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A SUBOPTIMAL CONTROL LAW TO IMPROVE THE TRANSIENT STABILITY OF POWER SYSTEMS

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ABSTRACT

The application of optimal control theory to damp the electromechanical oscillations associated with the transient conditions in power systems has been given little attention. A suboptimal control law which minimizes the sum of the performance indices of the two linear systems given by a piecewise linear model of a power system improves the transient stability better than the usual " optimal " control law obtained from a linearized model. Further, the suboptimal control law presented in this paper assures stability of the system for all disturbances resulting in rotor oscillations of magnitudes less than a preselected limit.

INTRODUCTION

In spite of the work [1 - 5] that has been done to apply optimal control theory for the purpose of synthesizing a linear feedback controller to damp small electromechanical oscillations, which occur during steady state operation of power systems, little has been done to damp large oscillations associated with transient conditions. The reasons for this have been a lack of adequate physical means to tackle the problem and a lack of mathematical tools to deal conveniently with the nonlinearities associated with large oscillations. However, with the advent of fast acting valves for the inlets of prime movers [6], it is now necessary to have a computationally convenient formulation of the problem. The usual method of linearizing the nonlinear equations, which describes the oscillations of the machines about or near the steady state operating point, yields an optimistic solution (Fig. 1), hence, the method is not suitable for dealing with large oscillations. The purpose of this paper is to formulate the problem for damping large oscillations by using some necessary approximations and to provide a solution.

FORMULATION OF THE PROBLEM

In the following discussion, it is assumed that a power system can be adequately represented by a piecewise linear system between some preselected extremities of the oscillations, with the only point of inflection being at the point of operation in the steady state. Even if this cannot yield a very accurate representation of the actual nonlinear system, it can be shown, by using Popov's Criterion, that the two 'pieces' of the linear model can always be selected so that any control law that stabilizes this model also stabilizes the actual nonlinear system provided the oscillations lie within the preselected limits. Further it can be seen (Fig. 1) that this type of approximation of linearizing the system equations about the steady state operating point yields a pessimistic solution.

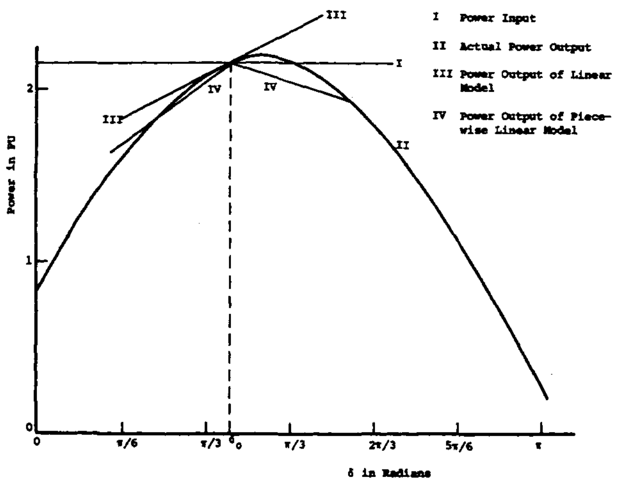

Fig. 1. Power Vs. Rotor Angle for the System Shown in Figure 2.

Now, the two 'pieces' of the piecewise linear system are represented by the first order linear differential equations given by (see appendix)

$$\dot{x} = A_1 x + B\ u \tag{1}$$

and

$$\dot{x} = A_2 x + B\ u \tag{2}$$

where x is an n vector of states; u is an m vector of controls; and A_1, A_2, and B are constant matrices of appropriate orders. Further the controls, u, are linear combinations of the states, x, and are given by

$$u = F\ x \tag{3}$$

where F is a constant matrix of an appropriate order.

It is known that for any system represented by Eq. (3) and

$$\dot{x} = A\ x + B\ u, \tag{4}$$

the expected value of the cost function, J, defined by

$$J = \int_{t=0}^{\infty} (x'Q\ x + u'R\ u)\ dt, \tag{5}$$

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where Q is positive semidefinite, and R is positive definite, is proportional to the tr.V (i.e., sum of the diagonal terms of V) where V is the solution of the equation

$$V(A+B\,F)+(A+B\,F)'\,V+Q+F'R\,F = 0 \qquad (6)$$

when it is assumed that the initial state vector, x_0, is a random vector uniformly distributed over the surface of a hypersphere [8].

Now, a suboptimal control law for the power system, described by Eqs. (1) to (3), is defined as that which minimizes $(tr.V_1 + tr.V_2)$ or $tr.(V_1 + V_2)$, where V_1 and V_2 are the solutions of the equations

$$V_1(A_1+B\,F)+(A_1+B\,F)'\,V_1+Q+F'R\,F = 0 \qquad (7)$$

and

$$V_2(A_2+B\,F)+(A_2+B\,F)'\,V_2+Q+F'R\,F = 0 \qquad (8)$$

SOLUTION OF THE PROBLEM

The problem now reduces to finding the matrix F such that $tr.(V_1 + V_2)$ is minimized subject to the constraints given by Eqs. (7) and (8). Therefore, the Hamiltonian,

$$\begin{aligned} h = tr.\{&(V_1 + V_2) \\ &+ P_1'[V_1(A_1+B\,F)+(A_1+B\,F)'\,V_1+Q+F'R\,F] \\ &+ P_2'[V_2(A_2+B\,F)+(A_2+B\,F)'\,V_2+Q+F'R\,F]\} \end{aligned} \qquad (9)$$

is selected. Now, the condition for F being an extremum of h is that the gradient matrices $\partial h/\partial P_1$, $\partial h/\partial P_2$, $\partial h/\partial V_1$, $\partial h/\partial V_2$, and $\partial h/\partial F$ should vanish [8]. Applying this condition to Eq. (9) yields Eqs. (7) and (8) and also

$$P_1(A_1+B\,F)'+(A_1+B\,F)P+I = 0, \qquad (10)$$

$$P_2(A_2+B\,F)'+(A_2+B\,F)P+I = 0, \qquad (11)$$

and

$$F = -R^{-1}B'(V_1P_1+V_2P_2)(P_1+P_2)^{-1}. \qquad (12)$$

The Eqs. (7), (8), (10), (11), and (12) are solved iteratively in the following steps:

Step 1: Assume an initial value of F such that all eigenvalues of $(A_1+ B\,F)$ and $(A_2+B\,F)$ have negative real parts.

Step 2: Use this value of F to solve Eqs. (7), (8), (10), and (11) for V_1, V_2, P_1, and P_2.

Step 3: Find a new value of F by substituting these values of V's and P's in Eq. (12).

Step 4: By using the value of F from Step 3, repeat Step 1.

If this iterative procedure converges, it only yields a local but not the absolute optimal solution for F because Eqs. (7), (8), (10), (11), and (12) are only necessary conditions.

In some sample problems, this procedure converges faster when the average of the values of Steps 1 and 3 are taken as the new value of F.

Example 1

A schematic diagram of the system considered is shown in Fig. 2. The machine and system constants as well as the steady state operating values of the various variables are given below.

$M = 0.06$	$D = 0.06$
$x_d = 0.320$	$x_d' = 0.084$
$Y_{11} = 0.266 - j\,1.53$	$Y_{12} = 0.18 + j\,1.08$
$T_d = 5.0$ sec.	$T_G = 0.3$ sec.
$V = 1.0$	$E_0 = 1.482$
$P_0 = 2.179$	$\delta_0 = 70^{\circ}$

The oscillations to be damped are assumed to be within the range of 45 to 110 degrees of load range, δ.

The two sets of first order, linear, differential equations representing the system are found to be

$$\begin{bmatrix} \Delta\dot{E} \\ \Delta\dot{\delta} \\ \Delta\dot{\omega} \\ \Delta\dot{p} \end{bmatrix} = \begin{bmatrix} -0.312 & 0.0 & 0.388 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -28.09 & -9.992 & -1.0 & 16.66 \\ 0.0 & 0.0 & 0.0 & -3.3 \end{bmatrix} \begin{bmatrix} \Delta E \\ \Delta\delta \\ \Delta\omega \\ \Delta p \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

and

$$\begin{bmatrix} \Delta\dot{E} \\ \Delta\dot{\delta} \\ \Delta\dot{\omega} \\ \Delta\dot{p} \end{bmatrix} = \begin{bmatrix} -0.312 & 0.0 & 0.349 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -28.945 & 4.435 & -1.0 & 16.66 \\ 0.0 & 0.0 & 0.0 & -3.3 \end{bmatrix} \begin{bmatrix} \Delta E \\ \Delta\delta \\ \Delta\omega \\ \Delta p \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Q is taken to be a diagonal matrix of the weighting factors 1, 10, 10 and 3 for ΔE, $\Delta\delta$, $\Delta\omega$ and Δp respectively. Engineering experience is necessary to determine a proper value for these weighting factors. F is taken to be a unit matrix of the appropriate order.

The optimal control for small oscillations [1] is obtained as

$$u_1 = -11.041\,\Delta E+1.105\,\Delta\delta+2.393\,\Delta\omega+4.551\,\Delta p \qquad (13)$$

and

$$u_2 = 4.551\,\Delta E-0.558\,\Delta\delta-1.243\,\Delta\omega-2.582\,\Delta p. \qquad (14)$$

The suboptimal control for large oscillations is found to be

$$u_1 = -9.464\ \Delta E+3.797\ \Delta\delta+3.135\ \Delta\omega+4.216\ \Delta p \tag{15}$$

and

$$u_2 = 4.218\ \Delta E-1.851\ \Delta\delta-1.701\ \Delta\omega-2.638\ \Delta p. \tag{16}$$

Figures 3 to 6 show the response of the system to a three-phase short circuit to ground at the machine terminals when cleared in 0.08 seconds in the presence of no control, the optimal control for small oscillations, and the suboptimal control for large oscillations.

The critical fault clearing times with these controls are found to be about 0.078 sec., 0.125 sec. and 0.235 sec. respectively.

Example 2

The schematic diagram of the system considered is given in Fig. 7, and the machine and system constants as well as the steady state operating values are given

$M = 0.06$		$D = 0.06$	
$x_d = 0.2$		$x_d' = 0.1$	
$T_{d_0} = 5.0$ sec.		$T_G = 0.3$ sec.	
$Y_{11} = Y_{22} = 0.3363 - j\ 1.076$			$= 1.128\ \underline{/-73^\circ}$
$Y_{12} = Y_{21} - 0.06375 + j\ 0.563$			$= 0.547\ \underline{/83.2^\circ}$
$V = 1\ \underline{/0^\circ}$		$P_1 = P_2 = 0.8$	
$E_{1_0} = E_{2_0} = 1.44 + j\ 0.42$			$= 1.498\ \underline{/16.2^\circ}$

The minimum norm approximation [7] of the optimal control for small oscillations at each machine in this system is found to be

$$u_1 = -6.998\ \Delta E-3.6311\ \Delta\delta+1.913\ \Delta\omega+4.318\ \Delta p \tag{17}$$

and

$$u_2 = 4.318\ \Delta E+2.730\ \Delta\delta-1.657\ \Delta\omega-3.769\ \Delta p \tag{18}$$

The minimum norm approximation of the suboptimal control for large oscillations is found to be

$$u_1 = -6.197\ \Delta E-3.346\ \Delta\delta+1.804\ \Delta\omega+3.956\ \Delta p \tag{19}$$

and

$$u_2 = 3.909\ \Delta E+2.005\ \Delta\delta-2.048\ \Delta\omega-4.630\ \Delta p \tag{20}$$

CONCLUSION

A linear suboptimal control law for damping large electromechanical oscillations associated with transient conditions in power systems has been synthesized, and the two examples given illustrate the improvement in the transient stability of power systems with this control as compared to the optimal control law for small oscillations, obtained from the linearized models of power systems.

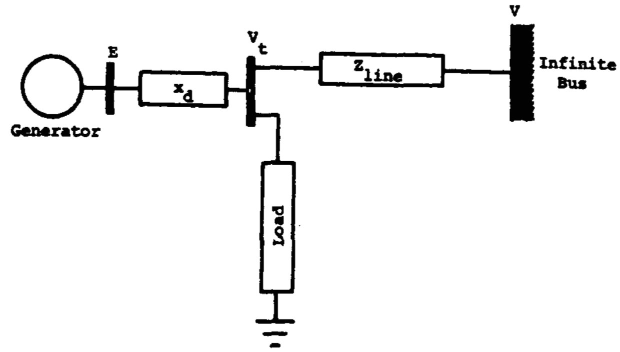

Fig. 2. A Single Machine System with a Load at the Terminals Connected to an Infinite Bus.

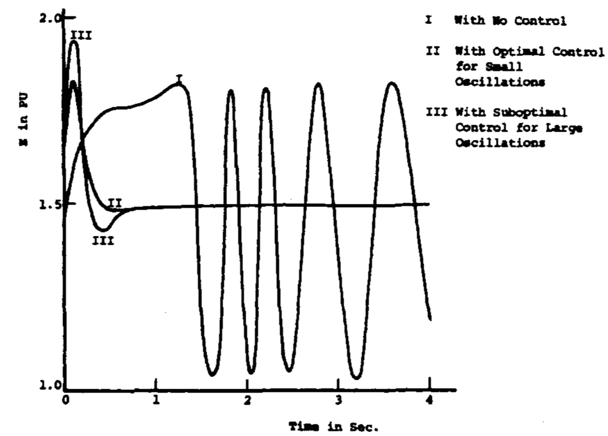

Fig. 3. Response of the Machine in Figure 1 to a Three Phase Short Circuit at its Terminals, Cleared in 0.08 Sec. - E Vs. Time.

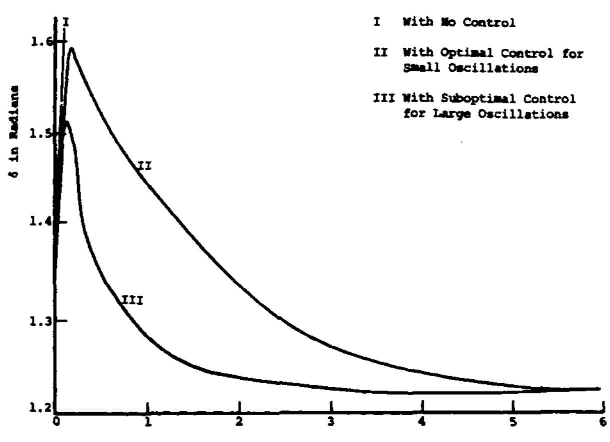

Fig. 4. Response of the Machine in Figure 1 to a Three Phase Short Circuit at its Terminals Cleared in 0.08 Sec. - δ Vs. Time.

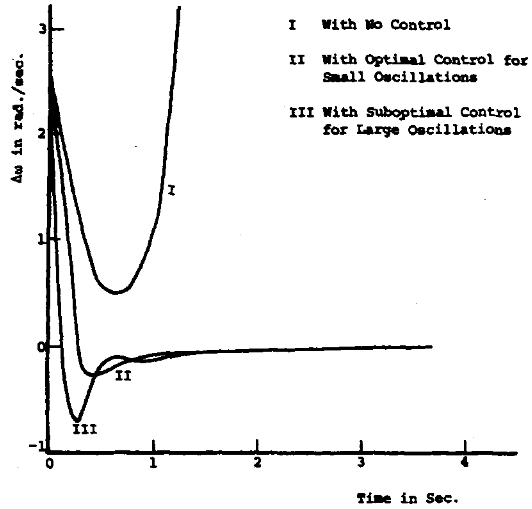

Fig. 5. Response of the Machine in Figure 1 to a Three Phase Short Circuit at its Terminals, Cleared in 0.08 Sec. - ω Vs. Time.

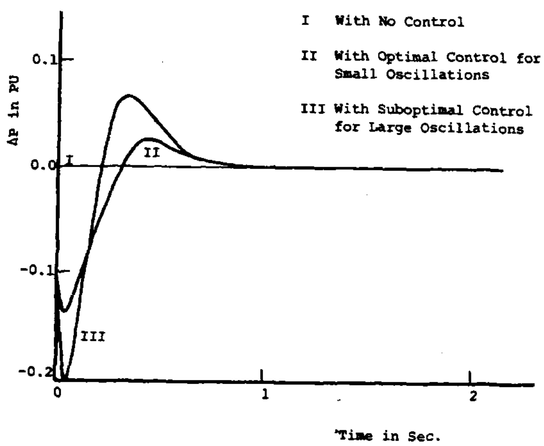

Fig. 6. Response of the Machine in Figure 1 to a Three Phase Short Circuit at its Terminals, Cleared in 0.08 Sec. - P Vs. Time.

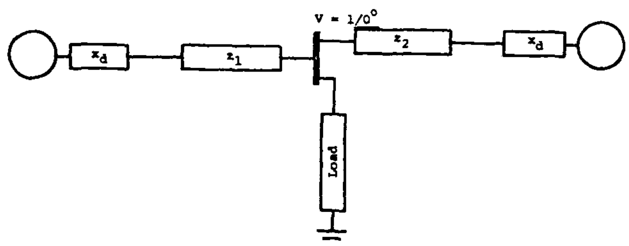

Fig. 7. A Two Machine System Considered for Example 2.

APPENDIX

The State Variable Formulation of the Power System Model

For the single machine system shown in Fig. 2, the system differential equations are

$$M\,\Delta\ddot{\delta}+D\,\Delta\dot{\delta}+\Delta P_e = P \qquad (21)$$

(Equation of Electromechanical Oscillation)

where

$$\Delta P_e = C_1\,\Delta\delta+b_1\,\Delta E$$

and

$$C_1 = \frac{\partial P_e}{\partial \delta} = -E\,V\,Y_{12}\sin(\delta-\theta_{12})$$

and

$$b_1 = \frac{\partial P_e}{\partial E} = 2E\,Y_{11}\cos\theta_{11}+V\,Y_{12}\cos(\delta-\theta_{12})$$

because

$$P_e = E^2\,Y_{11}\cos\theta_{11}+E\,V\,Y_{12}\cos(\delta-\theta_{12}).$$

Further we know that

$$\Delta\dot{\delta} = \Delta\omega \qquad (22)$$

By neglecting the time lag in the exciter, we have

$$\Delta E + \frac{d}{dt}\left[T_{d_0}\,\Delta E'\right] = U_1 \qquad (23)$$

where

$$E' = E - (x_d - x_d')I_d$$

$$I_d = -E\,Y_{11}\sin\theta_{11}-V\,Y_{12}\sin(\theta_{12}-\delta)$$

$$I_q = E\,Y_{11}\cos\theta_{11}+V\,Y_{12}\cos(\theta_{12}-\delta).$$

Therefore,

$$\Delta E' = C_2\,\Delta\delta+b_2\,\Delta E$$

where

$$C_2 = \frac{\partial E'}{\partial \delta} = -(x_d-x_d')V_0V_{12}\cos(\theta_{12}-\delta)$$

and

$$b_2 = \frac{\partial E'}{\partial E} = 1+(x_d-x_d')Y_{11}\sin\theta_{11}.$$

Further,

$$\Delta P + \frac{d}{dt}(T_G\,\Delta P) = U_2 \qquad (24)$$

Equations (21) to (24) can be expressed in the first order state variable form given by Equation 25.

Among the parameters in Equation 25, only C_2, b_1, and C_1 vary with the load angle, δ, during the oscillations. The values of these variables at $\delta = 45^\circ$, 70°, 110° are calculated. Among the values of each variable at $\delta = 45^\circ$ and 70°, the one which results in a less stable system is selected and substituted in Eq. (25) to yield the state equations representing the system in the range of $\delta = 45^\circ$ to 70°.

The equations for the range of $\delta = 70^\circ$ to 110° are obtained similarly.

$$\begin{bmatrix} \Delta \dot{E} \\ \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{P} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{d_0} b_2} & 0 & -\frac{c_2}{b_2} & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{b_1}{M} & -\frac{c_1}{M} & -\frac{D}{M} & \frac{1}{M} \\ 0 & 0 & 0 & -\frac{1}{T_G} \end{bmatrix} \begin{bmatrix} \Delta E \\ \Delta \delta \\ \Delta \omega \\ \Delta P \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{25}$$

Nomenclature:

M	Inertia constant of the machine
D	Damping coefficient
x_d	Direct axis synchronous reactance
x_d'	Direct axis transient reactance
Y_{11}	Self-admittance of the network at the internal bus of the machine
Y_{12}	Mutual admittance of the network between the internal buses of the two machines
T_d	Time constant of direct axis field
T_G	Gate time constant for turbine inlet
V	Reference bus voltage
E	Open circuit voltage of the machine
E'	Voltage behind transient reactance
δ	Rotor angle of the machine in electrical radians
ω	Angular velocity
P	Mechanical power input
P_e	Electrical power output
Δ	Increment operator
I_d, I_v	Direct and quadrature axes current of the machine
$[\cdot]_0$	Steady state operating value of $[\cdot]$

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Discussion

N.D. Rao (The University of Calgary, Calgary, Alberta, Canada): This paper proposes a novel method for the synthesis of a linear suboptimal control law for damping large electromechanical oscillations associated with transient conditions in synchronous power systems. In the opinion of this discussor, the central innovative feature of this work is the introduction of a piecewise linear model into the framework of linear optimal regulator theory as applied to power systems. Accordingly, this paper represents an improvement in technique as well as approach over the previous papers on a related theme. The digital simulation results provided by the author demonstrate very well the significant improvement in performance achieved by piecewise linearization, rather than outright linearization employed in earlier publications. The authors certainly deserve to be congratulated on such a worthwhile and significant contribution.

The value of this paper would be greatly enhanced if the authors could provide some additional light on the following points:

1. In any method employing piecewise linearization, problems of matching conditions at the boundary would naturally arise. Would this introduce any computational difficulties in the base of multi-machine systems?

2. In the case of several machines, what would be the additional degree of complication introduced by piecewise linearization vis-a-vis global linearization? Would the improvement in dynamic performance achieved by the authors' method offset the additional work called for by piecewise linearization? If the authors have carried out any multi-machine studies, a report of their experience in this regard will be helpful.

3. Finally, is it feasible to incorporate sensitivity minimization schemes with respect to parameter variations into the authors' synthesis procedure?

The discussor looks forward to seeing further progress made in this area by the authors.

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Carl E. Grund (General Electric Company, Schenectady, New York): The increase in critical clearing time of the large signal suboptimal control system compared to the small signal optimal control system is certainly quite dramatic. I believe that the industry would appreciate a comparison of the piecewise linear controller with a typical existing governor-prime mover, excitation system.

One extension of the ideas presented in the paper appears to be one of using an n-piece linear approximation of the transient torque-angle curve. The question to be answered then is regarding the improvement in critical clearing time, assuming an n-piece linear approximation is feasible. For a large transient swing the accuracy of a two piece linear approximation can be improved using a three or four piece approximation. Since the effectiveness of a controller is limited by the accuracy of the model, a more accurate model should yield a more effective controller.

Judging by the curve of mechanical power versus time of Fig. 6, the speed of response required by the governor-prime mover system is quite rapid based on presently available fast valving power response. What would be the effectiveness of the controller if the power response due to fast valving were limited to that presently available?

Manuscript received July 11, 1975.

P.K. Dash and **O.P. Malik** (The University of Calgary, Calgary, Alberta, Canada): The authors have presented an interesting formulation of optimal control technique for improving the transient stability of a power system. Instead of obtaining a single optimal control law for the entire region of excursion of state variables, the paper attempts to divide the trajectory of a nonlinear system into two regions and obtains the control law based on the two piecewise linear systems using the standard linear regulator theory. Although the technique in this paper yields a suitable optimal control law for the improvement of transient stability, there are some questions, however, which need clarification:

Manuscript received August 8, 1975.

1. In order to derive the control law it is necessary to solve four matrix equations at a time rather than one in the normal linear optimal control theory. Also the selection of the weighting matrices Q and R and the feedback matrix F are arbitrary. It has been recently shown by Y_uA that by properly choosing Q and R matrices by eiganshifting technique the linear control law can stabilise the nonlinear system. The discussers feel that by solving a set of matrix equations for obtaining control law for the piecewise linear systems, the computation time will increase considerably and therefore will be difficult to implement either for a large multimachine power system including several control loops or an online control scheme.

2. The discussers feel that the transient stability will improve considerably if the control law is obtained by choosing the weighting matrix Q constant during δ varying 45° to 70° and a function of some of the output states during δ swinging from 70° to 120°. The authors are requested to please comment on this.

3. From the figures presented in the paper it seems that with this two region control, rotor angle oscillations are damped out faster while other states like speed terminal voltage and power output of the system are bought back to normal faster by the single region linear optimal control law based on small oscillations. Also it may be pointed out here that the system chosen by authors does not have an excitation or governor control and therefore the results could not be representative of realistic systems.

Finally the authors are pleased to note that an increasing interest is continuing in the use of optimal control theory in stabilising nonlinear power systems and thus the authors contributions are worthwhile.

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A.B.R. Kumar and **Earl F. Richards**: The authors would like to thank the discussers for their interesting comments and suggestions. Particularly interesting were the comments relating to the scope and limitations on the technique of piecewise linearization in synthesizing an optimal control law for power systems.

In response to the comments of Dr. N.D. Rao, we would like to make the following remarks.

1. In our technique, we approximated the nonlinear "power-angle relationship" by a piecewise linear relationship. However, while carrying on the optimization procedure, we considered two independent linear systems and the total cost is minimized subject to the constraint that both have the same F matrix in $u = Fx$. Therefore, there were no problems of matching conditions at the boundary.

2. In the case of several machines, the computational needs of the technique will be at least twice as much as that of the usual optimal control techniques. However, the procedure yields an assurance of stability, as long as the system oscillations are within some pre-selected limits. In our opinion, this is the main advantage of this technique.

3. It is feasible to incorporate sensitivity minimization schemes with respect to parameter variations into the synthesis procedure. But the utility of these schemes will be marginal, because whatever amount of uncertainty is present in the nominal values of the parameters, that can be accounted for, by properly selecting the parameters of the two linear systems.

P.K. Dash and O.P. Malik have some queries to which we would like to respond.

1. The authors agree with the discussers in saying that the computational effort required by their technique can be 2 to 4 times as much as required in normal linear optimal control theory. Therefore, this technique is not presently suitable for on line control schemes. However, the authors believe that the computational requirements of this technique can be decreased in the future by virtue of a "similarity" in the equations for the V's and P's.

2. The authors believe that choosing the weighting matrix Q constant during δ variations from 45° to 70°, and a function of some of the output states during δ variations from 70° to 120°, is a good idea for on line control of power systems. But in contrast with the piecewise linear approach, this can not assure the stability of the system when the system swings from 45° to 120°.

Manuscript received September 22, 1975.

3. The relative speeds of damping the oscillations of various output variables can be modified by choosing appropriate values for the elements of Q. Whenever the performance with two reasonably synthesized control laws are compared against each other, invariably oscillations in some variables are damped faster by one control law and others by another control law.

4. The authors agree that the excitation and governor control are over simplified in their model. However, this can in no way reduce the significance of the technique of piecewise linearization in increasing the stability of power systems.

In response to the comments of Carl E. Grund, the following remarks are appropriate.

1. The increase in critical cleaning time of the large signal sub-optimal control system compared to the small signal optimal control system is about the same for the cases of $\delta_o = 70°$ and $45°$, and is about 0.1 sec.; i.e., from 0.125 sec. to 0.235 sec. in the first case and from 0.269 sec. to 0.265 sec. in the second case. (In both cases, the control laws used were as calculated at $\delta_o = 70°$.)

2. By increasing the turbine gate time constant T_G to 1.0 sec. from 0.3 sec. used in the paper, the critical clearing time increased when the control law calculated by our method was used, but decreased when the usual optimal control law was used. For the control law calculated by our method, maximum rate of change of power input was about the same for $T_G = 1$ sec. and $T_G = 0.3$ sec. If this is too much for the existing governor-prime mover systems, a practically feasible control law can be obtained by appropriate choice of the matrices Q and R. However, it may decrease the critical clearing time. (Here, it may be appropriate to point out that no mathematical technique of optimal control is adequate to replace the systems. The aim of optimal control theory is only to use those capabilities efficiently.)

3. Though an n-piece linear approximation of the transient torque-angle curve appears, at first sight, to be a possible extension of the ideas presented in our paper, it cannot be a direct extension. Consider for example, a three piece linear approximation obtained by joining four points on the torque-angle curve. Both knee points, in this case, cannot be the steady state operating points; and if we pick only one of them as the steady state operating point, it can be seen that the control law calculated in this case will be more pessimistic than in the case of the two piece approximation, which is itself pessimistic.