1952

Analysis of magnetic amplifiers based on assumed analytical expressions of B-H characteristics

Wiley Talley Ruhl

ANALYSIS OF MAGNETIC AMPLIFIERS BASED ON ASSUMED ANALYTICAL EXPRESSIONS OF B-H CHARACTERISTICS

BY

WILEY TALLEY RUHL

A

THESIS

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the

Degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Rolla, Missouri

1952

Approved by - Gabriel G. Skitek
Professor of Electrical Engineering

ACKNOWLEDGMENT

The writer of this thesis is deeply grateful to Mr. G. G. Skitek, Professor of Electrical Engineering, University of Missouri, School of Mines and Metallurgy, for his valuable suggestions and assistance in the preparation of this thesis.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

INTRODUCTION

In the past 10 years, magnetic amplifiers have received much attention in many different fields of engineering. Although the principles of this amplifier have been known since 1915, it has been forced into the background by the development of the vacuum tube amplifier, the need of better core material, and better dry disk rectifiers.

The basic magnetic amplifier, or saturable reactor, is nothing more than an iron core reactance coil whose a-c impedance is controlled by a d-c premagnetization of the core. This d-c premagnetization may be obtained in a number of ways, the simplest of which is a separate d-c source.

The name amplifier applies because, by a small variation of the d-c control current, a larger variation of the a-c output current may be obtained. The ratio of these currents is known as current gain and is a very important quantity in the study of magnetic amplifiers. It will be shown that with certain refinements, namely that of feedback, it is possible to greatly increase this gain.

The main purpose of this thesis is to analyze a series type magnetic amplifier in two different ways and then to compare these analyses.

More specifically, a half wave magnetic amplifier circuit will first be considered in which an attempt is made to consider all aspects of the problem. This analysis will be shown to be impractical because of mathematical complications and will then be abandoned. A new approach to the problem with certain simplifying assumptions will be made that will lead to an approximate mathematical solution.

A full-wave series magnetic circuit will then be analyzed assuming a power series approximation of the B-H characteristic. This type of approach has its shortcomings, however, but serves quite nicely to yield information which other approaches fail to give.

Finally, an approach to the problem will be made by assuming straight line magnetization curves. This analysis gives quite a lot of useful information as will be shown later in this thesis. These analyses will then be compared as to accuracy of results, assumptions made, and validity of the initial assumptions.

A short discussion on magnetic amplifiers used as mixers and as audio amplifiers will be given.

REVIEW OF LITERATURE

Although much material has been published in periodicals under the heading of magnetic amplifiers, very little has been released in the analytical approach to the problem. The great majority of material being published at the present time is qualitative.

Some of the articles professing to be of an analytical nature are in fact a collection of empirical formulas and experimental results which were obtained from a particular set-up and specific operating conditions.

A. G. Milnes(1)(2) of the Royal Aircraft Establishment

(1) Milnes, A. G. A New Theory on the Magnetic Amplifier. Proceedings of the Institute of Electrical Engineers. Vol. 97, pp. 460-497 (1950)

(2) Milnes, A. G. Magnetic Amplifiers. Proceedings of the Institute of Electrical Engineers. Vol. 96 pp. 329-338 (1949)

has published some articles which are quite good as far as giving an accurate picture of the mechanics of operation is concerned.

In any mathematical discussion of a non-linear element, such as the magnetic amplifier, it becomes necessary to make certain simplifying assumptions if any results are to be obtained which are useful.

Dr. Uno Lamm has published a book on the magnetic amplifier which, unfortunately, is not published in the United States and is not written in English.

The only book on magnetic amplifiers which is presently being published in English[3] is by Mr. J. H. Reynor and,

(3) Reynor, J. H. The Magnetic Amplifier. 1st ed. London, Stuart and Richards, 1948. 120 p.

unfortunately, contains no analytic discussion of the subject.

Several articles[4][5] have been written, giving an ac-

(4) Cohen, Sidney B. Analysis and Design of Self-Saturable Magnetic Amplifiers. Proceedings of the Institute of Radio Engineers. Vol. 39. pp. 1009-1020 (1951)

(5) Smith, E. J. Self-Saturating Magnetic Amplifiers. Transactions of the American Institute of Electrical Engineers. Vol. 69. pt. 2. pp. 1309-1317 (1950)

curate picture of the half-wave circuit. One, the article by S. B. Cohen, has been used in this thesis.

At the present time, the Carnegie Institute of Technology is actively engaged in investigating magnetic amplifiers but is not releasing very much for publication.

A number of companies, such as Westinghouse and General Electric, are doing theoretical research on this problem but are not releasing any information because of the military use to which these amplifiers are being put.

LIST OF SYMBOLS

N = number of turns on coil, specified by subscript; a for output circuit, b for feedback circuit, and c for control circuit.

ϕ = total flux in core as caused by winding, specified by subscript (lines or maxwells).

L = inductance (henrys).

i = instantaneous current in winding, specified by subscript; a for output circuit, b for feedback circuit, and c for control circuit. (amperes).

R_L = load resistance (ohms).

R_c = control circuit resistance (ohms).

l = effective length of flux path in each core (meters).

ω = angular frequency (radians/second).

X = reactance of a-c circuit $= \frac{2 N_a \omega A \mu}{l \cdot 10^8}$ (ohms).

μ = absolute permeability of core.

$I_{a_{avg}}$ = average value of current in a-c circuit (amperes).

I_c = d-c component of current in control circuit (amperes).

A = cross section of a single core (square meters).

R' = total resistance in output circuit of half-wave magnetic amplifier (ohms).

i_1 = a specific d-c current in control winding of half-wave circuit (amperes).

K_p = average power amplification.

α_1 = angular position at which core α saturates. (radians).

θ = angular position at which the current in the output circuit is zero (radians).

ϕ_s = saturation flux per single core (lines or maxwells).

E_m = maximum value of a-c supply voltage (volts).

B = flux density, $B(t)$ as a function of time, B_0 as evaluated at $t = 0$ (lines/square meter).

K = is defined as $\frac{E_m \cdot 10^8}{2 N_a \omega \phi_s}$

H = magnetic intensity (ampere turns per meter)

DISCUSSION OF SIMPLE HALF-WAVE SERIES MAGNETIC AMPLIFIER CIRCUIT

The magnetic amplifier in its most elementary form is the well-known saturable circuit shown in Figure 1a. The output of this simple series magnetic circuit consists of a source of alternating voltage in series with a load resistance, dry disk rectifier, and one side of a reactor. The other side of the reactor is in series with a source of d-c voltage, a control resistance, and a large inductor (L) which serves to smooth out the d-c control current and to keep the output of the reactor from inducing current into the control winding.

There will be only one assumption to begin with, namely that the B-H curve can be represented by a single curve as shown in Figure 1b. This means that the hysteresis effects are neglected. This is not an unreasonable assumption considering the high-permeability core materials in present-day use. During the past twenty years, much advancement has been made(6)(7) in the processing of magnetic materials making it

(6) Storm, H. F. Series Connected Saturable Reactor with Control Source of Comparatively Low Impedance. Proceedings of the American Institute of Electrical Engineers. Vol. 69. pt. 2. pp. 756-765 (1950)

(7) Ver Blanck, D. W., Finzi, L. A., Beaumariage, D. C. Analytical Determination of Characteristics of Magnetic Amplifiers with Feedback. Proceedings of the American Institute of Electrical Engineers. Vol. 68, pt. 1. pp. 565-570 (1949)

possible to assume a single-valued B-H curve with very little error.

The assumed B-H curve shown in Figure 1b can be represented by a sinh curve quite well for all practical purposes. The excitation, H, is directly proportional to the sum of the control current and the output current. The equation of the B-H curve may now be written as

$$i = a \sinh b\phi \quad \ldots\ldots\ldots\ldots (1)$$

where a and b are constants of the particular core material.

Writing the differential equation for the output circuit gives

$$L\frac{di}{dt} + R'i = E_m \sin \omega t \quad \ldots\ldots\ldots (2)$$

Solving for ϕ from ~~equation~~ (1)

$$\phi = \frac{1}{b} \sinh^{-1} \frac{i}{a} \quad \ldots\ldots\ldots\ldots (3)$$

$L\frac{di}{dt}$ is the voltage drop across the reactor and may also be written as $N\frac{d\phi}{dt} \cdot 10^{-8}$. Taking the time derivative of equation (3) and multiplying by $N \cdot 10^{-8}$ gives

$$\frac{N \cdot 10^{-8}}{b\sqrt{i^2+a^2}} \frac{di}{dt} \quad \ldots\ldots\ldots\ldots (4)$$

Substituting equation (4) into equation (2).

$$\frac{N \cdot 10^{-8}}{b\sqrt{i^2+a^2}} \frac{di}{dt} + R'i = E_m \sin \omega t \quad \ldots\ldots (5)$$

This equation is valid only for $0 < \omega t < \pi$ because the rectifier prohibits current flow on the negative half-cycle of the supply voltage. It should be noted that R' is the total forward resistance of the output circuit. This includes the resistance of the rectifier which, strictly speaking, is a function of the current through it.

The only omission in the previous discussion is the effects of the control circuit. The added effect of the control circuit will be simply to shift the operating point along the B-H curve. The currents induced into the control winding are assumed too small for consideration due to the inductor, L.

The difficulty of this type of approach is that the resultant differential equation is unsolvable by ordinary methods and a point-by-point or mechanical integration is the only type of solution possible.

Because of this, certain simplifying assumptions must be made in order to obtain a satisfactory solution. An analysis based on these assumptions is given in the following section.

DISCUSSION OF SIMPLE HALF-WAVE SERIES MAGNETIC AMPLIFIER CIRCUIT WITH SIMPLIFYING ASSUMPTIONS

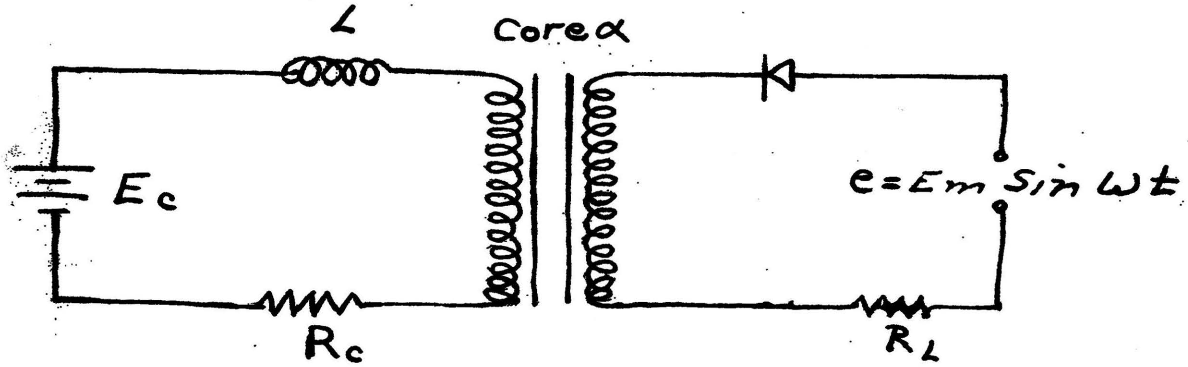
As discussed in the preceding analysis, simplifying assumptions are necessary to gain an approach to the problem of magnetic amplifiers. The following assumptions will be made in the analysis of this circuit.

1. The rectifier has zero forward resistance and infinite backward resistance.
2. Hysteresis effects and eddy currents are neglected.
3. The reactor winding resistances are small and are lumped in with the load resistance (R_L) and control resistance (R_c).
4. All magnetic leakage effects are neglected.
5. A B-H curve is assumed as shown in Figure 1c.

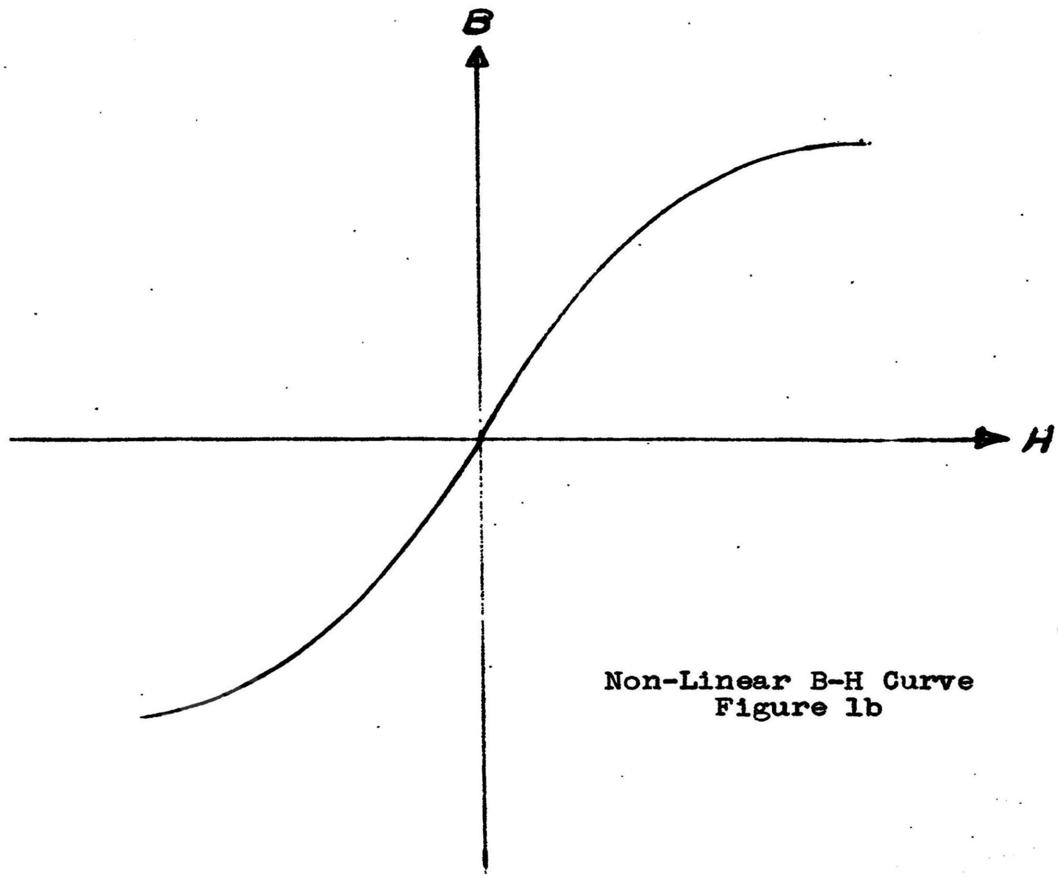
It is obvious from the B-H curve that the reactor can exhibit two different values of inductance. The reason for this is shown below. From Lenz's Law, $e = L\frac{di}{dt} = N\frac{d\phi}{dt}\cdot 10^{-8}$ where L is inductance in henrys and N is the number of turns. From this relation, $L = N\frac{d\phi}{di \cdot 10^{8}}$; but $d\phi = A\,dB$ where A is the area of the iron and B is the flux density. Now

$$L = N\frac{d\phi}{di\,10^{8}} = NA\frac{dB}{di \cdot 10^{8}} = NA\frac{dB}{dH}\cdot\frac{dH}{di\,10^{8}}$$

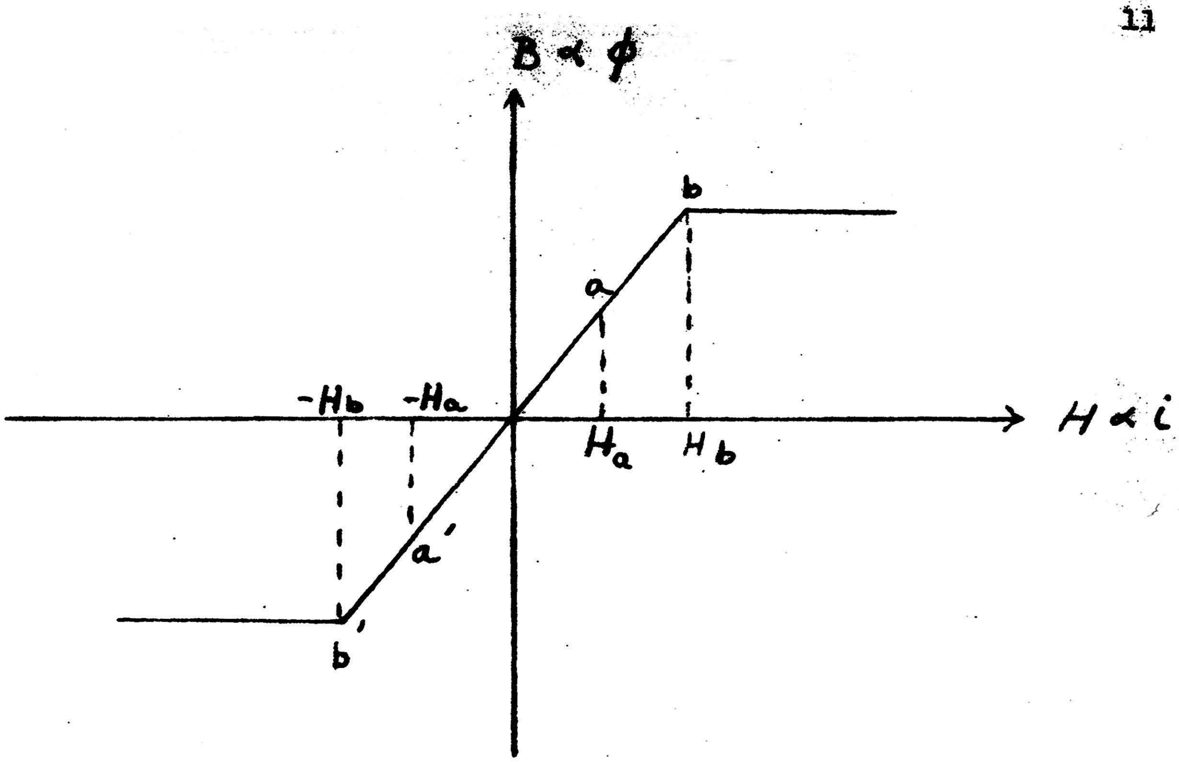
where H is the magnetic intensity in the iron core. The ratio $\frac{dB}{dH}$ is the slope of the B-H curve and is the permeability of the magnetic material. When the reactor operates to the right of point b in Figure 1c, the inductance is zero since the core permeability is zero. When the reactor is operated in the region between b' and b, then the reactance is large and depends upon the number of turns on the reactor, frequency of applied voltage, and core dimensions.



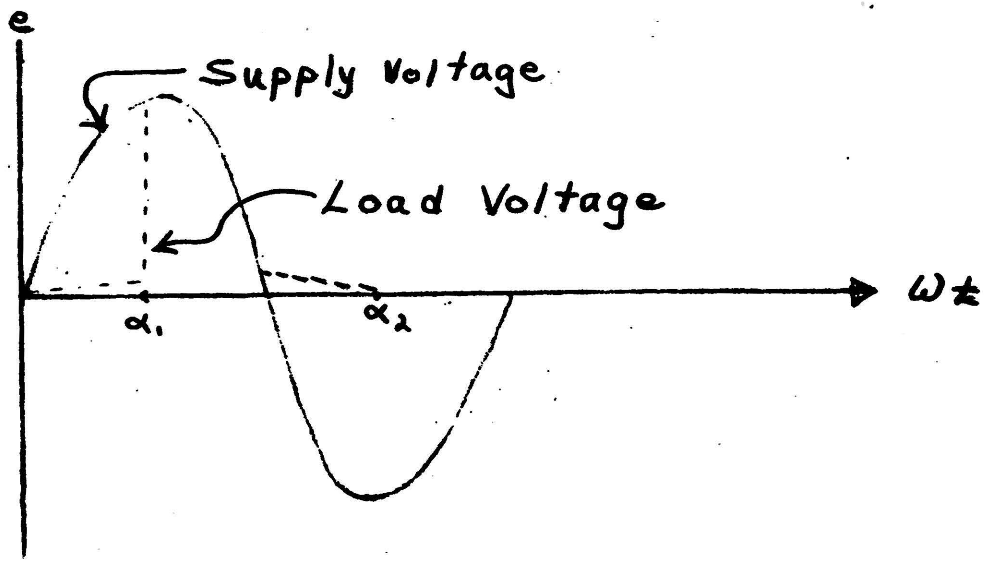
Half-Wave Magnetic Amplifier
Figure 1a



Non-Linear B-H Curve
Figure 1b



Straight Line B-H Curve
Figure 1c



Load Voltage and Supply Voltage
Waveforms for Circuit of Figure 1a
Figure 1d

When e is negative, the entire supply voltage must necessarily appear across the rectifier, assuming infinite backward resistance, and hence no output current will flow. When the supply voltage, e, is positive, it will either appear across the reactor as an induced voltage $N\frac{d\phi}{dt}$ or as an $i_a R_L$ drop across the output resistor, depending on the value of i_a.

Let us now assume that the reactor is pre-excited (biased) by a control current i_1 such that $N_c i_1 = -H_a$. The output circuit supply voltage, e, is assumed to be a sine wave and is shown in Figure 1d. Before the point 0 is reached, the a-c supply is negative and, therefore, is completely absorbed by the rectifier, the output current being zero. As ωt proceeds from 0 to α_1, the time rate of flux change is very great and the majority of the supply voltage will appear across the reactor as $N\frac{d\phi}{dt}$ with only a small value of output current flowing. This small value of output current will cause an additional flux in the core until at $\omega t = \alpha_1$ the knee or point b of the B-H curve, Figure 1c, reached and most of the supply voltage will appear across the output resistor as $i_a R_L$ drop. From α_1 to slightly before π, the reactor is saturated and can no longer support a voltage across itself. At $\omega t = \pi$ the supply voltage reverses polarity and appears across the rectifier as inverse voltage. The output voltage wave form is shown also in Figure 1d.

A small current flows from π to α_2 since the energy stored in the magnetic field during the interval $0-\alpha_1$ must be returned to the circuit. After $\omega t = 2\pi$ is reached, the same cycle of events again takes place.

The point α_1 is often called the point of firing or the firing angle and can be varied by the magnitude of the control current, i_c. If the foregoing events were assumed to take place when the core is biased at $H = -H_a$, then when it is biased at $+H_a$ the point of firing, α_1, will take place sooner in the cycle. The output current needs only to supply an intensity of $H_b - H_a$ before the core is saturated while previously it had to supply $H_b + H_a$.

The point α_2 will be called the extinction angle since it is the point where the load current and hence the load voltage return to zero. It also is dependent upon the current in the control winding. The extinction angle is obviously equal to or greater than 180°.

The firing angle may be calculated as follows: The differential equation for the output circuit is:

$$L\frac{di_a}{dt} + R_L i_a = E_m \sin \omega t$$

From $0 - \alpha_1$, the majority of the supply voltage is across the reactor, i.e. $L\frac{di_a}{dt} \gg R_L i_a$ so it may be said

$$L\frac{di_a}{dt} \cong E_m \sin \omega t = N \frac{d\phi_\alpha}{dt} 10^{-8}$$

$$d\phi_\alpha \cong \frac{E_m \cdot 10^8}{N} \sin \omega t \, dt \qquad \text{but } d\phi_\alpha \cong A \, dB \text{ so}$$

$$dB = \frac{E_m \cdot 10^8}{AN} \sin \omega t \, dt$$

$$B\Big]_{B_0}^{B(t)} = \frac{E_m \cdot 10^8}{AN} \int_0^t \sin \omega t \, dt \qquad \text{where } 0 < \omega t < \alpha_1$$

$$B(t) - B_0 = \frac{E_m \cdot 10^8}{NA\omega} (1 - \cos \omega t)$$

The B_0 term is the value of flux density evaluated when time is zero. This term is composed of two parts. The first is the flux density caused by the d-c in the control winding, and the second is the d-c component of current which flows in the output circuit. When $\omega t_1 = \alpha_1$, the firing angle, $B(t)$, is equal to the flux density of firing, $B(f)$. So we have $B_f = B_o + \frac{E_m \cdot 10^8}{NA\omega}(1 - \cos\alpha_1)$. Let the quantity $\frac{E_m \cdot 10^8}{NA\omega}$ be called B_m which is the maximum value of flux density. Now $B_f = B_o + B_m(1 - \cos\alpha_1)$ or $\alpha_1 = \cos^{-1}\left[\frac{B_m + B_o - B_f}{B_m}\right]$ Because of the rectifier action $0 \leq \alpha_1 \leq \pi$

In closing the analysis of this half-wave circuit type of magnetic amplifier, an expression for current will be derived which will be good until the point of firing, i.e. for $0 \leq \omega t \leq \alpha_1$. The general differential equation is

$$L\frac{di_a}{dt} + R_L i_a = E_m \sin\omega t \qquad \ldots\ldots\ldots(6)$$

where the rectifier forward voltage drop (if any is assumed) is included in $R_L i_a$ along with any assumed drop in the coil resistance.

The solution for equation (6) is $i_a(t) = Ae^{-\frac{R_L}{L}t} + i_p$ where A is a constant to be determined and i_p is the particular solution which will be solved for now:

$$i_p = C_1 \cos\omega t + C_2 \sin\omega t \qquad \ldots\ldots\ldots(7)$$

$$\frac{di_p}{dt} = -C_1\omega \sin\omega t + C_2\omega \cos\omega t \qquad \ldots\ldots(8)$$

where C_1 and C_2 are constants of integration. Substituting equation (8) and (7) into equation (6) gives

$$\left[-C_1\omega \sin\omega t + C_2\omega\cos\omega t + \frac{R_L C_1}{L}\cos\omega t + \frac{R_L C_2}{L}\sin\omega t\right] = \frac{E_m}{L}\sin\omega t$$

Equating like coefficients of the terms gives $-C_1\omega + \frac{R_L C_2}{L} = \frac{E_m}{L}$ and $C_1\omega + \frac{R_L}{L}C_1 = 0$ from which $C_1 = \frac{-L\omega E_m}{R_L^2 + \omega^2 L^2}$

$$C_2 = \frac{R_L E_m}{R_L^2 + \omega^2 L^2}$$

Finally $i_a(t) = Ae^{-\frac{R}{L}t} - \frac{L\omega E_m}{R_L^2+\omega^2L^2}\cos\omega t + \frac{RE_m}{R_L^2+\omega^2L^2}\sin\omega t$

when $t=0$, $i_a(t) = 0$ so $A = \frac{L\omega E_m}{R_L^2+\omega^2L^2}$

$$i_a(t) = \frac{L\omega E_m}{R_L^2+\omega^2L^2}\left[e^{-\frac{R}{L}t} - \cos\omega t\right] + \frac{RE_m}{R_L^2+\omega^2L^2}\sin\omega t$$

$$i_a(t) = \frac{L\omega E_m}{R_L^2+\omega^2L^2}\left[e^{-\frac{R_L}{L}t} + \frac{R_L}{\omega L}\sin\omega t - \cos\omega t\right]$$

let $I_m \triangleq \frac{E_m}{\sqrt{R_L^2+\omega^2L^2}}$ and $\theta' \triangleq \tan^{-1}\frac{\omega L}{R_L}$

then $\sin\theta' = \frac{\omega L}{\sqrt{R_L^2+\omega^2L^2}}$ and $\cos\theta' = \frac{R_L}{\sqrt{R_L^2+\omega^2L^2}}$,

so $i_a(t) = I_m\left[\sin(\omega t-\theta') + \sin\theta' e^{-\frac{R}{L}t}\right]$.

This expression gives the output current for any time until $\omega t = \alpha_1$, where the inductance (L) is a constant.

A continuous increase of control current cannot produce an unlimited increase of output current. When the control excitation completely saturates the core, the output voltage will appear across the output resistor at the very beginning of the cycle so the maximum power will be delivered to the output resistor. If a further increase in control current is realized, it is obvious that the maximum output power cannot be increased further since the amplifier cannot fire sooner than $\omega t = 0$. At this point, the load current is limited only by the value of circuit resistance. This region is called the resistance limited region. Actually, the value of current will be slightly lower because of air-core reactance.

The value of load current from α, to π is $i_a = \frac{E_m}{R_L} \sin \omega t$ since the drop across the reactance part of the reactor is now zero.

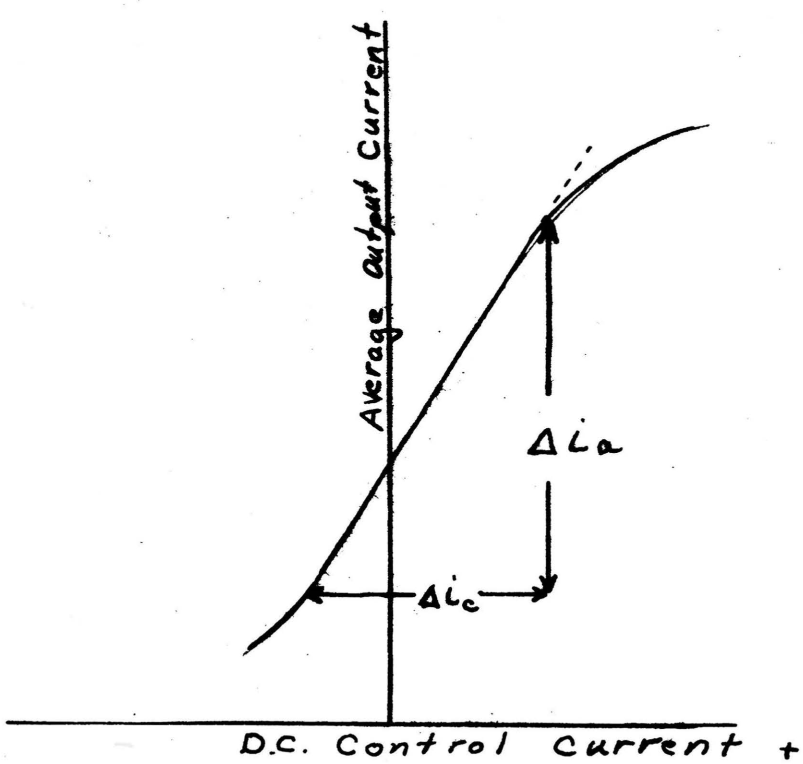
The foregoing analysis was taken in part from two articles.(8)(9) The results of these articles have been expanded

(8) Smith, E. J. Steady State Performance of Magnetic Amplifiers. Proceedings of the American Institute of Electrical Engineers. Vol. 69. pt. 2. pp. 1309-1317 (1950)

(9) Cohen, op. cit., pp. 1009-1020

in the previous section.

The figure below shows a typical transfer curve for the half-wave magnetic circuit just considered and was obtained by experimental methods.



As can be seen from the figure, the output vs input is linear over a considerable portion of the curve.

Although this half-wave magnetic circuit is simple to analyze, it is very seldom used in practice. One reason for this is the necessity for using the large series inductor in the control winding. If this were not used, the amplifier would act somewhat like a short-circuited transformer since the control circuit is generally of very low impedance. The two-core reactor is in general use today since it can be made to handle greater amounts of power and because the sensitivity can be improved. Also with two reactors, the control circuit can be so wound as to cancel out all odd harmonics of the supply frequency which eliminates the series inductor in the control circuit thus lowering cost.[10]

---

(10) Esselman, W. H. Steady State Analysis of Self-Saturating Magnetic Amplifiers Based on Linear Approximations of the Magnetization Curve. Transactions of the American Institute of Electrical Engineers. Vol. 70. pt. 1. p. 455 (1951)

DISCUSSION OF A SERIES MAGNETIC AMPLIFIER CIRCUIT WITH FEEDBACK ASSUMING A POWER SERIES B-H CURVE

Consider the magnetic amplifier shown in Figure 2a. This amplifier consists of two saturable reactors, each wound with an a-c and a d-c winding. The d-c control windings are wound in series opposition to cancel out the odd harmonic voltages. This does away with the series inductor (L) in the control circuit. In the first portion of this analysis the load resistance will be neglected since considerable difficulty is encountered if an attempt is made to include it.

With a d-c excitation applied to the d-c coils and an a-c voltage (sinusoidal) applied to the output windings, the wave of flux density in the α core can be written as

$$B_m \sin \omega t + B_o \quad \ldots\ldots\ldots\ldots (9)$$

and the wave of flux density in the β core as

$$B_m \sin \omega t - B_o \quad \ldots\ldots\ldots\ldots (10)$$

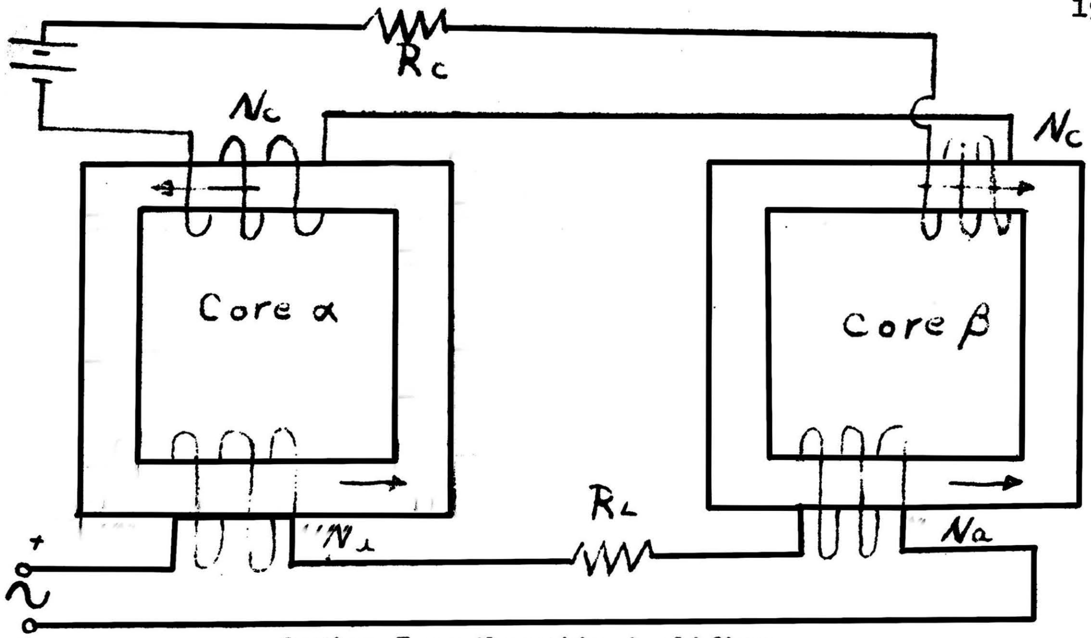
since the d-c winding is in series opposition. A plot of the flux density vs time is shown in Figure 2b.

Assume now that the B-H characteristic can be expressed as

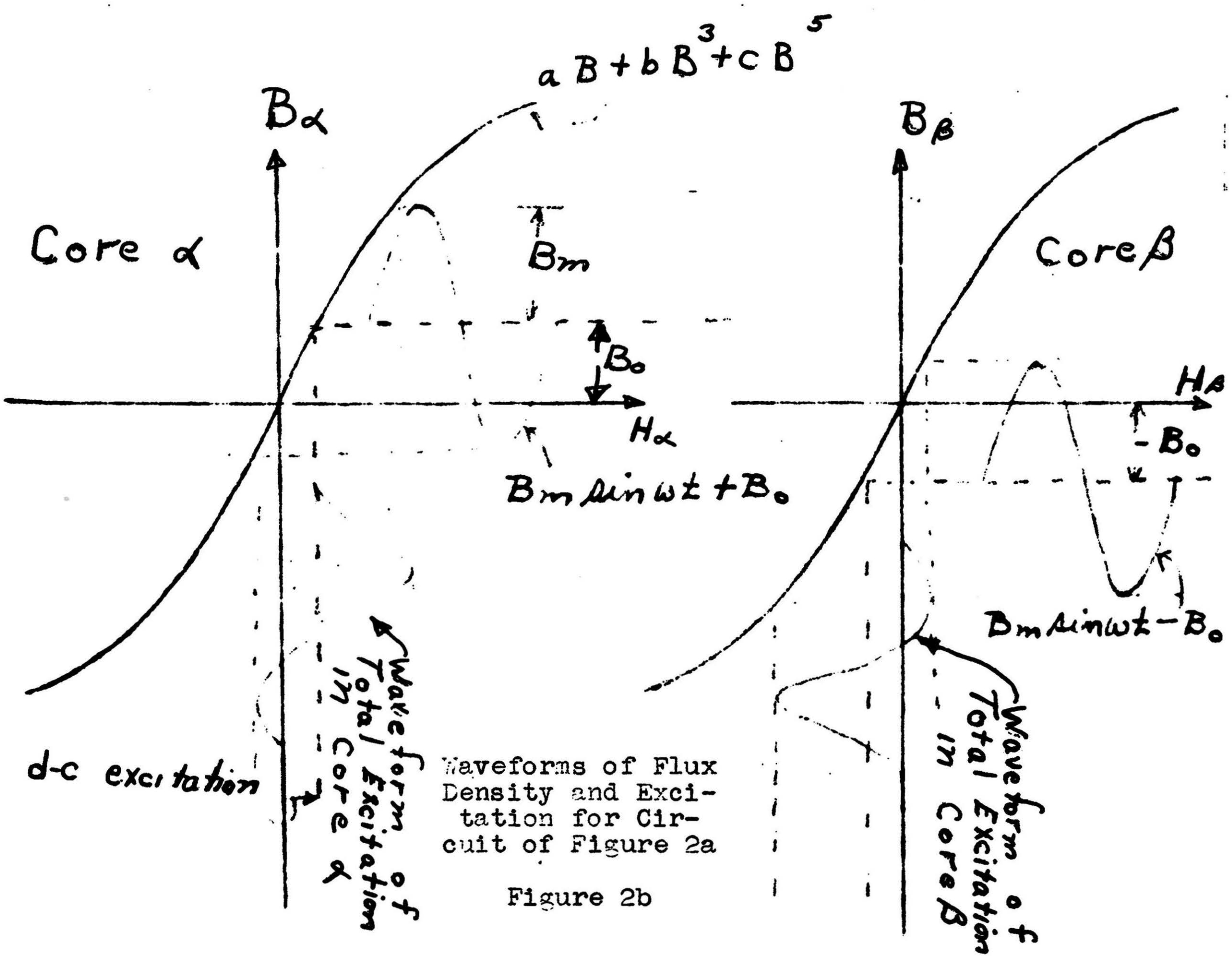
$$H = a'B + b'B^3 + c'B^5 \quad \ldots\ldots\ldots\ldots (11)$$

All the previous assumptions are made in this analysis; i.e. perfect rectifiers, negligible control circuit impedance, etc. By negligible control circuit impedance, it is meant that the control circuit is supplied by a generator of negligible impedance.

Substituting equation (9) into (11) we get the equation for the excitation in the α core as



Series Type Magnetic Amplifier
Figure 2a



Waveforms of Flux Density and Excitation for Circuit of Figure 2a

Figure 2b

$$H\alpha = a'(B_m \sin\omega t + B_0) + b'(B_m \sin\omega t + B_0)^3 + c'(B_m \sin\omega t + B_0)^5 \qquad (12)$$

This gives (see Appendix 1):

$$\begin{aligned}
H\alpha = {} & (a'B_0 + b'B_0^3 + c'B_0^5 + \tfrac{3}{2}b'B_0B_m^2 + 5c'B_0^3B_m^2 + \tfrac{15}{8}c'B_0B_m^4) \\
& + (a'B_m + 3b'B_0^2B_m + 5c'B_0B_m^4 + \tfrac{3b'}{4}B_m^3 + \tfrac{15}{2}c'B_0^2B_m^3 \\
& + \tfrac{5}{8}c'B_m^5)\sin\omega t \\
& - (\tfrac{3}{2}b'B_0B_m^2 + 5c'B_0^3B_m^2 + \tfrac{5}{2}c'B_0B_m^4)\cos 2\omega t \\
& - (\tfrac{b'}{4}B_m^3 + \tfrac{5}{2}c'B_0^2B_m^3 + \tfrac{5}{16}c'B_m^5)\sin 3\omega t \\
& + (\tfrac{5c'}{8}B_0B_m^4)\cos 4\omega t \\
& + (\tfrac{c'B_m^5}{16})\sin 5\omega t
\end{aligned} \qquad . \; (13)$$

Writing the equation relating H_α and ampere turns gives

$$H\alpha\, l = N_a i_a + N_c i_c \qquad . . . . (14)$$

and

$$H\beta\, l = N_a i_a - N_c i_c \qquad . . . . (15)$$

The constants a', b', and c' depend upon the particular core characteristics.

The expression for $H\beta$ is similiar to that of $H\alpha$ except that B_0 is replaced everywhere by $-B_0$. The expression for $H\beta$ is given in Appendix 1.

Adding equations (14) and (15) and solving for i_a gives

$$i_a = (H\alpha + H\beta)\frac{l}{2N_a} \qquad . . . . (16)$$

Subtracting equation (15) from equation (14) and solving for i_c gives

$$i_c = (H_\alpha - H_\beta)\frac{l}{2N_c} \quad \ldots\ldots\ldots\ldots \quad (17)$$

Substituting for H_α and H_β in equation (16) we have

$$i_a = \frac{l}{N_a}\Big[a' B_m + 3b' B_m B_0^2 + 5c' B_0^4 B_m + \frac{3b'}{4} B_m^3$$

$$+ \frac{15c'}{2} B_0^2 B_m^3 + \frac{5c'}{8} B_m^5\Big] \sin \omega t$$

$$- \frac{l}{N_a}\Big[\frac{b'}{4} B_m^3 + \frac{5}{2} c' B_0^2 B_m^3 + \frac{5}{16} c' B_m^5\Big] \sin 3\omega t$$

$$+ \frac{l}{N_a}\Big[\frac{c' B_m^5}{16}\Big] \sin 5\omega t \quad \ldots\ldots\ldots \quad (18)$$

Similarly, from equation (17) we have

$$i_c = \frac{l}{N_c}\Big[a' B_0 + b' B_0^3 + c' B_0^5 + \frac{3}{2} b' B_0 B_m^2 + 5c' B_0^3 B_m^2$$

$$+ \frac{15}{8} c' B_0 B_m^4\Big]$$

$$- \frac{l}{N_c}\Big[\frac{3b'}{2} B_0 B_m^2 + 5c' B_0^3 B_m^2 + \frac{5c'}{2} B_0 B_m^4\Big] \cos 2\omega t$$

$$+ \frac{l}{N_c}\Big[\frac{5c'}{8} B_0 B_m^4\Big] \cos 4\omega t \quad \ldots\ldots \quad (19)$$

Equation (19) shows that the current in the d-c circuit consists of a d-c component, a negative second harmonic, and a forth harmonic. Thus, all odd harmonics are canceled in this winding by virtue of the manner in which it is wound. The current in the output circuit consists of a fundamental, a negative third harmonic, and a fifth harmonic.

The d-c component of i_c could only have been caused by a d-c current in the control winding, which will be called I_c. Therefore, the d-c excitation for the α core is

$$I_c N_c = l\left[a'B_o + b'B_o^3 + c'B_o^5 + \frac{3}{2}b'B_oB_m^2 + 5c'B_o^3B_m^2 + \frac{15}{8}c'B_oB_m^4\right] \quad (20)$$

It is easily seen that this d-c excitation is influenced by the maximum value of a-c flux, B_m. This is because of the non-linearity of the assumed B-H characteristic. Equation (20) is actually the average value of equation (13) between $0-2\pi$. This is true because $\frac{1}{2\pi}\int_0^{2\pi} \sin n\omega t \, d(\omega t) \equiv 0$

and $\frac{1}{2\pi}\int_0^{2\pi} \cos n\omega t \, d(\omega t) \equiv 0$ where $n = 1, 2, 3 \cdots \infty$

Also $\frac{1}{2\pi}\int_0^{2\pi} I_c N_c \, d(\omega t) = \frac{1}{2\pi}\left[I_c N_c\right](\omega t)\Big|_0^{2\pi} = I_c N_c$

Referring to Figure 2b, it may be seen that the excitation curve of the α core during the interval $0-\pi$ is identical to that of the β core during the interval $\pi-2\pi$. The output current excitation can now be obtained by taking the average of equation (13) over a period of $0-\pi$ and subtracting the d-c excitation. The same result may be obtained by averaging the total excitation in the β core from $\pi-2\pi$ and then subtracting the d-c excitation.

$$I_{a\,avg} N_a = \frac{l}{\pi}\int_0^{\pi} H_\alpha \, d(\omega t) - I_c N_c \quad \ldots . (21)$$

But, as was just shown, $I_c N_c$ is the average of H_α over a period of $0-2\pi$. That is

$$I_c N_c = \frac{1}{2\pi}\int_0^{2\pi} H_\alpha d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} H_\alpha d(\omega t) + \frac{1}{2\pi}\int_{\pi}^{2\pi} H_\alpha d(\omega t). \quad (22)$$

Substituting the value of

$$\frac{1}{\pi}\int_0^{\pi} H_\alpha d(\omega t)$$

of equation (22) into equation (21) gives

$$I_{a\,avg} N_a = 2 N_c I_c - \frac{1}{\pi}\int_{\pi}^{2\pi} H_\alpha d(\omega t) - I_c N_c \ldots$$

$$I_{a\,avg} N_a = N_c I_c - \frac{1}{\pi}\int_{\pi}^{2\pi} H_\alpha d(\omega t) \quad \ldots\ldots (23)$$

From Figure 2b it may be seen that the excitation of the α core from $\pi - 2\pi$ is much less than that from $0-\pi$. The term

$$\frac{1}{\pi}\int_{\pi}^{2\pi} H_\alpha d(\omega t)$$

is actually the product of H_α and ωt during the interval $\pi - 2\pi$. From Figure 2b it may also be seen that as the initial slope of the B-H curve increases, the integral will become very small. If it were possible to have an infinite slope, the integral would become equal to 0. At this condition an infinitesimal amount of excitation would cause the reactor to saturate. A B-H curve of infinite slope can never be realized, of course; but in most magnetic amplifier materials such as Mu Metal, Hipersil, and Hipernik, the initial slope is very steep so that equation (23) reduces to

$$I_{a\,avg} N_a \cong I_c N_c \quad \ldots\ldots\ldots (24)$$

If a third winding is added, the result is a circuit of the type shown in Figure 3. The third winding is called a feedback winding. The output current is rectified by the bridge-type full-wave rectifier and fed back through the feedback windings. The feedback winding on each reactor has N_b turns and is wound so that its mmf aids the output circuit mmf. The equivalence of ampere turns in this amplifier circuit also holds true, i.e.

$$I_{a\,avg} N_a = I_b N_b + I_c N_c \qquad \text{. . . . . (25)}$$

where I_c was defined previously and I_b is the average value of current in the feedback circuit. Rearranging equation (25) gives

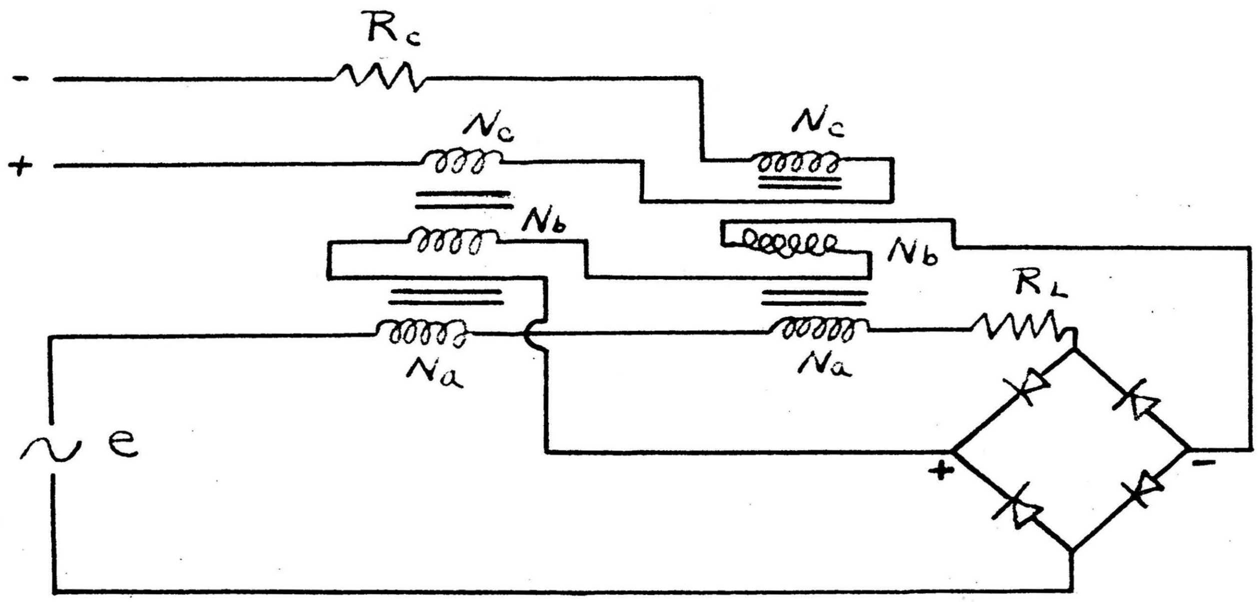
$$I_c N_c = I_{a\,avg} N_a - I_b N_b = I_{a\,avg} N_a (1-a) \qquad \text{. . . (26)}$$

where a is $\dfrac{I_b N_b}{I_{a\,avg} N_a}$

Now

$$\frac{I_{a\,avg}}{I_c} = \frac{N_c}{N_a(1-a)} \qquad \text{. . . . . (27)}$$

With the addition of the feedback winding, the ratio of output current (average) to control current has been increased by the factor $\frac{1}{1-a}$. The current gain can easily be made infinite by making $a = 1$ although the circuit tends to become unstable under these conditions. It will be noticed that equation (27) is a general equation since the quantity, a, is a ratio of feedback ampere turns to output ampere turns. In the case of the circuit in Figure 3 the current through the feedback winding is the same as the current through the output winding, i.e. $i_a = i_c$ and $I_{a\,avg} = I_c$ assuming perfect rec-



Series Type Magnetic Amplifier
with Feedback

Figure 3

tifiers. In this case the feedback factor, a, reduces to $a = \frac{N_b}{N_a}$ which will be defined as λ.

If a resistive load is assumed in the output circuit, the average power amplification can be defined as

$$K_p = \frac{R_L}{R_c}\left[\frac{I_{a\,avg}}{I_c}\right]^2 \quad \ldots\ldots\ldots \quad (28)$$

This is the ratio of average power out to average power in. Putting equation (27) into (28) and setting $a = \lambda$ gives

$$K_p = \frac{N_c^2}{N_a^2}\frac{R_L}{(1-\lambda)^2 R_c} \quad \ldots\ldots\ldots \quad (29)$$

where $\lambda = \frac{N_b}{N_a}$. This equation shows that $\frac{N_b}{N_a}$ should be large (close to unity) for large power amplifications. It is also seen that for the special case where $N_b = N_a$, this equation gives infinite power gain. In general, the feedback turns N_b on a magnetic amplifier are less than the output turns N_a. A value of $\lambda = 0.9$ is the general order of magnitude in actual amplifier circuits.(11)

---

(11) Milnes, A. G. In reply to a discussion of his article, Magnetic Amplifiers. Proceedings of the Institute of Electrical Engineers. Vol. 96. pp. 362-363 (1949)

It will be noted that the second term on the right hand side of equation (23), i.e.

$$\frac{1}{\pi}\int_{\pi}^{2\pi} H_a \, d(\omega t)$$

was neglected. If, however, this were not neglected, the expression for current and power gain would not give a value of infinity for 100% feedback. Actual experiments have shown that this term cannot be neglected at high values of λ.[12]

---

(12) Milnes, ibid.

DISCUSSION OF A SERIES TYPE MAGNETIC AMPLIFIER CIRCUIT ASSUMING A STRAIGHT LINE B-H CURVE

Consider the circuit shown in Figure 4. This is identical to the circuit of Figure 3.

The principal difference in this analysis and the previous analysis is in the assumed B-H curve. In this analysis a straight line B-H characteristic will be assumed.

If a careful study of actual operating conditions is made, it will be found, in general, that the reactance of the output circuit in the unsaturated portion of the B-H curve is much greater than the output (or load) resistance. This is an assumption which will be used presently. It will also be assumed that the impedance of the source of control signal is negligible. If this is true, only a very small a-c voltage is required in the output circuit to induce even harmonic voltages into the control circuit.

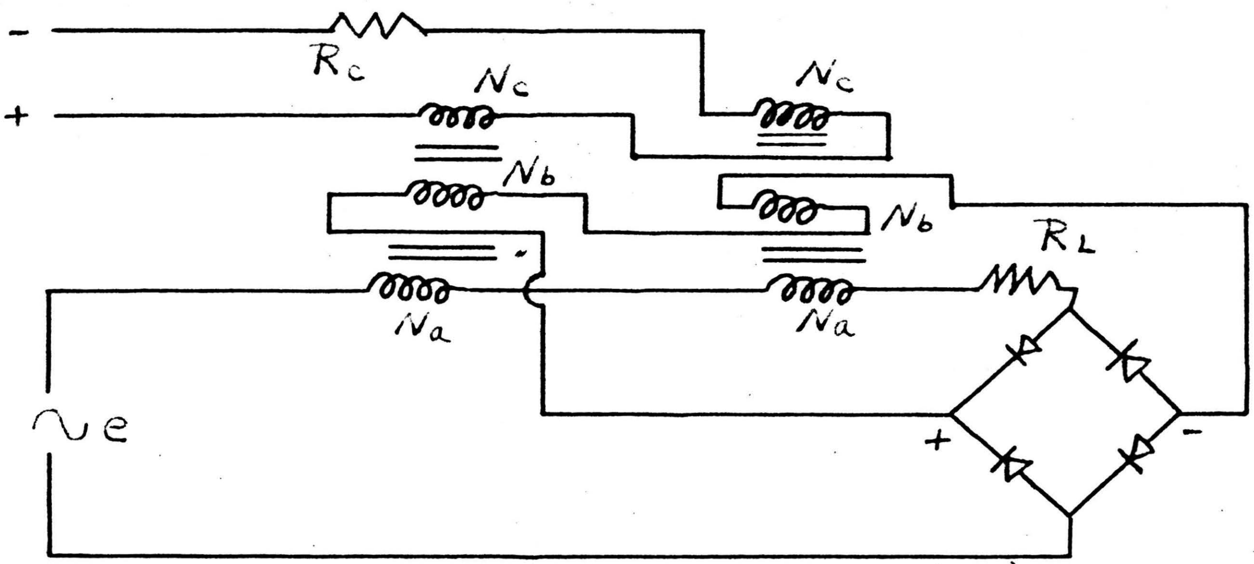
The straight line B-H curve will be similar to the one shown in Figure 1c. The first portion has a very steep slope which represents a high inductance. This is followed by a horizontal portion which represents the saturated condition.

The feedback circuit is coupled closely to the control circuit, which has negligible impedance, and is fed by the "perfect" rectifiers. Because of the low impedance of these circuits, even harmonic currents will circulate freely through them. This was shown for the control circuit in the previous analysis. This circulation of even harmonic currents results in voltage waveforms as shown in Figure 5a and 5b.

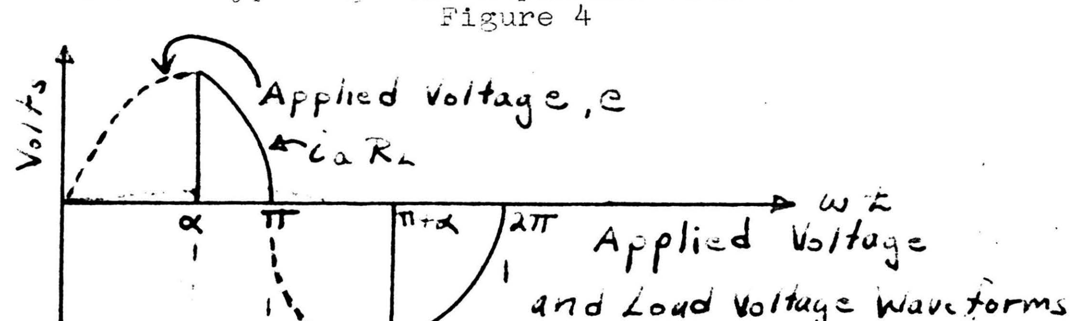
Since the control circuit is connected in series opposition, this causes core α to saturate at $\omega t = \alpha_1$, and core β to saturate in the opposite direction 180° later at $\omega t = \alpha_1 + \pi$ (see Figure 5). During the interval $0 < \omega t < \alpha_1$, the supply voltage applied to core α produces an output current which has an mmf in the same direction as the control mmf. During the same interval, this output current produces an mmf in core β which is in opposition to the control mmf. During this interval, the supply voltage is increasing until core α saturates at $\omega t = \alpha_1$. Core α has no impedance when saturated; and, therefore, all windings on this core are effectively short circuited. The total a-c induced currents during this interval, $0 < \omega t < \alpha_1$, are then induced across the windings on core β which has the effect of reducing the impedance of all windings on that core. Thus, in effect, the whole supply voltage between α_1 and π appears across the output resistor, R_L.

Thus, in the interval 0 to π, the only useful output current which flows is from α_1 to π. The firing angle, α_1, may be decreased or increased by increasing or decreasing the d-c control current respectively. This condition is within keeping of the original assumption that from 0 to α_1, the reactance is large as compared with the resistance, while from α_1 to π, the resistance is much greater than the reactance (which is essentially zero).

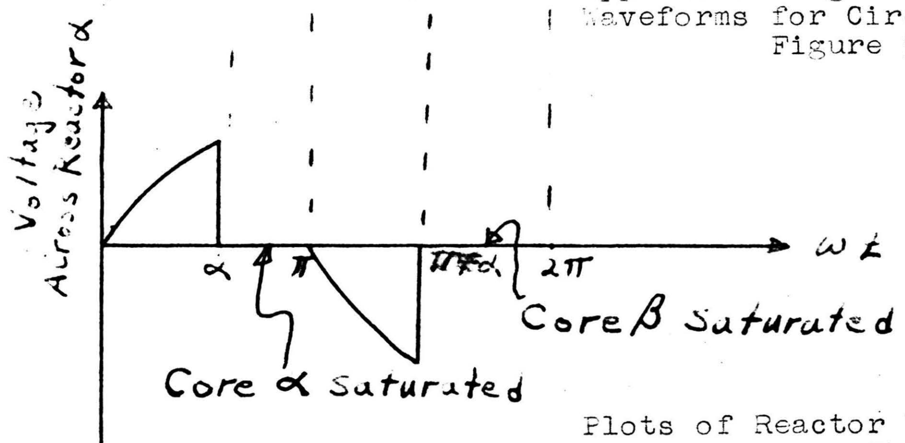
Between the interval $\pi < \omega t < 2\pi$, the same cycle of events is repeated, this time with core β saturating and in the opposite direction.



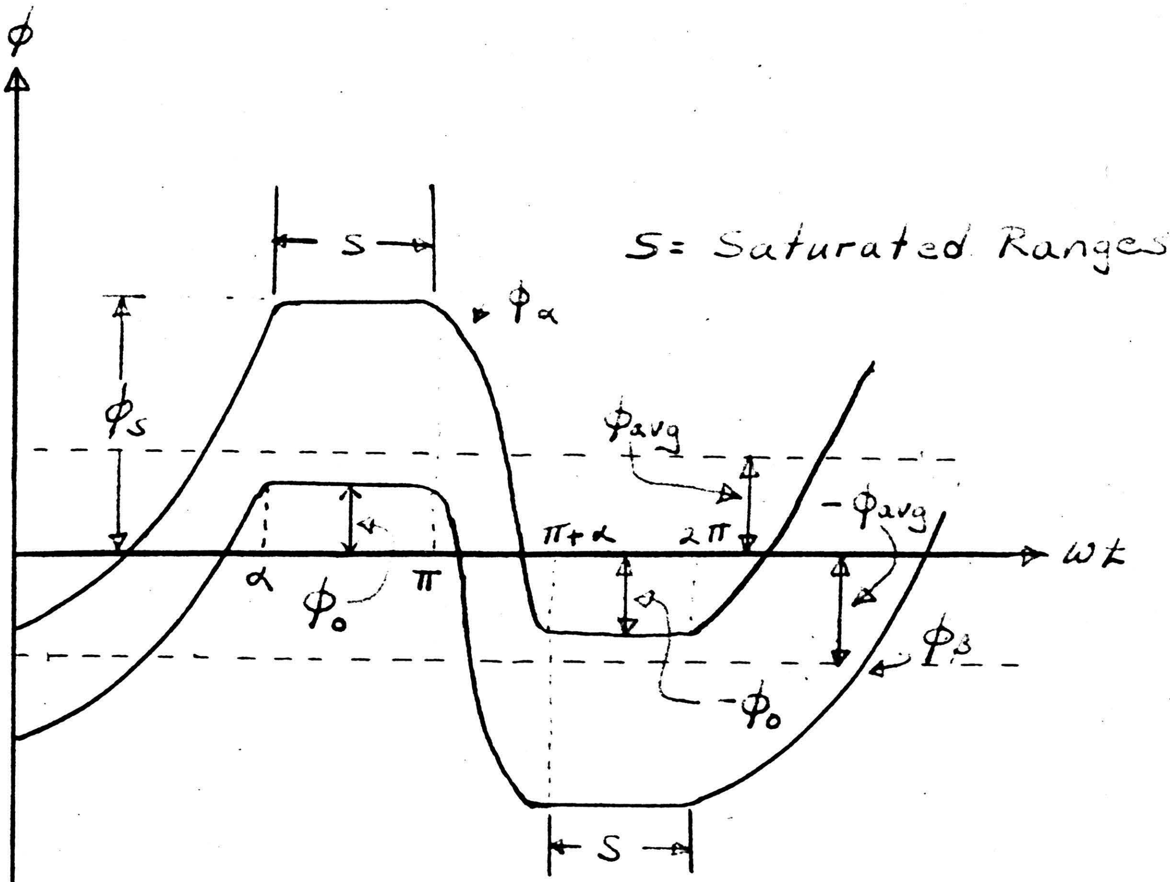
Series Type Magnetic Amplifier With Feedback
Figure 4



Applied Voltage and Load Voltage Waveforms for Circuit of Figure 4
Figure 5a



Plots of Reactor Voltages
Figure 5b



Flux Waveforms in the Cores

Figure 6

The analysis of this circuit will be made by breaking each half-cycle into two parts. The first portion is $0<\omega t<\alpha_1$, $\pi<\omega t<\pi+\alpha_1$, etc. The second portion is $\alpha_1<\omega t<\pi$, $\pi+\alpha_1<\omega t<2\pi$, etc. For the interval $0<\omega t<\alpha_1$ assuming $X \gg R_L$, it can be written

$$10^{-8}\left[N_a \frac{d\phi_\alpha}{dt} + N_a \frac{d\phi_\beta}{dt}\right] = E_m \sin\omega t \quad . \; . \; . \; . \; (30)$$

and

$$N_c\left[\frac{d\phi_\alpha}{dt} - \frac{d\phi_\beta}{dt}\right] = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; (31)$$

where ϕ_α and ϕ_β are the total instantaneous values of flux in the α and β cores respectively. Equation (30) is simply a loop equation relating the applied voltage to the voltage drops. Equation (31) states that the rate of flux change linking the control winding on the α core is the same as the rate of flux change linking the control winding on the β core.

It will be found convenient in the following discussion to let

$$K = \frac{E_m \cdot 10^8}{2\,\omega\,\phi_s}$$

purely for the sake of brevity. ϕ_s is the value of flux required to saturate either core.

The values of ϕ_α and ϕ_β may now be found from equation (30) by noting that

$$\frac{d\phi_\alpha}{dt} = \frac{d\phi_\beta}{dt}$$

during the interval $0<\omega t<\alpha_1$.

$$\frac{2N_a}{10^8}\frac{d\phi_\alpha}{dt} = E_m \sin \omega t \qquad \ldots\ldots\ldots (32)$$

Integration of equation (32) gives

$$\frac{2N_a \phi_\alpha}{10^8} = -\frac{E_m}{\omega}\cos\omega t + k_1 \qquad \ldots\ldots (33)$$

where k_1 is a constant of integration to be determined. It was assumed originally in this analysis that ϕ_α is at the saturation value, ϕ_s , when $\omega t = \alpha_1$. With this information, the constant of integration in equation (33) may be determined.

$$\frac{2N_a \phi_s}{10^8} = -\frac{E_m}{\omega}\cos\alpha_1 + k_1 \qquad \ldots\ldots (34)$$

Solving for k_1 from equation (34), substituting in into equation (33), and solving for ϕ_α gives

$$\phi_\alpha = \left[-\frac{E_m \cdot 10^8}{2N_a\omega}\right]\cos\omega t + \phi_s + \frac{E_m \cdot 10^8 \cos\alpha_1}{2N_a\omega} \qquad \ldots (35)$$

but

$$\frac{E_m \cdot 10^8}{2N_a\omega} = K\phi_s$$

so

$$\phi_\alpha = -K\phi_s\cos\omega t + \phi_s + K\phi_s\cos\alpha_1 \qquad \ldots\ldots (36)$$

It has been shown also that $\phi_\beta = \phi_s$ when $\omega t = \pi + \alpha_1$. Since the flux waves are periodical, ϕ_β must also be equal to $-\phi_s$ at $\omega t = 0$. From equation (30)

$$\frac{2N_a}{10^8}\frac{d\phi_\beta}{dt} = E_m \sin\omega t \qquad \ldots\ldots\ldots (37)$$

Integration gives

$$\frac{2 N_a \phi_\beta}{10^8} = -\frac{E_m}{\omega} \cos \omega t + k_2$$

where k_2 is another constant of integration. When $\omega t = 0$, $\phi_\beta = -\phi_s$ so

$$k_2 = -\frac{2 N_a \phi_s}{10^8} + \frac{E_m}{\omega}$$

$$\frac{2 N_a \phi_\beta}{10^8} = -\frac{E_m}{\omega} \cos \omega t - \frac{2 N_a \phi_s}{10^8} + \frac{E_m}{\omega}$$

$$\phi_\beta = -K \phi_s \cos \omega t - \phi_s + K \phi_s \qquad \text{. . . . . . (38)}$$

Figure 6 shows the flux wave shapes in the two cores. The flux in core β starts from a negative saturation value when $\omega t = 0$, and the flux in core α is at a maximum half a cycle later. From equation (31) it may be seen that if either flux is not changing, the other flux must also remain constant. This accounts for the flat looking portion of the flux curves when either core is saturated.

From Figure 6 it may be seen that the average value of flux, ϕ_{avg}, in core α is

$$\tfrac{1}{2} (\phi_s - \phi_o) \qquad \text{. . . . . (39)}$$

where ϕ_s and ϕ_o are shown on the figure. If this is not obvious, it may be seen by integrating the ϕ_α wave over a period of 0 to 2π and dividing by 2π. To do this will require the derivation of ϕ_α during the interval

$\pi < \omega t < \pi + \alpha_1$; but knowing that $\phi_\alpha = \phi_s$ when $\omega t = \pi$, this is easily done.

The equation for ϕ_0 may easily be obtained either by letting $\omega t = 0$ in equation (36) or by letting $\omega t = \alpha_1$ in equation (38). Either way

$$\phi_0 = -K\phi_s \cos\alpha_1 - (1-K)\phi_s \quad \ldots\ldots (40)$$

ϕ_{avg} may be written as

$$\phi_{avg} = \frac{1}{2}\left[\phi_s + K\phi_s \cos\alpha_1 + (1-K)\phi_s\right]$$

$$= \frac{1}{2}\phi_s\left[(2-K) + K\cos\alpha_1\right] \quad \ldots\ldots (41)$$

A similar expression holds true for the average flux in core β

In the interval $0 < \omega t < \alpha_1$ it may be written for core α that

$$i_a N_a + i_c N_c + i_b N_b = H_\alpha l \quad \ldots\ldots (42)$$

where $H_\alpha = \dfrac{\phi_\alpha}{\mu A}$ and for core β

$$i_a N_a - i_c N_c - i_b N_b = H_\beta l \quad \ldots\ldots (43)$$

where $H_\beta = \dfrac{\phi_\beta}{\mu A}$. Adding equations (42) and (43) and solving for $i_a N_a$ gives

$$i_a N_a = \frac{l}{2}(H_\alpha + H_\beta) = \frac{l}{2\mu A}(\phi_\alpha + \phi_\beta) \quad \ldots (44)$$

By subtracting the same equations,

$$i_c N_c + i_b N_b = \frac{l}{2\mu A}\left[\phi_\alpha - \phi_\beta\right]$$

If, now, the expressions for ϕ_α and ϕ_β from equations (36) and (38) are substituted into equations (43) and (44), the result is,

$$i_a N_a = \frac{l}{\mu A}\left[\tfrac{1}{2}k\phi_s(1+\cos\alpha_1) - k\phi_s \cos\omega t\right] \quad \ldots (45)$$

and

$$i_c N_c + i_b N_b = \frac{l}{\mu A}\left[\phi_s - \tfrac{1}{2}k\phi_s(1-\cos\alpha_1)\right] \quad \ldots (46)$$

Equation (46) may be rewritten as

$$i_c N_c + i_b N_b = \frac{\phi_s l}{2\mu A}\left[(2-K) + K\cos\alpha_1\right]$$

but from equation (41)

$$\phi_{avg} = \frac{\phi_s}{2}\left[(2-K) + K\cos\alpha_1\right]$$

so equation (46) may be expressed as

$$i_c N_c + i_b N_b = \frac{\phi_{avg}\, l}{\mu A} \quad \ldots\ldots\ldots\ldots\ldots (47)$$

Thus, the combined mmf of the control and feedback circuit is constant during the interval $0 < \omega t < \alpha_1$, and the output circuit mmf is a cosine function of time.

The previous equations in this analysis apply only during the unsaturated regions, i.e. during the interval 0 to α_1, π to $\pi+\alpha_1$, etc. The equations for the saturated regions, i.e. α_1 to π, $\pi+\alpha_1$ to 2π, etc will now be developed.

When core α is saturated from α_1 to π, the total supply voltage must appear across the output resistor, R_L. The a-c output current during this interval is then

$$i_a = \frac{E_m}{R_L} \sin \omega t \qquad \text{. . . . . (48)}$$

or by multiplying both sides by N_a

$$i_a N_a = \frac{E_m N_a}{R_L} \sin \omega t \qquad \text{. . . . . (49)}$$

but

$$E_m = \frac{2 \omega N_a K \phi_s}{10^8}$$

so

$$i_a N_a = \frac{2 \omega N_a^2 K \phi_s}{R_L \cdot 10^8} \sin \omega t$$

Let H_m be defined as $\dfrac{2 \omega N_a^2 K \phi_s}{R_L \cdot 10^8}$

then

$$i_a N_a = H_m \sin \omega t \qquad \text{. . . . . (50)}$$

Since the rate of flux change in core α is the same as the rate of flux change in core β, by equation (31), both core fluxes are constant since core α is saturated. It will be noted that in the unsaturated region of the B-H curve that the magnetic intensity is directly proportional to the flux in the core, i.e. $H = \frac{\phi}{\mu A}$, while during the saturated portion the magnetic intensity is independent of the flux.

During the portion of the cycle from α_1 to π, the flux level in core β is equal to ϕ_0 while the flux in core α is the saturation flux, ϕ_s, see Figure 6. The flux in core β can remain constant only so long as all the mmf's acting on that core remain constant. Hence, for core β

$$H_m \sin \omega t - (i_b N_b + i_c N_c) = \frac{\phi_0 l}{\mu A} \quad . \; . \; . \; . \; . \; (51)$$

or

$$-(i_b N_b + i_c N_c) = -\left(H_m \sin \omega t - \frac{\phi_0 l}{\mu A}\right) \quad . \; . \; . \; . \; (52)$$

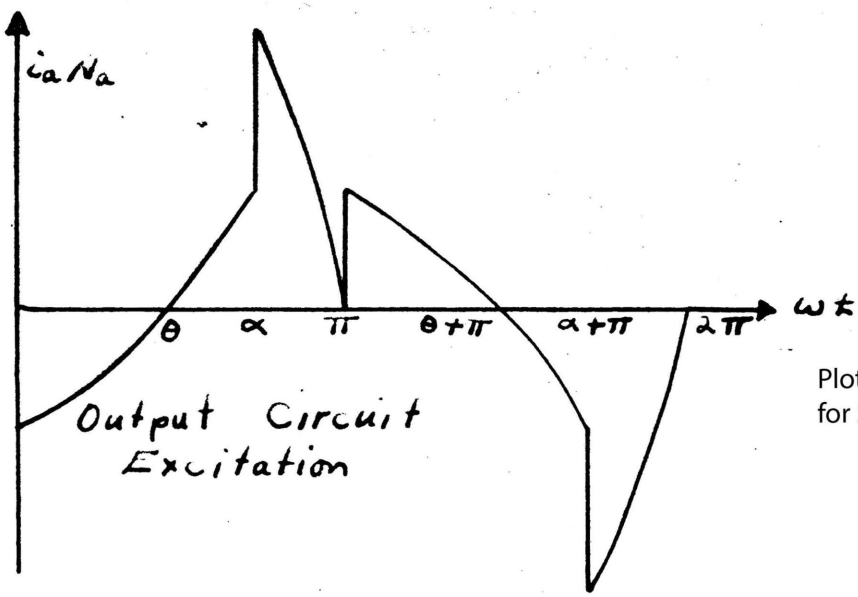
During the same period, the combined control and feedback ampere turns on core α is the same but of opposite sign and is

$$(i_b N_b + i_c N_c) = +\left(H_m \sin \omega t - \frac{\phi_0 l}{\mu A}\right) \quad . \; . \; . \; . \; (53)$$

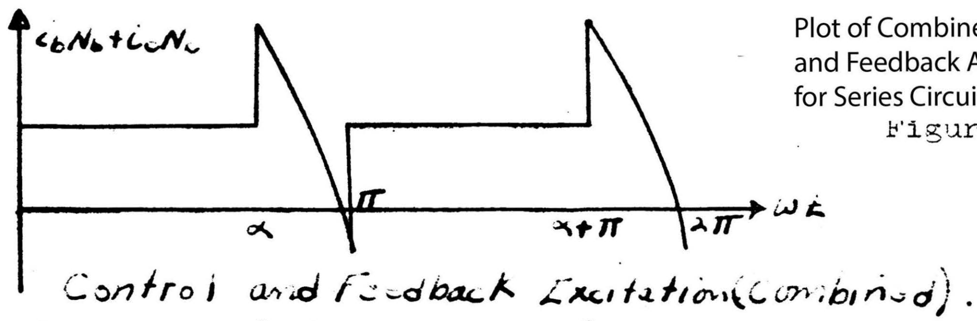
If the rectifiers are perfect and $N_b = N_a$, then the feedback mmf is at all times equal to the output circuit mmf. In general, though, the number of feedback turns in the feedback winding are not equal to the number of turns on the output winding. The ratio of $\frac{N_b}{N_a}$ has been defined as the feedback factor or degree of self-excitation and will be used in the following discussion.

From Figure 7a, the average value of the output circuit ampere turns is as follows:

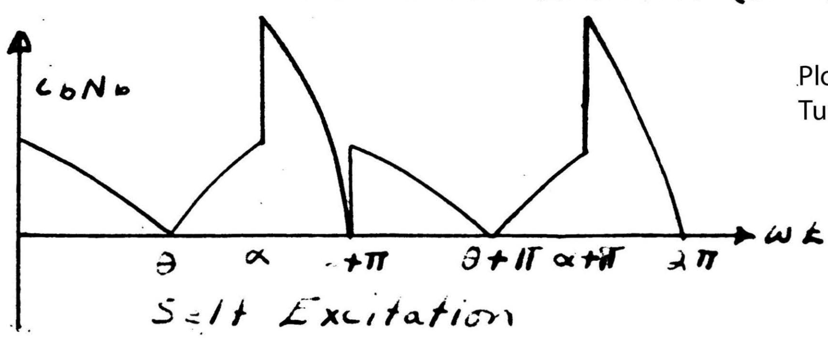
$$I_{a\,avg} N_a = \frac{1}{\pi} \int_0^{\pi} i_a N_a \, d(\omega t) \quad . \; . \; . \; . \; . \; (54)$$



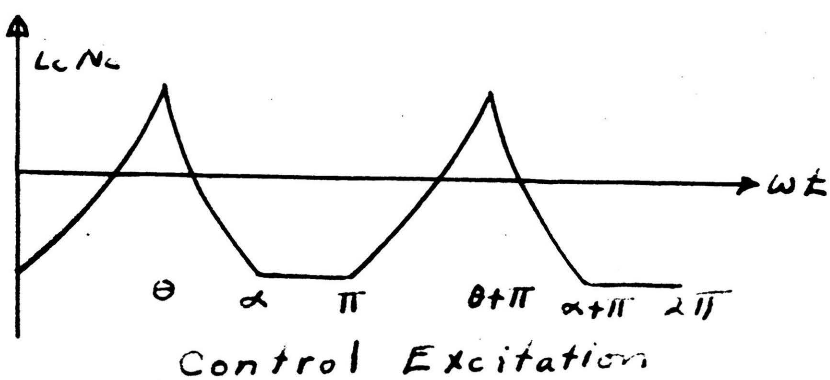
Plot of Output Ampere Turns for Series Circuit
Figure 7a



Plot of Combined Control and Feedback Ampere Turns for Series Circuit
Figure 7b



Plot of Feedback Ampere Turns for Series Circuit
Figure 7c



Plot of Control Ampere Turns for Series Circuit
Figure 7d

$$I_{a\,avg} N_a = \frac{1}{\pi}\int_0^{\theta} -i_a N_a \, d(\omega t) + \frac{1}{\pi}\int_{\theta}^{\alpha_1} i_a N_a \, d(\omega t)$$

$$+\frac{1}{\pi}\int_{\alpha_1}^{\pi} i_a N_a \, d(\omega t) \quad \ldots\ldots (55)$$

where θ is the value of ωt at which $i_a = 0$.

When equation (55) is integrated and the value

$$H_m = \frac{X}{R_L}\left[\frac{K \phi_s l}{A \mu}\right]$$

inserted, the result is

$$\left[\frac{\pi \mu A}{K \phi_s l}\right] I_{a\,avg} N_a = (2 \sin\theta - \sin\alpha_1) + (1 + \cos\alpha_1)\left(\frac{\alpha_1}{2} - \theta\right)$$

$$+ \frac{X}{R_L}(1 + \cos\alpha_1) \quad \ldots\ldots (56)$$

(see Appendix 2). It will be noted from equation (56) that the output current is independent of the feedback ratio, λ. The value of θ used in equation (56) may be obtained from equation (45) by letting $\omega t = \theta$ when $i_a N_a = 0$. This gives

$$2 \cos\theta = (1 + \cos\alpha_1) \quad \ldots\ldots (57)$$

If an expression for the average value of control ampere turns is written, the result is (see Appendix 3)

$$I_c N_c = \frac{1}{\pi}\int_0^{\pi} i_c N_c \, d(\omega t) \quad \ldots\ldots (58)$$

$$I_c N_c = \frac{1}{\pi}\int_0^{\theta}\left[\frac{\phi_{avg}}{\mu A}\, l - i_b N_b\right] d(\omega t) + \frac{1}{\pi}\int_{\theta}^{\alpha_1}\left[\frac{\phi_{avg}}{\mu A}\, l - i_b N_b\right] d(\omega t)$$

$$+ \frac{1}{\pi}\int_{\alpha_1}^{\pi}\left[(H_m \sin\omega t)(1-\lambda) - \frac{\phi_0\, l}{\mu A}\right] d(\omega t) \quad . \; . \; . \; (59)$$

By integration of equation (59) the result is

$$\frac{\mu A \pi}{K \phi_s l}\, I_c N_c = \left[\frac{\alpha_1}{2} - \pi\right]\left[1 - \cos\alpha_1\right] + \frac{\pi}{K}$$

$$- \lambda\left[\left(\frac{\alpha_1}{2} - \theta\right)(1 + \cos\alpha_1) + (2\sin\theta - \sin\alpha_1)\right]$$

$$+ (1-\lambda)\left[\frac{X}{R_L}(1 + \cos\alpha_1)\right] \quad . \; . \; . \; . \; . \; . \; . \; (60)$$

Equations (56) and (60) relate the average control current and the average output current. It should be noted that this is the average output current instead of the R.M.S. output current.

Some typical plots of output ampere turns, control ampere turns, and feedback ampere turns are shown in Figure 7.

There are two special cases of λ, namely $\lambda = 0$ and $\lambda = 1$, which represent the two extreme cases of self-excitation. When $\lambda = 0$, the case of no self-excitation exists. Conversely, when $\lambda = 1$, the number of turns in the feedback winding is the same as the number of turns on the output winding; and, since the currents in these two windings are equal, the mmf's are equal. When $\lambda = 0$, equation (60) reduces to

$$\left[\frac{\mu A \pi}{K \phi_s l}\right] I_c N_c = \left(\frac{\alpha_1}{2} - \pi\right)\left(1 - \cos\alpha_1\right) + \frac{\pi}{K} + \frac{X}{R_L}\left(1 + \cos\alpha_1\right) . \quad (61)$$

and when $\lambda = 1$

$$\left[\frac{\mu A \pi}{K \phi_s l}\right] I_c N_c = \Big[\sin\alpha_1 - 2 \sin\theta + (\pi + \theta - \alpha_1) \cos\alpha_1$$

$$- (\pi - \theta) + \frac{\pi}{K} \Big] \quad . . . . (62)$$

One method of expressing the circuit sensitivity is by the current amplification, i.e. $\frac{d I_{a\,avg}}{d I_c}$. This is done by taking the derivatives of equations (56) and (60) with respect to α_1 and dividing, i.e.

$$\frac{d[I_{a\,avg} N_a]}{d\alpha_1} \cdot \frac{1}{\frac{d[I_c N_c]}{d\alpha_1}} = \frac{d[I_{a\,avg} N_a]}{d[I_c N_c]}$$

This gives (see Appendix 4)

$$\frac{d\, I_{a\,avg} N_a}{d[I_c N_c]}$$

$$= \frac{\left(\frac{X}{R_L} + \frac{\alpha_1}{2} - \theta\right) \sin\alpha_1 - \frac{1}{2}(1 - \cos\alpha_1)}{\left[\frac{X}{R_L} + \pi - \frac{\alpha_1}{2}\right] \sin\alpha_1 + \frac{1}{2}(1 - \cos\alpha_1) - \lambda \left\{ \left[\frac{X}{R_L} + \frac{\alpha_1}{2} - \theta\right] \sin\alpha_1 - \frac{1}{2}(1 - \cos\alpha_1) \right\}} \quad . . . . (63)$$

It is obvious that this equation is much too complicated to be of much use. It would be very desirable if a more simplified equation expressing this ratio could be obtained.

By plotting equation (56) vs (60) it is easily seen that the plot of output ampere turns vs the control ampere turns for various values of K and X/R_L as parameters (see Figure 8) is nearly a straight line. By obtaining the mean slope of this curve an expression much less complicated than that given in equation (63) will result. It was stated previously that the angle α_1 is a function of the control current and may be increased or decreased at will by a variation of this current. It is obvious that the limits of α_1 are between 0 and π . For $\alpha_1 = 0°$, $\theta = 0°$ also, from the definition of θ given in equation (57). Similarly at $\alpha_1 = \pi$, $\theta = \frac{\pi}{2}$. Obtaining these end values from the two ampere turns equations: for $\alpha_1 = 0°$, $\theta = 0°$, in equation (60)

$$\left[\frac{\pi A \mu}{K \phi_s l}\right] I_c N_c = \frac{\pi}{K} + (1-\lambda)\frac{2X}{R_L} \triangleq X_2 \quad \ldots\ldots \quad (64)$$

and from equation (56)

$$\left[\frac{\mu A \pi}{K \phi_s l}\right] I_{a\,avg} N_a = \frac{2X}{R_L} \triangleq Y_2$$

For $\alpha_1 = \pi$ and $\theta = \frac{\pi}{2}$ we have

$$\left[\frac{\mu A \pi}{K \phi_s l}\right] I_c N_c = \frac{\pi}{K} - \pi - 2\lambda \triangleq X_1$$

and

$$\left[\frac{\mu A \pi}{K \phi_s l}\right] I_{a\,avg} N_a = 2 \triangleq Y_1$$

where the quantities X_1 , X_2, Y_1 , Y_2 are defined by their respective equations and are used to simplify the writing. These values are shown on Figure 8.

From Figure 8 the ampere turns ratio is obviously the slope of the line and is

$$\frac{y_2 - y_1}{x_2 - x_1}$$

This gives

$$\frac{I_{a\,avg} N_a}{I_c N_c} = \frac{2\frac{X}{R_L} - 2}{\left[\frac{\pi}{K} + (1-\lambda)\frac{2X}{R_L}\right] - \left[\frac{\pi}{K} - \pi - 2\lambda\right]}$$

$$= \frac{X/R_L - 1}{\left[X/R_L + \frac{\pi}{2}\right] + \lambda\left[1 - X/R_L\right]} \quad \ldots\ldots\ldots\ (65)$$

For the particular cases of $\lambda = 1$ and $\lambda = 0$ there results, for $\lambda = 0$

$$\frac{I_{a\,avg} N_a}{I_c N_c} = \frac{X/R_L - 1}{X/R_L + \frac{\pi}{2}} \quad \ldots\ldots\ldots\ (66)$$

and for $\lambda = 1$

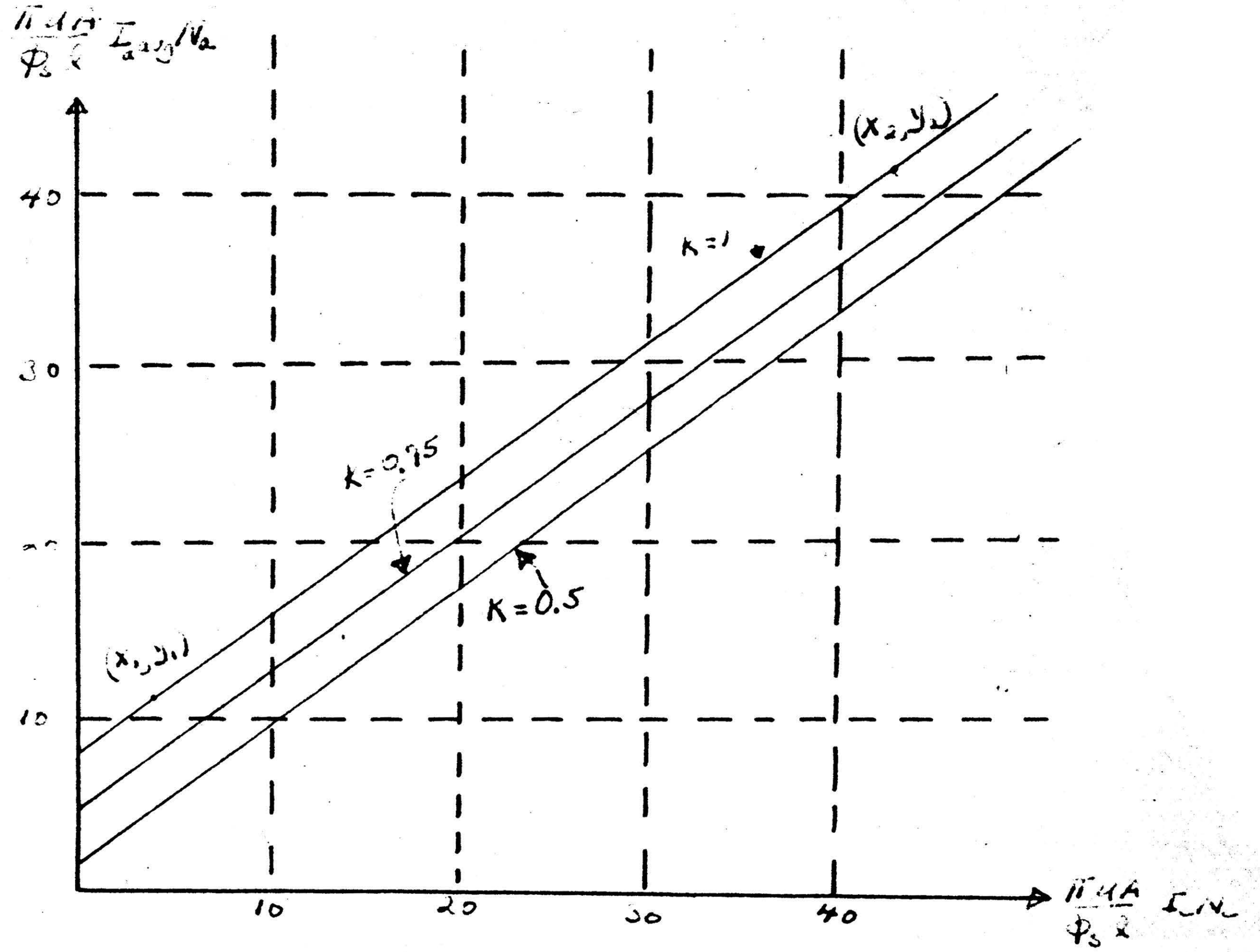
$$\frac{I_{a\,avg} N_a}{I_c N_c} = \frac{X/R_L - 1}{1 + \frac{\pi}{2}} \quad \ldots\ldots\ldots\ (67)$$

With a resistive load, R_L, in the output, the mean power gain, i.e. the ratio of average output power to average input power can be expressed as: for $\lambda = 0$

$$K_p = \frac{R_L}{R_c}\frac{N_c^2}{N_a^2}\left[\frac{X/R_L - 1}{X/R_L + \frac{\pi}{2}}\right]^2$$

and for $\lambda = 1$

$$K_p = \frac{R_L}{R_c}\frac{N_c^2}{N_a^2}\left[\frac{X/R_L - 1}{1 + \frac{\pi}{2}}\right]^2$$



Transfer Curve for Circuit
in Figure 4

Figure 8

MAGNETIC AMPLIFIERS AS FREQUENCY MIXERS

It is a well-known fact that most non-linear elements have mixing properties, the magnetic amplifier being no exception. Mixing is the ability of a device to convert two or more signals of different frequency into definite sums and differences of fundamental and harmonic frequencies. Every home radio, for instance, has a mixing device in it which reduces the higher frequency signals to lower frequency for the purpose of amplification.

The following is a duscussion illustrating the mixing properties of magnetic amplifiers. Consider a circuit like the one in the previous analysis where the B-H curve can now be represented by an expression such as

$$H = a'B + b'B^3 \quad \ldots\ldots \; (68)$$

It is obvious that a straight line B-H analysis would not yield any mixing characteristics since it is then a linear element, or $B \propto H$. Now assume that B for one core can be represented as

$$B_\alpha = B_0 + B_1 \sin \omega_1 t + B_2 \sin \omega_2 t \quad \ldots\ldots \; (69)$$

where ω_1 and ω_2 are two angular frequencies. B_0 is the mean value of flux density in the core, and B_1 and B_2 are the respective values of flux density associated with signals ω_1 and ω_2. That this assumption is not unreasonable may be seen from the previous analysis where the flux wave was a constant plus a sinusoidal variation. A possible block diagram of such a mixer is shown in Figure 9. The voltage,

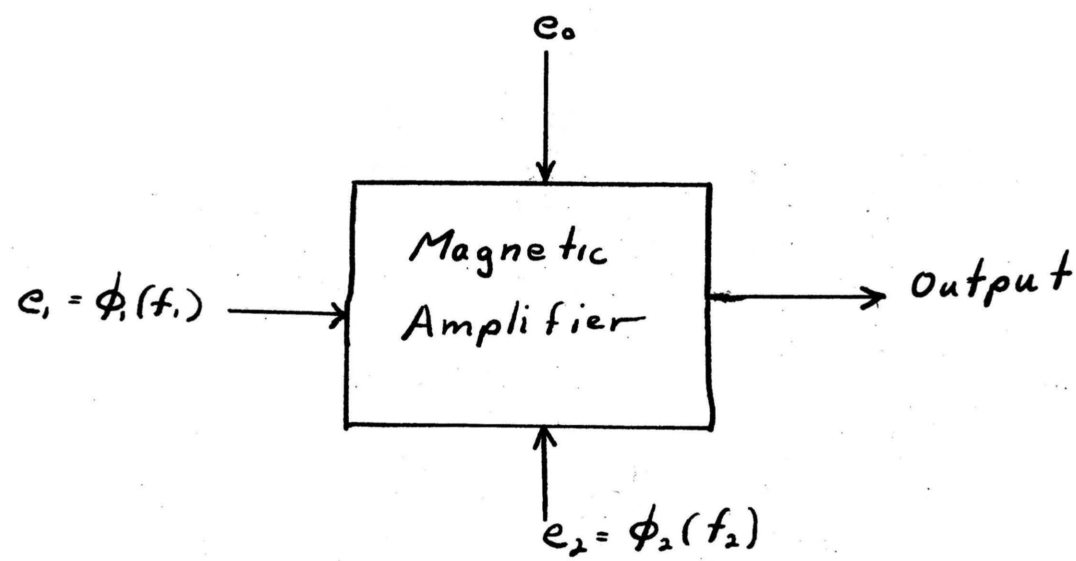


Figure 9

e_1, is a function, $\phi_1(f_1)$, of the frequency, f_1; and the voltage, e_2, is a function, $\phi_2(f_2)$, of the frequency, f_2. The voltage, e_o, is simply a d-c voltage associated with the B_o term and is a function of the d-c bias. Substituting equation (69) into equation (68) gives

$$H_\alpha = a'\left[B_o + B_1 \sin \omega_1 t + B_2 \sin \omega_2 t\right]$$

$$+ b'\left[B_o + B_1 \sin \omega_1 t + B_2 \sin \omega_2 t\right]^3 \quad \ldots \quad (70)$$

It will be sufficient to state that a mixing of frequencies must come about by a multiplication of terms rather than an addition or subtraction. With this in mind, equation (70) may be expanded as

$$
\begin{aligned}
H_\alpha = {} & a'B_0 + a'B_1 \sin \omega_1 t + a'B_2 \sin \omega_2 t \\
& + b'B_0^3 + b'B_1^3 \sin^3 \omega_1 t + b'B_2^3 \sin^3 \omega_2 t \\
& + 3b'B_0^2 B_1 \sin \omega_1 t + 3b'B_0 B_1^2 \sin^2 \omega_1 t \\
& + 3b'B_0^2 B_2 \sin \omega_2 t + 3b'B_0 B_2^2 \sin^2 \omega_2 t \\
& + 6b'B_0 B_1 B_2 \sin \omega_1 t \sin \omega_2 t \\
& + 3b'B_1^2 B_2 \sin^2 \omega_1 t \sin \omega_2 t \\
& + 3b'B_1 B_2^2 \sin \omega_1 t \sin^2 \omega_2 t \qquad \ldots\ldots (71)
\end{aligned}
$$

Only the last three terms of this expansion are products of different frequencies and have possibilities of mixing. These terms may be written separately as

$$6b'B_0 B_1 B_2 \sin \omega_1 t \sin \omega_2 t \qquad \ldots\ldots (72)$$

$$3b'B_1^2 B_2 \sin^2 \omega_1 t \sin \omega_2 t \qquad \ldots\ldots (73)$$

and

$$3b'B_1 B_2^2 \sin\omega_1 t \sin^2\omega_2 t \quad . \; . \; . \; . \; . \; . \; . \; (74)$$

By the use of certain trigonometric relations, equations (72), (73), and (74) may be rewritten respectively as equations (72a), (73a), and (74a). These are

$$6b'B_0 B_1 B_2 \left[\tfrac{1}{2}\cos(\omega_1-\omega_2)t - \tfrac{1}{2}\cos(\omega_1+\omega_2)t\right] \qquad .(72a)$$

$$3b'B_1^2 B_2 \left[\tfrac{1}{2}\sin\omega_2 t - \tfrac{1}{4}\left\{\sin(\omega_2+2\omega_1)t + \sin(\omega_2-2\omega_1)t\right\}\right] \qquad .(73a)$$

$$3b'B_1 B_2^2 \left[\tfrac{1}{2}\sin\omega_1 t - \tfrac{1}{4}\left\{\sin(\omega_1+2\omega_2)t + \sin(\omega_1-2\omega_2)t\right\}\right] \qquad .(74a)$$

If both signal windings are connected in the same way as the output winding of the previous analysis, then the total flux density for the other core may be written as

$$B_\beta = B_0 - B_1\sin\omega_1 t - B_2\sin\omega_2 t$$

and a similar expression for H_β may be derived.

The important thing is that with a certain type of flux wave, there exists definite mixing characteristics in this type of magnetic amplifier. It has just been shown that the frequencies, f_1 , f_2 , f_1+f_2 , f_1-f_2 , f_2+2f_1 , f_2-2f_1 , are present in the excitation wave of the core (where $\omega = 2\pi f t$). It may be said that, in general, with certain other types of circuits and inputs, different components of frequency will appear in different magnitudes.

MAGNETIC AMPLIFIERS USED FOR AUDIO AMPLIFICATION

It is possible to use magnetic amplifiers as audio amplifiers. The Navy is presently using one of these on one of their ships as a public-address system. It is a five-thousand watt amplifier, and a possible schematic diagram is shown in Figure 10.

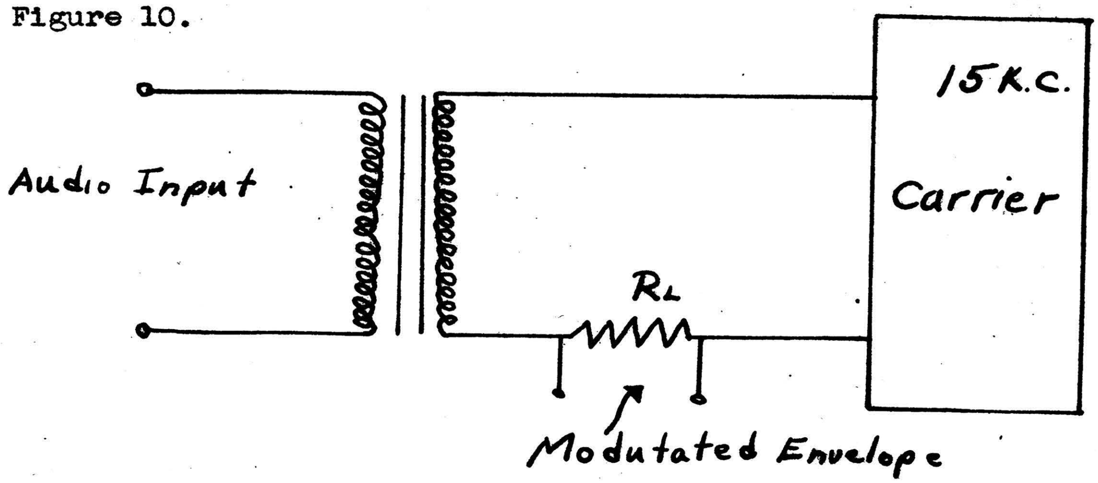


Figure 10

In the magnetic amplifiers so far considered, the control voltage has been assumed constant so that the envelope of the output voltage is a straight line. If, now, the control voltage is caused to vary at an audio rate, the envelope of the 15K.C. voltage across the output resistor will be varied at the audio rate. This, in principle, is the operation of a magnetic audio amplifier. The modulated output may be detected in the conventional method.

VACUUM TUBE VS MAGNETIC AMPLIFIER

A comparison of today's vacuum tubes with present-day magnetic amplifiers is no more justifiable than a comparison of the magnetic amplifiers of 1980 with vacuum tubes of 1920.

The progress of vacuum tubes is the result of many years of research and development. Magnetic amplifiers, as such, have just come into being in the past ten years and are awaiting similar years of research and development.

Magnetic amplifiers are not a suitable substitute for the vacuum tube; but they are, in many applications, a competitor.

Some special advantages of the magnetic amplifier are (1) a low stand-by power is possible, (2) no output transformer is needed, (3) no d-c power supply is required, (4) the amplifier is as rugged as a transformer, and (5) no warm-up time is required.

A COMPARISON OF THE POWER SERIES ANALYSIS WITH THE STRAIGHT LINE ANALYSIS

Comparing the results obtained in the two preceeding analyses, it is found that for a condition of $\lambda = 0$, or no self-excitation, the current gains compare quite favorably. The analysis for a power series B-H curve gives

$$\frac{I_{a\,avg}}{I_c} = \frac{N_c}{N_a}$$

for $\lambda = 0$ while the straight line analysis gives

$$\frac{I_{a\,avg}}{I_c} = \frac{N_c}{N_a}\left[\frac{X/R - 1}{X/R + \pi/2}\right]$$

However, the ratio $\frac{X}{R}$ is usually very large, equal to or greater than 10, so that the current gain, $\frac{I_{a\,avg}}{I_c}$, is equal to $\frac{N_c}{N_a}$ in the first case and almost so in the second case.

The current gain for a condition of $\lambda = 1$ is not as favorable since the power series gives a value of ∞ while the other does not. This is because at larger values of λ (close to 1) the magnetization current cannot be neglected as was done in the power series analysis. The magnetization current is that portion of the control ampere-turns which produces the working flux. The assumption of equal ampere turns is valid only if the saturating ampere turns is much smaller than the working ampere turns. As the control ampere turns approach zero, the output current can never be zero since this would require X to approach infinity. This, of course, is never realized. When the value of λ approaches one, this means that less and less control current is required since the majority of the d-c excitation is being produced by the feedback windings. The equivalence of ampere turns fails to hold at values of λ approaching 0.95.[13]

(13) Milnes, op. cit., p. 363.

The power gains for a condition of $\lambda = 0$ also show very close agreement. For the power series solution, a value of

$$K_p = \frac{R_L}{R_c}\left[\frac{N_c}{N_a}\right]^2$$

was obtained, while for the straight line B-H curve, a value of

$$K_p = \left[\frac{N_c}{N_a}\right]^2 \frac{R_L}{R_c}\left[\frac{X/R - 1}{X/R + \frac{\pi}{2}}\right]^2$$

was obtained. Again in this last equation if $X/R \geq 10$, the value of K_p in both cases becomes very nearly equal to

$$\frac{R_L}{R_c}\left[\frac{N_c}{N_a}\right]^2$$

Again the power gain for a condition of $\lambda = 1$ is unfavorable for the reason mentioned previously.

It is somewhat gratifying that there is any agreement whatsoever in these two types of analyses. There could hardly have been more difference in so far as the assumption of the original B-H curve is concerned. It will be noted, however, that the validity of the flux density variation which was assumed in the power series analysis holds true for the straight line analysis, namely that of a constant plus a sinusoidal variation. The difference, however, lies in the fact that it is impossible to determine just what components go into the making of B_0, the constant term of the power series solution, while the components of the constant term of flux are fully known in the straight line analysis.

In general, it seems that the assumption of a straight line B-H curve is more straightforward and yields more information than does the power series analysis. In the straight line assumption, it was possible to derive expressions showing flux and ampere-turn waveforms, while in the other solution it was not.

The power series analysis, however, should not be condemned as having no useful value. It was quite readily shown in this solution that all odd harmonics of output circuit supply frequency are rejected from the control circuit, while no even harmonics are allowed in the output circuit. This same condition holds true in any circuit of this type regardless of the B-H curve but is perhaps more easily seen in the power series B-H curve analysis.

Most present-day authors(14)(15)(16) assume a straight line B-H curve in any analytic discussion concerning magnetic amplifiers. Some(17) even go so far as to assume an infinite initial slope. This puts the saturation flux density at the value of zero excitation. This is the same as saying the core saturates with negligible excitation. In some special cases this may be quite accurate, depending, of course, on the core material and on what that particular analysis is supposed to show.

---

(14) Storm, op. cit., p. 756.

(15) Smith, op. cit., p. 1309.

(16) Lamm, A. Uno. Some Fundamentals of a Theory of the Transductor or Magnetic Amplifier. Transactions of the American Institute of Electrical Engineers. Vol. 66. p. 1078 (1947)

(17) Storm, op. cit., p. 756.

APPENDIX 1

$$H_\alpha = a'B + b'B^3 + c'B^5$$

$$B = B_m \sin \omega t + B_o$$

$$H_\alpha = a'(B_m \sin \omega t + B_o) + b'(B_m \sin \omega t + B_o)^3$$

$$+ c'(B_m \sin \omega t + B_o)^5$$

Multiplying and collecting like sin terms

$$H_\alpha = (a'B_o + b'B_o^3 + c'B_o^5) + (a'B_m + 3b'B_oB_m + 5c'B_o^4B_m) \sin \omega t$$

$$+ (3b'B_oB_m^2 + 10c'B_o^3B_m^2) \sin^2 \omega t$$

$$+ (b'B_m^3 + 10c'B_o^2B_m^3) \sin^3 \omega t + (5c'B_oB_m^4) \sin^4 \omega t$$

$$+ (c'B_m^5) \sin^5 \omega t$$

From trig. tables

$$\sin^2 \omega t = \tfrac{1}{2} - \tfrac{1}{2} \cos 2\omega t$$

$$\sin^3 \omega t = \tfrac{3}{4} \sin \omega t - \tfrac{1}{4} \sin 3\omega t$$

$$\sin^4 \omega t = \tfrac{3}{8} - \tfrac{1}{2} \cos 2\omega t + \tfrac{1}{8} \cos 4\omega t$$

$$\sin^5 \omega t = \tfrac{3}{8} \sin \omega t - \tfrac{1}{8} \sin 3\omega t - \tfrac{3}{16} \sin 3\omega t$$

$$+ \tfrac{3}{16} \sin \omega t + \tfrac{1}{16} \sin 5\omega t + \tfrac{1}{16} \sin \omega t$$

$$\sin^5 \omega t = \tfrac{5}{8} \sin \omega t - \tfrac{5}{16} \sin 3\omega t + \tfrac{1}{16} \sin 5\omega t$$

Collecting like terms

$$
\begin{aligned}
H_\alpha = {} & \left(a'\beta_0 + b'\beta_0^3 + c'\beta_0^5 + \tfrac{3}{2} b'\beta_0\beta_m^2 + 5c'\beta_0^3\beta_m^2 \right. \\
& \left. + \tfrac{15}{8} c'\beta_0\beta_m^4\right) \\
& + \left(a'\beta_m + 3b'\beta_0^2\beta_m + 5c'\beta_0^4\beta_m + \tfrac{3b'}{4}\beta_m^3 \right. \\
& \left. + \tfrac{15}{2} c'\beta_0^2\beta_m^3 + \tfrac{5c'}{8}\beta_m^5\right) \sin\omega t \\
& - \left(\tfrac{3}{2} b'\beta_0\beta_m^2 + 5c'\beta_0\beta_m^2 + \tfrac{5}{2} c'\beta_0\beta_m^4\right) \cos 2\omega t \\
& - \left(\tfrac{b'}{4}\beta_m^3 + \tfrac{5}{2} c'\beta_0^2\beta_m^3 + \tfrac{5}{16} c'\beta_m^5\right) \sin 3\omega t \\
& + \left(\tfrac{5}{8} c'\beta_0\beta_m^4\right) \cos 4\omega t + \left(\tfrac{c'\beta_m^5}{16}\right) \sin 5\omega t
\end{aligned}
$$

Similarly

$$
\begin{aligned}
H_\beta = {} & -\left(a'\beta_0 + b'\beta_0^3 + c'\beta_0^5 + \tfrac{3}{2} b'\beta_0\beta_m^2 + 5c'\beta_0^3\beta_m^2 \right. \\
& \left. + \tfrac{15}{8} c'\beta_0\beta_m^4\right) + \left(a'\beta_m + 3b'\beta_0^2\beta_m + 5c'\beta_0^4\beta_m \right. \\
& \left. + \tfrac{3b'}{4}\beta_m^3 + \tfrac{15}{2} c'\beta_0^2\beta_m^3 + \tfrac{5c'}{8}\beta_m^5\right) \sin\omega t \\
& + \left(\tfrac{3b'}{2}\beta_0\beta_m^2 + 5c'\beta_0^3\beta_m^2 + \tfrac{5c'}{2}\beta_0\beta_m^4\right) \cos 2\omega t \\
& - \left(\tfrac{b'}{4}\beta_m^3 + \tfrac{5}{2} c'\beta_0^2\beta_m^3 + \tfrac{5}{16} c'\beta_m^5\right) \sin 3\omega t \\
& - \left(\tfrac{5c'}{8}\beta_0\beta_m^4\right) \cos 4\omega t + \left(\tfrac{c'\beta_m^5}{16}\right) \sin 5\omega t
\end{aligned}
$$

APPENDIX 2

$$I_{a\,avg} N_a = \frac{1}{\pi}\int_0^{\theta} -i_a N_a \, d(\omega t) + \frac{1}{\pi}\int_{\theta}^{\alpha_1} i_a N_a \, d(\omega t)$$

$$+ \frac{1}{\pi}\int_{\alpha_1}^{\pi} i_a N_a \, d(\omega t)$$

The first and second integrals are evaluated by obtaining $i_a N_a$ from equation (45). The third integral is evaluated by substituting $i_a N_a$ from equation (50). For the first integral we have

$$\frac{1}{\pi}\int_0^{\theta} -\left\{\frac{l}{\mu A}\left[\frac{1}{2} k\phi_s (1+\cos\alpha_1) - k\phi_s \cos\omega t\right]\right\} d(\omega t)$$

$$= -\frac{l}{\mu A}\left[\frac{k\phi_s}{2}(1+\cos\alpha_1)(\omega t) - k\phi_s \sin\omega t\right]_0^{\theta}$$

$$= -\frac{l}{\mu A}\left[\frac{k\phi_s}{2}(1+\cos\alpha_1)\,\theta - k\phi_s \sin\theta\right]$$

For the second integral

$$\frac{1}{\pi}\int_{\theta}^{\alpha_1} i_a N_a \, d(\omega t) = \frac{l}{\mu A \pi}\int_{\theta}^{\alpha_1}\left[\frac{k}{2}\phi_s (1+\cos\alpha_1) - k\phi_s \cos\omega t\right] d(\omega t)$$

$$= \frac{l}{\mu A \pi}\left[\frac{K\phi_s}{2}(1+\cos\alpha_1)(\alpha_1 - \theta) - K\phi_s (\sin\alpha_1 - \sin\theta)\right]$$

For the third integral

$$\frac{1}{\pi}\int_{\alpha_1}^{\pi} H_m \sin\omega t \, d(\omega t)$$

$$= \frac{1}{\pi}\Big[-H_m \cos\omega t\Big]_{\alpha_1}^{\pi} = \frac{H_m}{\pi}\Big[1 + \cos\alpha_1\Big]$$

$$= \frac{X}{R_L}\left[\frac{K\phi_s l}{A\mu\pi}\right](1+\cos\alpha_1)$$

$$I_{a\,avg} N_a = \frac{X}{R_L}\frac{K\phi_s l}{A\mu\pi}(1+\cos\alpha_1) - \frac{K\phi_s l}{A\mu\pi}\frac{(1+\cos\alpha_1)\,\theta}{2}$$

$$+\frac{K\phi_s l}{A\mu\pi}\sin\theta + \frac{K\phi_s l}{A\mu\pi}\frac{(1+\cos\alpha_1)(\alpha_1-\theta)}{2}$$

$$-\frac{K\phi_s l}{A\mu\pi}(\sin\alpha_1 - \sin\theta)$$

$$\left[\frac{A\mu\pi}{K\phi_s l}\right] I_{a\,avg} N_a = \frac{X}{R_L}(1+\cos\alpha_1) - \frac{\theta}{2}(1+\cos\alpha_1) + \sin\theta$$

$$+\frac{(1+\cos\alpha_1)(\alpha_1-\theta)}{2} - (\sin\alpha_1 - \sin\theta)$$

$$= (2\sin\theta - \sin\alpha_1) + (1+\cos\alpha_1)\left(\frac{\alpha_1}{2}-\theta\right)$$

$$+\frac{X}{R_L}(1+\cos\alpha_1)$$

APPENDIX 3

$$I_c N_c = \frac{1}{\pi}\int_0^{\alpha_1}\left[\frac{\phi_{avg}\, l}{\mu A} - i_b N_b\right] d(\omega t)$$

$$+\frac{1}{\pi}\int_{\alpha_1}^{\pi}\left[(1-\lambda) H_m \sin\omega t - \frac{\phi_o l}{\mu A}\right] d(\omega t)$$

The portion $\frac{\phi_{avg}\, l}{\mu A}$ is equal to $i_c N_c + i_b N_b$ from equation (47) between $0-\alpha_1$; and when $i_b N_b$ is subtracted from this, it simply leaves $i_c N_c$. In the second integral, refering to equation (52), the value of $i_c N_c$ will be equal to

$$H_m \sin\omega t - \frac{\phi_o l}{\mu A} - i_b N_b$$

but, in general,

$$i_b N_b = \lambda (i_a N_a) = \lambda H_m \sin\omega t$$

The integral between $0-\alpha_1$ may be broken up into two integrals, one between $0-\theta$, the other between $\theta-\alpha_1$. So we can now write

$$I_c N_c = \frac{1}{\pi}\int_0^{\theta}\left[\frac{\phi_{avg}\, l}{\mu A} - i_b N_b\right] d(\omega t) + \frac{1}{\pi}\int_{\theta}^{\alpha_1}\left[\frac{\phi_{avg}\, l}{\mu A} - i_b N_b\right] d(\omega t)$$

$$+\frac{1}{\pi}\int_{\alpha_1}^{\pi}\left[(1-\lambda) H_m \sin\omega t - \frac{\phi_o l}{\mu A}\right] d(\omega t)$$

It may be seen from Figure 7a that from $0-\theta$, i_a is negative and from Figure 7c that i_b is positive. Since these currents are equal in magnitude but opposite in sign during the interval $0-\theta$, we may say that $i_a = -i_b$. Also, during $\theta - \alpha_1$, $i_a = i_b$. We may write now that

$$I_c N_c = \frac{1}{\pi}\int_0^{\theta}\left[\frac{\phi_{avg}\, l}{\mu A} + \lambda i_a N_a\right] d(\omega t) + \frac{1}{\pi}\int_{\theta}^{\alpha_1}\left[\frac{\phi_{avg}\, l}{\mu A} - \lambda i_a N_a\right] d(\omega t)$$

$$+ \frac{1}{\pi}\int_{\alpha_1}^{\pi}\left[(1-\lambda) H_m \sin\omega t - \frac{\phi_o l}{\mu A}\right] d(\omega t)$$

$$I_c N_c = \frac{l}{\mu A \pi}\int_0^{\theta}\left\{\left[\frac{\phi_s}{2}(2-K) + K\cos\alpha_1\right] + \lambda\left[\frac{K\phi_s}{2}(1+\cos\alpha_1) - K\phi_s\cos\omega t\right]\right\} d(\omega t)$$

$$+ \frac{l}{\mu A \pi}\int_{\theta}^{\alpha_1}\left\{\left[\frac{\phi_s}{2}(2-K) + K\cos\alpha_1\right] - \lambda\left[\frac{K\phi_s}{2}(1+\cos\alpha_1) - K\phi_s\cos\omega t\right]\right\} d(\omega t)$$

$$+ \frac{l}{\mu A \pi}\int_{\alpha_1}^{\pi}\left[\frac{K X \phi_s}{R_L}\sin\omega t + K\phi_s\cos\alpha_1 + (1-K)\phi_s\right] d(\omega t)$$

When this is integrated between the indicated limits

$$\left[\frac{\mu A \pi}{K\phi_s l}\right] I_c N_c = \left(\frac{\alpha_1}{2} - \pi\right)(1-\cos\alpha_1) + \frac{\pi}{K}$$

$$-\lambda\left[\left(\frac{\alpha_1}{2} - \theta\right)(1+\cos\alpha_1) + (2\sin\theta - \sin\alpha_1)\right]$$

$$+(1-\lambda)\left[\frac{X}{R_L}(1+\cos\alpha_1)\right]$$

APPENDIX 4

$$I_a \text{ avg } N_a \left[\frac{\mu A \pi}{K \phi_s l}\right] = (2 \sin\theta - \sin\alpha_1) + (1 + \cos\alpha_1)\left(\frac{\alpha_1}{2} - \theta\right)$$

$$+ \frac{X}{R_L}(1 + \cos\alpha_1)$$

$$\left[\frac{\mu A \pi}{K \phi_s l}\right] \frac{d[I_a \text{ avg } N_a]}{d\alpha_1} = -\cos\alpha_1 + \tfrac{1}{2}(1 + \cos\alpha_1)$$

$$+ \left(\frac{\alpha_1}{2} - \theta\right)(-\sin\alpha_1) + \frac{X}{R_L}(\sin\alpha_1)$$

$$= \frac{1}{2}(1 - \cos\alpha_1) + \left(\theta - \frac{\alpha_1}{2} - \frac{X}{R_L}\right)\sin\alpha_1$$

$$\left[\frac{\mu A \pi}{K \phi_s l}\right] I_c N_c = \left(\frac{\alpha_1}{2} - \pi\right)(1 - \cos\alpha_1) + \frac{\pi}{K}$$

$$- \lambda\left(\frac{\alpha_1}{2} - \theta\right)(1 + \cos\alpha_1)$$

$$- \lambda(2\sin\theta - \sin\alpha_1) + (1 - \lambda)\frac{X}{R_L}(1 + \cos\alpha_1)$$

$$\left[\frac{\mu A \pi}{K \phi_s l}\right] \frac{d[I_c N_c]}{d\alpha_1} = \left(\frac{\alpha_1}{2} - \pi\right)\sin\alpha_1 + \tfrac{1}{2}(1 - \cos\alpha_1)$$

$$- \lambda\left(\frac{\alpha_1}{2} - \theta\right)(-\sin\alpha_1) + (1 + \cos\alpha_1)\left(-\frac{\lambda}{2}\right)$$

$$+ \lambda\cos\alpha_1 - (1 - \lambda)\frac{X}{R_L}\sin\alpha_1$$

$$= \left(\frac{\alpha_1}{2} - \pi\right)\sin\alpha_1 + \frac{(1 - \cos\alpha_1)}{2} - \frac{X}{R_L}\sin\alpha_1$$

$$+ \lambda\left[\left(\frac{\alpha_1}{2} - \theta\right)\sin\alpha_1 - \frac{(1 + \cos\alpha_1)}{2}\right.$$

$$\left. + \cos\alpha_1 + \frac{X}{R_L}\sin\alpha_1\right]$$

$$\left[\frac{\mu A \pi}{K \phi_s l}\right] \frac{d(I_c N_c)}{d\alpha_1} = \left[\left(\frac{\alpha_1}{2} - \pi - \frac{x}{R_L}\right)\sin\alpha_1 + \frac{1}{2}(1-\cos\alpha_1)\right]$$

$$+ \lambda\left[\left(\frac{\alpha_1}{2} - \theta + \frac{x}{R_L}\right)\sin\alpha_1 - \frac{1}{2}(1-\cos\alpha_1)\right]$$

$$\frac{d(I_{a\,avg} N_a)}{d(I_c N_c)} =$$

$$= \frac{\left[\frac{x}{R_L} + \frac{\alpha_1}{2} - \theta\right]\sin\alpha_1 - \frac{1}{2}(1-\cos\alpha_1)}{\left\{\left[\frac{x}{R_L} + \pi - \frac{\alpha_1}{2}\right]\sin\alpha_1 - \frac{1}{2}(1-\cos\alpha_1)\right\} - \lambda\left\{\left[\frac{x}{R_L} + \frac{\alpha_1}{2} - \theta\right]\sin\alpha_1 - \frac{1}{2}(1-\cos\alpha_1)\right\}}$$

BIBLIOGRAPHY

Cohen, Sidney B. Analysis and Design of Self-Saturable Magnetic Amplifiers. Proceedings of the Institute of Radio Engineers. Vol. 39. pp. 1009-1020 (1951)

Dornhoefer, W. J. Self-Saturation in Magnetic Amplifiers. Transactions of the American Institute of Electrical Engineers. Vol. 68. pt. 2. p. 835 (1949)

Esselman, W. H. Steady State Analysis of Self-Saturating Magnetic Amplifiers Based on Linear Approximations of the Magnetization Curve. Transactions of the American Institute of Electrical Engineers. Vol. 70. pt. 1. p. 455 (1951)

Finzi, L. A., Beaumariage, D. C. General Characteristics of Magnetic Amplifiers. Transactions of the American Institute of Electrical Engineers. Vol. 69. pt. 2. p. 919 (1950)

Gale, H. M., Atkinson, P. D. Theoretical and Experimental Study of Series Connected Magnetic Amplifiers. Proceedings of the Institute of Electrical Engineers. Vol. 96. pt. 1. pp. 99-114 (1949)

Lamm, A. Uno. Some Fundamentals of a Theory of the Transductor or Magnetic Amplifier. Transactions of the American Institute of Electrical Engineers. Vol. 66. p. 1078 (1947)

Milnes, A. G. A New Theory on the Magnetic Amplifier. Proceedings of the Institute of Electrical Engineers. Vol. 97. pp. 460-497 (1950)

____________ Magnetic Amplifiers. Proceedings of the Institute of Electrical Engineers. Vol. 96. p. 329-338. (1949)

Reynor, J. H. The Magnetic Amplifier. 1st edition. Stewart and Richards, London. 1948. 120 p.

Smith, E. J. Self-Saturating Magnetic Amplifiers. Transactions of the American Institute of Electrical Engineers. Vol. 69. pt. 2. pp. 1309-1317 (1950)

Storm, H. F. Series Connected Saturable Reactor with Control Source with Comparatively Low Impedance. Proceedings of the American Institute of Electrical Engineers. Vol. 69. pt. 2. pp. 756-765 (1950)

Ver Planck, D. W., Finzi, L. A., Beaumariage, D. C. Analytical Determination of the Characteristics of Magnetic Amplifiers with Feedback. Proceedings of the American Institute of Electrical Engineers. Vol. 68. pt. 1. pp. 565-570 (1949)

VITA

The writer was born August 22, 1928 in Springfield, Missouri. He is the son of Mr. and Mrs. Wiley T. Ruhl, Sr. of the same city. His elementary education was obtained in the public schools of Springfield. He graduated from Springfield Senior High School in May, 1946 and enrolled in Southwest Missouri State College in September, 1946. In September, 1948 he enrolled in Missouri School of Mines and Metallurgy and received the degree of Bachelor of Science in Electrical Engineering in May, 1951.

During the summer months of 1951, the author was employed with the Westinghouse Corporation. He returned to Missouri School of Mines and Metallurgy in September, 1951 where he was employed as graduate assistant while completing the requirements for the Master of Science Degree in Electrical Engineering.