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An investigation of the acceleration of neutrons

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AN INVESTIGATION OF THE ACCELERATION OF NEUTRONS

BY

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A

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ABSTRACT

The acceleration of neutrons via their magnetic dipole moment in an axially symmetric magnetic field is considered. Conditions for bunching are derived and expressions for the off-axis forces are developed. It was found that neutrons off axis are forced back onto the axis and that neutrons slightly depolarized remain in their initial state. A calculation of the acceleration is made using the field of a current loop and the conclusion is that under present laboratory capabilities a neutron accelerator is not feasible.

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I. INTRODUCTION

The subject of this thesis is the theoretical investigation of the acceleration of neutrons via their magnetic dipole moment in the magnetic field of a thin, wire-wound solenoid. A magnetic field will be assumed to be directed along the principle axis of the solenoid. The conditions for the clustering of a polarized packet of neutrons will be shown and the state of neutrons which are slightly depolarized and of neutrons which are off the principle axis will be discussed.

The present sources of energetic neutrons include radioactive materials, charged-particle accelerators, nuclear reactors, and cosmic radiation.

The radioactive sources yield neutrons in the energy range of 0.5 Mev to 10 Mev. They generally consist of an alpha emitter and beryllium and the reaction is an (α,n) type.

The range of neutron energies that may be obtained from charged-particle accelerators is restricted to the upper limit of energy attainable by the accelerated particles. Two types of interactions are used in obtaining neutrons from accelerators. One is a direct transfer of energy by bombarding a neutron emitter and the other is by a stripping reaction. The stripping process is characterized by a (d,n) reaction.

The nuclear reactor is the best source of thermal neutrons. The neutron fluxes now readily attained from reactors are of the order of 10^{11} neutrons per square centimeter per second.

Cosmic radiation is the only source of neutrons in the upper Bev energy regions. The flux of neutrons in this energy region is, however, very limited.

The uses of neutrons include the measurements of cross-sections of nuclei, studies of the structures of crystals, elementary particle research, and biological and medical studies. A neutron accelerator would have two advantages over the other sources of neutrons. One, it would afford very monoenergetic beams of neutrons, and two, a continuous spectrum of neutron energies would be available for experimentation.

II. REVIEW OF THE LITERATURE

In a review of the literature, the author was unable to find any material that deals directly with the acceleration of neutrons. However, it was found that much effort is now being concentrated on high magnetic field research and on high energy accelerators.

H. P. Furth and R. W. Waniek[1] did experiments involving pulsed magnetic fields in the half-megagauss region and showed them to be practical. They developed some basic concepts regarding the construction of impact resistant solenoids and the selection of the proper power supply. S. Foner and H. H. Kolm[2] discussed a pulsed-field system of 750 kilogauss. The coil consisted of beryllium-copper helical winding having an inside diameter of 3/16 inch and a length of 1/2 inch. The power is supplied by a bank of surge capacitors. H. P. Furth, M. A. Levine, and R. W. Waniek[3] discussed high magnetic energy densities in coils of various geometries. 1.6 megagauss field pulses were produced in single turn coils. They observed that mechanical and thermal limitations apply to coils which depend only on the strength of materials. In an article by C. Fowler, W. B. Garn, and R. C. Caird[4], the production of very high magnetic fields using the technique of implosion is discussed. Magnetic fields are produced in the 10 to 15 megagauss range by surrounding the region of flux with a conducting shell and packing a high explosive around the shell. The explosion compressed the flux from initial fields of approximately 100 kilogauss. The fields occupied cylindrical volumes and were essentially axial. A paper by Wayne Garrett[5] discussed current carrying, wire-wound systems possessing an axis of revolution. An extensive use of the potential field expansion in Legendre polynomials is discussed for such systems. J. R. Barber[6] in his paper on magnetic fields inside a solenoid gave the most useful formulae together with numerical data for the calculations of field strengths for thin and thick solenoids.

A very good reference for the research accomplished in the study of high magnetic fields is the Proceedings of the International Conference on High Magnetic Fields held in 1961.[7] The papers given fall in the categories of the generation of high magnetic fields, the programs being conducted in high-magnetic field research, solid state and low temperature physics, and plasma and fusion physics in high magnetic fields. A standard reference for electromagnetic fields is a book by James C. Maxwell.[8] Field calculations are performed for coils of many geometries. An extensive treatment of magnetic fields in solenoids is given in the book by H. B. Dwight.[9] Many numerical problems based on practical cases are given throughout the book. Formulae are given for the axial magnetic fields and for radial components of fields of both thin and thick coils in a non-magnetic medium.

The Proceedings of the International Conference of High Energy Accelerators[10] gives a good up-to-date review of the research and development of high energy linear charged particle accelerators. A study of this book would shed light on some of the problems one would most surely encounter if a neutron accelerator were being developed.

III. DISCUSSION

A. Theory

The potential energy, U, of a magnetic dipole, $\vec{\mu}$, in a magnetic field, $\vec{B}$, is:

$$U = -\vec{\mu}\cdot\vec{B}.$$

The force, $\vec{F}$, exerted on the dipole in this field is then found by the usual gradient operation:

$$\vec{F} = -\text{ grad } U$$

or

$$\vec{F} = \text{grad } (\vec{\mu}\cdot\vec{B}). \tag{1}$$

If the magnetic field is axially symmetric, the axis is taken to be the z-axis, and the magnetic dipole is the magnetic dipole moment of the neutron which is assumed to be polarized in the direction of the z-axis, equation (1) reduces to the simple form

$$\vec{F} = \mu \, (dB/dz)\cdot\hat{k}$$

where $\hat{k}$ is the unit vector in the positive z-direction. Relaxing the vector notation, the acceleration of the neutron, a, is:

$$a = (\mu/m)dB/dz \tag{2}$$

where m is the mass of the neutron.

If the field gradient were to be changing from point to point then it would be desired to have the field gradient to increase as a function of distance along the axis. This would mean that the current used to produce the field would be increasing as a function of axial distance. In this situation, the upper limit of attainable energies depends on the current capacity of the field producing medium.[3]

The other alternative is to allow the field gradient to be constant at the position of the neutron. Under this condition, the neutron will be accelerated at a constant rate. This may be done by accelerating the field-form at the same rate the neutron is accelerating. Thus, the neutron will see a constant field-gradient.

In the reference frame of the neutron, the magnetic field has the form

$$B = B(z).$$

The magnetic field in the laboratory frame, z', is related to that in the neutron rest frame, z, through $R = 1/2at^2$ if these two frames are coincident at $t = 0$, by the relation

$$B(z) = B(z' - R)$$

since $$z' = z + R$$

Therefore $$B(z) = B(z' - 1/2at^2).$$

In the remainder of this discussion, the work will be done in the reference frame in which the neutron moving with the acceleration as given by equation (4) is at rest. The z-axis of this frame is to coincide with the axis of revolution of the solenoid and the origin of this frame is chosen so that the field is symmetric about the plane perpendicular to the z-axis and passing through the origin.

B. Bunching

In order to achieve higher currents of neutrons and also to insure that the beam be monoenergetic, the accelerated field must be of such a form that neutrons spatially in advance of the neutrons at rest in the accelerated frame as well as the neutrons spatially following these neutrons are brought to rest.

Denote the acceleration of the neutrons located at $z_o + \Delta z$ as a^+ and at $z_o - \Delta z$ as a^-. Then:

$$a^+ = (\mu/m)dB/dz\Big|_{z_o + \Delta z},$$

$$a = (\mu/m)dB/dz\Big|_{z_o} = \text{constant}.$$

$$a^- = (\mu/m)dB/dz\Big|_{z_o - \Delta z}.$$

For the sake of clarity, let $f(z) = dB(z)/dz$.

Then $$a^+ - a = (\mu/m)\frac{f(z_o + \Delta_z) - f(z_o)}{\Delta z}\,\Delta z.$$

If Δz is small, then

$$a^{+} - a = (\mu/m)(\Delta f/\Delta z)\Delta z$$

or $$a^{+} = a + (\mu/m)(\Delta f/\Delta z)\Delta z.$$

Similarly $$a^{-} = a - (\mu/m)(\Delta f/\Delta z)\Delta z.$$

Since μ is the magnetic moment of the neutron which is a negative quantity and since bunching is desired, it will be required that the slope of the field gradient be positive. Under this condition

$$a^{+} < a < a^{-}$$

and bunching is insured.

C. Off-axis forces

The current density is zero in the evacuated region of the solenoid. As a result

$$\text{curl } \vec{B} = 0.$$

When this is true the magnetic field in such regions can be written as the gradient of a scalar potential, V, or

$$\vec{B} = -\mu_o \text{grad } V \quad (3)$$

where μ_o is the permeability of free space.

Expressions for the magnetic scalar potential along the axis of both thin and thick wire-wound solenoids are known.[9]

Assume that these potentials may be expanded in a Taylor series about the point

$$z_o = 0,$$

then the axial potential, V_{ax}, will become

$$V_{ax} = \sum_{n=o}^{\infty} (V_o^{(n)} / n!) z^n$$

where $V_o^{(n)}$ is the nth derivative of the potential evaluated at the point $z_o = 0$.

Since we have assumed that there is axial symmetry and that the function is analytic in the region of the axis, then the potential function may be expanded[11] in the series of zonal harmonics as

$$V(r,\theta) = \sum_{n=o}^{\infty} A_n r^n P_n(\cos\theta)$$

where $$P_n(\cos\theta) = (1/2^n . n!) \cdot \frac{d^n(\cos^2\theta - 1)^n}{d(\cos\theta)^n},$$

θ is the angle between the z-axis and r, and r is the distance from the point z_o to the field point. This series of zonal harmonics must reduce to the series representing the axial potential. Therefore, the coefficients, A_n, are

$$A_n = V_o^{(n)}/n! \quad .$$

Then

$$V(r,\theta) = \sum_{n=o}^{\infty} (V_o^{(n)}/n!)\ r^n\ P_n(\cos\theta). \qquad (4)$$

The r- and θ-components of the magnetic field, B_r and B_θ respectively, as calculated from equations (3) and (4) are

$$B_r = -\mu_o \frac{\partial}{\partial r} V = -\mu_o \sum_{n=0}^{\infty} (V_o^{(n)}/n!) n r^{n-1}\ P_n(\cos\theta) \qquad (5)$$

and

$$B_\theta = -\mu_o \frac{1}{r} \frac{\partial}{\partial \theta} V = \mu_o \sum_{n=o}^{\infty} (V_o^{(n)}/n!) r^{n-1} (\sin\theta) P_n'(\cos\theta) \qquad (6)$$

where the prime indicates differentiation with respect to the arguement of the Legendre polynomials or with respect to cos θ.

To find the force acting on the neutron which is polarized in the direction of the z-axis, revert back to the force expression

$$\vec{F} = \text{grad}\ (\vec{\mu}\cdot\vec{B}). \qquad (7)$$

Let $\vec{\mu} = \mu\hat{k}$

and $\vec{B} = B_r\ \hat{n} + B_\theta\ \hat{1}$

where $\hat{k}$, $\hat{n}$, and $\hat{i}$ are unit vectors in the increasing z-, r-, and Θ-directions respectively. Then

$$\vec{\mu}\cdot\vec{B} = \mu\hat{k}(B_r\,\hat{n} + B_\Theta\,\hat{i})$$

or

$$\vec{\mu}\cdot\vec{B} = \mu(\cos\Theta\cdot B_r - \sin\Theta\cdot B_\Theta)$$

From equations (5) and (6)

$$\vec{\mu}\cdot\vec{B} = -\mu\mu_o \sum_{n=o}^{\infty} (V_o^{(n)}/n!)\; r^{n-1}(n\cdot\cos\Theta\cdot P_n(\cos\Theta) + \sin^2\Theta\cdot P_n'(\cos\Theta))$$

From equation (2) of Appendix 1 the above equation reduces to

$$\vec{\mu}\cdot\vec{B} = -\mu\mu_o \sum_{n=o}^{\infty} (V_o^{(n)}/n!)\cdot n\, r^{n-1}\, P_{n-1}(\cos\Theta) \quad (8)$$

For the sake of analysis, the gradient operation of equation (7) should be carried out in rectangular coordinates, that is,

$$\text{grad} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\,.$$

Since the magnetic field is assumed to be symmetric about the z-axis, generality will not be lost if the neutron is assumed to be situated in the x-z plane. Then the gradient reduces to

$$\text{grad} = \hat{i}\frac{\partial}{\partial x} + \hat{k}\frac{\partial}{\partial z}\,.$$

The force will then be given as

$$\vec{F} = F_x \hat{i} + F_z \hat{k} = \hat{i} \frac{\partial}{\partial x}(\vec{\mu}\cdot\vec{B}) + \hat{k} \frac{\partial}{\partial z}(\vec{\mu}\cdot\vec{B})$$

or $$F_x = \frac{\partial}{\partial x} (\vec{\mu}\cdot\vec{B})$$

and $$F_z = \frac{\partial}{\partial z} (\vec{\mu}\cdot\vec{B}).$$

Carrying out the above operations using equations (8) and equations (4) and (5) of Appendix 1 results in

$$F_x = \mu\mu_o \sum_{n=o}^{\infty} V_o^{(n)}/n!)n\cdot r^{n-2} \sin \Theta\cdot P_{n-2}^{\prime}(\cos \Theta) \tag{9}$$

$$F_z = -\mu\mu_o \sum_{n=o}^{\infty} (V_o^{(n)}/n!)n(n-1)\cdot r^{n-2} P_{n-2}(\cos \Theta). \tag{10}$$

Since the assumption was made that the field is symmetric, then $V_o^{(2i)} = 0$ for $i = 1,2, \cdots$. Thus each series starts with the term $n = 3$ and the coefficients will have the same sign. If $V_o^{(3)}$ is positive, then F_z will represent a force in the positive z-direction as the magnetic moment of the neutron is a negative quantity. F_x is, then, a force tending to push neutrons back onto the axis. If $V_o^{(3)}$ is negative, then it would be required to polarize the neutrons in the opposite direction to result in a positive acceleration in the positive z-direction.

D. Neutron slightly depolarized.

One case yet to be considered is that of a neutron which is depolarized. In the real situation, perfect polarization of all the neutrons cannot be realized. However, most of the neutrons will be either polarized or slightly depolarized.

The Hamiltonian for this system will be given as

$$H = g\beta\vec{S}\cdot\vec{B} \qquad (11)$$

where g is the g-factor for the neutron (g = 1.91315), β is the nuclear magneton and $\vec{S}$ is the spin. If the assumption is made that the spin is quantized in the z-direction, then the z-component of the magnetic field, B_z, does not contribute to a transition since its coefficient, S_z, the spin in the z-direction, has only diagonal components. The x-component of the field, B_x, will contribute energy for a possible transition as its coefficient, S_x, has only off diagonal elements. To first order

$$B_x = B_{xo} + (\partial B_x/\partial x)\Delta x + (\partial B_x/\partial z)\Delta z. \qquad (12)$$

Now $$\Delta x = V_x \Delta t \qquad (13)$$

and $$\Delta z = V_z \Delta t$$

where V_x and V_z are the x- and z-components of the velocity and t is the time. The z-component of the velocity will be negligible; therefore, the second term in equation (12) may be dropped. This introduces a time dependence into the

Hamiltonian. If the change in the Hamiltonian is very large compared to the Bohr period, $T_{n,m}$, between the states n and m, the neutron may undergo a transition. To be sure that a transition will not be likely to occur, the criterion that

$$\left|(\partial H/\partial t)_{n,m}\cdot T_{n,m}\right| << \left|E_n - E_m\right| \tag{14}$$

where $(\partial H/\partial t)_{n,m}$ is the matrix element of the time rate of change of the Hamiltonian between the states n and m and $E_n - E_m$ is the difference in energy between these states and is given by

$$E_n - E_m = \pm g\beta B_z. \tag{15}$$

Combining equations (11), (12), and (13) and taking the time derivative of the Hamiltonian results in

$$(\partial H/\partial t) = g\beta S_x(\partial B_x/\partial_x)\cdot v_x. \tag{16}$$

Substituting equations (15) and (16) into equation (14)

$$\left|g\beta S_x(\partial B_x/\partial x)\cdot\Delta v_x\cdot\hbar\right| << \left|g\beta B_z\right|^2$$

where $\hbar$ is Plank's constant divided by 2π. In cgs units, $\hbar \sim 10^{-27}$ erg-seconds, $g\beta \sim 10^{-23}$ erg/gauss and assume $v_x \sim 10^3$ centimeter/second, then

$$\left|\Delta B_x/\Delta x\right| \text{ X } 10^{-1} << \left|B_z^{\,2}\right|. \tag{17}$$

Looking at this criterion and using reasonable values for the field and slope to be expected shows that it is not likely that neutrons which are slightly depolarized would undergo a transition. This argument is based on the result of the adiabatic approximation.[12]

E. Application

As an application of the preceeding sections, consider the accelerator as a series of single loops of wire, all of radius b, placed side by side in a coaxial fashion to form a very long solenoid. A current, I, will be sent through each loop in such a manner that each loop will be activated at the rate of a as given by equation (2). That is, if the loop located at z' in the laboratory frame of reference is activated at time t = 0, then the loop at R will be activated at a time $t = (2R/a)^{1/2}$ later.

The magnetic scalar potential, V_{ax}, along the z-axis for a current loop is [13]

$$V_{ax} = 1/2I(1 - z(z^2 + b^2)^{-1/2}).$$

From this

$$V_o^{(0)} = 1/2I$$

$$V_o^{(1)} = -1/2I(1/b)$$

$$V_o^{(3)} = 1/2I(3!/2b^3)$$

$$V_o^{(2i)} = 0 \qquad \text{for } i = 1,2,3 \cdots$$

$$V_o^{(5)} = -1/2I(3 \cdot 5!/8b^5)$$

$$V_o^{(7)} = 1/2I(5 \cdot 7!/16b^7)$$

Taking the origin of the accelerating frame to be on the axis of the solenoid and in the plane of the loop that at that instant has the current passing through it, the force has a maximum value at $z = 1/2b$. The slope has a positive value in the region

$$1/2b < z < \infty$$

This is the range of values where equation (2) should be evaluated so that bunching will be insured.

Looking at the equations for the forces places another restriction on the point at which equation (2) is to be evaluated. The series will converge if $z \leq b$. Therefore, the range of values for z should become

$$1/2b < z < b. \qquad (18)$$

Also, it is more advantageous to use this range of values for z as for $z > b$, the slope is less at all points than the slope for the range of z as given in equation (18). The series representing the force components off the z-axis could be expanded using the proper coefficients as found from the magnetic scalar potential for the loop but the series does not converge fast enough to make a good approximation as to the magnitude of these forces. Qualitatively, however, the force off-axis is a force tending to push the neutrons towards the axis.

To get an idea of the magnitude of the acceleration that this type of accelerator can produce, assume that each loop is activated with 20 amperes of current. If equation (2) is evaluated at $z = (6/10)b$ and the radius of the loop is 1 centimeter, then the acceleration is

$$a = 60 \text{ centimeters/second}^2.$$

Using these values as a check on equation (17), the ratio of the slope to the square of the field is approximately 7×10^{-3} which is much less than one.

IV. CONCLUSIONS

The magnitude of the acceleration arrived at in the previous section indicates that this type of accelerator is not practical. Present laboratory solenoids can continuously sustain field gradients of the order of 200 kilogauss per meter.[2] With such field gradients, the acceleration of the neutron is of the order of 200 meters/second^2. If a neutron were accelerated from rest at this constant rate, it would require a distance of 2.2×10^{12} meters (1.4×10^{8} miles) to reach the velocity of one-tenth the speed of light which corresponds to about 5 Mev.

Assume that the accelerator were to be of a circular type. Let the radius of the accelerator be 6.4×10^{6} meters (the radius of the earth) and if the neutron were to have a velocity of one-tenth the velocity of light, then the field-gradient required to keep the neutron in orbit would have to have a magnetude of 1.4×10^{13} gauss/meter.

If a distance of 1 kilometer is used as an upper limit for the length of a linear accelerator and again desiring to achieve a 5 Mev neutron, the acceleration, neglecting relativity, would be 4.5×10^{11} meters/second^2. This requires a field gradient to be of the order of magnetude of 5×10^{14} gauss/meter. This requirement is a billion times larger than present laboratory capabilities.

With results such as these, it is readily evident that this principle of acceleration for neutrons will not be practicable in the near future.

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APPENDIX 1

Useful relationships between Legendre Polynomials.

$$u = \cos \Theta = z/r$$

(1) $P'_{n+1}(u) - u \cdot P'_n(u) = (n+1)P_n(u)$

(2) $(u^2 - 1)P'_n(u) - n \cdot u \cdot P_n(u) = n \cdot P_{n-1}(u)$

(3) $u \cdot P'_n(u) - P'_{n-1}(u) = n \cdot P_n(u)$

(4) $\partial/\partial x(r^n \cdot P_n(u)) = -(\sin \Theta) \cdot r^{n-1} P'_{n-1}(u)$

(5) $\partial/\partial z(r^n \cdot P_n(u)) = n \cdot r^{n-1} P^{o}_{n-1}(u)$

VITA

The author was born on March 26, 1940 in Ladd, Illinois. He received his primary education at Ladd Community Consolidated School and his secondary education at St. Bede Academy, Peru, Illinois. He attended Illinois Institute of Technology in Chicago, Illinois and was granted a Bachelor of Science Degree in Physics in January, 1963. He has been enrolled in the graduate school of the University of Missouri School of Mines and Metallurgy since September, 1962 and has held the position of Graduate Assistant in Physics during the period.