01 Jan 1984

A Three-spin Interaction Model

Harry A. Brown
Missouri University of Science and Technology

Journal of Magnetism and Magnetic Materials 43 (1984) L1–L2
North-Holland, Amsterdam

LETTER TO THE EDITOR

A THREE-SPIN INTERACTION MODEL

H.A. BROWN

Physics Department, University of Missouri-Rolla, Rolla, MO 65401, USA

Received 5 March 1984

A three spin interaction in the form of a box product, $\sigma_1 \times \sigma_2 \cdot \sigma_3$, is considered. The thermodynamic behavior is investigated using an effective field theory. A second-order phase transition is found and the magnetization and energy per spin appear to be very similar to those of an ordinary Heisenberg ferromagnet.

The simple scalar product Heisenberg exchange interaction has been found inadequate to explain some observed properties of magnetic materials and modifications of the Hamiltonian have been suggested recently in the form of multi-spin interactions [1–6].

In this note, we consider the thermodynamic behavior of a three-spin interaction in the form of a box product of spin-1/2 particles. The molecular-field-theory Hamiltonian for the three spins is assumed to be

$$H = -J[\sigma_1\sigma_2\sigma_3] - \mu H \sum_{i=1}^{3} \sigma_{iz}, \tag{1}$$

where

$$[\sigma_1\sigma_2\sigma_3] = \sigma_1 \cdot \sigma_2 \times \sigma_3.$$

From the 8×8 determinant for the eigenvalues, the partition function is found to be

$$Z = 4 \cosh c(1 + \cosh x), \tag{2}$$

where

$$c = \beta\mu H, \quad x = 2\sqrt{3b^2 + c^2}, \quad b = \beta J, \quad \beta = 1/kT. \tag{3}$$

The average values of the magnetization $(\mu\sigma)$ and the energy are found from

$$\sigma = -\tfrac{1}{3}\partial \ln Z/\partial c = \langle \sigma_3 \rangle \tag{4}$$

and

$$\epsilon = -\partial \ln Z/\partial\beta = \langle H \rangle \tag{5}$$

and are shown in fig. 1. The calculations involve the usual assumption that the internal field is proportional to the magnetization:

$$\mu H = (n-2) J\sigma, \tag{6}$$

where n is the coordination number of the lattice.

The magnetization and the energy behave like those of a Heisenberg ferromagnet. The transition temperatures are shown in table 1. The reciprocal susceptibility, as shown in the figure, is essentially a straight line obeying the Curie–Weiss law with an asymptotic Curie temperature given by

$$\theta = (n-2)J/k \approx T_c.$$

So, the present calculation leads to a very simple result: paramagnetism above T_c and ferromag-

Table 1

Reduced critical temperature, kT_c/J, for various lattices; n is the coordination number

n	kT_c/J
4	1.7
6	3.9
8	5.9
12	9.9

0304-8853/84/$03.00 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

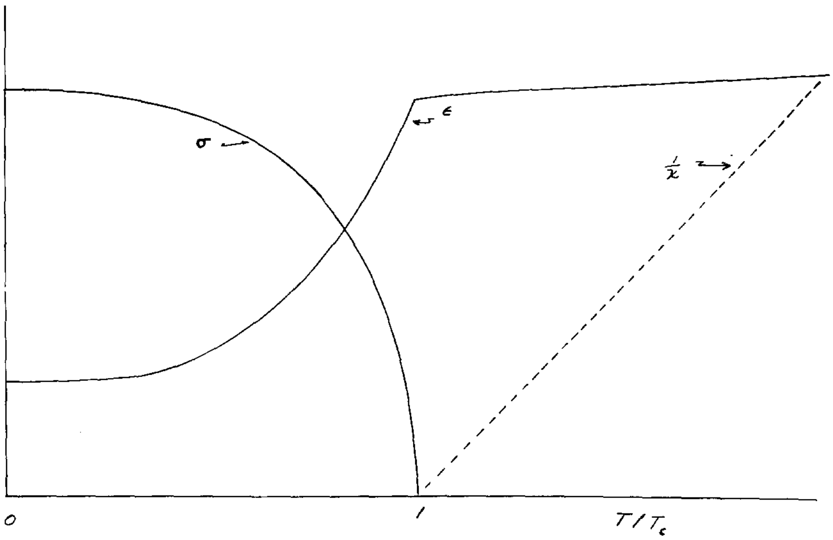

Fig. 1. The average magnetization per spin, σ, the average energy, ϵ, and the reciprocal susceptibility, $1/\chi$, as functions of reduced temperature, for a body-centered cubic lattice, $n = 8$, all in arbitrary units.

netism below with a second-order phase transition at T_c. (σ and ϵ are continuous at T_c, but their derivatives are not.) There is nothing in this behavior different from that of the usual Heisenberg ferromagnet, a rather unexpected result.

Extending to negative J to include the possibility of competing interactions changes the 8×8 matrix enough to make the resulting algebraic problem messy. The matter is under investigation.

Consideration is also being given to spin-1 interactions and the 27×27 Hamiltonian that arises for that case.

Using the present approximation, the Ising model shows no transitions for either positive or negative J. Transitions have been found using better approximations [3–6], but for somewhat different Hamiltonians than the Ising equivalent of (1), generally. These results may indicate the limitations of the molecular field theory applied to the Ising model. Of course, the Ising spin product $\sigma_1\sigma_2\sigma_3$ is quite different from the box product of Pauli spin matrices in (1), and it is that Hamiltonian which is the subject of this note.

So far as I know the box product interaction has not been reported in the literature. The three-spin interactions investigated by Penson [3,5] and Baxter and Wu [6] considered triple products of Ising spins. Other calculations [1,2] involve the three spins in two scalar products in the form $(S_1 \cdot S_2)(S_2 \cdot S_3)$.

It is interesting to note that the commutator $[(S_1 \cdot S_2),\ (S_2 \cdot S_3)]$ is proportional to the box product $[S_1 S_2 S_3]$ (with a factor i) and that such commutators arise in expansions of the exponential $e^{-\beta H}$, where $H = -2JS_i \cdot S_j$, in powers of βH.

References

[1] J. Aksamit and B. Westwanski, J. Phys. C 14 (1981) 255. J. Aksamit, J. Phys. C 13 (1980) L871.

[2] M. Rogers, J.H. Hetherington and J.M. Deinen, Rev. Mod. Phys. 55 (1983) 1.

[3] K.A. Penson, R. Jullien and P. Pfeuty, Phys. Rev. B 26 (1982) 6334.

[4] D.C. Mattis and R. Galler, Phys. Rev. B 27 (1983) 2894.

[5] K.A. Penson, Phys. Rev. B 29, in press.

[6] R.J. Baxter and F.Y. Wu, Phys. Rev. Lett. 21 (1973) 1294.