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Constitutive Modelling of Soils and Computation of Earthquake Damage and Liquefaction

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Constitutive Modelling of Soils and Computation of Earthquake Damage and Liquefaction

(State of the Art Paper)

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SYNOPSIS For realistic modelling of deformation and collapse of soil structures, an accurate constitutive model for the soil materials is necessary. In this paper we shall show
i) how such very successful models can be obtained by the use of generalised plasticity theory;
ii) the modification of such models for semi-saturated behaviour;
and finally
iii) how incorporation of such models into a two phase computer program allows the solution of complex problems. A possible mode of failure of the San Fernando dam is included.

This paper is divided into three parts according to the above.

INTRODUCTION

Accurate modelling of boundary value problems of soil mechanics using numerical (finite element) procedures requires a detailed knowledge of constitutive behaviour of the soil skeleton. This is particularly important in problems involving dynamics (e.g. earthquake engineering) where cyclic mobility or liquefaction may develop. Here the tendency of soils to 'densify' or reduce their volume when subject to continuing shear is responsible for pore pressure increases which weaken the maximum resistance progressively. For this reason in such problems the knowledge of full deformation history is essential to predict safety and simplified computations do not apply.

The whole problem of behaviour of soil structures is dependent on the soil skeleton - pore water pressure interaction. The classical work of Biot (1941) has, for the first time, formulated the governing equations for such phenomena but further development was needed to provide full forms suitable for non-linear finite element analysis (Zienkiewicz and Shiomi, 1985; Zienkiewicz, 1985). Today the whole problem is capable of solution with suitably specified material constitutive selection (Zienkiewicz et al., 1990). A general purpose code SWANDYNE X has been developed for this purpose.

In this paper we shall divide the presentation into three parts:

- Part I deals with the generalised plasticity form of constitutive models (Pastor et al., 1990);
- Part II deals with modifications to this constitutive model which are essential if semi saturated behaviour develops (or negative pore pressures exist);
- Part III, where some examples of computation solutions by SWANDYNE X are presented. We shall not go, however, into detailed formulation of the problem, which is available in Zienkiewicz et al. (1990).

PART 1: CONSTITUTIVE MODELS

1. Introduction

The basic feature of saturated soils under dynamic loading is their tendency to contract. If volume change is restricted by soil permeability, pore pressure increases and effective mean confining pressure decreases. Even if the total stress ratio amplitude is kept constant, the pore-pressure increases as the path approaches failure conditions. In the case of very loose granular soils, liquefaction may be triggered, while denser sands will exhibit cyclic mobility.

Classical plasticity models (Drucker et al., 1957; Nova and Wood, 1979; Roscoe and Burland, 1968) can reproduce basic trends of soils under monotonic loading, but they fail when applied to more complex situations such as described above.

Modified plasticity theories are able to introduce volumetric plastic strain under cyclic loading (Hashiguchi, 1980; Hirai, 1987; Mroz, 1967; Mroz et al., 1979), but often accuracy here is obtained by introducing considerable complexity. We believe that the bounding surface (Dafalias and Herrmann, 1982) and generalised plasticity models (Zienkiewicz and Mroz, 1984; Zienkiewicz et al., 1985; Pastor et al., 1985; Pastor and Zienkiewicz, 1986; Chan et al., 1988) provide a good compromise here.

We present here the general framework of generalised plasticity theory and two simple models for cohesive and granular soils under monotonic and transient loading.

2. Generalised Plasticity

If material behaviour is assumed to be independent of rate at which load is applied, but on the load increment itself, the relation between increment of strain and stress tensors may be given by

$$d\epsilon = \mathbf{C} : d\sigma' \tag{1}$$

where $\mathbf{C}$ is a fourth-order constitutive tension which depends on material microstructure, the state of stress and strain, past history and the direction of effective stress increment. To account for this dependence in a simple way, two different tensors, $\mathbf{C}_L$ and $\mathbf{C}_U$ may be postulated for stress increments corresponding to 'loading' and 'unloading' situations. A unit tensor $\mathbf{n}$ is now introduced to discriminate between loading and unloading,

$$\begin{aligned} \mathbf{n} : d\sigma' > 0 \quad &(\text{loading}) \\ \mathbf{n} : d\sigma' < 0 \quad &(\text{unloading}) \end{aligned} \tag{2}$$

This relation has of course to be modified slightly in case of strain softening behaviour, as shown in Pastor et al. (1990).

Continuity between both states requires that $\mathbf{C}_L$ and $\mathbf{C}_U$ are of the form (Pastor et al., 1985):

$$\begin{aligned} \mathbf{C}_L &= \mathbf{C}_e + \frac{1}{H_L} \mathbf{n}_{gL} \otimes \mathbf{n} \\ \mathbf{C}_L &= \mathbf{C}_e + \frac{1}{H_U} \mathbf{n}_{gU} \otimes \mathbf{n} \end{aligned} \tag{3}$$

where $\mathbf{C}_e$ is the constitutive tensor which characterises reversible material behaviour, $\mathbf{n}_{gL/U}$ are unit tensors and $H_{L/U}$ are scalar functions. It may be easily verified that continuity of deformation is now satisfied for neutral loading as then

$$\mathbf{n} : d\sigma' = 0 \tag{4}$$

Strain increments may be considered as an addition of reversible (elastic) and plastic parts, given by

$$\begin{aligned} d\epsilon^e &= \mathbf{C}^e : d\sigma' \\ d\epsilon^p &= (n_{gL/U} \otimes n) : d\sigma'/H_{L/U} \end{aligned} \tag{5}$$

In the above, it may be seen that $n_{gL/U}$ gives the direction of plastic flow, and can be determined from experiments. It is also important to note that plastic strain may be produced during unloading in this model.

So far, plastic strain has been assumed to be produced by a single mechanism. If several mechanisms exist, the above expressions can easily be generalised, giving

$$d\epsilon = \sum_{i=1}^{M} d\epsilon^{(i)}$$

where (6)

$$d\epsilon^{(i)} = \{\mathbf{C}_e^{(i)} + (n_{gL/U}^{(i)} \otimes n^{(i)}) H^{(i)}_{L/U}\} : d\sigma'$$

where M mechanisms have been considered. These are of 'series' type, subjected to the same stress, and can represent different phenomena such as crushing or rearrangement of grains, or behaviour on different physical microplanes.

Constitutive equations of isotropic materials can be cast in terms of invariants of stress and strain tensors:

$$\begin{aligned} I'_1 &= 1\mathrm{tr}(\boldsymbol{\sigma}') \\ J'_2 &= 1/2\ \mathrm{tr}(\mathbf{s}'^2) \\ J'_3 &= 1/3\ \mathrm{tr}\ (\mathbf{s}'^3) \end{aligned} \tag{7}$$

where $\mathbf{s}'$ is the deviatoric stress tensor

$$\mathbf{s}' = \boldsymbol{\sigma}' - I'_1/3\ \boldsymbol{\delta} \tag{8}$$

and $\boldsymbol{\delta}$ the identity tensor.

Alternatively, other sets of invariants may be used for convenience. Here (p', q, θ) will be used, to develop simple relations for clays and sands. This set of invariants is defined as follows:

$$\begin{aligned} p' &= 1/3\ I'_1 \\ q &= \sqrt{3J'_2} \\ \theta &= 1/3 \sin^{-1} [\ (3\sqrt{3}\ J'_3)\ /\ (2\ J'^{3/2}_2)\ ] \\ \theta &\le \pi/6 \end{aligned} \tag{9}$$

Constitutive relations derived in the space of stress invariants, can be generalised to three-dimensional situations. The procedure is described in Chan et al. (1988) and is summarized below.

We assume that $\hat{H}_{L/U}$, $\hat{\mathbf{n}}$ and $\hat{\mathbf{n}}_{gL/U}$ are known in (p', q, θ) space. From these, we find that

$$H_{L/U} = \hat{H}_{L/U}$$

$$n = \hat{n} \quad \hat{\sigma}'/ \quad \sigma'$$

$$n_g = \hat{n}_g \quad \hat{\sigma}'/ \quad \sigma' \qquad (10)$$

where

$$\hat{\sigma}' = (p', q, \Theta) \qquad (11)$$

3. Cohesive Soils

Plasticity models for clays are frequently based on the Critical State Model developed at the University of Cambridge (Schofield and Wroth, 1968; Roscoe et al., 1958), which postulates the existence of a unique line on p-q space, on which all residual states for a given clay should lie. This line is given by

$$q = Mp^1 \qquad (12)$$

and it is referred to as the Critical State Line.

Dilatancy of clays under virgin loading has been found to be dependent on the stress ratio ,

$$\eta = q/p' \qquad (13)$$

and can be assumed to be a linear function of it.

$$d_g = d\epsilon_v^p / d\epsilon_s^p = (1 + \alpha)(M - \eta) \qquad (14)$$

where $d\epsilon_v^p$ and $d\epsilon_s^p$ are the increments of volumetric and shear strain defined as

$$d\epsilon_v^p = tr(d\epsilon)$$

$$d\epsilon_s^p = 2/3 \ [\ 1/2 \ tr(de^2)]^{1/2} \qquad (15)$$

and M depends on the Lode's angle as

$$M = 6M_c/ (6 + M_c(1-\sin 3\Theta)) \qquad (16)$$

M_c being the value obtained in compression triaxial tests.

The direction of plastic flow in the space of invariants is therefore given by

$$n_{gL} = v_{gL}/ \ |v_{gL}| \qquad (17)$$

where

$$v_{gL} = (v_{gv}, v_{gs}, v_{g\theta})_L \qquad (18)$$

and

$$v_{gv} = (1 + \alpha)(M + \eta)$$

$$v_{gs} = 1$$

$$v_{g\theta} = -1/2 \ M \ q \ \cos 3\Theta \qquad (19)$$

An important issue is that of associativity of plastic flow. We will show later that in order to model some basic features of sand behaviour it is necessary to assume non-associativeness. However, experimental work carried out by Atkinson and Richardson (1985) shows that plastic flow of clays is associated, and, therefore, it will be assumed here that

$$n \equiv n_{gL} \qquad (20)$$

Plastic modulus will be taken as

$$H_L = H_0 \ p' \ \{f_1(\eta) + f_2(\xi)\} \ f_3(\zeta) \qquad (21)$$

where H_0 is a constant, ξ the accumulated plastic shear strain and ζ a mobilized stress function. Thus

$$\xi = \int |d\epsilon_s^p|$$

$$\zeta = p' \ \{1 - (1 + \alpha)/\alpha \quad \eta/M\}^{-1/\alpha} \qquad (22)$$

$$f_1(\eta) = [1-\eta/M]^{2.5} \ (1 + d_0^2)/(1+d^2) \ \mathrm{sign}[1-\eta/M]$$

$$f_2(\xi) = \beta \exp(-\beta \xi)$$

$$f_3(\zeta) = (\zeta_{MAX}/ \ \zeta)^{\gamma}$$

with

$$d_0 = (1+\alpha)d_0$$

$$\beta = \beta_0(1 - \xi / \xi_{MAX}) \qquad (23)$$

Unloading will be assumed to be purely elastic.

Clay reponse under cyclic load depends on four parameters (H_0, M, β_0, γ) in addition to the elastic constants.

The model predicts that residual conditions,

$$H = 0$$

$$\xi \rightarrow \infty \qquad (24)$$

will take place on the Critical State Line $q = Mp'$ as there

$$f_3 = 1$$
$$f_2 \rightarrow 0 \qquad (25)$$
$$f_1 = 0$$

Fig. 1 shows model predictions and experimental data obtained by Taylor and Bacchus (1969) on a saturated clay subjected to constant strain amplitude cycles under undrained conditions. As the number of cycles N increases, both p' and q decrease.

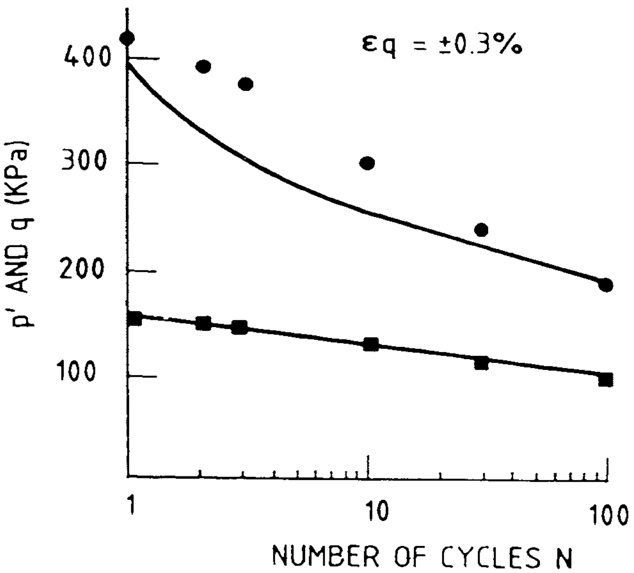


Fig. 1 Behaviour of clay under two-way strain controlled undrained triaxial loading (exp. from Taylor and Bacchus, 1969).

4. Granular Soils

Granular and cohesive soils have some common fundamental features. Dilatancy may be modelled by the same law given in Eq. (19), now written as

$$d_g = (1 + \alpha)(M_g - \eta) \qquad (26)$$

where Mg is the slope of the line on p'-q plane on which dilatancy is zero. This line has often been referred to as 'characteristic state' or 'phase transformation line'.

The direction of plastic flow is given by Eqs. (17) to (19), where M should now be replaced by M_g. As for clays, it depends on the Lode's angle, but a better approach to reality may be obtained by introducing a correcting factor f_g, such that

$$M_g(\Theta) = \frac{6\, M_{gc}}{6 + f_g\, M_{gc}(1 - \sin 3\Theta)} \qquad (27)$$

This factor is necessary to account for deviations from Mohr-Coulomb type of behaviour in extension. Fig. 2 shows some experimental results obtained by Yamada and Ishihara (1982), together with predictions with $f_g = 0.6$.

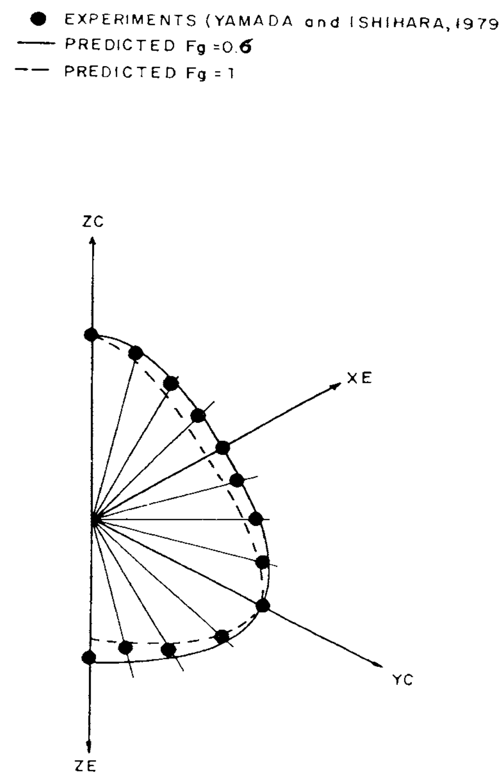


Fig. 2 Locus of zero dilatancy of constant p' (Yamada and Ishihara, 1982).

One of the basic features of sand behaviour is that of liquefaction of very loose sand under undrained monotonic loading. The existence of a peak in the deviatoric stress implies that

$$d\sigma' : d\epsilon^p < 0 \qquad (28)$$

and, therefore

$$d\sigma'(1/H\ \mathbf{n}_{gL} \otimes \mathbf{n})d\sigma' < 0 \qquad (29)$$

If we consider now that during the whole process the stress ratio is continuously increasing, which suggests that plastic modules H is positive, we arrive at the conclusion that, in order to fulfil Eq. (29), the flow must be non-associated:

$$n_{gL} \neq n \qquad (30)$$

Direction **n** may be taken is given by

$$\mathbf{n} = \mathbf{v}/|\mathbf{v}|$$
$$\mathbf{v} = (v_v, v_s, v_\theta) \qquad (31)$$

with

$$v_v = (1+\alpha)(M_f - \eta)$$
$$v_s = 1 \qquad (32)$$
$$v_\theta = -1/2\ M_f\ q \cos 3\theta$$

the plastic modulus is now assumed to be of the form

$$H_L = H_0\ p'\ H_f\ (H_v + H_s)\ H_{DM} \qquad (33)$$

where

$$H_f = (1 - \eta/\eta_f)^4$$
$$H_v = (1 - \eta/M_g)$$
$$H_s = \beta_0\ \beta_1\ \exp(-\beta_0 \xi) \qquad (34)$$
$$H_{DM} = (\zeta_{MAX}/\zeta)$$

and

$$\eta_f = (1 + 1/\alpha)M_f \qquad (35)$$

Again, both ξ and ζ are the accumulated plastic shear strain and mobilized stress functions, given by Eq. (22).

So far, only monotonic loading has been considered. The proposed model depends on seven parameters (M_f, M_g, H_0, β_0, β_1, γ and f_g) in addition to the elastic constants, and it is able to reproduce both drained and undrained behaviour of sands ranging from very loose to dense.

It is important to note that material softening is observed when very dense sands are tested under drained conditions. Before the peak is reached, one or more shear bands develop. Deformation localizes at these and the specimens are no longer homogenous. Therefore, quantitative results are not representative. However, it is logical to assume that some material softening may exist also, even if not so dramatic as that indicated by the tests. The model may predict this feature, producing a slight decrease in deviatoric strength after the peak is reached. This is obtained because the plastic modulus decreases after becoming zero at η_P

$$H_v + H_s = 0 \qquad (36)$$
$$\eta_P > M_g$$

If the test is run under loading control, H_s decreases while H_v does not, resulting in

$$H_v + H_s < 0$$

and, therefore (37)

$$H_L < 0$$

A basic feature of sand behaviour is that plastic strain develops during unloading, and two facts are usually found in experiments:

i) increment of plastic strain is of contractive nature;
ii) the importance of this effect increases with the stress ratio η_u from which unloading takes place.

$$\eta_u = (q/p')_u \qquad (38)$$

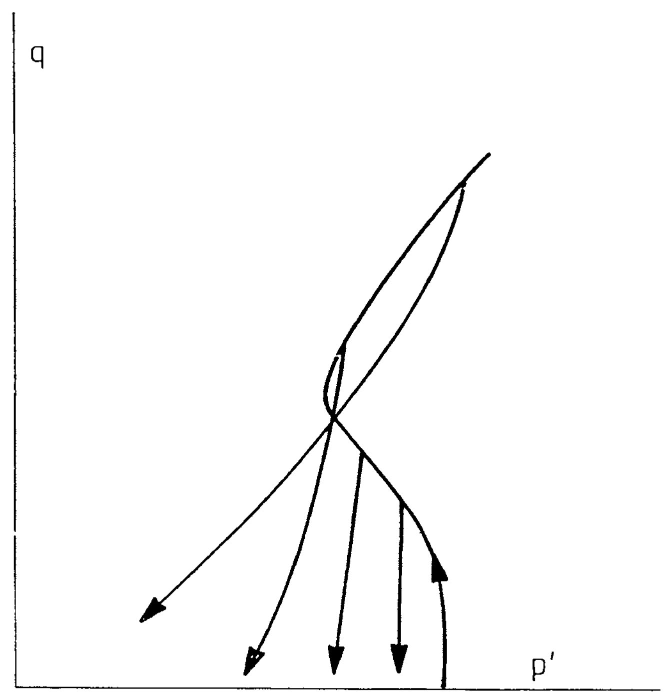


Fig. 3. Undrained unloading of a sand from different stress ratios (schematic).

This behaviour is sketched in Fig. 3, and may be taken into account by assuming

$$
\begin{aligned}
H_U &= H_{U0}(M_g/\eta_U)^{\gamma_U} && \text{for } |M_g/\eta_U| > 1 \\
&= H_{U0} && \text{for } |M_g/\eta_U| \le 1
\end{aligned} \qquad (39)
$$

$$
\begin{aligned}
n_{gU} &= (n_{guv},\ n_{gus},\ n_{gu\theta})^T \\
n_{guv} &= -\mathrm{abs}(n_{gv}) \\
n_{gus} &= n_{gs} \\
n_{gu\theta} &= n_{g\theta}
\end{aligned} \qquad (40)
$$

where n_{gv}, n_{gs} and $n_{g\theta}$ are the components of n_{gL}.

The proposed model is able to reproduce both liquefaction and cyclic mobility phenomena observed in very loose and loose sands under cyclic undrained loading, which are of course important in earthquake engineering. Fig. 4 shows model predictions and observed results for Banding sand (Castro, 1969), and it can be seen how liquefaction occurs after five cycles of loading, due to the pore pressure increasing.

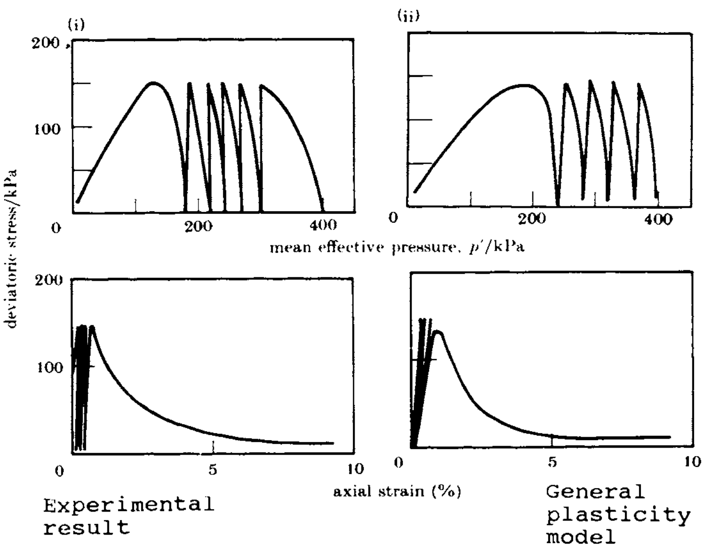


Fig. 4 Cyclic undrained (one way) tests on Banding sand (Castro, 1969).

Plastic strains produced during loading and unloading are also fundamental in explaining cyclic mobility phenomena. Fig. 5 shows both experimental results and model predictions for loose Niigata sand, and it can be seen how basic features are reproduced (Ishihara et al., 1975). It is quite remarkable how well the final mobility strain and p' variation are reproduced here.

In all of the preceding, we have assumed isotropic behaviour and therefore have ignored the well known densification phenomena which may occur due to mere rotations of the principal stress axes with the stress invariants kept constant. Further induced anisotropy was essentially suppressed. The modelling of such behaviour is of course more complex, and is probably best achieved by introduction of multiple mechanisms. Here again the generalised plasticity concepts can be used. However, we shall show later that exceptionally good prediction can be obtained with the above simple procedures.

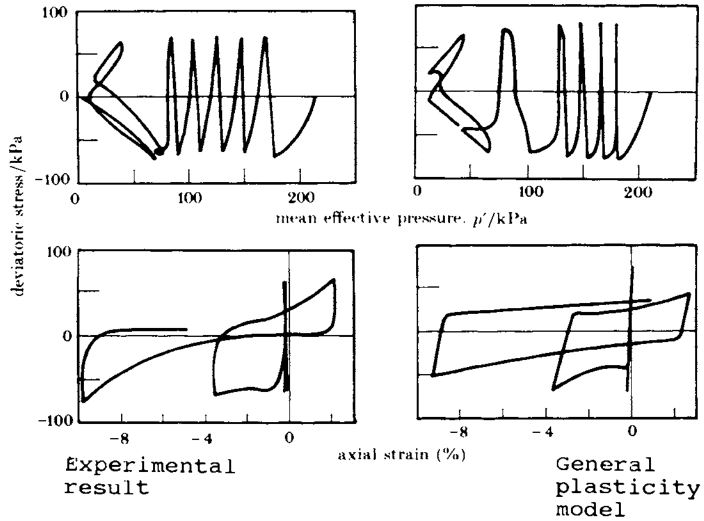


Fig. 5 Cyclic undrained (two way) test on Niigata sand (Ishihara et al., 1975).

PART II: SEMI-SATURATED BEHAVIOUR

5. The Effects of Semi-Saturation and Negative Pressures

The models so far discussed refer to the behaviour of fully saturated soils. In the analysis of such situations, as mentioned before and discussed in Zienkiewicz et al. (1990), the solution is achieved by considering the overall equilibrium and the coupled seepage flow equations. However, when the medium is semi- saturated the situation is more complex.

First, the definition of effective stress needs to be modified. Here it turns out reasonable that with an effective stress taken as (Bishop, 1959; Zienkiewicz et al., 1990)

$$\sigma' = \sigma + [S_w p_w + (1 - S_w)p_a]\ \delta \qquad (41)$$

where S_w stands for the degree of water saturation, the previous constitutive relations are valid (Zienkiewicz et al., 1990). In the above, p_w is the water pressure, p_a is the air pressure (which can be determined from an additional coupled equation or simply taken as zero, implying high air permeability).

Second, the fact that full saturation is not present implies automatically negative water pressure if $p_a = 0$ is assumed. This, of course, is a consequence of surface tensions or capillarity and with a free ingress of air results in a unique negative pressure-saturation relation of the type illustrated in Fig. 6.

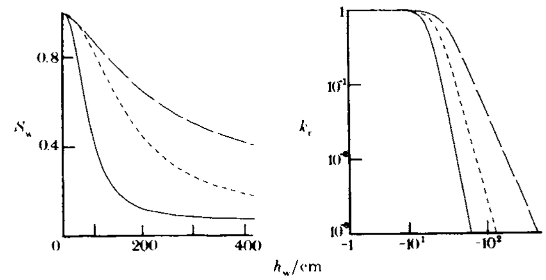


Fig. 6 Relation between negative water pressure $p_w = \gamma h_w$, saturation s_w and relative permeability.
_____ sand
- - - assumed in San Fernando dam analysis
— — — loam

This assumption of a unique relationship implies an immediate release of air (at atmospheric pressure) on application of tensile strains. In claylike materials, of course, some negative pressures of water may develop even in full saturation due to slow air ingress and the limit is set by vapour pressures.

In either case, the problem is strongly coupled with continuity of flow equations for the water and air where, as shown in Fig. 6, the permeability varies strongly with the degree of saturation (Zienkiewicz <u>et al</u>., 1990; Lloret and Alonso, 1980; Desai, 1976).

In Fig. 7 we show that the effects of negative pressures in semi-saturation can be reproduced by both calculation (Zienkiewicz <u>et al</u>., 1990; Schrefler and Simoni, 1988) and experiment (Liakopoulos, 1965).

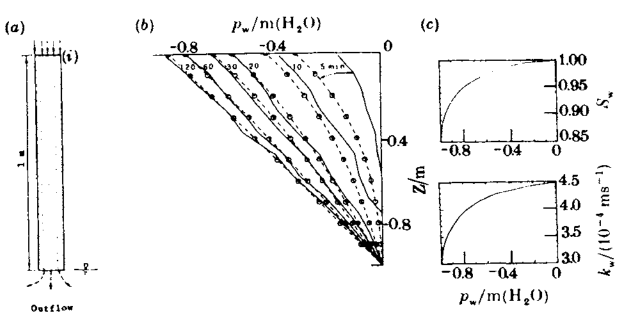


Fig. 7 Test example of semi-saturated flow experiment by Liakopoulos (1965). a) configuration of test: (i) uniform inflow interrupted at t=0; b) pressures - - -, computed; ——, recorded; c) data used (linear elastic analysis, E=3000 kPa).

The development of negative pressures due to incomplete saturation is very important in earthquake stability computation of embankments and other problems where a free water phreatic surface occurs. Here the negative pressures provide an effective cohesion in the zones above the phreatic line, which turns out to be vitally important in preventing localized failure.

PART III: SOME ILLUSTRATIVE COMPUTATIONS

6. A Test Example

To test both the model of constitutive behaviour and the computer code solving the coupled problem it is essential to compare the results with physical models. A very convenient basis is provided by centrifuge models and several such comparisons are reported in Zienkiewicz <u>et al.</u> (1990). Here we show a simple problem solved on the centrifuge by Venter (1987) and analysed by Xie (1990). Results are presented in Fig. 8 and the excellent prediction of pore pressure developments should be noted.

Such comparisons performed on various tests have given confidence in the modelling procedure suggested.

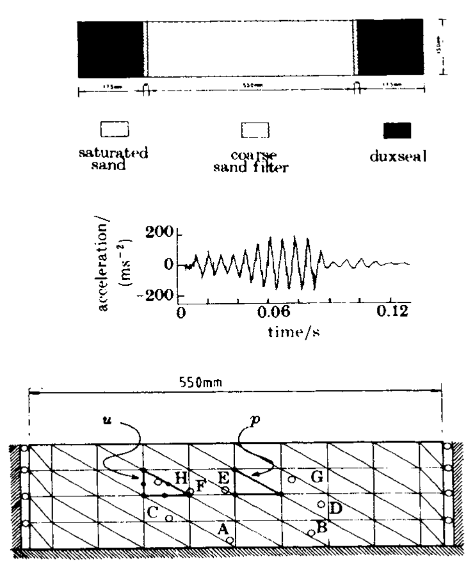
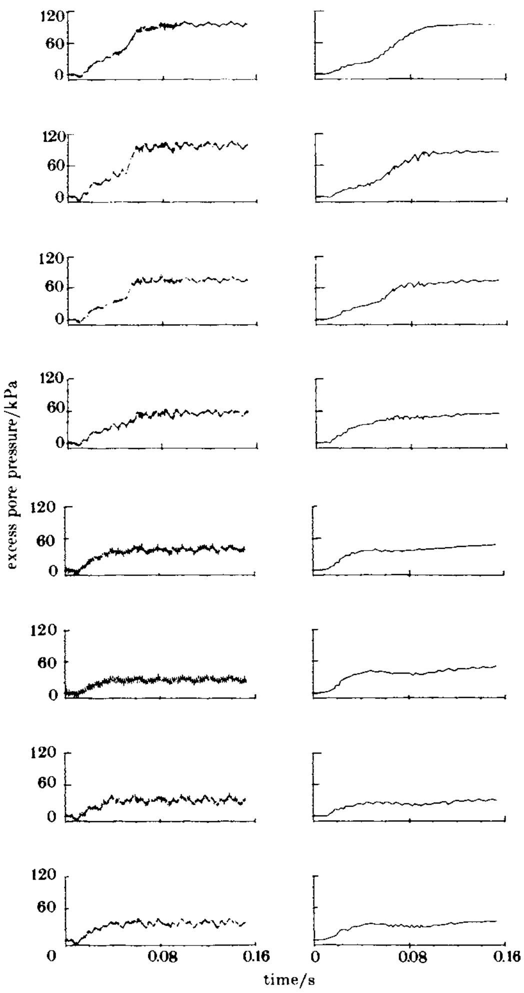


Fig. 8a An assessment of computation versus a centrifuge experiment: the problem showing acceleration input of base and finite element mesh.



Experiment computation

Fig. 8b An assessment of computation versus a centrifuge experiment: pressure development of various measurement points.

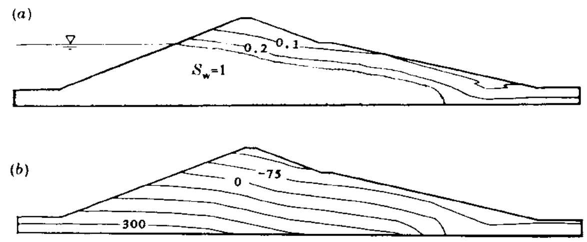
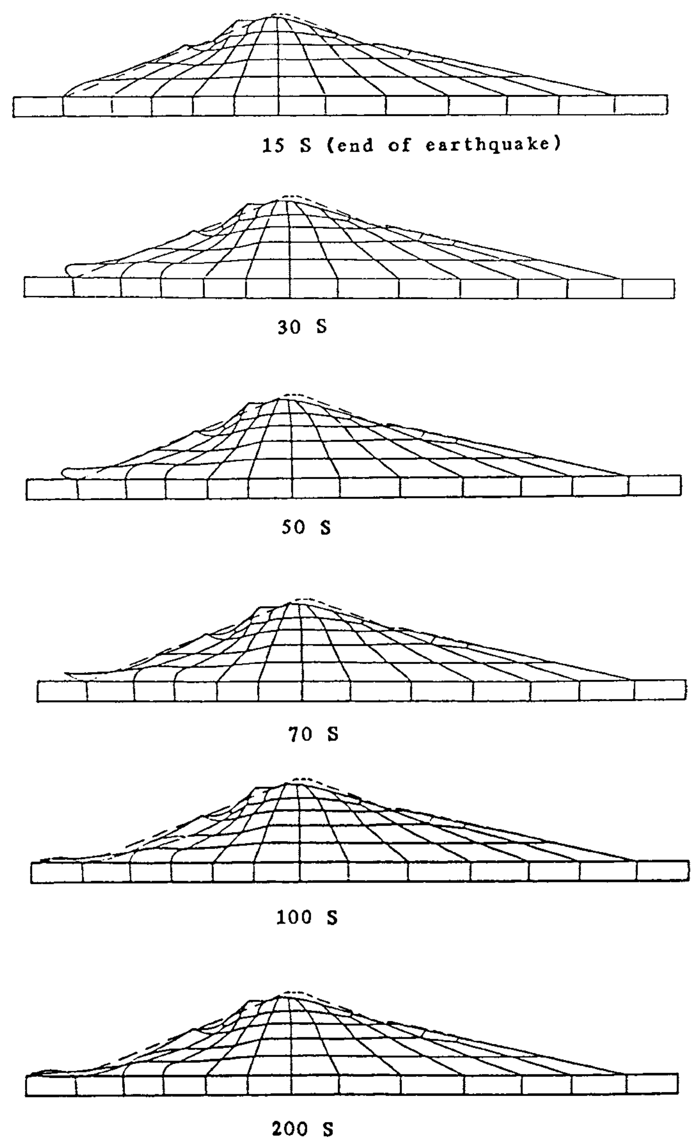


Fig. 9 San Fernando dam. Initial steady-state solution. Only saturation (a) and pressure contours (b) are shown. Contour interval in (b) is 75 kPa.



7. San Fernando Dam - a Re-analysis

To show the power of the modelling adopted, application to some realistic examples is of course required, even though full comparative measurements are not available. One such example of lasting interest is the failure of the lower San Fernando dam of 1971 (Seed et al., 1975). We have reported some of such computation in detail in (Zienkiewicz et al., 1990; Xie, 1990). Here we show in Figs. 9, 10 and 11 some results of these computations.

In Fig. 9 the initial pressure distribution and corresponding degree of saturation are shown. In Fig. 10 we show the deformed shape of the dam at various times after the earthquake, and in Fig. 11 the time history of pressure development at some points.

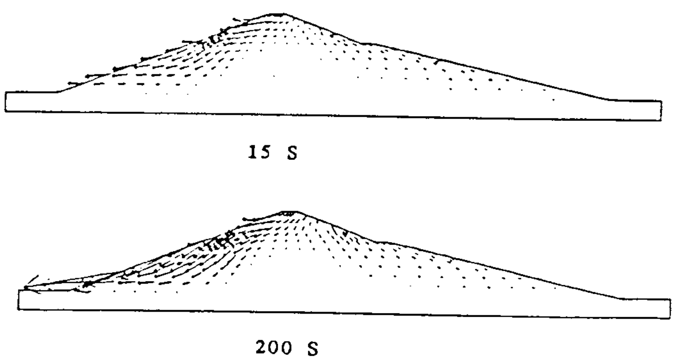


Fig. 10 San Fernando dam. Deformed shape of the dam at various times during and after the earthquake, and displacement vectors.

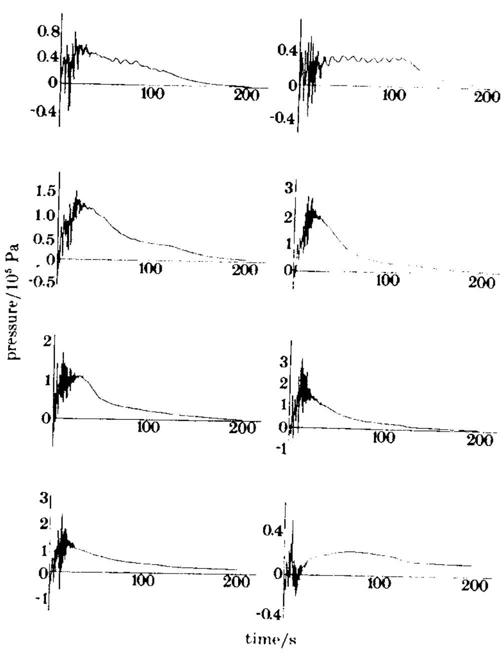


Fig. 11 San Fernando dam. Excess pressures at various points of the section. Note that pressure of some locations continues to rise after the end of the earthquake.

It is of interest to observe that pressure continues to rise in some locations well after the end of the earthquake. This indeed was conjectured by Seed (1979).

For fuller discussion of these results the reader is referred to the original papers (Zienkiewicz et al., 1990) where indeed the full formulation is described. The point we would like, however, to emphasise, is that the results presented show that realistic predictions of behaviour can today be achieved through the use of well proved equations, accurate constitutive modelling and efficient numerical discretization.

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