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## ANALYTICAL AND EXPERIMENTAL INVESTIGATION ON VIBRATION CONTROL OF PIEZOELECTRIC STRUCTURES

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### ABSTRACT

Active vibration control of smart structural materials (beam and plate) has been achieved by using distributed piezoelectric actuators. Numerical simulation and experimental testing have been conducted to investigate vibration suppression of the advanced structures. Results from simulation and testing are in satisfactory agreement.

### INTRODUCTION

Vibration control of flexible structures such as large spacecraft or satellite has been observed to be a good way to maintain structural integrity as well as to optimize performance. In recent years, very efficient manufacturing systems and micro machinery with tiny parts have begun rapid development. In order to overcome vibration problem during operation, application of this technique is necessary.

Lots of researches in the field of vibration control are concentrated on smart beam systems. In practical applications to large space structures or satellites, two dimensional elements or plates are the principal parts to form dynamic systems. However, only a few statements concerning with vibration control of a two dimensional structure have been presented up to now. In this paper, structures including beams and plates will be investigated. Both simulation and experiment will be performed to verify effectivenesses of several control systems.

There are many kinds of device system using for active vibration control, such as viscoelastic hydraulic and thermal gradient actuators, and so on. In this research, the piezo films made of piezoelectric polymer by Pennwalt Corporation under the name "Kynar" were used as sensors and actuators. The theory of piezoelectric isolator with general electric and mechanical boundary conditions was developed by Tzou [1]. An experiment was performed to validate the theory. A distributed active piezoelectric actuator was applied to directly constrain the motion of flexible structure by the same author [2]. The analytical, experimental and finite element method were used to evaluate the effectiveness of the control system.

The experimental results of active control of an intelligent structure with large number of active elements were presented by de Luis and Crawley [3]. A functional analysis formulation that assumed spatially continuously varying actuators and sensors was used to

develop optimal controllers. An intelligent composite beam with 32 embedded piezoelectric was constructed as the prototype testing structure. The results were in satisfactory agreement with those obtained from the finite element method.

An analytical technique with experimental results for using the piezo film as elements of intelligent structures was presented by Crawley, et al.[4,5], mechanical models for the interaction of piezoelectric with the structures as well as the coupling of electrical and mechanical properties were presented. The manufacturing technique for embedding piezoelectric materials inside the laminated composite was developed. The technique permitted the electrical insulation of the piezoelectric without reducing its effectiveness as an actuator.

The researchers of the Charles Stark Draper Laboratory and MIT have presented lots of papers concerning about active control of flexible structures like elements of scale model of a satellite or multi-component structures. The governing equation as well as the boundary conditions of a scale satellite component involved the control voltage generated by piezoelectric actuators has been derived by Bailey and Hubbard [6,7]. The Lyapunov second method was used to derive the control algorithm of this cantilever beam system, the tip angular velocity of beam was found the only control parameter to perform the all mode control without any truncation. The computer simulation and experimental results were presented to evaluate the effectiveness of the control system. It indicated that the control law derived was effective in damping out the vibration of a structure.

Active vibration control of smart beam systems were investigated through the numerical simulation and experimental testing by Wu and Tzeng [8]. The governing equation as well as boundary conditions were derived based on the equilibrium equation. This approach had some advantages to simplify the derivation and to make the numerical simulation more concise than other approaches. Properties of piezoelectric films as a sensor and an actuator were developed for experimental observations. Results from simulations as well as experimental analysis were in good agreement.

A vibration control strategy for multi-component structures has been developed by Miller and Hubbard [9]. Each component of the structure was an active controllable beam member, the control algorithm was developed by using Lyapunov direct method and was verified on a structure constructed from three rigidly joint autonomous smart components.

The investigation of beams with general boundary conditions and non-uniform spatial distributions was completed by Burke and Hubbard [10]. The analysis declared that while for most boundary conditions a spatially uniform control was appropriate, pinned-pinned, free-free, clamped-clamped, clamped-sliding beams required non-uniform spatial distribution to be controllable.

Practical consideration in the design as well as digital implementation of active vibration control by a proof-mass actuator has been investigated by Zimmerman and Cudney [11]. Control strategies were implemented by using a digital control system. The effect of quantization due to the finite wordlength of microprocessors, analog-to-digital, and digital-to-analog converters, on the desired control law was studied. Also, some experimental results of an active vibration control system were reported.

The category of vibration control of flexible structures was extended from one dimensional to two dimensional structures by Tzou [12,13]. The analytical and finite element study on distributed piezoelectric sensor and actuator coupled with flexible shell and plates were presented. Based on Maxwell's equation and Love's assumption, new theories on distributed sensing and active vibration control of a general shell were derived. These theories can easily be simplified for plates.

In this paper, a balanced analytical and experimental approach is undertaken. Active vibration control of smart beam and plate subjected to impulsive loading are simulated numerically and tested experimentally. Comparisons of results are presented.

## THEORETICAL ANALYSIS

For an unbonded piece of PVDF film, the effect of an applied voltage  $V(x,t)$  is to cause a strain  $\epsilon_p(x,t)$  which is given by

$$\epsilon_p(x,t) = \frac{d_{31}}{h_p} V(x,t) \quad (1)$$

where  $d_{31}$  is the strain constant and  $h_p$  is the thickness of the film. If the actuator is perfectly bonded to the surface of a beam, this piece of constrained piezo film cannot stretch freely. When the voltage  $V(x,t)$  is applied, instead of strain, a surface traction  $q(x,t)$  is exerted and can be expressed as

$$q(x,t) = E_p b d_{31} V(x,t) \quad (2)$$

where  $E_p$  is the Young's modulus,  $b$  is the width of the piezo film. By defining the constant  $C_1 = E_p b d_{31}$ , Equation (2) becomes

$$q(x,t) = C_1 V(x,t) \quad (3)$$

where  $C_1$  is a constant which stands for the surface traction exerted by the piezo film when a unit voltage is applied.

The governing equation of a flexible beam with a surface bonded piezoelectric actuator is derived based on the equilibrium equations and is expressed as

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = d \frac{\partial^2 q}{\partial x^2} \quad (4)$$

where  $d$  is the distance between neutral axis and the applied traction. The surface traction,  $q(x,t)$ , is generated by the bonded piezo film, so that the contribution of stiffness and mass from this film must be considered. Assuming that the bonding layer between the film and the beam is very thin when compared with the thickness of the film and the beam, then

$$\begin{aligned} EI &= E_1 I_1 + E_p I_p \\ \rho A &= \rho_1 A_1 + \rho_p A_p \\ d &= h_1 + h_p/2 - D \end{aligned} \quad (5)$$

where "1" stands for isotropic material and "p" stands for the piezo film,  $h$  is the thickness and  $D$  is the location of neutral axis. By introducing Equation (3) into Equation (4) yields.

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = C \frac{\partial^2 V}{\partial x^2} \quad (6)$$

where  $C$  is the torque produced per unit applied voltage, it can be expressed as

$$C = \frac{E_1 h_1 E_p d_{31} b (h_1 + h_p)}{2(E_1 h_1 + E_p h_p)} \quad (7)$$

The governing equation of a plate with surface bonded piezoelectric actuator is derived by using the equilibrium equations and can be expressed as

$$\bar{D}(\nabla^4 w) + \rho \frac{\partial^2 w}{\partial t^2} = -d \left( \frac{\partial^2 q_x}{\partial x^2} + \frac{\partial^2 q_y}{\partial y^2} \right) \quad (8)$$

where  $q_x$  and  $q_y$  are the surface tractions exerted by the piezo film on  $x$  and  $y$  direction, respectively. Assuming that the plate has the dimension with length  $a$  and width  $b$ , converting  $q_x$  and  $q_y$  into the applied voltage will give

$$q_x = E_p b d_{31} V \quad (9)$$

$$q_y = E_p a d_{32} V \quad (10)$$

Defining constants  $c_x = -d E_p b d_{31}$  and  $c_y = -d E_p a d_{32}$ , Equation (8) becomes the concise form as

$$\bar{D}(\nabla^4 w) + \rho \frac{\partial^2 w}{\partial t^2} = c_x \frac{\partial^2 V}{\partial x^2} + c_y \frac{\partial^2 V}{\partial y^2} \quad (11)$$

where Equation (11) is the governing equation of the smart plate which contains the terms contributed by the applied control voltage  $V(x,y,t)$ . In view of Equations (6) and (11), for uniformly distributed control systems where  $V(x,t)$  and  $V(x,y,t)$  are not spatial functions, the control effect results from boundaries only. However, for non-uniformly distributed systems with irregular piezo films, vibration control could be achieved locally.

## THE CONTROL ALGORITHM

Lyapunov's second or direct method was used to derive the control algorithm of this distributed control system. Assuming that there is some practical limit on the magnitude of control voltage  $V(t)$ , i.e.,

$$|V(t)| \leq V_{\max} \quad (12)$$

For an intelligent beam, one can define a functional  $F$  which represents the total energy of the system. This control was chosen to minimize the time rate of change of the function at every point in time. An appropriate function that may resemble the energy of the system as well as the boundary conditions is defined as

$$F(t) = \frac{1}{2} \int_0^l \left[ \left( \frac{\partial^2 y}{\partial x^2} \right)^2 + \left( \frac{\partial y}{\partial t} \right)^2 \right] dx + \frac{1}{2} M_t \left[ \frac{\partial y(\zeta, t)}{\partial t} \right]^2 + \frac{1}{2} I_t \left[ \frac{\partial^2 y(\zeta, t)}{\partial x \partial t} \right]^2 + \frac{1}{2} \kappa_t \left[ \frac{\partial y(\zeta, t)}{\partial x} \right]^2 + \frac{1}{2} K_t [\zeta, t]^2 \quad (13)$$

The terms outside the integral are lumped energy storage elements that can arise from the boundary conditions. The constants  $M_t$ ,  $I_t$ ,  $\kappa_t$ ,  $K_t$  stand for boundary point mass, point inertia, rotary spring stiffness and linear spring stiffness, respectively.  $\zeta$  represents the boundary point  $x = 0$  or  $x = l$ .

The time derivative of the functional expressed in Equation (13) incorporated with the governing equation and then integrated by parts twice yields

$$\frac{\partial F}{\partial t} = \int_0^l \frac{\partial^2 y(x, t)}{\partial x^2 \partial t} V(x, t) dx - \beta \left[ \frac{\partial^2 y(\zeta, t)}{\partial x \partial t} \right]^2 - \gamma \left[ \frac{\partial y(\zeta, t)}{\partial t} \right]^2 \quad (14)$$

where  $\beta$  is the rotary viscous damping coefficient and  $\gamma$  is the linear viscous damping coefficient.

For a system with spatially uniform control  $V(x, t) = V(t)$ , performing the integration of equation (14) yields

$$\frac{\partial F}{\partial t} = \frac{\partial^2 y(x, t)}{\partial x \partial t} \Big|_0^l V(t) - \beta \left[ \frac{\partial^2 y(x, t)}{\partial x \partial t} \right]^2 - \gamma \left[ \frac{\partial y(x, t)}{\partial t} \right]^2 \quad (15)$$

Substituting the boundary conditions into Equation (15), the control voltages to extract vibration energy from the system for beams with different boundary conditions are available. For example,

**Fixed - Fixed Beam:** when the boundary conditions were introduced into Equation (15), the control voltage is not presented in this equation. This means the system cannot be controlled by using a spatially uniform control, so nonuniformly distributed control must be considered.

**Fixed - Free Beam:** the control voltage  $V(t)$  is chosen to make Equation (15) as negative as possible. This will always extract energy from the system, i.e.,

$$V(t) = -\operatorname{sgn} \left( \frac{\partial^2 y}{\partial x \partial t} \Big|_{x=l} \right) V_{\max} \quad (16)$$

**Fixed - Simply Supported Beam:** By substituting the boundary condition into Equation (15), the control voltage  $V(t)$  can be chosen as Equation (16).

**Beam with Both Ends Simply Supported:** To make Equation (15) as negative as possible,  $V(t)$  can be chosen as

$$V(t) = -\operatorname{sgn} \left( \frac{\partial^2 y}{\partial x \partial t} \Big|_{x=l} - \frac{\partial^2 y}{\partial x \partial t} \Big|_{x=0} \right) V_{\max} \quad (17)$$

The term  $\frac{\partial^2 y}{\partial x \partial t} \Big|_{x=\zeta}$  is the angular velocity at the boundary of the beam. The control parameter listed in Equations (16) and (17) can be modified so that the control voltage can be generated by the feedback loop and will never exceed a practical limit. Assuming that a positive number  $G$  exist which stands for the system gain of the control loop, so that

$$V(t) = - \left( \frac{\partial^2 y}{\partial x \partial t} \Big|_{x=\zeta} \right) \cdot G \quad (18)$$

The control voltage  $V(t)$  can not exceed the practical limit as mentioned in Equation (12). So a proper gain  $G$  must be found by satisfying the follow equation:

$$V_{\max} = G \left( \frac{\partial^2 y}{\partial x \partial t} \Big|_{x=\zeta} \right)_{\max} \quad (19)$$

Since the proper control algorithm for plate system has not been derived yet, the similar control parameters as used for beam system will be utilized to perform the vibration suppression of the structure.

## NUMERICAL SIMULATION

A structure subjected to a distributed surface traction can be modified as a structure subjected to an axial loading and a distributed moment. The deflection of the beam in the axial direction due to axial loading is negligible because the stiffness is much greater than the transverse rigidity. Since the contribution of the geometric stiffness generated by this axial load is very slightly, it also will be neglected.

The equation of motion for this system can be expressed as

$$[M] \{y\} + [C] \{y\} + [K] \{y\} = [F] \quad (20)$$

where  $[M]$  is the global mass matrix;  $[C]$  is the passive structural matrix;  $[K]$  is the total stiffness matrix; and  $[F]$  is the forcing vector, combining the external excitation and the bending moment resulted from the piezoelectric surface traction.

In general, the damping matrix  $[C]$  in the equation of motion is determined experimentally. In order to incorporate the damping data into the analysis, the equation of motion is transferred from physical space to modal space.

The flowchart used to perform the computer simulation of this active control system is presented in Figure 1. To solve the modal equation of motion, the linear acceleration step-by-step method with Wilson- $\theta$  modification was used. In this simulation program, some of the subroutines are called from IMSL computer library (International

Mathematical & Statistical Library) which are very easy and convenient use. Since the rotational degree of freedom was involved, the control voltage to suppress the vibration of the system is equal to negative sign of the tip angular velocity times a constant based on Equation (19). The control voltage can create the surface traction  $q(x,t)$  which will generate the geometric stiffness and the moments at each nodal points. The proper control voltage for a specific system can be determined just by changing the amplification gain.

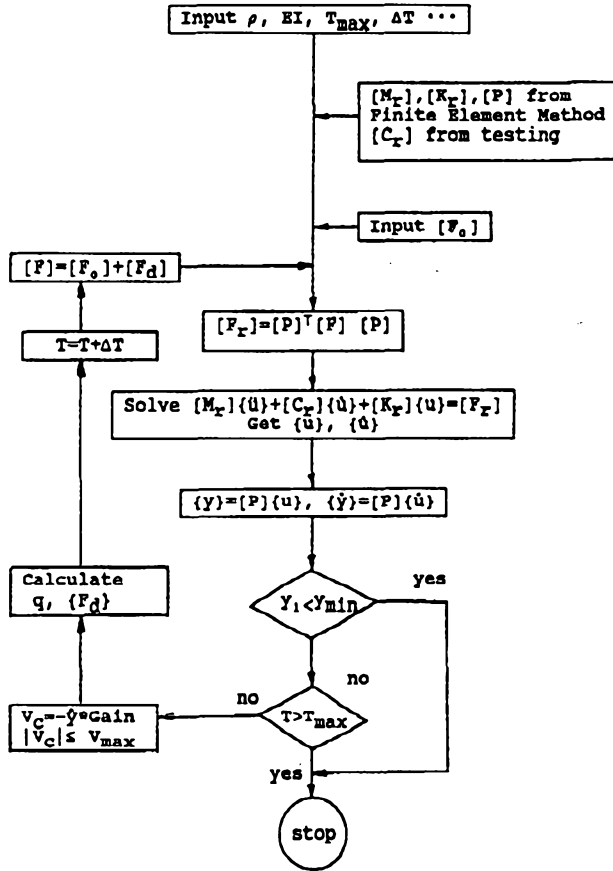


Figure 1. Flow Diagram for Computer Simulation.

Assuming that the active damper is uniformly distributed. The steel beam with dimensions 17 x 2.7 x 0.1 cm and aluminum plate with dimensions 12 x 7.5 x 0.04 cm will be used for investigation. The transient forces with magnitude of 10 Newtons and 0.2 Newton are applied to excite the beam and plate, respectively.

## EXPERIMENTAL ANALYSIS

In the research, piezo films named "Kynar" made by Pennwalt Corporation were employed as sensors and actuators. The thickness of the film is 28 $\mu$ m. A general purpose epoxy with low viscosity was used as the bonding agent. Due to the limitation of equipments in laboratory, the maximum control voltage can only reach 120 volts. From the results of numerical simulations, the cantilever beam and plate are found as the cases which has obvious control effect for control voltage not greater than 120 volts. For this reason, the cantilever beam and plate will be chosen to perform the vibration suppression in the experimental analysis.

Figure 2 is a schematic of the cantilever beam with PVDF(DT4-028K) bonded on both surfaces fixed on a stable fixture. An impact hammer was used to excite the structure. Either one of the piezo films was used to be the sensor or the actuator. For cantilever plate system, the

setup is exactly the same, but the specimen is substituted by an aluminum plate with same dimensions of surface bonded actuator. The effectiveness of this feedback control system is monitored by the noncontact Potonic Sensor. According to Lyapunov control law, the effective control parameter is the tip angular velocity, so that if the sensing signal sensed by this piezo film was differentiated by a differentiator, the tip angular velocity can be taken to be the control parameter. Applying this sensing voltage through the audio amplifier(Hafler 500) and charge amplifier (Kister 504E), the control voltage is achieved to suppress the vibration of the flexible structure.

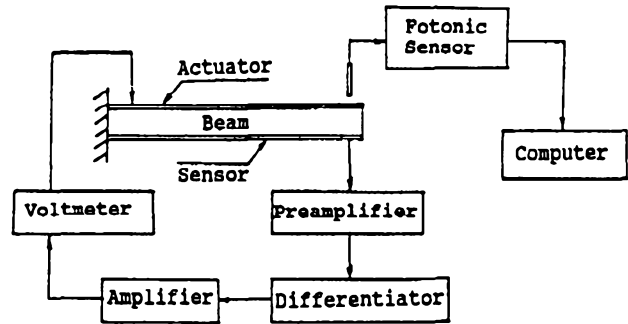


Figure 2. Schematic of Experimental Setup Used for Vibration Control.

## RESULTS AND DISCUSSIONS

### Numerical Results

Beams with different types of boundary conditions are controlled by Lyapunov control law numerically. Since the simulation program is not only suitable for Lyapunov control, so that other parameters such as linear/angular velocity and displacement will be used to simulate the vibration control of structures including beams and plates.

To acquire the damping matrix for simulation, the setup in Figure 3 is used. This process of extracting modal characteristics of a structure from a series of measured frequency response functions (FRFs) is called modal testing. An impact hammer with a piezoelectric force transducer driven by a low impedance voltage mode (LIVM) current source (Model 4102/Dytan) is used to excite the smart beam while the response is picked up by the piezoelectric sensor. An HP 3582 Spectrum analyzer which performs the Fast Fourier Transform is employed to measure the response mode of the structure in terms of FRFs. Having acquired the response data, the next major step of the process is the use of curve fitting techniques to identify the modal characteristics in a computing system.

**Lyapunov Control of Cantilever Beam:** To investigate the effectiveness of the control method, the peak of each oscillating cycle for the tip nodal point was connected to obtain the families of decay envelopes in the positive direction. Figure 4 shows the time histories of a cantilever beam under the excitation of a transient force applied on the 4th nodal point (40% length off the fixed end) of the beam with five different control gains, the results show a dramatic improvement when the maximum control voltage is increased from 0 to 1000 volts.

When the maximum control voltage is increased from 100 to 1000 volts, the time to decay the envelope to 5% of the maximum amplitude are all about 50%, 35%, 22%, and 12.5% of uncontrolled decay time, respectively. The passive effect of bonding piezo film is quite obvious, the decay time can be shortened up to 20%.

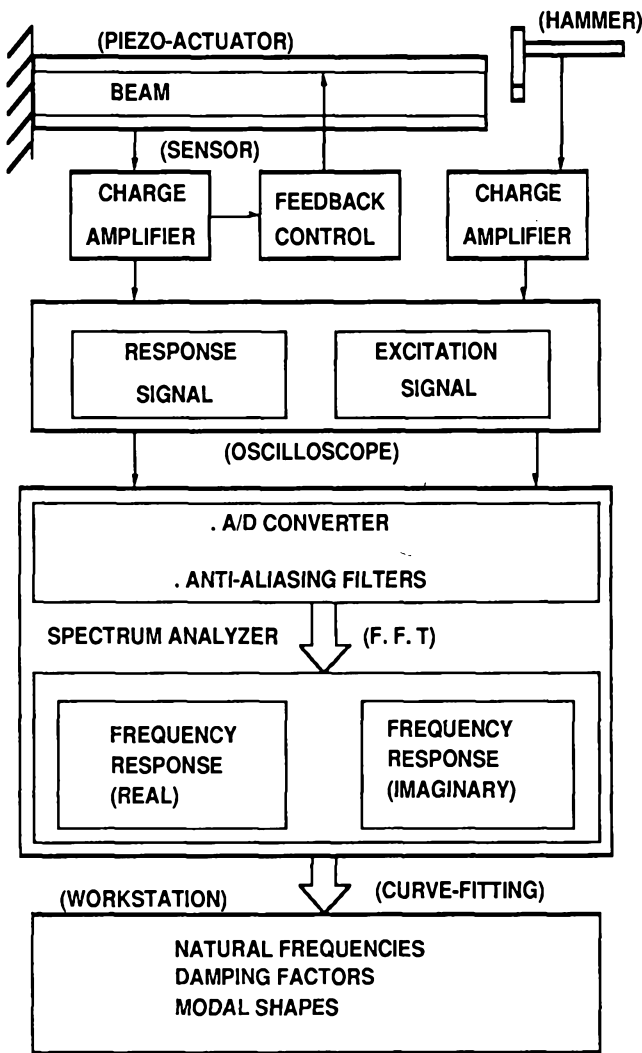


Figure 3. Modal Testing of a Smart Beam

**Lyapunov Control of Fixed - pinned Beam:** The family of decay envelopes of this beam which is subjected to the transient impact is illustrated in Figure 5. The effectiveness of the Lyapunov control law is evident when the maximum control voltage is increased from 0 volt to 2000 volts. Compared with the results of cantilever beam, more higher control voltage is necessary. The effect of passive damping is quite obvious which can shorten the decay time 15%, this is equal to the effect when a 1000 volts of maximum control voltage is applied.

**Lyapunov Control of pinned - pinned Beam :** Figure 6 is the family of decay envelopes of this beam which is subjected to the transient impact and under the action of Lyapunov control law. The effectiveness of the control system is proved by examining the decay time envelopes. When the maximum feedback control voltage is changed from 0 to 1000 volts, the decay time is shortened to 50 %. The effect of passive damping has been displayed in the figure, more than 10 % of decay time is reduced if the passive effect is considered.

**Linear Velocity Control of Beams:** When the linear velocity of the center point of the beam is selected as the control parameter, vibration of the beam is eligible to be suppressed. Figure 7 is the family of decay envelopes of a fixed-fixed beam which is subjected to linear velocity control. The effect of this control parameter is similar as controlled by Lyapunov law.

Linear velocity control of beams with boundary conditions as mentioned in cases 1-3 is accomplished. The difference of the effect by this method and Lyapunov control method is less than 3%. It can be judged from the output data but hard to identify from the plots.

**Angular Velocity Control of Cantilever Plate:** A transient impact is applied on the point  $(a/3, 0)$  to excited the cantilever plate. When the twisting angular velocity  $\Omega_x$  or bending angular velocity  $\Omega_y$  are used as the control parameter, the control method is quite similar to using Lyapunov control law for smart beam systems. Figures 8 and 9 are families of decay envelopes for the corner tip point of the cantilever plate using  $\Omega_x$  and  $\Omega_y$  as its control parameter, respectively. From the figures, one can see that when twisting angular velocity  $\Omega_x$  is used, the control effectiveness of this feedback system is very limited. But when bending angular velocity  $\Omega_y$  is chosen, a satisfactory effectiveness is obtained. This means bending angular velocity  $\Omega_y$  is a good control parameter for this cantilever plate but twisting angular velocity  $\Omega_x$  is not. The reason for this difference is that when this plate is excited by a transient impact, bending is the dominant mode, it will decay much slower than the twisting mode. So, when using  $\Omega_y$  as the control parameter, the feedback control voltage will maintain a greater magnitude than that of using  $\Omega_x$ . In some cases, if the twisting mode is the dominant one, then the twisting angular velocity must be chosen as the control parameter to obtain the higher magnitude of control voltages.

**Linear Velocity Control of Cantilever Plate:** Figure 10 is the family of time decay envelopes for the cantilever plate with corner tip linear velocity as its control parameters. The system can be effectively controlled. In beam system, the linear velocity almost works as effective as the angular velocity to be a control parameter. Normally the difference is less than 3%, so that it is very hard to be distinguished from the plots. For plat system, when corner tip linear velocity  $v_z$  is used as the control parameter, the control effectiveness is better than that when bending angular velocity  $\Omega_y$  is used (Figure 11).

When angular or linear velocity is applied as a control parameter, the magnitude of control voltage is oscillating. Connecting the peak points of each cycle, the distribution of control voltage for different control methods are achieved as shown in Figure 12. From this Figure, one can see that when using bending angular velocity,  $\Omega_y$  or linear velocity  $v_z$  as the control parameter, the distributions of feedback control voltage correspond to each other. When the twisting angular velocity  $\Omega_x$  is utilized to perform the vibration control, the feedback control voltage decays very rapidly. This is the reason why the effectiveness of this control method is so poor when compared with the previous two methods.

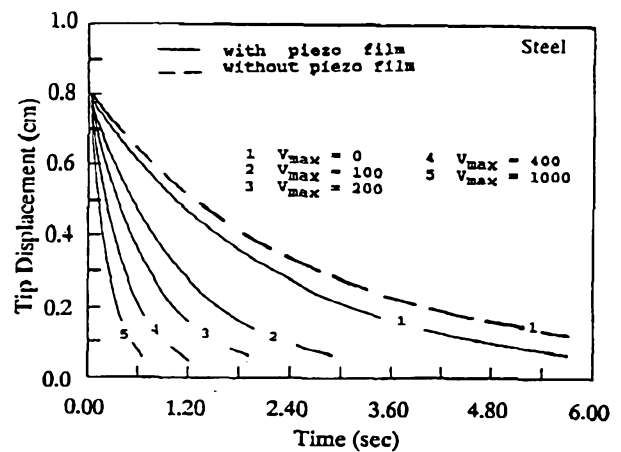


Figure 4. Family of Decay Envelopes of a Cantilever Beam with Lyapunov Control (Transient Impact).

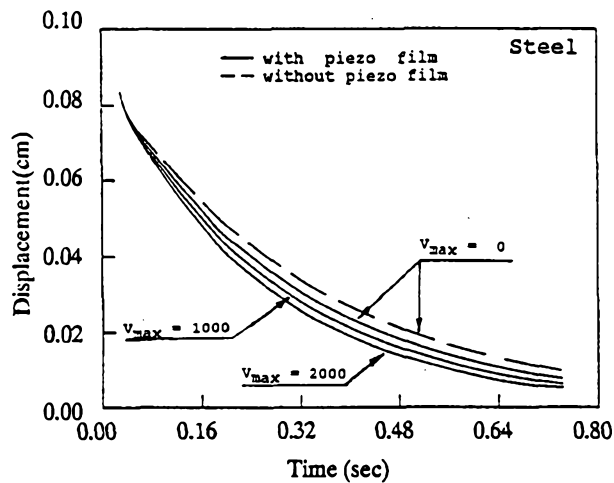


Figure 5. Family of Decay Envelopes of a Fixed-Simply Supported Beam with Lyapunov Control.

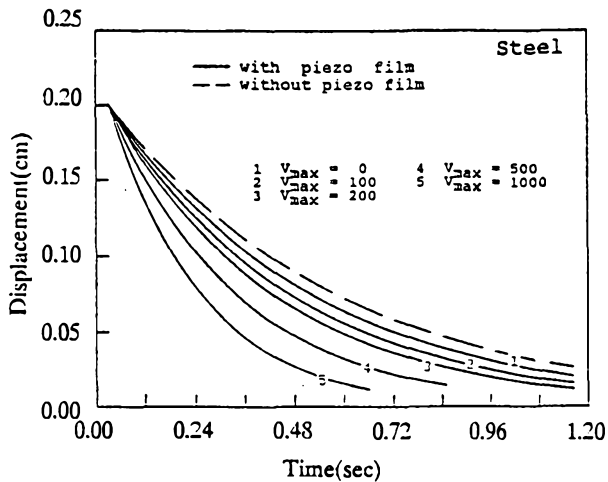


Figure 6. Family of Decay Envelopes of a Beam with Both Ends Simply-Supported by Lyapunov Control.

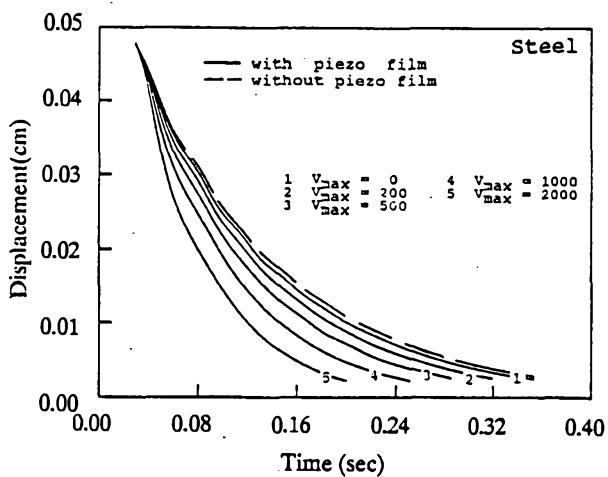


Figure 7. Family of Decay Envelopes of a Beam with Both Ends Fixed by Linear Velocity Control.

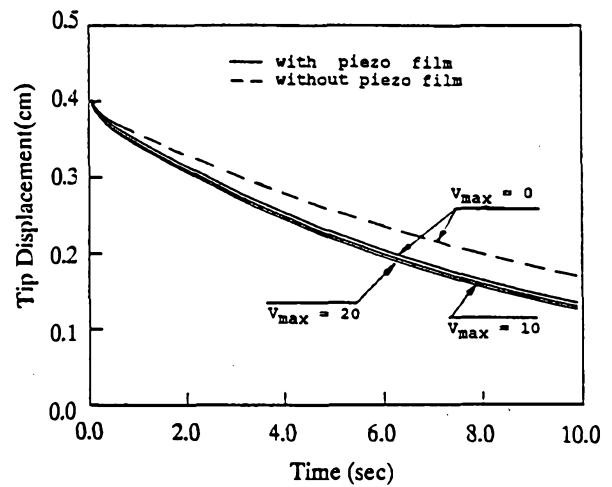


Figure 8. Family of Decay Envelopes of Plate with Twisting Angular Velocity Control.

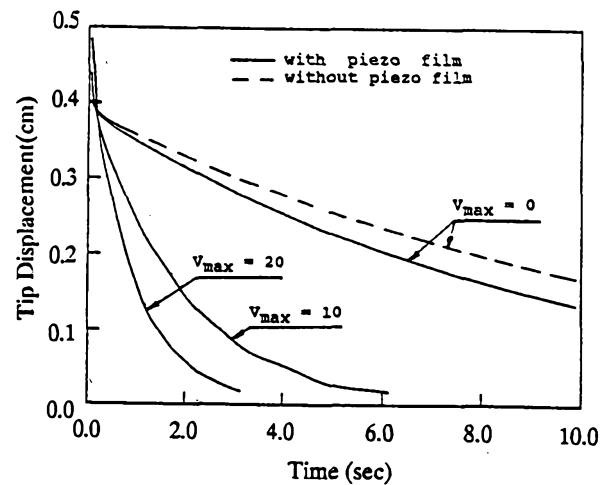


Figure 9. Family of Decay Envelopes of Plate with Bending Angular Velocity Control.

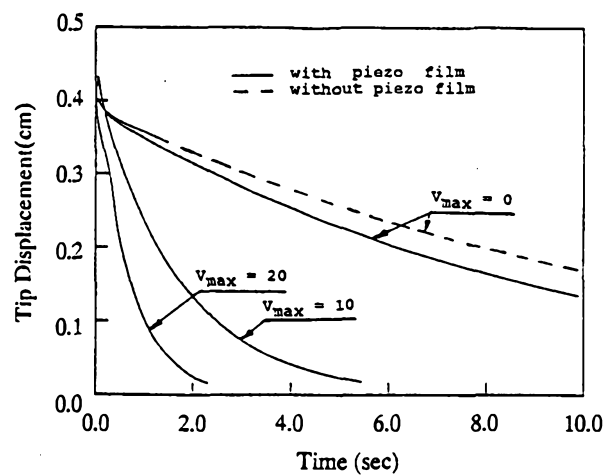


Figure 10. Family of Decay Envelopes of Plate with Linear Velocity Control.

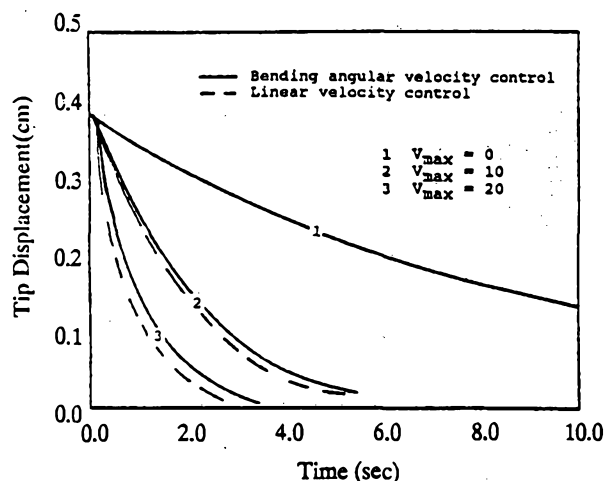


Figure 11. Distribution of Feedback Control Voltage for Different Control Methods.

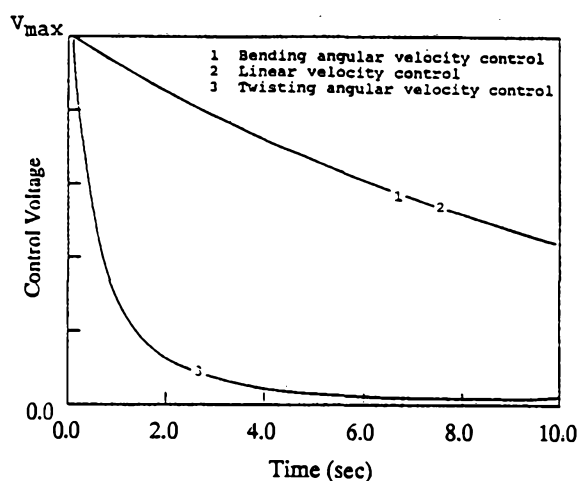


Figure 12. Distribution of Feedback Control Voltage for Different Control Methods.

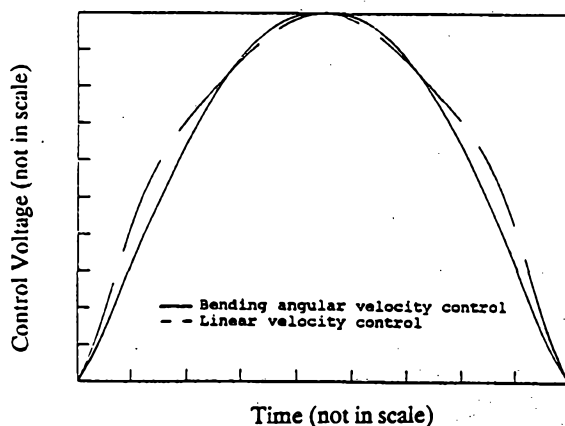


Figure 13. Feedback Control Voltage for Different Control Methods.(half cycle)

According to Figure 12, the distribution of feedback control voltage for bending angular velocity  $\alpha_y$  and linear angular velocity  $v_z$  are exactly the same. So, the same control effectiveness are expected for both control method. However, Figure 11 shows that the linear velocity control method is more efficient than the other which will cause some confusion. To explain this paradox, more detail observation for control voltage distribution plots is necessary. When a half cycle of control voltage distribution for both cases are plotted as shown in Figure 13, the reason of previous confusion is achieved. From Figure 13, one can see that the average feedback control voltage of the linear velocity control method is actually higher than the other methods. Even the peak values of both cycles are the same. The difference between those two plots is the contribution from twisting angular velocity.

Other types of control parameters, including the displacement (linear and angular), square of displacement have been used to perform the simulation of the vibration suppression for the beam and plate systems. The results show these the parameters can not be used to suppress vibration of the systems.

### Experimental Results

The experimental results of performing vibration suppression of a cantilever beam and plate will be presented. Different control parameters will be used. The families of time histories can show the decay times of the cantilever beams under the effect of various control voltages.

**Lyapunov Control of Cantilever Beam:** An impact force of 10 newtons was applied to the 4th nodal point (40% length off fixed end) of the cantilever beam. Since the constant impact force for each excitation is unavailable, a computerscope package "WAVEPAK" was used to measure the magnitude of each impact. If the error is less than 2% of the standard one, then this hit is acceptable.

The time decay envelopes of the beam with and without applying the control voltage are displayed in Figure 14. The top portion of this figure is the decay envelope of the beam without applying control voltage, while the bottom portion is the decay envelope of the beam controlled by a maximum feedback control voltage of 120 volts. Tracing the decay envelopes for each beam with various applied voltages, families of time histories are obtained and are displayed in Figures 15. From these figures, one can see that the time to decay the tip displacement within a deadband with control voltage increased from 30 volts to 120 volts are about 75%, 60%, 50%, and 45% of uncontrolled system, respectively. The case which can compare with the results from numerical simulation is when  $v_{max} = 100$  volts (Figure 16). From experimental results, the percentage of decay time is around 50%. It is quite correlated with the simulation results. The difference of decay envelopes between simulation and experimental results are found, which is due to the effects of air drag.

**Linear Velocity Control of Cantilever Beam:** Instead of using the angular velocity as the control parameter, the linear velocity, obtained by differentiating the linear displacement provided by the noncontact Fonic Sensor, was used as the control parameter. The time histories of the tip of the beam subjected to a transient force under the linear velocity control were displayed in Figure 17. The time to decay the displacement to a small range (5% of the maximum value) is equal to 80%, 70%, 60%, and 55% of the uncontrolled beam. The effectiveness of this control parameter is less than that using the Lyapunov's method, this may be caused by the effect of truncation of higher modes. The Fonic Sensor has its proper working range, when the beam oscillates, an overrange situation may occur. Also, the electric light may disturb the sensor, these are reasons to cause the error.

Comparisons of time decay envelopes of beams subjected to Lyapunov control as well as linear velocity control are illustrated in Figure 18, the effectiveness of Lyapunov control is 30% higher than linear velocity control.



**Bending Angular Velocity Control of Cantilever Plate:** Figure 19 is a time decay envelope of tip corner of the plate when a 0.2 Newton transient force is applied. While no feedback voltage is applied, the plate will oscillate with a very long decay time. In the same situation, if a feedback control voltage with maximum value of 50 volts is applied to the system, the vibration of the plate will suddenly be suppressed. By tracing the envelopes of various control voltages to form a family of decay envelopes, the control effectiveness of different control voltage can be judged. Figure 20 shows the time decay envelopes of the plate with bending angular velocity control. For  $V_{\max} = 10, 30, 50$  volts, the decay time will be shorten 30%, 45% and 70%, respectively.

**Linear Velocity Control of Cantilever Plate:** If linear velocity  $V_z$  is utilized as the control parameter, the decay times are shorten 15%, 30% and 40% when the maximum control voltage is equal to 10, 30 and 50 volts, respectively (Figure 21). In the numerical simulation, the linear velocity is better than bending angular velocity as the control parameter, but the plots as shown in Figure 22 indicate the contrary results.

Comparison of simulated and experimental results are illustrated in Figures 23 and 24. In the experiment, air drag is included, but not in the numerical simulation. When the plate is uncontrolled, it will decay sooner than expected. On the other hand, it is impossible to have a perfect bonding layer. But in simulation, perfect bonding is assumed and is the reason why control effectiveness for experiment is always poorer than expected.

Other types of control parameters, including linear displacement, and angular displacement were tried to be the control parameter, no effect was found to eliminate the vibration of the cantilever beam.

## CONCLUDING REMARKS

Numerical simulations show a dramatic improvement of the intelligent system using the Lyapunov control law when control voltages are increased from 0 to 1000 volts. Based on the same control algorithm, experimental results are less effective but in satisfactory agreement with those from numerical simulations.

For effective vibration control of cantilever beams and plates, the angular velocity and the linear velocity at the tip of the free end are proven to be suitable feedback control parameters. Damping analysis of smart structures through the modal testing provides essential material characteristics for vibration control.

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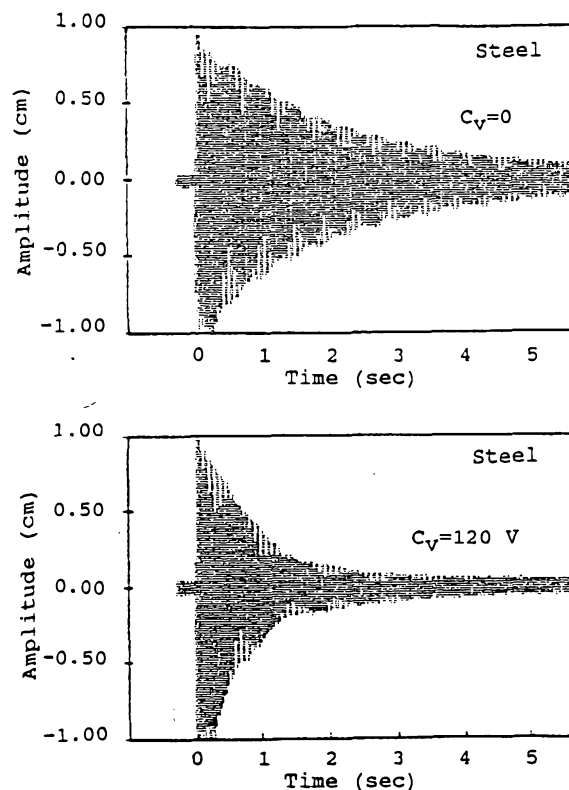


Figure 14. Time Decay Envelope of Beam with Lyapunov Control.

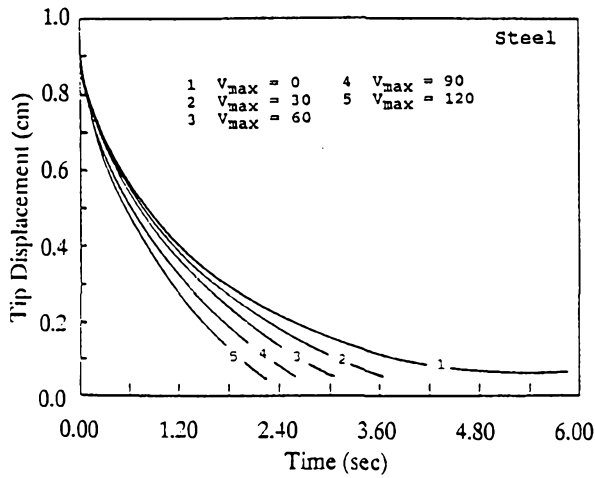


Figure 15. Family of Decay Envelopes of a Cantilever Beam with Lyapunov Control(Transient Impact).

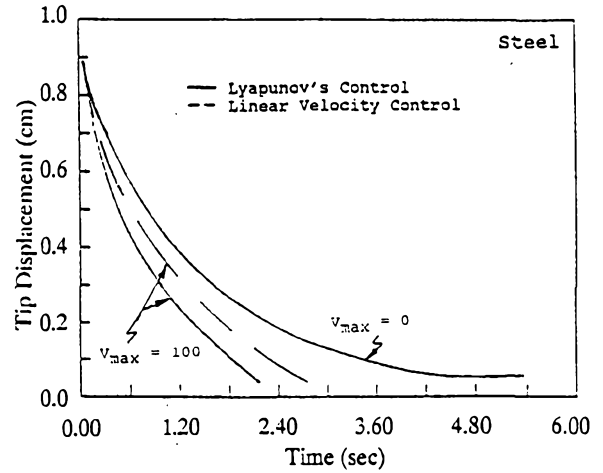


Figure 18. Experimental Results for Lyapunov Control and Linear Velocity Control(Transient Impact).

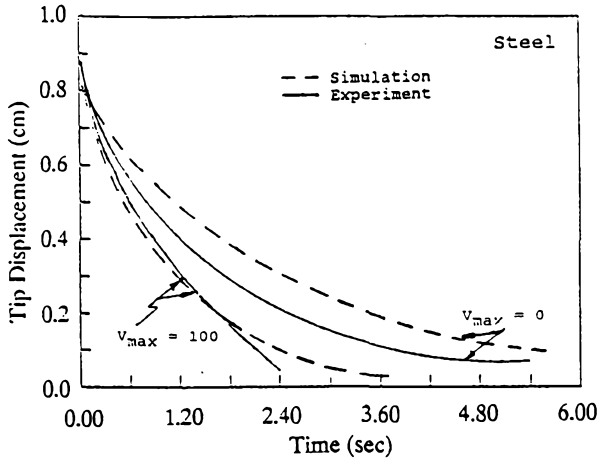


Figure 16. Simulated and Experimental Results for Cantilever Beam with Lyapunov Control(Transient Impact).

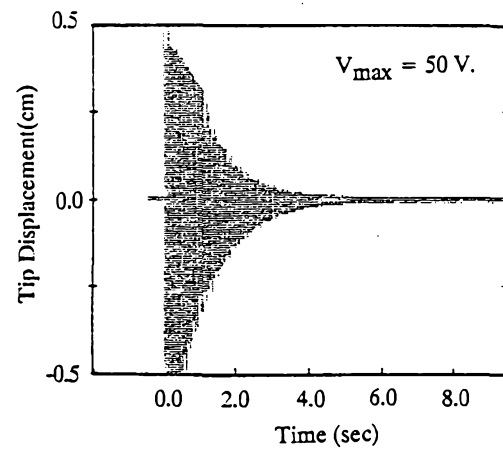
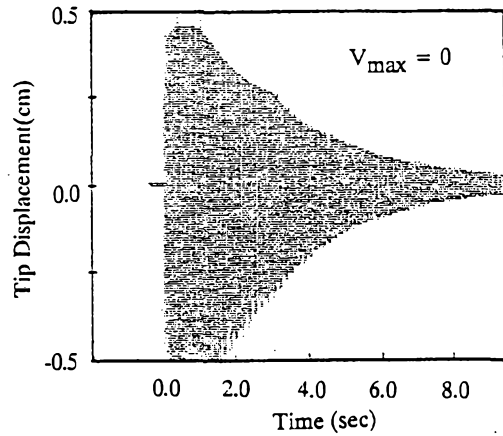


Figure 19. Time Decay Envelope of Cantilever Plate with Bending Angular Velocity Control.

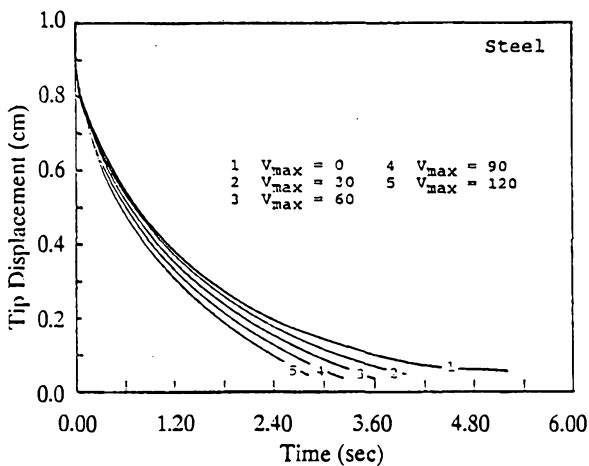


Figure 17. Time Histories of Beam with Linear Velocity Control(Transient Impact).

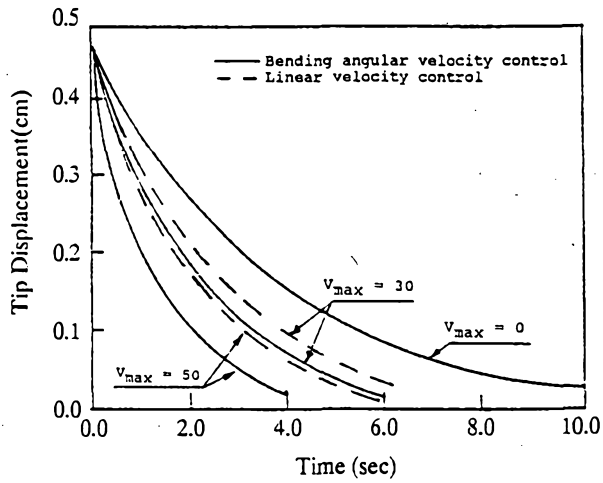


Figure 20. Family of Decay Envelopes of Plate with Bending Angular Velocity Control.

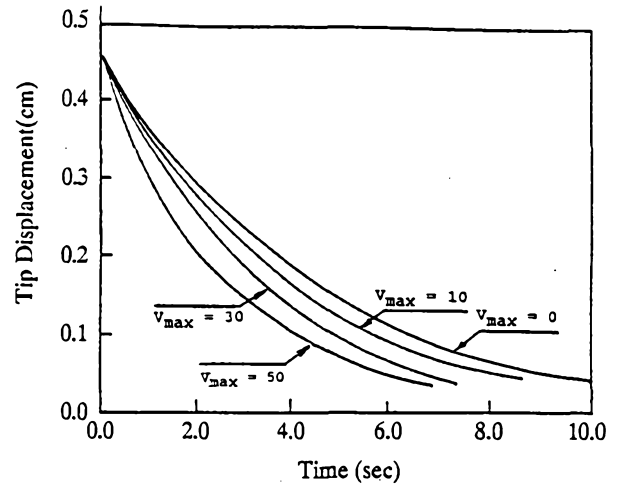


Figure 23. Simulated and Experimental Results for Cantilever Plate with Bending Angular Velocity Control.

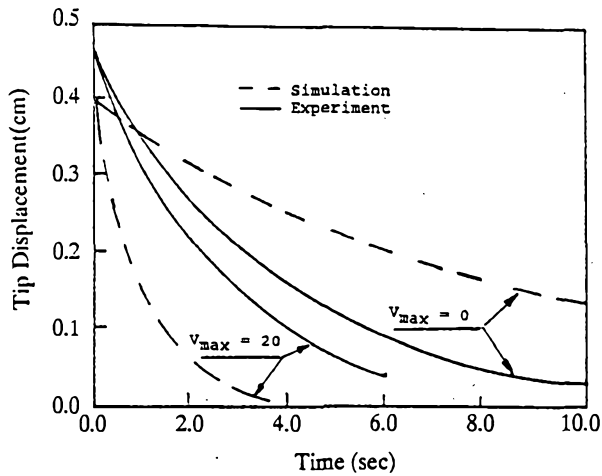


Figure 21. Family of Decay Envelopes of Plate with Linear Velocity Control.

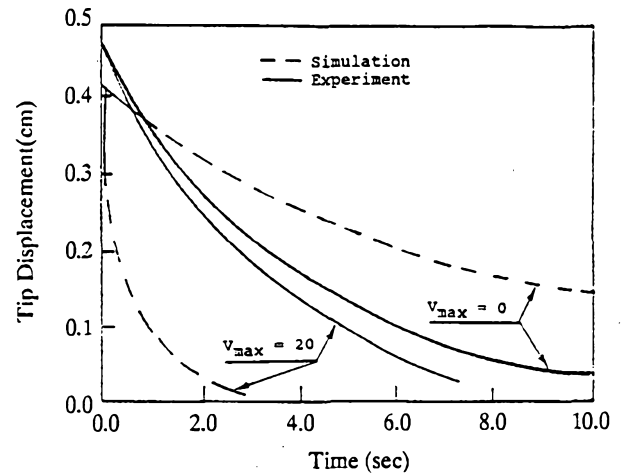


Figure 24. Simulated and Experimental Results for Cantilever Plate with Linear Velocity Control.

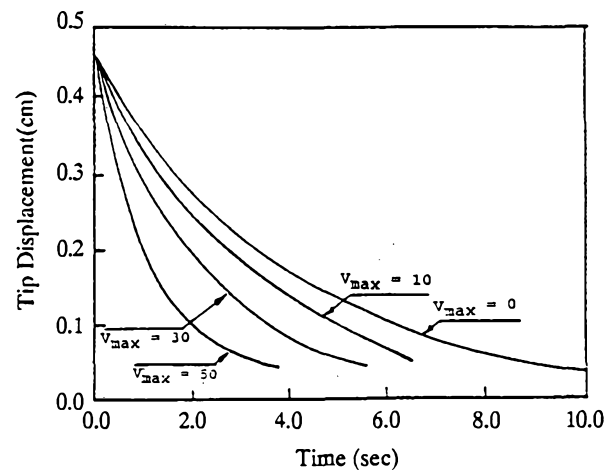


Figure 22. Experimental Results of Plate for Bending Angular Velocity and Linear Velocity Control.