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THE LOCATION OF UNDERGROUND POINTS BY
MEANS OF A MAGNETIC DIPOLE

BY
ANTHONY DEL PRETE, JR.

A
THESIS

submitted to the faculty of the
SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI
in partial fulfillment of the requirements for the

Degree of
MASTER OF SCIENCE, GEOLOGY MAJOR
Rolla, Missouri
1963

Approved by

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ABSTRACT

The object of this study was to design, construct, and test an electromagnet that could, from an underground location, generate a magnetic field which could be located at the surface, and that was portable enough to be carried through a cave. This could then be used to locate points in a cave relative to the surface.

A 20 foot solenoid composed of four five-foot sections was built and magnetized by a lightweight rechargeable four-cell eight volt battery. The total weight of the equipment was 30 pounds.

The magnet was calibrated by erecting it and measuring its vertical intensity with a Ruska magnetometer. These measurements were made at several distances and at different current strengths to determine the pole strength. Then the magnet was tested at a cave of known depth to check the results.

Theoretical consideration was given to the strength of the magnet and its effect at various distances. An IBM 1620 digital computer was used to calculate points of maximum vertical intensity of the magnetic field.

The results showed very good accuracy, but a depth penetration of 50 feet was the maximum obtainable. Changes needed to increase the depth penetration to 100 feet are also discussed.

ACKNOWLEDGMENTS

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I. INTRODUCTION

A. Purpose

This investigation was undertaken to locate points underground in relation to the surface, by magnetic means. Mapping an underground passage with Brunton compass and tape usually is a long and not very accurate method. The use of a transit involves even more work and sometimes is impossible because of small passages. For these reasons a new method was investigated which would facilitate accurate surface location of underground points.

B. Method

In magnetic prospecting, the depth to an unknown ore body can be determined approximately from the shape and magnitude of its magnetic anomaly. It was decided to build an electromagnet of known dimensions, which would be portable enough to be carried into a cave and operated by one man. This long cylindrical electromagnet would then be considered as a bar magnet and the field produced was to be detected by a magnetometer operated on the surface. The depth to the magnet could be determined from its vertical, horizontal, or total intensity anomaly. The vertical, rather than the horizontal or total, intensity was chosen because a vertical intensity magnetometer was the only one available.

C. Previous Work

No published references to use of this method were found by the writer, although the theory and equations have been known for many years. Charles A. Coulomb, in 1785, first established the inverse square law of attraction and repulsion of magnets. In 1820, Hans C. Oersted discovered that an electric current would affect a magnetic needle. A few years later, in 1825, the first electromagnet was built by William Sturgeon and, in 1873, J. C. Maxwell formulated systematically the laws of electricity and magnetism (Encyclopedia Britannica, 1962, p. 637).

The first use of magnetics in exploration for buried deposits probably began in Sweden, when the compass was used in the search for iron ores. Later, dip needles, which measure the inclination of the earth's magnetic field, were widely used. The first rugged and portable instrument capable of detecting small, local anomalies was the Schmidt field magnetometer developed in 1915 (Jakosky, 1950, p. 5). The Ruska magnetometer used in this work is of the Schmidt type.

The equations used in this study are standard in textbooks on exploration geophysics. The magnet is assumed to have all its effect concentrated at the poles and is equivalent to a magnetic dipole. Jakosky (1950) described the effects of a dipole in various positions and is the main source of the equations used.

II. THEORETICAL CONSIDERATION

A. General

The magnitude of the field produced by a solenoid with an air core is dependent on the current in the circuit and the number of turns of wire. If the solenoid has other than an air core, the field will also depend on the permeability of the core material. The intensity of the magnetic field in oersteds H , near the center of a closely wound solenoid, whose length is very much greater than its radius, is given by the equation (Handbook of Chemistry and Physics, 41st ed., p. 3105):

$$(1) \quad H = 4\pi Ni/10$$

N = number of turns of wire per centimeter

i = current in amperes

If the solenoid has an iron core, the flux density B , in the core, is given by,

$$(2) \quad B = uH$$

u = permeability of iron core

The permeability is not a constant and therefore the flux density B is not a linear function of the magnetic intensity H . This is discussed further in Chapter Four.

The region where the lines of induction emerge from the body is called the north pole and is determined by the direction of the current flow. If the right hand is placed around the solenoid with the fingers pointing in the direction of current flow, the thumb will point to the north pole of the magnet.

Equations 1 & 2 show that the diameter of the solenoid has no effect on the field intensity or the flux density in the core of a magnet, whose length is very much greater than its radius. The length has no effect either, except in calculating the number of turns of wire per centimeter. These two factors become important however, because the vertical field will be measured at a distance from the magnet, and will be discussed in the following section.

All the equations used apply to measurements in air, with a permeability of 1.0000004 c.g.s. units (Jakosky, 1950, p. 67). The susceptibility of dolomite varies from .0000009 to .000014 c.g.s. units, according to Jakosky. Using an average value of .0000075 c.g.s. units and calculating the permeability by the use of the relationship between susceptibility and permeability (Jakosky, 1950, p. 78),

$$(3) \quad u = 1 + 4\pi k$$

u = permeability

k = susceptibility

the permeability of dolomite is determined to be 1.000094 c.g.s. units.

This is essentially the same as that of air, so the measurements made through dolomite will be the same as those through air, within the sensitivity of the instrument used.

B. Vertical Intensity of a Buried Dipole

The concept of pole strength is useful in simplifying calculations involving magnets. The force between two magnetic poles varies inversely as the square of the distance between them. This force is also proportional to the product of the strength of the poles (Jakosky, 1950, p. 67).

$$(4) \quad F = m_1 m_2 / r^2$$

r = distance between poles

A system of units was established by defining a unit pole so that $F = 1$. Therefore a unit pole is one which when placed a distance of one centimeter from a similar pole will be acted upon with a force of one dyne. One dyne per unit pole is equivalent to one oersted. In the system of units used in this work pole strength is expressed in oersted-cm². The oersted is too large for field work and therefore the gamma, which is equal to 10^{-5} oersted is used.

Length and diameter of the magnet now become important, because the intensity of the field produced by each pole is proportional to the pole strength and inversely proportional to the cube of the distance to the pole.

The pole strength is affected in the following way, as shown by Jakosky (1950, p. 76),

$$(5) \quad m = IA$$

I = intensity in oersted

A = end area in cm^2

and will increase proportional to the square of the radius. The length is even more important, because the vertical intensity is inversely proportional to the cube of the distance to each pole.

The anomaly of the vertical intensity produced by a vertical dipole will have a maximum directly over the poles, while the vertical intensity anomaly produced by a horizontal dipole will have a maximum and a minimum. These will be off-set from the poles due to the effect of the opposite pole. The position of the maximum and minimum is dependent on the length of the magnet and its depth of burial.

The vertical component DZ , of the magnetic intensity anomalies produced by a vertical dipole is calculated by adding vectorially the effects of a north and south pole. The equation given by Jakosky (1950, p. 187) is

$$(6) \quad DZ = -m \left[\frac{D_1}{r_1^3} - \frac{D_2}{r_2^3} \right]$$

DZ = vertical intensity in oersteds

m = pole strength

D_1 = vertical distance to (-) pole

D_2 = vertical distance to (+) pole

r_1 = distance to (-) pole

r_2 = distance to (+) pole

If the dipole is horizontal, $D_1 = D_2$ and the equation is modified to

$$(7) \quad DZ = -m \left[\frac{D}{r_1^3} - \frac{D}{r_2^3} \right]$$

C. Determination of Point of Maximum Intensity

The original equation (7) on an (x, z) coordinate system yields the following (Jakosky, 1950, p. 189),

$$(8) \quad DZ = -m \left[\frac{D}{(x^2 + D^2)^{3/2}} - \frac{D}{(D^2 + (1-x)^2)^{3/2}} \right]$$

l = length of magnet

x = distance along the horizontal axis
measured from a point directly
over the (-) pole

To find the maximum and the minimum points of this equation, it is necessary to take the derivative DZ' , set it equal to zero, and solve for x .

$$(9) \quad DZ' = -mD \left[\frac{3x-3l}{(D^2 + (x-l)^2)^{5/2}} - \frac{3x}{(x^2 + D^2)^{5/2}} \right]$$

$$(10) \quad 0 = 3x \left[\frac{D^2 + (x-l)^2}{(x^2 + D^2)^{5/2}} - \frac{(3x-3l)(x^2 + D^2)^{5/2}}{(D^2 + (x-l)^2)^{5/2}} \right]$$

The Newton-Rhapson method (Hildebrand, 1956, p. 447) was used to solve equation (10) on an IBM 1620 digital computer (Appendix A). The following Newton-Rhapson equation approximates the value of x ,

$$(11) \quad X_{k+1} = X_k - f(X_k) / f'(X_k)$$

X_k = initial value of X

X_{k+1} = new value of X

$f(X_k)$ = function evaluated at X_k

$f'(X_k)$ = first derivative evaluated at X_k

The error in X_{k+1} tends to be proportional to the square of the error in X_k , as k approaches infinity (Hildebrand, 1956, p. 448). The error in equation (11) is very small, because the initial approximation of X_k was accurate to 0.5 cm. This means the error will become even smaller with each successive iteration. The distances, of the maximum vertical intensity, from the surface point directly over the (-) pole of a horizontal magnetic dipole of length 610 cm. are given in Table I for depths of 1000 to 3000 cm.

Equation (8) was solved for depths of 500 to 3000 cm. at intervals of 100 cm. (Appendix B). These data are shown in graph form for depths of 500 to 2100 cm. (Figures 1-3). These curves represent the standard anomalies for a magnet of 610 cm. length and 2331 oersted-cm² pole strength, and are used in comparison with the curve obtained in the field survey for the location of an underground point.

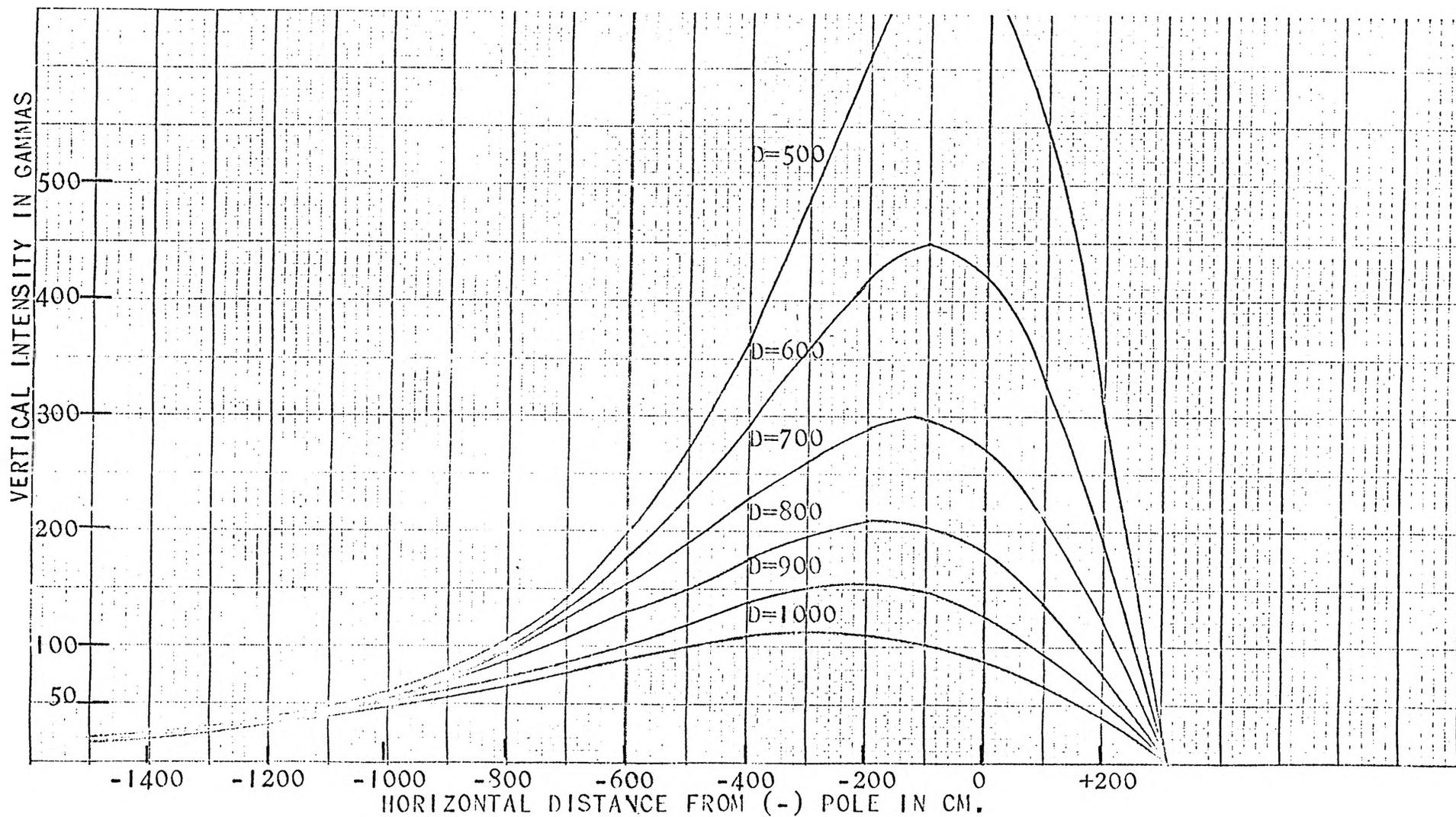


Figure 1. Theoretical Vertical Intensity Anomaly Over the Negative Portion of a Buried Horizontal Dipole

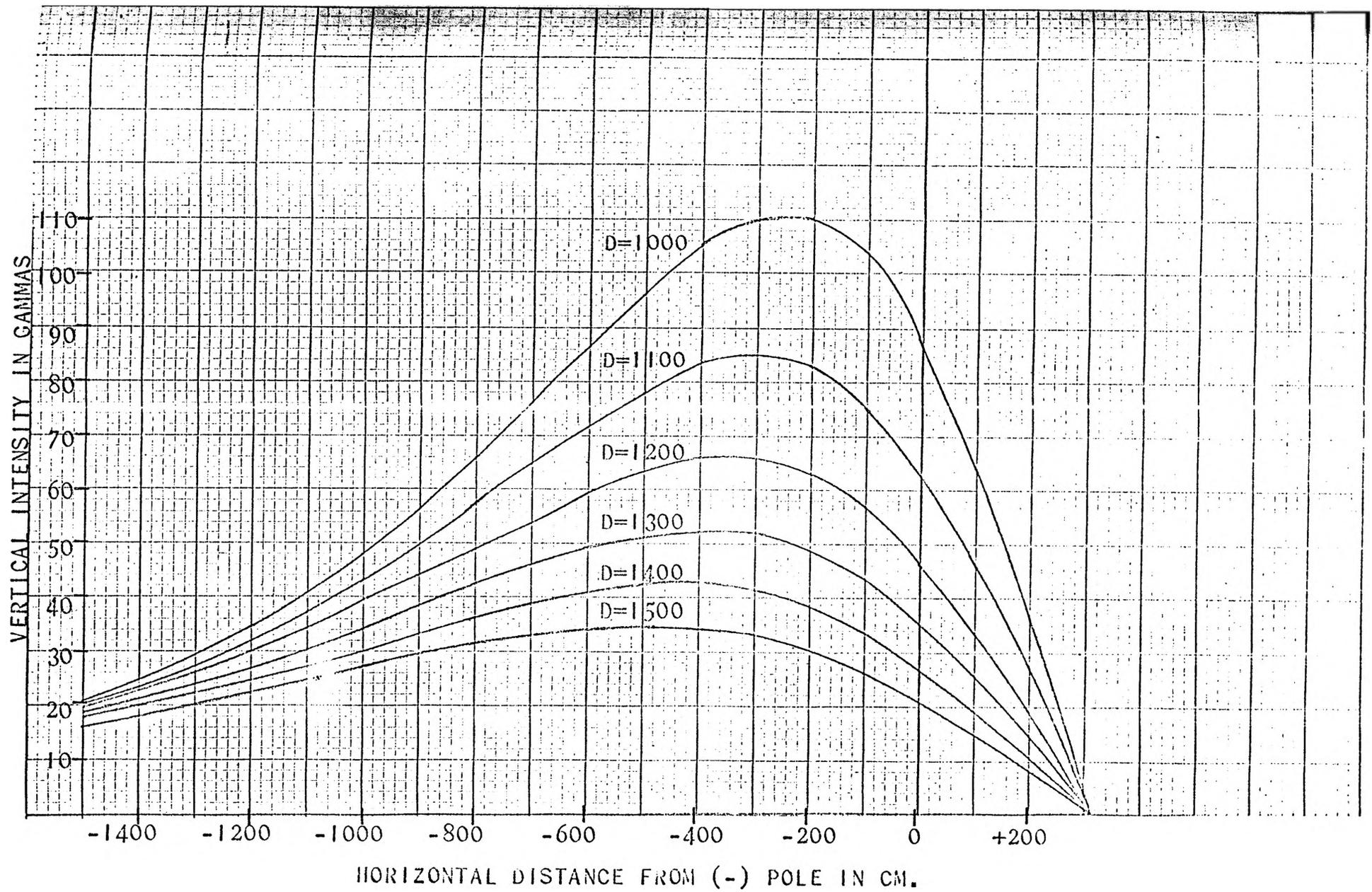


Figure 2. Theoretical Vertical Intensity Anomaly Over the Negative Portion of a Buried Horizontal Dipole

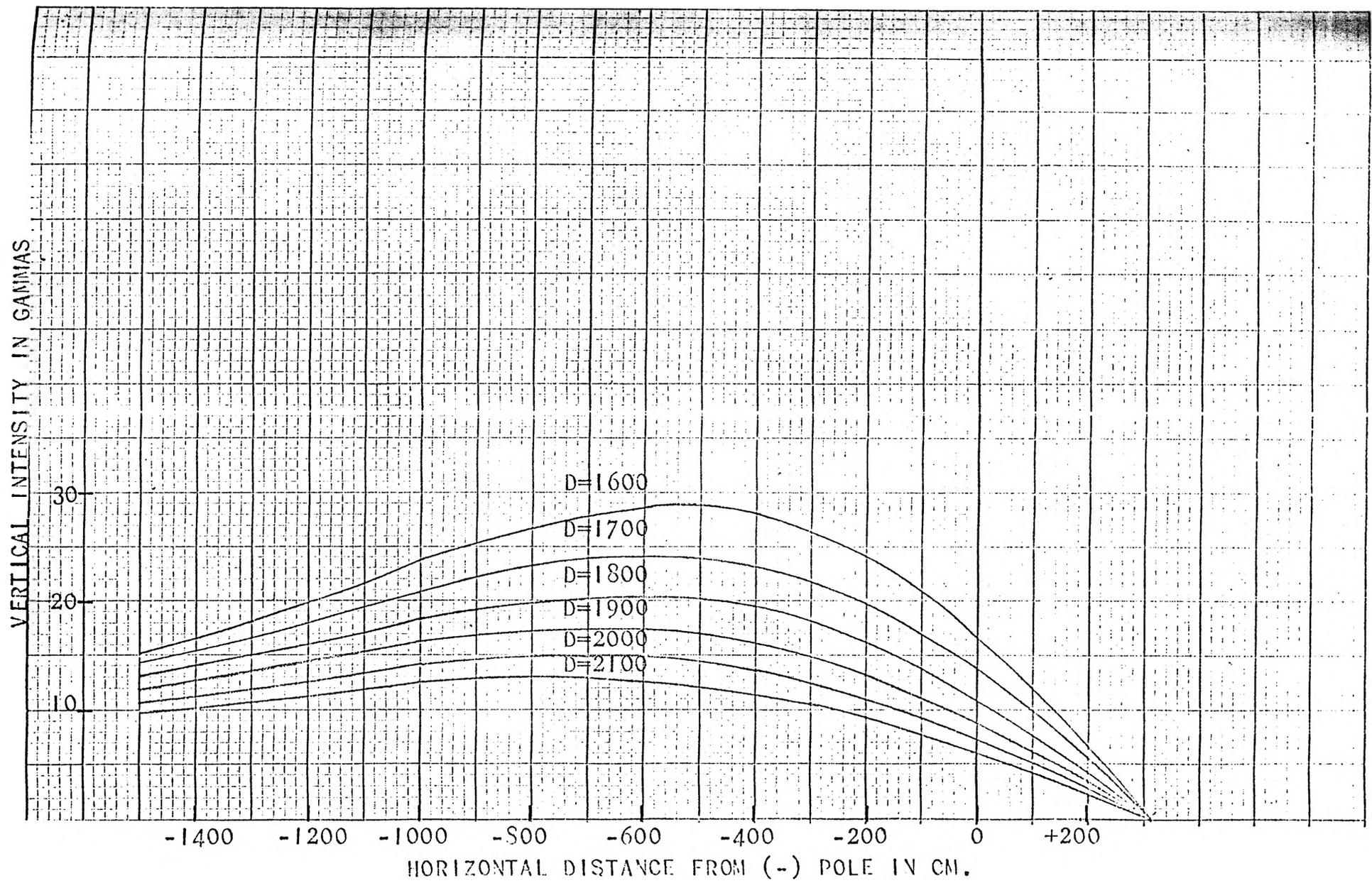


Figure 3. Theoretical Vertical Intensity Anomaly Over the Negative Portion of a Buried Horizontal Dipole

Table 1. Offset of Surface Maximum for Various Depths

DEPTH cm.	X cm.	DEPTH cm.	X cm.
1000	-251	2000	-723
1100	-296	2100	-771
1200	-341	2200	-820
1300	-388	2300	-869
1400	-435	2400	-918
1500	-482	2500	-967
1600	-530	2600	-1016
1700	-578	2700	-1066
1800	-626	2800	-1115
1900	-674	2900	-1164
		3000	-1214

X = distance along the horizontal axis
measured from a point directly
over the (-) pole.

III. DESCRIPTION OF EQUIPMENT

A. Magnet

Four iron rods, one half inch in diameter and five feet in length, were used for the core of the solenoid. This length and diameter of rod was chosen to satisfy the need for portability and lightness. One man could carry this equipment far back into a cave and set it up easily. The lengths of rod were connected by sleeves, firmly fastened to one end of each rod. This provided for a good connection between the rods and easy assembly and portability.

Solid copper bell wire of eighteen gauge was wound around each length of rod and was connected by small copper fasteners. The power source was four Willard plastic cased wet cells of two volts each. The wire was wound as tightly as possible, thereby producing a total of 3277 turns. This amount of wire had a resistance of four ohms and could carry two amperes at eight volts. See Figures 4 and 5 for a picture of the equipment.

B. Magnetometer

A ruska vertical intensity magnetometer was used for making all the measurements. This instrument measures the relative intensity of the vertical component of the magnetic field at successive points.

Adjustments and corrections were applied as outlined in the Ruska Field Manual (Ruska, 1957). The instrument constant was found to be 10.0 gammas per scale division and the temperature correction was -0.3 gammas per degree centigrade. Straight line variation was assumed at all stations.

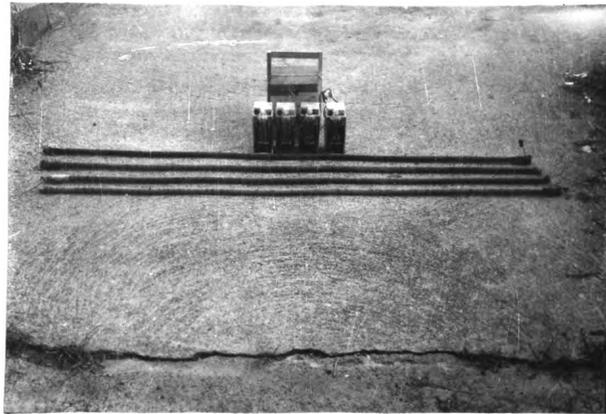


Figure 4. Solenoid and Batteries



Figure 5. Solenoid in Vertical Position for Determination of Pole Strength

IV. EXPERIMENTAL DATA

A. Field Procedure

All measurements were made relative to the vertical component of the earth's magnetic field. A reading was taken with the electromagnet off, and then with it turned on. The difference in readings gave the vertical intensity of the magnet. Measurements were made of the component of its field parallel to the length of the magnet. This was done by placing the magnet in a vertical position with three guy ropes and measuring the vertical field at different distances. To obtain the maximum vertical intensity at a point, it was necessary to measure the field in a plane through the mid-point of the magnet and normal to its axis. These readings were obtained by placing the magnet in a small gully and taking the readings on the adjacent hillside.

Five stations were occupied to measure the vertical intensity. Three were in the plane perpendicular to and bisecting the axis of the magnet, one was higher and one lower. The positions of these stations and the position of the magnet were determined by a transit and rod.

B. Pole Strength vs. Operating Current

The equivalent pole strength of the electromagnet was determined by using the above field data. The equation, as previously given, for a vertical dipole is,

$$(6) \quad DZ = -m \left[D_1/r_1^3 - D_2/r_2^3 \right]$$

Rearranging this equation to find the pole strength m gives the following.

$$(12) \quad m = DZ / [D_1/r_1^3 - D_2/r_2^3]$$

The value of m was determined for current strengths of 0.5, 1.0, 1.5, and 2.0 amperes, at each station. The horizontal distance to each station was measured and the distance to each pole was calculated from these figures. The measurements of DZ are shown in Appendix C and the calculated values of pole strength m , are shown below in Table II.

Table II. Pole Strength of Solenoid For Various Currents

Sta.	i	DZ total	Base	DZ-Base	m	mean m
A	0.5	468	365	103	1743	1911
B	0.5	390	364	26	2114	
C	0.5	436	362	74	1937	
D	0.5	448	372	76	1900	
E	0.5	524	371	153	1863	
A	1.0	482	365	117	1980	2069
B	1.0	388	364	24	1951	
C	1.0	446	362	84	2199	
D	1.0	454	372	82	2050	
E	1.0	546	371	178	2168	
A	1.5	498	365	133	2250	2206
B	1.5	390	364	26	2114	
C	1.5	448	362	86	2251	
D	1.5	458	372	86	2150	
E	1.5	557	371	186	2265	
A	2.0	506	365	141	2386	2331
B	2.0	392	364	28	2276	
C	2.0	452	362	90	2356	
D	2.0	462	372	90	2250	
E	2.0	567	371	196	2390	

The percent variation from the mean m , decreases from approximately ten percent at 0.5 amperes to approximately three percent at 2.0 amperes. The pole strength varies less as the current is increased, due to the saturation of the core. This can be seen in Figure 8.

Stations were occupied previous to the magnet set-up and immediately after it was taken down, in order to measure diurnal variation. These data are plotted in Figure 6. From these data it was decided to use the maximum current strength available of 2.0 amperes for all future work. The battery drain, even at this current, was negligible, because it was connected only a few seconds at each measurement. It appeared that additional batteries would increase the weight without appreciably adding to the magnetic penetration. The magnetic penetration of the magnet refers to its ability to be detected from a certain depth below ground level.

One section of the magnet was tested separately in the same manner as described above, but the current was increased to 6.2 amperes. A curve showing the relationship of the vertical intensity to the current strength is shown in Figure 7. This curve clearly shows the magnet reaching its saturation point. A current value of 6.2 amperes was the maximum used, but the curve has already flattened out and an increase of current would have little effect on the vertical intensity.

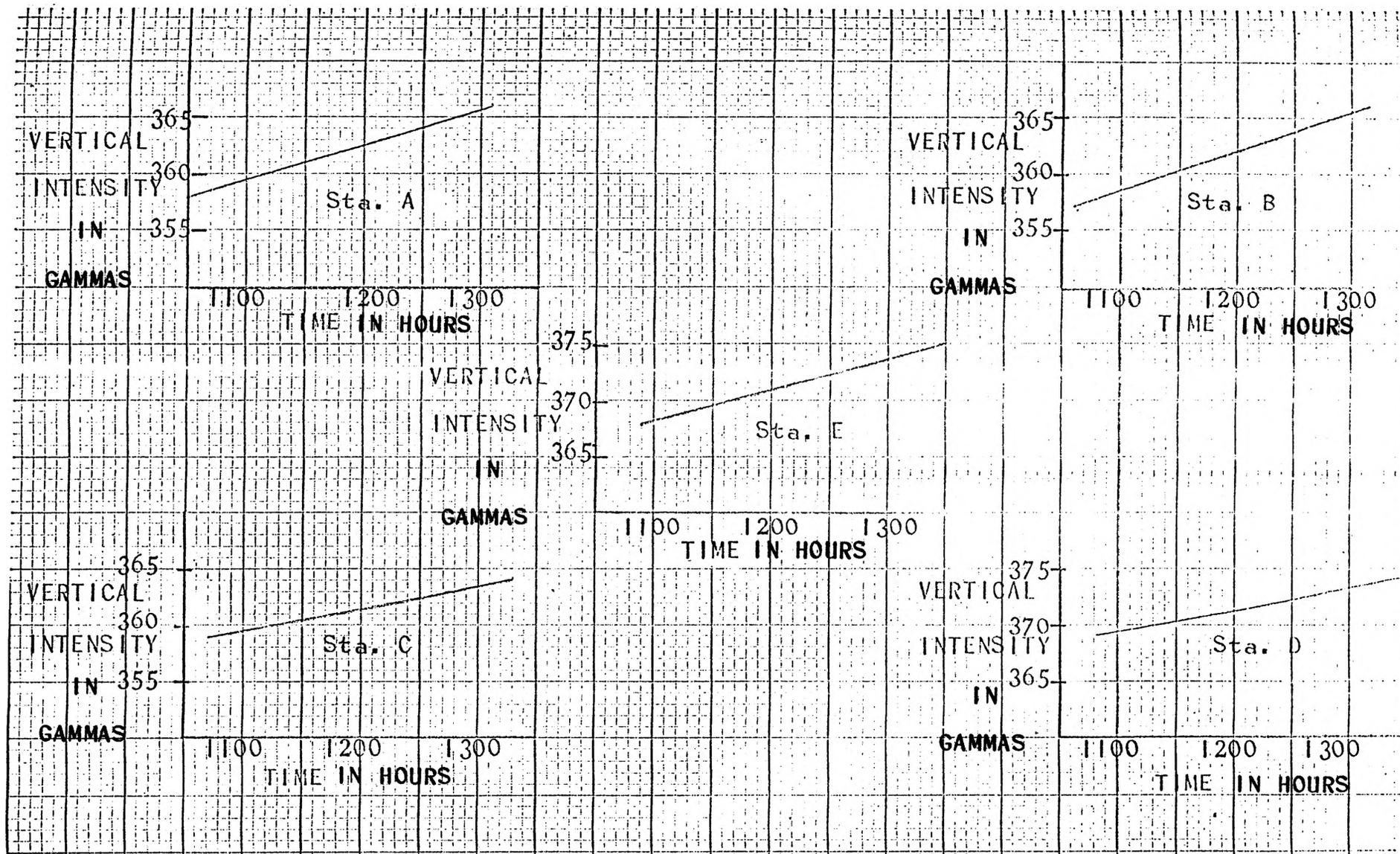


Figure 6. Diurnal Change at Calibration Test Site

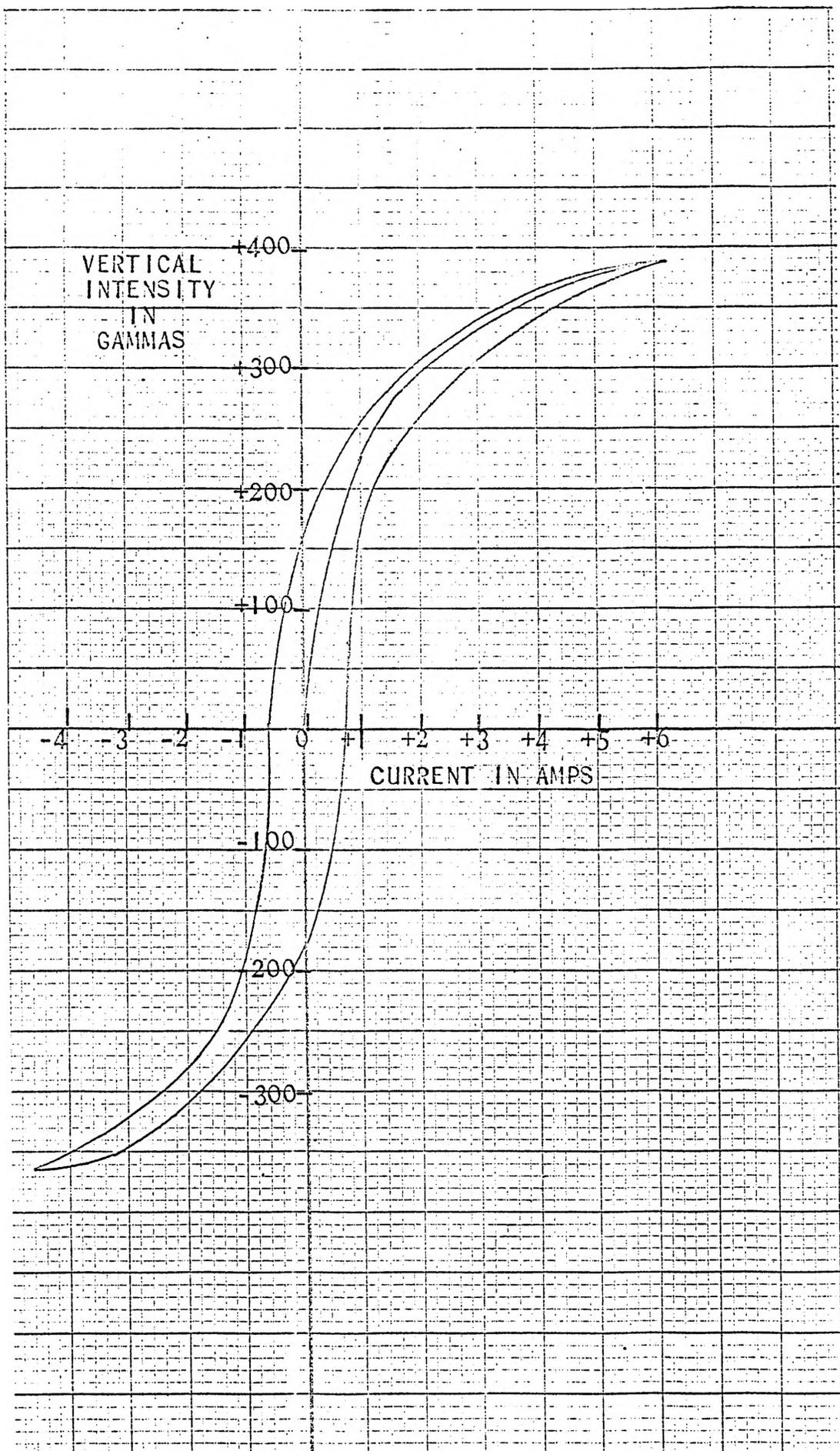


Figure 7. Relationship of Vertical Intensity to Current Strength

These readings were taken at a distance of 15.9 feet and the necessary corrections were made (Appendix D). Pole strengths were calculated and plotted in graph form with the results as shown (Figure 8). The curve varies from a straight line, as the current is increased, due to the saturation of the core.

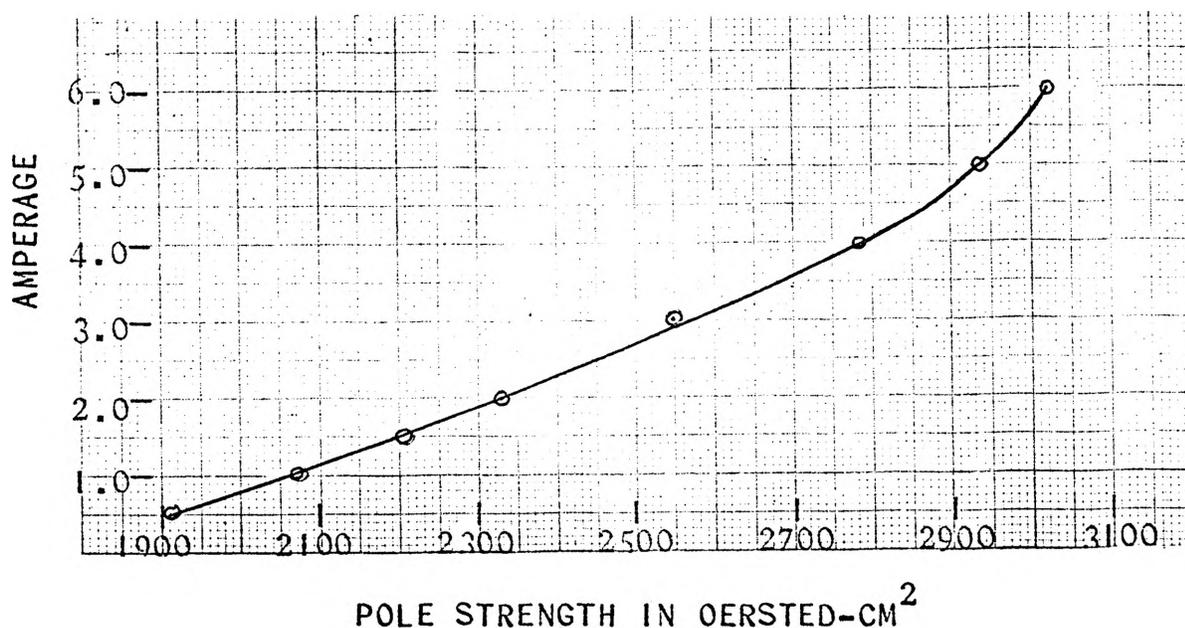


Figure 8. Pole Strength vs. Operating Current

C. Test at Schoolhouse Cave

Schoolhouse Cave is situated at the SE 1/4 NW 1/4 SE 1/4 of sec. 5, T. 35 N., R. 8 W., in Phelps County. The entrance is 2' x 2', but after a short distance the passage opens into a wide area with a low ceiling. The magnet was pulled into the cave in sections and assembled inside. When assembled it was placed in a north-south direction. It is important to know the orientation of the magnet to correctly determine its depth, because the depth determination magnetometer survey must be run in the vertical plane through the longitudinal axis of the magnet.

A preliminary east-west traverse was run over the vicinity of the magnet's south pole to find the point of maximum intensity and determine the vertical plane through the longitudinal axis of the magnet. Five stations were occupied at ten foot intervals. These data are tabulated in Appendix E and summarized in Table III. The vertical intensity is plotted in Figure 9 and the maximum is seen to be about two feet east of station three.

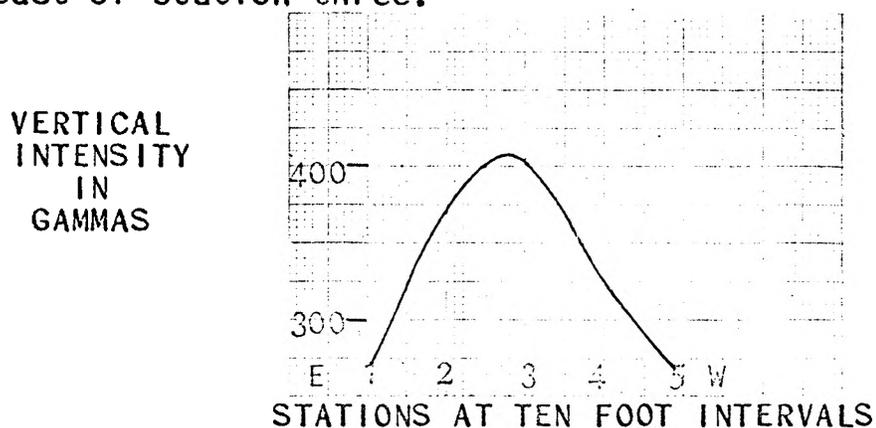


Figure 9. Vertical Intensity Anomaly in East-West Direction

A north-south line in the vertical plane through the longitudinal axis of the magnet was then laid out with the use of a Brunton compass and tape. Eight measurement stations were occupied along this line. Readings were taken at each station with the current in one direction and then with the current reversed. The two readings at each station were averaged to obtain a base value. These data are tabulated in Appendix E and summarized in Table III. The resultant curves are plotted in Figure 10.

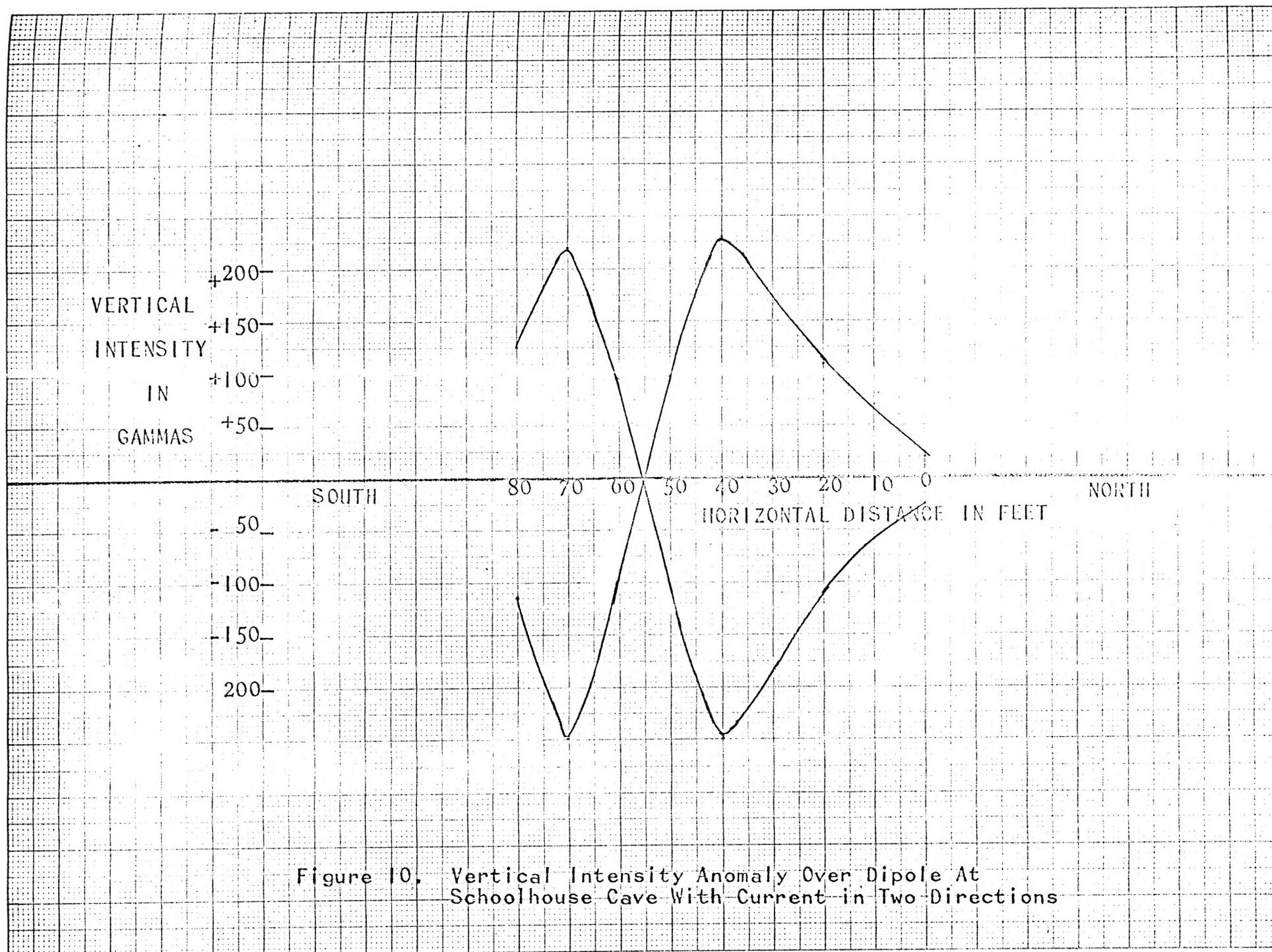
Table III. Vertical Intensity Over Schoolhouse Cave

Sta.	Vertical Intensity		Avg.	Corr. to Base of 171	
	+i	-i		+i	-i
E-W					
1			270		
2			370		
3			405		
4			328		
5			264		
N-S					
0	191	146	168	20	-25
1	242	112	177	71	-59
2	282	63	172	111	-108
4	400	-77	161	229	-248
5	270	66	168	99	-105
6	82	270	176	-89	99
7	-48	393	171	-250	220
8	56	297	176	-115	126

A Brunton compass, hand level, and tape were also used to determine the position and depth of the magnet. These data are tabulated in Table IV.

Table IV. Brunton and Tape Survey at Schoolhouse Cave

From	To	Elev. Diff.	Bearing	Distance
A	B	+6'	S 86° E	35'
B	C	+5'	N 86° W	15'
C	D	+5'	N 86° W	5'
D	E	+5'	N 86° W	8'
E	F	+5'	N 86° W	7'



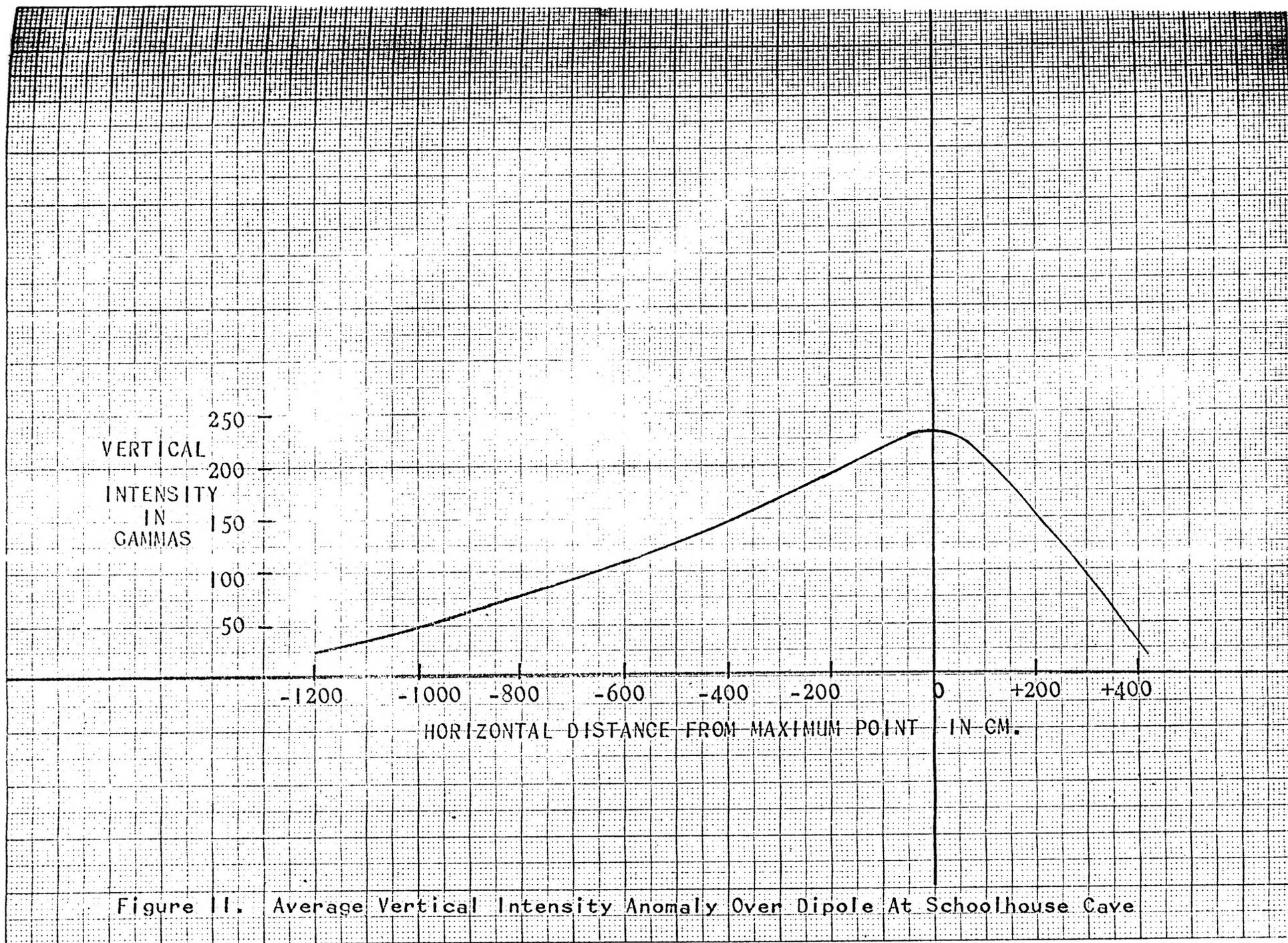


Figure 11. Average Vertical Intensity Anomaly Over Dipole At Schoolhouse Cave

V. RESULTS AND DISCUSSION

Results of the hand level survey indicated a depth of 26 feet. The average maximum value of vertical intensity as plotted in Figure 11 is 237 gammas. This was obtained by averaging the four maximum values of vertical intensity, regardless of sign, at stations four and seven. The values from stations five and six, zero, two, and eight were averaged respectively, to plot one curve similar to those in Figures 1 to 3. When this resultant curve (Figure 11) is compared to the standard curves (Figures 1 to 3), it is seen to resemble the 800 cm. depth curve, except for a slightly greater peak. The 800 cm. curve is for a depth of 26 feet and a 750 cm. curve would be for a depth of 25 feet. This result is the same as that obtained by the hand level survey, within one foot.

The percentage variation from the mean value of pole strength at 2.0 amperes was three percent. This three percent error in m , will produce a four percent error in the calculation of depth at 26 feet and 50 feet.

The difference in magnitude of vertical intensity, which would change the calculation of depth more than three feet, would have to be 90 gammas if the magnet were at a depth of 26 feet. It would have to be at least 15 gammas at a depth of 40 feet and only five gammas at a depth of 50 feet.

The curves for depths below 50 feet are very close together and the difference between the DZ's at different depths below 50 feet is not great enough to be distinguished by the magnetometer. Also, a slight mis-orientation of the magnetic survey traverse would have a large effect, because of the inverse cube relationship of the distance. It would be better to measure two profiles across the magnet and then measure a third, connecting the two maximum points.

The range of the magnetic field caused by the underground dipole will be a very local disturbance. With equipment similar to that used in this study the maximum range of detectable disturbance from the center of the magnet will be approximately 70 feet. This will vary very little with the depth below ground of the magnet.

VI. CONCLUSION AND RECOMMENDATIONS

This study showed that it is possible, with the standard magnetic exploration equations and the magnet used, to approximately locate points underground at depths down to 50 feet. By a more elaborate process one could define the observed effects with a higher precision, but this investigation was designed to test primarily a field method and not be a detailed study of magnetics.

A magnet with a larger effective pole strength and a greater separation between the poles would provide for greater depth penetration. The farther one pole is from the other the less cancelling effect it will have. An increase in current alone was discussed in Chapter Four and will not add substantially to the pole strength. An increase in the length alone will not help, because the current would be reduced because of the increased circuit resistance. However, an increase in both of these factors will greatly add to the penetrating power. An increase from 20 to 30 feet in length would decrease the current from 2.0 amperes to 1.3 amperes. It would take two more two-volt cells to maintain a current strength of 2.0 amperes and a total of twelve cells to reach 4.0 amperes.

If the magnet were 30 feet long, with the same effective pole strength, the maximum vertical intensity would be increased from 34 to 49 gammas at a magnet depth of 50 feet.

At 60 feet this increase in magnet length would cause an increase of maximum vertical intensity of approximately 5 gammas.

In order to penetrate 100 feet, a pole strength of approximately 3000 oersted-cm² would be needed. This value can be obtained with the present 20 foot magnet and a current value of 6.0 amperes. This would increase the vertical intensity readings by 50 percent. This is still too small, but a 40 foot magnet with a pole strength of 3000 oersted-cm² would suffice. The present system would require 24 cells to produce 6.0 amperes in a 40 foot magnet. This weight would severely restrict mobility in a cave, but a small hand generator might be a means of overcoming this battery weight problem.

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VIII. APPENDICES

APPENDIX A

PROGRAM FOR DETERMINATION OF POINT OF MAXIMUM INTENSITY

$$(10) \quad 0 = 3x[D^2+(x-1)^2]^{5/2} - [3x-31] [x^2+D^2]^{5/2}$$

$$(11) \quad X_{k+1} = X_k - f(X_k) / f'(X_k)$$

The derivative equation (10) was programed for an IBM 1620 digital computer in floating point FORGO. FORGO is a computer language for the 1620 with punched card input-output and a minimum of 40,000 positions of core storage. Floating point means that the decimal place will be retained.

Program

```

DIMENSION X(70)
ROD=610.
I=1
DO 4 J=1000,3000,100
DPTH=J
X(1)=-500
2 A=3.*X(I)*(DPTH**2+(X(I)-ROD)**2)**2.5
B=(3.*X(I)-3.*ROD)*(X(I)**2+DPTH**2)**2.5
C=(15.*X(I)**2-15.*X(I)*ROD)*(DPTH**2+(X(I)-ROD)**2)**1.5
D=3.*(DPTH**2+(X(I)-ROD)**2)**2.5-3.*(X(I)**2+DPTH**2)**2.5
E=(15.*X(I)**2-15.*X(I)*ROD)*(X(I)**2+DPTH**2)**1.5
X(I+1)=X(I)-(A-B)/(C+D-E)
P=X(I+1)-X(I)
R=ABS(P)
I=I+1
IF (R-1.) 4,4,2
4 PUNCH 6,X(I),DPTH
6 FORMAT (2E18.8)
STOP
END

```

APPENDIX B

PROGRAM FOR DETERMINATION OF THEORETICAL
VERTICAL INTENSITY ANOMALIES

$$(8) \quad DZ = -m \left[\frac{D}{(x^2 + D^2)^{3/2}} - \frac{D}{(D^2 - (1-x)^2)^{3/2}} \right]$$

This equation was programmed for an IBM 1620 digital computer in floating point FORGO.

Program

```

P=2331.
R=610.
DO 7 ID=500,3000,100
D=ID
X=-1600.
3 X=X+100.
Z=P*(D/(X**2+D**2)**1.5-D/(D**2+(X-R)**2)**1.5)
5 PUNCH 6,D,X,Z
6 FORMAT (3E18.8)
IF(X-300.)3,7,7
7 CONTINUE
STOP
END

```

Table V. Data From Computer For The Determination of Theoretical Vertical Intensity Anomalies

DEPTH (cm.)	X (cm.)	Z (gammas)	DEPTH (cm.)	X (cm.)	Z (gammas)
500	-1500	18.1	700	- 500	184.1
500	-1400	22.4	700	- 400	223.4
500	-1300	28.0	700	- 300	261.6
500	-1200	35.4	700	- 200	289.9
500	-1100	45.5	700	- 100	296.9
500	-1000	59.1	700	0	271.9
500	- 900	77.8	700	+ 100	210.3
500	- 800	104.0	700	+ 200	117.3
500	- 700	140.1	700	+ 300	5.8
500	- 600	192.7	800	-1500	21.7
500	- 500	265.1	800	-1400	26.1
500	- 400	362.5	800	-1300	31.4
500	- 300	483.8	800	-1200	38.1
500	- 200	611.2	800	-1100	46.4
500	- 100	701.1	800	-1000	56.7
500	0	694.9	800	- 900	69.4
500	+ 100	559.2	800	- 800	85.0
500	+ 200	315.2	800	- 700	103.7
500	+ 300	15.5	800	- 600	125.4
600	-1500	19.9	800	- 500	149.3
600	-1400	24.4	800	- 400	173.4
600	-1300	30.2	800	- 300	194.2
600	-1200	37.7	800	- 200	206.2
600	-1100	47.6	800	- 100	203.5
600	-1000	60.6	800	0	181.1
600	- 900	77.9	800	+ 100	137.5
600	- 800	110.0	800	+ 200	75.9
600	- 700	131.7	800	+ 300	3.7
600	- 600	172.2	900	-1500	21.8
600	- 500	223.9	900	-1400	25.9
600	- 400	286.7	900	-1300	30.8
600	- 300	355.3	900	-1200	36.8
600	- 200	416.3	900	-1100	44.0
600	- 100	447.3	900	-1000	52.7
600	0	424.2	900	- 900	63.1
600	+ 100	335.0	900	- 800	75.3
600	+ 200	188.4	900	- 700	89.3
600	+ 300	9.3	900	- 600	104.6
700	-1500	21.1	900	- 500	120.3
700	-1400	25.6	900	- 400	134.9
700	-1300	31.3	900	- 300	145.6
700	-1200	38.5	900	- 200	149.5
700	-1100	47.7	900	- 100	143.3
700	-1000	59.6	900	0	124.5
700	- 900	74.7	900	+ 100	93.0
700	- 800	94.0	900	+ 200	50.8
700	- 700	118.4	900	+ 300	2.5
700	- 600	148.5			

Table V. Data From Computer For The Determination of
Theoretical Vertical Intensity Anomalies (cont.)

DEPTH (cm.)	X (cm.)	Z (gammas)	DEPTH (cm.)	X (cm.)	Z (gammas)
1000	-1500	21.5	1200	- 500	63.3
1000	-1400	25.2	1200	- 400	65.7
1000	-1300	29.6	1200	- 300	65.9
1000	-1200	34.8	1200	- 200	63.2
1000	-1100	41.0	1200	- 100	57.0
1000	-1000	48.2	1200	0	47.2
1000	- 900	56.5	1200	+ 100	37.0
1000	- 800	65.9	1200	+ 200	18.2
1000	- 700	76.1	1200	+ 300	0.9
1000	- 600	86.7	1300	-1500	18.8
1000	- 500	96.9	1300	-1400	21.4
1000	- 400	105.4	1300	-1300	24.2
1000	- 300	110.5	1300	-1200	27.3
1000	- 200	110.4	1300	-1100	30.8
1000	- 100	103.3	1300	-1000	34.5
1000	0	88.1	1300	- 900	38.4
1000	+ 100	64.9	1300	- 800	42.2
1000	+ 200	35.1	1300	- 700	45.9
1000	+ 300	1.7	1300	- 600	49.1
1100	-1500	20.8	1300	- 500	51.5
1100	-1400	24.1	1300	- 400	52.5
1100	-1300	28.0	1300	- 300	51.8
1100	-1200	32.5	1300	- 200	48.8
1100	-1100	37.6	1300	- 100	43.5
1100	-1000	43.5	1300	0	35.6
1100	- 900	50.0	1300	+ 100	25.4
1100	- 800	57.1	1300	+ 200	13.5
1100	- 700	64.5	1300	+ 300	0.7
1100	- 600	71.7	1400	-1500	17.7
1100	- 500	78.2	1400	-1400	19.8
1100	- 400	82.9	1400	-1300	22.2
1100	- 300	84.9	1400	-1200	24.8
1100	- 200	82.9	1400	-1100	27.6
1100	- 100	76.0	1400	-1000	30.5
1100	0	63.8	1400	- 900	33.4
1100	+ 100	46.4	1400	- 800	36.2
1100	+ 200	24.9	1400	- 700	38.8
1100	+ 300	1.2	1400	- 600	40.8
1200	-1500	19.9	1400	- 500	42.1
1200	-1400	22.8	1400	- 400	42.3
1200	-1300	26.1	1400	- 300	41.1
1200	-1200	29.9	1400	- 200	38.3
1200	-1100	34.2	1400	- 100	33.7
1200	-1000	38.8	1400	0	27.3
1200	- 900	43.9	1400	+ 100	19.4
1200	- 800	49.2	1400	+ 200	10.3
1200	- 700	54.4	1400	+ 300	0.5
1200	- 600	59.3			

Table V. Data From Computer For The Determination of Theoretical Vertical Intensity Anomalies (cont.)

DEPTH (cm.)	X (cm.)	Z (gammas)	DEPTH (cm.)	X (cm.)	Z (gammas)
1500	-1500	16.5	1700	- 500	23.9
1500	-1400	18.3	1700	- 400	23.1
1500	-1300	20.3	1700	- 300	21.8
1500	-1200	22.4	1700	- 200	19.7
1500	-1100	24.6	1700	- 100	16.9
1500	-1000	26.9	1700	0	13.4
1500	- 900	29.1	1700	+ 100	9.4
1500	- 800	31.1	1700	+ 200	4.9
1500	- 700	32.8	1700	+ 300	0.2
1500	- 600	34.1	1800	-1500	13.0
1500	- 500	34.6	1800	-1400	14.0
1500	- 400	34.3	1800	-1300	15.1
1500	- 300	32.9	1800	-1200	16.2
1500	- 200	30.3	1800	-1100	17.3
1500	- 100	26.4	1800	-1000	18.3
1500	0	21.3	1800	- 900	19.1
1500	+ 100	15.0	1800	- 800	19.8
1500	+ 200	7.9	1800	- 700	20.2
1500	+ 300	0.4	1800	- 600	20.3
1600	-1500	15.3	1800	- 500	20.0
1600	-1400	16.8	1800	- 400	19.2
1600	-1300	18.5	1800	- 300	17.9
1600	-1200	20.2	1800	- 200	16.1
1600	-1100	21.9	1800	- 100	13.7
1600	-1000	23.6	1800	0	10.8
1600	- 900	25.3	1800	+ 100	7.5
1600	- 800	26.7	1800	+ 200	3.9
1600	- 700	27.8	1800	+ 300	0.2
1600	- 600	28.5	1900	-1500	11.9
1600	- 500	28.7	1900	-1400	12.8
1600	- 400	28.1	1900	-1300	13.6
1600	- 300	26.7	1900	-1200	14.5
1600	- 200	24.3	1900	-1100	15.3
1600	- 100	21.0	1900	-1000	16.1
1600	0	16.8	1900	- 900	16.7
1600	+ 100	11.8	1900	- 800	17.1
1600	+ 200	6.2	1900	- 700	17.3
1600	+ 300	0.3	1900	- 600	17.2
1700	-1500	14.1	1900	- 500	16.8
1700	-1400	15.4	1900	- 400	16.1
1700	-1300	16.7	1900	- 300	14.9
1700	-1200	18.1	1900	- 200	13.2
1700	-1100	19.5	1900	- 100	11.2
1700	-1000	20.1	1900	0	8.8
1700	- 900	22.0	1900	+ 100	6.1
1700	- 800	23.0	1900	+ 200	3.2
1700	- 700	23.7	1900	+ 300	0.2
1700	- 600	24.0			

Table V. Data From Computer For The Determination of
Theoretical Vertical Intensity Anomalies (cont.)

DEPTH (cm.)	X (cm.)	Z (gammas)	DEPTH (cm.)	X (cm.)	Z (gammas)
2000	-1500	10.9	2200	- 500	10.4
2000	-1400	11.6	2200	- 400	9.7
2000	-1300	12.3	2200	- 300	8.8
2000	-1200	13.0	2200	- 200	7.8
2000	-1100	13.6	2200	- 100	6.5
2000	-1000	14.2	2200	0	5.1
2000	- 900	14.6	2200	+ 100	3.5
2000	- 800	14.8	2200	+ 200	1.8
2000	- 700	14.9	2200	+ 300	0.1
2000	- 600	14.7	2300	-1500	8.3
2000	- 500	14.3	2300	-1400	8.7
2000	- 400	13.5	2300	-1300	9.0
2000	- 300	12.4	2300	-1200	9.3
2000	- 200	11.0	2300	-1100	9.6
2000	- 100	9.3	2300	-1000	9.8
2000	0	7.3	2300	- 900	9.8
2000	+ 100	5.0	2300	- 800	9.8
2000	+ 200	2.6	2300	- 700	9.7
2000	+ 300	0.1	2300	- 600	9.4
2100	-1500	9.9	2300	- 500	8.9
2100	-1400	10.5	2300	- 400	8.3
2100	-1300	11.1	2300	- 300	7.5
2100	-1200	11.6	2300	- 200	6.6
2100	-1100	12.1	2300	- 100	5.5
2100	-1000	12.5	2300	0	4.3
2100	- 900	12.8	2300	+ 100	2.9
2100	- 800	12.9	2300	+ 200	1.5
2100	- 700	12.8	2300	+ 300	0.1
2100	- 600	12.6	2400	-1500	7.5
2100	- 500	12.1	2400	-1400	7.8
2100	- 400	11.4	2400	-1300	8.1
2100	- 300	10.4	2400	-1200	8.4
2100	- 200	9.2	2400	-1100	8.5
2100	- 100	7.7	2400	-1000	8.7
2100	0	6.0	2400	- 900	8.7
2100	+ 100	4.2	2400	- 800	8.6
2100	+ 200	2.1	2400	- 700	8.4
2100	+ 300	0.1	2400	- 600	8.1
2200	-1500	9.1	2400	- 500	7.7
2200	-1400	9.5	2400	- 400	7.2
2200	-1300	10.0	2400	- 300	6.5
2200	-1200	10.4	2400	- 200	5.6
2200	-1100	10.8	2400	- 100	4.7
2200	-1000	11.0	2400	0	3.6
2200	- 900	11.2	2400	+ 100	2.5
2200	- 800	11.2	2400	+ 200	1.3
2200	- 700	11.1	2400	+ 300	0.1
2200	- 600	10.8			

Table V. Data From Computer For The Determination of
Theoretical Vertical Intensity Anomalies (cont.)

DEPTH (cm.)	X (cm.)	Z (gammas)	DEPTH (cm.)	X (cm.)	Z (gammas)
2500	-1500	6.9	2700	- 500	5.1
2500	-1400	7.1	2700	- 400	4.7
2500	-1300	7.3	2700	- 300	4.2
2500	-1200	7.5	2700	- 200	3.6
2500	-1100	7.6	2700	- 100	3.0
2500	-1000	7.7	2700	0	2.3
2500	- 900	7.7	2700	+ 100	1.6
2500	- 800	7.6	2700	+ 200	0.8
2500	- 700	7.4	2700	+ 300	0.0
2500	- 600	7.1	2800	-1500	5.2
2500	- 500	6.7	2800	-1400	5.3
2500	- 400	6.2	2800	-1300	5.4
2500	- 300	5.6	2800	-1200	5.5
2500	- 200	4.8	2800	-1100	5.5
2500	- 100	4.0	2800	-1000	5.5
2500	0	3.1	2800	- 900	5.4
2500	+ 100	2.1	2800	- 800	5.2
2500	+ 200	1.1	2800	- 700	5.1
2500	+ 300	0.1	2800	- 600	4.8
2600	-1500	6.3	2800	- 500	4.5
2600	-1400	6.5	2800	- 400	4.1
2600	-1300	6.6	2800	- 300	3.7
2600	-1200	6.7	2800	- 200	3.2
2600	-1100	6.8	2800	- 100	2.6
2600	-1000	6.8	2800	0	2.0
2600	- 900	6.8	2800	+ 100	1.4
2600	- 800	6.7	2800	+ 200	0.7
2600	- 700	6.5	2800	+ 300	0.0
2600	- 600	6.2	2900	-1500	4.8
2600	- 500	5.8	2900	-1400	4.9
2600	- 400	5.4	2900	-1300	4.9
2600	- 300	4.8	2900	-1200	4.9
2600	- 200	4.2	2900	-1100	4.9
2600	- 100	3.5	2900	-1000	4.9
2600	0	2.7	2900	- 900	4.8
2600	+ 100	1.8	2900	- 800	4.7
2600	+ 200	0.9	2900	- 700	4.5
2600	+ 300	0.0	2900	- 600	4.2
2700	-1500	5.7	2900	- 500	3.9
2700	-1400	5.9	2900	- 400	3.6
2700	-1300	6.0	2900	- 300	3.2
2700	-1200	6.1	2900	- 200	2.8
2700	-1100	6.1	2900	- 100	2.3
2700	-1000	6.1	2900	0	1.7
2700	- 900	6.0	2900	+ 100	1.2
2700	- 800	5.9	2900	+ 200	0.6
2700	- 700	5.7	2900	+ 300	0.0
2700	- 600	5.4			

Table V. Data From Computer For The Determination of
Theoretical Vertical Intensity Anomalies (cont.)

DEPTH (cm.)	X (cm.)	Z (gammas)
3000	-1500	4.4
3000	-1400	4.4
3000	-1300	4.5
3000	-1200	4.5
3000	-1100	4.5
3000	-1000	4.4
3000	- 900	4.3
3000	- 800	4.2
3000	- 700	4.0
3000	- 600	3.8
3000	- 500	3.5
3000	- 400	3.2
3000	- 300	2.8
3000	- 200	2.4
3000	- 100	2.0
3000	0	1.5
3000	+ 100	1.0
3000	+ 200	0.5
3000	+ 300	0.0

X = distance along the horizontal axis
measured from a point directly
over the (-) pole

Z = vertical intensity

APPENDIX C
FIELD DATA FOR POLE STRENGTH DETERMINATION

APPENDIX D
DATA FOR 'HYSTERESIS' CURVE

APPENDIX E
FIELD TEST AT SCHOOLHOUSE CAVE

IX. VITA

Anthony Del Prete, Jr. was born May 6, 1935, in Brooklyn, New York, the son of Anthony and Josephine Del Prete. He received his elementary and high school education in St. Albans and Amityville, respectively.

In September 1952 he enrolled at Hofstra College. In November 1954 he entered the United States Army and attended radio operator's school. After a tour in Korea, he was discharged from active service in November, 1956.

In January, 1957 he returned to Hofstra College and in September he transferred to the Missouri School of Mines and Metallurgy. He completed the requirements for a B.S. degree in Geology, in May 1960. In September he entered the University of Utah as a graduate student in geophysics with a Pan American Fellowship. In July 1961 he went to work as a geophysicist with the U. S. Coast and Geodetic Survey.

On June 27, 1962 he was united in marriage to Patricia Ann Knight of Salt Lake City, Utah.

In September 1962 he returned to the Missouri School of Mines and Metallurgy. He is a member of Sigma Gamma Epsilon and the American Geophysical Union.