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THE MAGNETIC MONOPOLE

BY

THOMAS E. KEMPLE

---

AN

ABSTRACT

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the

Degree of

MASTER OF SCIENCE

Rolla, Missouri

1961

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## THE MAGNETIC MONOPOLE

Since Dirac has shown that the concept of a magnetic monopole follows from the fundamental laws of quantum mechanics and that it imposes quantization of electric charge, one has reason for taking the magnetic monopole seriously. Such particles, if they exist, would also give symmetry to Maxwell's equations.

A comprehensive review of the literature is presented and all known properties of magnetic poles are discussed. Scattering of monopoles by heavy nuclei is analyzed in detail and definite mathematical relations are established. All previous experimental work known is outlined and discussed. Definite criteria for identification of monopole tracks in a Wilson cloud chamber are determined. Data from a limited search with this apparatus is presented. No monopoles appeared in a body of photographs containing approximately 1600 tracks from cosmic radiation and natural radioactivity.

It is concluded that if monopoles exist, they are either very rare or they are in a form in which they cannot be easily detected.

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Approved by

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S. J. Pagan Ralph E. Lee



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## CHAPTER I

### STATEMENT OF THE PROBLEM

1. Introduction. Within recent years many new particles have been discovered. All of these particles are either neutral or possess electric charge. It is reasonable to ask if isolated magnetic poles also exist in nature. Prior to Dirac's work very little attention was given to this matter. In 1931 Dirac<sup>1</sup> published a paper in which he presented a convincing theoretical argument for belief in magnetic monopoles. In 1948 he published another article<sup>2</sup> giving a more refined treatment of his previous work. Since Dirac's first publication several other investigators have worked on the problem both from a theoretical and experimental viewpoint and research is currently being done. The present paper is concerned with a search which has been conducted by the author.

2. Definition. Before delving further into this topic it is well to define precisely what we mean by a magnetic monopole. It is a hypothetical particle of unknown mass or dimensions which possesses magnetism of only one polarity, north or south. We shall speak of such particles as possessing magnetic charge. Throughout the remainder of the discussion we shall refer to these particles as magnetic monopoles or simply as monopoles.

3. Theoretical Consideration. There exists some theoretical grounds for belief in the existence of monopoles. We will consider a few of these in the paragraphs that follow.

a. Charge Quantization. Dirac has shown that "Quantum Mechanics does not preclude the existence of isolated magnetic poles", but "when developed naturally without the imposition of arbitrary assumptions leads inevitably to wave equations whose only physical interpretation is the motion of an electron in the field of a single pole".<sup>3</sup> He further shows that the wave equations which he sets up to describe this motion do not exist unless the electronic charge is quantized.<sup>4</sup> At present this appears to be the best explanation for the quantization of electronic charge. There has been at least one other theory advanced since Dirac's publications which infers charge quantization. On the basis of a model of a charged sphere containing electrons, Neugebauer<sup>5</sup> shows that by varying the value of the elementary charge, the actually observed value of this constant is energetically the most favorable one.

b. Symmetry of Maxwell's Equations. If Maxwell's equations could be written in symmetric form, they would appear as follows:

$$\nabla \cdot \bar{E} = 4\pi\rho \quad \nabla \cdot \bar{H} = 4\pi\sigma$$

$$\nabla \times \bar{E} = -\frac{1}{c} \frac{\partial \bar{H}}{\partial t} - \frac{4\pi}{c} \sigma \bar{w}$$

$$\nabla \times \bar{H} = \frac{1}{c} \frac{\partial \bar{E}}{\partial t} + \frac{4\pi}{c} \rho \bar{v}$$

where  $\sigma$  is the magnetic pole density,  $\sigma \bar{w}$  is the magnetic current density, and the other symbols have their usual meanings. It is obvious that symmetry can be achieved only by introducing free magnetic poles. This kind of symmetry has been extended to other electrodynamic equations, and it appears that there are no difficulties.



c. Possibilities for Pole Pair Production. The energy required for pole pair production is  $m_g c^2$ , where  $m_g$  is the rest mass. Assuming the mass to be of the order of magnitude of the proton mass, this would amount to about one billion electron volts. Energies of this order are available in cosmic radiation. For sufficiently high energies, the cross-section for the pair production of monopoles in the field of a nucleus is given by Bauer<sup>6</sup> to be  $\frac{\pi Z^2 r_g^2}{137}$ , where  $r_g$ , the monopole radius is  $\frac{g^2}{2m_g c^2}$ . The cross-section for an annihilation reaction is  $\pi r_g^2$ .

Assuming the monopole radius to be about the same as the electron radius. These cross-sections are approximately the same as for the corresponding electron positron reactions.

d. Other Considerations. Dirac<sup>7</sup> has suggested that the reason monopoles have not been observed may be that they are bound together in pairs by their large attractive force. This force would be about 4700 times as great as the corresponding force between the electron and the proton. The binding energy was estimated by Dirac to be of the order of  $5 \times 10^8$  electron volts. He suggested looking in atomic processes where energies of this magnitude are available.

Saha and Konthari<sup>8</sup> have attempted to explain the neutron mass in terms of monopoles. According to their hypothesis the neutron is a dipole composed of two oppositely charged free magnetic poles ... When two magnetic poles combine to form a neutron, nearly eighty per cent of the energy is radiated away ..., hence it is almost impossible

to dissociate the neutron".<sup>9</sup> This could explain why isolated monopoles have not been found. There does not, however, appear to be any support given to this theory at present.

Porter<sup>10</sup> in a recent article has noted that extremely high energy particles are present in cosmic ray showers. Energies have been estimated to be as high as  $5 \times 10^{18}$  electron volts. It is difficult to explain energies of this magnitude on the basis of acceleration of protons and nuclei through the galaxy. Porter suggests that this phenomena can be explained by assuming that a small fraction of cosmic radiation consists of magnetic monopoles. Assuming the unit pole strength to be  $3.3 \times 10^{-8}$  in cgs units as calculated by Dirac, a monopole could be accelerated to these energies by interstellar magnetic fields. A flux of  $10^{-14}$  particles per square centimeter per second was deemed adequate to produce the effects observed. Intensities of this order would be very difficult to detect directly.

4. Scope of the Present Work. The present work is divided into three parts. They are as follows: (1) An exhaustive study of the literature to determine what theoretical and experimental work has been done and to find out what is known about the properties of a monopole. (2) Calculations of the trajectory and scattering of monopoles in the field of a heavy nucleus. (3) An experimental search for monopoles with a Wilson Cloud Chamber. The chapters that follow will be devoted to a discussion of the results of these efforts.

## CHAPTER II

## PREDICTED PROPERTIES OF MAGNETIC MONOPOLES

1. Magnetic Charge of the Monopole. Dirac<sup>11</sup> has shown, by means of a relativistic quantum mechanical treatment of the interaction of electric and magnetic charges in an electromagnetic field, that free magnetic poles are possible only if the pole strength in magnetic units is an integral multiple of the quantity  $\frac{hc}{4\pi e}$ , where  $e$  is the electronic charge,  $h$  is Planck's constant, and  $c$  is the velocity of light, all measured in cgs units. This means that the unit magnetic charge should be about  $\frac{137}{2}$  times as large as the electron charge.

Dirac's result can also be obtained from classical electromagnetic theory. Several years prior to Dirac's work J. J. Thompson<sup>12</sup> was able to show that the angular momentum of a charged particle and a monopole separated by a finite distance is independent of the distance of separation and is equal to  $\frac{eg}{c}$  where  $g$  is the pole strength in magnetic units. Saha<sup>13</sup> assumed that  $e$  and  $g$  have their elementary values when the angular momentum of the field is  $\frac{1}{2} \frac{h}{2\pi}$  which is the smallest possible value according to the quantum theory. Combining this with the Thompson result he obtains

$$eg = \frac{nhc}{4\pi} \quad (2-1)$$

which is the Dirac result.

A more refined treatment of this problem has been given by Eldridge.<sup>14</sup> He assumes that two unit magnetic poles of opposite charge are infinitely far apart, and a particle with an electronic charge is situated at a distance  $r$  from the line joining the two poles. The angular momentum associated with the cross products of the electric and magnetic field is:

$$\vec{L} = e\vec{r} \times \frac{\vec{A}}{c}$$

where  $\vec{A}$  is the vector potential, but

$$A = \frac{2g}{r}, \text{ therefore:}$$

$$L = \frac{2eg}{c}$$

The quantum expression for angular momentum used by Eldridge is:

$$L = \frac{nh}{2\pi}$$

Combining this with the preceding expression, we again obtain Dirac's result.

2. Mass of the Monopole. The mass of the monopole has not yet been estimated with any degree of certainty. Various authors have given different estimates, depending upon their viewpoint. Richardson<sup>15</sup> believed, from an argument based on classical ideas, that the monopole mass must be about 500 times the electron mass. Langer estimated the rest mass of the monopole to be about 4700 times that of the electron.<sup>16</sup> Bauer<sup>17</sup> assumed the latter value in making his calculations. He reasoned

that there were monopoles of electronic mass, they would be created in considerable numbers by pair production processes. If this were true, they would certainly have been detected because of their large radiative effects. Bradner and Isbell<sup>18</sup> gave a lower limit for the mass of the monopole as approximately equal to that of the  $\pi$  meson mass. They believed that poles lighter than this would have resulted in a noticeable change in the Lamb shift. No upper limit was set.

The mass of the monopole can be calculated directly from assumptions about its radius. Saha<sup>19</sup> assumes that the dimensions of the monopole are equal to those of the electron. This assumption is also made by Tuve<sup>20</sup> and by Bauer.<sup>21</sup> There is, admittedly, no firm justification for these assumptions and the mass estimates cannot be relied upon.

One form of the classical electron radius is given by:

$$r_e = \frac{e^2}{2m_e c^2}$$

By exact analogy the monopole radius would be:

$$r_g = \frac{g^2}{2m_g c^2}$$

If we combine these two relations, assuming that  $r_g = r_e$ , we find:

$$\frac{m_g}{m_e} = \frac{g^2}{e^2} \approx 4700 \quad (2-2)$$

If the radius of the monopole were appreciably less than the electron radius its mass would be much greater. Fundamental particles with masses of this order of magnitude are not known.

3. Magnetic Atoms. If monopoles exist it should be possible for them to be bound together by their attractive forces to form magnetic atoms. This possibility has been investigated by Richardson.<sup>22</sup> He calculated possible energy states for different types of magnetic atoms and found that only states corresponding to high quantum numbers are possible. For lower quantum numbers the wave equations are not well behaved. These atoms are very much smaller than ordinary atoms. Their radii were estimated to be of the order of  $10^{-14}$  to  $10^{-15}$  centimeters.

The frequencies of the spectral lines emitted are about  $10^{10}$  times as great as the corresponding frequencies of ordinary atoms. This work suggests another possible approach in searching for monopoles. It might be possible to identify magnetic atoms by means of spectral analysis.

Peculiarities of these results are due to the comparatively large value of the pole strength. There are some uncertainties including mass of the monopole and the extent of nuclear forces. Large differences in assumptions about the mass would not appreciably alter the model of the magnetic atom. However nuclear forces, if any, at such close ranges might have an appreciable effect.

4. Binding with Matter. It may also be possible for monopoles to form bound states with charged particles. The electron-monopole system has been investigated by Banderet.<sup>23</sup> He showed that an electron cannot be bound to a monopole in the absence of an electric field. Other possibilities were investigated by Malkus.<sup>24</sup> He calculates Eigenstructures for three separate cases.

a. An Atomic Nucleus in the Field of a Monopole. The monopole can be bound to the proton only if its mass is comparable to the proton



mass. Bound states with other atomic nuclei cannot exist.

b. An Electron in the Combined Field of a Monopole and an Atomic Nucleus Situated Close Together. The presence of a monopole near the nucleus of an atom increases the energies of the electrons. This is due to diamagnetic effects which cause the electrons to be repelled out to higher energy levels.

c. Considerations with the Monopole at some Distance from the Atomic Nucleus. The interaction between a monopole and a many-electron configuration is such that a monopole may be bound to matter with energies comparable to the chemical bond but not significantly greater.

The binding energy of a monopole in a magnetic material is considerably greater. This has been calculated by Goto.<sup>25</sup> He found that for paramagnetic materials, the binding energy is between 1.28 and 12.8 electron volts and for ferromagnetic materials, it is between 160 and 700 electron volts. The actual value depends upon the specific material under consideration. The probability of escape for a monopole trapped in a magnetic material is very small due to the large binding energy. For this reason, Goto asserted that magnetic materials are the most likely source for monopoles.

5. Scattering by Nuclei. A Rutherford scattering formula for monopoles in the field of heavy nuclei has been calculated by Bauer.<sup>26</sup> His result may be expressed as follows:

$$N = \frac{Qnd}{R^2} \left( \frac{Zeg}{2m_g Vc} \right)^2 \frac{1}{\sin^4 \frac{\psi}{2}} \quad (2-3)$$

where  $N$  is the number of monopoles striking a detection screen per square centimeter per second,  $Q$  is the number of monopoles per square centimeter per second in the incident beam,  $n$  is the number of atoms per cubic centimeter of the scattering material,  $d$  is the uniform thickness of scattering medium,  $R$  is the distance of the scattering medium from the detection screen,  $\psi$  is the scattering angle, and the other symbols have their usual meaning. This expression is somewhat similar to the Rutherford scattering formula for charged particles. Ruark<sup>27</sup> pointed out that Bauer's formula can be obtained from the Rutherford scattering formula for electrons merely by substituting  $\frac{gV}{c}$  for  $e$ .

A more detailed analysis of scattering by nuclei has been given by Ford and Wheeler<sup>28</sup> on a semiclassical basis. Their results show that scattering is more pronounced at certain definite angles, which are called rainbow angles. This is perhaps the most distinguishing feature of monopole scattering. The rainbow angles can also be calculated classically, and turn out to be independent of all parameters of the problem. The values of these angles are:  $\psi = 140.1^\circ, 156.7^\circ, 163.5^\circ, \dots$ . They form an infinite series, coming closer and closer together as  $\psi$  approaches  $\pi$ .

The problem of scattering will be analyzed in more detail in Chapter IV.

6. Radiation. Energy losses due to radiation have been calculated by Bauer.<sup>29</sup> The expression, he gives for the amount of energy lost per centimeter of track length is:

$$\frac{-dT_R}{dX} = \frac{n\pi^2}{2} \left( \frac{Zeg^2}{m_g c^2} \right)^2 \frac{\beta}{4\alpha} \quad (2-4)$$



where  $\alpha = \frac{2\pi e^2}{hc} = \frac{1}{137}$  and  $\beta = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$

$\beta$  becomes very large as the velocity ( $v$ ) of the monopole approaches the velocity of light  $c$ . Consequently radiation losses become very important in the relativistic region. For low velocities radiation effects are small only if the mass is large.

7. Energy Loss in Traveling through Matter. The energy loss of a monopole in traveling through matter has been calculated by Cole<sup>30</sup> and independently by Bauer.<sup>31</sup> Cole's work was on a classical basis, while Bauer obtained both the classical and the wave mechanical result for the sake of comparison. The results given by Cole and Bauer agree very closely. To avoid repetition we shall state only the results obtain by Cole. His expression for the mean energy loss per centimeter of track length is:

$$\frac{\Delta T}{\Delta X} = \frac{4\pi n g^2 e^2}{m_e c^2} \sum_{r=1}^n \ln \left( \frac{K^2 V^2 m^2 c}{f_r g e} \right) \quad (2-5)$$

where  $m$  is the reduced mass of the monopole and the electron,  $f_r$  is the natural frequency of the  $r$ th electron in its orbit,  $K = 1.61$ ,  $V$  is the initial velocity of the monopole, and the other symbols are as previously defined. Taking relativistic modifications into account, this expression becomes:

$$\frac{\Delta T}{\Delta X} = \frac{4\pi n g^2 e^2}{m_e c^2} \sum_{r=1}^n \ln \left( \frac{K^2 \beta^2 V^2 m^2 c}{f_r g e} \right) \quad (2-6)$$

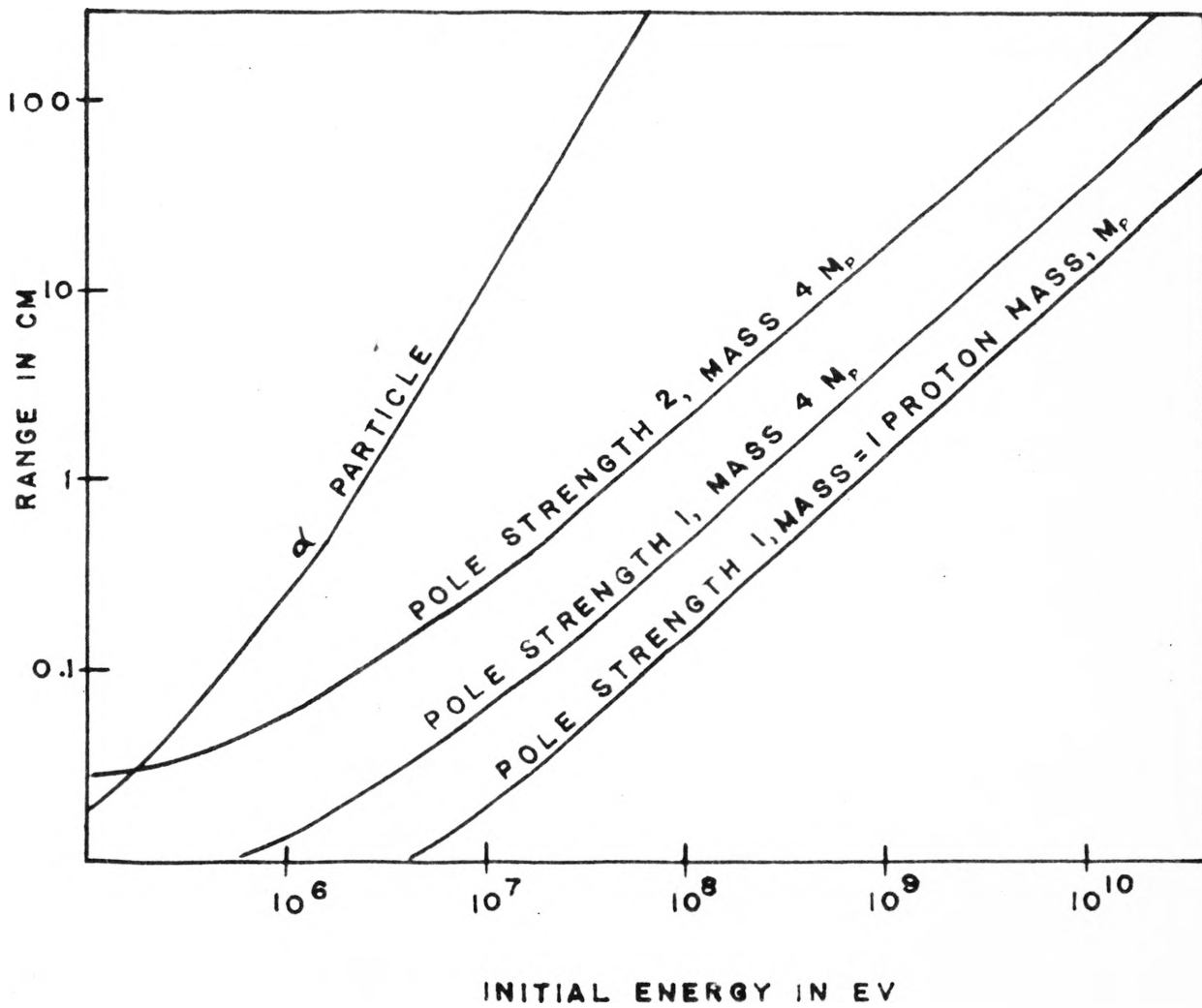


FIG. 1 RANGE IN AIR

8. Range. The range  $R$  can be calculated from the formula,  $R = \frac{T}{\frac{\Delta T}{\Delta X}}$

where  $T$  is the initial kinetic energy of the monopole. Figure 1 gives the range for monopoles with masses and pole strengths as indicated. The range of an alpha particle is shown for comparison. The range of the monopole is found to be comparatively small, especially in dense media. This introduces a difficulty in designing detection apparatus. Malkus<sup>32</sup> believed that the protective covering of most conventional apparatus is sufficient to stop a monopole.

9. Ionization. Ionization properties of a monopole have also been calculated by Cole.<sup>33</sup> The expression he gives for the number of ion pairs produced in traveling a distance  $\Delta X$  is:

$$\Delta I = \frac{2 \pi n g^2 e^2 \Delta X}{m_e W_1 c^2} \sum_r \ln \left( \frac{2m_e^2 v^2}{K^2 m_e W_r} \right) \quad (2-7)$$

where  $W_1$  is the ionization energy of the least bound electron,  $W_r$  is the ionization energy of the  $r$ th electron, and  $K = 0.618$ .

The essential features of ionization can be seen in Figure 2, where the ionization of poles with masses and pole strengths as indicated is compared with that of oxygen, carbon, and beryllium nuclei, and  $\alpha$  particles. Figure 3 is added to show the ionization pattern near the end of the path. Ionization mentioned in this section refers to total ionization.

A comparative analysis of the ionization properties of monopoles and charged particles were given by Fritz, Good, Kassner and Ruark<sup>34</sup> from considerations of the electric fields associated with these particles.

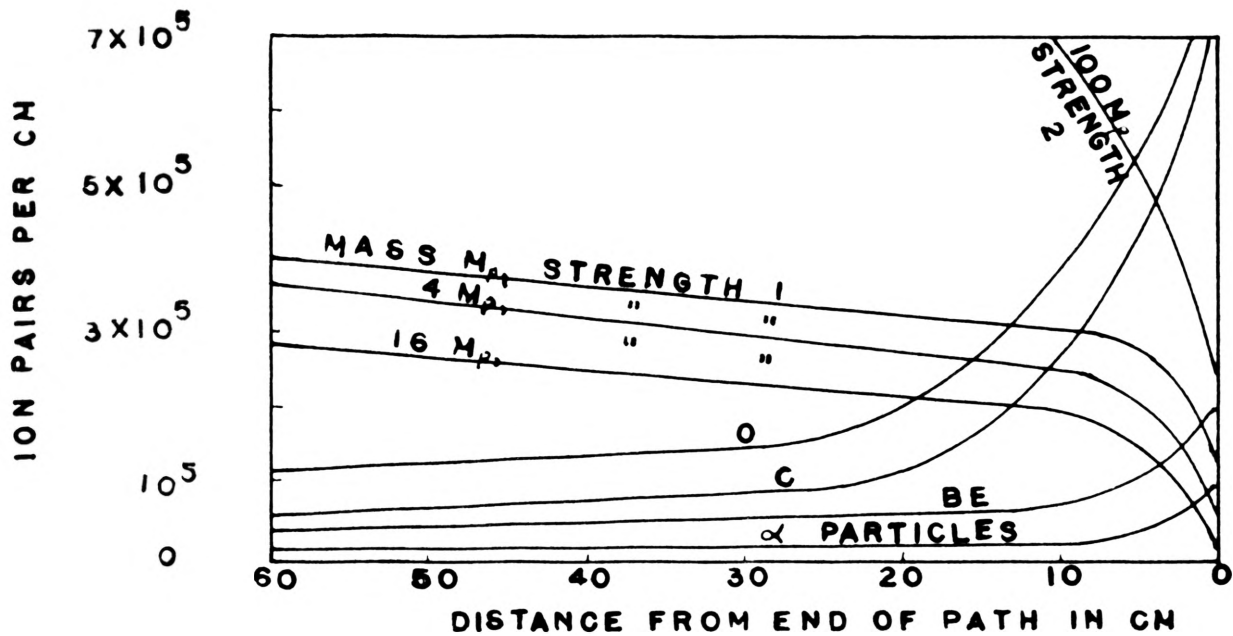


FIG. 2 IONIZATION OVER LONG RANGES OF PATH

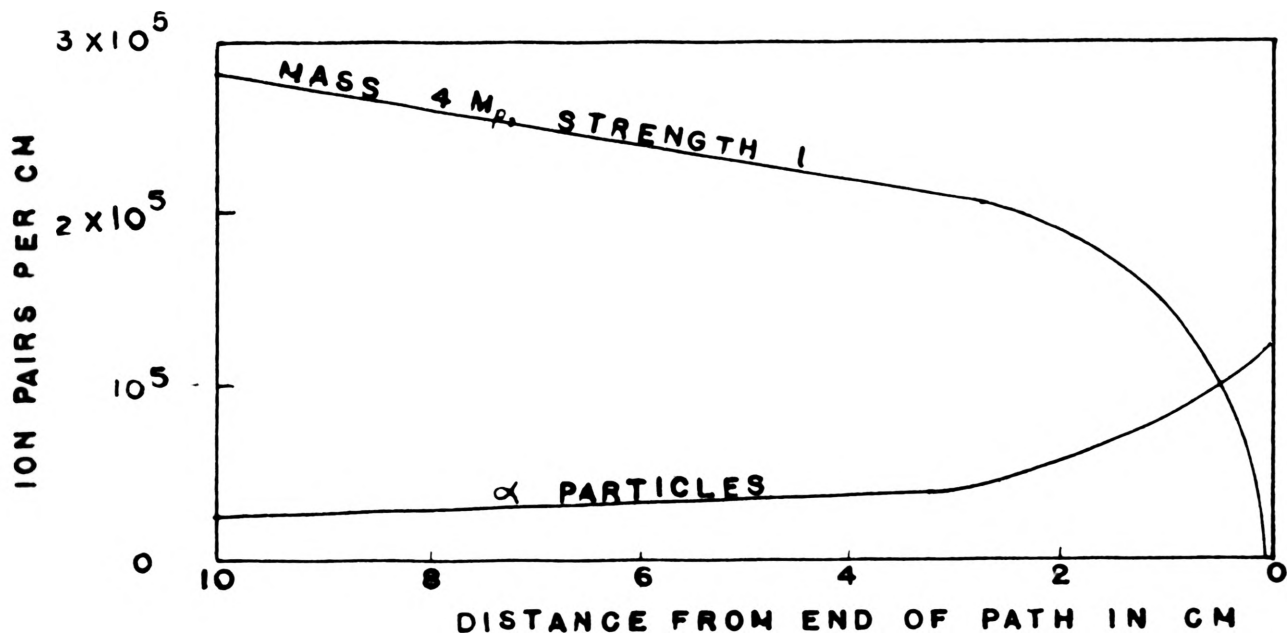


FIG. 3 IONIZATION NEAR END OF PATH

The electric field of a moving pole is given by  $|\bar{E}| = \frac{gV}{cr^2}$ . In the

relativistic region where  $\frac{V}{c} \approx 1$ , this reduces to  $|\bar{E}| = \frac{g}{r^2}$ . This

expression is the same as for a charged particle. Therefore, at relativistic velocities, the pattern of ionization due to a monopole is approximately the same as that due to a charged particle. The corresponding tracks differ only in intensity of ionization. The ratio of the ionization intensity of a monopole track compared to that of an electron track is  $\frac{g^2}{e^2}$ .

Below relativistic velocities, the situation is somewhat different. In the case of a charged particle, the ionization is inversely proportional to the square of the velocity and increases rapidly as the particle comes to rest. In the case of a monopole, the ionization is very intense, decreasing slowly along the path, but dropping off, rapidly to zero as the particle comes to rest. From the foregoing discussion it appears likely that, if monopoles exist, ionization properties will be important in their identification.

Bauer<sup>35</sup> has observed that the track of a monopole would be similar to that of a heavy nuclear fragment whose ionization pattern is due to recombination processes. Some tracks which have been explained on this basis, could possibly be due to free magnetic poles.

Criteria for determining the differences between these two types of tracks have been discussed by Katz and Parnell.<sup>36</sup> According to them, this tapering off of a heavy ion track cannot always be explained

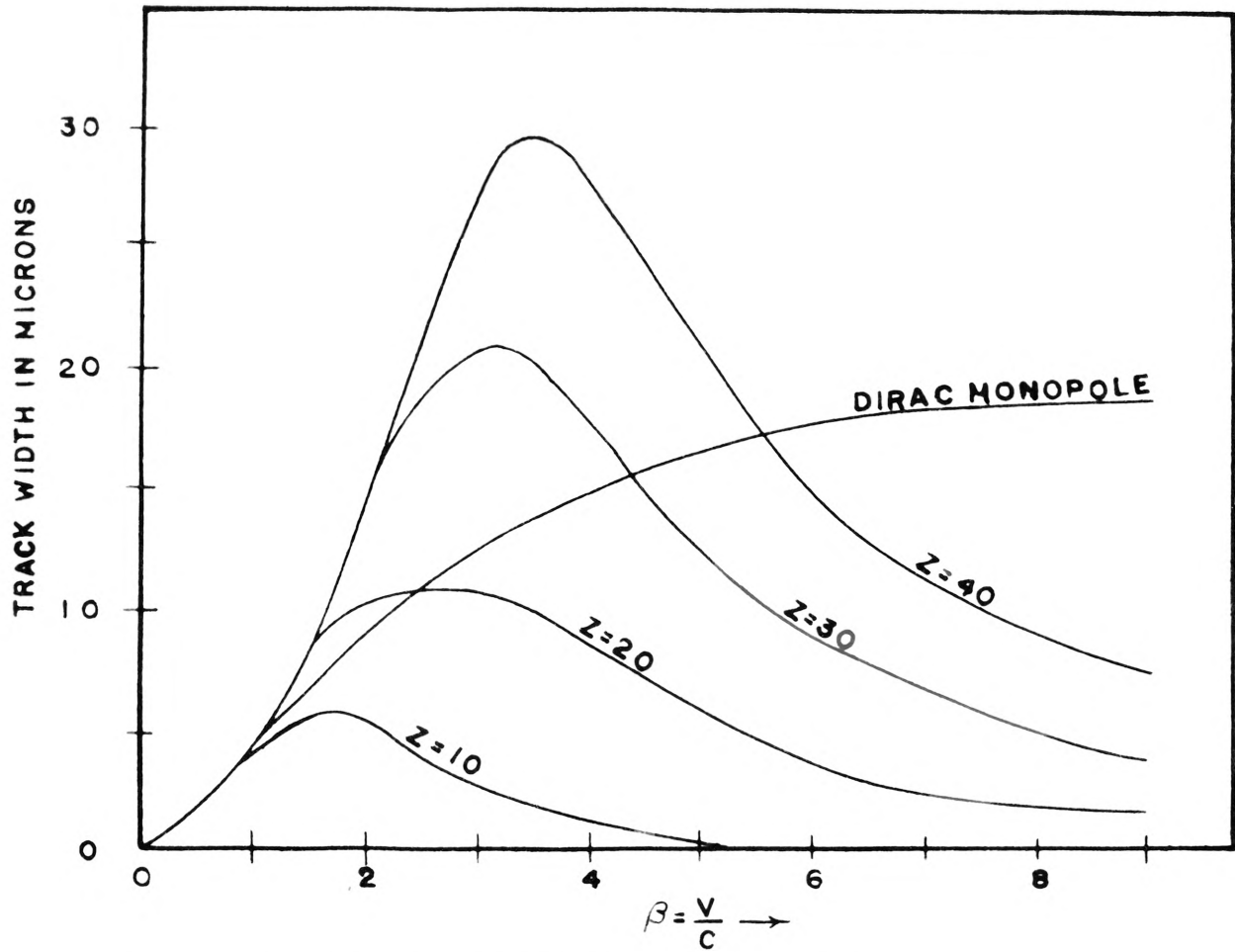


FIG. 4 TRACK WIDTH IN G-5 EMULSION AS A FUNCTION OF VELOCITY FOR NUCLEI COMPARED WITH THAT OF A MONOPOLE

by recombination processes. They refer to Lonchamp's<sup>37</sup> theory which ascribes track width to delta rays. At low velocities, delta rays have insufficient range to broaden a track and the track appears to thin down. Lonchamp's theory has been experimentally verified by himself and by Shjeggstad<sup>38</sup> from measurements of taper length and track width in this region. Katz and Parnell extended the theory to monopoles. By carrying out the calculation they are able to determine the exact way in which a monopole track tapers off toward the end. Figure 4 shows the track width of a monopole as a function of its velocity compared with that of certain heavy nuclei. Katz and Parnell give these very specific instructions to investigators hunting for monopoles: "Look for wedge-shaped tracks whose trunk achieves a thickness of about 15 microns and which are at least 500 microns long. Such a track may be the Dirac monopole".<sup>39</sup> These figures are for tracks formed in emulsions.

## CHAPTER III

### CLASSICAL SCATTERING THEORY FOR A MAGNETIC MONOPOLE IN THE FIELD OF AN ATOMIC NUCLEUS

In this section, we will attempt to determine the classical scattering properties of magnetic monopoles in the vicinity of a fixed charged particle. The author has decided to work the problem from a purely classical point of view. The details of the calculations presented in this section take a somewhat different form from that indicated in previous works.

The problem is to find the rainbow scattering angles and the Rutherford scattering formula for the scattering of a magnetic monopole in the field of a charged particle. In order to do this, the equation of motion of a monopole of strength  $g$  and mass  $m$  in the field of a fixed charged particle having a charge  $Ze$  must first be found. For simplicity radiation effects are assumed to be small enough to be neglected.

1. Derivations of the Equations of Motion. The monopole will initially be at a great distance from the charged particle approaching it with a velocity  $V$  and an initial impact parameter  $b$  as shown in Figure 5. As the magnetic monopole enters the field of the charged particle, it will experience a force given as

$$\bar{F} = m \lambda \frac{\bar{V} \times \bar{r}}{r^3}, \text{ where } \lambda = \frac{Zeg}{mc} \quad (3-1)$$

The bar above the symbol indicates that it is a vector quantity. It shall be assumed that the mass of the charged particle is large compared to the mass of the magnetic monopole.



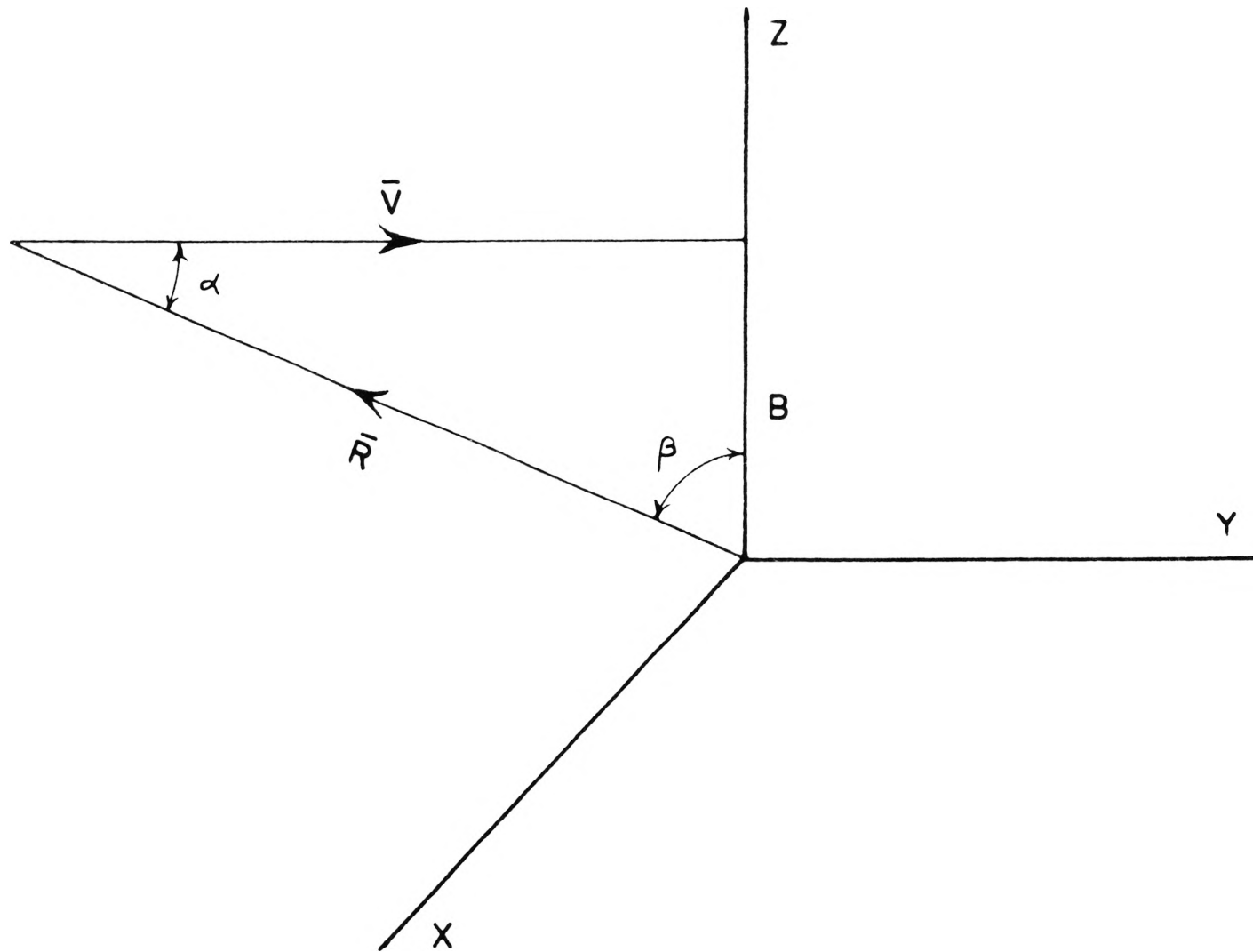


FIG 5 THE MONOPOLE A GREAT DISTANCE AWAY, APPROACHING A CHARGED PARTICLE SITUATED AT THE ORIGIN

Then, the acceleration experienced by the magnetic monopole takes the following form:

$$\frac{d^2 \bar{r}}{dt^2} = \frac{d\bar{v}}{dt} = \lambda \frac{\bar{v} \times \bar{r}}{r^3} \quad (3-2)$$

Our first impulse might be to break this equation up into its cartesian components and solve the resulting set of differential equations. These equations, however, appear to be very difficult to solve. Consequently, another method of approach here will be employed.

Before proceeding further, the conservation of angular momentum must be proved. The angular momentum is given by

$$\bar{L} = m \bar{r} \times \bar{v}$$

$$\bar{L} \cdot \bar{L} = L^2 = m^2 (\bar{r} \times \bar{v}) \cdot (\bar{r} \times \bar{v})$$

$$\frac{d}{dt}(L^2) = 2m^2 (\bar{r} \times \bar{v}) \cdot \frac{d}{dt} (\bar{r} \times \bar{v})$$

$$= 2m^2 (\bar{r} \times \bar{v}) \cdot \left( \bar{r} \times \frac{d\bar{v}}{dt} \right)$$

Applying equation 2, we obtain

$$\frac{d}{dt}(L^2) = 2m^2 \lambda (\bar{r} \times \bar{v}) \cdot \left[ \frac{\bar{r} \times (\bar{v} \times \bar{r})}{r^3} \right] = 0$$

Therefore,  $L$  is constant and the magnitude of the angular momentum is conserved.

If  $p$  is the impact parameter at any time  $t$ ,  $L_0$  is the initial angular momentum, and  $L$  the angular momentum at any time  $t$ , the following relations hold:

$$L_o = mVb \text{ and } L = mVp \quad (3-3)$$

Thus, the conservation of angular momentum requires that  $p = b$ . It is apparent that all tangents to the trajectory will pass the charged particle at a perpendicular distance  $b$  and the distance of closest approach is  $b$ . This is an important feature of the scattering of a magnetic monopole in the field of a charged particle.

We are now in a position to find  $r(t)$ . Referring to Figure 5 and making use of fundamental vector relations, we have

$$\bar{r} \cdot \bar{V} = \bar{r} \cdot \frac{d\bar{r}}{dt} = r \frac{dr}{dt} = V \sqrt{r^2 - b^2}$$

Integrating, and taking as our boundary condition,  $t = 0$  at  $r = b$  we obtain

$$r = \sqrt{V^2 t^2 + b^2} \quad (3-4)$$

Our boundary condition implies that the time  $t$  is negative before the distance of closest approach is reached and positive afterwards.

In order to obtain the rest of the solution we must make some vector transformation:

$$\bar{r} \times \frac{d\bar{V}}{dt} = \frac{d}{dt} (\bar{r} \times \bar{V}) = - \frac{d}{dt} (\bar{V} \times \bar{r})$$

But, from equation 1:

$$\begin{aligned} \bar{r} \times \frac{d\bar{V}}{dt} &= \frac{\lambda}{r^3} \bar{r} \times (\bar{V} \times \bar{r}) = \frac{\lambda}{r^3} \left[ (\bar{r} \cdot \bar{r})\bar{V} - (\bar{r} \cdot \bar{V})\bar{r} \right] \\ &= \frac{\lambda}{r^3} \left[ r^2 \frac{d\bar{r}}{dt} - r \frac{dr}{dt} \bar{r} \right] = \lambda \frac{d}{dt} \left( \frac{\bar{r}}{r} \right) \end{aligned}$$

Therefore

$$-\frac{d}{dt} (\bar{V} \times \bar{r}) = \lambda \frac{d}{dt} \left( \frac{\bar{r}}{r} \right)$$

Integrating we obtain

$$\bar{V} \times \bar{r} = -\lambda \frac{\bar{r}}{r} + \bar{A} \quad (3-5)$$

where  $\bar{A}$  is some constant vector.

From Figure 5, we see that

$$|\bar{V} \times \bar{r}| = Vr \sin \alpha = Vr \frac{b}{r} = Vb \quad (3-6)$$

In order to simplify our boundary conditions we will orient our axes

$$\text{at } t = -\infty, \bar{V} \times \bar{r} = Vb\bar{i}, \text{ and } \frac{\bar{r}}{r} = -\bar{j}$$

Substituting this back into equation 3-5, we find that

$$\bar{A} = Vb\bar{i} - \lambda \bar{j}$$

Therefore equation 5 becomes

$$\bar{V} \times \bar{r} = -\lambda \frac{\bar{r}}{r} + Vb\bar{i} - \lambda \bar{j} \quad (3-7)$$

This is another way of writing the equation of motion of the monopole and it is much easier to solve than equations 3-2.

Our next step is to find the equation of the surface of the surface containing the trajectory. From equation 3-7 and from vector identities, we have

$$\bar{r} \cdot (\bar{V} \times \bar{r}) = -\lambda r + Vbx - \lambda y = 0$$

Letting  $r = \sqrt{X^2 + y^2 + Z^2}$  and rearranging we have

$$\left(1 - \frac{V^2 b^2}{\lambda^2}\right) X^2 + \frac{2Vb}{\lambda} Xy + Z^2 = 0 \quad (3-8)$$

This is the equation of the surface containing the trajectory. We can identify this surface if we rotate the axis so that the  $Xy$  term is eliminated. We know from analytic geometry that

$$\tan 2w = \frac{\frac{2Vb}{\lambda}}{1 - \frac{V^2 b^2}{\lambda^2}}$$

and  $X = X' \cos w - y' \sin w$

$$y = X' \sin w + y' \cos w$$

where  $w$  is the angle through which we rotate our axes. Substituting these relations into equation 3-8 and simplifying, we obtain

$$X^2 + Z^2 - \frac{V^2 b^2}{\lambda^2} y^2 = 0 \quad (3-9)$$

We may drop the prime since we will be working in the rotated system for the remainder of the problem.

Equation 3-9 represents the surface of a right circular cone, whose position is shown in Figure 6.

From equation 3-9 the angle  $\theta$  of the cone is given by

$$\tan \theta = \frac{Vb}{\lambda} \quad (3-10)$$

This angle is shown in Figure 7.

These results agree with Fierz<sup>40</sup> who described the motion as the trace of a straight line on a cone as it rolls on a plane containing the line. He expressed the angle of the cone in terms of the vector constant  $\bar{A}$ . The method used by Fierz in solving this problem is not clear since details are omitted.

From Figures 6 and 7 and equation 3-4 we see that

$$y = -r \cos \theta = -\lambda \frac{\sqrt{v^2 t^2 + b^2}}{\sqrt{v^2 b^2 + \lambda^2}} \quad (3-11)$$

This is one of the component equations of motion of the monopole.

To obtain the rest of the solution we must first express equation 3-5 in the rotated system. Let us apply the boundary conditions at the turning point. We may rotate our axes about the y axis without upsetting the symmetry of the problem or changing any of our previous results. For convenience, we choose to rotate our axes so that the point of closest approach is in the yz plane. Thus at  $r = b$

$$\bar{v} = v\bar{i}, \quad \bar{r} = b \sin \theta \bar{k} - b \cos \theta \bar{j}$$

$$\bar{v} \times \bar{r} = -v b \sin \theta \bar{j} - v b \cos \theta \bar{k}$$

Substituting these boundary conditions into equation 3-5 and using the relations of Figure 7, we find that

$$\bar{A} = -\frac{\sqrt{v^2 b^2 + \lambda^2}}{v} \bar{j}$$

$$\text{Therefore, } \bar{v} \times \bar{r} = -\lambda \frac{\bar{r}}{r} - \sqrt{v^2 b^2 + \lambda^2} \bar{j} \quad (3-12)$$

This is the vector equation of motion of the monopole in the rotated system. We can now find  $X(t)$  and  $Z(t)$  by equating the components of equation 3-12 and solving. The component equations are

$$Z \frac{dy}{dt} - Y \frac{dZ}{dt} = -\frac{\lambda Z}{r} \quad (3-13a)$$

$$X \frac{dZ}{dt} - Z \frac{dX}{dt} = -\frac{\lambda Y}{r} - \sqrt{v^2 b^2 + \lambda^2} \quad (3-13b)$$

$$Y \frac{dX}{dt} - X \frac{dy}{dt} = -\frac{\lambda Z}{r} \quad (3-13c)$$

We need to make use of only the second equation. Substituting equation 3-4 and equation 3-11 into equation 3-13b and simplifying we obtain

$$Z dX - X dZ = \frac{v^2 b^2}{\sqrt{v^2 b^2 + \lambda^2}} dt$$

We can integrate this equation if we divide both sides by  $(Z^2 + X^2)$  and find  $(Z^2 + X^2)$  as function of  $t$ . From the geometry of the problem (see Figure 6), from equation 3-4, and from figure 7, we see that

$$Z^2 + X^2 = r^2 \sin^2 \theta = \frac{v^2 b^2 (v^2 t^2 + b^2)}{v^2 b^2 + \lambda^2} \quad (3-14)$$

Therefore

$$\frac{ZdX - XdZ}{Z^2 + X^2} = \frac{\sqrt{v^2 b^2 + \lambda^2}}{v^2 t^2 + b^2} dt$$

Integrating, and apply the boundry condition  $X = 0$  at  $t = 0$ , this expression becomes:

$$\tan^{-1} \frac{X}{Z} = \frac{\sqrt{v^2 b^2 + \lambda^2}}{vb} \tan^{-1} \frac{vt}{b} \quad (3-15)$$

Solving this equation simultaneously with equation 3-14, we find that

$$X = Vb \frac{\sqrt{v^2 t^2 + b^2}}{\sqrt{v^2 b^2 + \lambda^2}} \sin \left( \frac{\sqrt{v^2 b^2 + \lambda^2}}{Vb} \tan^{-1} \frac{vt}{b} \right) \quad (3-16)$$

$$Z = Vb \frac{\sqrt{v^2 t^2 + b^2}}{\sqrt{v^2 b^2 + \lambda^2}} \cos \left( \frac{\sqrt{v^2 b^2 + \lambda^2}}{Vb} \tan^{-1} \frac{vt}{b} \right) \quad (3-17)$$

These two equations together with equation 3-11 completely describe the path of a monopole in the field of a fixed charged particle.

## 2. Calculation of a Rutherford Scattering Formula for Monopoles.

To determine the scattering of monopoles we must calculate another property of its motion, the amount of spiraling. If we let,

$$\phi = \frac{\sqrt{v^2 b^2 + \lambda^2}}{Vb} \tan^{-1} \frac{vt}{b} \quad (3-18)$$

we see from equation 3-15, that  $\phi$  is the angle through which the pole has revolved about the axis of the cone. This angle is shown in Figure 6.

To find the angle  $\epsilon$  through which the monopole revolves throughout its entire motion we proceed as follows:

$$\lim_{t \rightarrow -\infty} \phi = - \frac{\sqrt{v^2 b^2 + \lambda^2}}{Vb} n \frac{\pi}{2} \quad n = 1, 3, 5, \dots$$

$$\lim_{t \rightarrow \infty} \phi = \frac{\sqrt{v^2 b^2 + \lambda^2}}{Vb} n \frac{\pi}{2}$$

$$\epsilon = \lim_{t \rightarrow \infty} \phi - \lim_{t \rightarrow -\infty} \phi = n\pi \frac{\sqrt{v^2 b^2 + \lambda^2}}{Vb} \quad (3-19)$$





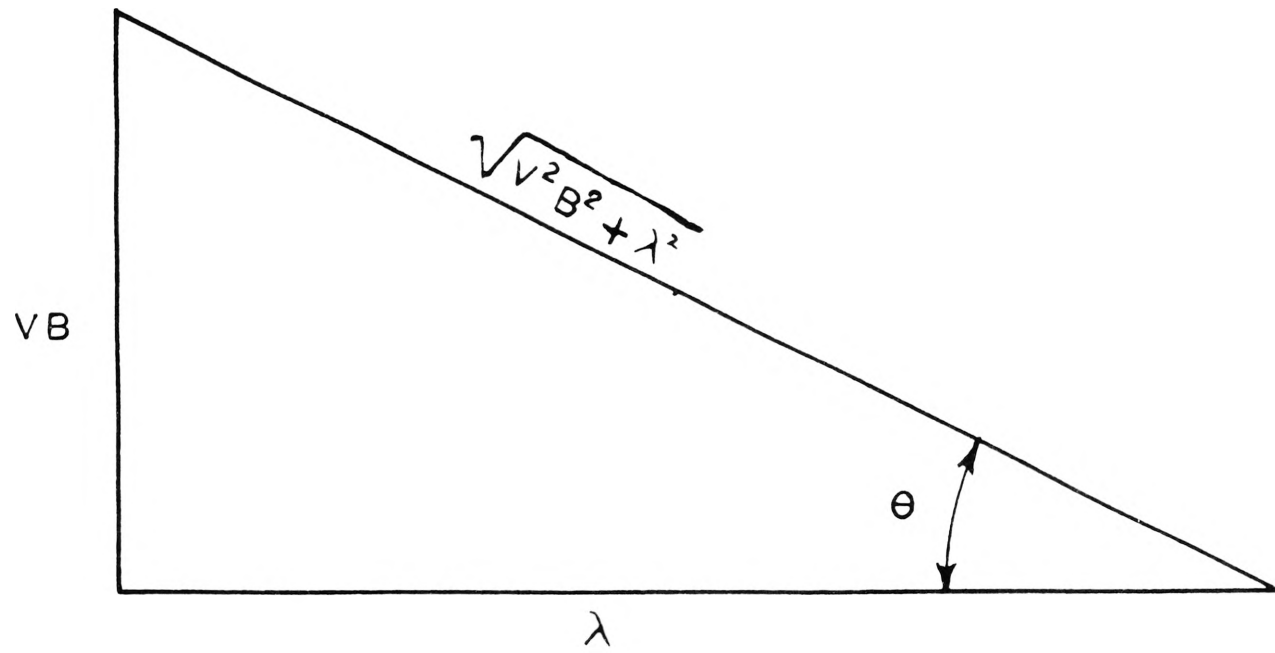


FIG 7 THE ANGLE OF THE CONE

We have shown that the angle of revolution  $\phi$  approaches a definite limit as  $t$  approaches infinity. This is the same as saying that in the limit the motion becomes parallel to the lateral surface of the cone. The relation between  $\epsilon$  and the scattering angle  $\psi$  is shown in Figure 6.

It may be shown from the geometry of Figure 6 and from equation 3-19 that

$$\cos \frac{\psi}{2} = \sin \theta \sin \frac{\epsilon}{2} = \frac{Vb}{\sqrt{V^2 b^2 + \lambda^2}} \sin \left( \frac{n\pi \sqrt{V^2 b^2 + \lambda^2}}{2 Vb} \right) \quad (3-20)$$

This equation cannot be solved easily for the impact parameter  $b$  in terms of the scattering angle  $\psi$ , consequently we employ the impulse, momentum method. The change in momentum is expressed as follows:

$$\Delta P = 2 m V \cos \frac{\pi - \psi}{2} = 2 m V \sin \frac{\psi}{2} \quad (3-21)$$

But the change in momentum is equal to the impulse:

$$\begin{aligned} \Delta P &= \left[ (\Delta P_x)^2 + (\Delta P_y)^2 + (\Delta P_z)^2 \right]^{1/2} \\ &= \left[ \left( \int_{-\infty}^{\infty} \bar{F}_x dt \right)^2 + \left( \int_{-\infty}^{\infty} \bar{F}_y dt \right)^2 + \left( \int_{-\infty}^{\infty} \bar{F}_z dt \right)^2 \right]^{1/2} \end{aligned} \quad (3-22)$$

Making use of equation 3-1, 3-12, 3-4, and 3-16, we see that

$$\begin{aligned} F_x &= \bar{F} \cdot \bar{i} = \frac{m\lambda}{r^3} (\bar{V} \times \bar{r}) \cdot \bar{i} = -\frac{m\lambda^2 x}{r^4} \\ &= \frac{m\lambda V^2 b^2}{\sqrt{V^2 b^2 + \lambda^2} (V^2 t^2 + b^2)^{3/2}} \sin \left( \frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} \tan^{-1} \frac{Vt}{b} \right) \end{aligned} \quad (3-23)$$

In the same manner it can be shown that

$$F_z = \frac{m \lambda^2 v b}{\sqrt{v^2 b^2 + \lambda^2} (v^2 t^2 + b^2)^{3/2}} \cos \left( \frac{\sqrt{v^2 b^2 + \lambda^2}}{v b} \tan^{-1} \frac{v t}{b} \right) \quad (3-24)$$

and

$$F_y = \frac{-m \lambda v^2 b^2}{\sqrt{v^2 b^2 + \lambda^2} (v^2 t^2 + b^2)^{3/2}} \quad (3-25)$$

We can now calculate the components of the momentum change.

$$\begin{aligned} \Delta P_x &= \int_{-\infty}^{\infty} F_x dx \\ &= \frac{m \lambda v^2 b^2}{\sqrt{v^2 b^2 + \lambda^2}} \int_{-\infty}^{\infty} \frac{1}{(v^2 t^2 + b^2)^{3/2}} \sin \left( \frac{v b}{\sqrt{v^2 b^2 + \lambda^2}} \tan^{-1} \frac{v t}{b} \right) dt \end{aligned}$$

If we let  $U = \tan^{-1} \frac{v t}{b}$ , then  $t = \frac{b}{v} \tan U$

$$dt = \frac{b}{v} \sec^2 U dU$$

and

$$\begin{aligned} \Delta P_x &= \frac{m \lambda^2}{\sqrt{v^2 b^2 + \lambda^2}} \int_{-\frac{n\pi}{2}}^{\frac{n\pi}{2}} \cos U \sin \left( \frac{\sqrt{v^2 b^2 + \lambda^2}}{v b} U \right) dU \\ &= \frac{m \lambda^2}{\sqrt{v^2 b^2 + \lambda^2}} \left\{ \frac{-\cos \left( \frac{\sqrt{v^2 b^2 + \lambda^2}}{v b} - 1 \right) U}{2 \left( \frac{\sqrt{v^2 b^2 + \lambda^2}}{v b} - 1 \right)} \right. \\ &\quad \left. - \frac{\cos \left( \frac{\sqrt{v^2 b^2 + \lambda^2}}{v b} + 1 \right) U}{2 \left( \frac{\sqrt{v^2 b^2 + \lambda^2}}{v b} + 1 \right)} \right\} \Bigg|_{-\frac{n\pi}{2}}^{+\frac{n\pi}{2}} = 0 \quad (3-26) \end{aligned}$$

In exactly the same way the expression for  $P_z$  and  $P_y$  turns out to be

$$\Delta P_z = \frac{m\lambda^2}{\sqrt{V^2 b^2 + \lambda^2}} \left[ \frac{\sin\left(\frac{\sqrt{V^2 b^2 + \lambda^2} - 1}{Vb} \frac{n\pi}{2}\right)}{\frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} - 1} + \frac{\sin\left(\frac{\sqrt{V^2 b^2 + \lambda^2} + 1}{Vb} \frac{n\pi}{2}\right)}{\frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} + 1} \right] \quad (3-27)$$

$$\Delta P_y = \frac{2mV\lambda}{\sqrt{V^2 b^2 + \lambda^2}} \quad (3-28)$$

The expression for  $\Delta P_z$  is in a rather awkward form. We now proceed to reduce it to a form which we can handle. We may rewrite this expression as follow:

$$\Delta P_z = \frac{m\lambda^2}{\sqrt{V^2 b^2 + \lambda^2}} \left[ \frac{-\cos\left(\frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} \frac{n\pi}{2}\right)}{\frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} - 1} + \frac{\cos\left(\frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} \frac{n\pi}{2}\right)}{\frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} + 1} \right] \quad (3-29)$$

We now make use of equation 3-20:

$$\sin\left(\frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} \frac{n\pi}{2}\right) = \frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} \cos \frac{\psi}{2}$$

$$\cos\left(\frac{\sqrt{V^2 b^2 + \lambda^2}}{Vb} \frac{n\pi}{2}\right) = \sqrt{1 - \frac{V^2 b^2 + \lambda^2}{V^2 b^2}} \cos \frac{\psi}{2} \quad (3-30)$$

Substituting equations 3-21, 3-26, 3-28, and 3-30 into equation 3-22 and simplifying we obtain

$$\Delta P_z = -2mVb \sqrt{\frac{V^2 b^2}{V^2 b^2 + \lambda^2} - \cos^2 \frac{\psi}{2}} \quad (3-31)$$

Substituting equations 3-21, 3-26, 3-28, and 3-31 into equation 3-22 and simplifying we obtain

$$\sin \frac{\psi}{2} = \sqrt{\frac{V^2 b^4}{V^2 b^2 + \lambda^2} - b^2 \cos^2 \frac{\psi}{2} + \frac{\lambda^2}{V^2 b^2 + \lambda^2}}$$

Solving this equation for  $b$  as a function of  $\psi$ , we obtain

$$\cot \frac{\psi}{2} = \frac{Vb}{\lambda} \quad (3-32)$$

Through use of equation 3-32, we obtain the Rutherford scattering formula by the usual procedure. The result is

$$N = \frac{Qnd}{R^2} \left( \frac{Zeg}{2mVc} \right)^2 \frac{1}{\sin^4 \frac{\psi}{2}} \quad (3-33)$$

which is in exact agreement with Bauer's result given on page 9.

Bauer's result, however, was obtained from quantum mechanics.

If we compare equation 3-10 with equation 3-32, we see that

$$\tan \theta = \cot \frac{\psi}{2} = \tan \frac{\pi - \psi}{2}$$

Therefore:  $2\theta = \pi - \psi$  (3-34)

It follows then from fundamental theorems of solid geometry that the velocity vector at  $t$  - and the velocity vector at  $t$  become parallel to directrices on opposite sides of the cone. These velocity vectors are along the direction of the asymptotes of a hyperbola formed by the intersection of the cone and a plane parallel to the axis of the cone. Equation 3-34 states that the angle between these asymptotes is  $2\theta$ . It necessarily follows that

$$\epsilon = m\pi \quad \text{where } m = 1, 3, 5, \dots$$

Comparing this with equation 19 and with Figure 7

$$m\pi = \pm n\pi \frac{\sqrt{v^2 b^2 + \lambda^2}}{vb} = \frac{n\pi}{\sin \theta}$$

Therefore  $\sin \theta = \frac{n}{m}$

where  $m$  and  $n$  are any two odd numbers. No restrictions are placed on  $n$  and  $m$  except  $\frac{n}{m}$  must be less than one. If we arbitrarily choose  $n = 1$

$$\sin \theta = 1, 1/3, 1/5, \dots$$

Combining this result with equation 3-34 we obtain

$$\psi = 0, 141.2^\circ, 157^\circ, 163.8^\circ, 167.2^\circ, \dots \quad (3-35)$$

These agree very closely with the values for the rainbow angles given by Ford and Wheeler in chapter two.

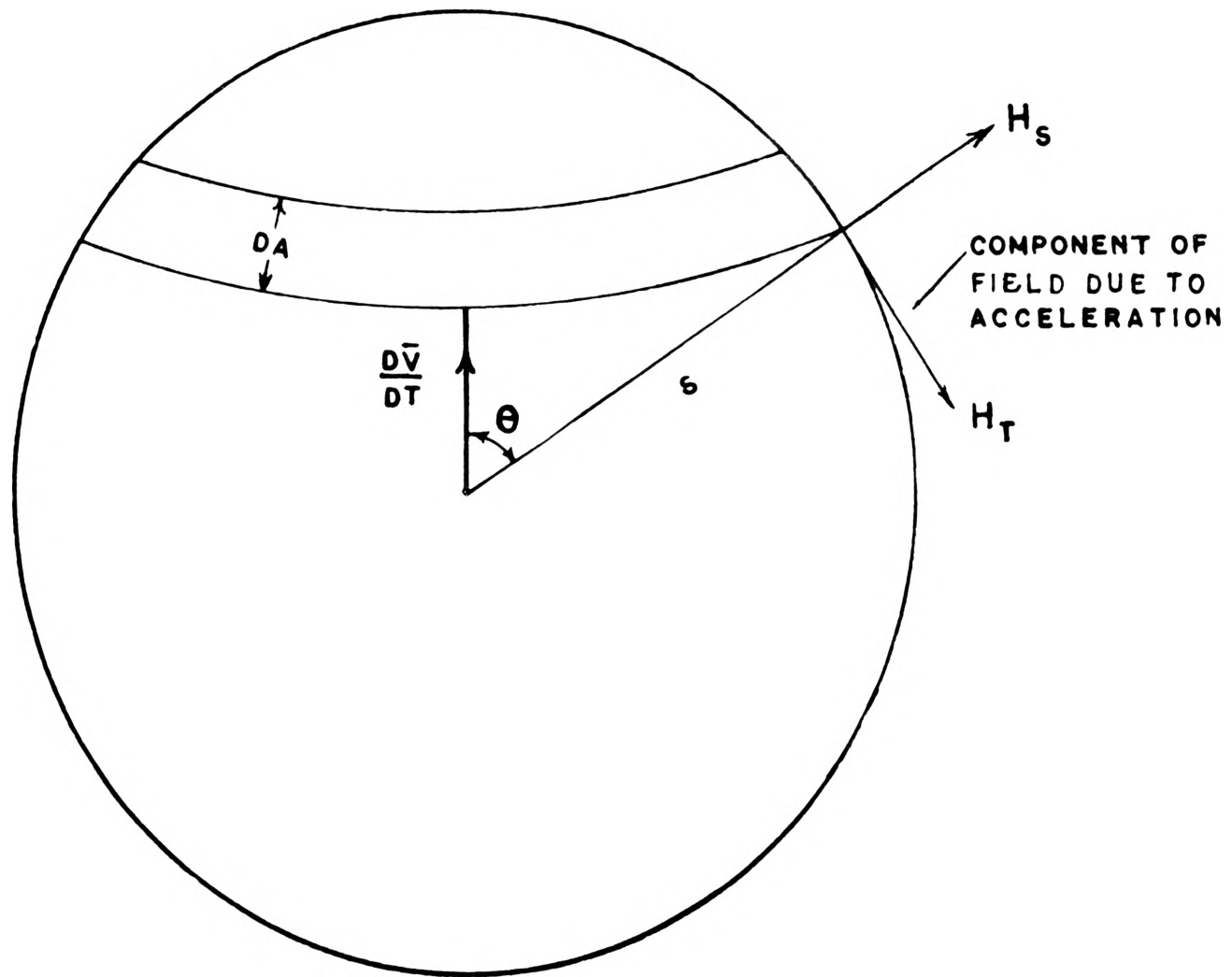


FIG. 8 RADIATION LOSS BY A MONOPOLE TRAVELING  
IN THE VICINITY OF A CHARGE



4. Effect of Radiation. All of the above results are true only if radiation effects are very small. But radiation is small only if the mass is large. We now compute the time rate at which energy is lost  $\frac{dT}{dt}$  by a monopole traveling in the field of a fixed charged particle. We may treat this problem in a manner exactly analogous to the case of radiation loss of an accelerated charged particle.

The radiation loss of an accelerated charged particle is

$$E = \frac{Ze \left| \frac{d\vec{V}}{dt} \right| \sin \sigma}{c^2 r} \quad (3-36)$$

where E is the field due to the radiation,  $\sigma$  is angle between the direction of the acceleration and line joining the particle with the field point which we are considering and s is the length of this line. By strict analogy,

$$H_t = \frac{g \left| \frac{d\vec{V}}{dt} \right| \sin \sigma}{c^2 s} \quad (3-37)$$

where H is the magnetic field due to radiation. Combining this result with equation 3-2 and equation 3-6, we have

$$H_t = \frac{g \lambda V b \sin \sigma}{c^2 s^4} \quad (3-38)$$

From the definition of the Poynting's vector we may write:

$$S = cH_t^2 = \frac{g^2 \lambda^2 V^2 b^2 \sin^2 \sigma}{c^3 s^8} \quad (3-39)$$

Referring to Figure 8, we see that

$$d \left( \frac{dT}{dt} \right) = S dA = \frac{2\pi g^2 \lambda^2 V^2 b^2 \sin^3 \sigma \theta d\theta}{c^3 s^6} \quad (3-3)$$

Integrating this expression and substituting the value of  $\lambda$  we obtain

$$\frac{dT}{dt} = \frac{8}{3} \frac{\pi Z^2 e^2 g^4 V^2 b^2 \text{ ergs}}{m^2 c^5 r^6} \quad (3-40)$$

Thus we see that radiation is negligible only when the mass m of the monopole is large which is what we inferred from Bauer's work.

## CHAPTER IV

### PREVIOUS EXPERIMENTAL WORK

Having discussed the theoretical properties of monopoles, the next step is to see what experimental work has been done. All experimental work to date can be divided into three classes: (1) Search for monopoles in static situations, (these include primarily the works of Ehrenhaft), (2) search for fast moving poles in cosmic radiation, (these include experiments with nuclear emulsions and with cloud chambers), and (3) attempts to produce monopoles by use of the bevatron, (this work was conducted by Bradner and Isbell). A detailed discussion of these experimental works follow.

1. The Work of Ehrenhaft. For a number of years, Ehrenhaft has advocated the existence of free magnetic poles. His claims to the discovery of these entities are based on his interpretations of extensive experiments which he and his collaborators performed. We will discuss some of his experimental work and compare it with information from other sources.

a. Polar Motion of Suspended Particles. Tiny particles with a high magnetic susceptibility were suspended in a gaseous medium between the poles of a magnet and illuminated with a strong light. When the magnetic field was turned on, the particles immediately began to move along the lines of force of the magnetic field. The following observations have been made in regard to the motions of these particles:

(1) Motion begins immediately when the field is turned on and ceases

instantly when it is turned off. This motion has also been observed in the magnetic field of the earth. It could be stopped by applying a magnetic field to counteract the earth's field. (2) Regardless of the direction from which the light is coming, the particles always move along the direction of the lines of force of the magnetic field. Some move toward the north pole, others move toward the south, often crossing the center between the two poles in both directions. A few particles, however, do not acquire motion at all. (3) The direction of motion of these particles reverses with the magnetic field. At low field intensity the particle speed is a linear function of the field intensity. As the field intensity increases the velocity levels off to a limiting value.

The above phenomena has been interpreted by Ehrenhaft<sup>41</sup> as decisive evidence for the existence of isolated magnetic poles. According to him, these particles behave as they do because they possess magnetic charge.

These observations of Ehrenhaft have been thoroughly investigated by Kane.<sup>42</sup> He indicates that the greater part of these motions are due to light, as shown by the fact that nearly all of the motion ceases as soon as the light is turned off.

A detailed explanation of the observations of Ehrenhaft has been given by Ford and Wheeler as follows: The suspended particles are very asymmetrical in shape. They will be oriented so that their long axis is parallel to the magnetic field. When illuminated with light the surface will be heated. These gases stream off the surface of the particle imparting translational motion to it. Irregularity of heating over surface due to asymmetry of the particle will also produce rotational

motion about an axis parallel to the magnetic field. The recoil push of the gas can be resolved into two components, one parallel to the magnetic field, the other perpendicular to the field. The perpendicular component will average to zero because of the spin. Consequently the particle is propelled in the direction of the magnetic field. It's residual magnetism will cause it to turn over and reverse its direction when the magnetic field is reversed. According to Ford and Wheeler:

"This existence of simple explanation would seem to make it entirely out of place for one to regard Professor Ehrenhaft's beautiful observations as evidence for free magnetic poles".<sup>43</sup>

Sometimes suspended particles exhibit polar motion without the action of light. Some particles will move against the gravitational field when the magnetic field is oriented vertically even when the illumination is too small to have any appreciable effect. Kane<sup>44</sup> has shown, however, that this phenomena does not indicate that the particles are magnetically charged. He explains that a changing magnetic field induces a current in the particles. If this current is more than ninety degrees out of phase with the current of the electromagnet, it will experience a repulsive force toward the farther pole. He was able to verify his interpretations in two ways: (1) Finely divided particles were sprinkled on the lower pole face. When a very weak magnetic field was applied, some of the particles would rise a short distance and then return to the lower pole face. If the particles possessed magnetic charge they would continue to move up until they reached the upper pole face. (2) Finely divided particles were suspended in a gaseous medium between two poles of a magnet. When a magnetic field was applied very gradually, no such motions as reported by Ehrenhaft could be detected.

Another experiment to test Ehrenhaft's hypothesis was reported by Hopper.<sup>45</sup> Fine particles of nickel were introduced between the poles of a magnet with field intensity of about 1000 gauss and allowed to settle under the action of gravity. If these particles possessed magnetic charge some of them should be deflected across the center toward the farther pole as they pass through the magnetic field. No such deflection were observed. All deflections were toward the nearer pole. The experiment was repeated with finely divided particles of iron with the same negative result.

b. Phenomena in Liquids. Ehrenhaft<sup>46</sup> also investigated the influence of a magnetic field on tiny particles suspended in a liquid medium. Suspensions of different materials were placed between the poles of a magnet and a field applied. In some of these suspensions, a rotational motion of the particles was observed upon application of the magnetic field. The particles were traveling in helical paths along the direction of the magnetic field. The direction of rotation of the particles always reversed with the magnetic field and the speed of rotation was found to be a function of the field intensity. Sometimes dual rotations were observed; particles of the same kind were seen to be simultaneously rotating in opposite directions at the same place in the liquid. These same spiraling motions have also been observed for bubbles produced by chemical action at the pole faces and for tiny particles suspended in a gaseous medium.

These spiraling motions were offered by Ehrenhaft as evidence for the existence of a magnetic current. According to his hypothesis a



magnetic current flows between the pole faces of the magnet. An electric field encircles this magnetic current, just as a magnetic field encircles an electric current. These tiny particles travel in spiral paths because they possess both electric and magnetic charge.

Perls<sup>47</sup> made an experimental investigation of the rotations described by Ehrenhaft and decided that they are due to the heating effects of the light which sets up convection currents within the liquid. Kane and Reynolds,<sup>48</sup> however, reported seeing these motions with dark field illumination.

These rotational motions of suspended particles were further investigated by Kendall.<sup>49</sup> He found that the motions of the particles were due to the motion of liquid itself, as determined from refractive index striations. This movement of the liquid results from the tendency of portions of the liquid containing high concentrations of ferrous ions or other ions of high magnetic susceptibility to move into regions where the magnetic field is strongest. Confirmation of this view point comes from the following experiments. Two soft iron pole pieces of an electromagnet were immersed in dilute hydrochloric acid and a magnetic field was applied. Bubbles were formed at the pole face and some of the iron was dissolved into the solution. The resulting nonuniformity of ionic concentrations set up rotary currents in the liquid. Streams of bubbles were carried along with this movement. The experiment was repeated using pole pieces heavily plated with cadmium to prevent ferrous ions from entering the solution. Bubbles were formed as before but their motions were not affected by the magnetic field. Another experiment was performed with the pole faces waxed and immersed in a solution of

ferrous chloride. Under these conditions no reactions could occur at the poles and no movement of the liquid in the magnetic field could be observed. However, when water was poured into the solution, destroying the uniformity of concentration, movement of the liquid in the magnetic field did occur. After the pouring of the water was stopped the movement gradually died away as the concentration was restored to uniformity. Various other experiments were conducted with various types of electrolytic solutions. Movements were observed only in those solutions containing ferrous ions or other ions of high magnetic susceptibility.

Rotation of uniform electrolytic solutions in constant homogeneous magnetic fields have been reported by Kane and by Reynolds.<sup>50, 51</sup> A satisfactory electrochemical explanation of these phenomena has been given by Swartz and Van der Grinten.<sup>52</sup> According to them these rotations are due to current flow in the solution caused by the presence of the pole faces which are immersed in the solution. Very slight differences in potential are set up at different places on the surface of the same piece of metal. These potential differences have been measured with a special probe and potentiometer arrangement. This shows that the metal acts as many small voltaic cells. Application of ferroxy indicators show the regions of anodes and cathodes in agreement with the potentiometer readings.

Another series of experiments were devised by Benedikt and Leng<sup>53</sup> to test Ehrenhaft's hypothesis of the existence of single magnetic poles. A well insulated cylindrical copper conductor was immersed in a colloidal suspension of ferrous oxide. Three thousand amperes of current were passed through the conductor. If some of these particles possessed a

magnetic charge they should be deflected by the magnetic field. Several observations were made but no such deflections were observed. The experiment was repeated with colloidal suspensions of iron and nickel with the same negative results. The suspensions were also placed between the poles of an electromagnet applying 10,000 gauss. No motion of the particles could be observed aside from slight Brownian motions. On the basis of this experiment, Benedikt and Leng estimated that a magnetic charge of  $1.5 \times 10^{-12}$  electromagnetic units on any of the particles that were observed could have been detected.

c. Magnetolysis. Ehrenhaft<sup>54</sup> has reported that it is possible to decompose water by application of a magnetic field. Two soft iron pole pieces of an electromagnet were immersed in acidulated water. Before the magnetic field was applied only hydrogen was liberated at the poles. As soon as the magnetic field was turned on both oxygen and hydrogen were given off. Oxygen came mostly from the north pole and the hydrogen came mostly from the south. The rate of evolution of the gases was found to be proportional to the magnetic field intensity. These results were brought forth in support of his argument for the existence of magnetic currents.

Others have attempted to determine the extent of magnetolysis and their findings do not concur with those of Ehrenhaft. A very careful experiment was conducted by Millest.<sup>55</sup> The acid to be used was first boiled to drive off dissolved gases. The pole faces were coated with tin to prevent direct interaction. The poles were then immersed in the acid and a magnetic field applied. The experiments were conducted for



several hours at a time; the composition of the evolved gases being checked intermittently. Only very small amounts of gas were ever collected and sometimes none at all. It was found that the evolution of gases did not occur in any regular and consistent manner as would be expected if it were due to magnetolysis. The small amounts of gas that were collected were probably dissolved gas driven off by the heating effects of the magnetic field. Confirmation of this came from the observation that small bubbles were formed in the interior of the liquid and not altogether at the pole faces. The experiment was repeated, exposing the naked pole faces to the acid solution. Only about one half of one per cent of the gas liberated was oxygen and the difference between the amount collected at the two poles was too small to be determined by the apparatus. Experiments in magnetolysis were also attempted by Kendall<sup>56</sup> and by Hoff, Naughton, Smoluchowski, and Ulig<sup>57</sup> working together. The same negative results were obtained.

Ehrenhaft<sup>58</sup> claimed that permanent magnets lose part of their magnetism during the magnetolytic process. This was supposedly determined by the deflection of a ballistic galvanometer before and after the experiment.

An experiment to check this claim was done by Goldman.<sup>59</sup> The field strength of an alnico permanent magnet was measured for five successive days before the experiment. During that time no changes were noted. The pole faces were then placed in contact with a four per cent sulfuric acid solution and later with a twelve per cent sulfuric acid solution. The total time of exposure to acid solutions was about 60 hours. No changes in the pole strength of the magnet could be detected either during the experiment or after.

d. Photomagnetism. As further confirmation of the existence of single magnetic poles, Ehrenhaft<sup>60</sup> has reported that matter can be magnetized by exposure to ultra-violet light. Small pieces of unmagnetized, annealed iron were placed normal to the earth's field and exposed to light rich in ultra-violet radiation. After short periods magnetic charges could be detected only on the irradiated side and on the surface. After longer periods of exposure, saturation values were reached. The polarity of the induced charge was mainly north magnetic.

The validity of the results were investigated by Frocken<sup>61</sup> and by Conner.<sup>62</sup> They irradiated small pieces of iron under carefully controlled conditions using an intense source of ultra-violet radiation. Several trials were made. Very sensitive detection instruments were used. The results were completely negative. No change in magnetization of the specimens could be detected during the exposure or afterwards.

It is impossible to say that Ehrenhaft has never observed a stable isolated pole. At present, however, his experimental work is not regarded as evidence in favor of their existence for two reasons:

(1) Other satisfactory explanations have been found for most of the observations which he has reported. (2) Other investigators have not always been able to obtain results consistent with his claims.

2. The Malkus Experiment. A search was conducted by Malkus<sup>63</sup> to determine the rate of arrival of Dirac monopoles at the earth's surface. According to him monopoles entering the earth's atmosphere in cosmic radiation would be slowed down very quickly to a low terminal velocity because of their high ionization loss. They will then drift along the

earth's magnetic field and diffuse into the earth. If monopoles were strongly bound to matter, they would remain in the earth's crust. If they have been accumulating in the earth's crust since the earth began, it should now be possible to detect them. There is, however, no measurable magnetic charge associated with surface matter. From these considerations, Malkus estimated that their rate of arrival at the earth's surface would be less than  $10^{-10}$  per square centimeter per second.

On the other hand, if the binding energy of a monopole is weak, this estimate would not be reliable since monopoles could then diffuse through the earth and be expelled by its magnetic field near the opposite pole. Malkus himself calculated, as we have seen in chapter two, that the binding energy is about the same as that of the chemical bond. This may not be sufficient to prevent diffusion.

Another consideration is that monopoles could be trapped in ferromagnetic materials. Goto<sup>64</sup> has shown that the binding in this case is great enough to prevent escape. If monopoles have been arriving at the earth for long periods of time, they should be present in ferromagnetic materials. The fact that no such charges can be detected indicates that their rate of arrival must be very small.

Malkus set up experimental apparatus to determine flux density of monopoles drifting along the earth's magnetic field. A schematic diagram of the apparatus is shown in Figure 9. Monopoles traveling along the earth's field would be drawn into the solenoid and accelerated toward a photographic emulsion. It should be possible to identify any monopole tracks from their ionization properties as given in chapter two. Careful

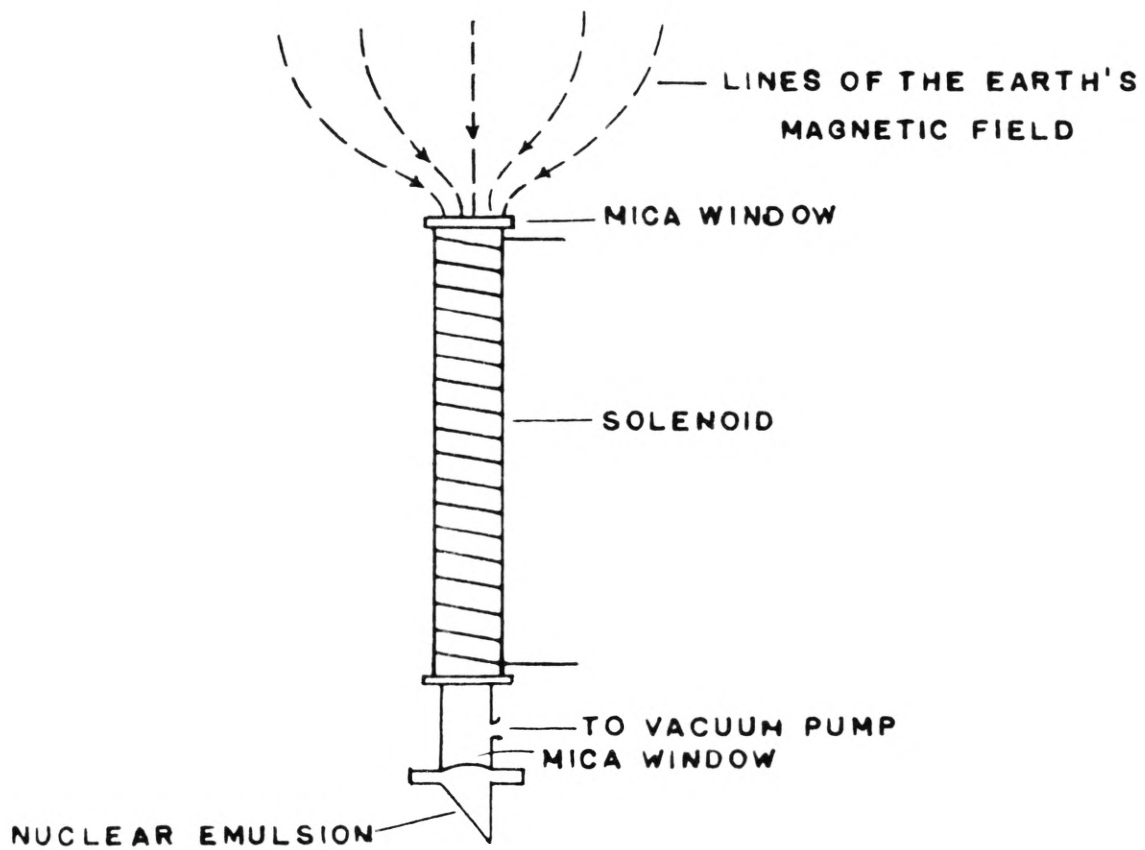


FIG. 9 APPARATUS USED BY MALKUS FOR  
DETECTION OF MONOPOLES

scanning of the emulsions for a period of two weeks showed no signs of monopole tracks. On the basis of this experiment, Malkus again concluded that their rate of arrival at the earth's surface must be less than  $10^{-10}$  per square centimeter per second.

3. The Experiment of Ruark and His Collaborators. Fitz, Good, Kassner and Ruark<sup>65</sup> conducted an experimental search for monopoles and subionizers with a very clean cloud chamber. The investigation was divided into two parts. In the first part, radium was used as a source of flux and in the second part cosmic radiation was used. Great pains were taken to insure optimum operating conditions. A bearden cloud chamber with a sensitive time of 1.5 seconds and very low background was used. It became possible to control this chamber so that in 15% of the expansions no background at all could be observed and in another 40% of the expansions only a few background drops could be seen. This enabled particles of very low ionization to be detected. Arrangements were also provided for taking a series of pictures of each expansion to enable accurate drop counts to be made. Definite criteria were set up for the selection of photographs and only the most favorable ones were used. These were carefully scrutinized for tracks of monopoles and subionizers. The search concerned not only Dirac type monopoles, but those having magnitudes other than that preferred by Dirac were considered. In the first part 900 tracks were examined and in the second 550 tracks were examined. No such entities were found, however.

4. Experiments with the Bevatron. Attempts were made by Bradner and Isbell<sup>66</sup> to produce monopoles by pair production process using the

Bevatron as a source of high energy particles. A target material was placed between the poles of a powerful electromagnet which were oriented vertically. Nuclear emulsion were placed between the target and the lower pole. An intense magnetic field was applied and the target was bombarded with 6.2 BeV protons. If any monopoles were produced in the collision processes, they would have been accelerated down the magnetic field striking the emulsion. Three experiments were performed as follows: (1) An aluminum target placed 13 centimeters above the emulsion was bombarded by  $5 \times 10^{12}$  protons in a 14,200 gauss field. (2) A copper target placed 7 centimeters above the emulsion was bombarded  $10^{12}$  protons in a 14,200 gauss field. (3) A polyethylene target placed 2.5 centimeters above the emulsion was bombarded with  $10^{17}$  protons in a 200,000 gauss field. No monopoles were found in these experiments and very low maximum values were set for the pair production cross sections, being as low as  $10^{-40}$  square centimeter per nucleon in the case of polyethylene.

One uncertain factor in this experiment is the monopole mass. The apparatus was set up to detect monopoles having masses ranging from the  $\pi$  meson mass to the proton mass. If their mass is appreciably greater than the proton mass, they would not have been produced by the Bevatron at all.

5. An Experiment at high Altitudes. Katz and Parnell<sup>69</sup> exposed five sets of Ilford G-5 emulsions, two inches by four inches each, at an altitude of 100,000 feet. Twenty nine tracks suitable for measurement were found. All turned out to be tracks of heavy nuclei. None could be attributed to monopoles.

This constitutes all known experimental work done so far. In every case negative results were obtained. Some physicists are still not entirely satisfied that these results are conclusive and are currently in the process of developing more highly refined equipment to continue the search.



## CHAPTER V

### AN EXPERIMENTAL SEARCH FOR MAGNETIC MONOPOLES

1. General Considerations. A limited search for magnetic monopoles was made with a Wilson Cloud Chamber. The experiment consisted of photographing tracks from cosmic radiation and examining these photographs for possible monopole tracks. The Wilson Cloud Chamber is considered desirable for this work since it is highly sensitive to individual ions. The advantages of the apparatus used in this particular search are: attainment of low background levels, long sensitive time, and multiple photographs of individual expansions. A discussion of each follows.

a. Low Background Level. The achievement of well developed tracks with a minimum of background depends primarily upon the skill and patience of the operator in adjusting the cloud chamber parameters. The problem is to find the proper expansion ratio such that condensation can occur on ions but ~~not~~ neutral nuclei. This margin is very small and can be attained only by careful adjustment. It is easy to produce a light background by a very slight over-expansion. Under the conditions of the present experiment, condensation on ions begins at an expansion ratio of about 1.085. The difference between this and the onset of condensation on aggregates of vapor molecules is difficult to determine precisely but can be safely estimated to be less than .005. For this reason the expansion ratio must be carefully adjusted.



A possible source of background is remnants of old droplets left over from previous expansions due to the incomplete re-evaporation of some of the droplets. These have been almost entirely eliminated, however, by continuing the expansion beyond the sensitive period allowing droplets to grow to full size and fall out so that re-evaporation cannot take place. Further precaution is taken by having an intermediate cleaning expansion. The cleaning expansion must be checked periodically to insure that additional droplets are not formed during this part of the cycle.

Another possible source of background is photomucleation from the flash units, although this has been practically eliminated by use of filters which eliminate ultraviolet light below  $4500 \text{ \AA}$ .

The amount of background that can be tolerated depends on the type of track. If the magnetic monopole is as heavily ionizing as Cole has predicted, a moderate amount of background is not objectionable. Dense background is undesirable, however, because excessive condensation creates vapor poverty and tracks cannot develop properly. Also, the heat of condensation will compress the chamber and make it less sensitive to ionizing particles.

b. Long Sensitive Time. The sensitive time of the chamber depends upon the type of liquid and gas used in the chamber. The longest sensitive time was attained with helium and a 2:1 ethyl alcohol, water combination. This is due to the low expansion ratio necessary to produce condensation and to the low viscosity of the gas. A useful sensitive time of about 3.5 seconds can

be achieved. The main limiting factors of the sensitive time are turbulence and vapor depletion due to earlier tracks.

A longer sensitive time is important because it allows one to follow the growth of tracks appearing early in the expansion until they are fully developed. It also provides a longer observation time and consequently a greater probability of finding rare particles. Care must be taken to insure that uniform sensitivity is maintained throughout the data taking period. This is controlled by a system of electronically operated valves which must be carefully adjusted and checked periodically.

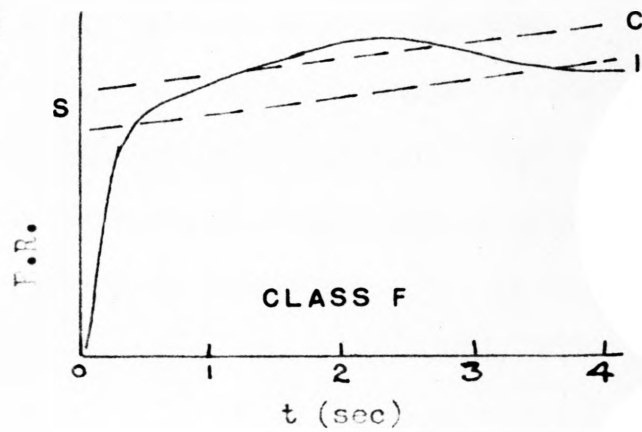
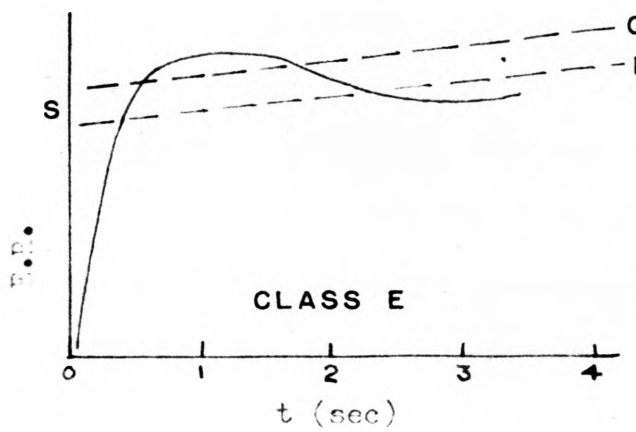
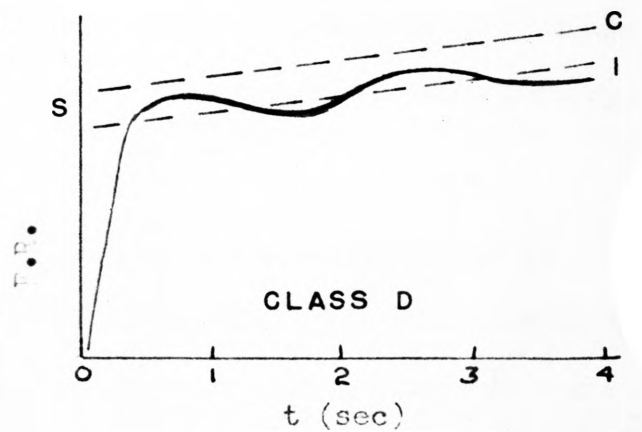
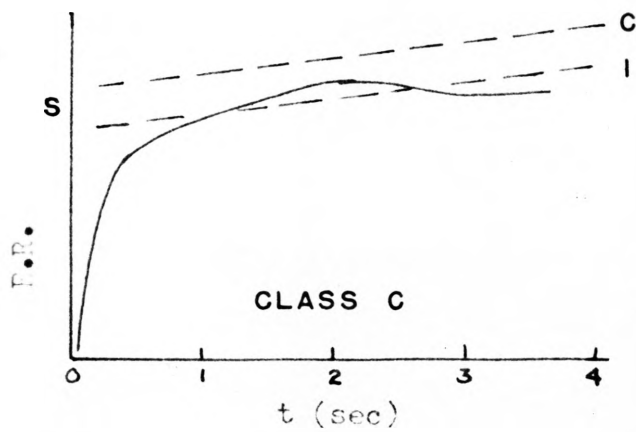
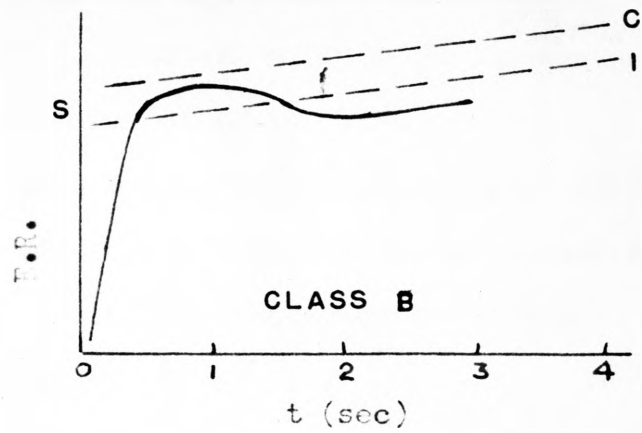
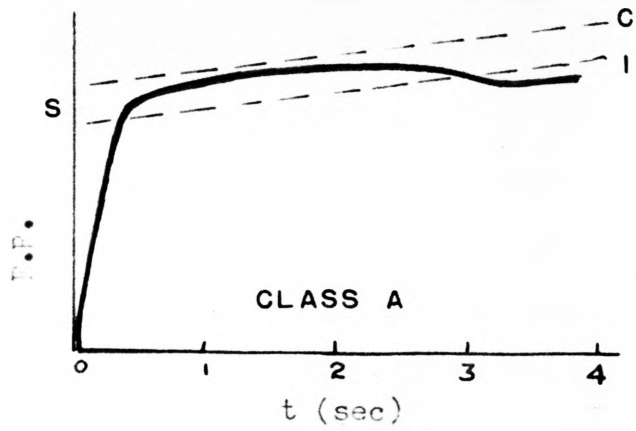
c. Multiple Photography. The achievement of a long sensitive time makes it possible to take a number of successive photographs of an individual expansion. In the present experiment twenty-two pictures of each expansion are taken. This enables one to make a detailed study of the growth and development of tracks and background and to properly identify the ionization along a trajectory. Stereoscopy is also employed to assist in determining the exact position of a track in the chamber.

d. Selection of Helium. In addition to increased sensitive time, helium has certain other advantages in this particular search. These advantages are greater diffusion and lower ionization rates. Greater diffusion causes tracks to spread quickly increasing the possibility of making accurate drop counts. A lower ionization rate is desirable in searching for heavily ionizing poles because accurate drop counts would be feasible for a wider range of pole strengths. Ranges are also greater because of the lower rate of energy loss. This allows poles with small energies to enter the central portions of the chamber where sensitivity is highest.

e. Thickness of the Cloud Chamber Wall. The glass walls of the chamber are about one fourth inch thick. In order to penetrate these walls a Dirac monopole must possess about  $1.5 \times 10^9$  electron volts. For smaller pole strengths the energy required is less. Energies of this order of magnitude are available in cosmic radiation. Magnetic monopoles might also be produced by pair production processes within the glass walls or inside the chamber.

The details of operation and design of the apparatus and the determination of optimum cloud chamber parameters have been adequately discussed in a thesis written by Zin Aung,<sup>68</sup> a coworker of the author, and need not be repeated here.

2. Viewing Procedures. The first step in this search was to determine a criteria for selection of photographs and to classify them accordingly. The classification used in this experiment was established by Zin Aung<sup>69</sup> and is shown in Figure 10. All expansions were classified into six groups. Class A expansions are those which are sensitive throughout the expansion. Class B expansions are sensitive at first but prematurely go insensitive. Class C expansions are below the sensitive region except late in the expansion. Class D expansions are insensitive in the middle portion but becomes sensitive again in the late part of the expansion. Class E expansions are those with dense background and class F expansions are those in which heavy background comes in late in the expansion. These classifications were made by going through the photographs and observing which frames contained newly formed tracks. Regions containing newly formed tracks were



S - SENSITIVE REGION; I - ION LIMIT; C - CLOUD LIMIT

FIG. 10 CLASSIFICATION OF EXPANSIONS

considered sensitive. Lack of these tracks indicated insensitivity. Considerations of the overall pattern of tracks for a series of photographs of a given expansion enables one to classify it.

For lightly ionizing poles only class A expansions are suitable. However for poles of the Dirac type classes B, C, or D are satisfactory provided one uses only the sensitive portions. Classes E and F are considered unsuitable for the present search.

After the photographs were classified, they were scanned for suspects making use of expected properties of a monopole track. Suspects singled out in the preliminary search were then subjected to a more careful study to determine their true identity.

3. Identification of Tracks. The main criteria used in determining the identity of a track are its ionization properties, track width, and scattering. The ionization properties of a magnetic monopole in comparison with charged particles was adequately discussed in chapter two. These predictions were made by assuming that the mass of magnetic monopole is large compared to the mass of an electron. No known theory exists for the ionization of light poles.

In Figures 11 A-H the ionization is plotted as a function of the range for poles of varying masses and varying pole strengths. The pole strengths considered were  $e$ ,  $4e$ ,  $16e$ , and  $\frac{137e}{2}$ .  $e$  is the electronic charge in electro-static units and the pole strength is in electromagnetic units. Four different masses were considered:  $10m$ ,  $250m$ ,  $1836m$ , and  $4700m$ .  $m$  is the electronic mass. The values  $250m$  and  $1836m$  were chosen to correspond to the meson and proton masses.

The graphs were based on formulas derived by Cole and applied to poles traveling through helium gas. From equation 2-5 the average energy loss per unit length of path  $\frac{\Delta T}{\Delta X}$  is

$$\frac{\Delta T}{\Delta X} = \frac{4\pi n g^2 e^2}{mc^2} \sum_r \ln \left( \frac{K' V^2 m' c}{f_r g e} \right) \quad (5-1)$$

The range R is given by

$$R = \frac{T}{\frac{\Delta T}{\Delta X}} = \frac{T}{\frac{4\pi n g^2 e^2}{mc^2} \sum_r \ln \left( \frac{K' V^2 m' c}{f_r g e} \right)} \quad (5-2)$$

where T is the initial kinetic energy of the monopole. But

$$T = \frac{1}{2} MV^2 \text{ and } m' = \frac{Mm}{M+m}$$

Therefore

$$R = \frac{MV^2 mc^2}{8\pi n g^2 e^2 \sum_r \ln \left[ \frac{K' c V^2}{f_r g e} \left( \frac{mM}{M+m} \right) \right]} \quad (5-3)$$

We can now solve for R by substituting the appropriate values for the constants. These values are as follows:

$$m = 9.1 \times 10^{-28} \text{ grams}$$

$$c = 3 \times 10^{10} \text{ centimeters per second}$$

$$e = 4.8 \times 10^{-10} \text{ stat-coulombs}$$

$$K' = 1.61$$

$$n = 4.25 \times 10^{19} \text{ atoms per cubic centimeter}$$

$$f_r = 3.72 \times 10^{16} \text{ cycles per second}$$



The value of  $N$  is determined from the known values of temperature and pressure in the chamber. The temperature is  $20^\circ\text{C}$  at the beginning of the expansion and decreases only slightly during the expansion. The pressure is 129 centimeters of mercury.  $f_r$  was given by Bohr and assumed to be the same for each electron in the helium atom. Substituting these values into the expression for  $R$ , we find

$$R = \frac{3.32 \times 10^{-9} \text{ MV}^2}{g^2 \ln \left( 2.71 \times 10^3 \frac{v^2}{g} \frac{mM}{m+M} \right)} \quad (5-4)$$

From equation 2-7, the ionization of a magnetic monopole in ion pairs per centimeter is

$$\frac{\Delta I}{\Delta X} = \frac{2\pi n g^2 e^2}{nc^2 W_1} \sum_r \ln \left( \frac{2m^2 v^2}{K^2 m W_r} \right) \quad (5-5)$$

where  $K = .618$ ,  $W$  is the ionization energy of the least bound electron and  $W_r$  is the ionization energy of the  $r$ th electron. For the helium atom  $W_1 = 3.92 \times 10^{-11}$  ergs and  $W_2 = 8.66 \times 10^{-11}$  ergs. Substituting these values into the expression above we have

$$\frac{\Delta I}{\Delta X} = 7.68 \times 10^{18} g^2 \ln \left( 9.95 \times 10^{18} \frac{Mm}{M+m} v \right) \quad (5-6)$$

Graphs were made from this equation and equation 5-4.

These graphs show the general nature of ionization along the track of a monopole. All possible combinations of masses and pole strengths given were considered except for a few cases where the formulas of Cole do not hold. Care was taken to avoid the relativistic region since Cole's formula for ionization does not contain relativistic corrections.

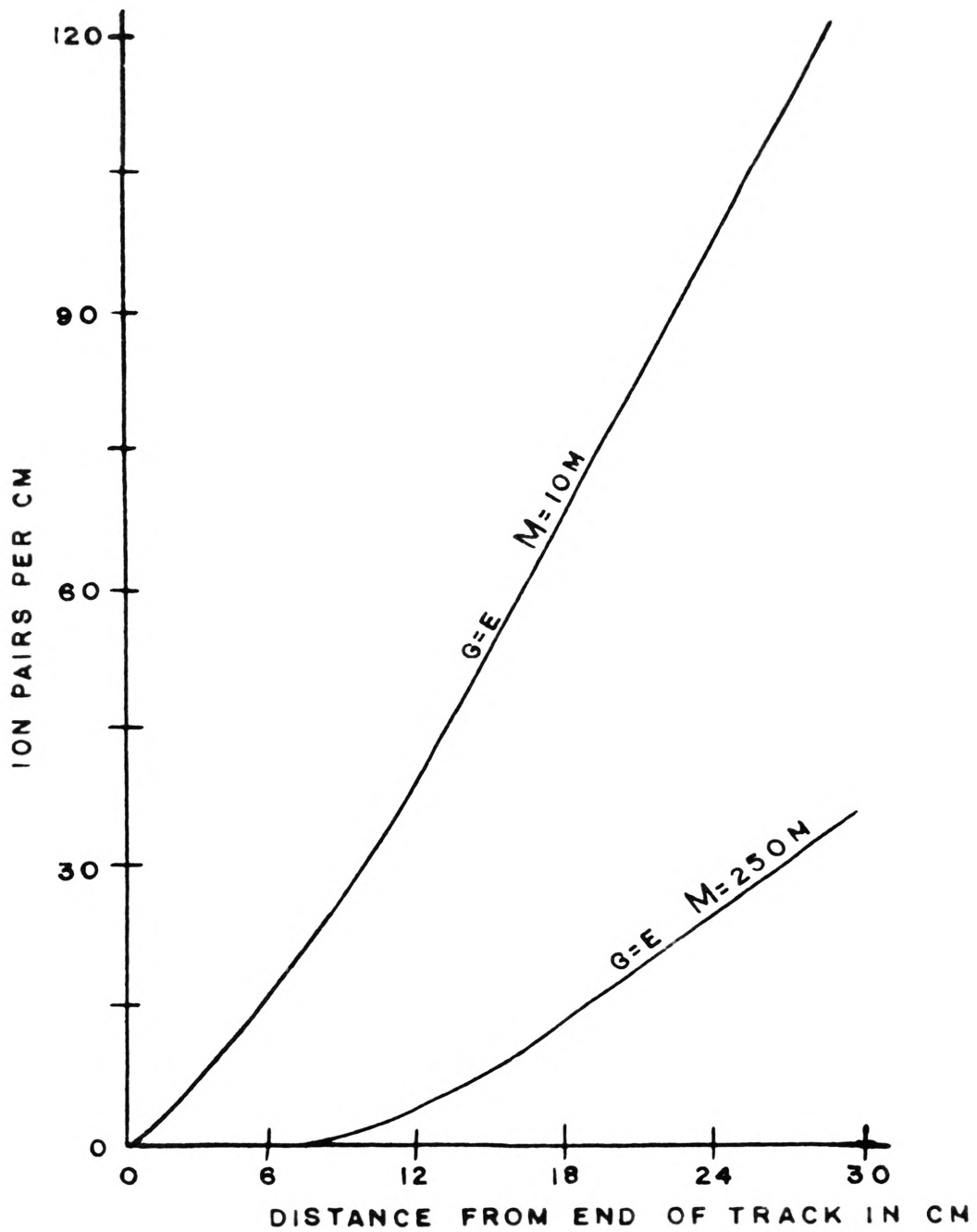


FIG. IIA IONIZATION AS A FUNCTION OF DISTANCE FROM END OF TRACK FOR MONOPOLES TRAVELING THROUGH HELIUM



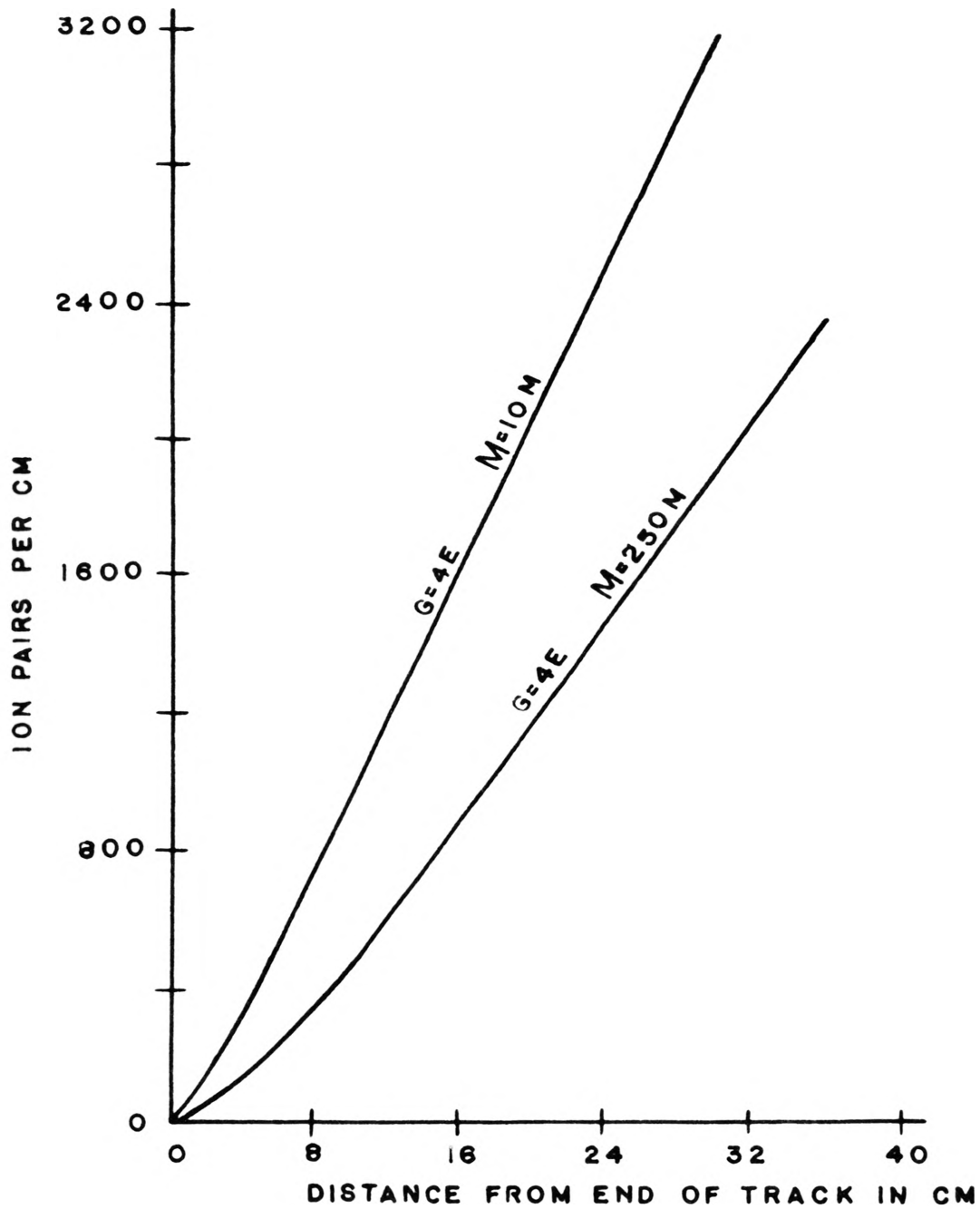


FIG. 11B

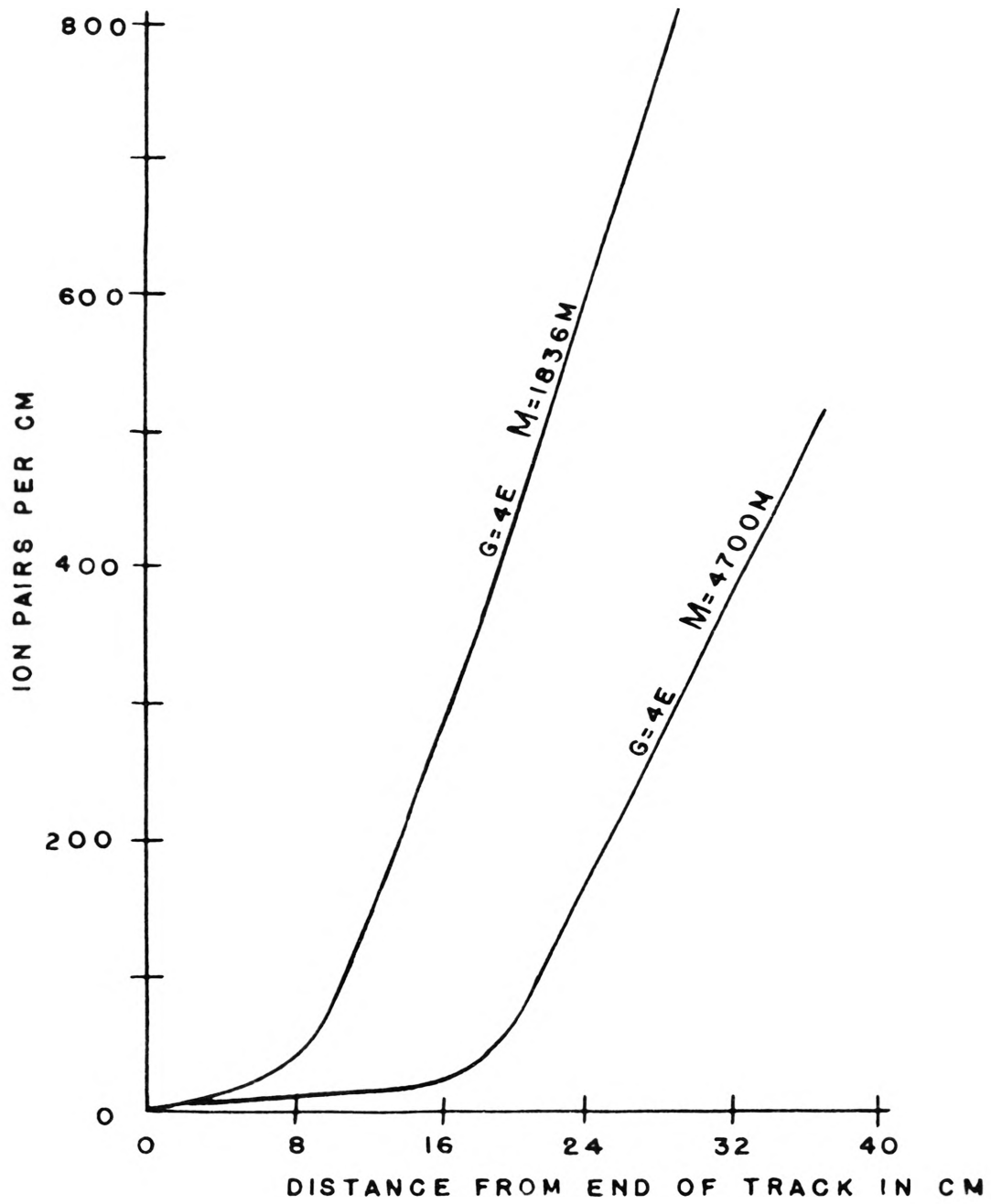


FIG. IIC

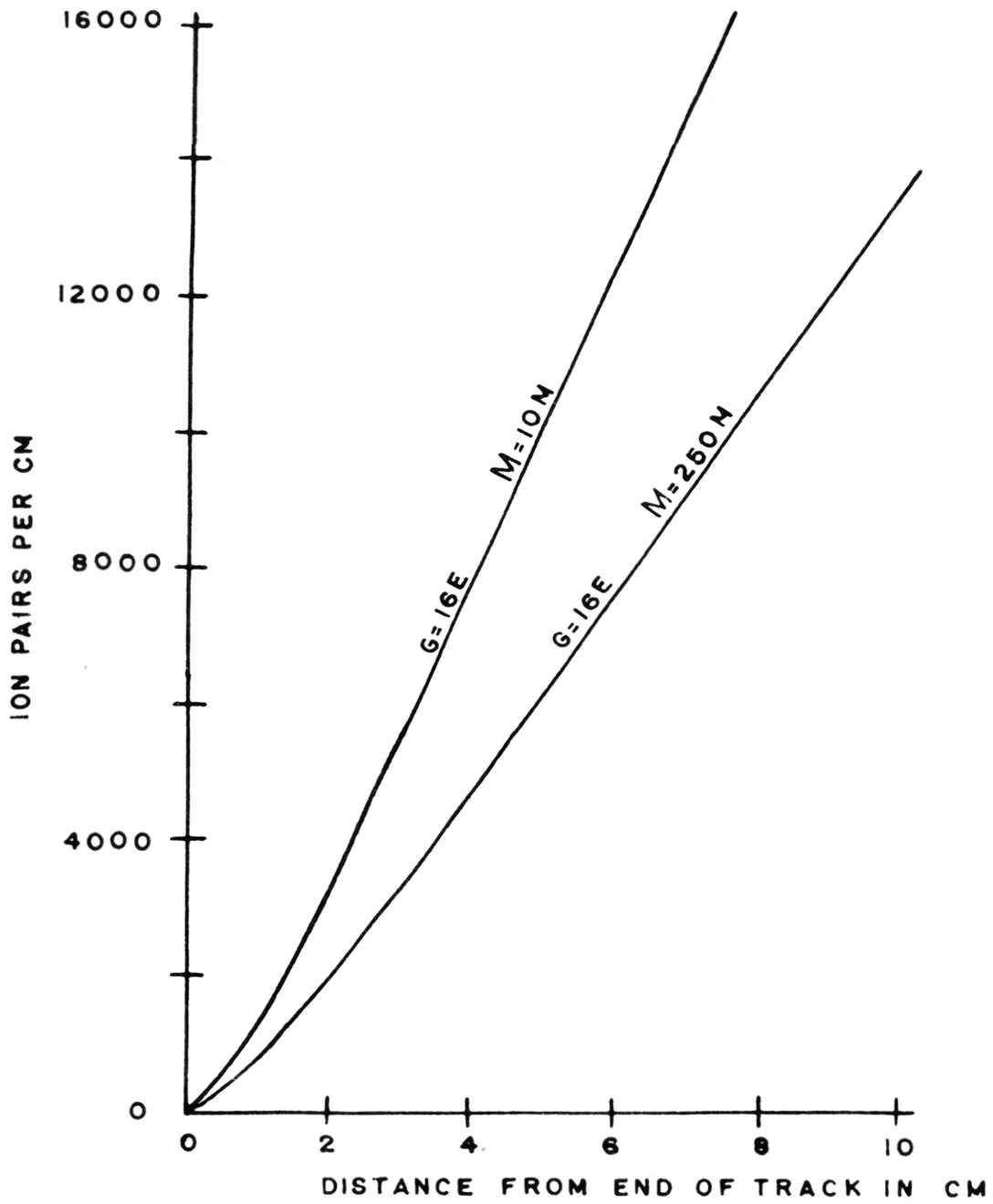


FIG. IID

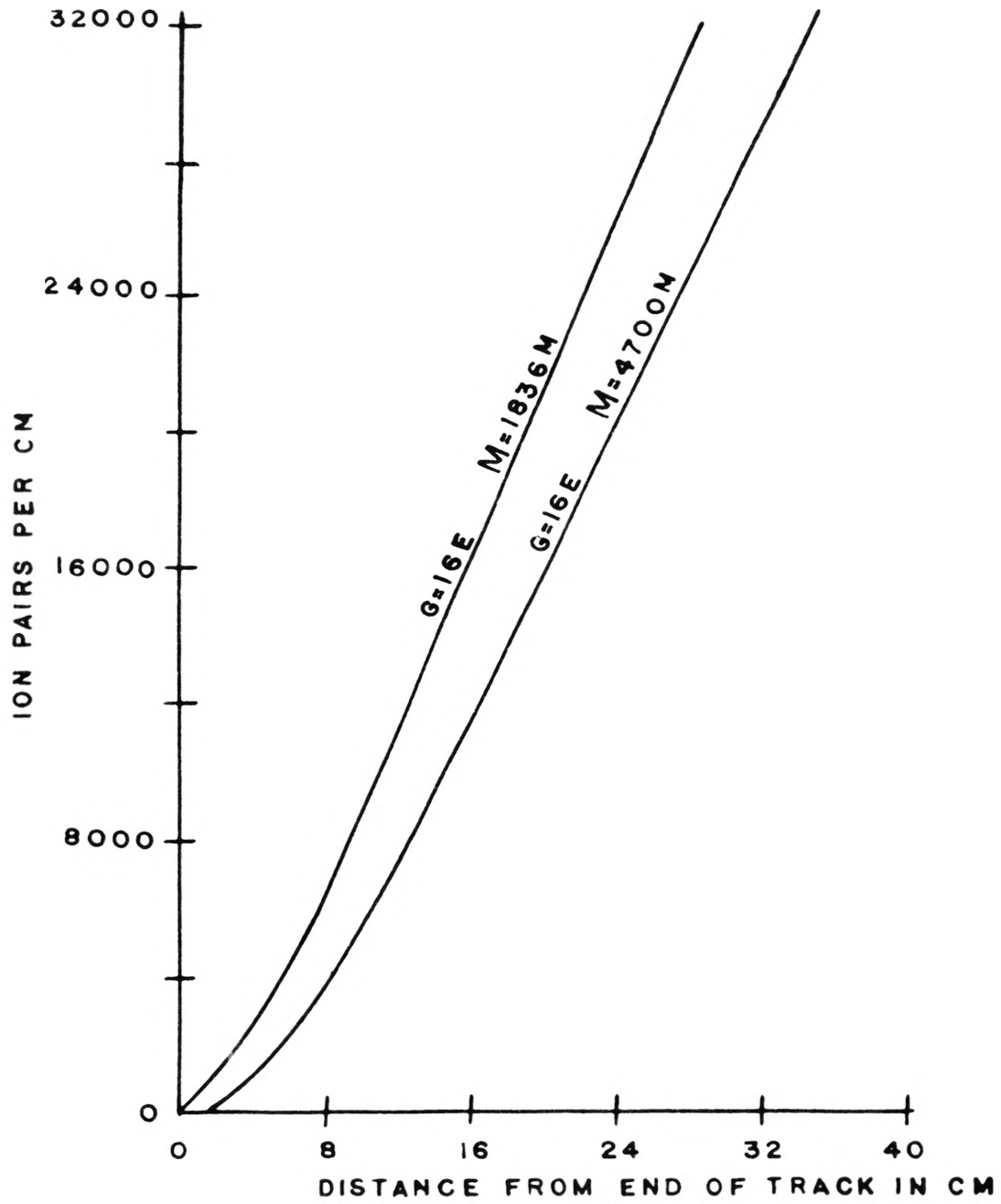


FIG. IIE

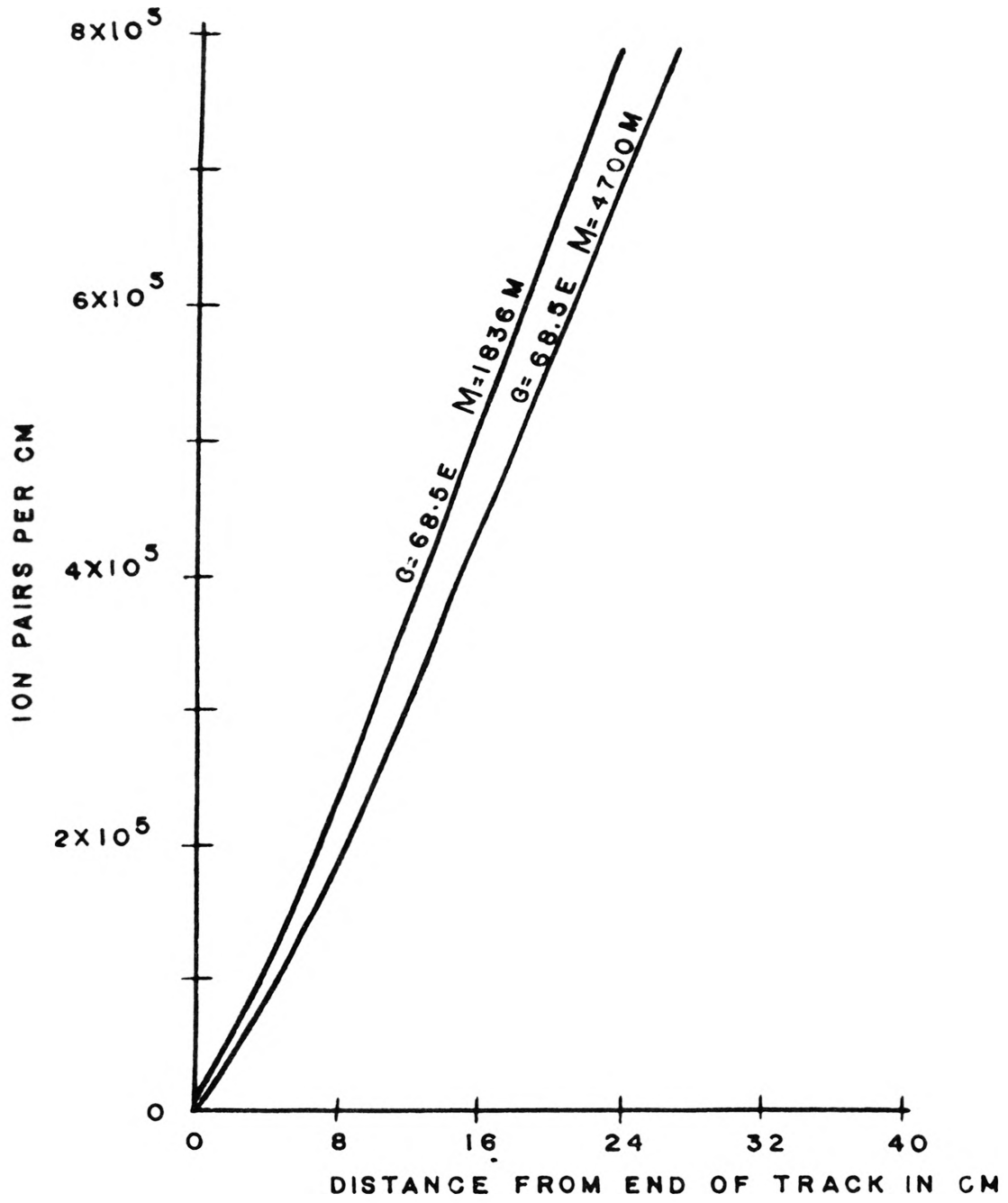


FIG. IIF

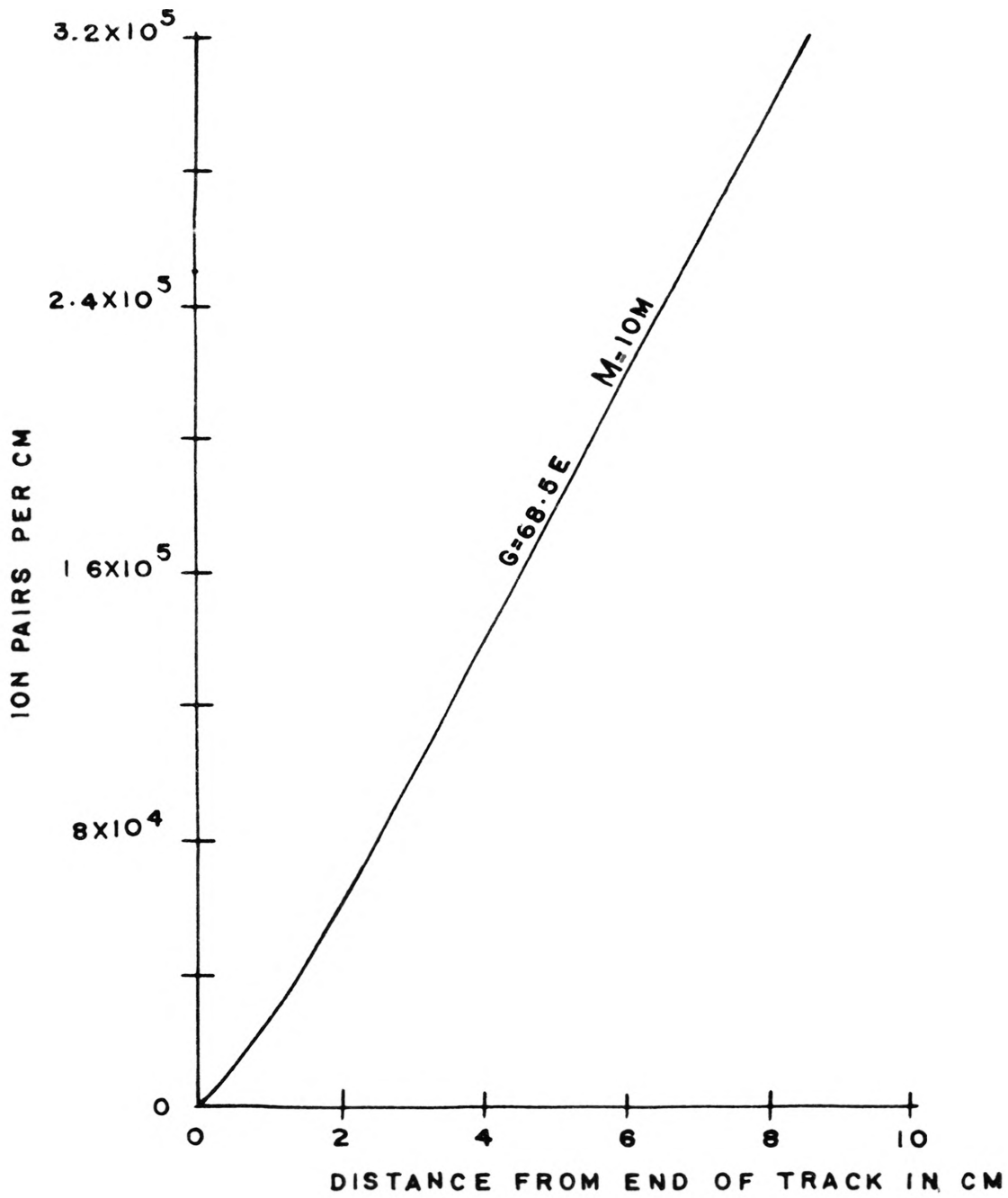


FIG. 11 G

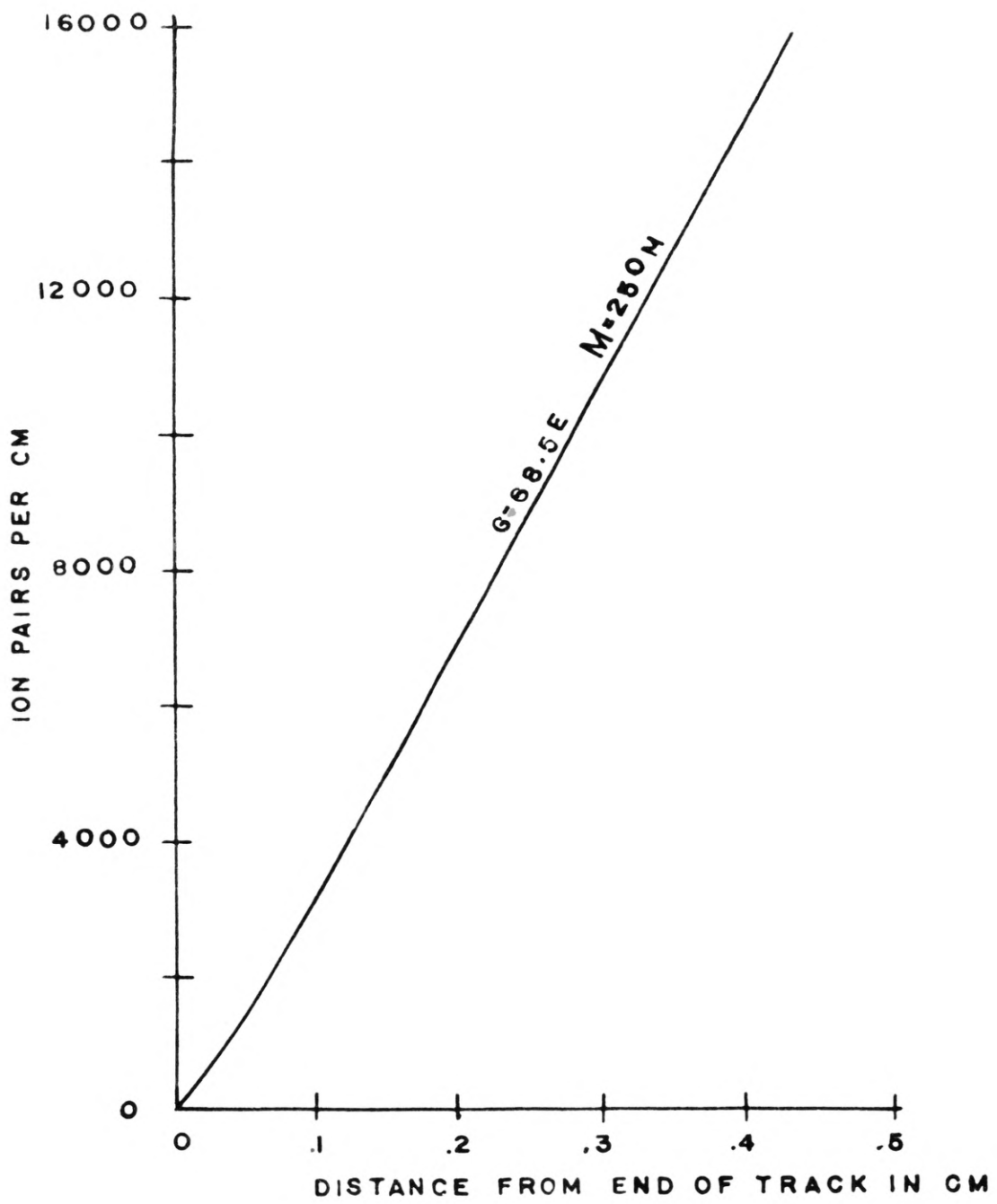


FIG. 11 H

The ionization alone is not enough to establish the identity of a monopole track since individual drop counts cannot be made if the ionization is too heavy. One can only observe the general shape of the track. Even then one must eliminate recombination processes. It was shown in chapter two that this can be done by measurements of width along the track.

Another aid in determining the identity of a track comes from considerations of scattering. Since the electric field of a moving pole depends upon the velocity, scattering should be much less at the end of the track than for a charged particle. It was shown quantitatively in chapter three that the scattering of a monopole is inversely proportional to the square of the velocity (see equation 3-33). On the other hand, the scattering due to a charged particle is inversely proportional to the fourth power of the velocity. Consequently tracks of charged particles with small masses become very curly toward the end of the track. An example of this is shown in plate 1. This would not be the case for a monopole track, particularly if the mass were large. It might be noted that a track of a monopole probably could not be identified unless it ended in the chamber. Its properties are such that it would be difficult to distinguish at any distance from the end of the track.

Before a track can definitely be identified one must be certain that the chamber is uniformly sensitive throughout. This can be determined by observations of other tracks. If ordinary tracks appear to die out or taper off in any particular region of the chamber, that region is probably insensitive and reliable information cannot be taken from it. Likewise, any track ending in regions near the chamber walls must be



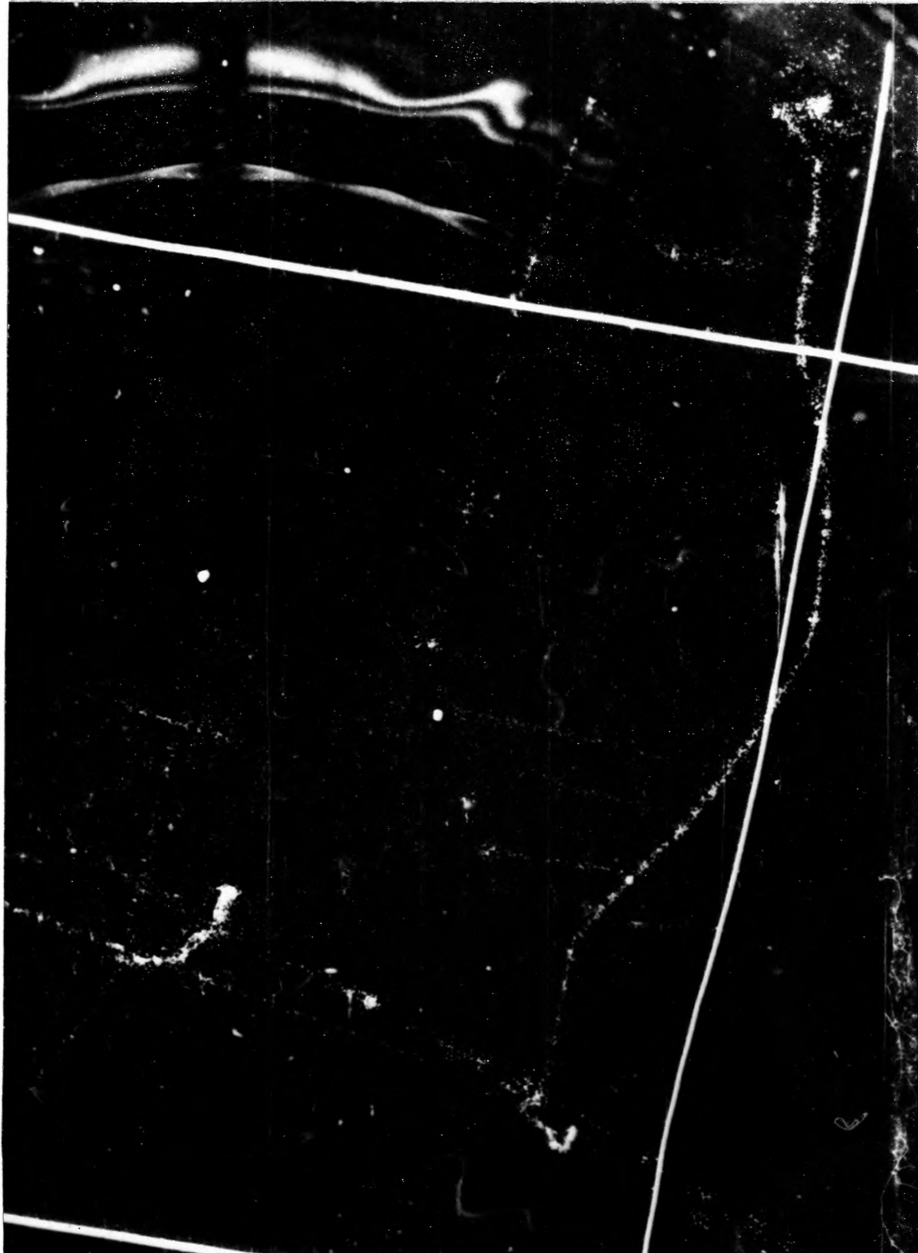


Plate 1. Example of Ionization and Scattering at the End of the  
Track of a light Charged Particle.



Plate 2. Example of a Tapering Track due to non-uniform  
Sensitivity

eliminated from consideration since the sensitivity of these regions is doubtful. Tracks ending near other heavily ionizing tracks such as alpha particle tracks and those ending in regions of heavy background must also be rejected since the sensitivity in these regions is usually not very high. Any tracks present at the beginning of an expansion cannot be considered since diffusion, fall out, and recombination processes may have had time to distort the properties of the track.

4. Data and Results. Observations of 110 expansions were made. The classification of these tracks is shown in Table I below. Expansions E and F were eliminated as unsuitable. A total of 1656 tracks were examined. This corresponds to a total sensitive time of about 130 seconds. None of the tracks observed could be attributed to any type of magnetic monopole. A preliminary search revealed several suspects which were eliminated in a more detailed study by application of the above criteria. A few short heavy tracks coming into the chamber appeared to resemble monopole tracks but are considered to be too close to the chamber walls to be reliable. Furthermore, their properties were consistent with those of alpha particles which frequently occur in helium gas. The results of this experiment can be classified with all previous experiments as negative.

CLASS OF EXPANSIONS	TOTAL NUMBER OF TRACKS
A	676
B	575
C	86
D	319
E	137
F	87
TOTAL	1880

Table I. Classification of Tracks Observed

## CHAPTER VI

### CONCLUSION

1. Results of Previous Work. In this paper an attempt has been made to analyze to a limited extent all known theoretical and experimental work done on monopoles. It is shown that there are some theoretical justifications for belief in the existence of monopoles but there is no experimental evidence in favor of this. If they do exist, it is probably safe to say that they are either very rare or they are in a form in which they cannot be easily detected. It is reasonable to ask what factors could have prevented their discovery. One possibility is that poles may be bound together in pairs by their large attractive forces as indicated in chapter one. Another possibility is that the probability of pair production may be extremely small. This would be the case if the mass were very large since the energy necessary for pair production is proportional to the mass. This would also be the case if the dimensions of the monopole were very small since the pair production cross section is proportional to the square of its linear dimensions. These factors and perhaps many others could explain why monopoles have not been discovered. Research work directed toward the detection of monopoles is still being carried on at other laboratories.

2. Suggestions for Further Work. The failure of all previous experimental work to demonstrate the existence of monopoles suggests two alternatives. Monopoles either do not exist or the problem of detection is rather special in nature and would probably require a

considerable amount of effort and planning. Such work cannot be attempted here. Only a few general comments can be made.

If one is going to use conventional methods, an accelerating magnetic field would seem desirable. This would increase the energy of the particles and make detection more likely. It would also seem desirable to employ magnetic fields for deflection purposes as suggested by Tuve.<sup>71</sup> The direction of deflection would enable one to distinguish between a magnetic monopole and a charged particle. Such a combination of magnetic fields could be used in conjunction with a cloud chamber like the one used in the present experiment. A strong solenoid of the type used by Malkus (see Figure 10) could be fastened to the side or top of the chamber. Two poles of a strong electromagnet could be placed horizontally across the chamber and oriented so that their magnetic field is perpendicular to the field of the solenoid. Particles coming into the chamber through the solenoid would pass through field of the electromagnet. The direction of deflection could be observed directly. No experimental work of this type is known to the author.

Katz and Parnell<sup>72</sup> suggests applying an electric field to a liquid helium bubble chamber. The monopole would travel in a helical path along the electric field. This would enable one to definitely identify its track. The field strength necessary to give a measurable deflection was reported to be about 1000 kilovolts per centimeter and it was stated that liquid helium can stand electric fields of this order of magnitude without breakdown.

Before any experimental work is done one should decide what are some possible sources for monopoles. Some possibilities which we have indicated

earlier in this work as follows: (1) monopoles might be present in cosmic radiation, (2) they might be drifting along the earth's magnetic field, (3) they might be found in ferromagnetic materials, and (4) they might be produced artificially in high energy processes. One would expect however that in any of these sources, their numbers would be extremely small and detection apparatus should be designed accordingly.

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## VITA

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