha + w_t = D_t, \\ E_t^U - q_t^r + q_t^p = E_{t+1}^U, \\ E_t^L + q_t^r - q_t^p = E_{t+1}^L, \\ \underline{W} \le w_t \le \bar{W}, \\ q_t^r \cdot q_t^p = 0. \end{cases} \tag{E92}$$

Proof of Discharging and Charging cannot happen at the same period.

Because a PSH can only pump/discharge or /generate/release at one period, we have the following non-convex complementary constraint from the perspective of ISO/merchant:

$$q_t^p \cdot q_t^r = 0 \tag{E93}$$

The KKT condition is then used to examine sufficient conditions for an exact relaxation of $q_t^p \cdot q_t^r = 0$ in Eq. (E93). The ISO's primal UCED problem and its corresponding dual variables for all constraints are:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{r}q_{t}^{r}+c^{p}q_{t}^{p}\right)+\sum_{t=1}^{T}c^{w}\cdot w_{t}\right)$$

$$\text{s.t.}\begin{cases}-q_t^p\le 0,\ q_t^p-\bar{Q}^p\le 0,\ q_t^p-E_t^L\le 0 & (\underline{\chi}1_t^p,\bar{\chi}1_t^{p1},\bar{\chi}1_t^{p2})\\ -q_t^r\le 0,\ q_t^r-\bar{Q}^r\le 0, q_t^r-E_t^U\le 0 & (\underline{\chi}1_t^r,\bar{\chi}1_t^{r1},\bar{\chi}1_t^{r2})\\ -E_t^U\le 0,\ E_t^U-\bar{E}^U\le 0 & (\underline{\theta}1_t^U,\bar{\theta}1_t^U)\\ -E_t^L\le 0,\ E_t^L-\bar{E}^L\le 0 & (\underline{\theta}1_t^L,\bar{\theta}1_t^L)\\ -g_{it}^h+\underline{G}_i^h\le 0, g_{it}^h-\bar{G}_i^h\le 0 & (\underline{\beta}1_{it},\bar{\beta}1_{it})\\ D_t-\left(\sum_{i=1}^{M}g_{it}^h+q_t^r\beta-q_t^p/\alpha-w_t\right)=0 & (\mu1_t)\\ E_{t+1}^U-E_t^U+q_t^r-q_t^p=0 & (\gamma1_{t+1}^U)\\ E_{t+1}^L-E_t^L-q_t^r+q_t^p=0 & (\gamma1_{t+1}^L)\\ -w_t\le-\underline{W}, w_t\le\bar{W} & (\underline{\omega}1_{it},\bar{\omega}1_{it})\\ \text{where}, \underline{\chi}1_t^p,\underline{\chi}1_t^r,\underline{\beta}1_{it},\underline{\theta}1_t^U,\underline{\theta}1_t^L,\bar{\chi}1_t^{p1},\bar{\chi}1_t^{p2},\bar{\chi}1_t^{r1},\bar{\chi}1_t^{r2},\bar{\beta}1_{it},\bar{\theta}1_t^U,\bar{\theta}1_t^L\ge 0.\end{cases}\tag{E94}$$

Similarly, we have the following Lagrangian functions:

$$\begin{aligned}L=&\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{r}q_{t}^{r}+c^{p}q_{t}^{p}\right)+\sum_{t=1}^{T}c^{w}\cdot w_{t}\\&+\sum_{t=1}^{T}\left(\mu1_t\left(D_t-\left(\sum_{i=1}^{M}g_{it}^h+q_t^r\beta-q_t^p/\alpha-w_t\right)\right)\right)\\&+\sum_{t=1}^{T}\left[\underline{\chi}1_t^p\cdot\left(-q_t^p\right)+\bar{\chi}1_t^{p1}\cdot\left(q_t^p-\bar{Q}^p\right)+\bar{\chi}1_t^{p2}\cdot\left(q_t^p-E_t^L\right)+\underline{\chi}1_t^r\cdot\left(-q_t^r\right)+\bar{\chi}1_t^r\cdot\left(q_t^r-\bar{Q}^r\right)+\bar{\chi}1_t^{r2}\cdot\left(q_t^r-E_t^U\right)\right]\\&+\sum_{t=1}^{T}\left[\underline{\theta}1_t^U\cdot\left(-E_t^U\right)+\bar{\theta}1_t^U\cdot\left(E_t^U-\bar{E}^U\right)+\underline{\theta}1_t^L\cdot\left(-E_t^L\right)+\bar{\theta}1_t^L\cdot\left(E_t^L-\bar{E}^L\right)\right]\\&+\sum_{t=1}^{T}\sum_{i=1}^{M}\left[\underline{\beta}1_{it}\cdot\left(-g_{it}^h+\underline{G}_i^h\right)+\bar{\beta}1_{it}\left(g_{it}^h-\bar{G}_i^h\right)\right]\\&+\sum_{t=1}^{T}[\gamma1_{t+1}^U\left(E_{t+1}^U-E_t^U+q_t^r-q_t^p\right)+\gamma1_{t+1}^L\left(E_{t+1}^L-E_t^L-q_t^r+q_t^p\right)+\underline{\omega}1_{it}\cdot\left(-w_t+\underline{W}\right)+\bar{\omega}1_{it}\left(w_t-\bar{W}\right)]\end{aligned}\tag{E95}$$

For the ISO's model, the first-order derivative of the Lagrangian function with respect to PHES pumping/releasing variables q_t^p and q_t^r must equal zero in the KKT condition for the primary issue, hence the following equation holds.

$$\begin{cases} \partial L/\partial q_t^r\Big|_{q_t^r=q_t^{r*(S)}} = c^r - u1_t^*\beta - \underline{\chi}1_t^{r*} + \overline{\chi}1_t^{r1*} + \overline{\chi}1_t^{r2*} + \gamma 1_{t+1}^{U*} - \gamma 1_{t+1}^{L*} = 0 \\ \partial L/\partial q_t^p\Big|_{q_t^p=q_t^{p*(S)}} = c^p + u1_t^*/\alpha - \underline{\chi}1_t^{p*} + \overline{\chi}1_t^{p1*} + \overline{\chi}1_t^{p2*} - \gamma 1_{t+1}^{U*} + \gamma 1_{t+1}^{L*} = 0 \end{cases} \tag{E96}$$

Assume that at decision time *t*, $q_t^{r*(S)} > 0$ and $q_t^{p*(S)} > 0$ exist in the ideal solution of the primal problem from the perspective of ISO. We have $\underline{\chi}1_t^{r*} = 0$ and $\underline{\chi}1_t^{p*} = 0$ by using the complementary slackness conditions. The following equation is obtained by adding two sub-equations in Eq.(E96).

$$c^r + c^p + \overline{\chi}1_t^{r1*} + \overline{\chi}1_t^{r2*} + \overline{\chi}1_t^{p1*} + \overline{\chi}1_t^{p2*} - u1_t^*\beta + u1_t^*/\alpha = 0 \tag{E97}$$

Because $\left\{\overline{\chi}1_t^{r1*}, \overline{\chi}1_t^{r2*}, \overline{\chi}1_t^{p1*}, \overline{\chi}1_t^{p2*}\right\} \geq 0$ are holding, equation (E97) can be rewritten as

$$c^r + c^p < u1_t^*\beta - u1_t^*/\alpha = -u1_t^*(1/\alpha - \beta) \tag{E98}$$

Equation (E98) describes the necessary condition for $q_t^{r*(S)} > 0$ and $q_t^{p*(S)} > 0$ to hold. Hence, the sufficient condition for the exact relaxation of the complementary constraint of equation (E99) is

$$c^r + c^p > -u1_t^*(1/\alpha - \beta) \tag{E99}$$

which is true when $u1_t^* \geq 0$ for all t∈T. Similarly, the primal problem from the perspective of the merchant with its corresponding constraint coefficients is shown below:

$$\min\left(\sum_{t\in T}\left(\left(c^r - P_t\beta\right)q_t^r + \left(c^p + P_t/\alpha\right)q_t^p\right) + \left(c^w - P_t\right)w_t\right)$$

$$\text{s.t.}\begin{cases} -q_t^p \le 0,\ q_t^p - \bar{Q}^p \le 0,\ q_t^p - E_t^L \le 0 & (\underline{\chi}2_t^p, \bar{\chi}2_t^{p1}, \bar{\chi}2_t^{p2}) \\ -q_t^r \le 0,\ q_t^r - \bar{Q}^r \le 0, q_t^r - E_t^U \le 0 & (\underline{\chi}2_t^r, \bar{\chi}2_t^{r1}, \bar{\chi}2_t^{r2}) \\ -E_t^U \le 0,\ E_t^U - \bar{E}^U \le 0 & (\underline{\theta}2_t^U, \bar{\theta}2_t^U) \\ -E_t^L \le 0,\ E_t^L - \bar{E}^L \le 0 & (\underline{\theta}2_t^L, \bar{\theta}2_t^L) \\ E_{t+1}^U - E_t^U + q_t^r - q_t^p = 0 & (\gamma 2_{t+1}^U) \\ E_{t+1}^L - E_t^L - q_t^r + q_t^p = 0 & (\gamma 2_{t+1}^L) \\ -w_t \le -\underline{W}, w_t \le \bar{W} & (\underline{\omega}1_{it}, \bar{\omega}1_{it}) \\ \text{where}, \underline{\chi}2_t^p, \underline{\chi}2_t^r, \underline{\theta}2_t^U, \underline{\theta}2_t^L, \bar{\chi}2_t^{p1}, \bar{\chi}2_t^{p2}, \bar{\chi}2_t^{r1}, \bar{\chi}2_t^{r2}, \bar{\theta}2_t^U, \bar{\theta}2_t^L \ge 0. \end{cases} \quad \text{(E100)}$$

We have the following Lagrange functions as shown in equation (E101):

$$\begin{aligned} L = &\sum_{t=1}^{T}\left[\begin{array}{l}\left((c^r - P_t\beta)q_t^r + (c^p + P_t/\alpha)q_t^p + (c^w - P_t)w_t\right) \\ +\gamma 2_{t+1}^U(E_{t+1}^U - E_t^U + q_t^r - q_t^p) + \gamma 2_{t+1}^L(E_{t+1}^L - E_t^L - q_t^r + q_t^p)\end{array}\right] \\ &+\sum_{t=1}^{T}\left[\begin{array}{l}\underline{\chi}2_t^p\cdot(-q_t^p) + \bar{\chi}2_t^{p1}\cdot(q_t^p - \bar{Q}^p) + \bar{\chi}2_t^{p2}\cdot(q_t^p - E_t^L) \\ +\underline{\chi}2_t^r\cdot(-q_t^r) + \bar{\chi}2_t^r\cdot(q_t^r - \bar{Q}^r) + \bar{\chi}2_t^{r2}\cdot(q_t^r - E_t^U)\end{array}\right] \\ &+\sum_{t=1}^{T}\left[\begin{array}{l}\underline{\theta}2_t^U\cdot(-E_t^U) + \bar{\theta}2_t^U\cdot(E_t^U - \bar{E}^U) + \underline{\theta}2_t^L\cdot(-E_t^L) + \bar{\theta}2_t^L\cdot(E_t^L - \bar{E}^L) \\ +\underline{\omega}1_{it}\cdot(-w_t + \underline{W}) + \bar{\omega}1_{it}(w_t - \bar{W})\end{array}\right]\end{aligned} \quad \text{(E101)}$$

In the KKT condition for the primal problem from perspectives of ISO, the first-order derivative of the Lagrangian function with respect to PHES releasing/pumping variables q_t^r and q_t^p must equal to zero, hence the following equation holds ($\forall t \in T$)

$$\begin{cases} \partial L/\partial q_t^r\big|_{q_t^r = q_t^{r*(M)}} = \left(c^r - P_t\right)\beta - \underline{\chi}2_t^{r*} + \bar{\chi}2_t^{r1*} + \bar{\chi}2_t^{r2*} + \gamma 2_{t+1}^{U*} - \gamma 2_{t+1}^{L*} = 0 \\ \partial L/\partial q_t^p\big|_{q_t^p = q_t^{p*(M)}} = \left(c^p + P_t\right)/\alpha - \underline{\chi}2_t^{p*} + \bar{\chi}2_t^{p1*} + \bar{\chi}2_t^{p2*} - \gamma 2_{t+1}^{U*} + \gamma 2_{t+1}^{L*} = 0 \end{cases} \quad \text{(E102)}$$

Assume there exist $q_t^{r*(M)} > 0$ and $q_t^{p*(M)} > 0$ at time t in the optimal solution of the primal problem from perspectives of Owner. We have $\underline{\chi}2_t^{r*} = 0$ and $\underline{\chi}2_t^{p*} = 0$ because of the complementary slackness conditions. Summing two sub-equations (E102) yields the following equation

$$\left(c^{r}-P_{t}\beta\right)+\left(c^{p}+P_{t}/\alpha\right)+\bar{\chi}2_{t}^{r1*}+\bar{\chi}2_{t}^{r2*}++\bar{\chi}2_{t}^{p1*}+\bar{\chi}2_{t}^{p2*}=0 \qquad \text{(E103)}$$

Because there are $\left\{\bar{\chi}2_{t}^{r1*},\bar{\chi}2_{t}^{r2*},\bar{\chi}2_{t}^{p1*},\bar{\chi}2_{t}^{p2*}\right\}\geq 0$, thus, Eq.(E103) can be rewritten as

$$c^{r}+c^{p}<-P_{t}/\alpha+P_{t}\beta=-P_{t}(1/\alpha-\beta) \qquad \text{(E104)}$$

The equation (E104) describes the necessary condition for $q_{t}^{r*(M)}>0$ and $q_{t}^{p*(M)}>0$. Hence, the sufficient condition for the exact relaxation of equation (E101) is

$$c^{r}+c^{p}>-P_{t}(1/\alpha-\beta) \qquad \text{(E105)}$$

which is true when $P_{t}\geq 0$ for all $t\in T$. Therefore, for all positive electricity prices, discharging/releasing and charging/pumping cannot happen simultaneously.

The relationship of optimal economics dispatch between ISO and merchant

From the respective of ISO: The primal model from the perspective of ISO is:

$$\min\left(\sum_{t=1}^{T}\sum_{i=1}^{M}C_{it}^{h}\cdot g_{it}^{h}+\sum_{t=1}^{T}\left(c^{r}q_{t}^{r}+c^{p}q_{t}^{p}\right)+\sum_{t=1}^{T}c^{w}w_{t}\right)$$

$$\text{s.t.}\begin{cases}
q_t^p \ge 0,\ \ q_t^p \le \bar{Q}^p, q_t^p \le E_t^L, & (\underline{\chi}^p{}_t, \bar{\chi}^{p1}{}_t, \bar{\chi}^{p2}{}_t) \\
q_t^r \ge 0,\ q_t^r \le \bar{Q}^r, q_t^r \le E_t^U, & (\underline{\chi}^r{}_t, \bar{\chi}^{r1}{}_t, \bar{\chi}^{r2}{}_t) \\
E_t^U \ge 0, E_t^U \le \bar{E}^U & (\underline{\theta}_t^U, \bar{\theta}_t^U) \\
E_t^L \ge 0, E_t^L \le \bar{E}^U & (\underline{\theta}_t^L, \bar{\theta}_t^L) \\
g_{it}^h \ge \underline{G}_i^h,\ \ g_{it}^h \le \bar{G}_i^h, & (\underline{\beta}_{it}, \bar{\beta}_{it}) \\
\sum_{i=1}^{M} g_{it}^h + q_t^r\beta - q_t^p/\alpha = D_t, & (\mu_t) \\
E_t^U - q_t^r + q_t^p = E_{t+1}^U, & (\gamma_{t+1}^U) \\
E_t^L + q_t^r - q_t^p = E_{t+1}^L, & (\gamma_{t+1}^L) \\
\underline{W} \le w_t \le \bar{W}, & (\underline{\nu}_t, \bar{\nu}_t)
\end{cases} \tag{E106}$$

Here, on the right-side hand of constraints, $\left\{\underline{\chi}_t^p, \bar{\chi}_t^{p1}, \bar{\chi}_t^{p2}, \underline{\chi}_t^r, \bar{\chi}_t^{r1}, \bar{\chi}_t^{r2}, \underline{\theta}_t^U, \bar{\theta}_t^U, \underline{\theta}_t^L, \bar{\theta}_t^L, \underline{\beta}_{it}, \bar{\beta}_{it}, \mu_t, \gamma_2^U, \gamma_{t+1}^U, \gamma_{T+1}^U, \gamma_2^L, \gamma_{t+1}^L, \gamma_{T+1}^L, \underline{\nu}_t, \bar{\nu}_t\right\}$ denotes the corresponding dual variables based on the constraints in Eq.(E106). Therefore, based on the relaxed primal problem in Eq. (E106), by using strong duality theory the duality problem is shown as:

$$\max\sum_{t=1}^{T}(\bar{Q}^{p}\cdot\bar{\chi}_{t}^{p1}+E_{t}^{L}\cdot\bar{\chi}_{t}^{p2}+\bar{Q}^{r}\cdot\bar{\chi}_{t}^{r1}+E_{t}^{U}\cdot\bar{\chi}_{t}^{r2}+\bar{E}^{U}\cdot\bar{\theta}_{t}^{U}+\bar{E}^{L}\cdot\bar{\theta}_{t}^{L}+\underline{W}\cdot\underline{\nu}_{t}+\bar{W}\cdot\bar{\nu}_{t})$$
$$+\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot\underline{\beta}_{it}+\bar{G}_{i}^{h}\cdot\bar{\beta}_{it}\right)+\sum_{t=1}^{T}\mu_{t}\cdot D_{t}-E_{1}^{U}\cdot\gamma_{2}^{U}+E_{T+1}^{U}\cdot\gamma_{T+1}^{U}-E_{1}^{L}\cdot\gamma_{2}^{L}+E_{T+1}^{L}\cdot\gamma_{T+1}^{L}$$
$$\text{s.t.}\begin{cases}\text{for } q_{t}^{p}\text{: }\underline{\chi}_{t}^{p}+\bar{\chi}_{t}^{p1}+\bar{\chi}_{t}^{p2}-\mu_{t}/\alpha+\gamma_{t+1}^{U}-\gamma_{t+1}^{L}=c^{p}/\alpha,\\ \text{for } q_{t}^{r}\text{: }\underline{\chi}_{t}^{r}+\bar{\chi}_{t}^{r1}+\bar{\chi}_{t}^{r2}+\mu_{t}\beta-\gamma_{t+1}^{U}+\gamma_{t+1}^{L}=c^{r}\beta,\\ \text{for } g_{it}^{h}\text{: }\underline{\beta}_{it}+\bar{\beta}_{it}+\mu_{t}=C_{it}^{h},\\ \text{for } E_{t}^{U}:-\bar{\chi}_{t}^{r2}+\underline{\theta}_{t}^{U}+\bar{\theta}_{t}^{U}-\gamma_{t}^{U}+\gamma_{t+1}^{U}=0, \qquad \forall t\in\{1,2,\cdots,T\}\\ \text{for } E_{t}^{L}:-\bar{\chi}_{t}^{p2}+\underline{\theta}_{t}^{L}+\bar{\theta}_{t}^{L}-\gamma_{t}^{L}+\gamma_{t+1}^{L}=0,\\ \text{for } w_{t}:\ \underline{\nu}_{t}+\bar{\nu}_{t}+\mu_{t}=c^{w},\\ \text{Where, }\underline{\chi}_{t}^{p},\underline{\chi}_{t}^{r},\underline{\beta}_{it},\underline{\theta}_{t}^{U},\underline{\theta}_{t}^{L},\underline{\nu}_{t}\geq 0;\ \bar{\chi}_{t}^{p1},\bar{\chi}_{t}^{p2},\bar{\chi}_{t}^{r1},\bar{\chi}_{t}^{r2},\bar{\beta}_{it},\bar{\theta}_{t}^{U},\bar{\theta}_{t}^{L},\bar{\nu}_{t}\leq 0.\end{cases} \tag{E107}$$

From the respective of electricity merchant: The primal objective function of electricity merchant with a PSH is shown as follows:

$$\max\sum_{t=1}^{T}\left(LMP_{t}\cdot\left(q_{t}^{r}\beta-q_{t}^{p}/\alpha\right)-\left(c^{g}\cdot q_{t}^{r}+c^{p}\cdot q_{t}^{p}\right)+\left(LMP_{t}-c^{w}\right)w_{t}\right)$$
$$\Leftrightarrow\min\left(\sum_{t\in T}\left(\left(c^{r}-P_{t}\beta\right)q_{t}^{r}+\left(c^{p}+P_{t}/\alpha\right)q_{t}^{p}\right)+\sum_{t=1}^{T}\left(c^{w}-P_{t}\right)w_{t}\right)$$
$$\text{s.t.}\begin{cases}q_{t}^{p}\geq 0,\ q_{t}^{p}\leq\bar{Q}^{p},q_{t}^{p}\leq E_{t}^{L}, & (\underline{\chi}_{t}^{p},\bar{\chi}_{t}^{p1},\bar{\chi}_{t}^{p2})\\ q_{t}^{r}\geq 0,\ q_{t}^{r}\leq\bar{Q}^{r},q_{t}^{r}\leq E_{t}^{U}, & (\underline{\chi}_{t}^{r},\bar{\chi}_{t}^{r1},\bar{\chi}_{t}^{r2})\\ E_{t}^{U}\geq 0,E_{t}^{U}\leq\bar{E}^{U} & (\underline{\theta}_{t}^{U},\bar{\theta}_{t}^{U})\\ E_{t}^{L}\geq 0,E_{t}^{L}\leq\bar{E}^{U} & (\underline{\theta}_{t}^{L},\bar{\theta}_{t}^{L})\\ E_{t}^{U}-q_{t}^{r}+q_{t}^{p}=E_{t+1}^{U}, & (\gamma_{t+1}^{U})\\ E_{t}^{L}+q_{t}^{r}-q_{t}^{p}=E_{t+1}^{L}, & (\gamma_{t+1}^{L})\\ \underline{W}\leq w_{t}\leq\bar{W}, & (\underline{\nu}_{t},\bar{\nu}_{t})\end{cases} \tag{E108}$$

In Eq. (E108), on the right-side hand of the constraints,

$\left\{\underline{\chi}_t^p, \overline{\chi}_t^{p1}, \overline{\chi}_t^{p2}, \underline{\chi}_t^r, \overline{\chi}_t^{r1}, \overline{\chi}_t^{r2}, \underline{\theta}_t^U, \overline{\theta}_t^U, \underline{\theta}_t^L, \overline{\theta}_t^L, \gamma_2^U, \gamma_{t+1}^U, \gamma_{T+1}^U, \gamma_2^L, \gamma_{t+1}^L, \gamma_{T+1}^L, \underline{\nu}_t, \overline{\nu}_t\right\}$ are the corresponding dual variables. Therefore, for the primal problem in Eq.(E108), the duality problem is obtained below:

$$\max\left(\begin{array}{c}\sum_{t=1}^{T}\left(\overline{Q}^p \cdot \overline{\chi}_t^{p1} + E_t^L \cdot \overline{\chi}_t^{p2} + \overline{Q}^r \cdot \overline{\chi}_t^{r1} + E_t^U \cdot \overline{\chi}_t^{r2} + \overline{E}^U \cdot \overline{\theta}_t^U + \overline{E}^L \cdot \overline{\theta}_t^L + \underline{W} \cdot \underline{\nu}_t + \overline{W} \cdot \overline{\nu}_t\right) \\ -E_1^U \cdot \gamma_2^U + E_{T+1}^U \cdot \gamma_{T+1}^U - E_1^L \cdot \gamma_2^L + E_{T+1}^L \cdot \gamma_{T+1}^L\end{array}\right)$$

$$\text{s.t.}\begin{cases}\text{for } q_t^p\text{: } \underline{\chi}_t^p + \overline{\chi}_t^{p1} + \overline{\chi}_t^{p2} + \gamma_{t+1}^U - \gamma_{t+1}^L = \left(c^p + P_t\right)/\alpha, \\ \text{for } q_t^r\text{: } \underline{\chi}_t^r + \overline{\chi}_t^{r1} + \overline{\chi}_t^{r2} - \gamma_{t+1}^U + \gamma_{t+1}^L = \left(c^r - P_t\right)\beta, \\ \text{for } E_t^U: -\overline{\chi}_t^{r2} + \underline{\theta}_t^U + \overline{\theta}_t^U - \gamma_t^U + \gamma_{t+1}^U = 0, \\ \text{for } E_t^L: -\overline{\chi}_t^{p2} + \underline{\theta}_t^L + \overline{\theta}_t^L - \gamma_t^L + \gamma_{t+1}^L = 0, \\ \text{for } w_t:\ \underline{\nu}_t + \overline{\nu}_t = c^w - P_t, \\ \text{Where, } \underline{\chi}_t^p, \underline{\chi}_t^r, \underline{\theta}_t^U, \underline{\theta}_t^L, \underline{\nu}_t \ge 0;\ \overline{\chi}_t^{p1}, \overline{\chi}_t^{p2}, \overline{\chi}_t^{r1}, \overline{\chi}_t^{r2}, \overline{\theta}_t^U, \overline{\theta}_t^L, \overline{\nu}_t \le 0.\end{cases} \quad \forall t \in \{1, 2, \cdots, T\} \qquad \text{(E109)}$$

Compare these two duality problems

Similarly, when the $\mu_t = \mu_t^*$ is fixed, the new duality problem can be broken into two subproblems:

Subproblem one:

$$\max\left[\begin{array}{l}\sum_{t=1}^{T}(\bar{Q}^{p}\cdot\bar{\chi}_t^{p1}+E_t^{L}\cdot\bar{\chi}_t^{p2}+\bar{Q}^{r}\cdot\bar{\chi}_t^{r1}+E_t^{U}\cdot\bar{\chi}_t^{r2}+\bar{E}^{U}\cdot\bar{\theta}_t^{U}+\bar{E}^{L}\cdot\bar{\theta}_t^{L}+\underline{W}\cdot\underline{\nu}_t+\bar{W}\cdot\bar{\nu}_t)\\ +\sum_{t=1}^{T}\mu_t^{*}\cdot D_t-E_1^{U}\cdot\gamma_2^{U}+E_{T+1}^{U}\cdot\gamma_{T+1}^{U}-E_1^{L}\cdot\gamma_2^{L}+E_{T+1}^{L}\cdot\gamma_{T+1}^{L}\end{array}\right]$$

$$\Leftrightarrow\max\left[\begin{array}{l}\sum_{t=1}^{T}(\bar{Q}^{p}\cdot\bar{\chi}_t^{p1}+E_t^{L}\cdot\bar{\chi}_t^{p2}+\bar{Q}^{r}\cdot\bar{\chi}_t^{r1}+E_t^{U}\cdot\bar{\chi}_t^{r2}+\bar{E}^{U}\cdot\bar{\theta}_t^{U}+\bar{E}^{L}\cdot\bar{\theta}_t^{L}+\underline{W}\cdot\underline{\nu}_t+\bar{W}\cdot\bar{\nu}_t)\\ -E_1^{U}\cdot\gamma_2^{U}+E_{T+1}^{U}\cdot\gamma_{T+1}^{U}-E_1^{L}\cdot\gamma_2^{L}+E_{T+1}^{L}\cdot\gamma_{T+1}^{L}\end{array}\right]$$

$$\text{s.t.}\begin{cases}\underline{\chi}_t^{p}+\bar{\chi}_t^{p1}+\bar{\chi}_t^{p2}-\mu_t^{*}/\alpha+\gamma_{t+1}^{U}-\gamma_{t+1}^{L}=c^{p}/\alpha,\\ \underline{\chi}_t^{r}+\bar{\chi}_t^{r1}+\bar{\chi}_t^{r2}+\mu_t^{*}\beta-\gamma_{t+1}^{U}+\gamma_{t+1}^{L}=c^{r}\beta,\\ -\bar{\chi}_t^{r2}+\underline{\theta}_t^{U}+\bar{\theta}_t^{U}-\gamma_t^{U}+\gamma_{t+1}^{U}=0, & \forall t\in\{1,2,\cdots,T\}\\ -\bar{\chi}_t^{p2}+\underline{\theta}_t^{L}+\bar{\theta}_t^{L}-\gamma_t^{L}+\gamma_{t+1}^{L}=0,\\ \underline{\nu}_t+\bar{\nu}_t+\mu_t=c^{w},\\ \text{Where},\underline{\chi}_t^{p},\underline{\chi}_t^{r},\underline{\theta}_t^{U},\underline{\theta}_t^{L}\geq 0;\bar{\chi}_t^{p1},\bar{\chi}_t^{p2},\bar{\chi}_t^{r1},\bar{\chi}_t^{r2},\bar{\theta}_t^{U},\bar{\theta}_t^{L}\leq 0;\ \gamma_t^{U},\gamma_t^{L}\in\mathbb{R}.\end{cases}\tag{E110}$$

Subproblem two:

$$\max\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_i^{h}\cdot\underline{\beta}_{it}+\bar{G}_i^{h}\cdot\bar{\beta}_{it}\right)$$

$$\text{s.t.}\begin{cases}\underline{\beta}_{it}+\bar{\beta}_{it}+\mu_t^{*}=C_{it}^{h}\\ \underline{\beta}_t\geq 0,\bar{\beta}_t\leq 0;\end{cases}\tag{E111}$$

Let $P_t=\mu^{*}_t$, which means the electricity merchant can make perfect prices prediction, or the ISO sends the LMPs to the merchant. Similar with that above, it can be found that subproblem one of the new duality problems from the ISO perspective is equivalent to the duality problems from the electricity merchant perspective due to the same objective function and the same constraints. Thus

$$\begin{Bmatrix} \underline{\chi}_t^{p*(S)}, \overline{\chi}_t^{p1*(S)}, \overline{\chi}_t^{p2*(S)}, \underline{\chi}_t^{r*(S)}, \overline{\chi}_t^{r1*(S)}, \overline{\chi}_t^{r2*(S)}, \\ \underline{\theta}_t^{U*(S)}, \overline{\theta}_t^{U*(S)}, \underline{\theta}_t^{L*(S)}, \overline{\theta}_t^{L*(S)}, \underline{\nu}_t^{*(S)}, \overline{\nu}_t^{*(S)} \end{Bmatrix} = \begin{Bmatrix} \underline{\chi}_t^{p*(M)}, \overline{\chi}_t^{p1*(M)}, \overline{\chi}_t^{p2*(M)}, \underline{\chi}_t^{r*(M)}, \overline{\chi}_t^{r1*(M)}, \overline{\chi}_t^{r2*(M)}, \\ \underline{\theta}_t^{U*(M)}, \overline{\theta}_t^{U*(M)}, \underline{\theta}_t^{L*(M)}, \overline{\theta}_t^{L*(M)}, \underline{\nu}_t^{*(M)}, \overline{\nu}_t^{*(M)} \end{Bmatrix}$$

$$= \left\{ \underline{\chi}_t^{p*}, \overline{\chi}_t^{p1*}, \overline{\chi}_t^{p2*}, \underline{\chi}_t^{r*}, \overline{\chi}_t^{r1*}, \overline{\chi}_t^{r2*}, \underline{\theta}_t^{U*}, \overline{\theta}_t^{U*}, \underline{\theta}_t^{L*}, \overline{\theta}_t^{L*}, \underline{\nu}_t^{*}, \overline{\nu}_t^{*} \right\}$$

Use the duality theorem to compare the optimal solution for $q_t^{*(S)} = \left\{ q_t^{p*(S)}, q_t^{r*(S)}, w_t^{*(S)} \right\}$ and $q_t^{*(M)} = \left\{ q_t^{p*(M)}, q_t^{r*(M)}, w_t^{*(M)} \right\}$ from the ISO and the electricity merchant perspectives, respectively.

From ISO's perspective:

$$\begin{cases} \text{Primal: } \min\left(\sum_{t=1}^{T} \sum_{i=1}^{M} C_{it}^{h} \cdot g_{it}^{h} + \sum_{t=1}^{T} \left(c^r q_t^{r(S)} + c^p q_t^{p(S)} \right) + \sum_{t=1}^{T} c^w w_t^{(S)} \right) \\ \text{Dual: } \max \begin{bmatrix} \sum_{t=1}^{T} \left(\overline{Q}^p \cdot \overline{\chi}_t^{p1} + E_t^L \cdot \overline{\chi}_t^{p2} + \overline{Q}^r \cdot \overline{\chi}_t^{r1} + E_t^U \cdot \overline{\chi}_t^{r2} + \overline{E}^U \cdot \overline{\theta}_t^U + \overline{E}^L \cdot \overline{\theta}_t^L + \underline{W} \cdot \underline{\nu}_t + \overline{W} \cdot \overline{\nu}_t \right) \\ -E_1^U \cdot \gamma_2^U + E_{T+1}^U \cdot \gamma_{T+1}^U - E_1^L \cdot \gamma_2^L + E_{T+1}^L \cdot \gamma_{T+1}^L \end{bmatrix} \end{cases}$$

From merchant's perspective:

$$\begin{cases} \text{Primal: } \min\left(\sum_{t\in T} \left(\left(c^r - P_t\beta \right) q_t^{r(M)} + \left(c^p + P_t / \alpha \right) q_t^{p(M)} \right) + \left(c^w - P_t \right) w_t^{(M)} \right) \\ \text{Dual: } \max \begin{bmatrix} \sum_{t=1}^{T} \left(\overline{Q}^p \cdot \overline{\chi}_t^{p1} + E_t^L \cdot \overline{\chi}_t^{p2} + \overline{Q}^r \cdot \overline{\chi}_t^{r1} + E_t^U \cdot \overline{\chi}_t^{r2} + \overline{E}^U \cdot \overline{\theta}_t^U + \overline{E}^L \cdot \overline{\theta}_t^L + \underline{W} \cdot \underline{\nu}_t + \overline{W} \cdot \overline{\nu}_t \right) \\ -E_1^U \cdot \gamma_2^U + E_{T+1}^U \cdot \gamma_{T+1}^U - E_1^L \cdot \gamma_2^L + E_{T+1}^L \cdot \gamma_{T+1}^L \end{bmatrix} \end{cases}$$

From the optimality theorem of the duality problem (that is, the optimal objective function value of the original problem and the duality problem are identical), we can get:

$$\begin{cases} \sum_{t=1}^{T}\sum_{i=1}^{M} C_{it}^{h}\cdot g_{it}^{h*} + \sum_{t=1}^{T}\left(c^{r} q_{t}^{r*(S)} + c^{p} q_{t}^{p*(S)}\right) + \sum_{t=1}^{T} c^{w} w_{t}^{*(S)} \\ = \begin{pmatrix} \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot \bar{\chi}_{t}^{p1*} + E_{t}^{L}\cdot \bar{\chi}_{t}^{p2*} + \bar{Q}^{r}\cdot \bar{\chi}_{t}^{r1*} + E_{t}^{U}\cdot \bar{\chi}_{t}^{r2*} + \bar{E}^{U}\cdot \bar{\theta}_{t}^{U*} + \bar{E}^{L}\cdot \bar{\theta}_{t}^{L*} + \underline{W}\cdot \underline{\nu}_{t}^{*} + \bar{W}\cdot \bar{\nu}_{t}^{*}\right) \\ + \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot \underline{\beta}_{it}^{*} + \bar{G}_{i}^{h}\cdot \bar{\beta}_{it}^{*}\right) + \sum_{t=1}^{T}\mu_{t}^{*}\cdot D_{t} - E_{1}^{U}\cdot \gamma_{2}^{U*} + E_{T+1}^{U}\cdot \gamma_{T+1}^{U*} - E_{1}^{L}\cdot \gamma_{2}^{L*} + E_{T+1}^{L}\cdot \gamma_{T+1}^{L*} \end{pmatrix} \\ \sum_{t\in T}\left(\left(c^{r} - P_{t}\beta\right) q_{t}^{r*(M)} + \left(c^{p} + P_{t}/\alpha\right) q_{t}^{p*(M)}\right) + \left(c^{w} - P_{t}\right) w_{t}^{*(M)} \\ = \begin{pmatrix} \sum_{t=1}^{T}\left(\bar{Q}^{p}\cdot \bar{\chi}_{t}^{p1*} + E_{t}^{L}\cdot \bar{\chi}_{t}^{p2*} + \bar{Q}^{r}\cdot \bar{\chi}_{t}^{r1*} + E_{t}^{U}\cdot \bar{\chi}_{t}^{r2*} + \bar{E}^{U}\cdot \bar{\theta}_{t}^{U*} + \bar{E}^{L}\cdot \bar{\theta}_{t}^{L*} + \underline{W}\cdot \underline{\nu}_{t}^{*} + \bar{W}\cdot \bar{\nu}_{t}^{*}\right) \\ -E_{1}^{U}\cdot \gamma_{2}^{U*} + E_{T+1}^{U}\cdot \gamma_{T+1}^{U*} - E_{1}^{L}\cdot \gamma_{2}^{L*} + E_{T+1}^{L}\cdot \gamma_{T+1}^{L*} \end{pmatrix} \end{cases}$$

We also have the following energy balance equation: $\sum_{i=1}^{M} g_{it}^{h*} + q_{t}^{g*(S)}\beta - q_{t}^{p*(S)}/\alpha + w_{t}^{*(S)} = D_{t}$, that is, electricity supply matches demand.

Similarly, by the equation above, there are the following conclusion for $\forall i \in \{1,2,\cdots,M\}, \forall t \in \{1,2,\cdots,T\}$ holding:

$$\begin{cases} \text{1) If } C_{it}^{h} < P_{t}=\text{LMP}_{t} \text{ and } g_{it}^{h*} = \bar{G}_{i}^{h}; \\ \text{2) or If } C_{it}^{h} = P_{t}=\text{LMP}_{t} \text{ and } \underline{G}_{i}^{h} \le g_{it}^{h*} \le \bar{G}_{i}^{h}; \\ \text{3) or If } C_{it}^{h} > P_{t}=\text{LMP}_{t} \text{ and } g_{it}^{h*} = \underline{G}_{i}^{h}; \end{cases} \tag{E112}$$

there are $\sum_{t=1}^{T}\left(\sum_{i=1}^{M} C_{it}^{h} g_{it}^{h*} - \sum_{i=1}^{M} P_{t} g_{it}^{h*}\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot \underline{\beta}_{it}^{*} + \bar{G}_{i}^{h}\cdot \bar{\beta}_{it}^{*}\right)$.

When $\sum_{t=1}^{T}\left(\sum_{i=1}^{M} C_{it}^{h} g_{it}^{h*} - \sum_{i=1}^{M} P_{t} g_{it}^{h*}\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\left(\underline{G}_{i}^{h}\cdot \underline{\beta}_{it}^{*} + \bar{G}_{i}^{h}\cdot \bar{\beta}_{it}^{*}\right)$,we can achieve

$$\begin{aligned} &\sum_{t=1}^{T}\left(\left(c^{r} - P_{t}\beta\right) q_{t}^{r*(S)} + \left(c^{p} + P_{t}/\alpha\right) q_{t}^{p*(S)} + \left(c^{w} - P_{t}\right) w_{t}^{*(S)}\right) \\ &= \sum_{t\in T}\left(\left(c^{r} - P_{t}\beta\right) q_{t}^{r*(M)} + \left(c^{p} + P_{t}/\alpha\right) q_{t}^{p*(M)} + \left(c^{w} - P_{t}\right) w_{t}^{*(M)}\right) \end{aligned},$$

which is the objection function of primal problem from merchant perspective. In this way, the merchant will get the optimal profit. The relations between the *optimal actions* $q_t^{*(S)}=\left\{q_t^{p*(S)},q_t^{r*(S)},w_t^{*(S)}\right\}$ *of the ISO and* $q_t^{*(M)}=\left\{q_t^{p*(M)},q_t^{r*(M)},w_t^{*(M)}\right\}$ *of the electricity merchant can be drawn:*

(1) For the merchant, if the forecasted LMPs equal the actual LMPs, and *when the primal problem from individual profit-maximizing in Eq.(E108) has a unique optimal solution:*

$$\begin{cases}1)\text{ If } C_{it}^h < P_t = LMP_t \text{ and } g_{it}^{h*}=\bar{G}_i^h,\ \left\{q_t^{p*(S)},q_t^{r*(S)},w_t^{*(S)}\right\}=\left\{q_t^{p*(M)},q_t^{r*(M)},w_t^{*(M)}\right\};\\ 2)\text{ If } C_{it}^h = P_t = LMP_t \text{ and } \underline{G}_i^h \le g_{it}^{h*} \le \bar{G}_i^h,\ \left\{q_t^{p*(S)},q_t^{r*(S)},w_t^{*(S)}\right\}=\left\{q_t^{p*(M)},q_t^{r*(M)},w_t^{*(M)}\right\};\\ 3)\text{ If } C_{it}^h > P_t = LMP_t \text{ and } g_{it}^{h*}=\underline{G}_i^h,\ \left\{q_t^{p*(S)},q_t^{r*(S)},w_t^{*(S)}\right\}=\left\{q_t^{p*(M)},q_t^{r*(M)},w_t^{*(M)}\right\}.\end{cases} \quad \text{(E113)}$$

(2) For the merchant, if the forecasted LMPs align with the actual LMPs, and *when the primal problem from individual profit-maximizing in Eq.(E108) has multiple optimal solutions:*

$$\begin{aligned}&\max\sum_{t=1}^{T}\left(LMP_t\cdot\left(q_t^r\beta-q_t^p/\alpha\right)-\left(c^r\cdot q_t^r+c^p\cdot q_t^p\right)+\left(LMP_t-c^w\right)w_t\right)\\ &=\sum_{t=1}^{T}\left(LMP_t\cdot\left(q_t^{r*(S)}\beta-q_t^{p*(S)}/\alpha\right)-\left(c^r\cdot q_t^{r*(S)}+c^p\cdot q_t^{p*(S)}\right)+\left(LMP_t-c^w\right)w_t\right) \qquad \text{(E114)}\\ &=\sum_{t=1}^{T}\left(LMP_t\cdot\left(q_t^{r*(M)}\beta-q_t^{p*(M)}/\alpha\right)-\left(c^r\cdot q_t^{r*(M)}+c^p\cdot q_t^{p*(M)}\right)+\left(LMP_t-c^w\right)w_t\right)\end{aligned}$$

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VITA

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