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Harry A. Brown
Missouri University of Science and Technology

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On the Ising model for amorphous ferromagnets

H A Brown
Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65401, USA

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Abstract. A calculation is made of the effect on the critical temperature of a ferromagnet whose exchange integrals are taken to be random variables. The Bethe–Peierls–Weiss approximation is used to find the partition function, Z, for an Ising system and then a configuration average is taken to find $\langle \ln Z \rangle$, from which the resulting decrease in the critical temperature can be found. Other results are discussed and comparisons made with the present result.

There are several approaches to the theory of amorphous ferromagnets. An extensive discussion of the bond-disordered Ising model has been given by Harris (1974). His results show that the effect of treating the exchange integral as a random variable is to lower the critical temperature by an amount proportional to the second moment of the distribution of the bond strengths.

Qualitatively similar results have been found from other theories (Kobe and Handrich 1972, Katsura and Shimada 1980, Schreiber *et al* 1975).

In general, these approximate treatments show a decrease in the critical temperature as the amorphous nature of the material increases. A rigorous result by Falk and Gehring (1975) shows that this must be the case for bond-disordered Ising ferromagnets.

The quantitative expression for this decrease is commonly written as

$$\delta T_c / T_c^0 = -A\Delta^2.$$

where T_c^0 is the critical temperature of the ordered system and $\delta T_c = T_c - T_c^0$ while $\Delta^2 = \langle \Delta J^2 \rangle / \langle J \rangle^2$ represents the mean square deviation in the distribution of the random exchange integrals $J_{ij} = \langle J \rangle + \delta J_{ij}$, the angular brackets indicating a configuration average.

Of the works mentioned above when applied to the simple cubic lattice, Harris finds $A = 0.073$, Kobe and Handrich, and Katsura and Shimada find $A = 0.04$, and Schreiber *et al* find a value $A = 0.025$, all small but with the decrease in T_c required by Falk and Gehring's work.

The methods employed were various: 'a resummation of perturbation theory' using cumulants by Harris. The constant-coupling approximation by Kobe and Handrich and Katsura and Shimada while Schreiber *et al* used the high-temperature expansion results of Rushbrooke and Wood (1968) with random Js.

Another approximation, the Bethe–Peierls–Weiss method, has been applied to the problem of the amorphous Ising ferromagnet (Bokák 1980) and the critical temperature is found to increase ($A = -0.008$ for a simple cubic) contradicting the other results.

Bokák's result follows from a peculiar configuration average taken of the exchange and internal field parts of the Hamiltonian instead of the free energy (Brout 1959, Brown and Jursich 1979, Brown 1979).

In this brief note, we would like to present the result of taking the proper configuration average of $\ln Z$ for an amorphous Ising system using the BPW method and show that δT_c is indeed negative.

The BPW Hamiltonian for an amorphous Ising ferromagnet is

$$H = -\tfrac{1}{2}\sum_{i=1}^{n} J_{0i}\sigma_0\sigma_i - \tfrac{1}{2}\mu \sum H_i\sigma_i - \tfrac{1}{2}\mu H_0\sigma_0$$

where the σs $= \pm 1$, J_{0i} is the random exchange integral between the central site in the cluster and its ith nearest neighbour, H_i an also random internal field acting on that site, H_0 an applied field, and n the coordination number.

The partition function that immediately follows is

$$Z = e^{\lambda_0}\prod_{i=1}^{n} 2\cosh(\lambda_i + b_i) + e^{-\lambda_0}\prod_{i=1}^{n} 2\cosh(\lambda_i - b_i),$$

where

$$\lambda = \tfrac{1}{2}\beta\mu H \qquad b = \tfrac{1}{2}\beta J \qquad \beta = 1/kT.$$

Now we form $\langle \ln Z\rangle$, the configuration average indicated by the angular brackets:

$$\langle \ln Z\rangle = \langle \ln Z\rangle_0 + \langle \partial \ln Z/\partial\beta\rangle_0 \delta\beta + \tfrac{1}{2}n(\beta/2)^2 \times [(\partial^2 \ln Z/\partial b^2)_0 \langle \delta J^2\rangle + \mu^2(\partial^2 \ln Z/\partial\lambda^2)_0\langle \delta H^2\rangle]$$

using the fact that $\langle \delta J_{0i}\rangle = 0 = \langle \delta H_i\rangle$, and $\langle \delta J_{0i}\delta H_i\rangle = 0$, the latter because of the assumed independence of the variations in J and H. In this Taylor expansion the subscript zero indicates that those quantities are evaluated for all variations zero in J_{0i} or H_i.

We make the assumption that $\langle \delta J^2\rangle/\langle J\rangle^2 = \langle \delta H^2\rangle/\langle H\rangle^2$ or that whatever distribution of Js is assumed, a similar distribution of Hs obtains. Since the internal fields are the result of the exchange interactions between the first shell sites and those more distant, these should be assumed to have a random behaviour similar to that in the cluster.

After a considerable amount of algebra, we find that

$$\frac{\delta T_c}{T_c^0} = -\frac{n}{2}\left(\frac{b_c^0}{n-1}\right)^2 \left(\frac{2n(n-1) - (6n-8)}{(2n^2-1) - 4b_c^0(n-1)}\right)\frac{\langle \delta J^2\rangle}{\langle J\rangle^2} = -A\Delta^2$$

$$= -0.0115\Delta^2 \qquad \text{for } n = 6$$

$$= -0.0068\Delta^2 \qquad \text{for } n = 8$$

and $b_c^0 = \langle J\rangle/2kT_c^0$ is the root of the consistency condition which fixes the critical temperature for the uniform material:

$$\frac{\delta\langle \ln Z\rangle_0}{\partial\lambda_0} = \frac{1}{n}\frac{\delta\langle \ln Z\rangle_0}{\partial\lambda}, \qquad \text{for } H_0 = 0$$

namely, $\tanh b_c^0 = 1/(n-1)$.

So the critical temperature of the amorphous Ising ferromagnet does indeed show a decrease according to the BPW approximation if the configuration average is taken in the proper way and this decrease is also about the right size.

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