30 Mar 2001, 4:30 pm - 6:30 pm

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M. R. Massimino
University of Catania, Italy

M. Maugeri
University of Catania, Italy

D. Novità
University of Catania, Italy

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M. R. Massimino
Department of Civil and
Environmental Engineer
University of Catania, Italy

M. Maugeri
Department of Civil and
Environmental Engineer
University of Catania, Italy

D. Novità
Department of Civil and
Environmental Engineer
University of Catania, Italy

ABSTRACT

A well-controlled test was carried out on a Leighton Buzzard sand-shallow foundation system by means of the six-degree-of-freedom shaking table available at the University of Bristol. The foundation consists of a concrete block located into a flexible shear-stack (Taylor et al., 1994) filled up to 1.00 m with the sand. During the test the block was subjected to a centred vertical load and to one direction sine dwell-type acceleration applied at the base of the shear stack.
The static and dynamic sand properties were evaluated through different laboratory tests, among them resonant column tests, cyclic and monotonic loading torsional shear tests were performed (Mazzarella, 1999). A comprehensive network of accelerometers and displacement transducers was used to check the static and dynamic soil-foundation interaction (Maugeri et al., 1999a).
The impedance functions (Gazetas, 1991) were evaluated and then compared with the experimental results. Finally, the experimental results were compared with the numerical ones obtained by means of a FEM code (Massimino, 1999) developed at the University of Catania.

INTRODUCTION

Several studies have been developed on the impedance approach for the evaluation of the static and, above all, the dynamic behaviour of shallow foundation (Lysmer and Richardt, 1966). Nevertheless, a very few examples of comparison between theoretical and experimental results are reported in literature (Carrubba and Maugeri, 1995). The comparison between the experimental and theoretical results is very important to estimate the accuracy of the theoretical formulation; the comparison of the experimental results with the numerical ones point out the grade of applicability of the numerical approaches, world widely used. Unfortunately, in literature there are not so many systematic comparisons between theoretical, numerical and experimental results, due essentially to the high cost of full scale forced vibration tests. Thus, the reported shaking table test, performed on a shallow foundation embedded in dry sand deposit, was very useful to make this kind of comparison. The experimental test, carried out by means of the shear-stack (Taylor et al., 1994) with horizontal moveable boundaries, enables us to avoid interference at the boundaries between the soil and the shear-stack walls.

SOIL-FOUNDATION CHARACTERIZATION

The utilized foundation is a concrete block of 4,2 kN, 0,40 m high *(2H)*, 0,40 m wide *(2B)* and 0.95 m long *(2L)*, with Young modulus E equal to 28.500N/mm^2 and Poisson ratio υ equal to 0.15.
During the test performed on the six-degree of freedom shaking table available at Bristol University, the foundation was located into a flexible shear-stack (Taylor et al., 1994), filled up to 1.00 m with the Leighton Buzzard sand. The foundation block was embedded of 0,1m (D) in the sand, so that the distances of the block from the bottom and from the walls of the shear-stack were respectively 0,90m and 2,20m for each side.
In order to analyse the static behaviour of the soil-foundation system and to evaluate the vertical stiffness, the block was loaded, during different steps, with three steel plates of 10kN each. The load was set in a centred position with respect to the foundation. The static settlements, due to the applied load, were evaluated by means of the displacement transducers located in the four corners of the concrete block *(Phase 1)*. *Phase 2* is related to the dynamic test performed applying a one direction sine dwell-type acceleration to the shaking table along the transversal axis of the foundation (Maugeri et al., 1999a). By means of the displacements measured in seismic condition, the horizontal and rocking stiffness were evaluated.
As far as the soil properties are concerned, the geotechnical characterisation of the Leighton Buzzard sand was possible by means of laboratory tests. Among the static tests, the shear one, carried out both by the large Casagrande box (10x10x2cm), must be mentioned; while among the dynamic tests, resonant column tests (RC) and cyclic loading torsional shear tests (CLTS) were

performed (Mazzarella, 1999). With reference to the shear tests, a linear relationship was observed between the height of deposition H_{dep} and the relative density Dr, such as between Dr and the shear resistance angle ϕ' (Cavallaro et al., 2001). The appropriate value of ϕ' was evaluated using the graphs above mentioned and considering the real height H_{dep} of the sand into the shear-stack, that was kept constant and equal to 60cm during the whole deposition. The Dr value, estimated by Cavallaro et al. (2001), appeared in good agreement with that evaluated experimentally, that was 48.5%. With reference to Fig. 1, ϕ' was estimated equal to 40°.

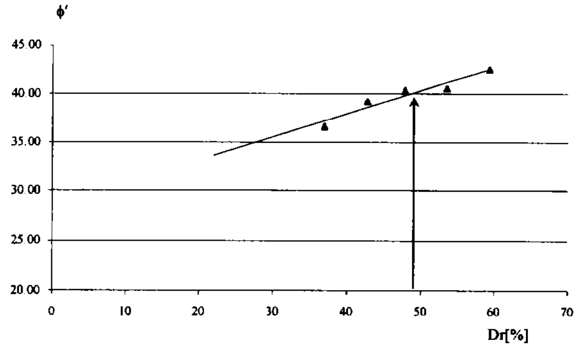

Fig. 1. Large Casagrande box shear test: Dr versus ϕ' (after Cavallaro et al., 2001).

The behavior of the soil under loads variable with the time, such as the seismic ones, can be analyzed studying the strain characteristics of the soil itself, particularly the shear modulus and the damping ratio variation with the deformation level.
In this sense, RC and CLTS tests were carried out and the initial shear modulus G_0 was evaluated under small deformation level ($10^{-4} < \gamma < 10^{-3}$ %). By means of the results of the laboratory tests, an average value of G_0 equal to 141MPa was estimated for a confining pressure σ'_c equal to about 150kN/m^2.
Actually, the shear modulus G_0 depends on different factors and among them it depends of Dr and on the stress and strain level. In order to consider the dynamic soil parameters, which take into account the real properties of the sand into the shear-stack, it was observed that: i) the test on the shaking table was performed with a confining pressure much more smaller in comparison with that applied during the RC and CLTS tests (σ'_c=4,4 kN/m^2); ii) the deformation level achieved during the test cannot be classified as a small deformation, since it is much more bigger than $\gamma < 10^{-3}$ %. For the confining pressure σ'_c=4,4 kN/m^2, used in the experiment, G_0 was evaluated through the correlation given by Seed & Idriss (1970):

$$G_0 = 1000 \cdot k_2 \cdot \sqrt{\sigma'_c} \qquad (1)$$

where k_2, equal to 4,77, depends on Dr and γ. By this correlation G_0 = 2197 kN/m^2 was obtained, which is considerable less than that evaluated by the laboratory tests performed at a much higher confining pressure. The shear wave velocity V_s was evaluated by means of the mass density ρ and the last value of G_0 through the expression $G_0 = \rho \cdot V_s^2$ (being $\rho = \gamma/g$=1,57kNs^2/m^5). The result obtained was V_s = 37 m/s.

STATIC AND DYNAMIC EXPERIMENTAL RESULTS

During the test performed on the shaking table, the instrumentation adopted (Maugeri et a., 1999a) allowed the measurement of the settlements along the vertical z axis and of the horizontal displacements along the y axis, transversal to the block (Fig. 2).

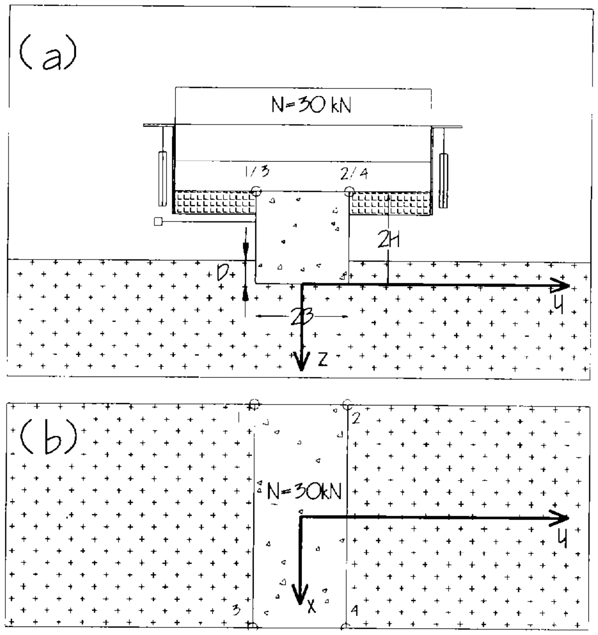

Fig. 2. Soil-foundation-super structure system: (a) front view; (b) plan view.

As far as the horizontal displacements along the longitudinal x axis are concerned, they were neglected since the length of the block was near the full width of the shear-stack; however the block was separated from it by thin strata of sponge, so no movement was possible.
Figures 3a and 3b show the static and dynamic displacements that took place when respectively a vertical central load N=30kN (*Phase 1*) and an unidirectional horizontal excitation (*Phase 2*) were applied. The horizontal excitation was a quasi-harmonic function, characterised by constant frequency and variable amplitude.
In Figs. 3a and 3b the dynamic input is represented by $H_{dyn.}$, evaluated taking into account the weight of the block and the weight of the steel plates. The shear-stack base acceleration was equal to 0.150 g and 0,265 g respectively in the first step (*Run I*) of the dynamic phase and in the second step (*Run II*) of the dynamic phase.
In *Run I*, with reference to the accelerations measured in the foundation (a_F = 0.10 g) and in the steel plates ($a_{S\text{-}P}$ = 0.20 g), the horizontal force $H_{dyn.}$ was evaluated equal to 6.42 kN. In *Run II*, (a_F = 0.13 g and $a_{S\text{-}P}$ = 0.26 g) $H_{dyn.}$ was equal to 8.37 kN.
The final vertical and horizontal displacements (w and u_y) and residual rotation (ϑ) measured at the static and dynamic phases of the experimental test (Maugeri et al., 1999b), are summarised in Tab.1.

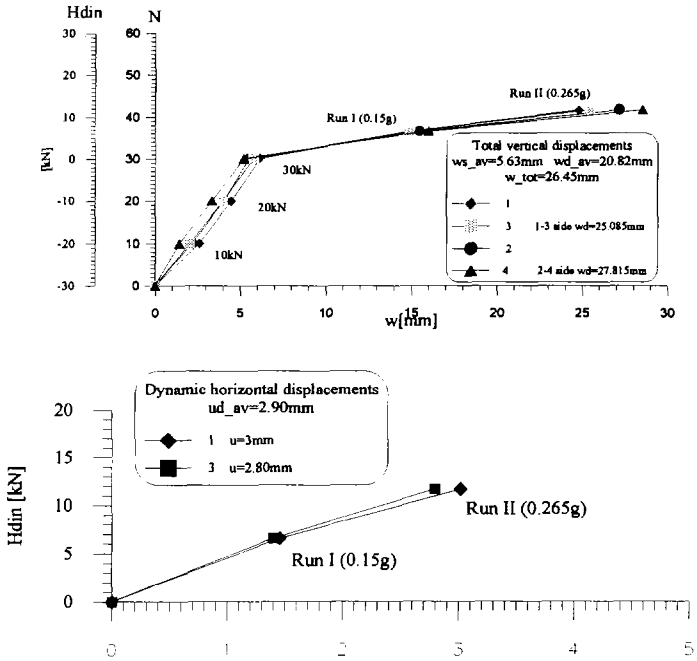

Fig. 3. (a) Total settlements; (b) Horizontal dynamic displacements.

Tab. 1. Shaking Table test results

Static Phase		Dynamic Phase				
N [kN]	w_{st} [mm]	H_{dyn} [kN]	$wtot_{1\text{-}3}$ [mm]	$wtot_{2\text{-}4}$ [mm]	u_y [mm]	ϑ [°]
30	5.63	8.37	25.09	27.82	2.91	0.39

IMPEDENCE FUNCTION EVALUATION

Two different mechanisms of interaction can take place between the structure, the foundation and the soil when a seismic excitation occurs: an *inertial interaction* and a *kinematic interaction* (Stewart et al., 1999).

In this paper, just the soil-structure interaction was focused by means of the impedance approach. In particular, the elastic-linear approach proposed by Gazetas (1991) was taken into account. The approach considers the equivalent lumped mass-spring-dashpot system in order to reproduce the behaviour of the half-space and allows the computation of the six dynamic displacements and rotations of a rigid block due to harmonic excitation.

For each mode of vibration, the dynamic impedance $k(\omega)$ can be expressed in the form:

$$k(\omega)= \bar{K}_d(\omega)+ i\,\omega\, C(\omega) \tag{2}$$

where the "dynamic stiffness" $\bar{K}_d$, which reflects the stiffness and the inertia of the supporting soil, and the "dashpot coefficient" C, which represents the radiation and material damping, are both not constant but functions of the circular frequency ω.

The approach developed by Gazetas (1991) is based on easy-to-use expressions and graphs drawn for different foundation shapes, partially or fully embedded and for homogeneous or inhomogeneous soil profiles. Thus, taking into account the geometrical and mechanical properties of the experimental system, the six theoretical impedances can be evaluated.

In the present paper only the following dynamic stiffnesses were estimated: the vertical stiffness $\bar{K}_{d,z}$; the longitudinal and lateral stiffnesses for horizontal motion in the long and in the short direction of the foundation, $\bar{K}_{d,x}$ and $\bar{K}_{d,y}$; the rocking impedances $\bar{K}_{d,rx}$ and $\bar{K}_{d,ry}$ for rotation motion applied in the long and in the short axis of the foundation; and the torsional stiffness $\bar{K}_{d,t}$, for rotational oscillation around the vertical axis (Tab 3). As an example, the expression proposed by Gazetas (1991) to compute the vertical static stiffness for a shallow foundation resting on a homogeneous half-space is the following:

$$K_{z,surf,st}=\frac{2GL}{1-\nu}(0.73+1.54\ ^{0.75}) \tag{3}$$

Moreover, for a partially embedded foundation, the expression (3) becomes:

$$K_{z,emb,st}=K_{z,surf,st}\left[1+\frac{1}{21}\frac{D}{B}(1+1{,}3\chi)\right]\cdot\left[1+0{,}2\left(\frac{A_w}{A_b}\right)^{2/3}\right] \tag{4}$$

being $\chi=A_b/4L^2$ with $A_b=2Lx2B$ and A_w the actual area sidewall-soil contact surface.

The vertical dynamic stiffness $\bar{K}_{z,emb,d}(\omega)$ is equal to $K_{z,emb,st}\cdot k_z$, being k_z the vertical dynamic coefficient; in turn k_z depends on L/B, ν and a_o. The a_o parameter must be computed by means of the expression $a_o=(\omega\cdot B)/V_s$, where ω is the circular frequency of the external force. For the quasi-harmonic function applied along the longitudinal axis of the shear stack ω is equal to 31,4Hz and then a_o is equal to 0,021. The correspondent dynamic coefficient k_z is equal to 1 (Gazetas,1991). Then, according to Gazetas (1991), the other dynamic stiffnesses were evaluated (Table 3).

Tab. 3. The evaluation of $\bar{K}_{emb,d}$ according to Gazetas (1991)

Imped.	$K_{emb,st}$	k	$\bar{K}_{emb,d}=K_{emb,st}*k$
k_z	5519 kN/m	1	5519 kN/m
k_y	5744 kN/m	1	5744 kN/m
k_x	5331 kN/m	1	5331 kN/m
k_{rx}	295 kNm	0.975	287 kNm
k_{ry}	1094 kNm	0.962	1053 kNm
k_t	4331 kNm	0.982	4254 kNm

NUMERICAL ANALYSIS

The experimental results were, finally, compared with the numerical ones carried out by means of the finite element SOFIA code (Massimino, 1999). The soil-foundation scheme is reported in Fig. 4, where it is possible to see a soil mesh characterised by 1831 nodes and 572 elements. The isoparametric quadratic soil elements are variable in size, moving from the shear stack boundaries toward the foundation block. The nodes of the lowest horizontal soil boundary are completely fixed, while the nodes of the two vertical soil boundaries are free in the vertical direction. Over the foundation block, embedded of 0.10 m, an overload is applied.

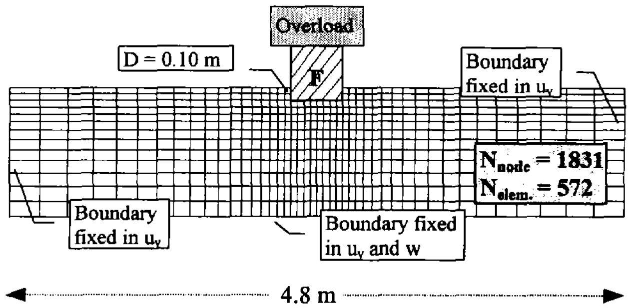

Fig. 4 Soil-foundation scheme developed with the SOFIA code

The numerical analysis was performed in two different and subsequent phases: in the first one (*Phase 1*) only the vertical load due to the foundation weight and an overload of 30 kN were taken into account; in the second one (*Phase 2*) the dynamic shaking of the system was considered by means of the pseudo-static approach, widely used in the routine design, such as in different numerical and experimental analyses (Oliva et al., 1990; Carrubba et al., 2000). Horizontal forces were applied to the foundation and to the overload, as the product of each weight for the corresponding critical acceleration, a_{max}, recorded during the experimental test.
As far as the constitutive laws are concerned, the foundation behaviour was considered linear-elastic, while the soil behaviour was considered non-linear, according to a revised Duncan and Chang (Duncan and Chang, 1970) stress-strain relationship (Massimino, 1999). Particularly, during the loading path the Young modulus value is updated step by step following a hyperbolic trend and starting from an initial value E_0 determined by the theory of elasticity from G_0. The G_0 value was evaluated according to expression (1), where k_2 was estimated for very low γ level and for D_r = 48 % (Seed & Idriss, 1970).
Besides, even if the analyses were performed in plain strain condition, the real tridimensionality of the experimental system was taken into account by means of an approximate procedure included in the SOFIA code. This procedure, on the basis of the Boussinesq theory, modifies conveniently the Young modulus of each soil element.
The SOFIA code output allows us to investigate on the stress and strain level and distribution reached in the soil and in the foundation. In particular, in the present case the strain condition was carefully analysed and compared to the experimental results.
In Figs. 5a and 5b the vertical and horizontal movement inside the soil due to the weight of the foundation (4.2 kN) and to the overload of 30 kN are reported respectively (*Phase 1*). Both figures show a perfect symmetry of the soil behaviour in respect to the *z* axis. In particular, the foundation settlement, useful to compute the vertical stiffness $K_{z,emb,st}$, is equal to 5.7 mm.
The horizontal soil movements are very small and are essentially due to the vertical lowering of the foundation block.
When the pseudo-static horizontal forces are considered during *Phase 2* (Figs. 6a and 6b), a strong asymmetric soil behaviour was analysed, with a concentration of the vertical and horizontal movements inside the failure area shown by the experiment (Maugeri et al., 1999a). More precisely, as far as the vertical soil movements are concerned, the applied loads gave the formation of a settlement area lightly shifted in respect to the vertical load axis. On the other hand, the horizontal soil movements achieved very high values, showing the greatest concentration in the failure area. Simulating the dynamic experimental test, the foundation horizontal movement estimated numerically was of about 3,15mm, while the rotation around the *x* axis was of about 0,90°. In this case, two main foundation movements, related to each other, took place: the horizontal one and the rocking one, as it is possible to note analysing the vertical and horizontal soil movement distribution. Finally, an average foundation vertical displacement of about 5.1 mm was evaluated.
The last vertical displacement must be increased to take into account the compaction effect due to the dynamic input, that is not possible to consider in the pseudo-static approach. The compaction is not negligible in the reported experimental test, considering the dry sand nature of the soil deposit and the low value of the initial relative density. On sand deposit dry or partially saturated the settlement due to the dynamic compaction can represent the greatest part of the total settlement, as observed during different earthquakes and during several laboratory tests performed with the shaking table or the simple shear apparatus (D'Apppolonia, 1968; Silver and Seed, 1971).
Because of this, the additional foundation vertical displacement was evaluated through the approach proposed by Silver and Seed (1971). According to these Authors, there is a deep relation between the vertical displacements, due to the dynamic compaction of the sand during the application of the seismic action, the number of cycle, the initial relative density and the vertical stress.
Among all these factors, the effect of the confining pressure is negligible; the phenomenon is essentially governed by the shear

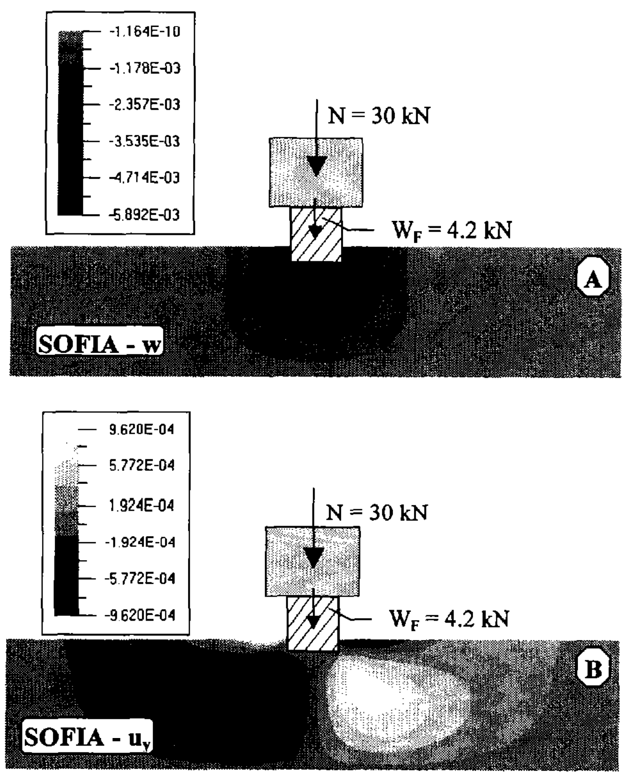

Fig. 5. Vertical and horizontal movement inside the soil for the static condition (Phase 1).

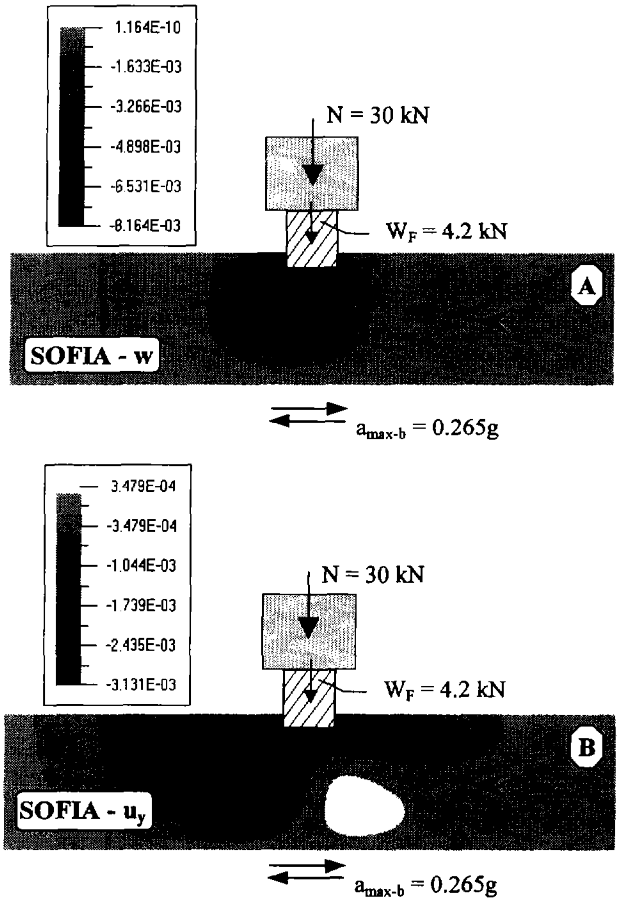

Fig. 6. *Vertical and horizontal movement inside the soil for the dynamic condition (Phase 2).*

strain and the initial relative density (Maugeri and Carrubba, 1991).

Because of this, the additional foundation vertical displacement was evaluated through the approach proposed by Silver and Seed (1971). According to these Authors, there is a deep relation between the vertical displacements, due to the dynamic compaction of the sand during the application of the seismic action, the number of cycle, the initial relative density and the vertical stress. Among all these factors, the effect of the confining pressure is negligible; the phenomenon is essentially governed by the shear strain and the initial relative density (Maugeri and Carrubba, 1991).

For the reasons above mentioned, considering the real number of cycles applied in the shear-stack during the test and the shear strain (Maugeri et al., 1999a), the vertical strain ε_c due to the compaction was 0,8% (Fig. 7). Taking into account the multidirectional shaking that takes place under an earthquake loading, the volumetric strain must be doubled (Tokimatsu and Seed, 1987), so that $\varepsilon_{c\text{-}tot}$ was equal to 1,6% and the vertical displacements due to the seismic compaction was 14,4mm. Finally, adding the settlement determined by the pseudo-static numerical analysis, equal to 5,1 mm, a total average foundation settlement of 19,5 mm is reached. It is important to underline that the estimated total settlement of 19,5 mm should be increased, considering that Fig.7 is valid for Dr = 60 %, while in the experimental test the sand relative density was of 48,5 %.

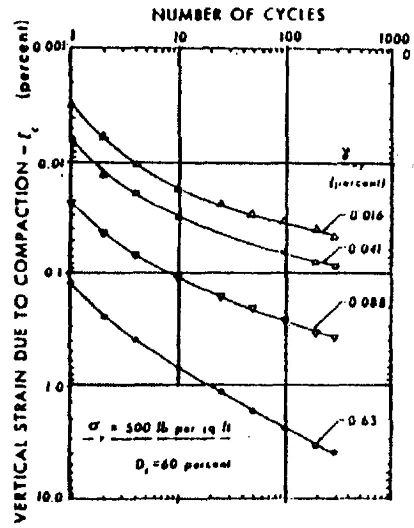

Fig 7 Relation between vertical strain, cycly number and shear strain for a siliceous sand (Silver and Seed, 1971)

COMPARISON BETWEEN EPERIMENTAL, THEORETICAL AND NUMERICAL RESULTS

The comparison between the experimental, theoretical and numerical results (Table 4) is very useful, since it allows us to evaluate the grade of applicability of both theoretical and numerical approaches. With this aim, even if the reported experimental test presents some limitations due to the scale effects and the system to simulate earthquakes, it is a very precious test, considering also the great costs necessary to carry out full-scale model.

Particularly, in the present paper, the vertical, horizontal and rocking stiffnesses evaluated by the shaking table test were compared with the theoretical (Gazetas, 1991) and the numerical ones. As far as the experimental values are concerned, the static stiffness along the z axis and the dynamic stiffness along the y axis were evaluated as the ratio of the applied load, respectively equal to the vertical and the pseudo-static horizontal one, over the corresponding displacement. No dynamic stiffness along the transversal axis was evaluated since no displacement was allowed in this direction. The rocking dynamic stiffness $\bar{K}_{rx,d}$ was estimated considering the rotation which arose at the application of the maximum acceleration. For the $\bar{K}_{y,d}$ and $\bar{K}_{rx,d}$ evaluation, both *Run I* and *Run II* were considered, to emphasise the impedance degradation with the increase of the dynamic input.

The results reported in Table 4 show that, as regards the vertical stiffness, both the theoretical and numerical values are in good agreement with the experimental ones, while about the horizontal and rocking stiffnesses, it is possible to note some divergences. The better result achieved for $K_{z,st}$, i. e. for static condition, in comparison with $\bar{K}_{y,d}$ and $\bar{K}_{rx,d}$, i. e. for dynamic condition, is mainly due to the absence in static condition of the dynamic compaction and to the movement coupling effect. Moreover, the experimental and numerical values of $K_{z,st}$, very close to the Gazetas's one, underline substantially for the *Phase 1* an elastic-linear behaviour, confirmed also by the experimental *N-w* curve (Fig. 3).

As far as the $\bar{K}_{y,d}$ and $\bar{K}_{rx,d}$ are concerned, it is possible to note a small divergence between the experimental and theoretical values since *Run I*. This divergence increases in *Run II*. It could be due to the gradual approaching the failure condition, which cannot be investigated by the elastic-linear Gazetas's procedure (1991). Finally, the divergence existing between the experimental and numerical values can be due to the pseudo-static approach used for the numerical simulation.

Tab. 3. Comparison between experimental, theoretical and numerical results

	Static cond.		Dynamic condition				
	w_{st} [mm]	$K_{z,st}$ kN/m		u_y [mm]	$\bar{K}_{y,d}$ kN/m	ϑ [°]	$\bar{K}_{rx,d}$ kN m
Exper.	5.63	5329	*Run I*	1.43	4490	1°	197
			Run II	2.91	2877	1.46°	157
Theor.	-	5519	-	-	5744	-	287
Numer.	5.70	5263	*Run II*	3.15	2657	0,9°	281

CONCLUSION

The experimental analysis of the static and dynamic soil-foundation system behaviour, analysed by means of a shaking table test, allow us to investigate on the soil-foundation interaction theoretical (Gazetas, 1991) and numerical (Carrubba et al., 2000) approaches, through the impedance function concept. First of all it must be underlined that both the theoretical and numerical procedures offer in this case values close enough to the experimental ones. Besides, the experimental tests are necessary to validate theoretical and numerical procedures. Nevertheless the theoretical procedures are very interesting for their fast applicability above all in dynamic condition; the numerical procedures are particularly useful to capture the global mechanisms, investigating also the strain and stress distribution inside the soil bed.

REFERENCES

Carrubba P., Maugeri M. [1995]. "Formulation and validation of the vertical impedance of a full-scale shallow foundation", Proc. 7nt Italian National Conf. L'Ingegneria Sismica in Italia, Siena (Italy), September 25-28, 1995, pp. 165-174 (in Italian).

Carrubba P., Massimino M.R., Maugeri M. [2000]. "Strain dependent impedance in shallow foundations" Proc. 12th World Conf. on Earth. Eng., January 30 - February 4, 2000, Auckland, New Zealand.

Cavallaro A., Maugeri M., Mazzarella R. [2001]. "Static and dynamic properties of Leighton Buzzard sand from laboratory tests", Proc. 4th International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, San Diego, California, USA March 26-31.

D'Appolonia E. [1968]. "Dynamic loading", State of the Art Report. Presented at the Aug. 1968, ASCE, Spec. Conf., Cambridge Mass.

Duncan J.M. and Chang C.Y. [1970]. Non linear analysis of stress and strain in soils", JSMFD, ASCE, 96, pp. 1629-1653.

Gazates, G. [1991]. "Foundation Vibrations.", in *Foundation Engineering Handbook*, (H.Y. Fang, ed.) Van Nostrand Reinhold, New York, NY, pp. 553-593.

Lysmer J., Richardt F.E. Jr. [1966]. "Dynamic response of footings to vertical loading", Journ. of Soil Mech. and Found. Div., Proc. ASCE, SM1, January 1966, pp. 65-91.

Massimino M. R. [1999]. "Non-linear analysis of soil-foundation-superstructure interaction by means of a new FEM code", PhD Thesis, University of Catania 1999 (in Italian).

Massimino M.R. and Maugeri M. [1999]. "Non-linear analysis of soil-structure interaction for a building on sand", Proc. 2nd Int. Symp. On Pre-failure Deformation Characteristics of Geomaterials, 27-29 September, Torino, Italy.

Maugeri M., Carrubba P. [1991]. "Soil and foundation dynamic", In Argomenti di Ingegneria Sismica, Augusta March-May 1991 (in Italian).

Maugeri M., Musumeci G., Novità D., Taylor C. A. [1999a]. "Shaking table test of a shallow foundation", Proc. Int. Conf. on Earthquake Resistant Engineering Structures (ERES99), Catania, Italy, June 15-17, pp. 595-604.

Maugeri M., Musumeci G., Novità D., Taylor C.A [1999b], "Shaking table test of a failure of a shallow foundation subjected to an eccentric load", 9th International Conference on SDEE'99, 9-12 August, Bergen, Norway.

Mazzarella R. [1999]. "Static and dynamic characterization of the Leighton Buzzard sand", Degree Thesis, University of Catania 1999 (in Italian).

Oliva M., Gavrilovic P. and Clough R.W. [1990]. "Seismic testing of large panel precast walls: comparison of pseudostatic and shaking table tests", Jour. of Earth. Eng. and Struct. Dyn., Vol. 19, pp. 859-875.

Richard R., Elms D.G., Budhu M.[1993]. "Seismic bearing capacity and settlements foundations ", Journal of Geotechnical and Engineering, Vol. 119 No.4, April, pp. 662-674.

Seed H.B. and Idriss I.M.[1970]."Soil moduli and damping factors for dynamic response analyses", Research Report EERC70-10, University of California, Berkeley.

Silver M.J., Seed H.B.[1971]."Volume changes in sands during cyclic loading", Journal of Soil Mech. and Found. Div., SM9, Sept., pp 1171-1182.

Stewart J.P., Fenves G.L, Seed R.B. [1999]. "Seismic soil-structure interacion in buildings. I: Analytical Methods", Journal of Geotechnical and Environmental Engineering, January 1999, Vol. 125 No.1, pp. 26-37.

Taylor C.A. , Dar A.R. and Crewe A.J. [1994]. "Shaking table modelling of seismic geotechnical problems", 10th European Conf. On Earthquake Engineering, Vienna, pp.441-446.

Tokimatsu K., Seed H.B. [1987]. "Evaluation of settlements in sands due to earthquake shaking", Journal of Geotechnical Engineering, Div., ASCE, Vol. 113, No.8, pp. 861- 878.