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Series capacitors

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SERIES CAPACITORS

BY

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A

THESIS

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INTRODUCTION

Static capacitors as an element of transmission and distribution circuits first appeared in the form of shunt capacitors. These shunt capacitors were primarily used in power factor correction. They can be regarded as a load in the circuit which draws a leading current. This leading current flowing through the reactive line produces a voltage rise, thus reducing the voltage drop between sending and receiving ends. This phenomenon helps the voltage control of the feeder as well as power factor correction. The disadvantage of the shunt capacitor is its inflexibility. The shunt capacitor is connected across the line; hence the voltage across its terminals is constant. The idea of connecting the capacitor in series with the line rather than parallel, thus making the capacitor an element of the line, opened a new field in transmission and distribution engineering. The voltage across the terminals of the series capacitor is not constant but is directly proportional to the line current. This characteristic of the series capacitor makes it extremely valuable in some applications; however, it is the same characteristic which in some cases makes its application difficult. The series capacitor as a voltage regulator in distribution systems [(1), (2), (3), (4)], proved very use-

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(1) Arnold, E. P. Series capacitor proves economical. Electrical West. pp. 48-50 (December, 1936)

(2) Dudley, C. L., and Snyder, E. H. Why not series capacitors for distribution systems. Electrical World. pp. 942-945 (June 30, 1934)

(3) John, J. L. Series capacitor wipes out feeder flicker. Electrical World pp. 3688-3690 (November 7, 1936)

(4) Smith, C. H. Series capacitor smoothes jagged voltages on long feeder. Electrical World. Vol. 114, pp 32-33 (December 28, 1940)

ful and economical. It found a large application field associated with electric arc furnaces (5), (6) and electric

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(5) Black, P. M., and Lischer, L. F. The application of a series capacitor to a synchronous condenser for reducing voltage flicker. A.I.E.E. Technical paper. (51-20)(December, 1950)

(6) Witzke, R. L., and Michelson, E. L. Technical problems associated with the application of a capacitor in series with a synchronous condenser. A.I.E.E. Technical paper. (51-88)(December, 1950)

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welders. (7) The first difficulty in the application of

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(7) James, F. D. The application of series capacitors to flash welders. A.I.E.E. Trans. Vol. 65, pp. 686-689(1946)

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series capacitor showed itself in its protection. An air gap, which flashes over at a certain voltage above the rated voltage of the capacitor, and short circuits the capacitor unit under abnormal conditions, constitutes the basic principle of the protection device of the series capacitor. (8), (9), (10)

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(8) Bloomquist, W. C., and Schroeder, T. W. Selection and application of automatic control for capacitors. General Electric Review. Vol. 48, pp. 37-40 (1946)

(9) Johnson, A. A., Marbury, R. E., and Arthur J. M. Design and protection of 10,000 kva series capacitor for 66 kv transmission line. A.I.E.E. Trans. Vol. 67, pp. 363-367 (1948)

(10) Marbury, R. E., and Owens, J. B. New series capacitor protective device. A.I.E.E. Trans. Vol. 65, pp. 142-146 (1946)

Series capacitors should also be protected against dielectric failure and continuous operation under over-load conditions. Furthermore from the standpoint of transient stability of power systems, series capacitors should be reinserted in the circuit as soon as the fault is cleared. These operations are provided by fast acting relays.

During recent years, the necessity of transmitting large blocks of power over long distances and the comparatively high costs, losses and difficulties associated with the construction of large synchronous condensers, opened an application field for series capacitors in transmission lines. The series capacitor proved itself to be very useful and economical in this field. However new problems associated with the various application of series capacitors arose.[11]

(11) Butler, J. W., and Concordia, C. Analysis of series capacitor application problems. A.I.E.E. Trans. Vol. 56, pp. 975-988 (1937)

Self-excitation of synchronous machines, self-excitation of induction motors, hunting of synchronous machines, and ferro-resonance in transformers are some of the important problems which are discussed in the body of the thesis. A solution for and methods of correcting the difficulty are established.

In this thesis the author has made an attempt to cover as many problems associated with the use of series capacitors as possible emphasizing some important points, and briefly explaining others. One other thing which interested the author in this subject is that in his country, Turkey, natural resources available for obtaining electric energy are located long distances from load centers thus making the transmission of large amounts of power over long distances necessary. Furthermore, in Turkey large industrial and residential lighting loads are both concentrated in big cities raising the problem of voltage regulation. Since series capacitors prove useful and economical both in voltage regulation and in transmission of large blocks of power over long distances, the author found this subject worthy of serious study. In this survey type of thesis the author tried to present some of the important principles explained in different articles with some minor contributions of his own.

METHODS OF LINE REACTANCE COMPENSATION

Good voltage regulation and lower transmission cost are the most important basic factors in transmission and distribution engineering. The regulation which exists between the generating station bus and the user's premises depends upon the circuit impedance the major portion of this impedance, being the reactance. Many devices have been produced to compensate the voltage drop due to the line reactance, but unfortunately a device which can neutralize the voltage drop due to the line resistance has not yet been found.

Sometimes due to the natural resources largescale hydroelectric developments are located in relatively remote districts and have raised the question of delivering power of the order of 300,000-400,000 kw over a distance more than 250 miles.

A large value of power developed and a long distance it is transmitted result in the increase of line drop and in dissipated energy in the line. As it is stated above, there are several devices produced to secure economical transmission of power. A recent development achieved in this field is related to the series capacitor. However in order to understand the factor of series capacitor in transmission lines it is advisable to obtain a bird's eye view of other devices.

If overhead line construction is to be considered, four different alternatives are available for reducing the reactance of the line to a desired value:

1.- Line impedance conversion
2.- Multiple circuit operation
3.- Reduction of system frequency
4.- Line reactance compensation.

1.- Line impedance conversion:

In this case the reduction of the equivalent series impedance is accomplished by transformation to high voltages. Now, the equivalent impedance of the high tension side referred to the low tension side is equal to the actual impedance of the high tension side divided by the transformer turn ratio squared. Thus the equivalent impedance between the low voltage busses is lower. On the other hand when voltages are increased, transmission structures become larger, separations and clearances become necessarily greater, line insulation, transformers and high voltage substation equipment become more costly.

2.- Multiple circuit operation:

If two or more transmission circuits are operated in parallel the impedance of the system is decreased. Furthermore if such multiple circuits are sectionalized at certain points remarkable improvement in operating characteristics and transient stability limits can be obtained. Moreover, with multiple circuits a flexibility of operation is obtained. However, the excessive cost of multiple circuits with sectionalizing facilities compared with the little gain in reduction of the impedance suggests the possibility of other means of providing economic loading status of long high voltage circuits.

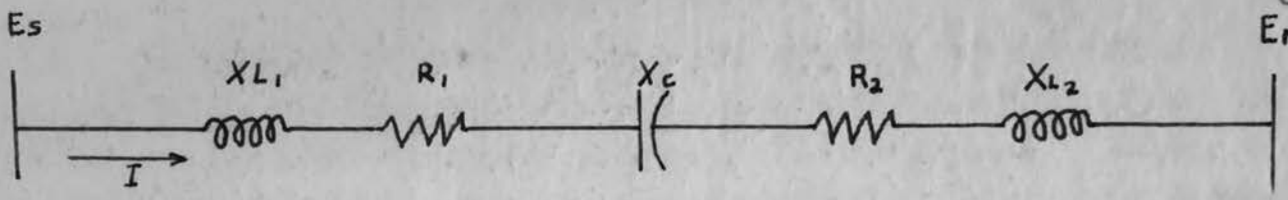
3.- Reduction of the system frequency:

Since reactance is directly proportional to the frequency ($X_L = 2\pi fL$) a reduction in the system frequency will result in a corresponding reduction in the line reactance. Again the necessity of numerous frequency changers, increased size and cost of induction equipment and incandescent lamp flicker at low frequencies constitute some very undesirable disadvantages.

4.- Line reactance compensation:

The compensation of line reactance is accomplished by inductance regulators, synchronous condensers, shunt capacitors and series capacitors. The comparison between the series capacitors and any of the other types will be made as the outstanding features of series capacitor are made clear. From the standpoint of maintenance, the static capacitor is sturdy and has few moving parts. The internal losses of the capacitor are much less than those of the regulator. The synchronous condenser with control equipment is more complicated than the static capacitor. The average losses of a static capacitor are about 0.25% of its rating where the losses of a synchronous condenser is sometimes from 5% to 8% of its rating. The shunt capacitor is similar in operation to a synchronous condenser, namely both improve voltage regulation by improving power factor. However shunt capacitors are inflexible; if they are left on light loads they cause a voltage rise at distribution points drawing leading current. Series capacitors, unlike shunt capacitors, conduct full line current and hence the voltage drop across a series capacitor is instantaneous and completely dependent of load conditions. This characteristic of series capacitor makes it very useful in the compensation of line reactance.

It is known that capacitances in alternating current circuits draw leading currents. In the same way a series capacitor connected in series with a transmission line draws a leading current thereby introducing a negative or leading reactance. If the proper capacity of the series capacitor is chosen, the leading reactance introduced by it completely eliminates the effect of the lagging reactance of the transmission line itself. This is illustrated in Fig.1.



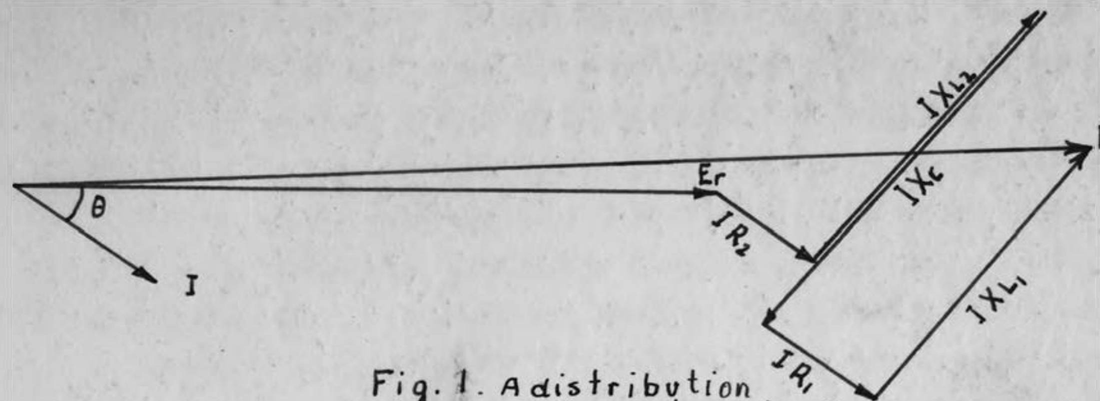


Fig. 1. A distribution line and its vector diagram

Sometimes when the feeder resistance is relatively high overcompensation is employed. On the other hand if overcompensation is employed, a lagging current due to the starting of a large motor may cause an excessive voltage rise which is harmful to lighting equipment. Furthermore when the load power factor is leading it is obvious that for this condition series capacitor reduces the receiving end voltage, hence it is undesirable under such circumstances.

It can be easily observed from Fig. 1 that the overall impedance of the line or, in other words, the effective length of the line is greatly reduced. This phenomenon makes possible the transmission of more power over a compensated transmission line than over an uncompensated line. However, up to now, the use of series capacitors has been confined mostly to distribution systems. The function of series capacitor in distribution systems consists of overcoming the bad voltage fluctuations caused by starting of motors, variation of motor loads, electric welders and furnaces. This is accomplished by the outstanding characteristic feature of series capacitors namely, their instantaneous response to changes in load. As was pointed out before series capacitor conducts the full line current. Hence the leading voltage drop across its terminals depends upon the load conditions. An instantaneous change in load current will result in an instantaneous change in the voltage drop along the line and an instantaneous change in the voltage across the series capacitor. Whence the change in the lagging voltage drop along the line will at once be compensated by the leading voltage drop across the terminals of the series capacitor. Thus not only the gradual voltage drop due to slowly increasing load but also the transient voltage drop due to motor starting, electric welders etc. will be almost instantaneously reduced.

THE USE OF SERIES CAPACITORS IN DISTRIBUTION SYSTEMS

The main duty of series capacitors in distribution circuits is to reduce fluctuations in feeder voltage thus providing a smooth voltage at the load end of the line. The series capacitor is a part of the impedance of the circuit. Its effect is to compensate the reactive drops in the circuit. When the reactive drop of the circuit is compensated, a voltage rise will appear on the load side of the series capacitor. In order to investigate the function of a series capacitor in a circuit the following example can be considered.

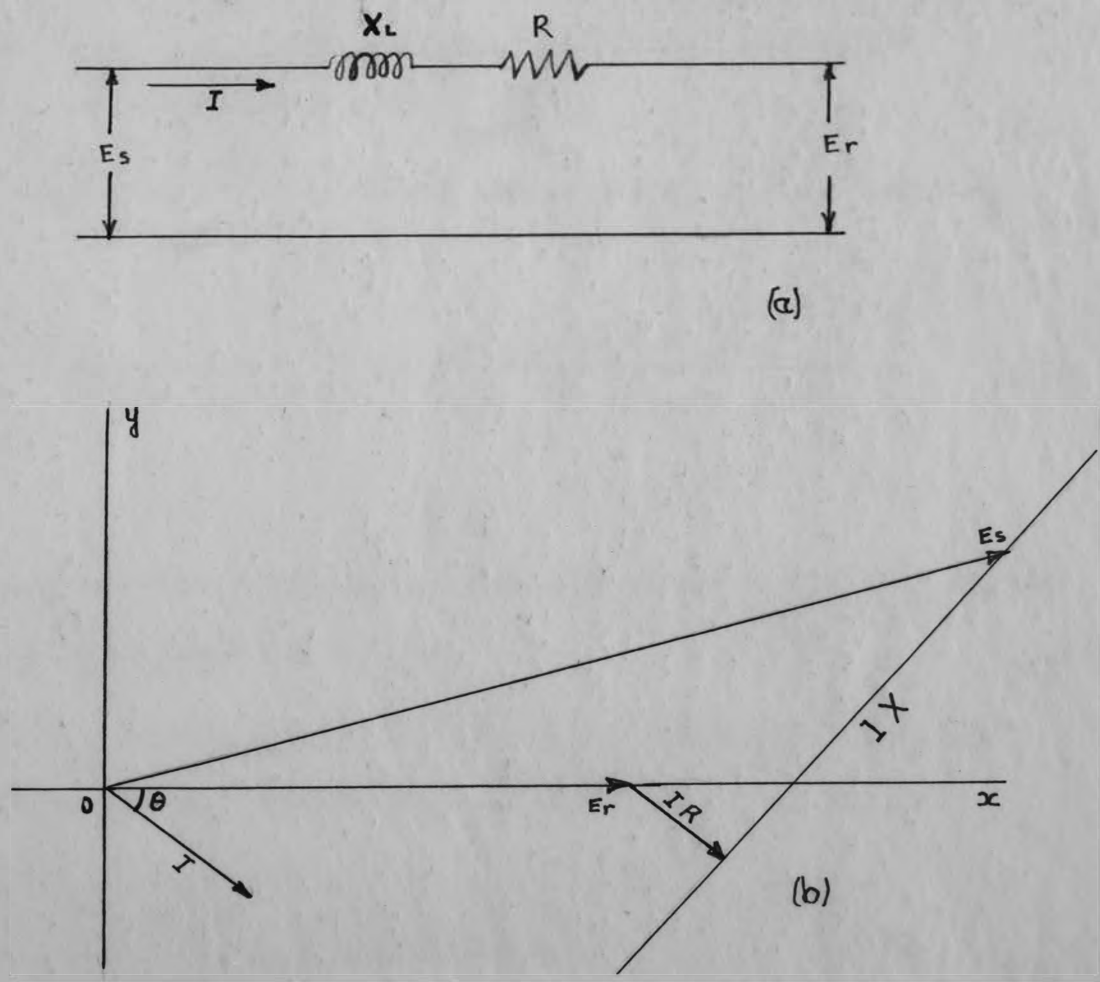


Fig.2. A distribution line and its vector diagram.

Fig. 2a indicates a circuit having a resistance R and a reactance X_L. E_s and E_r refers to sending end voltage and receiving end voltage respectively. The current flowing in the circuit is designated as I. The power factor of load is indicated by cos θ. The corresponding vector diagram of this circuit is shown in Fig. 2b. In order to compensate the reactance drop IX_L a series capacitor is inserted in the circuit. The value of the capacitive reactance of the series capacitor may be less than, equal to, or greater than the inductive reactance of the circuit in ohms.

The equation of the line represented by the reactive drop IX in Fig. 2b is found easily to be:

$$y = \operatorname{ctn}\theta \left[x - E_r - \frac{IR}{\cos\theta} \right] \qquad (1)$$

Then E_s, which is the distance between origin and any point on the line is given by the following formula:

$$E_s = \sqrt{x^2 + \left[\operatorname{ctn}\theta \left(x - E_r - \frac{IR}{\cos\theta} \right) \right]^2} \qquad (2)$$

Taking the square of both sides and arranging the terms:

$$\frac{E_s^2}{\left(E_r + \frac{IR}{\cos\theta}\right)^2 \left(\sin^2\theta - \cos^4\theta\right)} - \frac{\left[x - \cos^2\theta \left(E_r + \frac{IR}{\cos\theta} \right) \right]^2}{\left(E_r + \frac{IR}{\cos\theta}\right)^2 \left(\sin^2\theta - \cos^4\theta\right)} \qquad (3)$$

The equations of the asymptotes are as given by the ± sign:

$$E_s = \mp \left[x - \cos^2\theta \left(E_r + \frac{IR}{\cos\theta} \right) \right] \qquad (4)$$

The plot of the curve given by Eq. (3) is shown in Fig. 3.

Any value of the reactive drop as given by IX is:

$$IX = \frac{x - \frac{IR}{\cos\theta}}{\sin\theta} + IR \tan\theta \qquad (5)$$

This may be simplified to:

$$IX = \frac{1}{\sin\theta}\, x - \frac{1}{\sin\theta}\left(E_r + \frac{IR}{\cos\theta} \right) + IR\tan\theta \qquad (6)$$

The line given by Eq.(6) is shown in Fig. 4. The minus values of IX indicate that the impedance of the transmission line is capacitive, and plus values of IX indicate that it is inductive although it may be compensated.

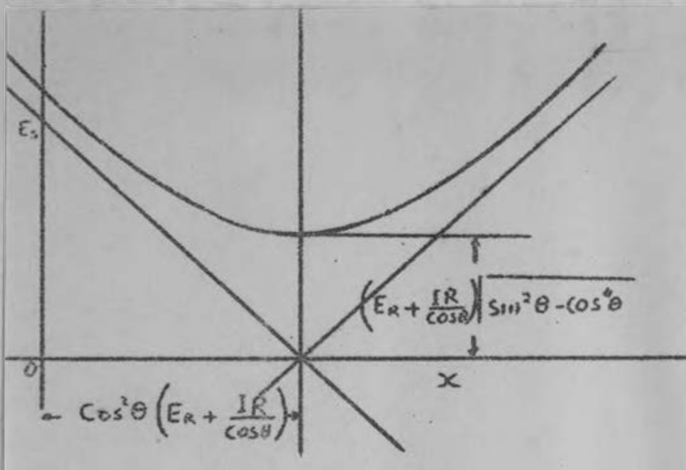


Fig. 3 Plot of Es against x

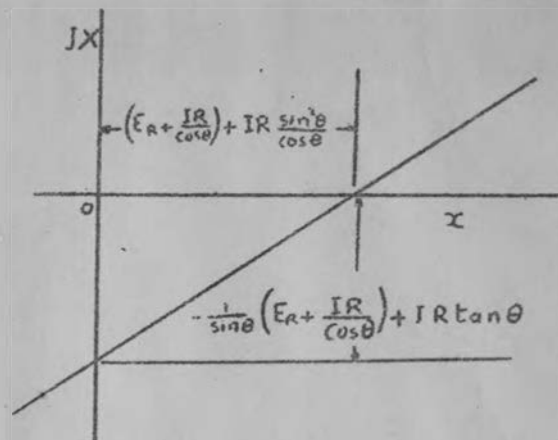


Fig 4 Plot of IX against x

The amount of compensation needed for a certain value of E_s can be found in the following manner:
The value of x is found from Fig. 3 for a given value of E_s; then the value of IX drop corresponding to this x is found from Fig. 4. The amount of compensation needed is the difference between the actual IX_L drop of the transmission line and the IX drop found as explained above.

When the value of X_c is smaller than the value of X_L the circuit is said to be under-compensated. When the value of X_c is larger than the value of X_L the circuit is said to be over-compensated. From the above discussions it is clear that in over-compensating the circuit the value of the sending end voltage is reduced more than in the under-compensated case. Over-compensation is used sometimes when the feeder resistance is large. On the other hand there are two main disadvantages of over-compensation:

1.-When an induction motor is started at the receiving end of the line the motor will draw a lagging current. If the sending end voltage is kept constant this lagging current will result in a voltage rise at the receiving end. If the motor starts frequently as is the case in rock crushers, saw mills etc. obviously a flicker in the lights will occur. Naturally this is undesirable. Hence to employ overcompensation should be strictly avoided under such circumstances.

2.-The angle between the sending end and receiving end voltages (β) is greater when the circuit is over-compensated than that when the circuit is under-compensated. This point is important from the standpoint of stability. This point will be discussed in the discussion of series capacitors in transmission lines.

From the above discussion it is seen that the change in the receiving end voltage due to a certain change in the load is very much less if there is a series capacitor in the circuit than the change in the receiving end voltage due to the same change in the load if there is no series capacitor in the circuit. By a proper selection of the magnitude of the series capacitor this voltage change can further be reduced. Hence series capacitor plays a very important role in voltage regulation. If the change of voltage in the receiving end occurs frequently due to frequent motor starts a so called voltage flicker takes place. Series capacitor by improving the voltage regulation keeps this undesirable voltage flicker within allowable limits.

The choice of the series capacitor for a certain circuit requires a careful study of several factors. The principal considerations involved in selecting and applying a series capacitor to a distribution system can be stated as follows:

1.-The voltage rise caused by series capacitor is naturally proportional to the ohmic value of the capacitor and the current through the capacitor. To reduce damage to lamps of customers on the load side of the series capacitor, this voltage must not be allowed to rise beyond 4 or 5 percent above normal with maximum load current. Also to avoid undesirable overvoltage flicker due to motor starting currents the limit for this voltage rise should be 2 or 3 percent above normal for the maximum starting current which can be expected due to the motors on the feeder.

2.-After the determination of the ohmic value of the capacitor the location of the capacitor must be specified. Most of the distribution circuits have loads distributed along the line. Hence the location of the capacitor should be such that the over-voltage to the customer receiving the maximum overvoltage is equal to the under-voltage to the customer receiving the maximum under-voltage under the condition of maximum current, and furthermore these values of the maximum over-voltage and maximum under-voltage should not go beyond the limit stated in the discussion of the first consideration. The best location of the capacitor is usually determined by cut and try method, taking local factors, such as load distribution, accessibility, etc., into account. In general, where the distribution circuits have distributed loads the best location is about one-third of the electrical distance (impedance) from the source of supply. To reduce flicker further, sometimes more than one series capacitor may be used.

3.-The next consideration is the establishment of the thermal capacity of the series capacitor which is to be used. To do this a 24-hour current chart or several current charts should be obtained. The capacitor has some short-time over-load capacity. It is customary, however, to provide continuous current carrying capacity equal to the maximum demand indicated on the current chart. This practice also leaves an open door for load growth. On the other hand to provide too much thermal capacity is uneconomical since the kva rating of capacitor for a certain number of ohms is directly proportional to the square of the r.m.s value of the current. In the protection of series capacitors the gap-setting as explained later is a limiting factor.

4.-The final consideration is the protective equipment. This will be discussed later in detail. The main part of the protective equipment is a gap which is across the terminals of the series capacitor. Now assume that the current rating and gap-setting of the series capacitor is established. A check should be made to insure that a starting current of a motor or a sudden fluctuation in load current will not be able to spark-over the gap. This point is important from the standpoint of the objective of the series capacitor. This objective is to reduce fluctuations in voltage due to these fluctuations in load current. In order to provide this it is customary to follow the rule given below. The voltage across the capacitor unit with maximum normal load plus the starting current of the largest motor on the circuit flowing through it, should be made less than twice normal rating of the capacitor, if the capacitor is placed directly in the line and should be made less than 1.5 times normal if a transformer combination is used.

After all these factors are considered carefully, as a last step, the magnitude and the duration of the maximum short-circuit current possible at the capacitor location should be checked to see whether the transformer and gap can carry this maximum short circuit current until the fault is cleared out.

As for the connection of the series capacitors, the following points should be considered. It is known that the relation of capacitive reactance, X_c, to capacitance C is given by the following formula:

$$X_c = \frac{1}{2\pi FC} \qquad (7)$$

It is evident from the above formula that the smaller the ohmic value of capacitor required, the larger and hence the more expensive the installation becomes. Furthermore where short lines of low inductive reactance are under consideration the compensating effect of series capacitor is less pronounced. However, if it is desired to use a series capacitor on such a line, an addition of an inductive reactor to the circuit will increase the inductive reactance of line making necessary the use of a capacitor of larger capacitive reactance and hence smaller capacitance, thereby reducing the installation cost.

If large current carrying capacity is required, it may be necessary to connect the units in series-parallel. However, this condition is not usually necessary in distribution circuits. From a practical standpoint it is preferable to use standard voltage units for series capacitor applications. For example, units of 5 or 10 kva capacity rated at 230, 460 or 575 volts connected in series, parallel or series-parallel provide such a wide range of ohmic values and current ratings that it should rarely be necessary to use special ratings.

In the manufacture of power capacitors, the tolerance in microfarads is allowed in the positive direction. For example, a 10 kva, 460-volt capacitor is 125 microfarads with a manufacturing tolerance of plus 15%. But in series capacitors one is interested in the most probable ohmic value. The probable maximum variation is given plus or minus some certain percentage of the ohmic values of the capacitor.

The following example illustrates the above considerations as applied in a specific case:

Assume that a 2000 volt feeder serves a uniformly distributed load of 100 kva at 0.866 power factor. At the end of the feeder there is a concentrated load of 50 kva at 0.8 power factor. The starting kva of the motor at the end of the feeder is 92 kva at 0.5 power factor. This motor starts frequently and causes voltage fluctuations which annoys very much the lighting customers. A maximum 3% change in voltage is allowed. In order to remedy the situation the best thing to do is to install series capacitors. To raise the feeder voltage or to increase the size of the copper conductors will cost more and will result in less improvement in voltage regulation than that expected from the installation of series capacitor. Voltage regulators are not so fast in operation as series capacitors. Hence, the installation of a series capacitor is the best alternative.

To determine the size and the location of the series capacitor, first the percentage voltage drop due to the motor start is determined:

$$\% \text{ Drop} = \frac{kva \times L\,(R\cos\theta + X\sin\theta)}{10\,kv^2} \qquad (8)$$

where:

kva = three phase starting kva = 92kva

L = length of line in miles = 5 miles

R = resistance of the line in ohms per mile = 1.2

X = reactance of the line in ohms per mile = 0.8

Cosθ=power factor of the motor during its start = 0.5

kv = line voltage in kv = 2kv.

Substituting the numerical values in above equation gives:

$$\% \text{ Drop} = \frac{92 \times 5\,(1.2 \times 0.5 + 0.8 \times 0.866)}{10 \times 2^2}$$

$$= 15$$

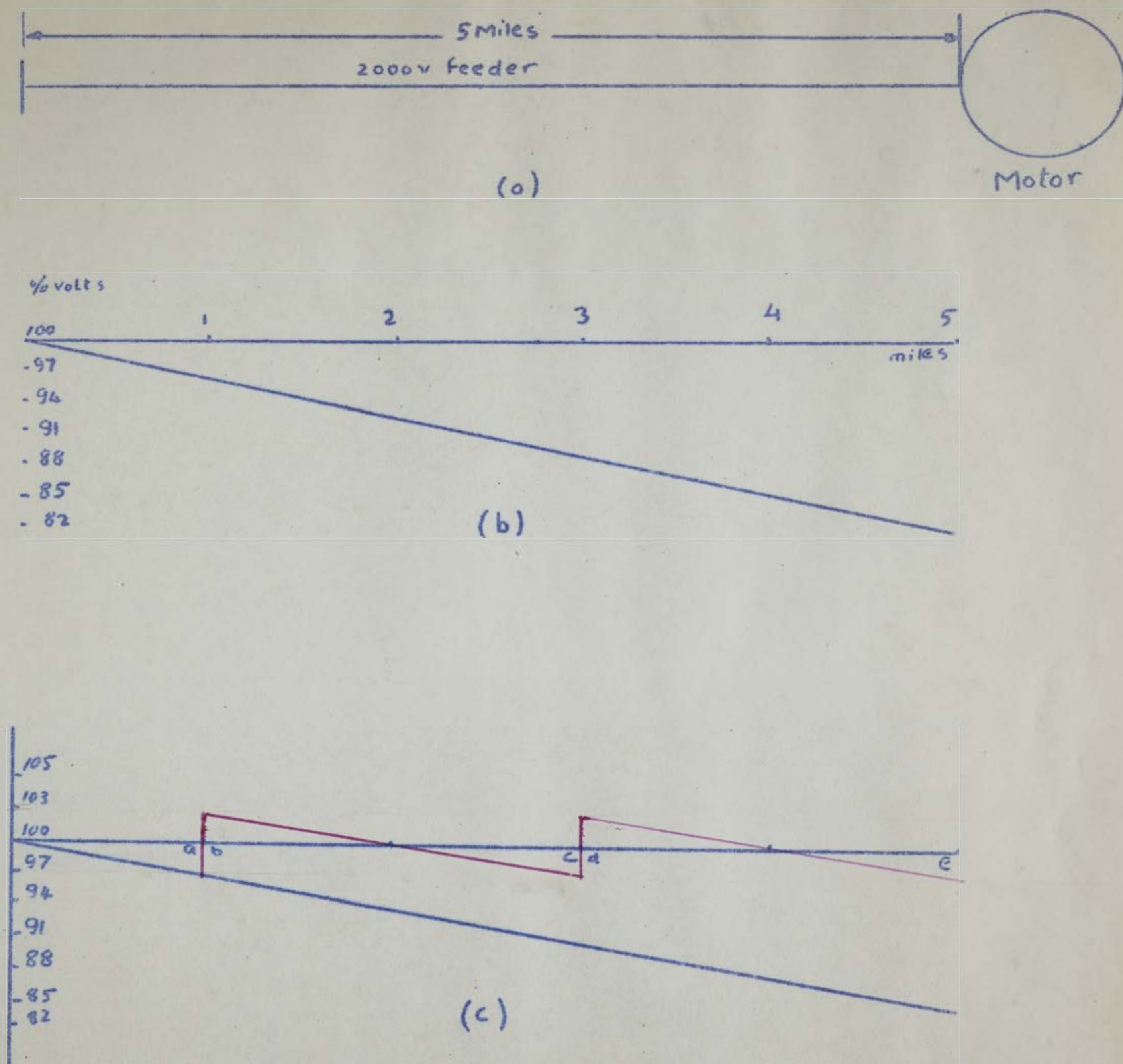


Fig. 5. (a) An uncompensated feeder; (b) voltage drop in percent plotted against distance; (c) effect of series compensation on voltage regulation.

The feeder and this voltage drop is shown in Fig. 5a and Fig. 5b respectively. Since it is desired to have a maximum voltage change in percent which does not exceed 3 percent two series capacitors must be used. If two series capacitors are used the line will be divided into five parts as shown in Fig.5c. The voltage change in percent occurring at the extremities of each part should not exceed 3 percent. In other words the voltage rise to customers b and d, and the voltage dips to customers a, c, and e should be equal to each other and all of them should be equal to or less than 3 percent.

The location of the capacitors must be so chosen that the above requirements are fulfilled. It is determined graphically from Fig.5. Fig. 5c shows the location of the capacitors. First unit is installed at a place which is one mile from the sending end, and the other unit is installed two miles away from the first unit towards the end of the line. The red line in Fig. 5c shows the variation of the percent volt change with the series capacitor units in operation when the motor at the end of the line starts. From Fig. 5c it is seen that none of the customers has a voltage variation due to the motor start, greater than 3 percent.

In order to determine the size of the unit, first it is observed that the voltage difference between customers a and b is 6 percent. Hence the capacitor itself must have an ohmic value such that 92 kva, which is the starting kva of the motor at 0.5 power factor, produces a voltage rise of 6 percent. The ohmic value of the capacitors is,

$$X_c = \frac{10 \times \% \text{ volts} \times kv^2}{kva \times \sin\theta} \qquad (9)$$

$$= \frac{10 \times 6 \times 2^2}{92 \times 0.866}$$

$$= 3 \text{ ohms each per phase}$$

In order to determine the ampere rating of the capacitors twenty-four-hour current charts should be obtained at the location of the capacitors. Normally, these values would establish the ampere rating of the capacitors. In this case, however, a rather high motor starting current is superimposed on these normal currents. The maximum line current, therefore, may be considered to be the normal peak when the motor is not running, plus the starting current of the motor. From the twinty-four-hour current charts the peak value of current obtained includes also the running current of the motor. In this case maximum value of line current can be found by subtracting the running current of the motor from the normal peak value of current and adding the starting current of the motor to this value. Naturally the running and starting currents and their respective power factors can readily be determined from the characteristics of the motor.

Gap-setting for short circuits is determined in the following manner:

The gap usually is set to spark-over at approximately twice the continuous current rating of the capacitor. About 50 % margin is allowed between the peak currents which are discussed above and spark-over currents. Naturally the kva ratings found in this way do not necessarily be equal to the kva ratings of the standard units. Hence the standard unit which has a kva rating which is very close to the calculated kva rating is chosen. Usually with some certain combinations of the standard units such as, series, parallel or series-parallel, it is possible to obtain a wide range of ohmic values and current ratings.

SELF-EXCITATION OF INDUCTION MOTORS WITH SERIES CAPACITORS

When an induction motor is started through a line having a series capacitor the motor may self excite and become a generator of low frequencies. The capacitive reactance of the series capacitor in conjunction with the inductive reactance of the motor may have a resonant frequency which is lower than that of the power supply. Under some certain conditions as for example when the induction motor is started a transient current having lower frequency components than normal current exists. This is true for every circuit containing resistance, inductance, and capacitance. For instance, consider the circuit below. When a sudden change occurs in the circuit a transient current exists.

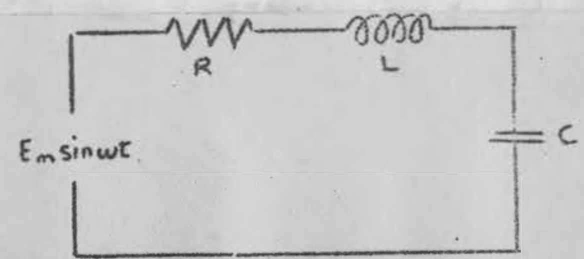


Fig 6 An ordinary a-c circuit containing R, L, C

Suppose that at t = 0 an alternating voltage is applied to the circuit. The differential equation for this case is given below:

$$E_m \sin \omega t = iR + \frac{1}{C}\int i\,dt + L\frac{di}{dt} \qquad (10)$$

Taking the Laplace of both sides and assuming zero initial conditions gives:

$$\frac{E_m w}{s^2 + w^2} = R\,i(s) + \frac{1}{Cs}\,i(s) + sL\,i(s) \qquad (11)$$

$$i_{(s)} = \frac{sE_m \omega}{(s^2+\omega^2)(s^2L + Rs + \frac{1}{C})} \qquad (12)$$

The roots of the denominator are:

$$s_1 = j\omega$$
$$s_2 = -j\omega$$

$$s_{3,4} = -\frac{R}{2L} \mp \sqrt{\frac{R^2}{4L} - \frac{1}{LC}}$$

If

$$\frac{1}{LC} > \frac{R^2}{4L}$$

then

$$s_{3,4} = -\frac{R}{2L} \mp j\sqrt{\frac{1}{LC} - \frac{R^2}{4L}}$$

Hence the system function of the circuit is:

$$\frac{1}{(s-j\omega)(s+j\omega)\left(s+\frac{R}{2L}+j\sqrt{\frac{1}{LC}-\frac{R^2}{4L}}\right)\left(s+\frac{R}{2L}-j\sqrt{\frac{1}{LC}-\frac{R^2}{4L}}\right)} \qquad (13)$$

It can be noticed that s_1 is conjugate of s_2 and s_3 is conjugate of s_4. Therefore it can be predicted that a transient current which consists of two components, one component having a frequency equal to that of the source and the other one having a damping factor of $-\frac{R}{2L}$ and a frequency of $\sqrt{\frac{1}{LC} - \frac{R^2}{4L}}$ exists. This frequency, for certain values of the R, L and C, may be lower than that of the source. If a negative resistance equal to the resistance of the circuit is added to the circuit the currents of the natural frequency flows continiously. It can be seen from the above equations if R is zero, then, time constant of the circuit becomes infinite or in other words damping factor becomes zero. Furthermore the frequency of the second component of the transient current for $R = 0$ becomes simply $\frac{1}{\sqrt{LC}}$. It is well known that this frequency is the natural frequency of the circuit.

The same thing can happen in an induction motor which is supplied through a line containing series capacitor. Under these conditions the induction motor becomes an induction generator of low frequency. Now consider the circuit below which represents the scematic diagram of induction motor with series capacitor.

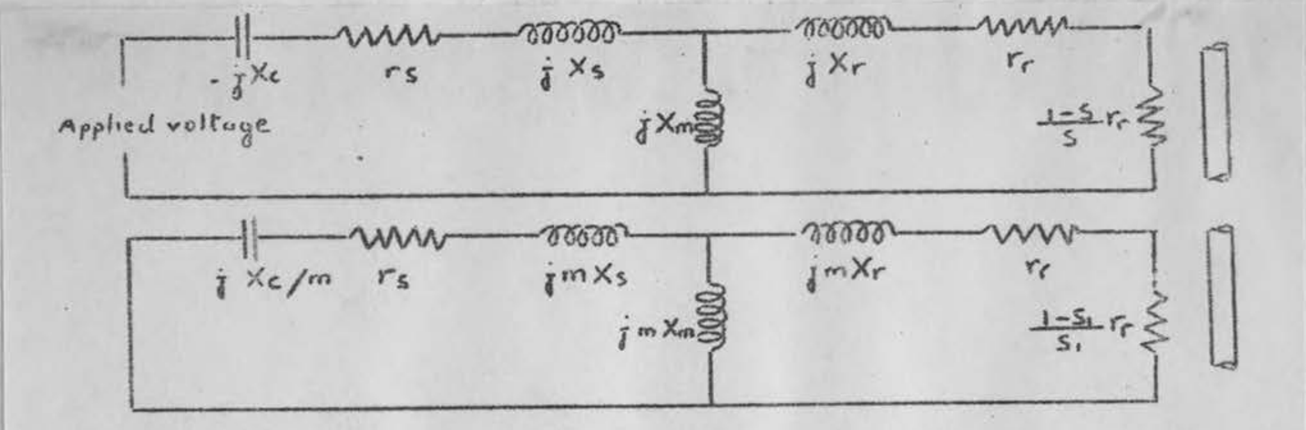


Fig 7. Diagram of an induction motor with series capacitor

Where:

X_c = capacitive reactance of the series capacitor
r_s = resistance of stator
X_s = leakage reactance of stator
X_m = reactance of the magnetizing branch
X_r = leakage reactance of rotor
r_r = resistance of rotor
s = slip.

(All reactances and resistances are expressed in per unit on a certain base.)

The first circuit of Fig 7 is the schematic diagram of an induction motor. The second circuit is the same as the first one except that all inductances and the capacitance are multiplied by m thus establishing a circuit of which resonant frequency is m times the system frequency. This circuit is used to explain how induction motor functions as an induction generator of low frequency current. After system voltage is applied to the motor the speed increases. When the speed goes beyond the synchronous speed with respect to the second circuit its slip becomes negative. When s_1 becomes negative, resistance $\frac{1-s_1}{s_1} r_r$ becomes also negative. If this resistance becomes negative enough just to cancel the positive resistances in the circuit, the circuit self-excites and the currents of resonant frequency flows continuously.

In order to find the conditions which must be satisfied to allow oscillation of natural frequency the impedance of the circuit is set equal to zero. The total resistance in the rotor circuit is:

$$r_r + \frac{1-s_1}{s_1} r_r = \frac{r_r}{s_1} \tag{14}$$

The impedance viewed from the rotor side is:

$$\frac{jmX_m\left[r_s + jmX_s - j\frac{X_c}{m}\right]}{r_s + jmX_m + jmX_s - j\frac{X_c}{m}} + jmX_r + \frac{r_r}{s_1}$$

$$= \frac{mX_m r_s\left(mX_m + mX_s - \frac{X_c}{m}\right) - mX_m r_s\left(mX_s - \frac{X_c}{m}\right) + \frac{r_r}{s_1}\left[r_s^2 + \left(mX_m + mX_s - \frac{X_c}{m}\right)^2\right]}{r_s^2 + \left(mX_m + mX_s - \frac{X_c}{m}\right)^2} \tag{15}$$

$$+ j\frac{mX_m r_s^2 + mX_m\left(mX_s - \frac{X_c}{m}\right)\left(mX_m + mX_s - \frac{X_c}{m}\right) + mX_r\left[r_s^2 + \left(mX_m + mX_s - \frac{X_c}{m}\right)^2\right]}{r_s^2 + \left(mX_m + mX_s - \frac{X_c}{m}\right)^2}$$

Equating the real component to zero gives:

$$mX_m^2 r_s = -\frac{r_r}{s_1}\left[r_s^2 + \left(mX_m + mX_s - \frac{X_c}{m}\right)^2\right]$$

$$\frac{r_r}{s_1} = -\frac{m^2X_m^2 r_s}{r_s^2 + \left(mX_m + mX_s - \frac{X_c}{m}\right)^2}$$

$$\frac{r_r}{s_1} = - \frac{m^2 X_m^2 r_s}{r_s^2 + \frac{1}{m^2}\left(m^2 X_m + m^2 X_s - X_c\right)} \tag{16}$$

Equating the imaginary part to zero gives:

$$m X_m r_s^2 + m X_m \left(m X_s - \frac{X_c}{m}\right)\left(m X_m + m X_s - \frac{X_c}{m}\right) + m X_r r_s^2 + m X_r \left(m X_m + m X_s - \frac{X_c}{m}\right) = 0$$

$$r_s^2 (m X_m + m X_r) = -\left(m X_m + m X_s - \frac{X_c}{m}\right)\left[m X_m \left(m X_s - \frac{X_c}{m}\right) + m X_r \left(m X_s - \frac{X_c}{m}\right) + m^2 X_m X_r\right]$$

$$= -\left(m X_m + m X_s - \frac{X_c}{m}\right)\left[m X_m \left(m X_s - \frac{X_c}{m}\right) + m X_r \left(m X_s - \frac{X_c}{m}\right) + m^2 X_m X_r\right]$$

$$= -\left(m X_m + m X_s - \frac{X_c}{m}\right)\left[(m X_m + m X_r)\left(m X_s - \frac{X_c}{m}\right) + m^2 X_m X_r\right]$$

$$r_s^2 = -\left(m X_m + m X_s - \frac{X_c}{m}\right)\left[m X_s + \frac{m^2 X_m X_r}{m X_m + m X_r} - \frac{X_c}{m}\right] \tag{17}$$

Since X_m, X_s, and X_r are the constants of the machine for the sake of simplicity let:

$$X_m + X_s = X_1$$

$$X_s + \frac{X_m X_r}{X_m + X_r} = X_2$$

then Eq. (17) becomes:

$$r_s^2 = -\left(m X_1 - \frac{X_c}{m}\right)\left(m X_2 - \frac{X_c}{m}\right)$$

or

$$r_s^2 = \frac{1}{m^2}\left(m^2 X_1 - X_c\right)\left(m^2 X_2 - X_c\right) \tag{18}$$

Using the same simplification Eq. (16) becomes:

$$\frac{r_r}{s_1} = - \frac{m^2 X_m^2 r_s}{r_s^2 + \frac{1}{m^2}\left(m^2 X_1 - X_c\right)} \tag{19}$$

Starting with Eq. (18) and rearranging the terms gives:

$$r_s^2 = -\frac{1}{m^2}(m^2X_1 - X_c)(m^2X_2 - X_c) \tag{21}$$

As according to its definition m has to be a real number; self-excitation is only possible if Eq. (21) has at least one real root.

Real roots are possible only if:

$$(r_s^2 - X_1X_c - X_2X_c)^2 > 4X_1X_2X_c^2 \tag{22}$$

and also:

$$r_s^2 - X_1X_c - X_2X_c < 0 \tag{23}$$

Taking the square root of both sides of Eq. (22) gives:

$$|r_s^2 - X_1X_c - X_2X_c| > 2\sqrt{X_1X_2}\,X_c$$

Taking the square root of both sides of Eq. (23) gives:

$$-[X_1X_c + X_2X_c - r_s^2] < 2\sqrt{X_1X_2}\,X_c$$

or

$$X_1X_c - 2\sqrt{X_1X_2}\,X_c + X_2X_c = (\sqrt{X_1X_c} - \sqrt{X_2X_c})^2 > r_s^2$$

or

$$r_s < |\sqrt{X_1X_c} - \sqrt{X_2X_c}| \tag{24}$$

Consequently self-excitation is only possible if Eq. (24) is satisfied. The result is that if Eq. (24) is not satisfied then self-excitation will not occur. Sometimes it is impossible to have a value of r_s which satisfies above equations. In this case a resistance R is shunted across the capacitor.

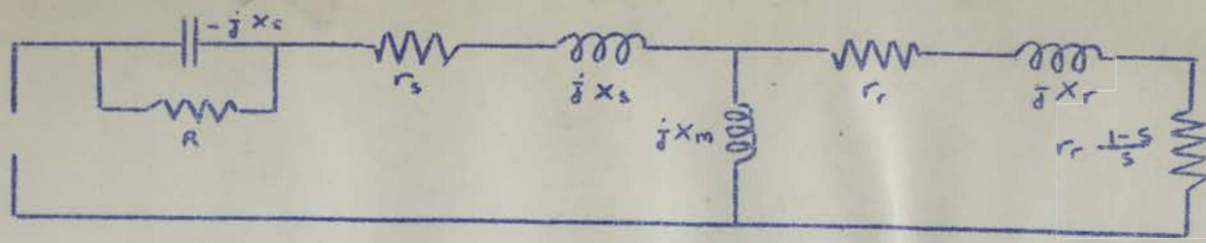


Fig. 8. Shunt resistance across the series capacitor

The above circuit can be simplified and made similar to the circuit of Fig. 7 in the following manner:

let:

$$r_s' = r_s + \frac{R X_c^2}{R^2 + X_c^2}$$

$$X_c' = X_c + \frac{R^2 X_c}{R^2 + X_c^2}$$

then, Fig. 8 becomes:

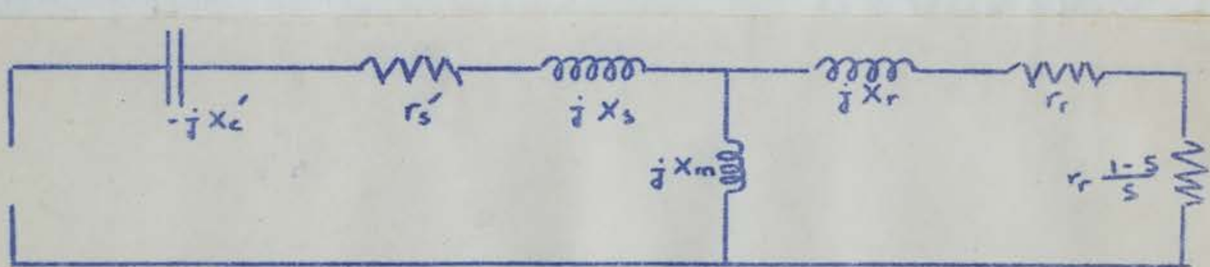


Fig. 9. Simplified form of Fig. 8

Again following exactly the same procedure an equation similar to Eq. (24) can be obtained:

$$r_s' > x_c' (x_1 + x_2)$$

or

$$r_s' > \sqrt{x_c' x_1} - \sqrt{x_c' x_2} \qquad (25)$$

Hence the value of R which satisfies Eq. (25) can be found. If this resistance is shunted across the capacitor self-excitation can be eliminated.

Sometimes especially when motors are started infrequently the use of a shunt resistance R and hence all these calculations can be avoided. Since the reactance of the capacitor is inversely proportional to frequency whereas that of an inductor is directly proportional, the reactance of the capacitor is small and consequently the voltage drop across it is large at small frequencies. This large voltage drop across the capacitor may cause the protective gap in parallel with the capacitor bank to flash-over, thus short circuiting the capacitor. This prevents the resonant condition and enables the motor to accelerate normally to full speed. After a certain time the capacitor is automatically restored to the circuit.

HUNTING OF SYNCHRONOUS MACHINERY

Any synchronous machine connected to a power system is subject to disturbances caused by several reasons. These disturbances tend to set up rotor oscillations of the synchronous machine. Usually the inherent damping of the system is sufficient to prevent hunting resulting from the above effects. However, under certain conditions this damping may be very small and even negative. The effect of the armature resistance and reactance together with the line resistance and reactance determines whether or not a negative damping exists. As the ratio between the above mentioned resistande and inductance $\frac{R}{X}$ increases, the tendency toward hunting also increases. In a system of which the inductive reactance is compensated with the capacitive reactance of the series capacitor the ratio between resistance and inductance is high and system is subject to hunting. The effect of the armature resistance upon the hunting of the machinery can be made clear in the following manner.

It is a well known fact that the fictitious voltage e_d' is equal to the arithmetic sum of the direct axis component of the terminal voltage and the product of the direct axis component of current and transient reactance x_d'. Hence the difference between e_d and e_d' can be written as:

$$e_d - e_d' = i_d X_d - i_d X_d' \qquad (26)$$

or

$$e_d' = e_d - (x_d - x_d')\, i_d \qquad (27)$$

From the above equations it can be easily seen that the greater the demagnetizing component of current is, the smaller is e_d' assuming a constant excitation. Now assume a synchronous machine connected to an infinite bus with a line which contains only reactance as shown in Fig. 10.

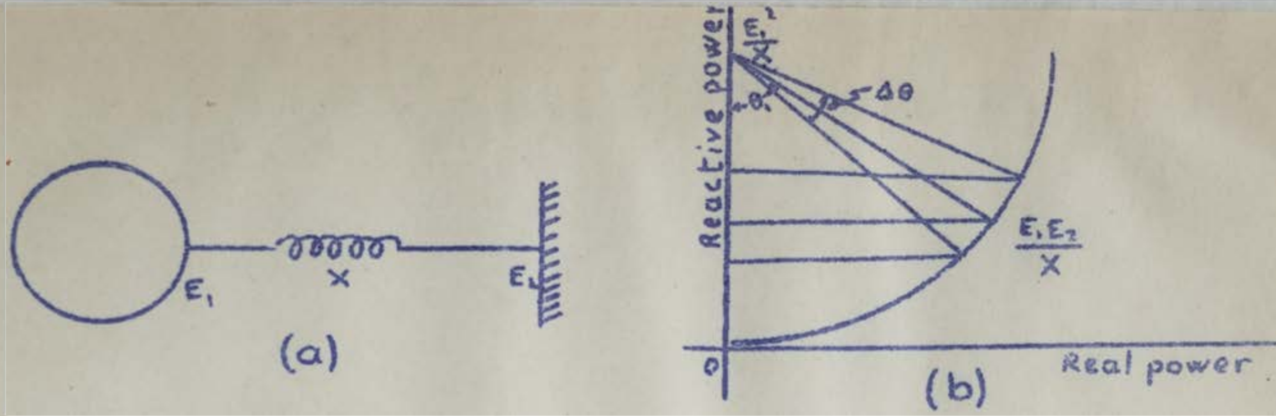


Fig. 10a. A synchronous machine connected to an infinite bus through an inductance, (b) power-circle diagram of (a).

The power equation for the sending end of the system shown in Fig. 10a is:

$$P = \frac{E_1^2}{X}\angle{-90^\circ} - \frac{E_1E_2}{X}\angle{\theta-90^\circ} \tag{28}$$

where θ is the angle between the generated voltage and the voltage of the infinite bus at the receiving end, and E_1 and E_2 are generating end voltage and receiving end voltage respectively. The corresponding power-angle diagram is shown in Fig. 10b. Now assume that for some reason the rotor began to oscillate sinusoidally about the mean angle θ with a frequency f and an amplitude $\Delta\theta_m$. From Fig. 10b it can be seen that as the angle increases from θ_1 to θ_2 the demagnetizing reactive kva and consequently the demagnetizing current i_d also increases. As i_d increases e_d decreases so that a positive incremental change $\Delta\theta$ in θ corresponds to a negative incremental change $\Delta e_d'$ in e_d'. However, due to the inductance associated with the field circuit, e_d' cannot change rapidly. Hence, if the sinusoidal change of $\Delta\theta$ is plotted on a horizontal axis the corresponding change $\Delta e_d'$ will follow an elliptical curve as shown in Fig. 11a. Then, as the angle increases, e_d' decreases, and $\Delta e_d'$ will follow the elliptical path in a clockwise direction. In Fig. 11b

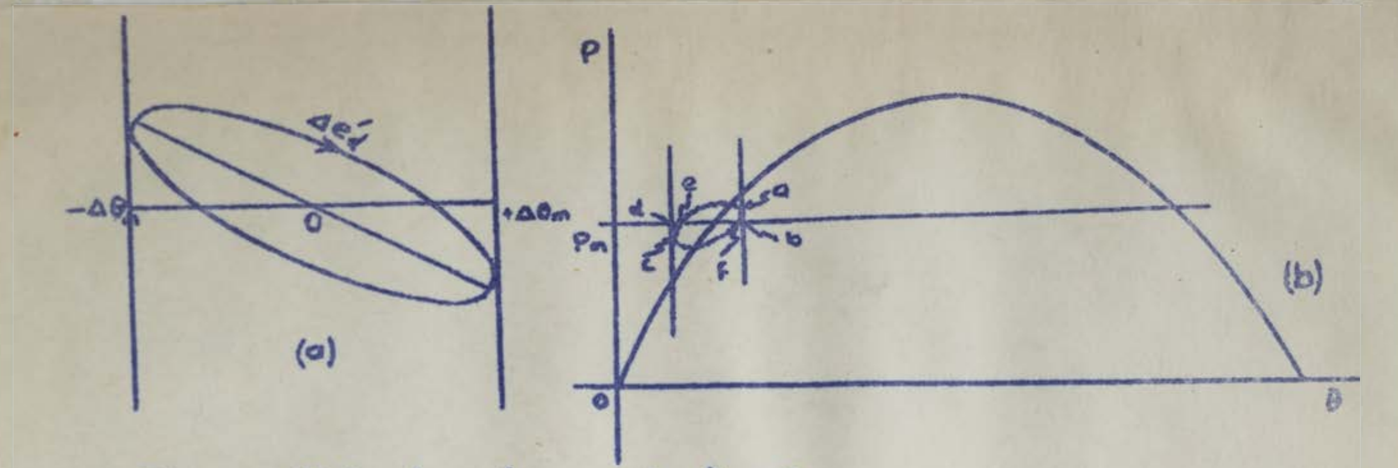


Fig. 11 Plot of $\Delta e'_d$ against $\Delta\theta_m$.

power is plotted against angle. As power increases with an increase in e_d', the instantaneous power curve also will follow a similar elliptical path in a clockwise direction. Assuming a constant mechanical input, P_m, the acceleration torque at any instant will be the difference between P_m and the corresponding point on the instantaneous electrical power curve. From Fig. 11b it can be seen that going from c to a, the decelerating torque represented by the area (eab) is greater than the area (cde) by an amount (eaf) since (cde) is equal to (abf). Under these conditions the system is stable, namely there is a positive damping torque.

Now assume that the same synchronous machine is connected to the same infinite bus with a line having a resistance and reactance. The system and the corresponding power diagram is shown in Fig. 12a and Fig. 12b respectively. The power equation for this case is:

$$S = \frac{E_1^2}{Z} \angle{-\beta} - \frac{E_1 E_2}{Z} \angle{\theta-\beta} \qquad (29)$$

where β is the arctangent of the ratio of line reactance to line resistance.

From Fig. 12b it can be observed that an increase in angle will result in an increase in i_d and consequently in e_d', and a decrease in angle will result in a decrease in i_d and e_d'. This is the reverse of the previous case. It

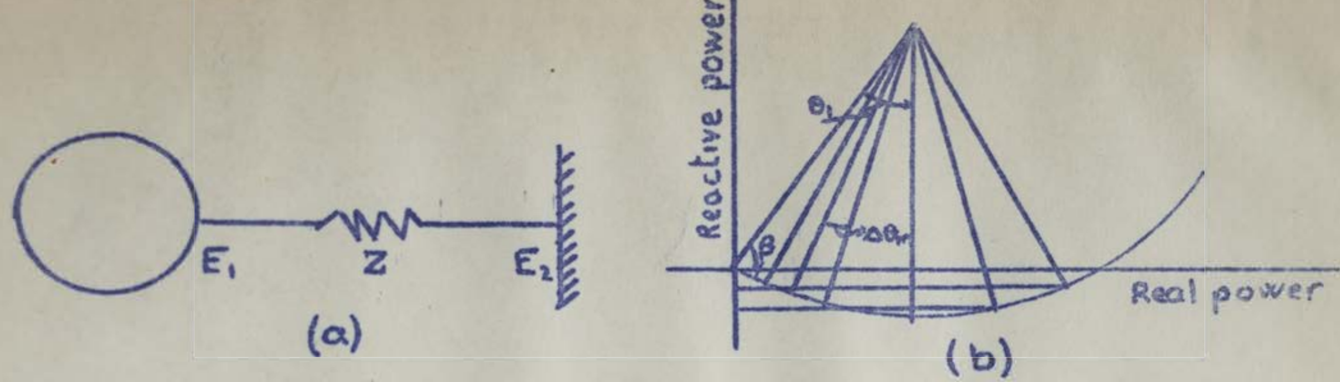


Fig.12a. An synchronous machine connected to an infinite bus through an impedance, (b) power-circle diagram of (a).

is obvious that under this condition the incremental change $\Delta e_d'$ will follow the eliptical path in a counterclockwise sense. The same thing is also true for the power curve. The natural result of this is that now the accelerating torque will be greater than the decelerating torque and the system will be unstable. However, again from Fig. 12b it can be seen that for values of θ greater than θ_s, an increase in angle will result in a decrease in i_d and e_d', as was the case discussed previously, and the system will be stable.

The angle θ_1 is the angle at which the reactive power has a minimum value. This angle can be found by differentiating the reactive power equation and setting it equal to zero. However, this need not be done as this angle can be easily determined from Fig. 12b to be equal to $90-\beta$.

When the line between the synchronous machine and the infinite bus is purely reactive it was shown that the decelerating torque represented by the area (edb) was greater than the accelerating torque represented by the area (cde) by an amount which is represented by the area (eaf) when instantaneous incremental value of electrical torque travels from c to a. When it travels from a to c the decelerating torque is greater than the accelerating torque by an amount represented by the area (fce). Hence in one complete cycle the excess energy to produce damping is the area represented by the loop. When the line between the synchronous machine and the infinite bus contains resistance as well as reactance it was shown that the decelerating torque was greater than the accelerating torque. Using the same reasoning it can be easily shown that in one complete cycle the deceleration torque is greater than the acceleration torque by an amount represented by the area of the loop. If clockwise direction is taken as reference this area is negative. Hence making use of this fact a criterion can be established to determine a limit for resistance below which the danger of hunting does not exist. To do this the excess energy represented by the area of the loop is found and a relation between resistance and inductance can be found which makes this area positive. For this purpose the power loop is enlarged and redrawn in Fig. 13.

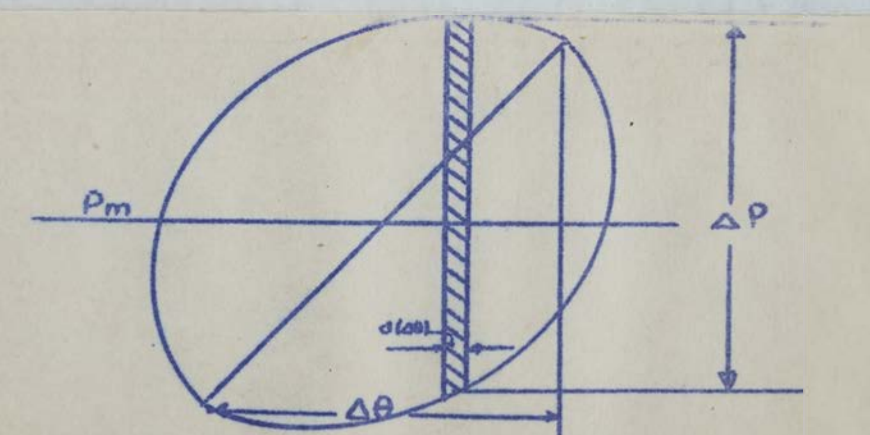


Fig. 13. Elliptical path of $\Delta e'_d$

The incremental area shown in Fig. 13 is:

$$\Delta P \times d(\Delta\theta)$$

where

$$\Delta P = \frac{dP}{de'_d}\,\Delta e'_d$$

At the beginning of this discussion it was assumed that

$$\Delta\theta = \Delta\theta_m \sin 2\pi f t$$

$\Delta e_d'$ will also form a sinusoidal variation but due to the inductance associated with the field circuit, e_d' can not change instantaneously; there will be a lagging angle φ. Hence $\Delta e_d'$ can be written as:

$$\Delta e_d' = \Delta e_{dm}' \sin(2\pi f t + \varphi) \tag{30}$$

where $\Delta e_{dm}'$ is the amplitude of the oscillation. Hence,

$$\Delta P \times d(\Delta\theta) = \frac{dP}{de_d'} \times \Delta e_{dm}' \sin(2\pi f t + \varphi) \times d(\Delta\theta)$$

$$= \frac{dP}{de_d'} \Delta e_{dm}' \sin(2\pi f t + \varphi) \Delta\theta_m 2\pi f \cos 2\pi f t \, dt \tag{31}$$

Since the frequency of oscillation is f, the period T of the oscillation will simply be $\frac{1}{f}$. Integrating this area for one complete cycle gives:

$$\text{Area} = \int_0^T \frac{dP}{de_d'} \Delta e_{dm}' \sin(2\pi f t + \varphi) \cdot \Delta\theta_m 2\pi f \cos 2\pi f t \, dt$$

$$= 2\pi f \Delta\theta_m \times \frac{dP}{de_d'} \Delta e_{dm}' \int_0^{1/f} \sin(2\pi f t + \varphi) \cos 2\pi f t \, dt \tag{32}$$

Making use of the following trigonometric relation

$$\sin\alpha \cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

Eq. (32) becomes:

$$\text{Area} = 2\pi f \Delta\theta_m \frac{dP}{de_d'} \Delta e_{dm}' \times \frac{1}{2}\left[\int_0^{1/f} (\sin(4\pi f t + \varphi) + \sin\varphi) \, dt\right]$$

$$= 2\pi f \Delta\theta_m \frac{dP}{de_d'} \Delta e_{dm}' \left[-\frac{\cos(4\pi f t + \varphi)}{4\pi f} + t \sin\varphi\right]_0^{1/f}$$

$$= 2\pi f \Delta\theta_m \frac{dP}{de_d'} \Delta e_{dm}' \times \frac{1}{f} \sin\varphi \times \frac{1}{2}$$

$$= \pi \Delta\theta_m \frac{dP}{de_d'} \Delta e_{dm}' \times \sin\varphi . \tag{33}$$

From Eq. (33) it is seen that the sign of the area which represents power depends upon the two factors:

1.- The sign of $\frac{dP}{de'_d}$

2.- The sign of $\Delta e'_{dm} \sin \varphi$

Now $\frac{dP}{de'_d}$ can be found in the following way:

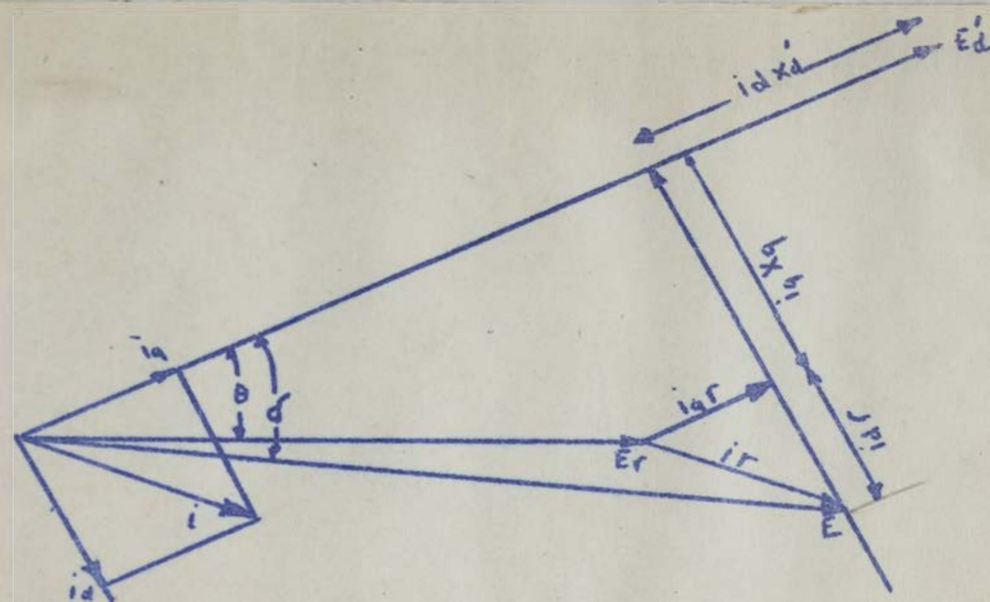


Fig. 14. Vector diagram of a synchronous machine

Reffering to Fig. 14:

$$E_T \cos\theta + i_q r + i_d x'_d = e'_d \qquad (34)$$

$$E_T \sin\theta + i_d r = i_q x_q \qquad (35)$$

Solving above equations simultaneously for i_d and i_q gives:

$$i_q = \frac{1}{x_q}\left(E_T \sin\theta + i_d r \right) \qquad (36)$$

$$i_d = \frac{1}{x'_d}\left(e'_d - E_T \cos\theta - i_q r \right) \qquad (37)$$

Substituting the value of i_q in Eq. (36) and Eq. (37) gives:

$$i_d = \frac{1}{x'_d}\left(e'_d - E_T \cos\theta - \frac{1}{x_q}(E_T \sin\theta + i_d r) r \right)$$

$$i_d = \frac{x_q e'_d - x_q E_T \cos\theta - r E_T \sin\theta}{r^2 + x'_d x_q} \qquad (38)$$

and

$$i_q = \frac{1}{X_q}\left(E_T \sin\theta + \frac{X_q e'_d - X_q E_T \cos\theta - r E_T \sin\theta}{r^2 + x'_d X_q} r \right)$$

$$i_q = \frac{x'_d E_T \sin\theta + r e'_d - r E_T \cos\theta}{r^2 + x'_d X_q} \tag{39}$$

The power which also includes the loss in the resistance can be expressed as follows:

$$P = \text{Real Part}\left[i E^* \right]$$

$$P = i_q E_T \cos\delta + i_d E_T \sin\delta$$

$$P = i_q (e'_d - x'_d i_d) + i_d x_q i_q = i_q e'_d - x'_d i_d i_q + x_q i_d i_q \tag{40}$$

Substituting the values of i_q and i_d in Eq. (40) gives:

$$P = (x_q - x'_d) \frac{x_q e'_d - X_q E_T \cos\theta - r E_T \sin\theta}{r^2 + x'_d X_q} \times \frac{x'_d E_T \sin\theta + r e'_d - r E_T \cos\theta}{r^2 + x'_d X_q}$$

$$+ \frac{x'_d E_T \sin\theta + r e'_d - r E_T \cos\theta}{r^2 + x'_d X_q} \times e'_d \tag{41}$$

Differentiating P with respect to e'_d gives:

$$\frac{dP}{de'_d} = (x_q - x'_d) \frac{X_q}{r^2 + x'_d X_q} \times \frac{x'_d E_T \sin\theta + r e'_d - r E_T \cos\theta}{r^2 + x'_d X_q}$$

$$+ (x_q - x'_d) \frac{r}{r^2 + x'_d X_q} \times \frac{x_q e'_d - X_q E_T \cos\theta - r E_T \sin\theta}{r^2 + x'_d X_q} \tag{42}$$

$$+ \frac{x'_d E_T \sin\theta + 2 r e'_d - r E_T \cos\theta}{r^2 + X'_d X_q}$$

In order to investigate the conditions for $\frac{dP}{de'_d} > 0$, let:

$$(x_q - x'_d) \frac{1}{r^2 + x'_d X_q} \left[\frac{(x'_d E_T \sin\theta + r e'_d - r E_T \cos\theta) x_q}{r^2 + x'_d X_q} * \frac{\{ x_q e'_d - X_q E_T \cos\theta - r E_T \sin\theta) r}{r^2 + x'_d X_q} \right]$$

$$+ \frac{x'_d E_T \sin\theta + 2 r e'_d - r E_T \cos\theta}{r^2 + x'_d X_q} > 0$$

If $\theta = 0$ it can be observed that every term in the above equation is greater than zero:

$$X_q - x'_d > 0$$

$$r^2 + x'_d x_q > 0$$
$$re'_d - rE_T > 0$$
$$x_q e'_d - x_q E_T > 0$$
$$2re'_d - rE_T > 0$$

It can also be seen that as the angle increases in the positive direction the above equation which is equal to $\frac{dP}{de'_d}$ becomes more and more positive. When $\theta = \frac{\pi}{2}$ this equation becomes:

$$\frac{x_q - x'_d}{(r^2 + x'_d x_q)^2}\left[x_q (x'_d E_T + re'_d) + r(x_q e'_d - rE_T)\right] + \frac{x'_d E_T + 2re'_d}{r^2 + x'_d x_q} > 0 \qquad (43)$$

It is quite obvious that the value of the equation is positive. As the angle continues to increase, approaching π, since the value of $\cos\theta$ is negative for $\frac{\pi}{2} \leqslant \theta \leqslant \frac{3\pi}{2}$ the terms involving $(-\cos\theta)$ will be positive and the value of the whole equation will have even a greater positive value than before. Since in practice θ is never greater than π it can be said that $\frac{dP}{de'_d}$ is always positive. Hence, the sign of the area which represents the value of the power depends upon the sign of $\Delta e'_{dm} \sin\varphi$. The value of $\Delta e'_{dm} \sin\varphi$ can be determined in the following manner.

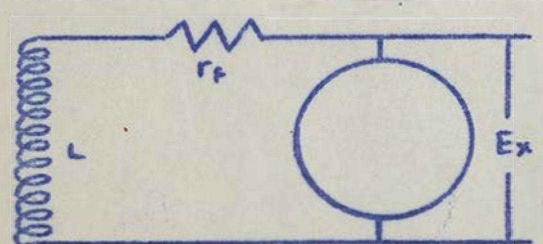


Fig. 15. Excitation voltage applied to the field of a synchronous machine.

Suppose that a constant excitation E_x is applied to the field of the machine. As far as the transient conditions are considered excitation voltage, E_x, can be written in the following manner:

$$E_x = ri + L\frac{di}{dt} \qquad (44)$$

The above equation can also be written in per unit quantities in the following manner:

$$E_x = e_d + T_{do} \frac{de'_d}{dt} \tag{45}$$

where T_{do} is the time constant of the machine. In incremental values Eq. (45) becomes:

$$\Delta e_d + T_{do} \frac{d\Delta e'_d}{dt} = 0$$

or

$$\Delta e_d = - T_{do} \frac{d\Delta e'_d}{dt} \tag{46}$$

since excitation voltage, E_x, is constant. On the other hand it was previously shown that:

$$e_d = e'_d + i_d (x_d - x'_d)$$

and

$$i_d = \frac{x_q e'_d - x_q E_T \cos\theta - r E_T \sin\theta}{r^2 + x'_d x_q}$$

Hence:

$$e_d = e'_d + \frac{(x_q e'_d - x_q E_T \cos\theta - r E_T \sin\theta)(x_d - i_d)}{r^2 + x'_d x_q}$$

$$e_d = \frac{(r^2 + x_d x_q) e'_d - (x_d - x'_d)(r E_T \sin\theta + x_q E_T \cos\theta)}{r^2 + x'_d x_q}$$

Therefore:

$$\Delta e_d = \frac{(r^2 + x_d x_q)\Delta e'_d - (x_d - x'_d)(r E_T \cos\theta - x_q E_T \sin\theta)\Delta\theta}{r^2 + x'_d x_q} \tag{47}$$

On the other hand it was assumed previously that:

$$\Delta\theta = \Delta\theta_m \sin 2\pi f t$$

$$\Delta e'_d = \Delta e'_{dm} \sin (2\pi f t + \varphi)$$

Substituting all these values in Eq. (46) gives:

$$\frac{(r^2 + x_d x_q)\Delta e'_{dm} \sin(2\pi f t + \varphi) - (x_d - x'_d)(r E_T \cos\theta - x_q E_T \sin\theta)\Delta\theta_m \sin 2\pi f t}{r^2 + x'_d x_q} \tag{48}$$

$$= - T_{do}\, 2\pi f t\, \Delta e'_{dm} \cos(2\pi f t + \varphi)$$

When $t = 0$,

$$\frac{r^2 + x_d x_q}{r^2 + x'_d x_q} \Delta e'_{dm} \sin\varphi = - T_{do}\, 2\pi f t\, \Delta e'_{dm} \cos\varphi$$

When $2\pi ft = \frac{\pi}{2}$,

$$\frac{(r^2 + x_d x_q)\Delta e'_{dm}\cos\varphi - (x_d - x'_d)(rE_T\cos\theta - x_q E_T\sin\theta)\Delta\theta_m}{r^2 + x'_d x_q} = T_{d0}\, 2\pi f\, \Delta e'_{dm}\sin\varphi$$

Solving these two simultaneous equations for $\sin\varphi$ gives:

$$\frac{(r^2 + x_d x_q)^2}{(r^2 + x'_d x_q)^2} \times \frac{\Delta e'_{dm}\sin\varphi}{T_{d0}\, 2\pi f} + T_{d0}\, 2\pi f\, \Delta e'_{dm}\sin\varphi$$

$$= \frac{(x_d - x'_d)\left[rE_T\cos\theta - x_q E_T\sin\theta\right]\Delta\theta_m}{r^2 + x'_d x_q} \qquad (49)$$

Hence:

$$\Delta e'_{dm}\sin\varphi = -\frac{(x_d - x'_d)(rE_T\cos\theta - x_q E_T\sin\theta)\Delta\theta_m}{r^2 + x'_d x_q} \times \frac{1}{\dfrac{(r^2 + x_d x_q)^2}{T_{d0}\, 2\pi f\, (r + x'_d x_q)^2} + T_{d0}\, 2\pi f} \qquad (50)$$

The sign of $\Delta e'_{dm}\sin\varphi$ depends only on the sign of

$$rE_T\cos\theta - x_q E_T\sin\theta$$

Hence the condition for $\Delta e'_{dm}\sin\varphi > 0$ is:

$$x_q E_T\sin\theta > rE_T\cos\theta$$

or

$$\tan\theta > \frac{r}{x_q} \qquad (51)$$

In the limit,

$$\tan\theta = \frac{r}{x_q}$$

or

$$\theta = \tan^{-1}\frac{r}{x_q} \qquad (52)$$

Considering the line reactance and resistance Eq. (52) becomes:

$$\theta = \tan^{-1}\frac{r + R}{x_q + X} \qquad (53)$$

Where R is the line resistance and X is the line reactance. For operating conditions the above equation puts a limit to the degree of compensation of line inductive reactance by the capacitive reactance of the series capacitor.

The result obtained above by analytical analysis conforms with the result of the qualitative analysis previously discussed on page (28)

If a series capacitor with a shunt resistance across its terminals is considered to be in the armature circuit of a synchronous machine, then the Laplace transform of the phase voltages will be as follows since the Laplace transform of the equivalent impedance of the series capacitor in parallel with the shunt resistance is $\frac{1}{\frac{1}{R}+\frac{s}{X_c}}$:

$$e_a = s\psi_a - \left(r + \frac{X_c}{s+\frac{X_c}{R}}\right) i_a \tag{54}$$

$$e_b = s\psi_b - \left(r + \frac{X_c}{s+\frac{X_c}{R}}\right) i_b \tag{55}$$

$$e_c = s\psi_c - \left(r + \frac{X_c}{s+\frac{X_c}{R}}\right) i_c \tag{56}$$

where:

ψ_a, ψ_b, ψ_c are phase flux linkages

r is armature circuit resistance

X_c is, $\frac{1}{C}$, capacitive reactance of the series capacitor

R is the shunt resistance

i_a, i_b, i_c, are phase currents

e_a, e_b, e_c are phase voltages.

(These quantities are all per unit quantities.)

On the other hand, direct axis and quadrature axis armature voltages, currents and flux linkages are given by the following relations:

$$e_d = \frac{2}{3}\left[e_a\cos\theta + e_b\cos(\theta-120) + e_c\cos(\theta+120)\right] \tag{57}$$

$$e_q = -\frac{2}{3}\left[e_a\sin\theta + e_b\sin(\theta-120) + e_c\sin(\theta+120)\right] \tag{58}$$

$$i_d = \frac{2}{3}\left[i_a\cos\theta + i_b\cos(\theta-120) + i_c\cos(\theta+120)\right] \tag{59}$$

$$i_q = -\frac{2}{3}\left[i_a\sin\theta + i_b\sin(\theta-120) + i_c\sin(\theta+120)\right] \tag{60}$$

$$\psi_d = \frac{2}{3}\left[\psi_a \cos\theta + \psi_b \cos(\theta-120) + \psi_c \cos(\theta+120)\right] \tag{61}$$

$$\psi_q = -\frac{2}{3}\left[\psi_a \sin\theta + \psi_b \sin(\theta-120) + \psi_c \sin(\theta+120)\right] \tag{62}$$

Rearranging the terms of Eq. (54), Eq. (55), and Eq. (56) the following equations are obtained:

$$s e_a = s^2 \psi_a + \frac{X_c}{R} s \psi_a - r s i_a - r \frac{X_c}{R} i_a - X_c i_a - \frac{X_c}{R} e_a \tag{63}$$

$$s e_b = s^2 \psi_b + \frac{X_c}{R} s \psi_b - r s i_b - r \frac{X_c}{R} i_b - X_c i_b - \frac{X_c}{R} e_b \tag{64}$$

$$s e_c = s^2 \psi_c + \frac{X_c}{R} s \psi_c - r s i_c - r \frac{X_c}{R} i_c - X_c i_c - \frac{X_c}{R} e_c \tag{65}$$

Differentiating Eq. (57) gives:

$$\frac{de_d}{dt} = \frac{2}{3}\left[\frac{de_a}{dt}\cos\theta + \frac{de_b}{dt}\cos(\theta-120) + \frac{de_c}{dt}\cos(\theta+120) - e_a \sin\theta - e_b \sin(\theta-120) - e_c \sin(\theta+120)\right] \tag{66}$$

Since

$$e_q = -\frac{2}{3}\left[e_a \sin\theta + e_b \sin(\theta-120) + e_c \sin(\theta+120)\right]$$

Eq. (66) can be written in Laplace transform in the following way:

$$s e_d - e_q = \frac{2}{3}\left[s e_a \cos\theta + s e_b \cos(\theta-120) + s e_c \cos(\theta+120)\right] \tag{67}$$

Substituting the values of s_{ea}, s_{eb}, and s_{ec} from Eq. (63), Eq. (64), and Eq. (65) the following equation is obtained:

$$\begin{aligned} s e_d - e_q = {} & \frac{2}{3}\left[s^2 \psi_a \cos\theta + s^2 \psi_b \cos(\theta-120) + s^2 \psi_c \cos(\theta+120)\right] \\ & + \frac{2}{3}\frac{X_c}{R}\left[s \psi_a \cos\theta + s \psi_b \cos(\theta-120) + s \psi_c \cos(\theta+120)\right] \\ & - \frac{2}{3} r \left[s i_a \cos\theta + s i_b \cos(\theta-120) + s i_c \cos(\theta+120)\right] \\ & - \frac{2}{3} r \frac{X_c}{R}\left[i_a \cos\theta + i_b \cos(\theta-120) + i_c \cos(\theta+120)\right] \\ & - \frac{2}{3} X_c \left[i_a \cos\theta + i_b \cos(\theta-120) + i_c \cos(\theta+120)\right] \\ & - \frac{2}{3}\frac{X_c}{R}\left[e_a \cos\theta + e_b \cos(\theta-120) + e_c \cos(\theta+120)\right] \end{aligned} \tag{68}$$

Differentiating Eq. (61) gives:

$$\frac{d\varphi_d}{dt} = \frac{2}{3}\left[\frac{d\varphi_a}{dt}\cos\theta + \frac{d\varphi_b}{dt}\cos(\theta-120) + \frac{d\varphi_c}{dt}\cos(\theta+120)\right] - \frac{2}{3}\left[\varphi_a\sin\theta + \varphi_b\sin(\theta-120) + \varphi_c\sin(\theta+120)\right] \tag{69}$$

Substituting Eq. (62) in Eq. (69) and taking Laplacian transform of both sides Eq. (69) becomes:

$$s\varphi_d - \varphi_q = \frac{2}{3}\left[s\varphi_a\cos\theta + s\varphi_b\cos(\theta-120) + s\varphi_c\cos(\theta+120)\right] \tag{70}$$

Differentiating Eq. (69) gives:

$$\frac{d^2\varphi_d}{dt^2} = \frac{2}{3}\left[\frac{d^2\varphi_a}{dt^2}\cos\theta + \frac{d^2\varphi_b}{dt^2}\cos(\theta-120) + \frac{d^2\varphi_c}{dt^2}\cos(\theta+120)\right] - \frac{2}{3}\left[\frac{d\varphi_a}{dt}\sin\theta + \frac{d\varphi_b}{dt}\sin(\theta-120) + \frac{d\varphi_c}{dt}\sin(\theta+120)\right] + \frac{d\varphi_q}{dt} \tag{71}$$

Differentiating Eq. (62) gives:

$$\frac{d\varphi_q}{dt} = -\frac{2}{3}\left[\frac{d\varphi_a}{dt}\sin\theta + \frac{d\varphi_b}{dt}\sin(\theta-120) + \frac{d\varphi_c}{dt}\sin(\theta+120)\right] - \frac{2}{3}\left[\varphi_a\cos\theta + \varphi_b\cos(\theta-120) + \varphi_c\cos(\theta+120)\right] \tag{72}$$

Substituting Eq. (61) in Eq. (72), Eq. (72) becomes:

$$\frac{d\varphi_q}{dt} + \varphi_d = -\frac{2}{3}\left[\frac{d\varphi_a}{dt}\sin\theta + \frac{d\varphi_b}{dt}\sin(\theta-120) + \frac{d\varphi_c}{dt}\sin(\theta+120)\right] \tag{73}$$

Substituting Eq. (73) in Eq. (71) gives:

$$\frac{d^2\varphi_d}{dt^2} = \frac{2}{3}\left[\frac{d^2\varphi_a}{dt^2}\cos\theta + \frac{d^2\varphi_b}{dt^2}\cos(\theta-120) + \frac{d^2\varphi_c}{dt^2}\cos(\theta+120)\right] + 2\frac{d\varphi_q}{dt} + \varphi_d \tag{74}$$

Taking the Laplace transform of both sides of Eq. (74) gives:

$$s^2\varphi_d - 2s\varphi_q - \varphi_d = \frac{2}{3}\left[s^2\varphi_a\cos\theta + s^2\varphi_b\cos(\theta-120) + s^2\varphi_c\cos(\theta+120)\right] \tag{75}$$

Differentiating Eq. (59) gives:

$$\frac{di_d'}{dt} = \frac{2}{3}\left[\frac{di_a}{dt}\cos\theta + \frac{di_b}{dt}\cos(\theta-120) + \frac{di_c}{dt}\cos(\theta+120)\right] - \frac{2}{3}\left[i_a\sin\theta + i_b\sin(\theta-120) + i_c\sin(\theta+120)\right] \tag{76}$$

Substituting Eq. (60) in Eq. (76) and again using Laplace transformation, Eq. (76) becomes:

$$s i_d - i_q = \frac{2}{3}\left[s i_a \cos\theta + s i_b \cos(\theta - 120) + s i_c \cos(\theta + 120)\right] \qquad (77)$$

Substituting Eq. (70), (75), and (77) in Eq. (68), Eq. (68) becomes:

$$s e_d - e_q = s^2 \varphi_d - 2s\varphi_q - \varphi_d + \frac{X_c}{R} s\varphi_d - \frac{X_c}{R}\varphi_q - r s i_d + r i_q - r\frac{X_c}{R} i_d$$

$$- X_c i_d - \frac{X_c}{R} e_d \qquad (78)$$

Rearranging terms, Eq. (78) can be rewritten in the following form:

$$\left(s + \frac{X_c}{R}\right)\left[e_d + r i_d + \varphi_q - s\varphi_d\right] + X_c i_d = e_q + r i_q - s\varphi_q - \varphi_d \qquad (79)$$

Starting with the differentiation of Eq. (58) and following exactly the same procedure another equation which is similar to Eq. (79) can be obtained:

$$\left(s + \frac{X_c}{R}\right)\left[e_q + r i_q - s\varphi_q - \varphi_d\right] + X_c i_q = -\left(e_d + r i_d - s\varphi_d + \varphi_q\right) \qquad (80)$$

Solving Eq. (79) and (80) simultaneously for i_d and i_q the following equations are obtained:

since

$$\varphi_d = -\frac{x'_d T_o s + X_d}{T_o + 1} i_d \qquad (81)$$

$$\varphi_q = -X_q i_q \qquad (82)$$

Substituting Eq. (81) and (82) in Eq. (79) gives:

$$\left(s + \frac{X_c}{R}\right)\left[e_d + r i_d - X_q i_q + s\frac{X'_d T_o s + X_d}{T_o + 1} i_d\right] + X_c i_d$$

$$= e_q + r i_q + s X_q i_q + \frac{X'_d T_o s + X_d}{T_o + 1} i_d$$

$$\left(s+\frac{X_c}{R}\right)\left[r+s\,\frac{X'_d T_o s+X_d}{T_o+1}\right]+X_c-\frac{X'_d T_o s+X_d}{T_o+1}$$

$$=e_q-\left(s+\frac{X_c}{R}\right)e_d+\left[r+sX_q+\left(s+\frac{X_c}{R}\right)X_q\right]i_q \qquad (83)$$

Substituting Eq. (81) and (82) in Eq. (80) gives:

$$\left(s+\frac{X_c}{R}\right)\left[e_q+ri_q+sX_q i_q+\frac{X'_d T_o s+X_d}{T_o+1}\,i_d\right]+X_c i_q$$

$$=-e_d-ri_d-s\,\frac{X'_d T_o s+X_d}{T_o+1}\,i_d+X_q i_q$$

$$\left[\left(s+\frac{X_c}{R}\right)(r+sX_q)+X_c-X_q\right]i_q=-e_d-ri_d-\left(s+\frac{X_c}{R}\right)\frac{X'_d T_o s+X_d}{T_o+1}\,i_d$$

$$-s\,\frac{X'_d T_o s+X_d}{T_o+1}\,i_d-\left(s+\frac{X_c}{R}\right)e_q \qquad (84)$$

Solving Eq. (84) for i_q gives:

$$i_q=-\frac{e_d+\left(s+\frac{X_c}{R}\right)e_q+\left[r+\left(2s+\frac{X_c}{R}\right)\frac{X'_d T_o s+X_d}{T_o+1}\right]}{\left[\left(s+\frac{X_c}{R}\right)(r+sX_q)+X_c-X_q\right]}$$

Substituting the value of i_q in Eq. (83) gives:

$$\left[\left(s+\frac{X_c}{R}\right)\left(r+s\,\frac{X'_d T_o s+X_d}{T_o+1}\right)+X_c-\frac{X'_d T_o s+X_d}{T_o+1}\right]i_d$$

$$=e_q-\left(s+\frac{X_c}{R}\right)e_d-\left[r+\left(2s+\frac{X_c}{R}\right)X_q\right]\frac{e_d+\left(s+\frac{X_c}{R}\right)X_q}{\left(s+\frac{X_c}{R}\right)(r+sX_q)+X_c-X_q}$$

$$-\left[r+\left(2s+\frac{X_c}{R}\right)X_q\right]\frac{i_d\left[r+\left(2s+\frac{X_c}{R}\right)\frac{X'_d T_o s+X_d}{T_o+1}\right]}{\left(s+\frac{X_c}{R}\right)(r+sX_q)+X_c-X_q}$$

Rearranging the terms:

$$i_d\left[\left(s+\frac{X_c}{R}\right)\left(r+s\,\frac{X'_d T_o s+X_d}{T_o+1}\right)+X_c-\frac{X'_d T_o s+X_d}{T_o+1}+\left[r+\left(2s+\frac{X_c}{R}\right)X_q\right]\right.$$

$$\left.\times\frac{r+\left(2s+\frac{X_c}{R}\right)\frac{X'_d T_o s+X_d}{T_o+1}}{\left(s+\frac{X_c}{R}\right)\left(r+sX_q\right)+X_c-X_q}\right]$$

$$=e_q-\left(s+\frac{X_c}{R}\right)e_d-\left[r+\left(2s+\frac{X_c}{R}\right)X_q\right]\frac{e_d+\left(s+\frac{X_c}{R}\right)X_q}{\left(s+\frac{X_c}{R}\right)\left(r+sX_q\right)+X_c-X_q}$$

Solving for i_d gives:

$$i_d=\frac{e_q-\left(s+\frac{X_c}{R}\right)e_d-\left[r+\left(2s+\frac{X_c}{R}\right)X_q\right]\frac{e_d+\left(s+\frac{X_c}{R}\right)e_q}{\left(s+\frac{X_c}{R}\right)\left(r+sX_q\right)+X_c-X_q}}{\left(s+\frac{X_c}{R}\right)\left(r+s\frac{X'_d T_o s+X_d}{T_o+1}\right)+X_c-\frac{X'_d T_o s+X_d}{T_o+1}+\frac{\left[r+\left(2s+\frac{X_c}{R}\right)X_q\right]\left[r+\left(2s+\frac{X_c}{R}\right)\frac{X'_d T_o s+X_d}{T_o+1}\right]}{\left(s+\frac{X_c}{R}\right)\left(r+sX_q\right)+X_c-X_q}} \qquad (85)$$

The denominator of i_d is:

$$\Delta=\left[\left(s+\frac{X_c}{R}\right)\left(r+sX_q+X_c-X_q\right)\right]\left[\left(s+\frac{X_c}{R}\right)\left(r+s\,\frac{sX'_d T_o+X_d}{T_o+1}\right)+X_c-\frac{sX'_d T_o+X_d}{T_o+1}\right]$$

$$+\left[r+\left(2s+\frac{X_c}{R}\right)X_q\right]\left[r+\left(2s+\frac{X_c}{R}\right)\frac{s X'_d T_o+X_d}{T_o+1}\right]$$

Multiplying by (T_o+1) gives:

$$\Delta=\left[\left(s+\frac{X_c}{R}\right)\left(r+sX_q+X_c-X_q\right)\right]\left[\left(s+\frac{X_c}{R}\right)\left\{\left(r+X_c\right)\left(T_o+1\right)+s\left(sX'_d T_o+X_d\right)\right.\right.$$

$$\left.\left.-\left(sX'_d T_o+X_d\right)\right\}\right]+\left[\left(2s+\frac{X_c}{R}\right)X_q+r\right]\left[r\left(T_o+1\right)+\left(2s+\frac{X_c}{R}\right)\left(sX'_d T_o+X_d\right)\right]$$

Multiplying the factors in the above equation gives:

$$\Delta=\left[\left(s^2R+s^3X_q+s^2X_c-s^2X_q+2\frac{X_c}{R}rs+2\frac{X_c}{R}X_q s^2+2\frac{X_c^2}{R}s-2\frac{X_c}{R}X_q s\right]\right.$$

$$\times\left[\left(r+X_c\right)\left(T_o+1\right)+s^2X'_d T_o+\left(X_d-X'_d T_o\right)s-X_d\right]+\left[r+2sX_q+\frac{X_c}{R}X_q\right]$$

$$\times\left[2s^3X'_d T_o+\left(2X_d+\frac{X_c}{R}X'_d T_o\right)s+r\left(T_o+1\right)+\frac{X_c}{R}X_d\right] \qquad (86)$$

Let:

$$\left(r + X_c - X_q + 2\frac{X_c}{R}X_q\right) = A$$

$$2\frac{X_c}{R}\left(r + X_c - X_q\right) = B$$

$$\left[(r + X_c)(T_o + 1) - X_d\right] = C$$

$$\left(X_d - X'_d T_o\right) = D$$

$$\left[2X_d + \frac{X_c}{R}X'_d T_o\right] = E$$

$$\left[r(T_o + 1) + \frac{X_c}{R}X_d\right] = H$$

Then rewriting Eq. (86):

$$\Delta = \left[s^3 X_q + A s^2 + B s\right]\left[s^2 X'_d T_o + D s + C\right] + \left[r + 2 s X_q + \frac{X_c}{R}X_q\right] \times \left[2 X'_d T_o s^3 + E s + H\right]$$

and multiplying the factors gives:

$$\Delta = s^5 X_q X'_d T_o + s^4 D X_q + s^3 C X_q + s^4 A X'_d T_o + s^3 A D + s^2 A C + s^3 B X'_d T_o + s^2 B D + s B C + s^3 2 r X'_d T_o + s r E + r H + s^4 4 X_q X'_d T_o + s^2 2 E X_q + s 2 X_q H + s^3 2 \frac{X_c}{R} X_q X'_d T_o + s \frac{X_c}{R} X_q E + \frac{X_c}{R} X_q H \qquad (87)$$

Again for simplicity let:

$$\left(D X_q + A X'_d T_o + 4 X_q X'_d T_o\right) = K$$

$$\left(C X_q + A \times D + B X'_d T_o + 2 r X'_d T_o + 2\frac{X_c}{R} X_q X'_d T_o\right) = M$$

$$\left(A \times C + B \times D + 2 E X_q\right) = N$$

$$\left(B \times C + r E + H X_q + \frac{X_c}{R} X_q E\right) = P$$

$$\left(r H + \frac{X_c}{R} X_q H\right) = R$$

$$X_q X'_d T_o = T$$

Then Eq. (87) becomes:

$$\Delta = Ts^5 + Ks^4 + Ms^3 + Ns^2 + Ps + R \tag{88}$$

Then the roots of the above equation may be analyzed in the following manner:

$$
\begin{array}{c|lll}
s^5 & T & M & P \\[2ex]
s^4 & K & N & R \\[2ex]
s^3 & \dfrac{KM-TN}{K} & \dfrac{KP-TR}{K} & \\[3ex]
s^2 & \dfrac{\dfrac{(KM-TN)N}{K} - \dfrac{(KP-TR)K}{K}}{\dfrac{KM-TN}{K}} & \dfrac{\dfrac{(KM-TN)R}{K} - K\times 0}{\dfrac{KM-TN}{K}} & \\[5ex]
s^1 & \dfrac{\dfrac{\dfrac{(KM-TN)N}{K} - \dfrac{(KP-TR)K}{K}}{\dfrac{KM-TN}{K}} \times \dfrac{KP-TR}{K} - \dfrac{(KM-TN)R}{K}}{\dfrac{\dfrac{(KM-TN)N}{K} - \dfrac{(KP-TR)K}{K}}{\dfrac{KM-TN}{K}}} & & \\[6ex]
s^0 & R & &
\end{array}
$$

Simplifying the above form gives:

$$
\begin{array}{l|lll}
s^5 & T & M & P \\
s^4 & K & N & R \\
s^3 & \dfrac{KM-TN}{K} & \dfrac{KP-TR}{K} & \\
s^2 & N-K\,\dfrac{KP-TR}{KM-TN} & R & \\
s^1 & \dfrac{KP-TR}{K} - \dfrac{\dfrac{KM-TN}{K}\,R}{N-K\,\dfrac{KP-TR}{KM-TN}} & & \\
s^0 & R & &
\end{array}
$$

Then not to have self excitation the following equations must be satisfied:

$$T > 0 \qquad (89)$$

$$K > 0 \qquad (90)$$

$$\frac{KM-TN}{K} > 0 \qquad (91)$$

$$N-K\,\frac{KP-TR}{K} > 0 \qquad (92)$$

$$\frac{KP-TR}{K} - \frac{\dfrac{KM-TN}{K}\,R}{N-K\,\dfrac{KP-TR}{KM-TN}} > 0 \qquad (93)$$

$$R > 0 \qquad (94)$$

Eq. (89), (90), and (94) are inherently satisfied. Hence not to have self-excitation only Eq. (91), (92), and (93) must be satisfied.

FERRORESONANCE IN TRANSFORMERS

If a voltage is applied to an unloaded transformer at about the zero point of the voltage wave, a high inrush current may result. This inrush current lasts for several cycles after which the transformer draws its normal exciting current. However, if the unloaded transformer is energized through a series capacitor an abnormal exciting current may persist in the steady state. This results in a badly distorted voltage wave form mostly composed of lower frequency components in the secondary of the transformer. This phenomenon is known as ferroresonance. Ferroresonance can be eliminated automatically in most cases by means of a parallel gap included in the protection equipment of the series capacitor. Sometimes the magnetizing inrush current is of sufficient magnitude to cause a voltage drop to appear across the capacitor high enough to break down the gap. Toward the end of the transient period the current in the gap decreases and consequently the arc across the gap breaks down, thus restoring the capacitor to the circuit.

If the gap alone cannot prevent ferroresonance a shunt resistor across the capacitor can be used to remedy the situation. The shunt resistor across the series capacitor used to prevent subsynchronous resonance of motors which was discussed previously in detail naturally also prevents ferroresonance of transformers.

Sometimes to prevent ferroresonance a certain minimum load on the transformer side of the capacitor is connected before the bank is energized.

PROTECTION OF SERIES CAPACITORS

A bank of series capacitors connected in series with each phase of a transmission or distribution line may be subjected, for short periods, to some multiple of its rated voltage. The dielectric of a capacitor can be made to withstand without damage these high voltages. On the other hand for a given reactance, the cost of capacitors increases approximately as the square of the rated voltage. Hence a voltage limiting device must be used. The best way to protect a series capacitor from over-voltage during fault is to make use of a gap. A simple capacitor protective device for low voltage protection making use of a parallel gap connected across the capacitor is shown in Fig. 16.

The secondary voltage of the transformer which is connected across the auxiliary capacitor is a high multiple of the primary voltage. The auxiliary gap is set to flash over approximately twice this voltage. The reactance of the choke coil is low at low frequencies and very high at high frequencies. This inductance provides a local oscillatory circuit when the auxiliary gap flashes over. This increases the high frequency voltage across the main gap to the flash-over voltage. The choke coil prevents the series capacitor from supressing this increase in high frequency voltage across the main gap. This simple protection is sufficient when the duration of fault is not long enough to cause the heat generated by the arc to damage the gap. The circuit shown in Fig. 17 eliminates this difficulty. In this case the gap breaks down if an abnormal voltage appears across the series capacitor. The current flowing through the gap energizes the coil and the coil closes the contactor, thus short circuiting the series capacitor, gap, and a part of itself. The remaining part of the coil is sufficient to keep the contactor closed until the fault is cleared. However, there

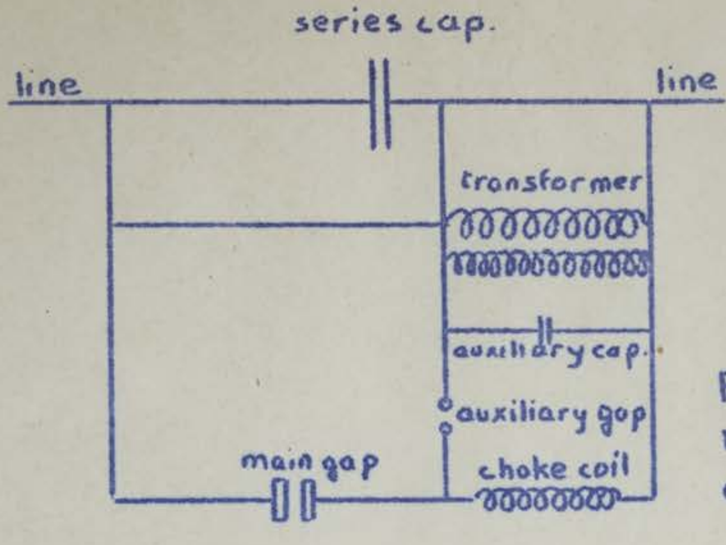


Fig.16. Low voltage protection device of a series capacitor.

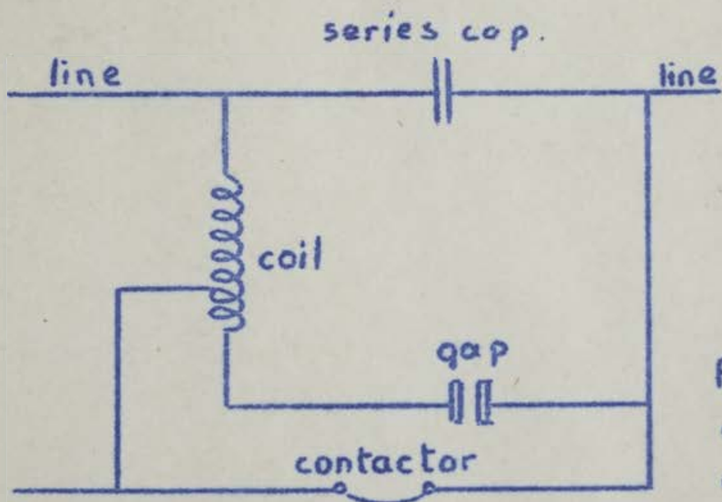


Fig.17. Simple protection device of a series capacitor.

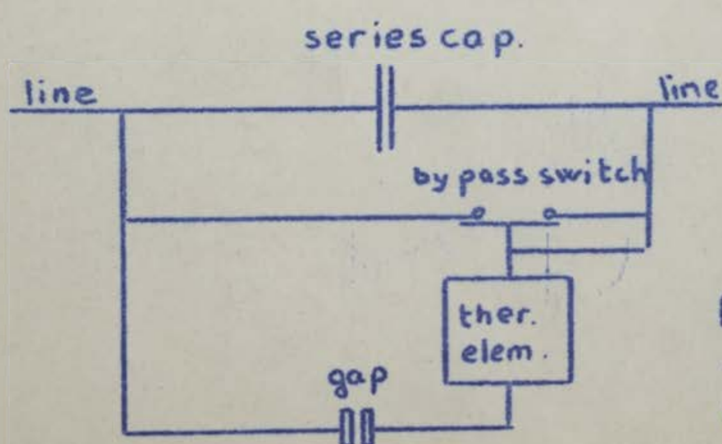


Fig.18. Protection device of a series capacitor equipped with a thermal element

are some difficulties also in this type of protection. First of all the coil must be very carefully adjusted for each equipment so that it can close the contactor at some suitable multiple of the rated capacitor current and open it again when the current drops to the rated value. Furthermore when the gap breaks down the stored energy of the capacitor is discharged into the coil. Hence the coil must be designed to withstand this. Sometimes due to the high discharge current of the capacitor the contactor moves so rapidly that its contacts may be closed while the discharge current is still high. The contacts must be carefully designed to prevent welding or excessive burning due to the above explained phenomenon. Naturally all these things make the design of both coil and contactor very expensive. The use of a thermal switch eliminates the above mentioned difficulties. The circuit of the protective device using the thermal switch is shown in Fig. 18. In this case the current, flowing through the gap when the gap breaks down due to a fault, passes through the thermal element and by means of the heat it develops actuates the by-pass switch thus short circuiting the gap, the series capacitor, and the thermal element. After a time since there is no current passing through the thermal element, the by-pass switch opens. However, during that time any fault can be cleared by means of the protective equipment of the line.

A series capacitor will also be damaged by continuous operation at more than 110% of its rated current. On the other hand over-voltage or over-current protection device may not operate at less than twice the rated voltage or current; therefore some other means must be found to protect the series capacitor against the probable damages resulting from the continuous operation at over-load. A thermal element connected in series with the series capacitor can actuate a by-pass switch which short circuits the

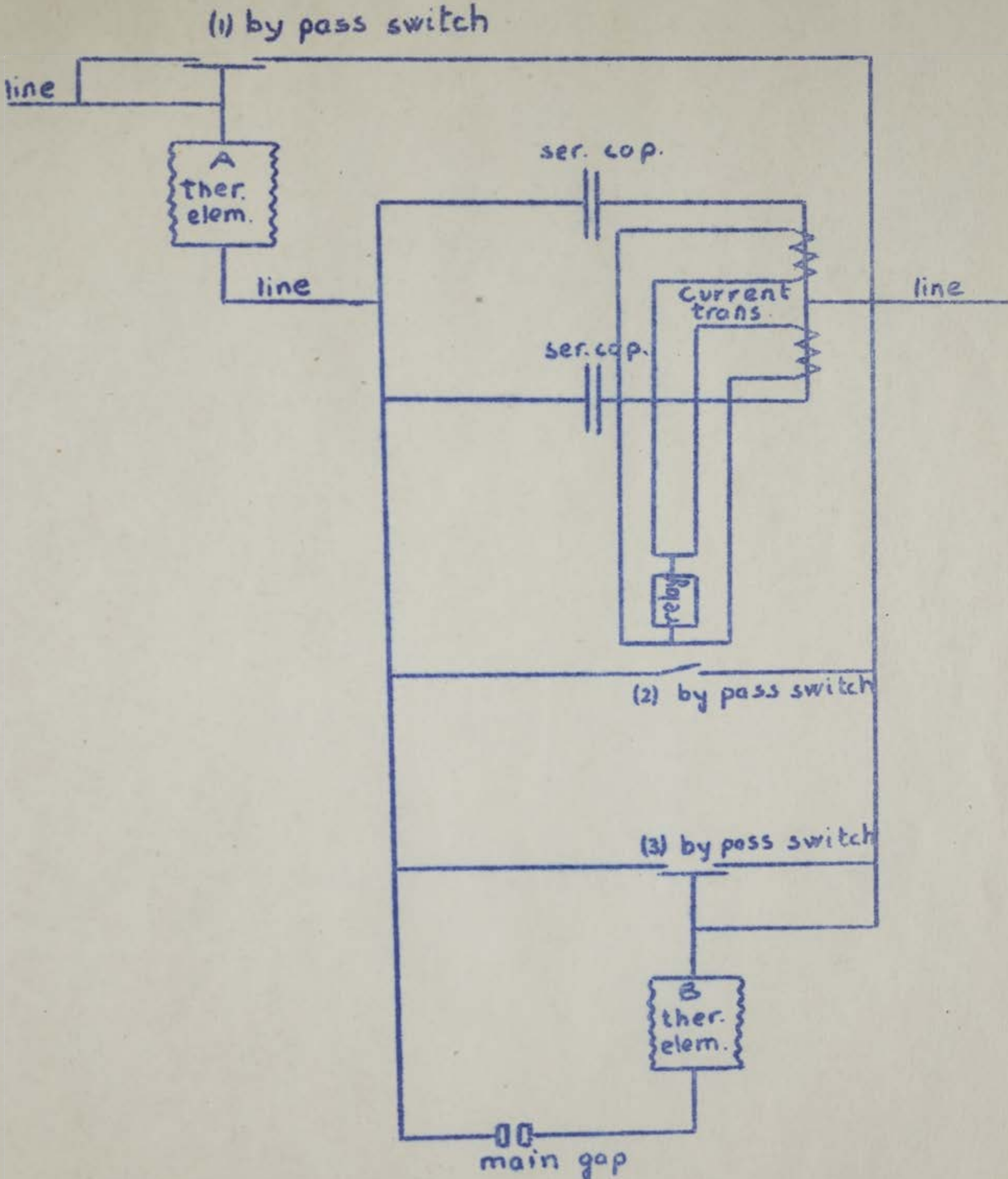


Fig.19. Complete diagram of the protective equipment of a series capacitor.

series capacitor before continuous over-load damages it. This is shown in Fig. 19.

Sometimes the dielectric of a series capacitor may fail. A failure of dielectric short circuits the capacitor. The short circuited capacitor unit may sustain an internal arc which causes gas to be generated in the unit. After a time the pressure of this gas may be high enough to rupture the case and even cause damage to other equipment. If a capacitor unit is equipped with a fuse, then the fuse will remove the unit in case of a dielectric failure. But the removal of a unit increases the reactance of the bank, as all units are connected in parallel, thus subjecting the other units to over-voltage. To prevent this a differential type relay is used as shown in Fig. 19. Under normal operation no current will flow through the differential branch, since all the units are identical. In case of the failure of one of the units then due to the unbalance of currents a current will flow through the differential branch thus closing the by-pass switch (2) which short circuits the bank of the capacitors. In Fig. 19 only two units are shown in parallel for the sake of simplicity.

For high voltages artificial means are used to cool the air-gap.

CONCLUSION

The conclusion which may be drawn from the previous discussions is that series capacitors are proving more and more useful and promising in transmission and distribution engineering. If all the difficulties encountered in the application of series capacitors are analyzed carefully, making an extensive study of their application considerations, they will be the strongest competitors in their field of application. From the standpoint of construction and maintenance, series capacitors have already proved very economical as compared with synchronous condensers.

In the near future the series capacitor will have a wide field of application in transmission and distribution engineering.

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