Classification of Analysis Methods for Dynamic Soil-Structure Interaction

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(State of the Art Paper)

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SYNOPSIS: The various methods to perform soil-structure-interaction analysis are classified. The first classification uses as criterion the behavior (linear or nonlinear) of the structure and of the unbounded soil. The second classification distinguishes between the direct method and the substructure method, which do not necessarily lead to identical results. Within each method, however, the various procedures are mathematically equivalent. In the substructure method the dynamic stiffness representing the interaction forces of the unbounded soil is determined based on the boundary element method in the time or frequency domain. In the latter case various so-called realizations in the time domain are distinguished using the extent of the frequency-domain calculations as a criterion.

INTRODUCTION

The various methods to analyse dynamic soil-structure interaction appear at first sight to be quite different from each other, although they all model the same feature: wave propagation in the unbounded soil towards infinity. As shown is this paper, certain differences between the results of the so-called direct method and of the substructure method do actually exist. Within each of these two methods, however, the various approaches, which are based on different concepts, are mathematically equivalent. They are thus not truly different and independent procedures. Using certain criteria, such as e.g. which steps are performed in the frequency domain and which in the time domain, a classification of the methods to analyse soil-structure interaction results.

To establish a firm base for the classification of the methods, the objective and the significant features of soil-structure-interaction analysis described in the two text books [33, 38] are summarized in the following. The dynamic system whose response is to be determined consists of two distinct parts with different properties: the (generalized) structure with bounded dimensions consisting of the actual structure and possibly an irregular adjacent soil region, and the unbounded soil extending to infinity (Fig. 1). The (generalized) structure is modelled straightforwardly with finite elements (the word "generalized" is dropped for the sake of conciseness in the following). To analyse the semi-infinite domain of the unbounded soil numerically, a surface forming the boundary of the numerical model is chosen which encloses the structure. The properties associated with the degrees of freedom of the nodes on this so-called interaction horizon [45] represent the significant features of the unbounded domain located on the exterior of this surface. In particular, the radiation condition has to be enforced which states that only outwardly propagating

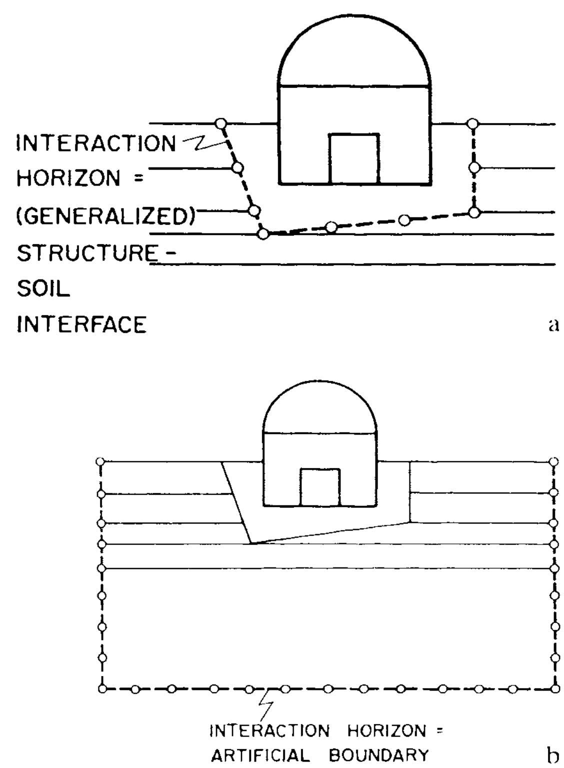

Fig. 1 Dynamic Models for (a) Substructure Method (b) Direct Method

waves exist for a load applied to the structure or for the scattered motion in the case of e.g. seismic excitation. A certain arbitrariness exists when selecting the location of the interaction horizon, which actually has no physical significance. The interaction horizon can coincide with the structure-soil interface leading to the substructure method; or it can be identical to an artificial boundary up to which the soil is modelled with, for example, finite elements, which results in the direct method. If the same rigorous radiation condition were formulated in both methods, the two methods would actually be the same, leading to identical results.

These concepts can be applied to other dynamic structure-medium interactions, such as fluid-structure interaction and structural acoustics.

Two procedures of classification are discussed. The first procedure examined in the next section is based on the linear versus nonlinear behavior of the structure and of the unbounded soil. The second procedure distinguishes between the direct method (third section) and the substructure method (fourth section).

Not all methods which have been developed can be discussed in this paper. Preference is given to those with which the author has had some contact. It is also not possible to provide an accurate historical review of the development of soil-structure-interaction analysis. The cited literature is restricted to a selection of historical references, some review papers which are still worthwhile to consult today and recent articles describing the latest progress.

LINEAR VERSUS NONLINEAR STRUCTURE AND UNBOUNDED SOIL

The first classification uses as criterion the behavior (linear or nonlinear) of the structure and of the unbounded soil (Table 1). In the first case both the structure and the unbounded soil remain linear, which applies to many analyses of nuclear power plants and machine foundations. Quite surprisingly, also for a transient excitation such as an earthquake loading, the analysis for this total linear system is routinely performed in the frequency domain and only exceptionally in the time domain. This is due to the fact that the formulation of the radiation condition (only outgoing waves at infinity) is as an analytical expression better known in the frequency than in the time domain (and to a lesser extent as it is straightforward to incorporate hysteretic material damping). But equivalent formulations in the time domain of the radiation condition expressing a mechanics feature do, of course, exist. Reviews of this so-called complex response analysis working in the frequency domain exist [22, 28, 19] as do general-purpose computer programs [20, 23]. This first case which has reached a very high level of development is not addressed any further.

STRUCTURE	UNBOUNDED SOIL	CALCULATIONAL DOMAIN
LINEAR NON LINEAR (NON)LINEAR	LINEAR LINEAR NONLINEAR	FREQUENCY (TIME) TIME TIME

Table 1 Calculational Domain Determined by Behavior of Structure and Unbounded Soil

In the second case the structure can behave nonlinearly while the unbounded soil will remain linear. The latter is normally justified, as for the three-dimensional spreading of the waves when propagating away from the structure the amplitudes decay. Examples are structures which perform in the nonlinear range for high seismic excitation; base-isolation systems with friction plates exhibiting strong nonlinear characteristics which have to be considered in design; local nonlinearities such as the partial uplift of the basemat and the separation occurring between the sidewalls of the base and the neighboring soil in the case of embedded structures; and the highly nonlinear soil behavior arising adjacent to the basemat. The analytical methods working in the time domain (with possibly certain steps performed in the frequency domain) are summarized in a text book [38].

In the third case the total dynamic system will be nonlinear (with as a special case linear behavior of the structure), which is analysed in the time domain. The nonlinearity of the unbounded soil can be caused by e.g. the two-dimensional propagation of surface waves, for which no decay of the amplitudes occurs. This case has hardly been addressed. An exception is Ref. 42 where as an approximation the far field is modelled based on one-dimensional elasto-plastic wave propagation with one stress component.

The direct and substructure methods discussed below will concentrate on the second case; i.e. the structure will be nonlinear (with as a special case linear behavior) while the unbounded soil will behave linearly.

DIRECT METHOD

How to formulate the radiation condition of the unbounded soil is the key issue in the analysis of soil-structure interaction. If the radiation condition (outwardly propagating waves only) is formulated at infinity, the rigorous boundary condition results,

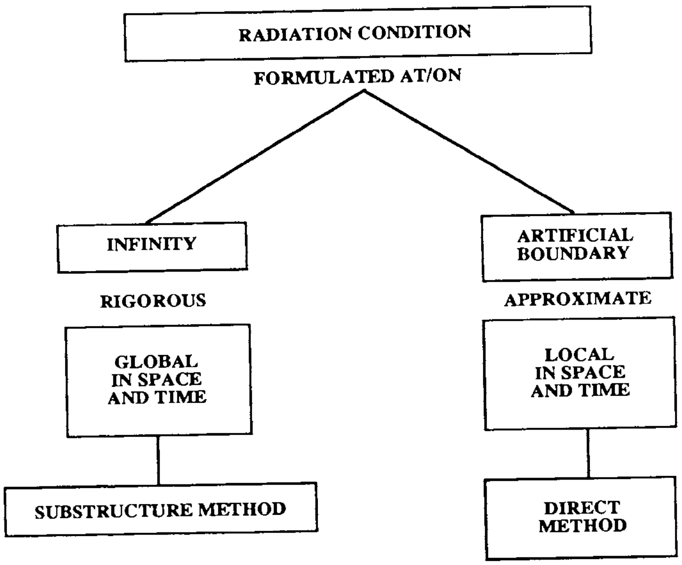

Fig. 2 Differences in Formulating the Radiation Condition for Substructure and Direct Methods

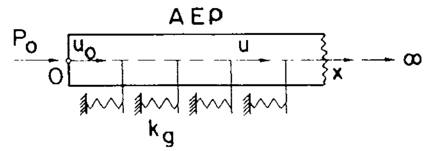

Fig. 3 Semi-Infinite Rod on Elastic Foundation

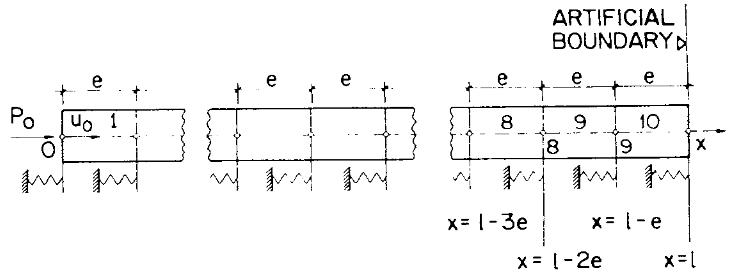

Fig. 4 Finite-Element Discretization up to Artificial Boundary

which is global in space and time (Fig. 2). That is, all degrees of freedom of the nodes located on the interaction horizon from the start of the excitation contribute to the interaction forces. The substructure method (with the structure-soil interface coinciding with the interaction horizon) is based on this concept. In the direct method approximations are introduced. The radiation condition is formulated directly on the interaction horizon (= artificial boundary) in such a way that a (frequency independent) highly absorbing boundary condition results which is local in space and time. These transmitting boundaries thus use information only from the node being addressed or from the nearly region of the mesh at the current time station or, at most, at a few recent time stations.

Based on various mathematical principles many apparently different local transmitting boundaries have been developed: the viscous damper [21], the paraxial approximation [11, 5], the extrapolation algorithm [18] and the superposition boundary [29, 8] to have just a few. Ref. [6] contains a detailed evaluation of some of these formulations. In Ref. [16] it is demonstrated that all the transmitting boundaries mentioned above, although they appear to be vastly different from each other, are actually all mathematically equivalent and thus are essentially alternative realizations of one and the same boundary mechanism. It is also demonstrated that the transmitting boundaries can be classified as being of first order, second order, etc. whereby higher-order schemes which would result in higher accuracy may lead to dynamic instabilities and can thus not be used. A real challenge exists to develop a transmitting boundary of higher accuracy which is local in time and space and which can handle (approximately) all types of waves without restrictions on the geometry and on the material properties such as Poisson's ratio. This is definitively the area of soil-structure-interaction analysis where the research efforts should be concentrated.

To demonstrate that improvement is possible, the wave propagation towards infinity in the one-dimensional semi-infinite rod on an elastic foundation [37, 39] is addressed (Fig. 3). This systems is dispersive and exhibits a cutoff frequency; properties which also arise in actual sites.

The semi-infinite rod with area A, modulus of elasticity E, mass density ρ and spring stiffness per unit length k_g is subjected to a prescribed axial support movement in the shape of a rounded triangular pulse at point 0

$$u_0(\bar{t}) = \frac{u_0}{2}\left[1 - \cos\left(2\pi\frac{\bar{t}}{\bar{t}_0}\right)\right], \quad 0 < \bar{t} < \bar{t}_0, \qquad (1a)$$

$$u_0(\bar{t}) = 0, \qquad \bar{t} > \bar{t}_0, \qquad (1b)$$

with the dimensionless time $\bar{t} = tc_\ell\kappa$ ($c_\ell = \sqrt{E/\rho}$), $\kappa = \sqrt{k_g/(EA)}$. $\bar{t}_0$ equals 2. The exact value of the dimensionless reaction force $\bar{P}_0(\bar{t}) = P_0(\bar{t})/(Ku_0)$ with the static stiffness coefficient $K = \sqrt{EAk_g}$ is plotted as a dashed line in Fig. 5. Between point 0 and the artificial boundary (Fig. 4) 10 one-dimensional finite elements of equal length e are chosen.

A potentially powerfull transmitting boundary can be based on the paraxial approximation [11, 5]. In this concept one constructs a differential equation similar to the wave equation, which allows in an approximate manner only outgoing waves to propagate. This differential equation is then used as the boundary condition enforced on the artificial boundary.

The differential equation of motion equals

$$u_{,xx} - \kappa^2 u - \frac{\ddot{u}}{c_\ell^2} = 0 \tag{2}$$

For harmonic motion with frequency ω the solution

$$u = e^{i(\omega t - kx)} \tag{3}$$

with the wave number k leads to the dispersion relation

$$k_2 + \kappa_2 - \frac{\omega^2}{c_\ell^2} = 0 \tag{4}$$

For waves propagating only in the positive x-direction (outward)

$$k = +\kappa\sqrt{a_0^2 - 1} \tag{5}$$

follows with $a_0 = \omega/(c_\ell\,\kappa)$. Eq. 5 represents the rigorous radiation condition, which should be modelled by a transmitting boundary mechanisms as accurately as possible.

To construct a linear differential equation for the paraxial approximation, eq. (5) is expanded for $a_0>1$ into a Taylor series. Keeping one and two terms results in

$$ik - i\kappa a_0 = 0 \tag{6a}$$

and

$$k a_0 - \kappa a_0^2 + \frac{\kappa}{2} = 0 \tag{6b}$$

For the solution specified in eq. 3, eq. 6 corresponds to

$$u_{,x} + \frac{\dot{u}}{c_\ell} = 0 \tag{7a}$$

and

$$\dot{u}_{,x} + \frac{\ddot{u}}{c_\ell} + \frac{\kappa^2 c_\ell}{2} u = 0 \tag{7b}$$

Eq. 7a corresponds to the first-order paraxial approximation which is equal to the viscous damper, eq. 7b to the second-order paraxial approximation.

The reaction forces determined from a finite-element analysis with the above boundary conditions enforced are shown in Fig. 5, whereby for the paraxial approximation a special interface element has to be introduced. Although the second-order paraxial approximation (eq. 7b) is a more accurate representation of the exact equation (eq. 5) than the viscous damper (eq. 7a), the results do not confirm this. Compared to the exact solution a significant

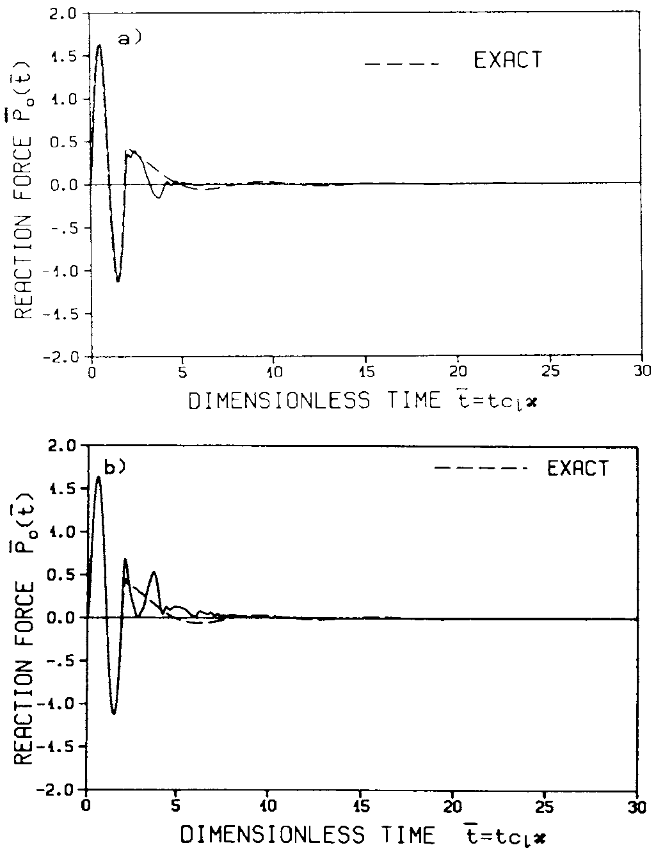

Fig. 5 Dynamic Response
a) Viscous Damper
b) Paraxial Approximation

difference still exists. The other transmitting boundaries (extrapolation algorithm and superposition boundary) do also not lead to a higher accuracy (results not shown in this paper).

Another possibility does, however, exist, which leads to a dramatic increase in accuracy. It is in this one-dimensional case equivalent to the procedure to develop systematically consistent lumped-parameter models described in Ref. [43], where the same example is solved. The reader is also referred to the discussion in connection with Fig. 11.

Multiplying both sides of the rigorous dispersion relation (eq. 5) by $EAu_0(a_0)$ leads to a force-displacement relationship which is formulated as

$$P(a_0) = iK\sqrt{a_0^2 - 1}\; u_0(a_0) \tag{8}$$

whereby $iK\sqrt{a_0^2 - 1)}$ represents the dynamic-stiffness coefficient S. Rewriting S as

$$\frac{S}{K} = i\left(\sqrt{a_0^2 - 1} - a_0\right) + ia_0 \tag{9}$$

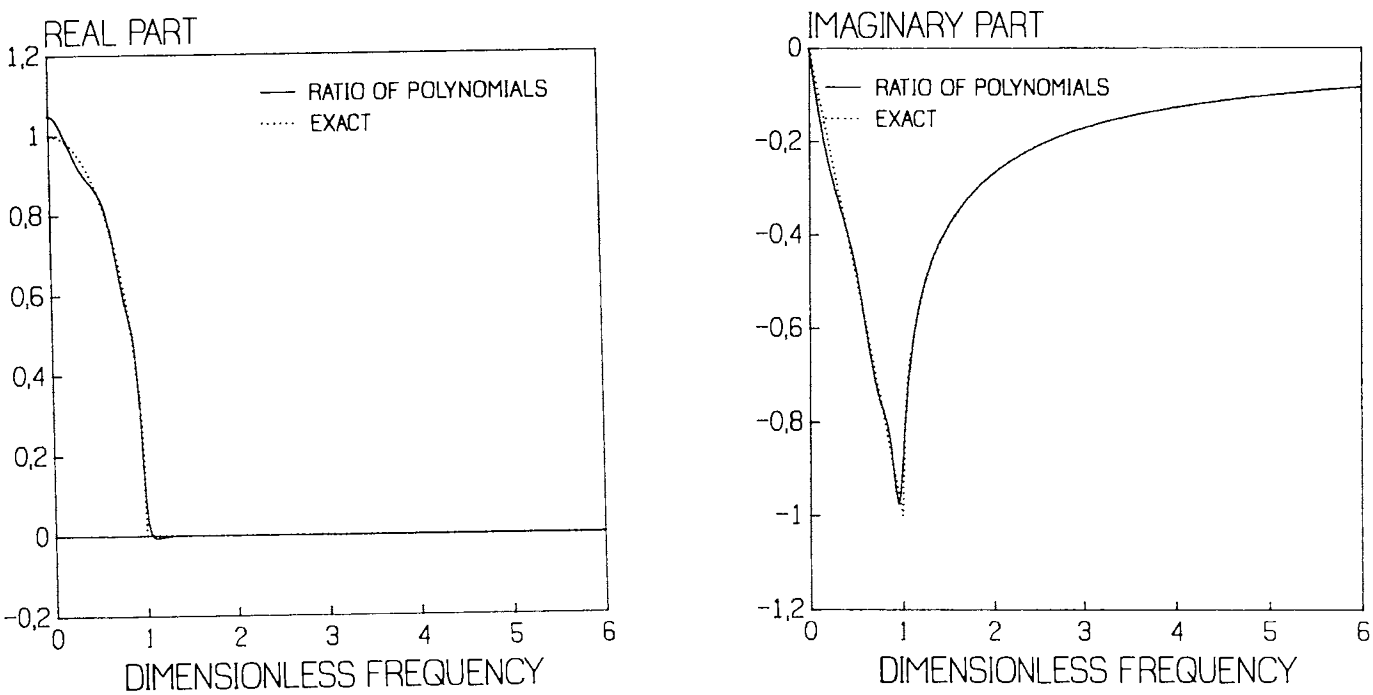

Fig. 6 First Part of Dynamic-Stiffness Coefficient

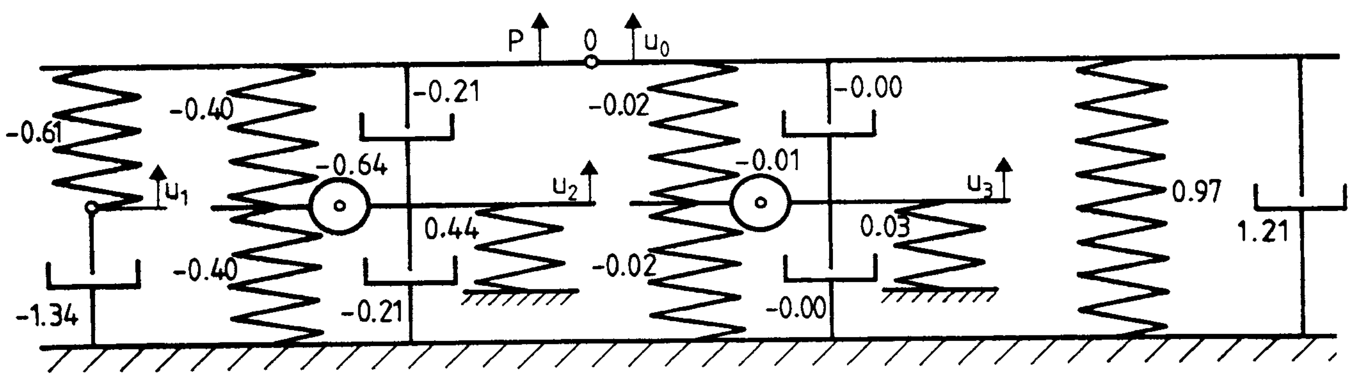

Fig. 7 Lumped-Parameter Model for Dispersion Equation (The Coefficients of the Springs, Dampers and Masses are to be Multiplied by K, $K/(\kappa c_\ell)$ and $K/(\kappa c_\ell)^2$, respectively)

two parts are formed. The second, ia_0, represents a damper. The first, $i\left(\sqrt{a_0^2 - 1} - a_0\right)$, is approximated as a ratio of a polynomial of 4th degree to a polynomial of 5th degree in ia_0 using a curve fitting procedure ($0<a_0<\infty$). As can be seen from Fig. 6, a very good agreement for the real and imaginary terms results. The ratio of the two polynomials is then rewritten as a partial-fraction expansion. For each term a physical model can be constructed [43] which forms the building block for the lumped-parameter model. The total lumped-parameter model consisting of springs, dampers and masses for both parts representing the dynamic-stiffness coefficient or the dispersion equation is shown in Fig. 7. Besides the displacement u_0 three internal displacements u_1, u_2, u_3 arise in the lumped-parameter model, which can easily be incorporated in a general-purpose finite-element program. The reaction force determined from a finite-element analysis for the prescribed displacement u_0 (eq. 1) coincides from a practical point of view with the exact value (comparison not shown).

The procedure outlined above could possibly be generalized to the two- and three-dimensional cases with more than one wave number. The dispersion relationship for the two-dimensional (x, y) scalar wave equation describing e.g. the out-of-plane (antiplane) motion (c_s = shear-wave velocity) equals

$$k_x^2 + k_y^2 - \frac{\omega^2}{c_s^2} = 0 \tag{10}$$

The exact dispersion relation (radiation condition) for an artificial boundary x = const. is formulated as

$$k_x = +k_y \sqrt{\frac{\omega^2}{c_s^2\, k_y^2} - 1} \tag{11}$$

Eq. 11 can again be interpreted as a force-displacement relationship with the right-hand side representing the dynamic-stiffness coefficient. Comparing eq. 11 with eq. 5 it follows that by identifying $\omega/(c_s k_y)$ as a_0 the same curve fitting procedure can be applied leading to the same lumped-parameter model. The parameter k_z is, however, not constant and would have to be approximately determined in each node on the artificial boundary based on the distribution of the displacement along the artificial boundary and this task would have to be performed at every time station.

SUBSTRUCTURE METHOD

Dynamic Stiffness Calculated with Boundary Element Method

The dynamic-stiffness matrix relating the displacements in the nodes on the structure-soil interface to the interaction forces of the unbounded soil (or in a computional algorithm the interaction forces at a specific time station) are calculated based on the boundary element method . Ref. [3] provides a review of this very effective numerical method, which is well suited to represent semi-infinite domains such as the unbounded soil. For this case the advantages of the boundary element method become significant, i.e. the discretization is performed only on the surface of the domain (reduction of the dimensionality of the problem by one) and the radiation condition can be enforced exactly at infinity by incorporating it in the fundamental solution.

The first time-domain boundary-element formulation for the unbounded soil was presented in Refs. [15] and [32], whereby in the latter a nonlinear application is included. For restricted cases such as the out-of-plane motion, earlier work e.g. Ref. [7] can be mentioned. Many different boundary-element formulations in the time domain exist (Refs. [36, 3]). Besides the direct method, which is applied widely, indirect methods, weighted residual procedures and others have been used.

Displacement, Velocity and Acceleration Convolutions

The interaction force-displacement relationship formulated in the frequency domain as the product of the frequency-domain dynamic-stiffness matrix and the displacement amplitudes is written in the time-domain as the convolution of the time-domain dynamic-stiffness matrix and the displacements. Besides this displacement convolution corresponding velocity and acceleration convolutions leading to the same interaction forces can be used. The following is based on Ref. [26], whereby velocity and acceleration convolutions for the unbounded soil are also addressed in Ref. [34].

In the frequency domain the interaction force (amplitudes $\{R(\omega)\}$) - displacement (amplitudes $\{u(\omega)\}$) relationship is formulated with the dynamic-stiffness matrix $[S(\omega)]$ as

$$\{R(\omega)\} = [S(\omega)]\,\{u(\omega)\} \tag{12}$$

To determine its Fourier transformation, $[S(\omega)]$ is decomposed into a singular part, which is equal to its asymptotic value at $\omega = \infty$, $[K_\infty] + i\omega[C_\infty]$, and the remaining regular part $[K_r(\omega)]$, which is absolutely integrable over the ω-axis [38]

$$[K_r(\omega)] = [S(\omega)] - [K_\infty] - i\omega[C_\infty] \tag{13}$$

The interaction force-displacement relationship in the time domain involving a displacement convolution is then equal to

$$\{R(t)\} = \int_0^t [K_r(t-\tau)]\,\{u(\tau)\}\,d\tau + [K_\infty]\,\{u(t)\} + [C_\infty]\,\{\dot{u}(t)\} \tag{14}$$

with $[K_r(t)]$ denoting the inverse Fourier transform of $[K_r(\omega)]$.

Alternatively, eq. 12 can be rewritten as

$$\{R(\omega)\} = \frac{[S(\omega)]}{i\omega}\, i\omega\{u(\omega)\} \tag{15}$$

where $i\omega\{u(\omega)\}$ are the velocity-amplitudes. $[S(\omega)]/(i\omega)$ is again decomposed into its singular part consisting of $[C_\infty]$ and of its real term at $\omega = 0$, $[K_0]/(i\omega)$, and the remaining regular part $[C_r(\omega)]$. $[K_0]$ is the static-stiffness matrix.

$$[C_r(\omega)] = \frac{[S(\omega)]}{i\omega} - \frac{[K_0]}{i\omega} - [C_\infty] \tag{16}$$

Eq. 14 can then be reformulated involving a velocity convolution with $[C_r(t)]$ denoting the inverse Fourier transform of $[C_r(\omega)]$ as

$$\{R(t)\} = \int_0^t [C_r(t-\tau)]\,\{\dot{u}(\tau)\}\,d\tau + [K_0]\,\{u(t)\} + [C_\infty]\,\{\dot{u}(t)\} \tag{17}$$

$[C_r(t)]$ is also the indefinite time integral of $[K_r(t)]$.

Finally, eq. 12 can be specified as

$$\{R(\omega)\} = \frac{[S(\omega)]}{(i\omega)^2}\ (i\omega)^2\ \{u(\omega)\} \tag{18}$$

The singular part of $[S(\omega)]/(i\omega)^2$ determined by its behavior at $\omega = 0$ equals $[K_0]/(i\omega)^2 + [C_0]/(i\omega)$. The Fourier transform $[M_r(t)]$ of the regular part

$$[M_r(\omega)] = \frac{[S(\omega)]}{(i\omega)^2} - \frac{[K_0]}{(i\omega)^2} - \frac{[C_0]}{i\omega} \tag{19}$$

allows the interaction forces involving an acceleration convolution to be formulated as

$$\{R(t)\} = \int_0^t [M_r(t-\tau)]\ \{\ddot{u}(\tau)\}\ d\tau + [K_0]\ \{u(t)\} + [C_0]\ \{\dot{u}(t)\} \tag{20}$$

Again, $[M_r(t)]$ is the indefinite time integral of $[C_r(t)]$.

The formulation involving the velocity convolution (eq. 17) is especially attractive, as for the calculation of the singular term only the static-stiffness matrix $[K_0]$ and the asymptotic value at $\omega = \infty$ of the damper $[C_\infty]$ are needed. The contribution of a unit area to $[C_\infty]$ equals the product of the mass density and the wave-propagation velocity.

The two alternative formulations (eq. 17 + 20) can also be derived in the time domain starting from eq. 14 and using integration by parts, whereby the singularities arising at the limits of integration have to be taken into account.

As an example a very simple one-dimensional case is examined, the spherical cavity with radius a embedded in a full space (shear modulus G, mass density ρ, dilatational-wave velocity c_p) with symmetric waves occurring caused by a uniform pressure p acting on the cavity's wall (Fig. 8). The dynamic-stiffness coefficient $S(a_0)$ relating the amplitude of the radial wall displacement $u(a_0)$ to $p(a_0)$ with $a_0 = \omega a/c_p$ equals [38]

$$S(a_0) = K_0\left[1 - \frac{c_p^2}{4c_s^2} + i a_0 \frac{c_p^2}{4c_s^2} + \frac{c_p^2}{4c_s^2}\ \frac{1}{1 + i a_0}\right] \tag{21}$$

where $K_0 = 4G/a$. With $K_\infty = K_0\left(1 - c_p^2/(4c_s^2)\right)$, $C_\infty = K_0 a c_p/(4c_s^2) = \rho c_p$ and with ($\bar{t} = tc_p/a$)

$$K_r(\bar{t}) = K_0 \frac{c_p^3}{4ac_s^2}\ e^{-\bar{t}} \tag{22}$$

the interaction force-displacement relationship (eq. 14) equals

$$p(t) = K_0 \frac{c_p^3}{4ac_s^2} \int_0^t e^{-c_p/a(t-\tau)}\ u_0(\tau)d\tau + K_0\left(1 - \frac{c_p^2}{4c_s^2}\right) u_0(t) + \rho c_p \dot{u}_0(t) \tag{23}$$

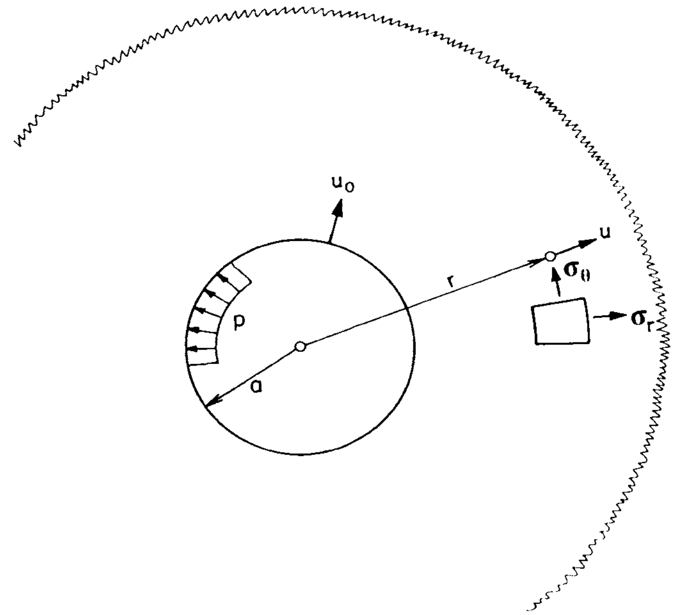

Fig. 8 Spherical Cavity with Uniform Pressure

With

$$C_r(\bar{t}) = -K_0 \frac{c_p^2}{4c_s^2}\ e^{-\bar{t}} \tag{24}$$

eq. 17 with the velocity convolution is formulated as

$$p(t) = -K_0 \frac{c_p^2}{4c_s^2} \int_0^t e^{-c_p/a(t-\tau)}\ \dot{u}_0(\tau)d\tau + K_0 u_0(t) + \rho c_p \dot{u}_0(t) \tag{25}$$

With $C_0 = 0$ and with

$$M_r(\bar{t}) = K_0 \frac{ac_p}{4c_s^2}\ e^{-\bar{t}} \tag{26}$$

eq. 20 with the acceleration convolution results in

$$p(t) = K_0 \frac{ac_p}{4c_s^2} \int_0^t e^{-c_p/a(t-\tau)}\ \ddot{u}_0(\tau)d\tau + K_0 u_0(t) \tag{27}$$

The interaction forces $\{R(t)\}$ expressed in eqs. 14, 17 or 20 have to be calculated at each time station. The evaluation of the convolution integrals is computionally expensive: the total number of operations is proportional to the square of the number of time steps and in addition the total time history of the displacement, velocity or acceleration has to be stored. As discussed in the next subsection, however, the recursive evaluation of the convolution integrals makes this time-domain analysis using the substructure method computationally competitive, as for a typical seismic excitation a reduction of one to three orders of magnitude results (Refs. [31, 24, 40

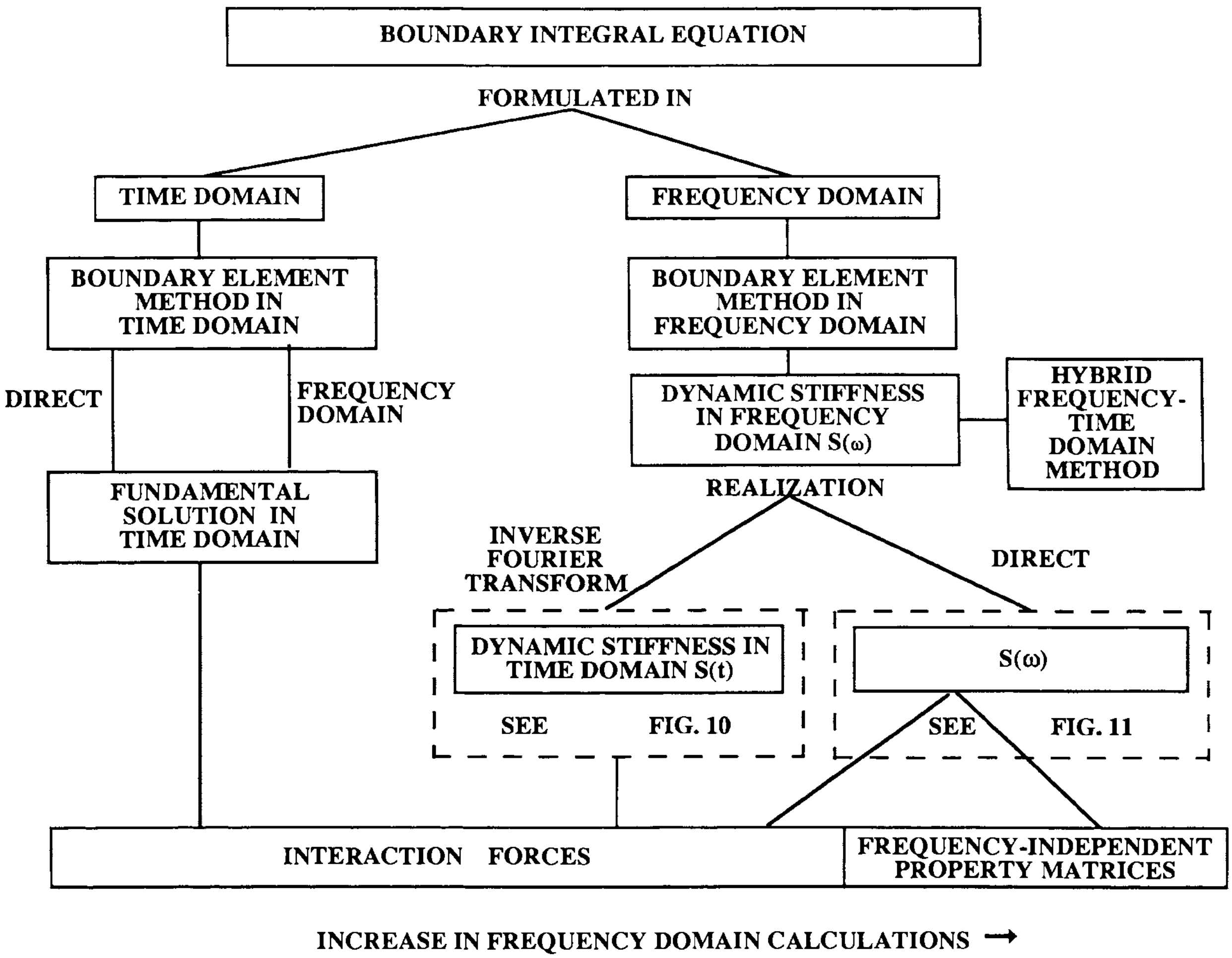

Fig. 9 Classification of Computational Procedures of Substructure Method to Model Unbounded Soil

41]). For instance, the recursive evaluation of the convolution integral in eq. 14 at time $t_n = n\Delta t$

$$\{R_r\}_n = \int_0^{t_n} [K_r(t-\tau)] \, \{u(\tau)\} \, d\tau \qquad (28)$$

leads to

$$\{R_r\}_n = \sum_{i=1}^{M} [a]_i \, \{R_r\}_{n-i} + \Delta t \sum_{i=0}^{L} [b]_i \, \{u\}_{n-i} \qquad (29)$$

where $[a]_i$ and $[b]_i$ are matrices which are independent of the time step. The interaction forces $\{R_r\}_n$ are thus computed from the n-th displacements $\{u\}_n$ and the M and L past values of the forces and displacements, respectively. The decrease in computational effort (the number of operations is proportional to the number of time steps) and storage requirement is especially large for dynamic-stiffness coefficients which do not return to zero immediately after the waves have passed. The velocity and acceleration convolution integrals, which are not addressed any further in this paper, can be treated analogously.

Computational Algorithms

A classification of the various methods to model the contribution of the unbounded soil to the equations of motion is shown in Figs. 9, 10 and 11. The further to the right a procedure is placed in Fig. 9, the more calculations are performed in the frequency domain. Many other methods with slight differences also exist.

As already pointed out, the unbounded soil is modelled using the boundary element method (Ref. [3]), which is based on a boundary integral equation. The latter e.g. in the form of a reciprocity relationship or of an application of the superposition principle can either

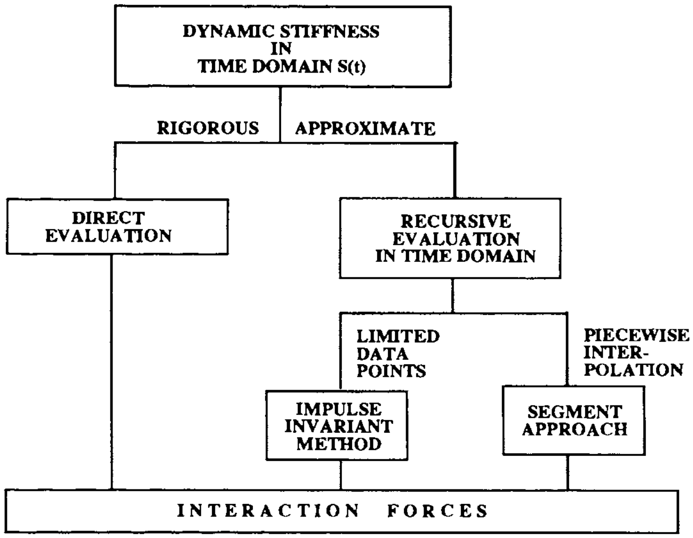

Fig. 10 Interaction Forces Determined Starting from Dynamic Stiffness in Time Domain

be formulated in the time domain or in the frequency domain.

The time-domain boundary-element method is addressed first (left part of Fig. 9). The fundamental solution (Green's function) of the full space is specified directly in the time domain, which leads to an additional discretization of the free surface of the site [15, 1]. Alternatively, as recommended for layered sites, the fundamental solution is first determined in the frequency domain and then using the inverse Fourier transformation calculated in the time domain [35]. In this case the discretization is limited to the structure-soil interface. The interaction forces of the unbounded soil acting in the nodes on the structure-soil interface can then be calculated, which is performed for each time station. A recursive evaluation of the convolution integrals appearing in the boundary integral equation should be possible. The time-domain boundary element method using the fundamental solution of the full space avoids all calculations in the frequency domain.

The frequency-domain boundary element method is examined next (right part of Fig. 9). The procedure leads to the dynamic-stiffness matrix in the frequency domain $[S(\omega)]$ of the unbounded soil referred to the nodes located on the structure-soil interface. This transfer function matrix describes the displacement amplitude (input) - interaction force amplitude (output) relationship in the frequency domain. The corresponding relationship in the time domain is called a realization, whereby many possibilities exist for a specific dynamic-stiffness matrix in the frequency domain [4]. It is appropriate to distinguish between those which, by first performing an inverse Fourier transformation, are based on the dynamic-stiffness matrix in the time domain $[S(t)]$ and those which start directly from $[S(\omega)]$. The former, which lead to the interaction forces at a specific time station, are classified as shown in Fig. 10. The latter which can result in addition in a set of differential equations for the interaction forces with initial values or in frequency-independent property matrices are classified as in Fig. 11. Finally, $[S(\omega)]$ of the unbounded soil can be assembled with the dynamic-stiffness matrix of the structure and the total dynamic system solved in the frequency domain. This leads to the hybrid frequency-time domain method [17, 9, 10], where a series of linear analyses are performed in the frequency domain iteratively with pseudo-loads taking the nonlinearities into account.

Turning to the realizations which start from the dynamic-stiffness matrix in the time domain $[S(t)]$ (Fig. 10), the convolution integral can be evaluated directly non-recursively. The equations for the displacement, velocity and acceleration convolution are specified in eqs. 14, 17 and 20. The recursive formulation (eq. 29) represents, in general, an approximation. Actually, the dynamic-stiffness matrix in the time domain $[S(t)]$ is approximated in some way. The choice of a recursive equation is not unique, and many possibilities exist. Two options are developed in Ref. [40]. The first called the impulse-invariant method [31] sets the approximate dynamic-stiffness matrix corresponding to the recursive formulation equal to the exact one at specified points in a certain time range. This results in a system of equations with the unknown $[a]_i$ and $[b]_i$. The approximate dynamic-stiffness matrix will, in general, deviate from the exact one in the other time ranges. In the second procedure, the segment approach, the dynamic-stiffness coefficients in the time domain are interpolated piecewise. Applying the so-called z-transformation then results in an explicit recursive equation without solving a system of equations.

The realizations which work directly from the dynamic-stiffness matrix in the frequency domain $[S(\omega)]$ and thus avoid the calculation of $[S(t)]$ are classified in Fig. 11. Various possibilities exist. The first quite inefficient procedure consists of performing at each time step a Fourier transformation of the displacement time history $\{u(t)\}$, which leads to $\{u(\omega)\}$. The interaction forces in the frequency domain $\{R(\omega)\}$ then follow as the product of $[S(\omega)]$ and $\{u(\omega)\}$; those in the time domain $\{R(t)\}$ are equal to the inverse Fourier transform. This procedure, consisting of successive Fourier transformations, is discussed in Ref. [25]. In the same reference a recursive evaluation of the amplitudes of the displacements in the frequency domain at the time station $t = n\Delta t$, $\{u(\omega)\}_n$ is described, using only the amplitudes of the previous time step $\{u(\omega)\}_{n-1}$ and the displacements at time $t = n\Delta t$, $\{u\}_n$ and at $t = (n-1)\Delta t$, $\{u\}_{n-1}$. An alternative derivation based on the z-transformation is possible which is addressed in Ref. [41]. It is important to

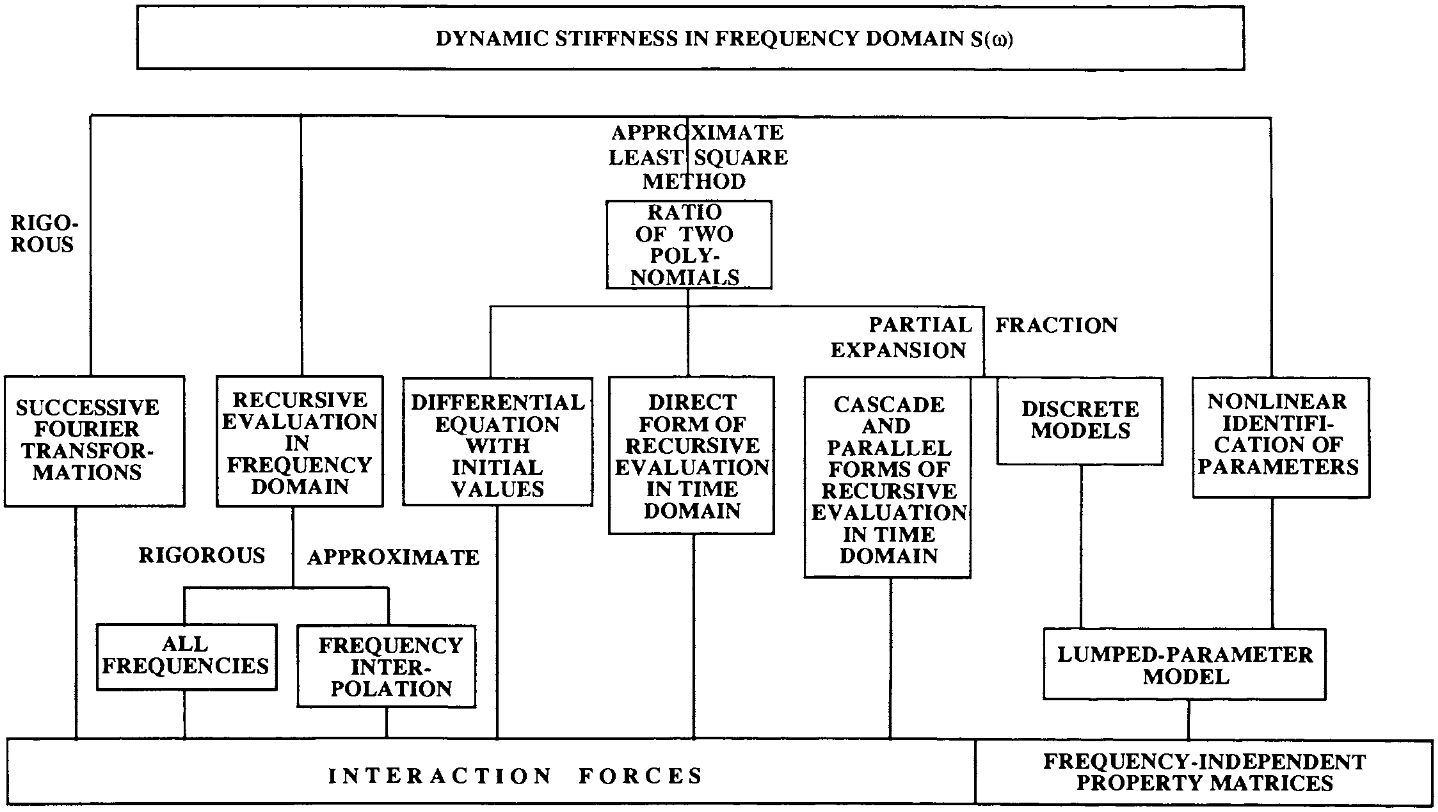

Fig. 11 Interaction Forces and Property Matrices Determined from Dynamic Stiffness in Frequency Domain

stress that the recursive evaluation calculated for all frequency components is rigorous. It corresponds to the exact calculation of the convolution integral in the frequency domain. It is customary in the standard complex response analysis performed for a total linear system in the frequency domain not to solve the system of equations for all frequencies, but also to make use of interpolation schemes. The same concept can of course also be used in the recursive evaluation which will now, however, only be approximate.

As an alternative each coefficient of the dynamic-stiffness matrix in the frequency domain [S(ω)] can be approximated as a ratio of two polynomials in iω using a curve-fitting technique based on the least-squares method which leads to the solution of a system of linear equations (right-hand side of Fig. 11). No other approximation is introduced. It is possible to transform the ratios of the two polynomials to ordinary differential equations which constant coefficients for the interaction forces together with the initial conditions, which can be solved directly [41]. Using the z-transformation, the so-called direct form of the recursive evaluation of the convolution integral can be derived [41]. Applying the partial fraction expansion to the ratio of the two polynomials and using the z-transformation the cascade [27] and parallel forms [41] of the recursive evaluation in the time domain of the interaction forces are derived. Alternatively, each term of the partial-fraction expansion can be rigorously represented by a discrete model consisting of frequency-independent springs, dampers and masses. They form the lumped-parameter model [43] which can be directly incorporated in a general-purpose computer program, or the corresponding frequency-independent property matrices (stiffness, damping, mass) [44] can be used as input. In this case the interaction force-displacement relationship follows as the realization. The latter can also be derived from [S(ω)] by a nonlinear identification of the parameters of the lumped-parameter model [2, 14].

CONCLUSIONS

The first classification uses as criterion the behavior (linear or nonlinear) of the structure and of the unbounded soil. When they both remain linear, unified highly-developed analysis procedures exist, which work mostly in the frequency domain. When the unbounded soil exhibits nonlinear behavior with the structure being linear or nonlinear, the only available analysis procedure is based on one-dimensional elasto-plastic wave propagation with one stress component in the far field. The analysis is performed in the time domain. In the remaining case the unbounded soil will behave linearly with the structure exhibiting nonlinear behavior (with linearity as a special case). For this case the second classification distinguishes between the direct method and the substructure method. It has been demonstrated before that all available local transmitting boundaries of the direct method working in the time domain are at least in the limit of the continuum formulation mathematically equivalent. In the substructure method the dynamic stiffness representing the interaction forces of the unbounded soil is calculated based on the boundary element method either in the time domain or in the frequency domain. In the former case it is possible to formulate the entire procedure in the time domain. In the latter case various realizations in the time domain are distinguished using the extent of the frequency-domain calculations as a criterion. The dynamic stiffness in the frequency domain can either be transformed to the time domain or used directly. Either interaction forces calculated recursively or frequency-independent property matrices corresponding to a lumped-parameter model of the unbounded soil are determined.

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