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ECONOMIC AND ENVIRONMENTAL COMPARISON OF DIFFERENT ORDERING POLICIES FOR AN INTEGRATED INVENTORY CONTROL AND SUPPLIER SELECTION PROBLEM

by

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ABSTRACT

This study analyzes an integrated inventory control and supplier selection problem in stochastic demand environment under carbon emissions regulations. In particular, a continuous review inventory model with multiple suppliers is investigated under carbon taxing and carbon trading regulations. We analyze and compare the optimal supplier selection and order splitting decisions with single sourcing and two alternative delivery structures for multi-sourcing, namely, sequential ordering and sequential delivery. For each of the three ordering policies, a solution method is proposed and these policies are compared in terms of their economic as well as environmental performances. A numerical study is conducted to demonstrate the efficiencies of the solution methods proposed. Further numerical studies analyze how the economic and environmental performances of different ordering policies vary as the supplier capacities and lead times change.

Keywords: Carbon emissions, Continuous Review Inventory

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1. INTRODUCTION

Global warming is a growing concern and carbon emissions are a leading contributor to global climate change which was created increasing pressure around the world to enact legislation to curb these emissions. Carbon emission regulations have emerged to address these issues and incentivize firms to curb greenhouse gas (GHG) emissions, primarily carbon-dioxide (other GHG emissions can be measured in terms of carbon-dioxide, see e.g., EPA 2014). Furthermore, the increased environmental awareness of consumers enforces firms to green their operations to stay competitive. Industry and transportation sectors are the largest contributors to GHG emissions. For instance, industry and transportation sectors generated 29% and 15% of the global GHG emissions in 2010 (ECOFYS, 2010). The U.S. Environmental Protection Agency (EPA) reports that industrial and transportation sectors contributed 20% and 28%, respectively, to national GHG emissions in 2012 (EPA, 2014). Thus, a very large fraction of carbon emissions are due to supply chain activities including inventory holding, freight transportation, and logistics and warehousing activities.

Inventory management is particularly important for a company as this determines not only the level of inventory carried and warehousing activities but also the amount and the frequency of freight shipments and logistical operations. The inventory control policy of a company, therefore, is inextricably linked with its environmental performance. There is a growing body of literature that analyzes inventory control models with environmental considerations. As will be reviewed in Section 2, these studies include environmental aspects of the inventory related operations by either associating direct costs with the environmental damage due to the inventory related operations or considering environmental objectives such as emissions minimization along with the classical economic objectives such as cost minimization (profit maximization) or modeling the inventory control policies under environmental regulations such as carbon cap, carbon tax, carbon trading, or carbon offsetting. In this study, we incorporate the environmental aspects of inventory related operations by formulating an inventory control model under carbon taxing and carbon trading policies. Specifically, under carbon taxing, a company pays taxes for the emissions it generates.

The tax per unit carbon emissions is defined by governmental agencies. European countries Denmark, Finland, Sweden, Netherlands, and Norway are among the first countries that implemented carbon taxing (Lin and Li, 2011). Under carbon trading, on the other hand, a company is subject a carbon emissions limit per unit time, which is known as carbon cap, and carbon emissions are tradable through an emissions trading system such as European and New Zealand Emissions Trading systems. That is, the company can buy extra carbon allowances or sell its excess carbon emissions.

Particularly, our focus is on a retailer's integrated inventory control and supplier selection problem under the aforementioned environmental regulations. We consider the case of stochastic demand and assume a continuous review inventory control system. The retailer can split his/her order among an arbitrary number of heterogeneous suppliers.

2. LITERATURE REVIEW

Sustainability has been considered in various operations and supply chain management settings (see, e.g., the reviews by Corbett and Kleindorfer, 2001a, b, Linton et al., 2007, Srivastava, 2007). In this study, we integrate sustainability in an inventory control model with multiple sources of supply. In case of multiple sources of supply, the supplier selection models have been introduced for companies to choose the suppliers to build relationships with. Supplier evaluation and selection models have been intensively studied in the literature. One may refer to Ho et al. (2010) for a review of supplier evaluation and selection studies. Generally, supplier selection models constitute multi-attribute decision making problems and various methods such as data envelopment analysis, mathematical programming, analytic hierarchy process, fuzzy set theory, and ranking methods have been utilized to help companies evaluate and select suppliers (Ho et al., 2010).

With increasing sustainability concerns along supply chains, environmental considerations have also been considered in supplier selection models. In particular, green supplier selection models take into account not only the supplier attributes considered in the classical supplier evaluation and selection models but also environmental/sustainability attributes of the suppliers. Igarashi et al. (2013) note that product- and company-related environmental attributes are mainly introduced in green suppliers' selection models. We refer the reader to Genovese et al. (2010), Govindan et al. (2013), and Igarashi et al. (2013) for reviews of the green supplier selection models.

Our study does not consider a multi-attribute supplier selection model with environmental considerations. We rather consider an inventory control model with multiple possible source of supply under environmental regulations. Therefore, in the following review, our focus is on the inventory control studies that account for environmental aspects of the inventory related operations. We distinguish such studies based on the demand characteristics (deterministic vs. stochastic demand), sourcing characteristics (single vs. multiple supply sources), and model characteristics. Most of the studies that integrate environmental aspects into inventory control models focus on well-known inventory control models such as the economic order quantity model, economic lot-sizing model, and single-period stochastic demand model, and their variations. Furthermore, environmental aspects of the inventory related operations are integrated into these models through either modeling environmental regulations such as carbon cap, taxing, trading, and offsetting or associating direct costs with environmental pollution generated from inventory control related operations or regarding environmental objectives along with the classical economic objectives. This study considers a stochastic demand continuous review inventory control model with multiple supply sources under environmental regulations. . In particular, Benjaafar et al. (2012), Absi et al. (2013), Palak et al. (2014), and Helmrich et al. (2015) study ELS problems under environmental regulations and, Mafakheri et al. (2011) and Azadnia et al. (2014) formulate a multi-objective EL with environmental considerations. Among these studies, while Absi et al. (2013) and Palak et al. (2014) account for different sources of supply by considering different transportation modes, Mafakheri et al. (2011) and Azadnia et al. (2014) directly integrate supplier selection decisions with ELS model and assess the supplier’s environmental performance in the selection. Unlike these studies, we consider a stochastic inventory control model over a long planning horizon instead of multi-period deterministic demand model. Furthermore, we model different delivery structures in case of multiple sourcing. Most of the stochastic inventory control models with environmental considerations revisit the classical single-period stochastic demand model, i.e., the Newsvendor model. The Newsvendor model maximizes the expected profits due to a single order by considering the costs associated with unsold items in case of overage and unmet demand in case of underage. Song and Leng (2012), Zhang and Xu (2013), Choi (2013a,b), Liu et al. (2013), Rosic and Jammernegg (2013), Hoen et al. (2014), and Arikan and Jammernegg (2014) study the Newsvendor model and its variations (including dual sourcing and multi-item settings) under environmental regulations. Among these studies, Hoen et al. (2014) consider different modes of transportation and Choi (2013a,b), Rosic and Jammernegg (2013), and Arikan and Jammernegg (2014) integrate different sourcing channels (dual sourcing with a local and an off-shore supplier) as alternative options to order from.

The most related study to ours is the one by Arikan et al. (2013). Arikan et al. (2013) numerically demonstrate how the costs and carbon emissions generated change with different transportation modes and delivery lead times when a cost- or emissions-minimizing order quantity-reorder point policy is used for ordering decisions. That is, they do not consider order splitting and environmental regulations.

PAPER

Economic and Environmental Comparison of Different Ordering Policies for An Integrated Inventory Control and Supplier Selection Problem

October 30, 2014

ABSTRACT

This study analyzes an integrated inventory control and supplier selection problem in stochastic demand environment under carbon emissions regulations. In particular, a continuous review inventory model with multiple suppliers is investigated under carbon taxing and carbon trading regulations. We analyze and compare the optimal supplier selection and order splitting decisions with single sourcing and two alternative delivery structures for multi-sourcing, namely, sequential ordering and sequential delivery. For each of the three ordering policies, a solution method is proposed and these policies are compared in terms of their economic as well as environmental performances. A numerical study is conducted to demonstrate the efficiencies of the solution methods proposed. Further numerical studies analyze how the economic and environmental performances of different ordering policies vary as the supplier capacities and lead times change.

Keywords: Carbon emissions, Continuous Review Inventory

1. INTRODUCTION

There is a growing consensus that carbon emissions are a leading contributor to global climate change which was created increasing pressure around the world to enact legislation to curb these emissions. Carbon emission regulations have emerged to address these issues and incentivize firms to curb greenhouse gas (GHG) emissions, primarily carbon-dioxide (other GHG emissions can be measured in terms of carbon-dioxide, see e.g., EPA 2014). Furthermore, the increased environmental awareness of consumers enforces firms to green their operations to stay competitive. Industry and transportation sectors are the largest contributors to GHG emissions. For instance, industry and transportation sectors generated 29% and 15% of the global GHG emissions in 2010 (ECOFYS, 2010). The U.S. Environmental Protection Agency (EPA) reports that industrial and transportation sectors contributed 20% and 28%, respectively, to national GHG emissions in 2012 (EPA, 2014). Thus, a very large fraction of carbon emissions are due to supply chain activities including inventory holding, freight transportation, and logistics and warehousing activities.

Inventory management is particularly important for a company as this determines not only the level of inventory carried and warehousing activities but also the amount and the frequency of freight shipments and logistical operations. The inventory control policy of a company, therefore, is inextricably linked with its environmental performance. There is a growing body of literature that analyzes inventory control models with environmental considerations. As will be reviewed in Section 2, these studies include environmental aspects of the inventory related operations by either associating direct costs with the environmental damage due to the inventory related operations or considering environmental objectives such as emissions minimization along with the classical economic objectives such as cost minimization (profit maximization) or modeling the inventory control policies under environmental regulations such as carbon cap, carbon tax, carbon trading, or carbon offsetting. In this study, we incorporate the environmental aspects of inventory related operations by formulating an inventory control model under carbon taxing and carbon trading policies. Specifically, under carbon taxing, a company pays taxes for the emissions it generates. The tax per unit carbon emissions is defined by governmental agencies. European countries Denmark, Finland, Sweden, Netherlands, and Norway are among the first countries that implemented carbon taxing (Lin and Li, 2011). Under carbon trading, on the other hand, a company is subject a carbon emissions limit per unit time, which is known as carbon cap, and carbon emissions are tradable through an emissions trading system such as European and New Zealand Emissions Trading systems. That is, the company can buy extra carbon allowances or sell its excess carbon emissions.

Particularly, our focus is on a retailer's integrated inventory control and supplier selection problem under the aforementioned environmental regulations. We consider the case of stochastic demand and assume a continuous review inventory control system. The retailer can split his/her order among an arbitrary number of heterogeneous suppliers. We note that inventory control models with order splitting among multiple sources of supply have been studied in the literature. In this study, the sources of the supply are defined as suppliers; hence, order splitting decisions also determine the supplier selection decisions. Nevertheless, the sources of supply can be not only different suppliers (distribution centers, manufacturers) but also different transportation modes available for shipment, or even different carriers of the same transportation mode such as different truck/vehicle types (see, e.g., Konur, 2014 and ?) or truckload and less-than-truckload carriers (see, e.g., Konur and Schaefer, 2014). The models and solution methods discussed in this paper, therefore, apply to the integrated stochastic inventory control and transportation mode selection and/or integrated stochastic inventory control and carrier selection problems.

One may refer Minner (2003) for a review of inventory control models with supplier selection. Our study considers stochastic demand and the inventory control models with supplier selection under stochastic demand are grouped into two classes: models with deterministic and stochastic lead times (Minner, 2003). Similar to Moinzadeh and Nahmias (1988), Moinzadeh and Schmidt (1991), Zhang (1996), Chiang and Gutierrez (1996), and Jain et al. (2010), we assume that suppliers have deterministic lead times, i.e., they are reliable. One may refer to Thomas and Tyworth (2006) for a review of stochastic inventory control models with order splitting in case of stochastic lead times.

The suppliers considered herein vary in their shipping specifications (delivery lead times and freight minimums) and shipping costs (unit procurement and fixed delivery setup costs) as well as environmental characteristics (per unit and fixed emissions generation from order shipments). Different suppliers can have different delivery lead times due to distinct points of origin or transportation modes used for delivery. Due to the same reasons, the suppliers might have varying unit procurement costs (which can include the unit purchasing/manufacturing and unit shipping cost) and fixed delivery costs as well as carbon emission generation characteristics. Therefore, similar to the most of the studies integrating inventory control and supplier selection, we consider heterogeneous suppliers. Furthermore, similar to Burke et al. (2007), Dai and Qi (2007), Awasthi et al. (2009), and Zhang and Zhang (2011), we account for supplier capacities and assume that different suppliers have different capacities. For instance, different transportation modes have different capacities or different vehicle types of the same transportation mode can have different capacities (various freight trucks have different volume/weight limits, see, e.g., Konur, 2014).

In most of the integrated inventory control and multi-sourcing models, the split orders are assumed to be delivered to the retailer sequentially. That is, after the retailer places the orders, the supplier with the lowest lead time (or the lowest realized lead time in case of stochastic lead times) delivers first, then the supplier with the second lowest lead time delivers second, and so on (in case of stochastic lead times, it is possible that different suppliers deliver simultaneously). As noted by Glock (2012) as well, delivery structure of the orders affects the inventory related costs. Furthermore, as is discussed in this study, different delivery structures have different environmental performances. Therefore, it is important to consider different delivery structures in integrated inventory control and supplier selection models.We note that different delivery structures are generally modeled for the supplier (or manufacturer) in twoechelon supply chains in the context of shipment consolidation (see, e.g., Çetinkaya (2005) for a review of consolidation policies) or multi-item inventory systems in the context of joint replenishment problem (see, e.g., Khouja and Goyal (2008) for a review of joint replenishment problems). Unlike shipment consolidation and joint replenishment problems, this study analyzes a single-echelon (retailer) and single-item inventory system with multiple supply sources (suppliers) in a stochastic demand environment. We consider three different ordering policies, namely single sourcing, sequential ordering, and sequential delivery, for the integrated inventory control and supplier selection problem of interest in this study under carbon taxing and carbon trading regulations. In particular, under single sourcing, the retailer does not consider order splitting; hence, he/she chooses the single supplier to order from. Given the selected supplier, the retailer's problem is then to determine the re-order point R (the on-hand inventory level to place an order) and the order quantity, q_i , if supplier i is the single selected supplier. On the other hand, in the case order splitting is considered as an option, the retailer can control the deliveries from different suppliers by changing the order release times to the suppliers. For instance, Kim and Goyal (2009) consider two different delivery options, which they refer to as lumpy and phased deliveries, in a single buyer-multiple suppliers setting. In case of lumpy deliveries, the orders from different suppliers are delivered simultaneously while different suppliers' orders are delivered alternately in case of phased deliveries. Glock (2012) defines six different delivery structures regarding the production cycles of two manufacturers and the delivery at the single buyer. Both of these studies consider the two-echelons (buyer and vendor) of the supply chain simultaneously and they assume deterministic demand. In this study, our focus is on the retailer only and the retailer is subject to stochastic demand. We, therefore, consider two structures for order splitting: sequential ordering and sequential delivery.

Under sequential ordering, the retailer starts ordering from the selected suppliers such that the orders from different suppliers are received simultaneously. Specifically, in the case the retailer enjoys less frequent warehousing activities such as unloading operations and inventory placement, sequential ordering can be preferred. Furthermore, all of the orders are delivered at once under sequential ordering; however, the retailer needs to carefully monitor the timing to release split orders to the suppliers.

On the other hand, under sequential delivery, the retailer places the orders from selected suppliers simultaneously, thus, receives the orders from different suppliers sequentially due to distinct lead times. Therefore, compared to sequential ordering, order placement is simpler under sequential delivery; however, there are more frequent shipments, i.e., more frequent warehousing operations and inventory placements can be required. Figure 1 illustrates the retailer's inventory over time with single sourcing when supplier 2 is selected, sequential ordering, and sequential delivery when an order is split among three suppliers such that $\tau 1 < \tau 2 < \tau 3$, where τi is the lead time of supplier i. This study contributes to the body of literature on inventory control models with environmental considerations by (i) integrating supplier selection decisions in continuous review inventory systems and (ii) regarding different delivery structures. To the best knowledge of the authors, integrated continuous review inventory control and supplier selection models under stochastic demand with environmental considerations have not been analyzed in the literature. Actually, as will be discussed in our literature review, while there is a growing body of literature on environmental inventory control models, most of these studies assume deterministic demand or stochastic demand in the single period. Furthermore, while integrated stochastic inventory control and supplier selection models have been analyzed extensively (see the reviews cited above and the references cited in those reviews), different delivery structures are not considered in such models. Most of the integrated stochastic inventory control and supplier selection studies adopt sequential delivery and focus on the economic comparison of single sourcing and order splitting. In this study, we compare not only single sourcing to order splitting but also two different delivery structures for ordering splitting. And, our comparison evaluates economic as well as environmental performance of the different ordering policies considered.

Specifically, we formulate the retailer's supplier selection and inventory control model under carbon trading regulation with the three ordering policies (it is discussed that carbon taxing is a special case of carbon trading regulation). For each model, a solution method is developed. Then, we compare these three ordering policies not only in terms of economic but also environmental aspects. It is noted that while a retailer can prefer order splitting to minimize costs under carbon trading, single sourcing can be a more environmental alternative. Also, when the two delivery structures for order splitting are compared, we note that there is no pure dominance between them in terms of economic objectives. This observation suggests that sequential ordering can be a better alternative in terms of costs compared to sequential delivery, which, as aforementioned, is the delivery structure commonly assumed in integrated stochastic inventory control and supplier selection models. Furthermore, when sequential ordering (sequential delivery) is a better policy in terms of economic performances, sequential delivery (sequential ordering) can be a better policy in terms of environmental performance. Thus, the retailer's preference for a delivery structure will depend on his/her economic as well as environmental goals. The tools provided in this study enable comparing different delivery structures for multiple sourcing and single sourcing from both economic and environmental aspects. Finally, we conduct a set of numerical studies to demonstrate the efficiency of the proposed solution methods. Further numerical studies are presented to illustrate the effects of supplier characteristics on the economic and environmental performances of the ordering policies.

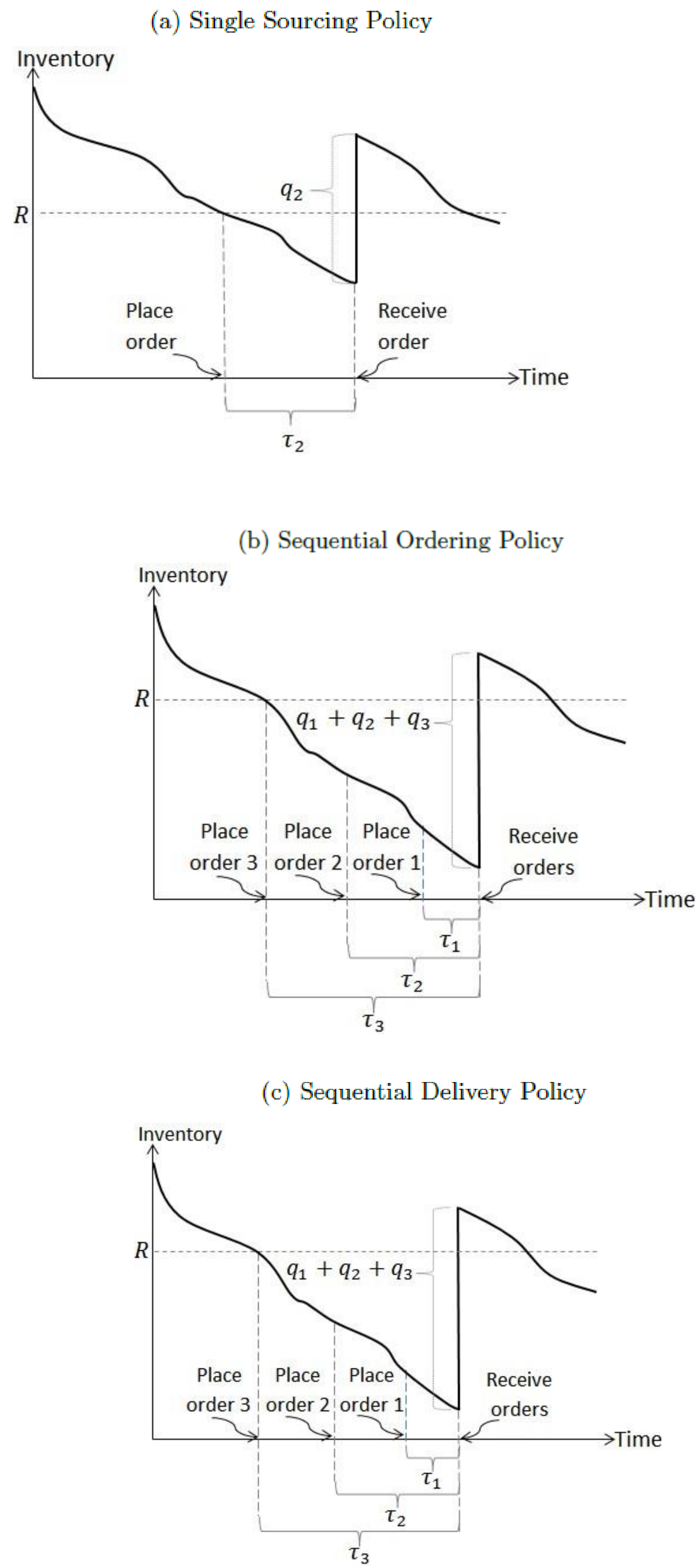

Figure 1: Inventory vs. Time

The rest of the paper is organized as follows. In Section 2, we review the inventory control models with environmental considerations. Section 3 discusses the settings of the problem and formulates the mathematical models of the retailer's optimization problems under single sourcing, sequential ordering, and sequential delivery policies. A solution method for each model is proposed in Section 4. Section 5 economically and environmentally compares the ordering policies. Numerical studies are presented in Section 6 and concluding remarks, summary of contributions, and future research directions are given in Section 7.

2. LITERATURE REVIEW

Sustainability has been considered in various operations and supply chain management settings (see, e.g., the reviews by Corbett and Kleindorfer, 2001a, b, Linton et al., 2007, Srivastava, 2007). In this study, we integrate sustainability in an inventory control model with multiple sources of supply. In case of multiple sources of supply, the supplier selection models have been introduced for companies to choose the suppliers to build relationships with. Supplier evaluation and selection models have been intensively studied in the literature. One may refer to Ho et al. (2010) for a review of supplier evaluation and selection studies. Generally, supplier selection models constitute multi-attribute decision making problems and various methods such as data envelopment analysis, mathematical programming, analytic hierarchy process, fuzzy set theory, and ranking methods have been utilized to help companies evaluate and select suppliers (Ho et al., 2010).

With increasing sustainability concerns along supply chains, environmental considerations have also been considered in supplier selection models. In particular, green supplier selection models take into account not only the supplier attributes considered in the classical supplier evaluation and selection models but also environmental/sustainability attributes of the suppliers. Igarashi et al. (2013) note that product- and company-related environmental attributes are mainly introduced in green suppliers' selection models. We refer the reader to Genovese et al. (2010), Govindan et al. (2013), and Igarashi et al. (2013) for reviews of the green supplier selection models.

Our study does not consider a multi-attribute supplier selection model with environmental considerations. We rather consider an inventory control model with multiple possible source of supply under environmental regulations. Therefore, in the following review, our focus is on the inventory control studies that account for environmental aspects of the inventory related operations. We distinguish such studies based on the demand characteristics (deterministic vs. stochastic demand), sourcing characteristics (single vs. multiple supply sources), and model characteristics. Most of the studies that integrate environmental aspects into inventory control models focus on well-known inventory control models such as the economic order quantity model, economic lot-sizing model, and single-period stochastic demand model, and their variations. Furthermore, environmental aspects of the inventory related operations are integrated into these models through either modeling environmental regulations such as carbon cap, taxing, trading, and offsetting or associating direct costs with environmental pollution generated from inventory control related operations or regarding environmental objectives along with the classical economic objectives. This study considers a stochastic demand continuous review inventory control model with multiple supply sources under environmental regulations.

Most of the deterministic inventory control models with environmental considerations revisit the classic Economic Order Quantity (EOQ) model. The EOQ model analyzes the trade-off between inventory holding and order setup costs for a product that has deterministic demand. Hua et al. (2011), Jaber et al. (2013), Arslan and Turkay (2013), Chen et al. (2013), Toptal et al. (2014), Konur and Schaefer (2014), Konur (2014), and He et al. (2014) study the EOQ model and/or its extensions (to additional decision variables or multi-item/multi-echelon settings) under carbon regulation policies such as carbon cap, taxing, trading, and offsetting. Among these studies, only Konur and Schaefer (2014) and Konur (2014) consider multiple sources of supply. In particular, while Konur and Schaefer (2014) model the EOQ model under four different carbon emissions regulations with less-than-truckload and truckload carriers, Konur (2014) considers different freight trucks for shipments under carbon cap regulation. On the other hand, Bonney and Jaber (2011), Wahab et al. (2011), Ritha and Martin (2012), Digiesi et al. (2012), Ritha and Vinoline (2013), and Battini et al. (2014) analyze inventory control models similar to the EOQ model by directly associating costs to the environmental pollution/carbon emissions generated from the inventory control related operations. Among these studies, Digiesi et al. (2012) and Battini et al. (2014) consider different sources of supply by including different modes of transportation in their models. Finally, Bouchery et al. (2012), Chan et al. (2013), and Bozorgi et al. (2014) integrate environmental aspects into the EOQ model and/or its extensions by considering environmental objectives in addition to the economic objectives and these studies consider single source of supply.

Other than the EOQ model, the economic lot-sizing (ELS) models with deterministic demand have been recently analyzed with environmental considerations. In particular, Benjaafar et al. (2012), Absi et al. (2013), Palak et al. (2014), and Helmrich et al. (2015) study ELS problems under environmental regulations and, Mafakheri et al. (2011) and Azadnia et al. (2014) formulate a multi-objective EL with environmental considerations. Among these studies, while Absi et al. (2013) and Palak et al. (2014) account for different sources of supply by considering different transportation modes, Mafakheri et al. (2011) and Azadnia et al. (2014) directly integrate supplier selection decisions with ELS model and assess the supplier's environmental performance in the selection. Unlike these studies, we consider a stochastic inventory control model over a long planning horizon instead of multi-period deterministic demand model. Furthermore, we model different delivery structures in case of multiple sourcing. Most of the stochastic inventory control models with environmental considerations revisit the classical single-period stochastic demand model, i.e., the Newsvendor model. The Newsvendor model maximizes the expected profits due to a single order by considering the costs associated with unsold items in case of overage and unmet demand in case of underage. Song and Leng (2012), Zhang and Xu (2013), Choi (2013a,b), Liu et al. (2013), Rosic and Jammernegg (2013), Hoen et al. (2014), and Arikan and Jammernegg (2014) study the Newsvendor model and its variations (including dual sourcing and multi-item settings) under environmental regulations. Among these studies, Hoen et al. (2014) consider different modes of transportation and Choi (2013a,b), Rosic and Jammernegg (2013), and Arikan and Jammernegg (2014) integrate different sourcing channels (dual sourcing with a local and an off-shore supplier) as alternative options to order from.

Brito and de Almeida (2012) model a multi-objective Newsvendor model with a single supply source, where one of the objectives is to minimize the environmental damage due to salvaged products in case of overage. In a recent study, Carrillo et al.

(2014) study environmental implications of different retail channels (such as classical channels and online channels) such that the retailer's decision in each channel is defined under the settings of the Newsvendor model. They associate a cost value, which can represent the unit environmental savings or premiums, for the online retailing channel. Similar to these studies, we consider multiple options for sourcing; however, we do consider an arbitrary number of options as the supply sources instead of dual sourcing. Furthermore, we directly integrate sourcing decisions with order decisions instead of analyzing the ordering decisions under each source and compare them. That is, the models we formulate jointly determine the optimal sourcing and ordering decisions under environmental regulations. Also, we consider a continuous review inventory control model instead of a single-period stochastic demand model.

To the best knowledge of the authors, environmental considerations are not directly integrated within continuous review inventory control models. The most related study to ours is the one by Arikan et al. (2013). Arikan et al. (2013) numerically demonstrate how the costs and carbon emissions generated change with different transportation modes and delivery lead times when a cost- or emissions-minimizing order quantity-reorder point policy is used for ordering decisions. That is, they do not consider order splitting and environmental regulations. In this study, we formulate and analyze a continuous review inventory control model under environmental regulations and we integrate supplier selection decisions in this model. Furthermore, different delivery structures are considered in case of order splitting. In the next section, we explain the details of the settings and formulation of the models analyzed in this study.

3. PROBLEM FORMULATION

We consider a retailer's inventory control problem for a single item which has stochastic demand. Let the demand per unit time for the item be a normally distributed random variable with mean λ and standard deviation υ. We therefore assume that the demand during a time period of t is normally distributed with a mean of λt and standard deviation of $\upsilon\sqrt{t}$ (see, e.g., Nahmias, 2009). Let $f_t(y)$ and $F_t(y)$ denote the probability density and cumulative probability functions, respectively, of the normally distributed random variable y with mean λt and standard deviation $\upsilon\sqrt{t}$. Due to the stochastic demand, there might be shortages and be the expected number of shortages and let $n(r,t)$ be the expected number of shortages over a time period t when the starting inventory is r. It then follows that $n(r,t) = \int_r^\infty (y-r)f_t(y)dy$. It is assumed that the inventory is continuously reviewed, i.e., the retailer knows the inventory level at any moment. In case of continuous inventory review, a common inventory control policy adopted is (Q, R) model, where Q denotes the order quantity and R denotes the re-order point to place an order. That is, whenever the inventory on hand is R, an order of Q units is placed. In the settings of the classical (Q, R) model, the retailer is subject to inventory holding, penalty, procurement, and order setup costs. Let $\tilde{h}$ denote the retailer's per unit per unit time inventory holding cost. It is assumed that all of the shortages are backordered and there is a penalty cost $\tilde{p}$ backordered. Furthermore, let A be the setup cost per order. In this study, we assume that the retailer can partially order his/her order quantity from a set of n suppliers, indexed by i such that $i \in S$ where $S = \{1,2,\dots,n\}$, i.e., we allow order splitting. As different suppliers might have distinct characteristics with regards to their locations, wholesale prices, and shipment requirements, we define $\hat{c}_i$ as the retailer's unit procurement cost from supplier i. Furthermore in addition to the retailer's major setup cost per order, we assume that the retailer is subject to fixed order setup cost $\hat{a}_i$, when an order is placed from supplier $i \in S$. Note that $\hat{c}_i$ can be defined to include supplier i's unit transportation cost in addition to the unit procurement cost; and, $\hat{a}_i$ can include the fixed transportation or delivery cost such as the truck driver's cost or loading/unloading charges for an order from supplier i. Furthermore, we assume that each supplier has a shipment capacity of w_i units per order due to limited supply or the capacity of the transportation mode used by supplier i. We define τ_i as the delivery lead time of supplier i and it is assumed that different suppliers might have different lead times due to different points of origin or transportation modes used.

As noted in Section 1, there is a significant amount of carbon emissions generated from inventory holding, freight transportation, and warehousing activities. Similar to , Hua et al. (2011), Chen et al. (2013), Toptal et al. (2014), Konur (2014), and Konur and Schaefer (2014), we assume that $\hat{h}$ units of carbon emissions generated from holding one unit inventory per unit time due to electricity used in the warehouse for cooling/heating/lighting operations and $\hat{A}$ as the emissions generated from each inventory replenishment due to material handling and unloading/loading operations. We also consider that $\hat{p}$ units of carbon emissions are generated from backordered shortages as the retailer might need to ship the backordered unit to the customer (see, e.g., Anderson et al., 2012) or the customer might need to re-travel to the retailer's store to pick the backordered unit (see, e.g., Cachon, 2014). A substantial amount of carbon emissions are due to freight transportation and the transportation emissions depend on the transportation mode selected, type of vehicles used, the load carried, and the shipment distance (Konur, 2014, Konur and Schaefer, 2014). As different suppliers can use different transportation modes, or even different vehicle types of the same transportation mode (such as different truck types or rail cars), we consider that each supplier's delivery to the retailer has different carbon emissions generation characteristics. In particular, we let $\hat{c}_i$ be the carbon emissions generated per unit shipped and $\hat{a}_i$ denote the fixed carbon emissions generated per shipment made by supplier $i \in S$. For instance, $\hat{a}_i$ can be considered as the carbon emissions generated due to the empty weight of the transportation unit (e.g., a truck) and $\hat{c}_i$ is the carbon emissions generated from each additional unit loaded to the truck (similar parameters are also defined in Hua et al., 2011, Chen et al., 2013, Konur, 2014, Konur and Shaefer, 2014).

In this study, we assume that the retailer is subject to one of the two most-common environmental regulations: carbon taxing and carbon trading. Under carbon taxing, the retailer is charged per unit of carbon emissions generated and let α denote the carbon tax per unit of carbon emissions generated. On the other hand, under carbon trading, the retailer is subject to a carbon cap and carbon emissions are tradable. As mentioned previously, if the retailer's carbon emissions per unit time is below the carbon cap, the retailer can sell his/her excess carbon emissions; whereas, if the retailer's carbon emissions per unit time is above the carbon cap, the retailer needs to buy the extra carbon allowances. Let β denote the carbon trading price per unit of carbon emissions and Φ be the carbon cap per unit time. Similar to Hua et al., 2011 and Toptal et al. (2014), we assume that there are sufficient demand and supply for carbon trading in the market; hence, the retailer can sell all of his excess carbon credits or buy unlimited carbon allowances. One can note that when $\Phi = 0$, carbon taxing and carbon trading regulations are identical if $\beta = \alpha$. Therefore, in the mathematical formulation and the solution analysis, we will only focus on carbon trading regulation as carbon taxing is the aforementioned special case of carbon trading.

The retailer's objective is to minimize his/her total expected costs per unit time by determining which suppliers to select, how much to ship from each supplier, and when to start ordering from the suppliers. Let

$$x_i = \begin{cases} 1 & \textit{is supplier i is selected,} \\ 0 & \textit{otherwise} \end{cases}$$

and x be the binary n-vector of x_i values. Furthermore, let q_i be the quantity ordered from supplier i at each replenishment and q denote the n-vector of q_i values. Note that if $x_i = 0$ then $q_i = 0$ and if $x_i = 1$ then $q_i \leq w_i$. As is defined previously, R is the re-order point.

We assume that the supplier can use one of the three policies for order splitting among the selected suppliers: (i) single sourcing, (ii) sequential ordering, and (iii) sequential delivery. In case of single sourcing, the retailer selects a single supplier to order from; hence, there is no need for order splitting. On the other hand, when multi-sourcing is allowed, we consider two different policies for order splitting, which are sequential ordering and sequential delivery. In sequential ordering, the retailer splits his/her order among different suppliers sequentially considering their lead times such that the split orders from different suppliers are received by the retailer at the same time. In sequential delivery, the retailer splits the order among different suppliers at the same time and the split orders from different suppliers are received by the retailer at different times due to varying supplier lead times. In sequential delivery, we assume that the next order will not be placed until the partial order of the last supplier (supplier 3 in Figure 1) has been delivered. In what follows, we mathematically formulate the retailer's inventory control and supplier selection problem with each order splitting policy. A table summarizing the notation and possible metrics is noted in the Appendix. Additional notation will be defined as needed.

3.1 SINGLE SOURCING

In the case the retailer adopts single sourcing policy, for any selected supplier, the retailer's inventory control policy is the classical (Q, R) model with an additional upper bound constraint on the order quantity due to supply limit. Suppose that supplier i is selected to be ordered from; hence, the lead time is τ_i . Then, the retailer's cost function is the cost function of the classical (Q, R) model. In particular, assuming that only supplier i is used under the settings of the classical (Q, R) model, one can derived that $\widetilde{C_i}\lambda$ expected procurement cost per unit time and $\tilde{h}\left(R-\lambda\tau_i+\frac{1}{2}q_i\right)$ is the expected inventory holding cost per unit time. Also, as the expected cycle length (the time between receiving two consecutive orders from supplier i) is equal to $\frac{q_i}{\lambda}$ the expected order setup cost per unit time and expected penalty cost per unit time amount to $\frac{(\tilde{A}+\tilde{a_i})\lambda}{q_i}$ and $\frac{\tilde{p}\lambda n(R,\tau_i}{q_i}$ respectively. It then follows that the retailers expected cost per unit time under single sourcing as a function of the decision variables R, q, and x, denoted by C^1(R,q,x) is

$$C^1(R,q,x)=\sum_{i\in S}x_i\left[\widetilde{c_i}\lambda+\tilde{h}\left(R-\lambda\tau_i+\frac{1}{2}q_i\right)+\frac{(\tilde{A}+\widetilde{a_i})\lambda}{q_i}+\frac{\tilde{p}\lambda n(R,\tau_i)}{q_i}\right] \tag{1}$$

The expected carbon emissions generated from inventory related operations under single sourcing can be defined similar to the expected inventory related costs given in Equation (1). Particularly, it can be shown that the retailer's carbon emissions per unit time under single sourcing as a function of the decision variables R, q, and x, denoted by E1 (R, q, x), reads

$$E^1(R,q,x)=\sum_{i\in S}x_i\left[\widehat{c_i}\lambda+\hat{h}\left(R-\lambda\tau_i+\frac{1}{2}q_i\right)+\frac{(\hat{A}+\widehat{a_i})\lambda}{q_i}+\frac{\hat{p}\lambda n(R,\tau_i)}{q_i}\right] \tag{2}$$

Where $\widehat{c_i}\lambda$, $\hat{h}\left(R-\lambda\tau_i+\frac{1}{2}q_i\right)$, $\frac{(\hat{A}+\widehat{a_i})\lambda}{q_i}$ and $\frac{\hat{P}\lambda n(R,\tau_i)}{q_i}$ define the expected carbon emissions generated per unit time from transportation, inventory holding, order setup and background operation respectively, when supplier i is selected.

Under a carbon trading policy with carbon cap of Φ, the total amount of traded carbon emissions is equal to E^1 (R, q, x) − Φ. Note that if E^1 (R, q, x) − Φ > 0, the retailer is buying extra carbon allowances at a cost of β per unit; and, if E1(R, q, x) − Φ < 0, the retailer is selling his/her excess carbon emissions at a price of β per unit. The retailer's optimization problem with single sourcing under carbon trading then can be formulated as follows:

$$(\mathrm{P1}):\ \min \prod^{2}(\mathrm{R},\mathrm{q},\mathrm{x}) = \mathrm{C}^2(\mathrm{R},\mathrm{q},\mathrm{x}) + \beta(\mathrm{E}^2(\mathrm{R},\mathrm{q},\mathrm{x}) - \phi)$$

$$\text{s.t} \frac{\widehat{\mathrm{P}}\lambda \mathrm{n}(\mathrm{R},\tau_\mathrm{i}}{\mathrm{q_i}} 0 \le \mathrm{q_i} \le \mathrm{x_i w_i} \qquad \forall\, \mathrm{i} \in \mathrm{S}$$

$$\sum \mathrm{x_i} = 1$$

$$\mathrm{x_i} \in \{0,1\} \quad \forall\, \mathrm{i} \in \mathrm{S}$$

$$\mathrm{R} > 0.$$

Π1(R, q, x) defines the total expected costs per unit time under single sourcing and the first constraint ensures that only a single supplier is selected. The second set of constraints guarantees that the retailer can only order from the selected supplier and the order quantity is less than or equal to the selected supplier's capacity. The third set of constraints is the binary definitions of xi values and the fourth constraint is the non-negativity of the re-order point. Let (R1, q1, x1) denote an optimal solution of P1.

3.2 SEQUENTIAL ORDERING

In the case retailer adopts sequential ordering policy, the effective lead time, i.e., the time between the retailer starts ordering from the suppliers until the orders are simultaneously received, is the maximum of the lead times of the selected suppliers. Let τ (x) denote the effective lead time when supplier selection decision is given by x. It then follows that

$$\tau(x) = max_{i\epsilon S}\{\tau_i x_i\} \tag{3}$$

The expected inventory level with sequential ordering is defined similar to the classical (Q,R) model and one can derive that $\tilde{h}\left(R - \lambda\tau(x) + \frac{1}{2}\sum_{i\epsilon S} q_i\right)$ is the expected inventory holding cost per unit time. Similarly it can be argued that the expected cycle length is $\frac{1}{\lambda}\sum_{\mathrm{i}\epsilon \mathrm{S}} q_{\mathrm{i}}$; thus the expected procurement cost per unit time $\frac{\sum_{i\in S} \tilde{\mathrm{c}}_{\mathrm{i}} q_i}{\sum_{i\in S} q_i}$ and $\frac{\lambda(\tilde{\mathrm{A}}+\sum_{i\in S}\tilde{\mathrm{a}}_{\mathrm{i}})}{\sum_{i\in S} q_i}$ the expected order setup cost per unit time amount to and respectively. Finally shortage can occur during the effective lead time and the expected number of shortages per cycle is$\mathrm{n}(\mathrm{R},\tau(\mathrm{x}))$ it then follows that the expected penalty cost per unit time is equal to $\frac{\tilde{\mathrm{p}}\lambda\mathrm{n}(\mathrm{R},\tau_{\mathrm{i}}}{\sum_{\mathrm{i}\epsilon \mathrm{S}} \mathrm{q}_{\mathrm{i}_{\mathrm{i}}}}$. These imply that the retailers expected cost per unit time under sequential ordering as a function of the decision variables R, q and x denoted by C^2(R,q,x) , is

$$C^2(R,q,x) = \frac{\lambda\sum_{i\in S}\tilde{c}_i q_i}{\sum_{i\in S} q_i} + \tilde{h}\left(R - \lambda\tau(x) + \frac{1}{2}\sum_{i\in S} q_i\right) + \frac{\lambda(\tilde{A}+\sum_{i\in S}\widetilde{a_i}x_i)\lambda}{\sum_{i\in S} q_i} + \frac{\tilde{p}\lambda n(R,\tau_i)}{\sum_{i\in S} q_i} \tag{4}$$

Where the first, second, third and the last terms are expected procurement. Inventory holding, order setup and penalty cost per unit time, respectively, such that τ(x) is defined in equation (3). The expected carbon generation from inventory related operations could be defined similar to the expected inventory related costs given in equation (4). Particularly it can be shown that retailers carbon emission put unit time under sequential ordering as function of the decision variables R, q and x denoted by E^2(R,q,x) , reads

$$E^2(R,q,x) = \frac{\lambda \sum_{i\in S} \widehat{c_i} q_i}{\sum_{i\in S} q_i} + \hat{h}\left(R - \lambda\tau(x) + \frac{1}{2}\sum_{i\in S} q_i\right) + \frac{\lambda(\hat{A} + \sum_{i\in S} \widehat{a_i} x_i)\lambda}{\sum_{i\in S} q_i} + \frac{\hat{p}\lambda n(R,\tau_i)}{\sum_{i\in S} q_i} \tag{5}$$

Where the first, second, third and the last terms are expected procurement. Inventory holding, order setup and penalty cost per unit time, respectively, such that $\tau(x)$ is defined in equation (3).

Similar to P1 , the retailer's optimization problem with sequential ordering under carbon trading such that carbon cap is $\phi$and carbon trading price is β , then can be formulated as follows:

$$(\mathrm{P2}):\ \min \prod\nolimits^{2} (\mathrm{R,q,x}) = \mathrm{C}^2(\mathrm{R,q,x}) + \beta(\mathrm{E}^2(\mathrm{R,q,x}) - \phi)$$

$$\text{s.t} \frac{\widehat{\mathrm{P}}\lambda\mathrm{n}(\mathrm{R},\tau_\mathrm{i}}{\mathrm{q_i}} 0 \le \mathrm{q_i} \le \mathrm{x_i w_i} \qquad \forall\ \mathrm{i} \in \mathrm{S}$$

$$\sum \mathrm{x_i} = 1$$

$$\mathrm{x_i} \ \in \{0,1\} \quad \forall\ \mathrm{i} \in \mathrm{S}$$

$$\mathrm{R} > 0.$$

Π2 (R, q, x) defines the total expected costs per unit time under sequential ordering and the first set of constraints guarantees that the retailer can only order from the selected suppliers and the order quantity from each selected supplier is less than or equal to the supplier's capacity. The second set of constraints is the binary definitions of xi values and the third constraint is the non-negativity of the re-order point. Let (R2, q2, x2) denote an optimal solution of P2.

3.3 SEQUENTIAL DELIVERY

In the case the retailer adopts sequential delivery policy, we define a cycle as the time between receiving two consecutive orders from the same supplier; therefore, the expected cycle length can be defined similar to the classical (Q, R) model. That is expected cycle length is $\frac{1}{\lambda}\sum_{i\epsilon S} q_i$. It then follows that the expected procurement per unit time is equal to $\frac{\sum_{i\epsilon S} \tilde{c}_i q_i}{\sum_{i\epsilon S} q_i}$ and the expected order setup cost per unit time is equal to $\frac{\lambda(\tilde{A}+\sum_{i\epsilon S}\tilde{a}_i)}{\sum_{i\epsilon S} q_i}$. Defining the expected inventory holding cost and expected penalty cost per unit time, on the other hand is different than the sequential ordering policy. To do so, without loss of generality, let us assume that the suppliers are sorted such that $\tau 1 < \tau 2 < \ldots < \tau n$. Given that $x_i = 1$ for $i \leq k$ and $x_i = 0$ for $i \geq k+1$such that $k+1 \leq n$, one can show that the expected inventory held during one cycle amounts to $R + \sum_{i=1}^{k} q_i(\tau_{n+1} - \tau_i) - \lambda/2\, \tau_{n+1}^2$, which does not depend on x_i values. It then can be concluded that the expected inventory holding cost per unit time for any x is equal to $h(R - \lambda\frac{\sum_{i\epsilon S}\tau_i q_i}{\sum_{i\epsilon S} q_i} + \frac{1}{2\sum_{i\epsilon S} q_i})$. Notice that we will guarantee that $q_i = 0$ if $x_i = 0$ by adding constraints in formulating the retailer's optimization problem. Now, let us focus on defining the expected penalty cost per unit time. To do so, we first calculate the expected number of shortages within one cycle. Shortages can occur during the time periods from the moment orders placed until the first order received, from the moment first order received until the second order received, and so on. Let e_i be the random variable defining the inventory right before receiving supplier i's order. Furthermore, let us define $z_{ij} = \max\{0, (\tau i - \tau j)/|\tau i - \tau j|\}$. That is,

$$z_{ij} = \begin{cases} 1 & if \tau_i > \tau_j \\ 0 & otherwise \end{cases}$$

Such that z_{ii}=0. Then, one can show that e_i is a normally distributed random variable with mean $\mu_i(R, q, x) = x_i(R + \sum_{i\epsilon S} z_{ij} q_j - \lambda\tau_i)$ and$\sigma_i(x) = x_i \vartheta\sqrt{\tau_i}$. By definition of $\mathrm{e_i}$, it follows that the expected number of shortages right after the moment the previous supplier's order received until right before the moment supplier I's order received is $n_i(R,q,x) = -\int_{-\infty}^{0} \mathrm{e_i} f^i(\mathrm{e_i})\mathrm{de_i}$, where $f^i(\mathrm{e_i})$ is the normal density function with mean $\mu_i(R,q,x)$ and standard deviation $\sigma_i(x)$. It then follows that

$$n_i(R,q,x) = -\mu_i(R,q,x) + \sigma_i(x)L\left(-\frac{\mu_i(R,q,x)}{\sigma_i(x)}\right), \tag{6}$$

Where L(z) is the standard loss function. That is, the expected number of total shortages within one replenishment cycle is $\sum_{i\in S} n_i(R,q,x)$.

The above discussion leads that the retailer's expected cost per unit time with sequential delivery as a function of the decision variables R, q, and x, denoted by $C^3(R,q,x)$,is

$$C^3(R,q,x) = \frac{\lambda\sum_{i\in S}\tilde{c}_i q_i}{\sum_{i\in S} q_i} + \tilde{h}\left(R - \lambda\frac{\sum_{\mathrm{i\in S}}\tau_{\mathrm{i}}\mathrm{q_i}}{\sum_{\mathrm{i\in S}}\mathrm{q_i}} + \frac{1}{2}\sum_{i\in S} q_i\right) + \frac{\lambda(\tilde{A}+\sum_{i\in S}\widetilde{a_i}x_i)}{\sum_{i\in S} q_i} + \frac{\tilde{p}\lambda n_i(R,q,x)}{\sum_{i\in S} q_i} \tag{7}$$

Where the first, second, third, and the last terms are the expected procurement, inventory holding, order setup, and penalty cost per unit time, respectively, such that ni (R, q, x) is defined in Equation (6).The expected carbon emissions generated from inventory related operations can be defined similar to the expected inventory related costs given in Equation (7). Particularly, it can be shown that the retailer's expected carbon emissions per unit with sequential delivery as a function of the decision variables R, q, and x denoted by

$$E^3(R,q,x) = \frac{\lambda\sum_{i\in S}\hat{c}_i q_i}{\sum_{i\in S} q_i} + \hat{h}\left(R - \lambda\frac{\sum_{\mathrm{i\in S}}\tau_{\mathrm{i}}\mathrm{q_i}}{\sum_{\mathrm{i\in S}}\mathrm{q_i}} + \frac{1}{2}\sum_{i\in S} q_i\right) + \frac{\lambda(\hat{A}+\sum_{i\in S}\widehat{a_i}x_i)}{\sum_{i\in S} q_i} + \frac{\hat{p}\lambda n_i(R,q,x)}{\sum_{i\in S} q_i} \tag{8}$$

Where the first, second, third, and the last terms are the expected carbon emissions generated per unit time from transportation, inventory holding, order setup, and backordering operations, respectively, such that ni (R, q, x) is defined in Equation (6).

Similar to P2, the retailer's optimization problem with sequential delivery under carbon trading, such that carbon cap is Φ and carbon trading price is β, can be formulated as follows:

$$(\mathrm{P3}):\ \min F^3(R,q,x) = C^3(R,q,x) + \beta(\mathrm{E}^3(\mathrm{R},\mathrm{q},\mathrm{x}) - \phi)$$

$$\text{s.t } 0 \leq \mathrm{q_i} \leq \mathrm{x_i w_i} \qquad \forall\ \mathrm{i} \in \mathrm{S}$$

$$\mathrm{x_i} \in \{0,1\} \quad \forall\ \mathrm{i} \in \mathrm{S}$$

$$\mathrm{R} > 0.$$

$F^3(R,q,x)$ defines the total expected costs per unit time under sequential delivery. The constraints are defined similar to P2. Let (R3, q3, x3) denote an optimal solution of P3.

4. SOLUTION ANALYSIS

In this section, we analyze models P1, P2, and P3, and propose a solution method for each model. We note that each model has different settings; hence, we analyze underlying characteristics of the models and develop solution methods accordingly. Prior to the analysis of each model, we next note a strain forward property of the optimal solutions of models P1, P2 and P3.

Property 1 For (R^j, q^j, x^j) for $\mathrm{j} = 1,2,3$, if $\mathrm{x}_j^i = 1, then \quad 0 < \mathrm{q}_j^i \leq w_i$.

Property 1 states that the retailer will order a positive amount from each selected supplier in optimal solutions of P1, P2, and P3. This is intuitive as the retailer will neither pay extra setup costs nor generate unnecessary carbon emission unless the order quantity from a selected supplier is positive. In the reminder of this section, let $c_i = \widetilde{c_\imath} + \beta\widehat{c_\imath}$, $c_i = \widetilde{c_\imath} + \beta\widehat{c_\imath}$, $h = \tilde{h} + \beta\hat{h}, A = \tilde{A} + \beta\hat{A}$ $a_i = \widetilde{a_\imath} + \beta\widehat{a_\imath}$, and $p = \tilde{p} + \beta\hat{p}$.

4.1 SOLUTION OF SINGLE SOURCING

Suppose that the retailer order from supplier I, i.e., $x_i = 1$ and $x_j = 0 \quad \forall \mathrm{j} \neq \mathrm{i}$; hence $0 < q_i \leq w_i$ and $q_j = 0 \quad \forall \mathrm{j} \neq \mathrm{i}$;. In this case, the retailer's total expected cost per unit time under carbon trading is equal

$$\pi_i(R, q_i) = c_i\lambda + h(R - \lambda\tau_i + \frac{1}{2}q_i) + \frac{(A + a_i)\lambda}{q_i} + \frac{p\lambda n(R, \tau_i)}{q_i} - \beta\varphi.$$

Therefore given $x_i = 1$ and $x_j = 0 \quad \forall \mathrm{j} \neq \mathrm{i}$; P1 reduce to

$$\begin{aligned}(P1\text{-}\ i):\quad & \min \quad \rho_i(R, q_i)\\ & \text{s.t} \quad 0 \text{ £ } \mathrm{q}_i \text{ £ } w_i\\ & \qquad R>0.\end{aligned}$$

Let (R_i^*, q_i^*) be the optimum solution of P1-I. Note that $\beta\varphi$ is a constant; thus, $\pi_i(R, q_i)$ is the expected cost function of the classical (Q,R) model. Let $(R^{(i)}, q^{(i)})$ be a minimizer of $\pi_i(R, q_i)$. An efficient heuristic method commonly used to approximate the minimizer of $\pi_i(R, q_i)$ is the iterative method proposed by Hadley and Whitin (1963). Particularly, Hadley and Whitin (1963) method, starting with the EOQ formula for q_i, iteratively solves the following first order conditions of equation (9) until two consecutive R and q_i values are close to each other within a specified value.

$$q_i = \sqrt{\frac{2\lambda\left[A + a_i + pn(R, t_i)\right]}{h}}$$

$$1 - F_{t_i}(R) = \frac{q_i h}{p\lambda}.$$

This method is an heuristic approach as the convexity of $\pi_i(R, q_i)$ is conditional (see, e.g., Brooks and Lu, 1969); however in most cases, Hadley and Whitin (1963) method is able to find the minimizer of $\pi_i(R, q_i)$ (see, e.g., XXX). Therefore in our analysis we accept the output of Hadley and Whitin (1963) method as $(R^{(i)}, q^{(i)})$.

Note that if $q^{(i)} \leq w_i$ then $(R_i^*, q_i^*) = (R^{(i)}, q^{(i)})$; on the other hand $(R^{(i)}, q^{(i)})$ is not feasible for P1-I if $q^{(i)} > w_i$. P1-I is a nonlinear programming model and interior point method (IPM) is a common method used to solve such models (see, e.g, Forsgren et.al., 2002). Nevertheless we utilize the Hadley and Whitin(1963) method in solving P1-I as detailed in the following algorithm.

Algorithm1 solving P1-i

1. Determine $(R^{(i)}, q^{(i)})$ using Hadley and Whitin (1963) method.
2. If $q^{(i)} > w_i$, let $q^{(i)} = w_i$ and calculate $R^{(i)}$ using equation (11).
3. Return $(R_i^*, q_i^*) = (R^{(i)}, q^{(i)})$.

Upon comparing algorithm 1 to IPM through a numerical study we observe that algorithm 1 finds the same solution with IPM and requires less computational time. The details of the numerical compression can be seen in section 5. Therefore we use algorithm 1 to find (R_i^*, q_i^*). Once (R_i^*, q_i^*) is found for each supplier i, (R^1, q^1, x^1) can be easily determined. Particularly let $j^1 = \operatorname{argmin}_{i\in S}\{\pi_i((R_i^*, q_i^*)\}$.then $R^1 = R_{j1}^*$, $q_{j1}^1 = q_{j1}^*$ and $q_i^1 = 0 \quad \forall i \neq j^1$ and $x_j^1 = 1$ and $x_i^1 = 1 \quad \forall i \neq j^1$.

4.2. SOLUTION OF SEQUENTIAL ORDERING

In this section we first analyze the retailers order quantity decision given the supplier selection decisions. Then using the order quantity analysis, we develop a local search method to find the supplier selection decisions. Given x, let (R_x^{2*}, q_x^{2*})denote a minimizer of $\Pi^2(R, q, x|x)$ subject to $q_i \le w_i x_i \,\forall i \in S$. One can use IPM to determine(R_x^{2*}, q_x^{2*}). However, in what follows, we use the properties of (R^2, q^2, x^2) in determining (R_x^{2*}, q_x^{2*}) and then, develop a local search heuristic to find the retailer’s supplier selection decision.

Now, suppose that the supplier selection decisions are known, i.e., x is given. Let S(x) and $\bar{S}(x)$ denote the set of selected and unselected suppliers, respectively, as indicated by x. That is, if$x_i = 1$, then$i \in S(x)$; else, $i \in \bar{S}(x)$(note that$S = S(x) \cup \bar{S}(x)$). Furthermore, let us define$j^{x^2} = argmax_{i\in S(x)}\{c_i\}$, i.e., $j^{x^2}$is the supplier with the maximum per unit purchase cost among the selected suppliers indicated by x. Next, we characterize an important property of (R^2, q^2, x^2).

Property 2 $q_i^2 = 0 \,\forall i \in \bar{S}(x^2) - \{j^{x^2}\}$, and $0 < q_{j^{x^2}}^2 < w_{j^{x^2}}$.

Let$Q^2 = \sum_{i\in S} q_i^2, i.e.,\ Q^2$ is the total order quantity in the optimal solution of **P2**. Property 2 implies that $\sum_{i\in S(x^2)} w_i - w_{j^{x^2}} < Q^2 < \sum_{i\in S(x^2)} w_i$ Particularly, once x^2 and Q^2 are known, one can determine q^2 using Property 2. It further follows from Property 2 that, given$x = x^2$, the retailer’s total expected costs per unit time under carbon trading is equal to

$g_x(R,Q) = c_{j^x}\lambda + h\left(R - \lambda\tau_{(x)} + \frac{1}{2Q}\right) + \lambda(\sum_{i\in S(x)}(c_i - c_{j^{x^2}})w_i + A +$
$\sum_{i\in S(x)} a_i x_i) / Q \quad + p\lambda n(R, \tau(x))/Q - \beta\phi$

(12)

Therefore, assuming that $x = x^2$, **P2** reduces to

(P2-x): min g_x(R,Q)

$$s.t. \sum_{i\in S(x^2)} w_i - w_{j^{x^2}} < Q^2 < \sum_{i\in S(x^2)} w_i$$

R > 0.

Let (R_x^{2*}, q_x^{2*}) be an optimal solution of P2-x. One can notice that $g_x(R,Q)$ is defined similar to $\pi_{xi}(R_i, q_i)$ when $\sum_{i\in S(x)}(c_i - c_{j^{x^2}})w_i + A + \sum_{i\in S(x)} a_i x_i \geq 0$. Note that the conditional joint convexity of the expected cost function of the (Q,R) model assumes the order setup cost to be non-negative. Specifically, for non-negative order setup cost, the expected cost function of the (Q,R)model is convex in Q for a given R, convex in R for a given Q, and jointly convex in Q and R given that R is greater than or equal to the expected lead time demand (i.e., safety stock is non-negative). Therefore Hadley and Whitin (1963) method can be used to determine a minimizer of $g_x(R,Q)$, denoted by $(R^{(x)}, Q^{(x)})$, when $\sum_{i\in S(x)}(c_i - c_{j^{x^2}})w_i + A + \sum_{i\in S(x)} a_i x_i \geq 0$. Similar to Equations (10) and (11), the first order conditions od equations 12 read as follows:

$$Q = \sqrt{2\lambda\left[\sum_{i\in S(x)}(c_i - c_{j^{x^2}})w_i + A + \sum_{i\in S(x)} a_i x_i + pn(R, \tau(x))\right]} / h \qquad (13)$$

$$1 - F_{\tau_{(x)}}(R) = \frac{Qh}{p\lambda}. \qquad (14)$$

Let ($R^{(x)}, Q^{(x)}$) be defined as the output of Hadley and Whitin (1963) method when $\sum_{i\in S(x)}(c_i - c_{j^{x^2}})w_i + A + \sum_{i\in S(x)} a_i x_i \geq 0$. If $\sum_{i\in S(x^2)} w_i - w_{j^{x^2}} < Q^2 < \sum_{i\in S(x)} w_i$,

$(R_x^{2*}, q_x^{2*}) = \left(R^{(x)}, Q^{(x)}\right)$. On the other hand, $\left(R^{(x)}, Q^{(x)}\right)$ can be feasible for P2-x in two cases: (i) $Q^{(x)} < \sum_{i\in S(x^2)} w_i - w_{j^x}$ and (ii) if $Q^{(x)} > \sum_{i\in S(x)} w_i$. Similar to Algorithm 1, Algorithm 2, stated below, first utilizes the Hadley and Whitin (1963) in to find (R_x^{2*}, Q_x^{2*}) when $\sum_{i\in S(x)}(c_i - c_{j^{x^2}})w_i + A + \sum_{i\in S(x)} a_i x_i \geq 0$. For the cases when $\sum_{i\in S(x)}(c_i - c_{j^{x^2}})w_i + A + \sum_{i\in S(x)} a_i x_i < 0$, Algorithm 2 uses IPM to determine (R_x^{2*}, Q_x^{2*}).

Algorithm2 Solving P2-x:

1. If $\sum_{i\in S(x)}(c_i - c_{j^{x^2}})w_i + A + \sum_{i\in S(x)} a_i x_i \geq 0$
2. Determine $\left(R^{(x)}, Q^{(x)}\right)$ using Hadley and Whitin (1963) method.
3. If $Q^{(x)} < \sum_{i\in S(x^2)} w_i - w_{j^x}$, let $Q^{(x)} = lim_Q \sum_{i\in S(x^2)} w_i - w_{j^x}$ and calculate $R^{(x)}$ using equation (14).
4. if $Q^{(x)} >, let\ Q^{(x)} = \sum_{i\in S(x)} w_i$ and calculate $R^{(x)}$ using equation (14).
5. Return(R_x^{2*}, Q_x^{2*})= $\left(R^{(x)}, Q^{(x)}\right)$.
6. If $\sum_{i\in S(x)}(c_i - c_{j^{x^2}})w_i + A + \sum_{i\in S(x)} a_i x_i < 0$
7. Return (R_x^{2*}, Q_x^{2*}) using IPM.

Upon comparing Algorithm 2 to IPM through a numerical study, we observe that Algorithm 2 finds the same solutions with IPM and requires less computational time. The details of the numerical comparison can be seen in section6. Therefore, we use Algorithm 2 to find(R_x^{2*}, Q_x^{2*}). Then, one can use Property 2 to determine(R_x^{2*}, q_x^{2*}). Particularly, given x andQ_x^{2*}, let q_x^{2*}= $0 \forall i \in \bar{s}(x)$, q_i^*(x) =w_i $\forall i \in s(x) - \{j^x\}$, and $q_{j^x}^*(x) = Q_x^{2*} - \sum_{i\in S(x)} w_i - w_{j^x}$. Then $q_x^{2*} = [q_1^*(x), q_2^*(x), \dots, q_n^*(x)]$.

Once (R_x^{2*}, q_x^{2*}) is determined for all possible binary x vectors, x^2 can be determined by comparing $\Pi^2(R_x^{2*}, q_x^{2*}, x)$ values. However, there are $2^n - 1$ binary x vectors; hence, total enumeration can be computationally cumbersome. Therefore, we next develop a local search heuristic to find a good selection vector. Prior to the details of the local search heuristic, we note another property ofx^2.

Property 3 if $Q_x^{2*} = lim_Q\left(\sum_{i\in S(x^2)} w_i - w_{j^x}\right)$, then$x \neq x^2$.

Property 3 eliminates those binary x vectors where $\mathrm{Q_x^{2*}}$ converges to the lowest cumulative capacity of the selected suppliers from the search of x^2. The local search heuristic that we explain next, therefore, disregards such vectors.

The local search heuristic method for solving **P2** works as follows. Suppose that x is given. First, using Algorithm 2, we determine (R_x^{2*}, Q_x^{2*}). If $Q_x^{2*} = \sum_{i\in S(x^2)} w_i - w_{j^x} + \epsilon$, where ϵ is a very small number, we let$\Pi^2(R_x^{2*}, q_x^{2*}, x) \approx \infty$ since x cannot be optimum for P2 in this case based on Property 3. Else using Property 2 we determine q_x^{2*} and calculate$\Pi^2(R_x^{2*}, q_x^{2*}, x)$. After that we seek the best neighbor of x, where a neighbor of x is another binary n vectors that differs from x with a single entry. Particularly, x has n neighbors and we define the i^{th} neighbor of x, denoted by $x^{[i]}$ by letting $x_i^{[i]} = 1$ if $x_i = 0$ otherwise, and $x_j^{[j]} = x_j\, \forall j \in s - \{i\}$. Similar to the calculation of $\Pi^2(R_x^{2*}, q_x^{2*}, x)$we calculate $\Pi^2(R^*_{x^{[i]}}, q^*_{x^{[i]}}, x^{[i]})$ for each i using Algorithm 2 and Properties 2 and 3. If the best neighbor of x is worse than x, i.e., if $\Pi^2(R_x^{2*}, q_x^{2*}, x) \leq min_i\left\{F^2(R^*_{x^{[i]}}, q^*_{x^{[i]}}, x^{[i]})\right\}$ x defines a local optimum and we stop the search. On the other hand, if $\Pi^2(R_x^{2*}, q_x^{2*}, x) > min_i\left\{F^2(R^*_{x^{[i]}}, q^*_{x^{[i]}}, x^{[i]})\right\}$, we define a new x, such that $x = argmin_{\,i}\left\{F^2(R^*_{x^{[i]}}, q^*_{x^{[i]}}, x^{[i]})\right\}$ and continuo the local search process. Now let $\bar{x}$ be the local optimum reached via the local search when the local search starts with x. we repeat the local search starting with m different x vectors to avoid returning a bad quality local optimum. The best local optimum returned is accepted as the solution of **P2**. Algorithm 3 states this local search heuristic with multiple starting solutions.

Algorithm 3 solving P2

0. Let $x^{1}, x^{2}, \dots, x^{m}$be m given starting x vectors.

1. For $l = 1{:}\,m$

2. Let $x = x^{l}$

3. Calculate (R_x^{2*}, Q_x^{2*}) using Algorithm 2

4. If $Q_x^{2*} = \sum_{i\in S(x^2)} w_i - w_{j^x} + \epsilon$, $\Pi^2(R_x^{2*}, q_x^{2*}, x) \approx \infty$

5. Else, determine q_x^{2*} using Property 2 and calculate $\Pi^2(R_x^{2*}, q_x^{2*}, x)$

6. For $i = 1{:}\,n$

7. Let $x^{[i]} = x$ and $x_i^{[i]} = 1$ if $x_i = 0$ and $x_i^{[i]} = 0$ $x_i = 1$

8. Calculate $(R_{x^{[i]}}^{2*}, Q_{x^{[i]}}^{2*})$ using Algorithm 2

9. If $Q_{x^{[i]}}^{2*} = \sum_{i\in S(x^{[i]})} w_i - w_{j^{x^{[i]}}} + \epsilon$, $\Pi^2(R_{x^{[i]}}^{2*}, q_{x^{[i]}}^{2*}, x^{[i]}) \approx \infty$

10. Else, determine $q_{x^{[i]}}^{2*}$ using Property 2 and calculate $\Pi^2(R_{x^{[i]}}^{2*}, q_{x^{[i]}}^{2*}, x^{[i]})$

11. End

12. If$\Pi^2(R_x^{2*}, q_x^{2*}, x) > min_i\{\Pi^2(R_{x^{[i]}}^{2*}, q_{x^{[i]}}^{2*}, x^{[i]})\}$,$x = argmin_i\{\Pi^2(R_{x^{[i]}}^{2*}, q_{x^{[i]}}^{2*}, x^{[i]})\}$ and go to 3.

13. Else $x^{l} = x$

14. End

15. Return $(R^2, q^2, x^2) = argmin_i\{\Pi^2(R_{\bar{x}^l}^{2*}, q_{\bar{x}^l}^{2*}, \bar{x}^{l})\}$

4.3 SOLUTION OF SEQUENTIAL DELIVERY

In this section, we propose an algorithm similar to Algorithm 3 to solve Model P3. However, due to the definition of the expected number of shortages in each shortage period, determining re-order point and order quantities from the selected suppliers is more complex. Particularly, given x let (R_x^{3*}, q_x^{3*}) denote a minimizer of $\Pi^3(R, q, x|x)$ subject to $q_i \le x_i w_i$ $\forall i \in s$. We use IPM to determine (R_x^{3*}, q_x^{3*}) for a given x. then, similar to Algorithm 3, a local search is used to find the selected suppliers. Algorithm 4 states this local search heuristic with multiple starting solutions, where IPM is used for solving the subproblems.

Algorithm 4 solving P3

0. Let $x^{1}, x^{2}, \dots, x^{m}$be m given starting x vectors.
1. For $l = 1{:}\, m$
2. Let $x = x^{l}$
3. Determine (R_x^{3*}, q_x^{3*}) using IPM
4. For $i = 1{:}\, n$
5. Let $x^{[i]} = x$ and $x_i^{[i]} = 1$ if $x_i = 0$ and $x_i^{[i]} = 0\ x_i = 1$
6. Determine $(R_{x^{[i]}}^{3*}, Q_{x^{[i]}}^{3*})$ using IPM
7. End
8. If$\Pi^3(R_x^{3*}, q_x^{3*}, x) > min_i\{\Pi^3(R_{x^{[i]}}^{3*}, q_{x^{[i]}}^{3*}, x^{[i]})\}$,$x =$ $argmin_{\,i}\{\Pi^3(R_{x^{[i]}}^{3*}, q_{x^{[i]}}^{3*}, x^{[i]})\}$ and go to 3.
9. Else $x^{l} = x$
10. End
11. Return $(R^3, q^3, x^3) = argmin_i\{\Pi^3(R_{\bar{x}^{l}}^{3*}, q_{\bar{x}^{l}}^{3*}, \bar{x}^{l})\}$

5. COMPARISONS OF THE ORDERING POLICIES

In this section, our focus is to discuss how the three ordering policies modeled compare to each other in terms of not only expected total costs but also expected carbon emissions per unit time. In particular, while environmental regulations are becoming more common worldwide, there are still many countries that do not have nationally legislated environmental regulations. For instance, there is no federal environmental regulation in the U.S. However, environmental regulations are not the only motivation for ompanies to green their operations. As it discussed in surveys by Loebich et.al. (2011) and Kiron et.al. (2012), recent motivation for companies become greener is rather to stay competitive in the market considering the increasing awareness of consumers on environment and/or brand image. Therefore, we next compare the three ordering policies in terms of not only expected total cost per unit time after carbon trading $(\Pi^j(R^j, q^j, x^j)$, denoted as Π^j and the expected costs per unit time $(C^j(R^j, q^j, x^j)$, denoted as C^j)but also expected carbon emission per unit time $(E^j(R^j, q^j, x^j)$, denoted as E^j)where $j = 1, 2, 3$defines single sourcing (SS), sequential ordering (SO), and sequential delivery (SD), respectively.

Based on the comparison of the total expected costs per unit time after carbon trading (i.e., the expected costs per unit time plus the expected costs due to buying carbon allowances or minus the expected revenues due to selling excess carbon emission), one can note that $\Pi^1 \geq \Pi^2$and $\Pi^1 \geq \Pi^3$. This simply follows from the fact that the optimal solution of model P1 is a feasible solution for model P2 and P3 for any given setting. Therefore, under carbon trading policy, the retailer will not prefer single sourcing unless other criteria are regarded. Furthermore, it can be noticed that when $\sum_{i \in S} x_i^1 = \sum_{i \in S} x_i^2 = \sum_{i \in S} x_i^3 = 1$, i.e., the retailer chooses to order from a single supplier even f order splitting is allowed$\Pi^1 = \Pi^2 = \Pi^3$,. Nevertheless, if $C^j$and $E^j$are also considered in comparing single sourcing to sequential ordering and sequential delivery, the following cases are possible:

SS vs. SO SS vs. SD

Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
$P^1 > P^2$	$P^1 > P^2$	$P^1 > P^2$	$P^1 > P^3$	$P^1 > P^3$	$P^1 > P^3$
$C^1 > C^2$	$C^1 > C^2$	$C^1 < C^2$	$C^1 > C^3$	$C^1 > C^3$	$C^1 < C^3$
$E^1 > E^2$	$E^1 < E^2$	$E^1 > E^2$	$E^1 > E^3$	$E^1 < E^3$	$E^1 > E^3$

Specially, in cases 1 and 3, sequential ordering not only reduces expected costs after carbon trading but also expected carbon emission compared to single sourcing. Similarly, in case 4 and 6, sequential delivery not only reduces expected costs after carbon trading but also expected carbon emission compared to single sourcing. That is, multiple sourcing can result in cheaper as well as greener inventory control for a company. On the other hand, in case 2 and 5, while the retailer would prefer sequential ordering and sequential delivery, respectively based on the expected costs after carbon trading, expected carbon emission are lower with single sourcing. The insights of these cases are as follows. In absence of carbon trading, if the retailer tries to minimize not only expected costs but also carbon emission (i.e., a multi-objective inventory control model similar to the one given in Bouchery et.al., 2012 is used by the retailer), depending on the retailors cost and emission targets, the retailer can prefer SS over SO and SS over SD or vice versa.

On the other hand, when the two delivery structures in case of order splitting are compared in terms of total expected costs per unit time after carbon trading, one cannot guarantee that the retailer will prefer one policy over the other for any given setting. That is, it is the both possible to have $\Pi^2(R^2, q^2, x^2) \leq \Pi^3(R^3, q^3, x^3)$ and $\Pi^2(R^2, q^2, x^2) \geq \Pi^3(R^3, q^3, x^3)$ depending on demand, retailer, and suppliers characteristics as well as regulation parameters. This then implies that, under carbon trading, sequential ordering can be a better policy compared to sequential delivery, which is the delivery structure generally assumed in the integrated stochastic inventory control and supplier selection models. We note that this result readily applies for the case when the retailer does not operate under any environmental regulation (i.e., when $b = 0$);

hence, considering sequential ordering as an alternative to sequential delivery can result in substantial cost savings for a retailer. Nevertheless, if C^j and E^j are also considered in comparing sequential ordering to sequential delivery, the following case are possible:

SO vs. SD   SD vs. SO

Case 7	Case 8	Case 9	Case 10	Case 11	Case 12
$P^2 < P^3$	$P^2 < P^3$	$P^2 < P^3$	$P^3 < P^2$	$P^3 < P^2$	$P^3 < P^2$
$C^2 < C^3$	$C^2 < C^3$	$C^2 > C^3$	$C^3 < C^2$	$C^3 < C^2$	$C^3 > C^2$
$E^2 < E^3$	$E^2 > E^3$	$E^2 < E^3$	$E^3 < E^2$	$E^3 > E^2$	$E^3 < E^2$

In case 7 and 9, sequential ordering not only reduces costs after carbon trading but also expected carbon emission compared to sequential delivery. Similarly, in case 10 and 12, sequential delivery not only reduce1s` expected costs after carbon trading but also expected carbon emissions compared to sequential ordering. That is, by considering different delivery structures in case of order splitting, the retailer can lower his/her costs as well as carbon emissions. On the other hand, in case 8, while the retailer would prefer sequential ordering over sequential delivery based on the expected costs after carbon trading, expected carbon emission are lower with sequential delivery. Similarly, in case 11, while the retailer would prefer sequential delivery over sequential ordering based on the expected costs after carbon trading, expected carbon emissions are lower with sequential ordering. Those observations suggest that, in case there is no environmental regulation in place, the retailers preference for delivery structure depends on the retailers cost and emission targets.

6. NUMERICAL STUDIES

This section focus on the two sets of numerical studies: (i) efficiency of the algorithms proposed and (ii) effects of the changes in supplier capacities and supplier lead times on the retailers expected costs, carbon emissions, and total costs. We do not evaluate how the changes in the carbon trading price and carbon cap will affect the retailers expected costs and carbon emissions per unit time one can easily discuss 21that the models presented in this study will imply observations similar to the ones given for the EOQ model in Hua et.al. (2011) and Chen et.al. (2013). Our focus is rather on the effects of multiple sourcing and delivery structures. The tools provided in this study can be used for analyzing the effects of regulation parameters as well as the retailer parameters such as inventory related costs and emissions and demand characteristics.

All of the algorithms are coded in Matlab2014a and the problem instances solved using a personal computer with 8GB RAM and 3.30GHz processor. The tables referred in this section are given in the Appendix. In the following analysis we assume that the retailer operates under a carbon trading regulation with carbon trading price $b = 0.1$ and $F = 20,000$ carbon cap. Unless stated otherwise, the following values are used for the other problem parameters to generate problem instances (similar values are used for inventory control models with environmental considerations, see, e.g., Hua et al., 2011, Chen et al., 2013, Toptal et al., 2014, Konur, 2014, and Konur and Schaefer, 2014):Retailer parameters: the retailers demand per unit time (year) is normally distributed with mean $/ = 10,000$ and standard deviation $u = 1,000$ and it assumed $\tilde{h} \approx U[2,4], \hat{h} \approx U[0.5,1], \tilde{A} \approx U[50,100], and\ \hat{A} \approx \mathrm{U}[25,50]$, where U[a, b] defines a continuous uniform distribution within the range [a, b]. Supplier parameters: Given suppliers,

$\tilde{c}_i \approx U[2,4], \hat{c}_i \approx U[1,2], \tilde{a}_i \approx U[20,40], \hat{a}_i \approx U[10,20], w_i \approx U[50,100], and\ \tau_i \approx \mathrm{U}[0.01,0.015]$, where w_i it is assumed that is rounded to the nearest multiplier of 10 for practical purposes.

6.1 EFFICIENCY OF THE ALGORITHMS

Recall that algorithms 1 and 2 are stated as alternatives to IPM for solving problems P1-I and P2-x, respectively, which are the subproblems analyzed in models P1 and P2. We, therefore, first compare algorithms 1 and 2 to IPM.

To compare algorithm 1 to IPM, for each $n = \{3,6,9,12,15\}$, we randomly generate 10 problem instances and solve each problem instance n times, one with each n for $R_i^*, q_i^* and\ p_x(R_x^*, q_x^*)$ supplier as the single source of supply, using both methods. Table 1 documents the averages over all problems solved with each values along with the computational times in seconds (denoted as CPU). As can be seen in table 1, algorithm 1 and IPM find the same solution for all problem instances solved. Furthermore, algorithm 1 is more efficient computationally. Thus, we use algorithm 1 to solve the retailers ordering decisions for a given supplier in case of single sourcing.

To compare algorithm 2 to IPM, for each $n = \{3,6,9,12,15\}$, we randomly generate 10 problem instances and solve each problem instance with n randomly generated n-binary x vectors. Similar to table 1, table 2 documents the average over all problems solved with each $R_x^{2*}, Q_x^{2*} and\ g_x(R_x^{2*}, Q_x^{2*})$ values along with the computational times in seconds. One can observe from table 2 that algorithm 2 to IPM find the same solutions for all the problem instances solved and algorithm 2 requires less than half of the solution time required by IPM on average. Therefore, we used algorithm 2 then property 2 to determine the retailers ordering decision for given supplier selections in case of sequential ordering.

Recall that algorithm 3 and 4 are local search heuristic methods proposed for models P2 and P3 respectively (for model P1, we solve each of the n options with algorithm 1). Total enumeration, where each of the possible binary n-vector is evaluated, can be used as an alternative method to algorithms 3 and 4 for solving problems P2 and P3. Therefore, we compare algorithms 3 and 4 for solving problems P2 and P3. Therefore, we compare algorithms 3 and 4 to total enumeration.

To compare algorithm 3 to total enumeration, for each $n = \{3,6,9,12,15\}$, we randomly generate 10 problem instances and solve each problem instance using both methods. Table 3 documents the averages over all problems solved with each n n for $\sum_{i \in s} x_i^2$ for (i.e., number of select suppliers with sequential ordering), $\sum_{i \in s} q_i^2$ $R^2, and\ \Pi^2(R^2, q^2, x^2)$ (denoted as Π^2) values along with the computational times in seconds. It can be seen in table 3 that algorithm 3 is able to find the optimal solution in all of the problem instances solved. Furthermore, while Algorithm 3 requires less than a second to solve the problem instances, total enumeration requires more than 800 seconds on average. That is, algorithm 3 can find the optimal solutions very efficiently.

Therefore, in analysis (ii), we use algorithm 3 to solve model P2.

To compare algorithm 4 to total enumeration, for each $n = \{3, 6, 9, 12, 15\}$We randomly generate 10 problem instances and solve each problem instance using both methods. Similar to table 3, table 4 documents the average over all problems solved with each n for $\sum_{i \in s} x_i^3$ (i.e., number of selected suppliers with sequential delivery), $\sum_{i \in s} q_i^3$ (i.e., the total order quantity with sequential delivery), $R^3, and\ \mathrm{P}^3(R^3, q^3, x^3)$(denoted as P^3) values along with the computational times in seconds. One can observe that algorithm 4 finds the same solutions with total enumeration. Furthermore, while for smaller n values (when n=3 and n-6) algorithm 4 takes longer time to solve the problem instances on average (specifically, due to evaluating same x vectors more than once), for larger n values, total enumeration requires longer computational times on average. In particular, for n=12 and n=15, algorithm 4 is drastically more efficient in terms of computation times compared to total enumeration. Based on these observations, in analysis (ii), we use algorithm 4 to solve model P3 instead of total enumeration.

6.2 EFFECTS OF SUPPLIERS

In this section, we numerically analyze how the multiple sourcing affects their retailers inventory control and supplier selection decisions as well as his/her expected costs, carbon emissions, and total costs per unit time with carbon trading under each of the ordering policies considered. Specifically, we focus on illustrating the changes in the number of selected suppliers, the total order quantity, and the re-order point (R^j) as well as the expected costs per unit time ($(C^j(R^j,q^j,x^j)$, denoted as C^j),expected carbon emission per unit time ($(E^j(R^j,q^j,x^j)$, denoted as E^j)and expected total cost per unit time after carbon trading ($(\mathrm{P}^j(R^j,q^j,x^j)$denoted as P^j) as the supplier capacities (w_i) and lead times (τ_i) increase for $j=1,2,3$. Note that under single sourcing $\sum_{i\in S} x_i^1 = 1$in all of the problem instances solved.

To analyze the effect of supplier capacities, with each n={3,6,9}, we randomly generate 10 problem instances with the range given for $w_i$values in table 5 and 6. Table 5 documents the averages overall 30 problem instances solved with each $w_i$range for $\sum_{i\in S} x_i^j$, $\sum_{i\in S} q_i^j$, and R^j for $j=1,2,3$. Similarly table 6 documents the average over all 30 problem instances solved within each w_i range for C^j ,E^j and $\tilde{\mathrm{O}}^j$for $j=1,2,3$. We have the following observations based on table 5 and 6.

• As expected and can observed in Table 5, the number of selected suppliers (except with single sourcing) and the re-order point tend to decrease while the total order quantity tends to increase with an increase in the suppliers' capacities with any ordering policy. Particularly, the retailer will prefer to use fewer suppliers in case the suppliers' capacities are larger. Furthermore, since the suppliers have larger capacities, the retailer can increase his/her order quantity while avoiding the extra setup costs and carbon emissions (it is even possible to decrease setup costs and carbon emissions while the order quantity increases as the retailer might prefer fewer suppliers with larger cumulative capacity). This increase in the order quantities, in turn, leads to lower re-order points. We note that the retailer will not continuously increase his/her order quantity with increasing supplier capacities since it will not be costly justifiable to order more than needed.

• As can be observed in Table 6, with any ordering policy, the retailer's expected costs, carbon emissions, and total costs per unit time after carbon trading decrease with an increase in the suppliers' capacities. These observations are expected since an increase in the supplier capacities without an increase in the supplier setup costs and carbon emissions imply cheaper and cleaner transportation capacity; hence, both expected costs and carbon emissions per unit time decrease. This then leads to decreased total expected costs per unit time after carbon trading.

To analyze the effects of supplier lead times, with each n={3,6,9}, we randomly generate 10 problem instances with the ranges given for t_i values in tables 7 and 8. Table 7 documents the averages over all 30 problem instances solved within each t_i range for $\sum_{i \in S} x_i^j$, $\sum_{i \in S} q_i^j$, and R^j for $j = 1, 2, 3$. Similarly table 8 documents the averages over all 30 problem instances solved within each range for each t_i range for C^j, E^j and Π^j for $j = 1, 2, 3$. We have the following observations based on table 7 and 8.

- As expected and can observed in Table 7, the retailer's re-order point increases while the number of selected suppliers (except single sourcing) and the total order quantity do not follow an increasing or decreasing pattern as the suppliers' lead times increase with any ordering policy.

As can be seen in Table 8, the retailer's expected costs, carbon emissions, and total costs after carbon trading per unit time show neither an increasing nor a decreasing trend with increased supplier lead times on average with any ordering policy. This follows from the fact that by increasing his/her re-order point, and selecting suppliers and order quantities accordingly, the retailer can avoid the drawbacks of longer lead times.

7. CONCLUSIONS

This paper studies an integrated stochastic inventory control and supplier selection model under environmental regulations. In particular, we formulate and analyze a continuous review inventory control model under carbon trading regulation with three ordering policies: single sourcing, sequential ordering, and sequential delivery. A solution method is discussed for each policy. A comparison of these policies

In terms of their economic and environmental performances is provided. A set of numerical studies is conducted to demonstrate the efficiency of the solution methods proposed. Further numerical studies illustrate the effects of supplier capacities and lead times on the retailer's ordering and supplier selection decisions as well as costs and carbon emissions.

The following results are documented. In case the retailer solely has economic objectives, preferring multiple sourcing instead of single sourcing will reduce the total expected costs after carbon trading. Furthermore, it is also possible that multiple sourcing will reduce expected carbon emissions. However, it might be the case that expected carbon emissions are lower with single sourcing; therefore, in case the retailer has economic as well as environmental objectives, single sourcing can be preferred over multiple sourcing depending on the retailer's economic and environmental targets. Furthermore, in case the retailer solely has economic objectives, any of the delivery structures considered for order splitting can be preferred depending on the settings. It is possible that sequential ordering (sequential delivery) reduces not only expected costs but also carbon emissions compared to sequential delivery (sequential ordering). Nevertheless, it might be the case that while one delivery structure outperforms the other economically, it can be outperformed by the other environmentally.

The contributions of this study are as follows. An integrated continuous review inventory control and supplier selection model is analyzed under environmental regulations with three ordering policies. We economically and environmentally compare single sourcing to multiple sourcing and, sequential ordering to sequential delivery. Even without environmental aspects of the models considered, it is a contribution of this study that sequential ordering is discussed to be a potentially better delivery structure. The models enable numerical analysis of the supplier capacities and delivery lead times on a retailer's ordering decisions, supplier selection decisions, expected costs, and expected carbon emissions with each ordering policy.

Future research directions include considering similar models with stochastic delivery lead times and analyze the effects of the variability of the lead times economically and environmentally. Furthermore, the literature review reveals that there are a limited number of studies that investigate multi-item inventory control systems with environmental considerations. Economic and environmental analyses of multi-item inventory systems under deterministic and stochastic demand with different delivery structures remain as future research questions.

SECTION

3. CONCLUSIONS

In case the retailer solely has economic objectives, preferring multiple sourcing instead of single sourcing will reduce the total expected costs after carbon trading. Furthermore, it is also possible that multiple sourcing will reduce expected carbon emissions. However, it might be the case that expected carbon emissions are lower with single sourcing; therefore, in case the retailer has economic as well as environmental objectives, single sourcing can be preferred over multiple sourcing depending on the retailer's economic and environmental targets. Furthermore, in case the retailer solely has economic objectives, any of the delivery structures considered for order splitting can be preferred depending on the settings. It is possible that sequential ordering (sequential delivery) reduces not only expected costs but also carbon emissions compared to sequential delivery (sequential ordering). Nevertheless, it might be the case that while one delivery structure outperforms the other economically, it can be outperformed by the other environmentally.

These are some of the contributions of this study. An integrated continuous review inventory control and supplier selection model is analyzed under environmental regulations with three ordering policies. We economically and environmentally compare single sourcing to multiple sourcing and, sequential ordering to sequential delivery. Even without environmental aspects of the models considered, it is a contribution of this study that sequential ordering is discussed to be a potentially better delivery structure. The models enable numerical analysis of the supplier capacities and delivery lead times on a retailer's ordering decisions, supplier selection decisions, expected costs, and expected carbon emissions with each ordering policy.

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APPENDIX

A. NOTATION AND POSSIBLE METRICS

Notation	Description	Metric
Demand Parameters		
λ:	expected demand rate	units/year
υ:	standard deviation of demand rate	units
τ:	lead time duration	year
y:	normally distributed random variable with λy and $\upsilon\sqrt{t}$	units
$f(y)$:	probability density function of y	
$F(y)$:	cumulative distribution function of y	
Retailer Parameters		
$\widetilde{h}$:	Inventory holding cost per unit per unit time	\$/unit/year
$\widetilde{A}$:	Fixed order setup cost	\$/order
$\widetilde{p}$:	Unit backorder cost	\$/unit
$\widehat{h}$:	Emissions due to inventory holding per unit per unit time	CO_2 lbs/unit/year
$\widehat{A}$:	Emissions due to order placement	CO_2 lbs/order
$\widehat{p}$:	Emissions due to per unit backorder	CO_2 lbs/unit
Retailer's Decision Variables		
x_i:	1 if supplier i is selected, 0 otherwise	
$\mathbf{x}$:	n-vector of x_i values	
q_i:	order quantity from supplier i	units
$\mathbf{q}$:	n-vector of q_i values	
R:	re-order point for starting ordering	units
Supplier Parameters		
τ_i:	delivery lead time of supplier i	year
w_i:	delivery capacity of supplier i per order	units
$\widetilde{c}_i$:	retailer's unit procurement cost from supplier i	\$/unit
$\widetilde{a}_i$:	setup cost due to order placement from supplier i	\$/order
$\widehat{c}_i$:	Emissions due to per unit procurement from supplier i	CO_2 lbs/unit
$\widehat{a}_i$:	Emissions due to order placement from supplier i	CO_2 lbs/order
Regulation Parameters		
β:	unit carbon emissions trading price	\$/CO_2 lbs
α:	unit carbon emissions tax	\$/CO_2 lbs
Φ:	carbon cap	CO_2 lbs/year

APPENDIX

B. PROOFS FOR PROPERTIES IN SECTION 4

Proof of Property 1: Suppose that there exists an optimal solution $(R^j, \mathbf{q}^j, \mathbf{x}^j)$ such that $x_i^j = 1$ and $q_i^j = 0$ for some $i \in S$. Now, consider another solution $(R^{j'}, \mathbf{q}^{j'}, \mathbf{x}^{j'})$ such that $R^j = R^{j'}$, $\mathbf{q}^j = \mathbf{q}^{j'}$ and $x_r^j = x_r^{j'}$ $\forall r \neq i$ and $x_i^{j'} = 0$. Note that $(R^{j'}, \mathbf{q}^{j'}, \mathbf{x}^{j'})$ is a feasible solution for models P1, P2, and P3 for $j = 1, 2, 3$. Then, it follows that $C^j(R^j, \mathbf{q}^j, \mathbf{x}^j) > C^j(R^{j'}, \mathbf{q}^{j'}, \mathbf{x}^{j'})$ and $E^j(R^j, \mathbf{q}^j, \mathbf{x}^j) > E^j(R^{j'}, \mathbf{q}^{j'}, \mathbf{x}^{j'})$;

therefore, $\Pi^j(R^j, \mathbf{q}^j, \mathbf{x}^j) > \Pi^j(R^{j'}, \mathbf{q}^{j'}, \mathbf{x}^{j'})$. This implies that $(R^j, \mathbf{q}^j, \mathbf{x}^j)$ is not an optimal solution. □

Proof of Property 2: Let R^2 and $\mathbf{x}^2$ be given. Furthermore, let $Q^2 = \sum_{i\in S} q_i^2$ be given. Then, it follows from Equations (4) and (5), and model **P2** that $\mathbf{q}^2$ is the optimal solution of P2-$(R^2, Q^2, \mathbf{x}^2)$ stated below:

$$\begin{aligned} (\mathbf{P2} - (R^2, Q^2, \mathbf{x}^2)) : \quad & \min \quad G(\mathbf{q}) = \frac{\lambda}{Q^2} \sum_{i\in S(\mathbf{x}^2)} c_i q_i + M(R^2, Q^2, \mathbf{x}^2) \\ & \text{s.t.} \quad \sum_{i\in S(\mathbf{x}^2)} q_i = Q^2 \\ & \qquad\; q_i \leq w_i \quad \forall i \in S(\mathbf{x}^2), \end{aligned}$$

where $M(R^2, Q^2, \mathbf{x}^2) = h\left(R^2 - \lambda\tau(\mathbf{x}^2) + \frac{1}{2}Q^2\right) + \frac{\lambda\left(A + \sum_{i\in S(\mathbf{x}^2)} a_i x_i^2\right)}{Q^2} + \frac{p\lambda n(R^2, \tau(\mathbf{x}^2))}{Q^2} - \beta\Phi$ such that $c_i = \widetilde{c}_i + \beta\widehat{c}_i$, $h = \widetilde{h} + \beta\widehat{h}$, $A = \widetilde{A} + \beta\widehat{A}$, $a_i = \widetilde{a}_i + \beta\widehat{a}_i$, and $p = \widetilde{p} + \beta\widehat{p}$. Note that given R^2, Q^2, and $\mathbf{x}^2$, both $M(R^2, Q^2, \mathbf{x}^2)$ and $\frac{\lambda}{Q^2}$ are constants. Let $\mathbf{q}^2$ be the optimal solution of **P2**-$(R^2, Q^2, \mathbf{x}^2)$ such that $0 < q_{j^{\mathbf{x}^2}}^2 \leq w_{j^{\mathbf{x}^2}}$ and there exists an $i \in S(\mathbf{x}^2)$ where $q_i^2 < w_i$ and $i \neq j^{\mathbf{x}^2}$. Now consider $\mathbf{q}^*$ such that $q_j^* = q_j^2$ $\forall j \in S(\mathbf{x}^2) - \{i, j^{\mathbf{x}^2}\}$, $q_i^* = q_i^2 + \Delta$ and $q_{j^{\mathbf{x}^2}}^* = q_{j^{\mathbf{x}^2}}^2 - \Delta$ where $0 < \Delta \leq \min\{q_{j^{\mathbf{x}^2}}^2, w_i - q_i^2\}$. Note that $\mathbf{q}^*$ is feasible for **P2**-$(R^2, Q^2, \mathbf{x}^2)$. Furthermore, since $c_i < c_{j^{\mathbf{x}^2}}$, it then follows that $G(\mathbf{q}^2) > G(\mathbf{q}^*)$. This contradicts that $\mathbf{q}^2$ is the optimal solution of **P2**-$(R^2, Q^2, \mathbf{x}^2)$. Therefore, $q_i^2 = w_i$ $\forall i \in S(\mathbf{x}^2) - \{j^{\mathbf{x}^2}\}$, and $q_{j^{\mathbf{x}^2}}^2 \leq w_{j^{\mathbf{x}^2}}$. □

Proof of Property 3: Let $\mathbf{x}$ be given such that $Q_{\mathbf{x}}^{2*} = \lim_{Q^-}\left(\sum_{i\in S(\mathbf{x}^2)} w_i - w_{j^{\mathbf{x}}}\right)$. Consider x' such that $x_i' = x_i$ $\forall i \in S$ and $x_{j^x}' = 0$. Then, once can show easily that $\Pi^2(R_{\mathbf{x}}^{2*}, \mathbf{q}_{\mathbf{x}}^{2*}, \mathbf{x}) > \Pi^2(R_{\mathbf{x}'}^{*}, \mathbf{q}_{\mathbf{x}'}^{*}, \mathbf{x}')$. It then follows that $\mathbf{x}$ cannot be optimum for **P2**, i.e., $\mathbf{x} \neq \mathbf{x}^2$. □

APPENDIX

C. TABLES IN SECTION 6

Table 1: Comparison of Algorithm 1 to Interior Point Method

	Algorithm 1				Interior Point Method			
n	R_i^*	q_i^*	$\pi_i(R_i^*, q_i^*)$	CPU	R_i^*	q_i^*	$\pi_i(R_i^*, q_i^*)$	CPU
3	364.3	76.7	45213	0.001	364.3	76.7	45213	0.048
6	381.5	73.5	47717	0.001	381.5	73.5	47717	0.046
9	365.2	75.8	46512	0.001	365.2	75.8	46512	0.051
12	374.7	73.8	46014	0.001	374.7	73.8	46014	0.060
15	378.9	75.4	45679	0.001	378.9	75.4	45679	0.071
avg	372.9	75.0	46227	0.001	372.9	75.0	46227	0.055

Table 2: Comparison of Algorithm 2 to Interior Point Method

	Algorithm 2				Interior Point Method			
n	R_{x}^{2*}	Q_{x}^{2*}	$g_{\mathrm{x}}(R_{\mathrm{x}}^{2*}, Q_{\mathrm{x}}^{2*})$	CPU	R_{x}^{2*}	Q_{x}^{2*}	$g_{\mathrm{x}}(R_{\mathrm{x}}^{2*}, Q_{\mathrm{x}}^{2*})$	CPU
3	359.7	109.3	41570	0.001	359.7	109.3	41570	0.024
6	348.5	199.4	40428	0.006	348.5	199.4	40428	0.025
9	331.7	299.3	37749	0.012	331.7	299.3	37749	0.026
12	329.6	395.1	36331	0.017	329.6	395.1	36331	0.026
15	294.3	514.7	35371	0.019	294.3	514.7	35371	0.024
avg	332.8	303.6	38290	0.011	332.8	303.6	38290	0.025

Table 3: Comparison of Algorithm 3 to Total Enumeration

	Algorithm 3					Total Enumeration				
n	$\sum_{i\in S} x_i^2$	$\sum_{i\in S} q_i^2$	R^2	Π^2	CPU	$\sum_{i\in S} x_i^2$	$\sum_{i\in S} q_i^2$	R^2	Π^2	CPU
3	2.20	178	336.3	35457	0.004	2.2	178	336.3	35457	0.392
6	3.50	255	341.6	36140	0.010	3.5	255	341.6	36140	2.785
9	3.40	262	326.0	32826	0.008	3.4	262	326.0	32826	32.176
12	3.60	273	338.2	31781	0.005	3.6	273	338.2	31781	338.455
15	3.90	292	336.9	31348	0.005	3.9	292	336.9	31348	3648.851
avg	3.32	252	335.8	33510	0.007	3.32	252	335.8	33510	804.532

Table 4: Comparison of Algorithm 4 to Total Enumeration

	Algorithm 4					Total Enumeration				
n	$\sum_{i\in S} x_i^3$	$\sum_{i\in S} q_i^3$	R^3	Π^3	CPU	$\sum_{i\in S} x_i^3$	$\sum_{i\in S} q_i^3$	R^3	Π^3	CPU
3	2.3	183	321.1	35421	1.060	2.3	183	321.1	35421	0.317
6	3.5	255	311.6	36076	12.122	3.5	255	311.6	36076	5.460
9	3.5	270	293.7	32750	41.183	3.5	270	293.7	32750	81.891
12	3.7	280	306.2	31712	101.435	3.7	280	306.2	31712	1104.981
15	4	297	307.9	31276	210.817	4	297	307.9	31276	14745.662
avg	3.4	257	308.1	33447	73.323	3.4	257	308.1	33447	3187.662

Table 5: Inventory Control and Supplier Selection Decisions vs. Supplier Capacities

	Single Sourcing			Sequential Ordering			Sequential Delivery		
w_i	$\sum_{i\in S} x_i^1$	$\sum_{i\in S} q_i^1$	R^1	$\sum_{i\in S} x_i^2$	$\sum_{i\in S} q_i^2$	R^2	$\sum_{i\in S} x_i^3$	$\sum_{i\in S} q_i^3$	R^3
[20, 40]	1.00	36.0	395.7	4.03	120.7	372.2	4.07	121.3	365.2
[40, 60]	1.00	56.0	379.0	3.30	168.7	353.7	3.33	170.7	332.5
[60, 80]	1.00	71.7	370.8	3.00	211.3	340.4	3.03	213.3	317.6
[80, 100]	1.00	93.0	358.5	2.43	217.7	338.1	2.43	217.7	314.3
[100, 120]	1.00	109.7	354.5	2.40	262.0	328.7	2.43	265.3	306.0
[120, 140]	1.00	128.7	343.7	2.00	259.7	322.5	2.10	272.7	300.2
[140, 160]	1.00	152.0	334.7	1.93	292.3	320.8	1.93	292.3	298.0
[160, 180]	1.00	169.0	345.0	1.87	315.7	321.4	1.87	315.7	303.2
[180, 200]	1.00	190.7	317.9	1.83	348.3	307.0	1.87	355.0	285.0
avg	1.00	111.9	355.5	2.53	244.0	333.9	2.56	247.1	313.6

Table 6: Expected Costs and Carbon Emissions vs. Supplier Capacities

	Single Sourcing			Sequential Ordering			Sequential Delivery		
w_i	C^1	E^1	Π^1	C^2	E^2	Π^2	C^3	E^3	Π^3
$[20, 40]$	57554	31325	58686	46124	24228	46547	46119	24199	46539
$[40, 60]$	44717	25000	45217	38269	21168	38386	38228	21106	38338
$[60, 80]$	39794	21989	39992	35687	19779	35665	35641	19691	35610
$[80, 100]$	36336	20053	36341	33694	18610	33555	33635	18582	33493
$[100, 120]$	33724	19420	33665	31362	18383	31200	31310	18325	31142
$[120, 140]$	32587	18344	32421	31161	17619	30923	31112	17553	30867
$[140, 160]$	30638	18631	30501	29337	17981	29135	29285	17958	29081
$[160, 180]$	30240	17601	30000	29353	17086	29062	29308	17067	29014
$[180, 200]$	29072	18115	28883	28394	17684	28162	28348	17633	28112
avg	37185	21164	37301	33709	19171	33626	33665	19124	33577

Table 7: Inventory Control and Supplier Selection Decisions vs. Supplier Lead Times

	Single Sourcing			Sequential Ordering			Sequential Delivery		
τ_i	$\sum_{i\in S} x_i^1$	$\sum_{i\in S} q_i^1$	R^1	$\sum_{i\in S} x_i^2$	$\sum_{i\in S} q_i^2$	R^2	$\sum_{i\in S} x_i^3$	$\sum_{i\in S} q_i^3$	R^3
$[0.0050, 0.0075]$	1.00	84.3	233.7	2.40	184.7	221.4	2.43	187.3	201.8
$[0.0075, 0.0100]$	1.00	82.0	293.4	2.80	203.3	271.4	2.80	203.3	256.3
$[0.0100, 0.0125]$	1.00	87.0	342.7	2.67	211.0	311.7	2.67	211.0	301.4
$[0.0125, 0.0150]$	1.00	84.3	394.5	2.77	208.3	359.1	2.77	208.3	355.0
$[0.0150, 0.0175]$	1.00	82.7	439.9	2.83	218.0	399.7	2.83	218.0	396.6
$[0.0175, 0.0200]$	1.00	83.7	479.8	2.83	214.7	437.5	2.87	218.0	437.4
$[0.0200, 0.0225]$	1.00	79.7	525.5	2.60	197.7	479.8	2.60	197.7	484.3
$[0.0225, 0.0250]$	1.00	80.3	569.3	2.73	206.3	517.9	2.73	206.3	523.0
$[0.0250, 0.0275]$	1.00	83.3	613.1	2.67	199.3	562.9	2.67	199.3	572.3
$[0.0275, 0.0300]$	1.00	82.0	654.6	2.70	199.3	602.7	2.70	199.3	612.4
avg	1.00	82.9	454.6	2.70	204.3	416.4	2.71	204.9	414.0

Table 8: Expected Costs and Carbon Emissions vs. Supplier Lead Times

	Single Sourcing			Sequential Ordering			Sequential Delivery		
τ_i	C^1	E^1	Π^1	C^2	E^2	Π^2	C^3	E^3	Π^3
$[0.0050, 0.0075]$	36411	20905	36502	33765	19393	33704	33717	19388	33656
$[0.0075, 0.0100]$	39296	20661	39362	35173	19160	35089	35135	19141	35049
$[0.0100, 0.0125]$	37090	22256	37315	33740	19707	33711	33716	19690	33685
$[0.0125, 0.0150]$	37887	21486	38035	34210	19310	34141	34200	19298	34129
$[0.0150, 0.0175]$	38491	20413	38533	34732	18573	34590	34725	18560	34581
$[0.0175, 0.0200]$	37560	22618	37822	34077	19817	34059	34087	19748	34062
$[0.0200, 0.0225]$	38422	22163	38639	34808	19679	34776	34821	19669	34788
$[0.0225, 0.0250]$	38659	21822	38841	35131	19550	35086	35145	19538	35099
$[0.0250, 0.0275]$	37756	20974	37854	34312	19056	34218	34340	19049	34245
$[0.0275, 0.0300]$	38643	21522	38795	35199	19334	35132	35227	19325	35160
avg	38022	21482	38170	34515	19358	34451	34511	19341	34445

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