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Decoupled explosive charge effects on blasting performance

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DECOUPLED EXPLOSIVE CHARGE EFFECTS ON BLASTING PERFORMANCE

by

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ABSTRACT

This investigation was performed to determine the burden at which hard rock would break utilizing decoupled and unstemmed charges when the free surface parallel to the borehole was the wall of an open hole. This open hole was created by burn cuts and by drilling holes of 7-7/8 in. and 12 in. diameter. Additional tests were performed using partially stemmed charges where the stemming was the same size as the explosive charge.

The explosive used was a 0.75 lb. slurry charge, 1.5 in. diameter and 10 in. long. The diameter and depth of the borehole were 2 in. and 3 ft., respectively. The borehole pressure, P_b, for cylindrical charges was found to vary with the explosion pressure, P_3, and the ratio of the radius of the charge R_c, to the radius of the hole R_h, by the relationship:

$$P_b = P_3(R_c/R_h)^5$$

This equation relates the borehole pressure and the explosion pressure for the symmetrical decoupled condition.

For the non-symmetrical decoupled condition the average borehole pressure obtained for top, side, and bottom holes around a central opening was 57%, 35% and 11.5% of P_3 respectively, with a decoupling ratio $D = R_h/R_c = 1.33$. The reflection theory of rock breakage was used to determine the decay exponent, m, in the transition zone and from

the experimental data, the maximum burden R_{tmax} was found to be a function of the radius of the hole, R_h, the radial stress in the rock at the boundary of the cavity, P_m^o, and the tensile strength of the rock, σ_t. These quantities are related by the equation

$$R_{tmax} = R_h/2(P_m^o/\sigma_t)^{1/m}$$

with 1/m = 0.66, 0.71 and 0.84 for top, side, and bottom holes, respectively. The burden as a function of the diameter of the opening, ϕ, for side holes was determined to be of the form:

$$R_\phi = R_{tmax}(1-3^{-0.41\phi})$$

The relationship between burdens, for partially stemmed R_t and unstemmed conditions R_{t_1} depends on the exponential decay time τ and τ_1 by the equation

$$\tau/\tau_1 = R_t/R_{t_1}$$

which was found to have a value of 1.113 in this investigation.

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I. INTRODUCTION

Fragmentation of rock by blasting in mines and quarries is a function of several variables, including explosive density, detonation velocity, the characteristic impedance of the explosive and of the rock, maximum available energy, borehole pressure, the condition and properties of the rock, hole diameter, charge length, type of stemming, point of initiation, and the decoupling. All these factors must be carefully controlled for optimum performance of an explosive. The burden or "line of least resistance", depends on these factors as well as others.

This investigation studied the variation of the burden due to the decoupling factor, which is defined as the ratio of the radius of the hole, R_h, to the radius of the charge, R_c. The decoupling factor is important because when an explosive charge does not fill the drill hole in which it is detonated then it is less efficient for fracturing rock than a charge of the same volume that does completely fill the borehole.

Another important parameter that affects the burden is the unstemming factor. In this experimental work the charges were normally unstemmed with only a few partially stemmed shots, which implies a decrease in the burden dimension as compared to stemmed shots.

A mathematical equation giving the burden directly as a function of the unstemming factor is very difficult to derive. However, its effect can be illustrated by adding constants to the theoretical expression developed in this thesis based on experimental data.

II. REVIEW OF LITERATURE

Information on the effect of decoupling in the literature is very limited. However, an excellent contribution towards a solution of this problem has been made by Cook[1] and Atchison, et. al.[2].

Cook found that the borehole pressure, P_b is related to the adiabatic or explosion pressure, P_3, by the following:

$$P_b = P_3 \Delta^n \qquad (1)$$

where n is approximately 2.5 and Δ is the loading density or the fraction of the borehole occupied by the explosive excluding the open hole above the charge. Thus for a cylindrical charge:

$$\Delta = (V_3/V_b) = (R_c/R_h)^2 \qquad (2)$$

where

V_3 = Volume of the explosive charge

V_b = Volume of borehole

R_c = Radius of the charge

R_h = Radius of the drill hole

Considering that the decoupling $D = R_h/R_c$, equation (1) becomes:

$$P_b = P_3 (D)^{-5} \qquad (3)$$

Equation (1) was obtained by Cook for loading densities varying from 1 to 0.84, 1 to 0.85 and 1 to 0.64 with 90 percent straight gelatin, 40 percent straight gelatin, and High Ammonia dynamite (10 percent NG), respectively (see Table I). Therefore according to equation (1), the maximum efficiency is attained when $\Delta = 1$ or $D = R_h/R_c = 1$, because $P_b = P_3$.

Atchison, et. al. employed short cylindrical charges with a length-to-diameter ratio of 8, in granite and limestone. They proposed a simplified mathematical theory to explain the decoupling effect considering three zones around a charge detonated in a cavity in rock; their mathematical model is illustrated in Fig. (1); thus, the three zones are: a) the source zone in the cavity, b) the transition zone surrounding the cavity, which is the region where non-elastic effects occur such as crushing and cracking the rock, and c) the seismic zone where the propagation of the stress pulse is nearly elastic and if reflective boundaries are not encountered no fragmentation occurs. They considered that the following equations represent the pressures and stresses in the three respective zones, assuming a spherical charge at the center of a spherical cavity. Thus in the source zone for $R_c < R < R_h$

$$P = P_c (R/R_c)^{-3\gamma} \qquad (4)$$

where P_c is the detonation pressure of the explosive, P the pressure in the cavity, R the distance from the center

Table I. Influence of Loading Density on Borehole Pressure P_b. (After Cook, 1958, p. 275)

Density (g/cc)	Borehole Density (lb/ft) 9" dia.	5" dia.	90% Straight-Gelatin (p_3=450 tons/sq in) Δ	P_b	40% Straight-Gelatin (p_3=265 tons/sq in) Δ	P_b	High Ammonia Dynamite, 10% NG (p_3=265 tons/sq in) Δ	P_b
1.35	37.2	11.5	-		1.0	265	-	
1.3	35.9	11.0	-		0.96	245	-	
1.25	34.5	10.6	1.0	450	0.925	220	-	
1.2	33.1	10.2	0.96	405	0.890	205	-	
1.15	31.7	9.8	0.92	368	0.85	185	-	
1.1	30.4	9.3	0.88	330	-		1.0	265
1.05	29.0	8.9	0.84	275	-		0.95	245
1.0	27.6	8.5	-		-		0.91	220
.9	24.8	7.6	-		-		0.82	175
.8	22.1	6.8	-		-		0.73	145
.7	19.3	5.9	-		-		0.64	115

$$P_b = P_3 \Delta^{2.5}$$

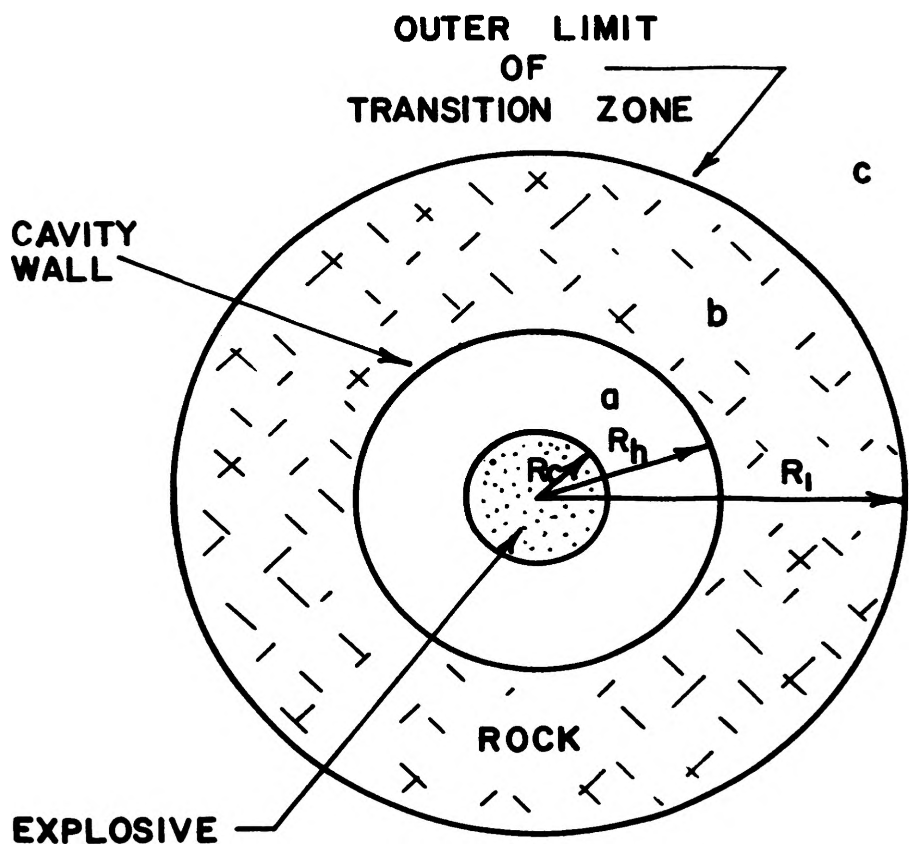

Fig. 1. Decoupling Model (After Atchison, et. al., 1964, p. 3.)

of the charge and γ is the ratio of the specific heat of explosion gases. According to Brown[3], for most explosives a value of $\gamma = 1.2$ is a good approximation. Since $R_g = C_p - C_v$ for one mole of a perfect gas, from the following relationship.

$$C_p/C_v = \gamma = 1 + R_g/C_v$$

The value of C_v is approximately 10 cal/mole ^oK where $\gamma = 1.2$ and $R_g = 1.987$ cal/mol ^ok.

If equation (4) is evaluated at $R = R_h$, the pressure at the outer limit of the cavity is

$$P_h = P_c(R_h/R_c)^{-3\gamma} \quad (5)$$

In the transition zone for $R_h < R < R_1$,

$$\sigma = \sigma_h(R/R_h)^{-m} \quad (6)$$

where σ is the radial stress in the rock, σ_h the pressure exerted on the cavity wall by the explosion gases, and m a constant which describes the stress decay in the transition zone. Also, they considered that the radial stress $\sigma_h = k\ P_h$, where k is a proportionality constant determined from the ratio of the characteristic impedance of the explosive to that of the rock. The characteristic impedance of the explosive is defined as the product of the explosive density and detonation velocity, similarly the characteristic impedance of the rock is the product of the rock density and the longitudinal propagation velocity.

Finally, in the elastic or seismic zone for $R_1 < R < \infty$,

$$\sigma = \sigma_1 (R/R_1)^{-n} \tag{7}$$

where σ_1 is the stress at the outer limit of the transition zone and n is a constant which describes the stress decay in the seismic zone. The principal results of these studies carried out by Atchison, et. al. were as follows:

1. The amplitude of the strain pulse decreases as the decoupling increases.
2. The period of the strain pulse decreases at first and then increases as decoupling increases.

Previous to the investigation mentioned above, again Atchison[4] describes the influence of decoupling on the explosive performance by measuring the amplitude of the strain pulse produced by the detonation of explosive charges in drill holes as the ratio of the radius of the hole diameter to the charge diameter was varied. The tests were made in limestone, with a total of 18 shots, and charges with a length-to-diameter ratio of 8, the drill hole diameter varied from 1-7/8" up to 4-1/2", with decoupling (R_h/R_c) of 1 up to 2.

Atchison found that the strain is approximately proportional to the 1.5 power of the ratio of the charge diameter to the hole diameter.

Equation (4) was obtained by assuming a reversible, adiabatic law, thus:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \tag{8}$$

Clark[5] gives a relationship between P_b and P_3 for a loading density less than one, of the form

$$P_b = \frac{P_3V_3}{\gamma V_b - V_3(\gamma-1)} \tag{9}$$

Clark assumed ideal behavior of gases as a first approximation; thus $C_v = 3/2R_g$, which implies that $\gamma = 1+R_g/C_v = 5/3$. Substitution of this value in equation (9) yields

$$P_b = \frac{3P_3}{5(V_b/V_3)-2} \tag{10}$$

again using $\gamma = 1.2$, equation (9) may be written as follows:

$$P_b = \frac{P_3}{1.2(V_b/V_3)-(0.2)} \tag{11}$$

Hino[6] derived an equation for the condition where pressure transmitted to the rock is reduced by the void space around the explosive charge and called this the "cushion effect". The estimation of the borehole pressure, P_b, given by Hino is as follows using the conventional approximation $P_2 = 2P_3$, where P_2 and P_3 are the detonation pressure and explosion pressure, respectively, and $T_3 = T_b$, where T_3 is the explosion temperature and T_b the borehole temperature. The last assumption is valid for values of decoupling close to unity. Thus the explosion pressure P_3 is:

$$P_3 = 1/2\ P_2 \tag{12}$$

For the explosion and borehole pressure:

$$\begin{aligned} P_3\ (v_3-\alpha_3) &= nR_gT_3 \\ P_b\ (v_b-\alpha_b) &= nR_gT_3 \end{aligned} \tag{13}$$

where

$v_b = V_b/M = \dfrac{\text{Volume of borehole}}{\text{mass of explosive}}$ = specific volume of borehole.

$v_3 = V_3/M = \dfrac{\text{Volume of explosive charge}}{\text{mass of explosive}}$ = specific volume of explosive charge.

R_g = Gas constant per mole.

n = Number of moles of gas per unit mass

α = Co-volume

Equation (13) is a modified form of Abel's equation of state which has been employed successfully by Cook[1] in his studies of the hydrodynamic theory of detonation. Thus,

$$P_3(v_3-\alpha_3) = P_b(v_b-\alpha_b)$$

or

$$P_b = 1/2\ P_2\left(\frac{v_3-\alpha_3}{v_b-\ _b}\right) \tag{14}$$

The numerical relation between the co-volume α and the specific volume v is given by Hino from the results obtained by Cook as follows:

$$\alpha = 0.92(1-1.07\ e^{-1.39v}) \tag{15}$$

Bergmann, et. al.[7] found that the peak shot-hole pressure for different decoupling ratios is related by the empirical equation

$$P = P_{Det} \times D^{-0.95} \qquad (16)$$

where

P = Peak shot-hole pressure

P_{Det} = Detonation pressure

D = Decoupling ratio = (effective Shot-hole volume/Explosive volume)

Equation (16) is an empirical relationship based on the acceleration and the change in pressure acting on explosively driven plates with varying air gaps between the explosive and the plates.

Additional work has been done by Nicholls[8] and Haas[9], to determine the energy effect of coupling explosives with rock.

Nicholls used the relation:

$$\varepsilon = k(d/W^{1/3})^n \qquad (17)$$

where

ε = Peak compressive strain

k = Strain intercept

n = Slope

$d/W^{1/3}$ = Scaled distance

d = Shot-hole gage distance

$W^{1/3}$ = The cube root of the charge weight

He also assumed that

$$\varepsilon \propto \frac{P_m}{\rho c^2} \left[\frac{d}{W^{1/3}}\right]^n \tag{18}$$

where

ρ = Density of the rock

c = Longitudinal propagation velocity in the rock

P_m = Applied pressure in the medium

Inasmuch as all tests were conducted in the same rock ρc^2 was a constant, thus combining the last two equations gives

$$k = A\ P_m \tag{19}$$

where A is a constant.

Nicholls pointed out, "if the detonation pressure is assumed to equal the pressure or stress in the medium at the cavity wall, a plot of peak strain intercept k, and detonation pressure P, should be linear", but it is interesting to note that the relation between k and P found by Nicholls was a curved line. Thus the effect of the ratio of the characteristic impedance of the explosive to that of the rock was used to linearize the stress-strain relationship.

Hence equation (19) becomes

$$k = APf\left[\frac{(\rho C)_e}{(\rho C)_r}\right] \tag{20}$$

where $f[(\rho C)_e/(\rho C)_r]$ = a function of the ratio of the characteristic impedance of the explosive to that of the rock.

The medium stress, P_m, was calculated from the following equation.

$$P_m = \frac{(K/P)_{Z=a}}{(K/P)_{Z=1}} P \qquad (21)$$

where

Z = Ratio of characteristic impedances

$(K/P)_{Z=a}$ = the value of (K/P) obtained graphically by Nicholls, where $Z=a$ represents the Z value of each explosive, and $(K/P)_{Z=1}$ = the value of (K/P), determined in the same way where $Z=1$.

Finally, the percentage of the calculated explosive energy transferred to the rock was expressed as the ratio of H to $NR_gT/\gamma-1$ where

H = Radial strain energy per pound of explosive

N = Moles of gaseous products of detonation per unit of weight of explosive

R_g = The gas constant

γ = Ratio of specific heats

T = Detonation temperature

Nicholls concluded that the maximum energy is transmitted to the rock by the detonation of an explosive if the charge diameter equals the drill hole diameter and if the characteristic impedance of the explosive equals that of the rock.

Haas[9] used specimens of 4x4x2 in., which were cut from slabs of Yule marble. In these experiments the air gap was varied from zero gap to 1 in. The explosive charges were 0.5 in. diameter by 1.5 in. long for all the tests. The explosive used had a small critical diameter and is ideal for laboratory studies where stable detonation is desired.

The peak stress in the 2 in. thick marble depended on the length of the air gap. Values ranged from 33,800 psi, for direct contact between the explosive and marble to 12,200 psi for a 1 in. air gap. Haas found that the fragmentation size decreased as the air gap increased. He also observed that the shock intensity near the explosion and the resulting fragmentation were strongly influenced by the contact area between the explosive and the rock.

A. Theoretical Approach Relating the Burden as a Function of the Decoupling Factor. For a cylindrical charge, $P_b = P_3 \ (R_c/R_h)^5$ according to equation (1).

Because the borehole pressure is a function of the explosion pressure, Cook[10] says, "Experimental studies show that the explosive cannot really couple the detonation shock wave with a solid except under perfect contact at the end-not along the sides of the charge. Lateral coupling does not even involve the detonation wave directly because the detonation head does not extend all the way to the interface".

Inasmuch as the detonation pressure is a function of the detonation wave and it is moving within the explosive charge parallel to the explosive-rock interface it is quite reasonable to assume that no direct relationship exists between the borehole pressure and the detonation wave.

An important characteristic of the typical blasthole is that because of its shape practically all of the available energy is utilized and is transmitted through the walls of the borehole into the rock. Therefore, the explosion pressure is more significant than the detonation pressure as a performance factor in boreholes.

According to equation (1) the borehole pressure and the explosion state pressure are equal only when the explosive completely fills the hole or the loading density, Δ, is one.

If the volume of the borehole is slightly greater than the volume of the gaseous products in the explosion state, the borehole pressure will be significantly lower than the explosion state pressure, which indicates that the exponent n (equation 1), must be larger.

An approximation of the exponent n, Appendix A, gives good agreement with the value obtained by Cook.

The initial borehole pressure is a very important factor in determining the rock breakage, providing of course that the borehole contains the necessary total explosive energy to break up the burden properly. The transmitted stress or initial peak stress in the rock, P_m^o, is obtained from the impedance mismatch equation from elastic theory. This equation, in general form, may be written as follows:

$$P_t = \frac{2(\rho v)_t}{(\rho v)_t + (\rho v)_i} P_i \tag{22}$$

where

P_t = The transmitted stress = P_m°

P_i = The incident stress = P_b

and ρ and v are the density of the material and propagation velocity of the pulse respectively, and the subscripts i and t denote the incident and transmitted medium. Thus $(\rho v)_i = (\rho v)_{explosive}$ = characteristic impedance of the explosive and $(\rho v)_t = (\rho v)_{rock}$ = characteristic impedance of the rock. Therefore equation (22) becomes:

$$P_m^{\circ} = \frac{2P_b}{Q + 1} \tag{23}$$

where $Q = (\rho v)_{exp}/(\rho v)_{rock}$ and is the relative effective (borehole) impedance, thus the condition $(\rho v)_{exp} = (\rho v)_{rock}$ has a special significance in blasting. It is the condition of maximum energy transfer from the incident medium to the transmitted medium; since no energy goes back into the incident medium, therefore, it is desirable to have the characteristic impedance of the explosive match that of the rock (Q = 1).

The value of Q may also be expressed as follows, according to Clay, et. al.[11]; by means of the principles of conservation of mass, momentum and energy, the detonation pressure is given by the equation

$$P_2 = \rho_e D W$$

where

ρ_e = Explosive density

D = Detonation velocity

W = Particle velocity of gases

In condensed explosives the particle velocity, W, is approximately D/4. If the conventional approximation $P_2 \simeq 2P_3$ is used then the last equation becomes

$$2P_3 = \rho_e \frac{D^2}{4} \tag{24}$$

or

$$2P_3\rho_e = (\rho_e D/2)^2 \tag{25}$$

Thus,

$$(\rho_e D) = (\rho V)_{explosive} = (8\rho_e P_3)^{1/2} \tag{26}$$

Substitution of eq. (1) into (26) yields:

$$(\rho v)_{explosive} = (8\rho_e P_b \Delta^{-n})^{1/2} \tag{27}$$

Therefore the effective (borehole) impedance Q may be given approximately by the relation:

$$Q = (8\rho_e P_b \Delta^{-n})^{1/2} / (\rho v)_{rock} \tag{28}$$

In the transition zone the radial stress σ in the rock is related, according to equation (6), as follows:

$$\sigma = \sigma_h (R/R_h)^{-m}$$

where $\sigma_h = P_m^{\circ}$.

The exponent (m) describes the stress decay in the transition zone and may be evaluated using the reflection theory of rock breakage. According to Duvall and Atchison[12] the primary cause of rock breakage is the reflection at the free surface of the compressive strain pulse generated in rock by the detonation of the explosive charge. Upon reflection this compressive strain pulse becomes a tensile strain pulse. As the strength of the rock in tension is much less than in compression, the reflected tensile pulse is able to break the rock in tension progressing from the free surface back towards the shot point.

Thus the "full crater" is obtained assuming that the intensity of the reflected tension wave at the center of the charge is:

$$\sigma_t = P_m^{\,o}(2R_t/R_h)^{-m} \tag{29}$$

where

σ_t = Tensile strength of the rock

and

R_t = Maximum burden.

This assumption may be true in the case of a single reflection but in the case of brittle solids, such as rock, primary fractures occur due to multiple reflections, therefore equation (29) represents only an approximation; inasmuch as σ_t, $P_m^{\,o}$ and R_h are known, and R_t is measured in the field, this implies that m must be obtained experimentally. Finally the maximum burden R_t is:

$$R_{tmax} = R_t = R_h/2(P_m^{\,o}/\sigma_t)^{1/m} \qquad (30)$$

For tunnel rounds, initially the burden is a function of the diameter of the opening ϕ, when the shot holes located around the opening are fired, both burden and opening increase in size, until the burden reaches a maximum value, which remains constant even though the opening continues to increase in diameter, (see page 24). Therefore, it is necessary to obtain a function of the form:

$$f[(1-k^{-u\phi})]$$

such that for any value of $\phi_1 \geq \phi$, $R_\phi = R_{tmax}$, and for $\phi=0$, $R_\phi=0$. Where

R_ϕ = The burden for tunnel rounds

ϕ = Diameter of opening

k and u = Constants to be evaluated from experimental data

Thus, equation (30) becomes:

$$R_\phi = R_{tmax}(1-k^{-u\phi}) \qquad (31)$$

1. Critical Decoupling. According to Atchison, et. al.[2], if the elastic limit of the rock is σ_t, there will exist a critical decoupling, D_o, for a given explosive and type of rock that results in a stress of σ_t at the boundary of the cavity. According to equation (23):

$$P_m^{\,o} = \frac{2P_b}{\frac{(\rho v)_{exp}}{(\rho v)_{rock}} + 1}$$

where $P_b = P_3 \ (R_c/R_h)^5$. By definition $D = R_h/R_c$, therefore $P_b = P_3 \ D_o^{-5}$ for critical decoupling. Substitution of P_m^o by σ_t and P_b as a function of P_3 and D_o, into equation (23) yields:

$$\sigma_t = \frac{2P_3 \ D_o^{-5}}{\frac{(\rho v)_{exp}}{(\rho v)_{rock}} + 1}$$

where

$$D_o = \left\{ \frac{2P_3}{\sigma_t [\frac{(\rho v)_{exp}}{(\rho v)_{rock}} + 1]} \right\}^{1/5} \qquad (32)$$

Equation (32) defines critical decoupling for cylindrical symmetry between the drill hole and the explosive charge.

2. The Unstemmed Factor. Stemming is used in the blasting process to confine the product gases for a sufficient time duration to allow the pressure to work against the borehole sidewall, fracturing the solid rock, prior to displacement and stemming ejection.

Experimental tests were made utilizing unstemmed charges and partially stemmed charges where the stemming was the same dimensions (length and diameter) as the explosive charge, resulting in a partially, decoupled stemming. The maximum burden is less for these conditions than for a completely stemmed shot.

Consequently, it is necessary to determine a mathematical relationship between burdens for both, unstemmed and partially stemmed charges.

The maximum velocity of the burden $V(R_t)_{max}$ may be estimated as follows[6]:

$$E = 1/2\ M_r\ V(R_t)^2_{max} \tag{33}$$

where

E = Available energy

and

M_r = Mass of rock to be blasted.

Taking into consideration a loss of about 10% of the total energy E, for leakage of gases through cracks and the borehole etc., equation (33) becomes

$$0.9\ E = 1/2\ M_r\ V(R_t)^2_{max}$$

Thus[10]

$$V(R_t)_{max} = (1.8\ E/M_r)^{1/2} \tag{34}$$

It is assumed that the effective force $F = P(t) \times A$, where $P(t)$ is the pressure and A the effective area equal to $\pi R_h h$, where R_h and h are the radius and the length of the drill hole containing explosive, respectively.

Further, assume that $F = F_i e^{-t/\tau}$, i.e., the force decreases exponentially in time (t), τ being an exponential decay time constant, such that for $t=\tau$, $F = F_i/e$, and F_i the initial force so that $F_i = P_b \times A$, where P_b is the borehole pressure. Then

$$F = P_b \times \pi \times R_h \times h \times e^{-t/\tau} \tag{35}$$

Applying Newton's equation

$$F = M_r \times \frac{d^2R_t}{dt^2} \tag{36}$$

or

$$F = M_r \times \frac{d}{dt}(\frac{dR_t}{dt}) \tag{37}$$

Thus

$$F = M_r \times \frac{dV(R_t)}{dt}) \tag{38}$$

and

$$\int_o^t F\ dt = M_r \int_o^{V(R_t)} dV(R_t) \tag{39}$$

where t = effective time of P(t) on the borehole sidewall. Substitution of equation (35) into (39) gives

$$\frac{P_b\ \pi\ R_h\ h}{M_r} \int_o^t e^{-t/\tau}\ dt = V(R_t)_{max} \tag{40}$$

and finally

$$\frac{P_b\ \pi\ R_h\ h}{M_r} \tau(1-e^{-t/\tau}) = V(R_t)_{max} \tag{41}$$

thus for $t=t_1$

$$\frac{P_b\ \pi\ R_h\ h}{M_r} \tau(1-e^{-t_1/\tau}) = V(t_1) \tag{42}$$

Since t_1 and $V(t_1)$ are unknown, the value of the constant τ, cannot be determined, except for the value of t where $V(t) = V(R_t)_{max}$, and for any time $t_2 > t$, $V(t_2) = V(R_t)_{max}$, therefore $e^{-t/\tau} \approx 0$, and $t >> \tau$, thus τ may be obtained as follows from equation (41)

$$\tau = \frac{V(R_t)_{max} \cdot M_r}{P_b \, \pi \, R_h \, h} \tag{43}$$

but $M_r = (R_t \cdot S \cdot h)\rho_r$, where ρ_r is the rock density, h is the hole depth, S the spacing between holes, and $S/R_t = 2.3$ to 3, the values obtained from this experimental work taking the first value of 2.3 equation (43) yields:

$$\tau = \frac{V(R_t)_{max} \; 2.3 \; R_t^{\,2} \; \rho_r}{P_b \, \pi \, R_h} \tag{44}$$

If τ is considered as the exponential decay time for partially stemmed charges, then for unstemmed charges it will be:

$$\tau_1 = \frac{V(R_{t_1})_{max} \; 2.3 \; R_{t_1}^{\,2} \; \rho_r}{P_b \, \pi \, R_h} \tag{45}$$

where P_b remains constant. Thus the ratio τ/τ_1 is:

$$\frac{\tau}{\tau_1} = \frac{V(R_t)_{max} \, R_t^{\,2}}{V(R_{t_1})_{max} \, R_{t_1}^{\,2}} \tag{46}$$

but

$$\frac{V(R_t)^2_{max}}{V(R_{t_1})^2_{max}} = \frac{2E/M_r}{2E/M_{r_1}} = \frac{M_{r_1}}{M_r} = \frac{R_{t_1}^{\,2}}{R_t^{\,2}} \tag{47}$$

thus

$$\frac{V(R_t)_{max}}{V(R_{t_1})_{max}} = \frac{R_{t_1}}{R_t} \tag{48}$$

Substitution of the (48) into equation (46) gives

$$\frac{\tau}{\tau_1} = \frac{R_t}{R_{t_1}} \tag{49}$$

Inasmuch as the geometric parameters, and characteristics of explosive and rock remain constant, the ratio τ/τ_1 for side holes, around the central opening in a tunnel round, must be the same as for top and bottom holes, respectively. Therefore, knowing τ/τ_1 and R_t it is possible to determine R_{t_1}, for any condition.

3. Experimental Investigation. The purpose of this experimental investigation was to determine the burden at which hard rock would break utilizing decoupled and unstemmed charges, when the free surface parallel to the borehole was the wall of an open hole. Also, additional tests were performed using partially stemmed charges where the stemming was the same length and diameter as the explosive charge and was also decoupled.

The tests were conducted in a granite quarry, approximately 90 miles from the UMR campus, located near Graniteville, Missouri. Some physical properties of this rock type are presented in Table II.

The explosive used in all the tests was a 0.75 lb. slurry charge, 1.5 in. diameter and 10 in. long. The relevant properties of the explosive used are given in Table III. The initial opening was created by burn cuts and by drilling holes of 7-7/8 in. and 12 in. diameter.

The burn cuts were formed by drilling six 2 in. diameter holes in a circular pattern with one in the center that was loaded with the explosive charges, that

Table II. Rock Properties.

Property	Missouri Red Granite
Specific gravity	2.60
Apparent porosity, %	0.40
Compressive strength, kg/cm^2	1620.00
Tensile strength, kg/cm^2	99.10
Ultimate strain, 10^{-6} cm/cm	2770.00
Ranked mechanical drillability, rate basis*	8.00
Ranked mechanical drillability, diameter basis*	5.00
Rebound hardness	53.00
Secant modulus of elasticity, 10^5 kg/cm^2	5.83
Compressional wave velocity, km/sec	4.51

* for 10 rock types

Table III. Explosive Properties

GEL POWER A-1*

GENERAL:	Theoretical density	=	1.57 gm/cc
	Actual density	=	1.18 gm/cc
	Oxygen balance	=	+0.30% (w/o package)
	Energy	=	982,446 ft-lbs/lb (calculated)
	Measured rate (1-1/2" diameter, unconfined)	=	4400 mps

DETONATION STATE (CALCULATED)

Velocity	=	5049 meters/sec
Pressure	=	61460 atmospheres
Temperature	=	2596 °K
Product gas	=	36.6 g moles/kg
Product density	=	1.4881 gm/cc

EXPLOSION STATE (CALCULATED)

Pressure	=	28930 atmospheres
Temperature	=	2322 °K
Product gas	=	36.6 g moles/kg
Product density	=	1.1800 gm/cc

*Trademark of Hercules, Inc.

when detonated, broke to the ring of unloaded holes creating the desired opening in the rock, then the test hole was drilled parallel to the opening, with different burdens depending on the diameter of the opening, and shot to obtain the relationship between the burden and opening diameter (fig. 2).

Two charges in the same hole, fired by a single blasting cap, were used in the first ring of holes to ensure pulling the full length of the 3 ft. deep hole for the small burdens required by the initial holes until the center hole was opened up to about a 2 ft. diameter opening.

Also, test holes were drilled and fired, one at a time, around the burn cut, the 7-7/8 in. and 12 in. diameter drilled holes until the burden and spacing relationships for a 15 ft. diameter tunnel round were determined.

Six tunnel test rounds were thus fired to further develop and duplicate the desirable conditions to apply this new technique to driving a 15 ft. diameter tunnel in granite. The relationship between burden and diameter of the opening as it increases due to subsequent shots are shown in fig. (3). The diameter and depth of the holes were held constant at 2 in. diameter and 3 ft. deep.

A charge diameter of 1.5 in. results in a decoupling factor, D of $R_h/R_c = 1.33$.

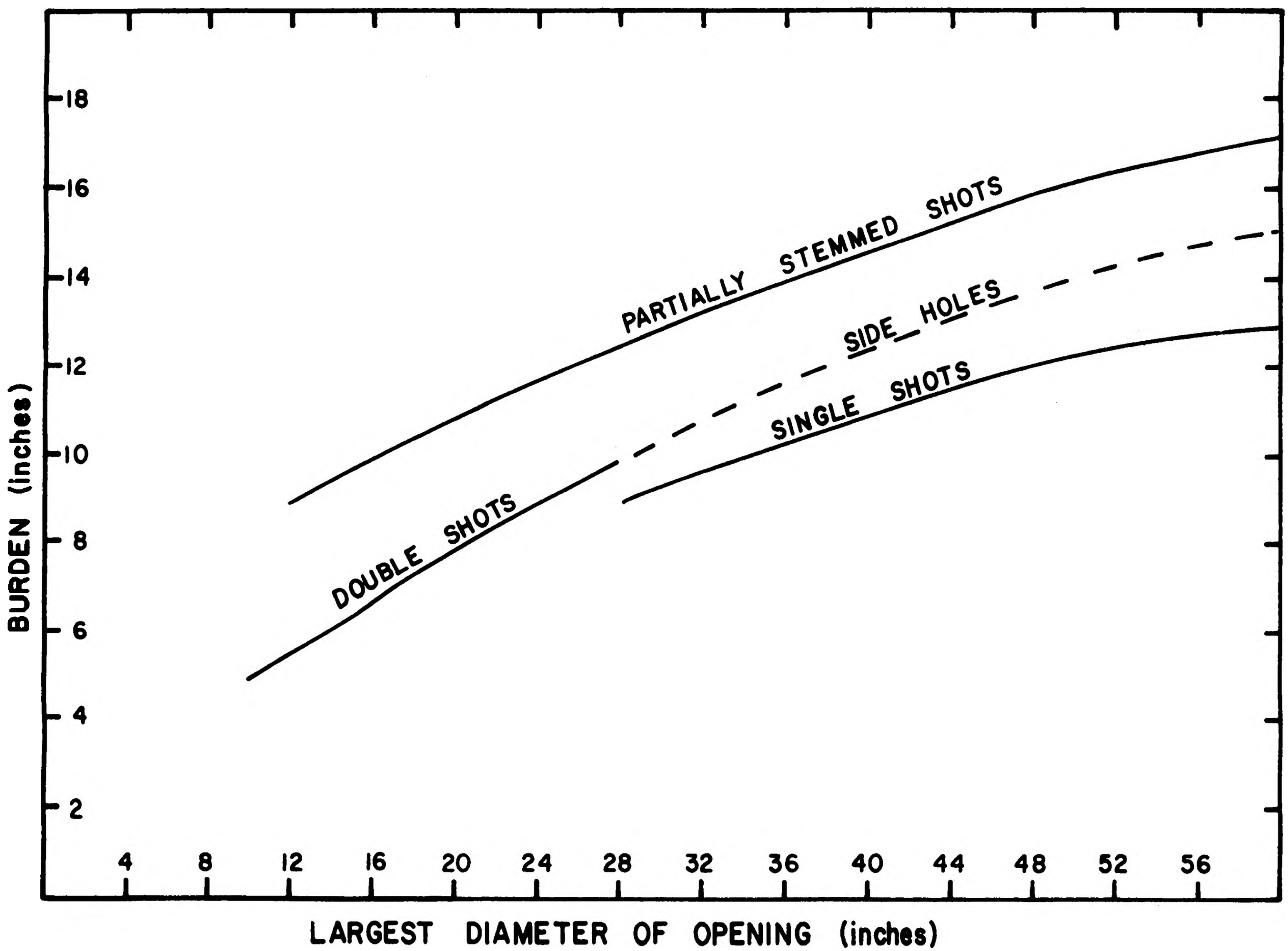

Fig. 2. Relationship between Burden and Largest Diameter of Opening.

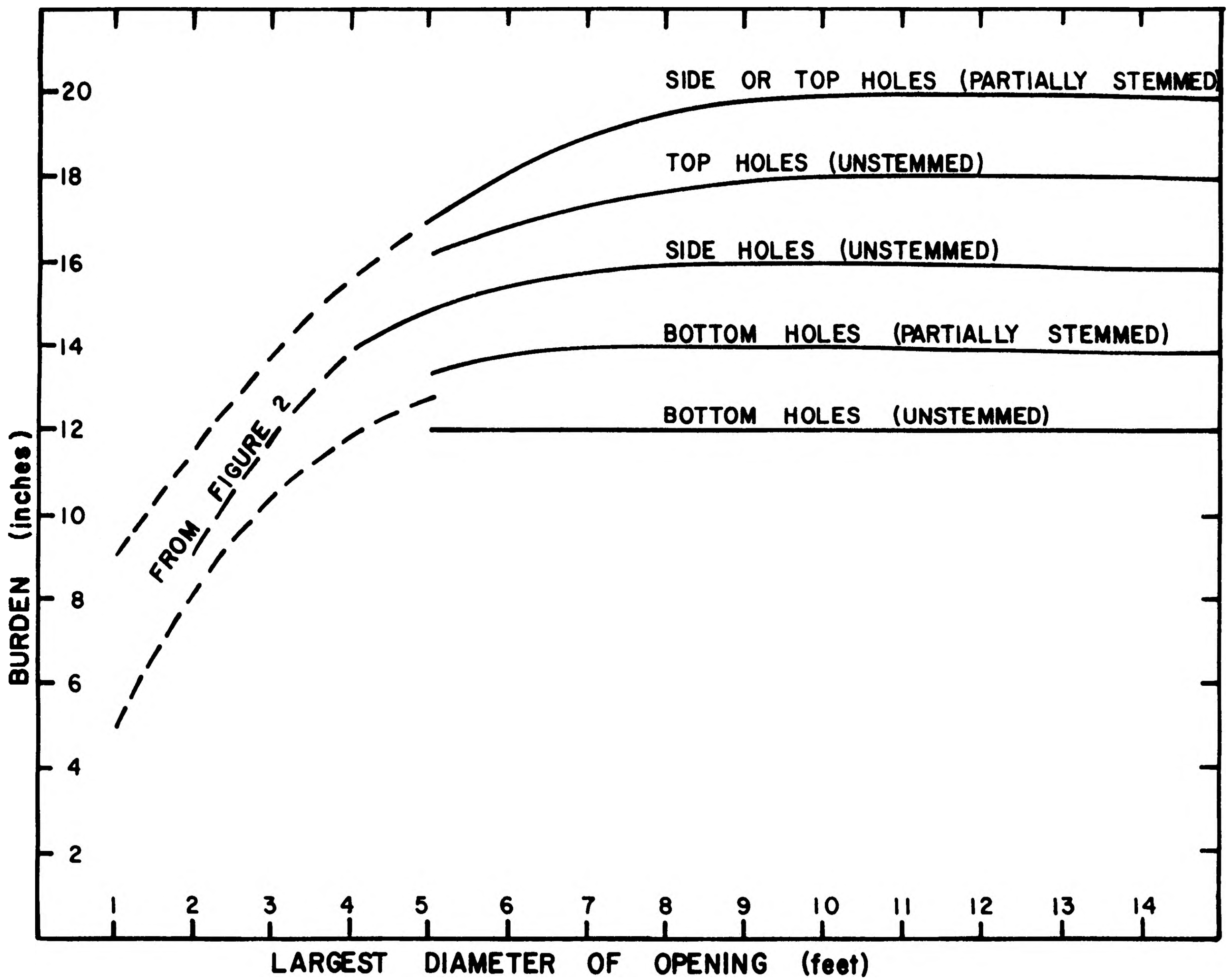

Fig. 3. Relationship between Burden and Largest Diameter of Opening.

III. RESULTS

A. Determination of the Average Borehole Pressure and Constants

The data obtained from side holes for unstemmed charges, (fig. 3), were used to determine the constants m, k and u; since the majority of the holes around the opening are side holes, as shown in fig. 4.

Also, it is important to point out that the borehole pressure, P_b, or the pressure at the outer boundary of the cavity, will be different at each point of the borehole sidewall, due to the lack of symmetry between the borehole and the position of the explosive charge. The best, intermediate and worst coupling conditions correspond to the top, side and bottom holes respectively.

Four possible loading conditions in blasting operations are illustrated in fig. 5. The two bottom conditions applies to this investigation. Therefore, it is necessary to obtain an average borehole pressure on the sidewall area. An approximation to this problem for side, top and bottom holes is developed in Appendix (B).

From equation (30):

$$R_{tmax} = R_h/2(P_m{}^o/\sigma_t)^{1/m}$$

where:

R_h = Radius of the hole

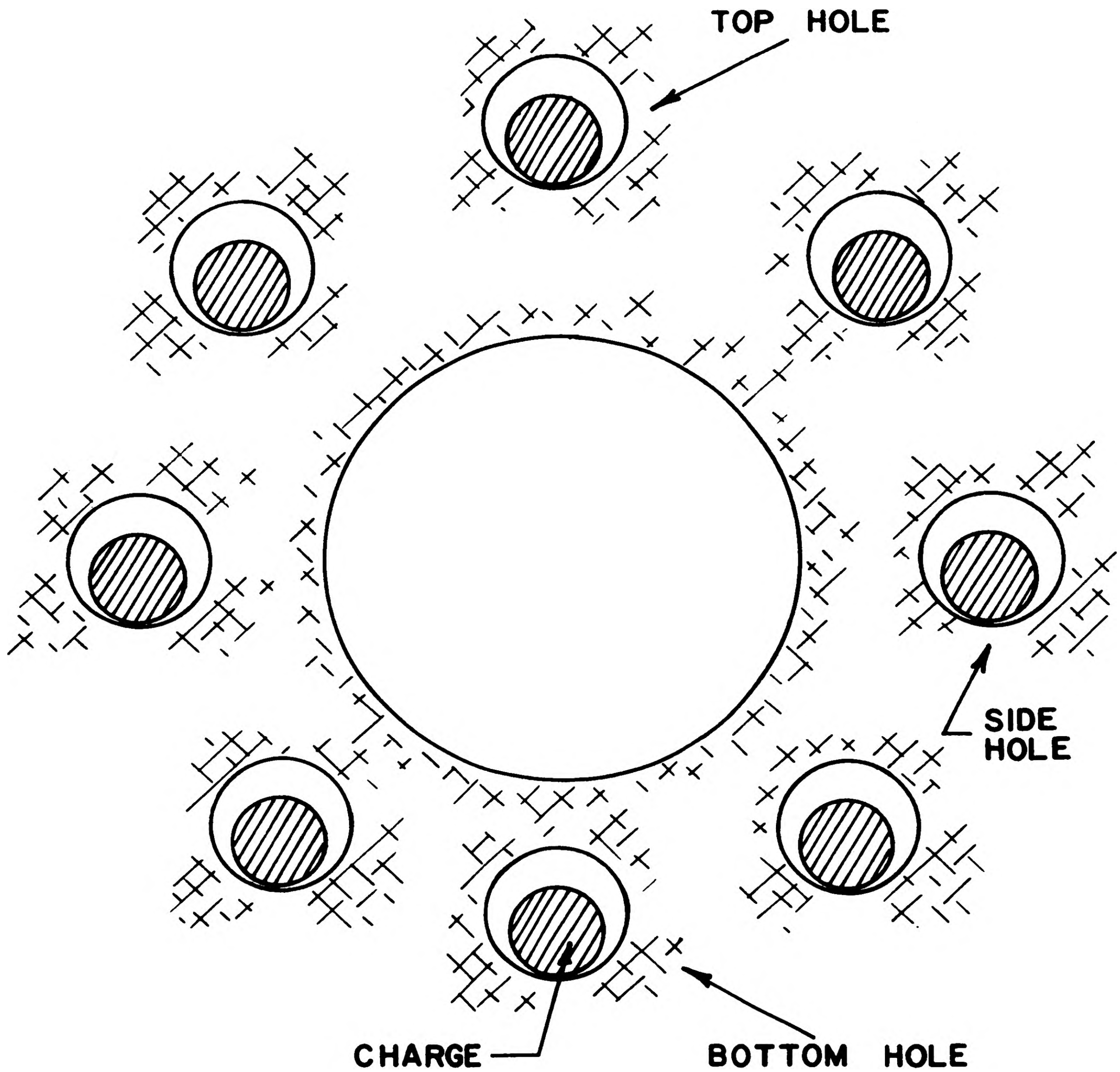

Fig. 4. Decoupled Charges around the Opening, Showing Burden and Spacing Relationships.

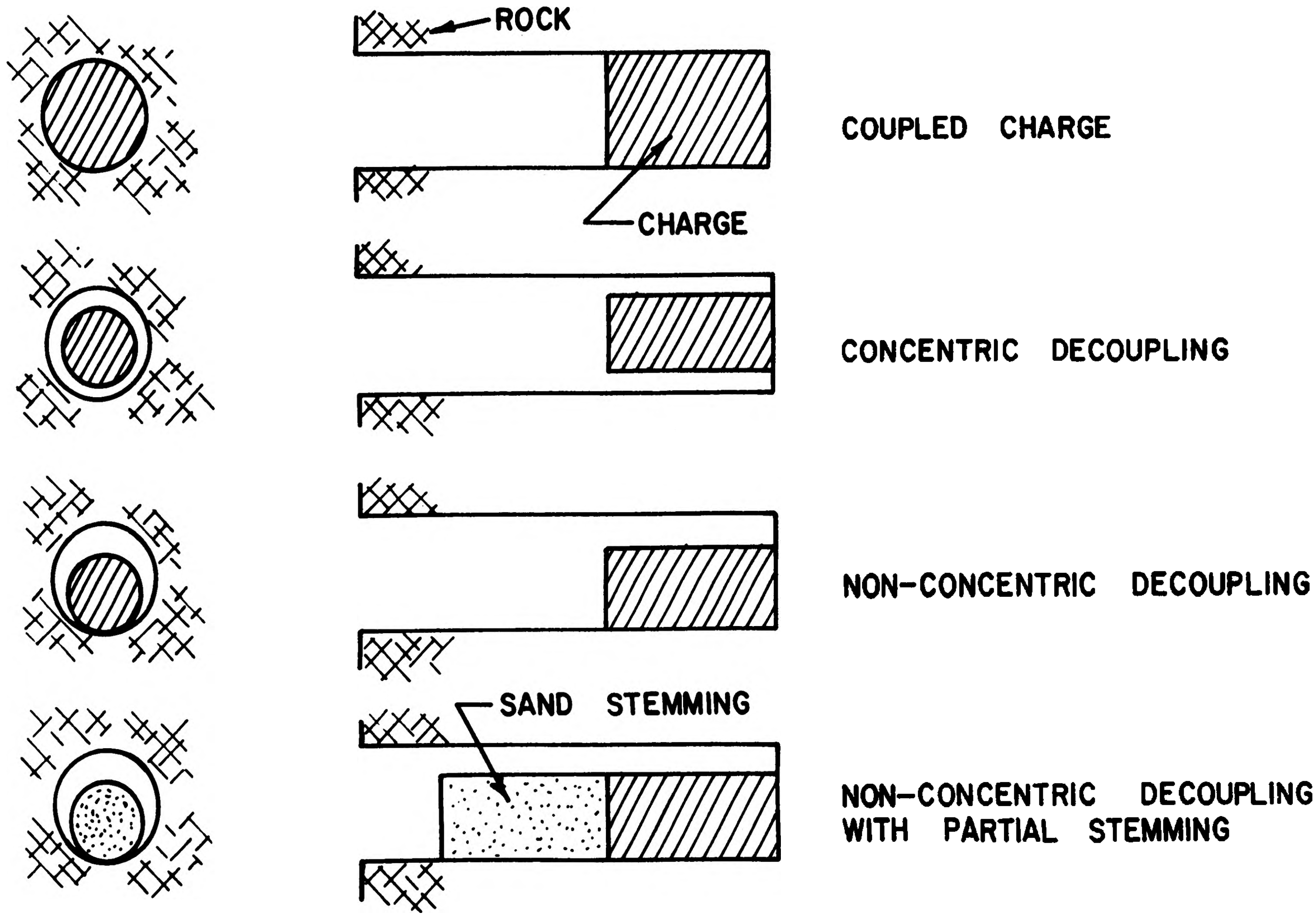

Fig. 5. Different Loading Conditions for Coupled and Decoupled Charges.

$P_m{}^o = \dfrac{2P_b}{\dfrac{(\rho V)_{exp}}{(\rho V)_{rock}} + 1}$ = radial stress in the rock at the cavity boundary

P_b = Borehole pressure

$(\rho V)_{exp}$ = Characteristic impedance of explosive

$(\rho V)_{rock}$ = Characteristic impedance of the rock

m = Decay exponent in the transition zone (to be evaluated)

σ_t = Static tensile strength of the rock

The static tensile strength of the solid rock, σ_t, may be considered constant only for homogeneous and isotropic solids, while in actual rocks this value depends on the probability of the existence of weakest points in the solid structure. This indicates that the tensile strength of the rock may have a range of values. A better choice would be the dynamic tensile strength, however this value is unknown for this rock. In general, the dynamic strength is believed to be higher than the static tensile strength.

The average borehole pressure, $P_{b(average)}$, for side holes is given by the equation

$$P_{b(average)} = \frac{P_3 R_c{}^5}{2R_h}\left[\frac{3(4a^2-2b)^2-4b^2}{8b^4} - \frac{(4a^2-2b)}{2b^3}\right] \qquad (50)$$

where

$$a = R_h - R_c$$

and

$$b = R_c{}^2 - 2R_hR_c$$

as compared to $P_b = P_3 \; (R_c/R_h)^5$ which applies to cylindrical symmetrical decoupling.

The numerical values for these variables are as follows:

R_h = 1 in.

R_c = 0.75 in.

ρ_r = Rock density = 2.6 gr/cc

V_r = Longitudinal velocity of the rock = 4,510 m/sec

ρ_e = Explosive density = 1.18 gr/cc

V_e = Detonation velocity = 5,049 m/sec

P_3 = explosion pressure = 28,930 atm ≃ 28,930 kg/cm^2

σ_t = Static tensile strength of the rock = 99.1 kg/cm^2.

From equation (50) $P_{b(average)}$ is 10,000 kg/cm^2 (∿35% P_3) and $\frac{(\rho V)_{exp}}{(\rho V)_{rock}} + 1 \simeq 1.51$, thus $P_m^{\,o}$ = 13,245 kg/cm^2. From figure (3), the maximum burden for side holes is R_t = 16 in. Therefore, the decay exponent (m) from equation (30) is calculated as:

$m = 1.41$

$1/m = 0.71$

and finally

$$R_{tmax} = R_t = \frac{R_h}{2} \left(\frac{P_m^{\,o}}{\sigma_t}\right)^{0.71} \qquad (51)$$

As an example for R_h = 1.25 in. and R_c = 0.75 in., the average borehole pressure according to equation (50) is:

$P_{b(average)}$ = 5,964 kg/cm^2

and

$$P_m^{o} = 7{,}900 \text{ kg/cm}^2$$

Therefore

$$R_{tmax} = 14 \text{ in.}$$

In the same way for R_h = 1.5 in. and R_c = 0.75 in., R_{tmax} = 13.3 in. For tunnel rounds, the burden dimension R_ϕ, is given by equation (31)

$$R_\phi = R_{tmax} \; (1-k^{-u\phi})$$

where the function $f(1-k^{-u\phi})$ fits the experimental data very closely.

Using the method of least squares, the constants k and u have been determined for side hole, unstemmed charges, fig. (3). The value of the constants thus obtained are: k = 3 and u = 0.41.

Therefore the above equation may be written:

$$R_\phi = R_{tmax}(1-3^{-0.41\phi}) \text{ for all } \phi \geq 2 \text{ ft.} \tag{52}$$

(below 2 ft. there was not complete breakage) with ϕ in ft. and R_ϕ in in.

The average borehole pressure for top hole (fig. 4) is given by the equation:

$$P_{b(average)} = \frac{P_3 R_c^{\,5}}{\pi\ R_h} \Bigg\{ \frac{\pi}{2} \left[\frac{3(4a^2-2b)^2-4b^2}{8b^4} - \frac{(4a^2-2b)}{2b^3} \right]$$

$$+ [-R^4+(4a^2-2b)R^2-b^2]^{1/2} \left[\frac{1}{b^2R^2} - \frac{3(4a^2-2b)^2+2b}{4b^3R^4} \right]$$

$$+ \text{arctg}\ \frac{(4a^2-2b)R^2-2b^2}{2b[-R^4+(4a^2-2b)R^2-b^2]^{1/2}} \left[\frac{(4a^2-2b)}{2b^3} \right.$$

$$\left. - \frac{3(4a^2-2b)^2-4b^2}{8b^4} \right] \Bigg\} \qquad (53)$$

where

$$a = R_h - R_c$$

$$b = R_c^{\,2} - 2R_hR_c$$

$$R = [R_h^{\,2} + (R_h - R_c)^2]^{1/2}$$

Thus, for $R_h = 1$ in. and $R_c = 0.75$ in.

$$a = 0.25$$

$$b = -0.9375$$

$$R = (1.0625)^{1/2}, \text{ and}$$

$$(4a^2-2b) = 4R_h^{\,2}+2R_c^{\,2}-4R_hR_c = 2.125$$

Substitution of these values into equation (53) yields:

$$P_{b(average)} = 16{,}675\ \text{kg/cm}^2\ (\sim 57\%\ P_3)$$

The constant 1/m must be theoretically the same as determined previously, for side holes 1/m = 0.71, since the geometric parameters and characteristics of the explosive and of the rock remain constant.

Thus, according to equation (30) we have:

$$R_{tmax} = R_t = R_h/2(P_m^o/\sigma_t)^{1/m}$$

for

$$R_h = 1 \text{ in.}$$

$$\sigma_t = 99.1 \text{ kg/cm}^2$$

and

$$P_m^o = 22{,}086 \text{ kg/cm}^2$$

The value of the burden for top holes (unstemmed charges) is R_t = 18 in. (fig. 3). Therefore:

$$1/m = 0.663 \simeq 0.66$$

Thus for top holes

$$R_{tmax} = R_t = R_h/2(P_m^o/\sigma_t)^{0.66}$$

The value of the decay constant for (side holes) is $1/m = 0.71$, whereas the decay constant for (top holes) is $1/m = 0.66$, a difference of 0.05, which indicates good agreement.

Following the same process, the average borehole pressure for a bottom hole (fig. 4) is given by the equation:

$$P_{b(average)} = \frac{P_3 R_c^{\ 5}}{\pi\ R_h} \left\{ \frac{\pi}{2} \left[\frac{3(4a^2-2b)^2-4b^2}{8b^4} - \frac{(4a^2-2b)}{2b^3} \right] \right.$$

$$+ [-R^4+(4a^2-2b)R^2-b^2]^{1/2} \left[\frac{3(4a^2-2b)+2b^2}{4b^3R^4} - \frac{1}{b^2R^2} \right]$$

$$+ \text{arctg} \frac{(4a^2-2b)R^2-2b^2}{2b[-R^4+(4a^2-2b)R^2-b^2]^{1/2}} \left[\frac{3(4a^2-2b^2-4b^2}{8b^4} \right.$$

$$\left. \left. - \frac{(4a^2-2b)}{2b^3} \right] \right\} \tag{54}$$

where

$$a = R_h - R_c$$

$$b = R_c^{\ 2} - 2R_hR_c$$

$$R = [R_h^{\ 2}+(R_h-R_c)^2]^{1/2}$$

Thus for $R_h = 1$ in., $R_c = 0.75$ in. and $P_3 = 28,930\ kg/cm^2$, we have

$$a = 0.25$$

$$b = -0.9375$$

$R = (1.0625)^{1/2}$, and

$$(4a^2-2b) = 4R_h^{\ 2}+2R_c^{\ 2}-4R_hR_c = 2.125$$

$P_{b(average)} = 3,320\ kg/cm^2$ ($\sim$11.5% P_3)

As before, the decay constant (1/m) to be determined, must be the same theoretically as previously obtained. Again from equation (30)

$$R_t = R_{tmax} = R_h/2(P_m^{\ o}/\sigma_t)^{1/m}$$

for

$R_h = 1$ in.

$\sigma_t = 99.1\ kg/cm^2$

$P_m^{\,o} = 4,397\ kg/cm^2$

The value of the burden for bottom holes (unstemmed charges) is $R_t = 12$ in. (fig. 3).

Therefore

$$1/m = 0.84.$$

Here, the decay exponent for bottom holes differs by 0.13 from that of side holes ($1/m = 0.71$). It is interesting to note that if the burden is 10 in. instead of 12 in., the decay exponent decreases to 0.79, and for 9 in. it equals 0.76. This shows that the decay exponent is very sensitive to small variations in rock properties, particularly the tensile strength σ_t which is assumed constant. Also relatively little data was obtained for bottom holes.

The critical decoupling D_o, taking into consideration perfect symmetry between the drill hole and the explosive charge, is as follows from equation (32):

$$D_o = \left[\frac{2P_3}{\sigma_t \left[\frac{(\rho V)_{exp}}{(\rho V)_{rock}} + 1 \right]} \right]^{1/5}$$

for

$P_3 = 28,930$ atm. $\approx 28,930\ kg/cm^2$

$\sigma_t = 99.1\ kg/cm^2$

and

$$\frac{(\rho v)_{exp}}{(\rho v)_{rock}} + 1 = 1.51$$

The critical decoupling D_o is:

$$D_o = 3.29 \approx (2.5/0.75)$$

Therefore, with a hole diameter of 5 in. and a charge diameter of 1.5 in., theoretically, the rock will not break.

B. Relationship between Burdens for Unstemmed and Partially Stemmed Charges

According to equation (34), the maximum velocity of the burden $V_{(R_t)max}$ is:

$$V_{(R_t)max} = (1.8\ E/M_r)^{1/2}$$

where

E = Available Energy

M_r = Mass of rock to be blasted

E = 982,446 ft-lbs/lb expl (see Table III)

$M_r = (R_t x S x h)\rho_r$

for

R_t = 20 in. = 0.508m for top holes, partially stemmed (fig. 3)

$S = 2.3\ R_t$ = 1.168m = spacing between holes

h = 36 in. = 0.914m = depth of hole

ρ_r = 2,600 kg/m^3 = rock density

Thus

$$M_r = 1{,}410 \text{ kg}$$

and for a mass of explosive of 0.75 lb.

$$E = 987{,}446 \text{ ft-lbs/lb expl} \times 0.75 = 736{,}834 \text{ ft-lbs}$$

$$= 999{,}147 \text{ Joules}$$

Therefore

$$V(R_t)_{max} = 26.62 \text{ m/sec}$$

and the decay exponent τ from equation (44) is:

$$\tau = \frac{V(R_t)_{max}\ 2.3\ R_t^{\ 2}\ \rho_r}{P_b\ \pi\ R_h}$$

where

$$P_b = 16{,}675 \text{ kg/cm}^2 \text{ (previously calculated)}$$

$$R_h = 1 \text{ in.} = 2.54 \text{ cm}$$

$$\rho_r = \frac{2{,}600 \text{ kg/m}^3}{980 \text{ cm/sec}^2} \text{ (expressed in force units)}$$

Thus

$$\tau = 3.15 \times 10^{-4} \text{ sec}$$

Similarly for top hole, unstemmed charge the value of R_{t_1} = 18 in. (fig. 3), and

$$V_{(R_{t_1})_{max}} = 29.6 \text{ m/sec}$$

$$\tau_1 = 2.83 \times 10^{-4} \text{ sec}$$

Therefore

$$\frac{\tau}{\tau_1} = 1.113 = \frac{R_t}{R_{t_1}}$$

or

$$R_t \text{ (partially stemmed)} = 1.113\ R_{t_1} \text{ (unstemmed)}$$

Thus, for (bottom holes)

R_t = 1.113 x 12 in. = 13.36 in., which is in good agreement with the value of R_t (partially stemmed) = 14 in. from fig. (3) determined experimentally.

IV. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The effect of decoupled and unstemmed charges for determining the burden at which rock would break when the free surface parallel to the borehole was the wall of a cylindrical opening was investigated. The following conclusions are indicated.

The peak borehole pressure is critically dependent on the ratio of the radius of the charge to the radius of the hole, and will be significantly lower than the explosion state pressure if the volume of the borehole is slightly greater than the volume of the gaseous products in the explosion state. Likewise, the co-volume exerts a great influence on the borehole pressure, inasmuch as the co-volume is an appreciable fraction of the borehole specific volume.

A good approximation for the borehole pressure can be obtained for cylindrical charges from the following expression:

$$P_b = P_3 \ (R_c/R_h)^{n_1}$$

where the value of the exponent, n_1, is approximately 5. This equation may be applied, with accuracy, for values of R_c/R_h near unity (0.6 - 1.0, approximately) where the exponent n_1 remains nearly constant.

The burden or "line of least resistance", is largely dependent on the borehole pressure and is one of the most important factors in determining rock breakage because almost all the area over which the pressure must act to perform work lies on the borehole sidewall. The ratios of burdens for unstemmed and partially stemmed charges are directly proportional to the ratios of the exponential decay times when an infinite reaction rate is assumed. The theoretical approach and test results together provide a guide for predicting the effects resulting from decoupled and unstemmed charges on the burden dimension in hard rock. Two areas in which further study is needed are:

1. Further verification of the theory of decoupling presented in this investigation requires additional tests extending the range of decoupling up to and above critical decoupling, preferably using cylindrical charges in cylindrical cavities. In this way it is possible to determine whether or not the exponent n_1 remains constant.

2. Similar experiments on other rock types.

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VITA

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APPENDIX A

Theoretical approach for determining the value of the exponent (n).

According to Cook[1] the borehole pressure P_b is related to the adiabatic or explosion pressure P_3 by a relation of the form

$$P_b = P_3 \Delta^n \qquad (1)$$

where n = approximately 2.5 and Δ = the loading density or the fraction of the borehole occupied by the explosive excluding the open hole above the explosive.

Thus

$$P_b = P_3 (V_3/V_b)^{2.5} = P_3 (v_3/v_b)^{2.5} \qquad (2)$$

where

V_b = Volume of borehole

V_3 = Volume of explosive charge

$v_b = V_b/M = \frac{\text{Volume of borehole}}{\text{mass of explosive}}$ = specific volume of borehole

$v_3 = V_3/M = \frac{\text{Volume of Explosive charge}}{\text{mass of explosive}}$ = specific volume of explosive charge.

Equation (2) may be written as follows

$$P_b v_b^{2.5} = P_3 v_3^{2.5} \qquad (3)$$

The relationship between pressure and volume in an adiabatic, reversible process is given by the equation:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \tag{4}$$

or

$$P_1v_1^{\gamma} = P_2v_2^{\gamma} \tag{5}$$

where γ is the ratio of the specific heat capacities of the explosion gases, and for most explosives a value of $\gamma = 1.2$ is a good approximation.

Thus, equation (5) becomes

$$P_1v_1^{1.2} = P_2v_2^{1.2} \tag{6}$$

or

$$P_bv_b^{1.2} = P_3v_3^{1.2} \tag{7}$$

Comparing equation (3) with (7), gives a difference of the exponent of a factor of two.

Because the expansion of gases is not truly reversible, a correction factor (ζ) will be used as an approximation. Therefore, equation (5) becomes:

$$P \cdot v^{\gamma\zeta} = K = \text{constant} \tag{8}$$

From the laws of thermodynamics it is known that a change in internal energy of gases is given by

$$dE = dq - dw \tag{9}$$

where q is heat and w work.

Equation (9) may be written as follows.

$$n\, C_v\, dT = dq - Pdv \tag{10}$$

For an adiabatic expansion dq = 0 and equation (10) becomes

$$n\ C_v\ dT = -Pdv \tag{11}$$

where n = number of moles of gases per unit of mass. T, P and v are temperature, pressure and specific volume, and C_v = specific heat capacity of gaseous products at constant volume.

For one mole of an ideal gas $R_g = C_p - C_v$, then

$$\frac{C_p}{C_v} = \gamma = 1 - \frac{R_g}{C_v} \tag{12}$$

where R_g = the gas constant and C_p = specific heat capacity of gaseous products at constant pressure. For $\gamma = 1.2$ and $R_g = 1.987 \frac{cal}{mol\ {}^oK}$, $C_v \simeq 10 \frac{cal}{mol\ {}^oK}$.

Substitution of P from equation (8) into (11) gives

$$n\ C_v\ dT + \frac{K}{v^{\gamma\zeta}}\ dv = 0 \tag{13}$$

Integration from the state T_3, v_3 to the state T_b, v_b gives

$$n\ C_v \int_{T_3}^{T_b} dT + \int_{v_3}^{v_b} \frac{K}{v^{\gamma\zeta}}\ dv = 0 \tag{14}$$

or

$$n\ C_v\ (T_b - T_3) + \left[\frac{Kv}{v^{\gamma\zeta}(1-\gamma\zeta)}\right]_{v_3}^{v_b} = 0 \tag{15}$$

but $P = \frac{K}{v^{\gamma\zeta}}$, therefore

$$n\ C_v\ (T_b - T_3) + \left[\frac{Pv}{(1-\gamma\zeta)}\right]_{v_3}^{v_b} = 0 \qquad (16)$$

and finally

$$n\ C_v\ (T_b - T_3) + \frac{P_b v_b - P_3 v_3}{(1-\gamma\zeta)} = 0 \qquad (17)$$

Using the equation of state

$$P_b(v_b - \alpha_b) = nR_g T_b \qquad (18)$$

Where α is the co-volume, and the equation

$$P_3 v_3{}^{\gamma\zeta} = P_b v_b{}^{\gamma\zeta} \qquad (19)$$

and also

$$P_3(v_3 - \alpha_3) = nR_g T_3 \qquad (20)$$

Subtracting (20) from (18) gives the following:

$$P_b(v_b - \alpha_b) - P_3(v_3 - \alpha_3) = nR_g(T_b - T_3) \qquad (21)$$

Dividing by R_g and multiplying by C_v gives:

$$\frac{C_v}{R_g}\left[P_b(v_b - \alpha_b) - P_3(v_3 - \alpha_3)\right] = n\ C_v(T_b - T_3) \qquad (22)$$

Substituting eq. (22) into (17) gives:

$$\frac{P_b v_b - P_3 v_3}{(1-\gamma\zeta)} + \frac{C_v}{R_g}\left[P_b(v_b - \alpha_b) - P_3(v_3 - \alpha_3)\right] = 0 \qquad (23)$$

or

$$\frac{P_b}{P_3} = \frac{[R_g v_3 + C_v(1-\gamma\zeta)(v_3-\alpha_3)]}{[R_g v_b + C_v(1-\gamma\zeta)(v_b-\alpha_b)]} \tag{24}$$

From (19) we have:

$$P_3 v_3^{\gamma\zeta} = P_b v_b^{\gamma\zeta}$$

Thus

$$\frac{P_b}{P_3} = (v_3/v_b)^{\gamma\zeta} = (V_3/V_b)^{\gamma\zeta} \tag{25}$$

and

$$\frac{[R_g V_3 + C_v(1-\gamma\zeta)(v_3-\alpha_3)]}{[R_g V_b + C_v(1-\gamma\zeta)(v_b-\alpha_b)]} = (v_3/v_b)^{\gamma\zeta} \tag{26}$$

where $\gamma = 1.2$, $C_v \simeq 10 \frac{\text{cal}}{\text{mol } ^{\circ}\text{k}}$ and $R_g = 1.987 \frac{\text{cal}}{\text{mol } ^{\circ}\text{k}}$. The explosive used in the experimental work was a 0.75 lb. slurry charge, 1.5 in. diameter, and 10 in. long, with a density $\rho_3 = 1.18$ gr/cc.

Thus $v_3 = \frac{1}{\rho_3} = 0.8474$.

The density in the borehole for a diameter of 2 in., is $\rho_b = 0.663$ gr/cc., then $v_b = \frac{1}{\rho_b} = 1.5083$. Thus for $v_3 = 0.8474$, $\alpha_3 = 0.624$[1] and $(v_3-\alpha_3) = 0.2234$ and $v_b = 1.5083$, $\alpha_b = 0.8117$ and $(v_b-\alpha_b) = 0.6966$.

With these data, the value of (ζ) from (26) is $\zeta \simeq 2.1$ then $n = \gamma\zeta = 2.52 \simeq 2.5$ and equation (25) becomes:

$$P_b = P_3 \; (V_3/V_b)^{2.5} \tag{27}$$

For cylindrical charges, this may be written as follows:

$$P_b = P_3 (R_c/R_h)^5 \qquad (28)$$

Values of ζ for different loading densities were obtained from (26) and the data in Table I giving a range of values between 2 and 2.2 and consequently $n \simeq 2.5$, gives an excellent agreement with Cook's approximation.

For values of Δ close to unity, the value of $n \simeq 2.5$, is a good approximation. The equation of state, $P(v-\alpha) = nR_gT$, gives excellent results over a range of v from 0.2 to 2 cc/gr. If v_b is too high and outside of this range this equation cannot be applied with accuracy, also the borehole pressure, P_b, will be low and another equation of state such as Van der Waal's must be used.

APPENDIX B

Theoretical determination of the average borehole pressure.

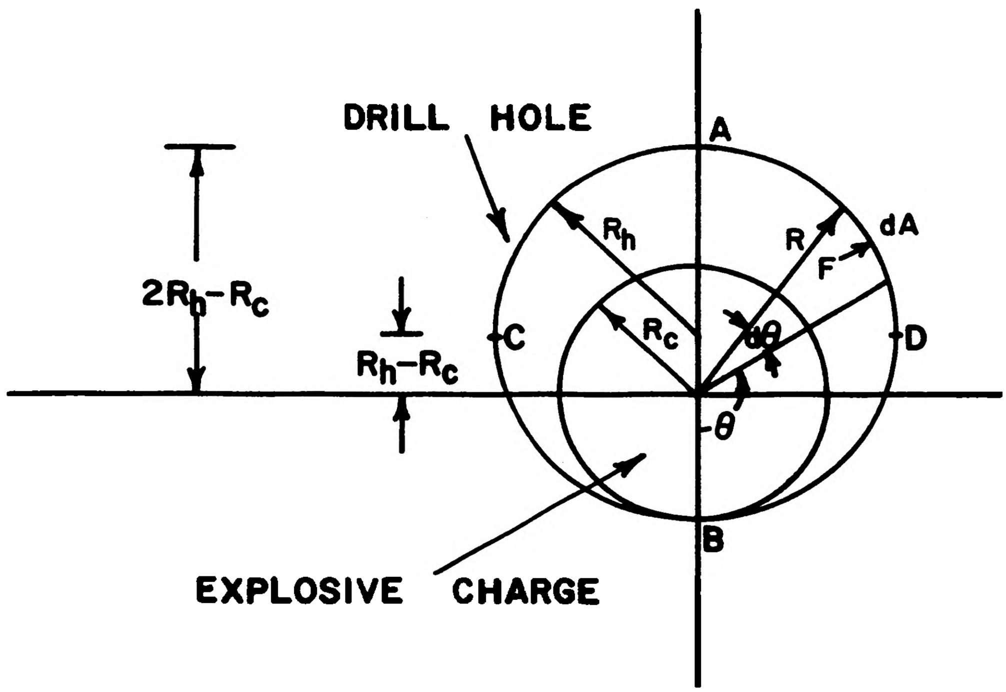

Fig. 6. Decoupled Model for Non-Concentric Charge.

The relationship for symmetrical decoupling of a cylindrical charge is:

$$P_b = P_3 \ (R_c/R_h)^5 \tag{1}$$

where P_b = borehole pressure, P_3 = explosion pressure, R_c = radius of the charge and R_h = radius of drill hole. For non-symmetrical decoupling, the radial distance from the center of the charge to the wall of the borehole, R, changes with respect to the angle θ (fig. 6). Thus, equation (1) must be modified as follows for non-symmetrical decoupling.

$$P_b = P_3 \ (R_c/R)^5 \tag{2}$$

where $R = f(\theta)$.

Taking a differential of area, dA, the relationship between force F and borehole pressure P_b is

$$dF = P_b \cdot dA \tag{3}$$

where dF = differential force acting over the area dA, and $dA = Rd\theta$ assuming a hole of unit depth. Substitution of equation (2) into (3) yields

$$dF = P_3 \ (R_c/R)^5 \ dA \tag{4}$$

$$F = \int P_3 \ (R_c/R)^5 \ dA. \tag{5}$$

According to the cosine law the relationship between R and θ is:

$$R^2+2R(R_h-R_c)\cos\theta+(R_c^2-2R_hR_c) = 0$$

or

$$R^2 + 2\,R\,a\cos\theta + b = 0 \tag{6}$$

where $a = R_h-R_c$ and $b = R_c^2-2R_hR_c$. From equation (6):

$$\cos\theta = -\frac{1}{2a}\left(\frac{R^2+b}{R}\right) \tag{7}$$

differentiation of both sides of eq. (7) gives:

$$\sin\theta \, d\theta = \frac{1}{2a}\left[\frac{R^2-b}{R^2}\right] dR \quad (8)$$

or

$$(1-\cos^2\theta)^{1/2} \quad d\theta = \frac{1}{2a}\left(\frac{R^2-b}{R^2}\right) dR \quad (9)$$

Substitution of eq. (7) into (9) yields:

$$[4a^2R^2-(R^2+b)^2]^{1/2} \, d\theta = \left(\frac{R^2-b}{R}\right) dR$$

and

$$d\theta = \frac{(R^2-b)}{R[4a^2R^2-(R^2+b)^2]^{1/2}} dR \quad (10)$$

thus $dA = R \, d\theta$ becomes:

$$dA = \frac{(R^2-b)}{[4a^2R^2-(R^2+b)^2]^{1/2}} dR \quad (11)$$

Therefore equation (5) yields:

$$F = \int P_3 \left(\frac{R_c}{R}\right)^5 \frac{(R^2-b)}{[4a^2R^2-(R^2+b)^2]^{1/2}} dR \quad (12)$$

or

$$F = P_3R_c^{\,5} \int \frac{(R^2-b)}{R^5[4a^2R^2-(R^2+b)^2]^{1/2}} dR \quad (13)$$

Considering the pressure acting on the sidewall for a half circumference AB:

$$P_b \text{ (average)} = \frac{F}{\pi R_h} \quad (14)$$

Thus:

$$P_b \text{ (average)} = \frac{P_3R_c^{\,5}}{\pi R_h} \int_{R_1}^{R_2} \frac{(R^2-b)}{R^5[4a^2R^2-(R^2+b)^2]^{1/2}} dR \quad (15)$$

And the solution of the integral is:

$$P_b\ (\text{average}) = \frac{P_3 R_c^{\,5}}{\pi R_h}\ \frac{1}{2}\Bigg[\frac{[-R^4+(4a^2-2b)R^2-b^2]^{1/2}}{b^2R^2}$$

$$+\ \frac{(4a^2-2b)}{2b^3}\ \text{arctg}\ \frac{(4a^2-2b)R^2-2b^2}{2b[-R^4+(4a^2-2b)R^2-b^2]^{1/2}}$$

$$-\ \frac{3(4a^2-2b)+2b^2}{4b^3R^4}\ [-R^4+(4a^2-2b)R^2-b^2]^{1/2}$$

$$-\ \frac{3(4a^2-2b)^2-4b^2}{8b^4}\ \text{arctg}\ \frac{(4a^2-2b)R^2-2b^2}{2b[-R^4+(4a^2-2b)R^2-b^2]^{1/2}}\Bigg]_{R_1}^{R_2} \qquad (16)$$

At the limiting condition when $R_h = R_c$, from equation (6) we have $R^2 = -b$ where $a = 0$ and $b = -R_c^{\,2}$. Thus $R_1 = -R_c(\theta = 0^\circ)$ and $R_2 = R_c(\theta = 180^\circ)$ and equation (16) becomes:

$$P_b\ (\text{average}) = \frac{P_3}{\pi}\ \frac{R_c^{\,4}}{2}\ \left(\frac{\pi}{R_c^{\,4}} + \frac{\pi}{R_c^{\,4}}\right) = P_3$$

The lower limit and upper limit of the definite integral considering the average borehole pressure acting on the half circumference AB (side holes in fig. 6) are according to equation (6) $R_1 = R_c$ $(\theta = 0^\circ)$ and $R_2 = 2R_h - R_c$ $(\theta = 180^\circ)$. Equation (16) then becomes:

$$P_b\ (\text{average}) = \frac{P_3R_c^{\,5}}{2\pi R_h}\Bigg[\frac{(4a^2-2b)}{2b^3}\ \text{arctg}(\infty) - \frac{3(4a^2-2b)-4b^2}{8b^4}$$

$$\text{arctg}(\infty) - \frac{(4a^2-2b)}{2b^3}\ \text{arctg}(-\infty) + \frac{3(4a^2-2b)-4b^2}{8b^4}\ \text{arctg}(-\infty)\Bigg]$$

Therefore

$$P_{b\ (average)} = \frac{P_3 R_c^{\ 5}}{2\pi R_h}\left[\frac{3(4a^2-2b)^2}{8b^4}\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) - \frac{(4a^2-2b)}{2b^3}\left(\frac{3\pi}{2} - \frac{\pi}{2}\right)\right]$$

and the average borehole pressure for (side holes) is:

$$P_{b\ (average)} = \frac{P_3 R_c^{\ 5}}{2\pi R_h}\left[\frac{3(4a^2-2b)^2-4b^2}{8b^4} - \frac{(4a^2-2b)}{2b^3}\right] \tag{17}$$

where $a = R_h - R_c$, $b = R_c^{\ 2} - 2R_hR_c$, and $(4a^2-2b) = 4R_h^{\ 2} + 2R_c^{\ 2} - 4R_hR_c$.

Again when $R_c = R_h$ (17) becomes $P_{b\ (average)} = P_3$. Following the same process, the average borehole pressure for (top holes) acting on the half circumference (DBC) (fig. 6) is:

$$P_{b\ (average)} = \frac{P_3 R_c^{\ 5}}{\pi R_h}\left\{\frac{\pi}{2}\left[\frac{3(4a^2-2b)^2-4b^2}{8b^4} - \frac{(4a^2-2b)}{2b^3}\right] + [-R^4+(4a^2-2b)R^2-b^2]^{1/2}\left[\frac{1}{b^2R^2} - \frac{3(4a^2-2b)+2b^2}{4b^3R^4}\right] + \operatorname{arctg}\frac{(4a^2-2b)R^2-2b^2}{2b[-R^4+(4a^2-2b)R^2-b^2]^{1/2}}\left[\frac{(4a^2-2b)}{2b^3} - \frac{3(4a^2-2b)^2-4b^2}{8b^4}\right]\right\} \tag{18}$$

where $R = [R_h^{\ 2} + (R_h - R_c)^2]^{1/2}$.

And for (bottom holes) considering the half circumference (DAC) (fig. 6) is:

$$P_b \text{ (average)} = \frac{P_3 R_c^{\,5}}{\pi R_h} \Bigg\{ \frac{\pi}{2} \left[\frac{3(4a^2-2b)^2-4b^2}{8b^4} - \frac{(4a^2-2b)}{2b^3} \right]$$

$$+ [-R^4+(4a^2-2b)R^2-b^2]^{1/2} \left[\frac{3(4a^2-2b)+2b^2}{4b^3R^4} - \frac{1}{b^2R^2} \right]$$

$$+ \operatorname{arctg} \frac{(4a^2-2b)R^2-2b^2}{2b[-R^4+(4a^2-2b)R^2-b^2]^{1/2}} \left[\frac{3(4a^2-2b)^2-4b^2}{8b^4} \right.$$

$$\left. - \frac{(4a^2-2b)}{2b^3} \right] \Bigg\} \qquad (19)$$

Likewise when $R_h = R_c$ equations (18) and (19) become P_b (average) $= P_3$.